

Statistical mechanics of multi-fractal walk models: fluctuating stock as REM in the Lee-Yang-Fisher phase

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We investigate multi-fractal random walk (MRW) model of financial markets. The model is considered at the maximal scale of observation where the multi-fractal properties are valid, as well as for the smaller scales of observation. We connect with the MRW model observed at given scale a statistical-mechanics model with some free energy. When the ratio of minimal and maximal scales of observation is large the free energy of this model is exactly equivalent to the free energy of the Random Energy Model (REM), where Boltzman weights are multiplied by random numbers with normal distribution. The model at the maximal scale describes the global properties of the model, as this scale is a natural scale of observation. While observed at the maximal scale, the financial market (assumed to be described by MRW) is identified to be in the high-temperature, Lee-Yang-Fisher (LYF) phase, far from the transition temperature. We describe a scenario for the financial crisis, where the high frequency fluctuations are frozen in the spin-glass (SG) phase, while the low frequency fluctuations are in the LYF phase. Thus in the ordinary situation the market is off critical phenomenon, lives in the LYF phase, while during the financial crisis the high frequency fluctuations undergo a phase transition into the SG phase.