Chaotic Behavior of Exchange Rate: Perspectives on Large Lyapunov Exponent of USD-TWD Time Series

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Abstract

The exchange rate market has become a major framework of the international trades. Recently, many studies related to the nonlinear behaviors of exchange rate show that the latter embodies the characteristics of chaos. We demonstrate potentially chaotic behaviors of USD-TWD (Taiwan Dollar) Dollar) exchange rate by investigating observed time series directly rather than modeling such behaviors. We investigate the relationship between exchange rate dynamics and chaotic behaviors by computing the largest Lyapunov exponent (LLE) of USD-TWD at different time intervals. According to the historical data, positive LLEs are shown for the USD-TWD exchange rate, which concludes that corresponding exchange rate time series is chaotic. Numerical simulations for exchange rate dynamics collected during the financial crisis are also shown.

1. Introduction

Economic globalization urges the financial market in each country to be closely integrated with the global financial system. (Foreign) exchange rate is a major trading mechanism for open financial markets nowadays. The United States dollar (USD) is the major benchmark index of currency trading for international financial markets. On the other hand, due to the financial crisis outburst in 2008, investors are more concerned about the influence of the USD exchange rate to the investment portfolio for the global financial markets.

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Most economists believe that the exchange rate dynamics is a random behavior. It will be fluctuated by oil prices, public policies, inflations, the interest rate, money supply, bond yield, and other factors. These factors makes exchange rate prediction difficult. Other economists believe that the exchange rate dynamics possesses the characteristics of nonlinear dynamics. De Grauwe et al. [1] suggested the exchange rate dynamics is driven by random shocks and chaotic factors. By analyzing exchange rate time series, we can explore some "chaotic characteristics" of the exchange rate dynamics. For example, Das and Das [2] proposed effective identification of chaotic exchange rate behaviors by the utilizing largest Lyapunov exponent (LLE) methodology. They also found there are many nonlinear behaviors in the exchange rate dynamics as different positive Lyapunov exponents (LEs) are shown in exchange rate time series of different countries. LEs represent the strength of the sensitivity to the initial conditions which are also related to the expanding or contracting nature of different directions in phase space. The positive LLE concludes that corresponding exchange rate time series is chaotic. Bask [3] tested the presence of a positive Lyapunov exponent in Swedish exchange rate. Ou et al. [4] modeled real exchange rate dynamics by some ordinary differential equation. chaotic behaviors of this model can be easily analyzed by computing LEs.

According to the numerical simulations, we observe that instability of exchange rate fluctuations cycle within financial crisis was due by smaller capital flow range, namely, narrow range of LLE values. The purpose of this study is to manifest the chaotic characteristics of exchange rate behaviors directly by time series rather than modeling it. The arrangement of this paper is as follows. In section 2, background knowledge of chaotic time series is given. In section 3, USD-TWD time series collected within global financial crisis is analyzed by the LLE methodology. Future work and conclusions will also be discussed.

2. Preliminary Knowledge

2.1. Phase Space Reconstruction of Time Series

The highly nonlinear, even chaotic nature of dynamical systems hinder efforts to explore time series behaviors. Such limitations have been greatly improved by the works of Whitney and Takens [5][6]. One of these methodologies, proposed by Ruelle and Takens [7], is to study the relationships between chaos and dimension and the equilibrium states in physical systems.

Takens proposed a method of phase space reconstruction using time delay coordinates [6], which has been treated as a basis of the phase space technologies. According to the Takens' Theorem, phase space of a sequence of observations, namely a time series, can be rebuilded.

Theorem 1 (Takens). *Let M be a d-dimensional manifold, $\varphi : M \to M$ is a smooth differmorphism, $y : M \to \mathbb{R}$ has second-order continuous derivatives, $\phi(\varphi, y) : M \to \mathbb{R}^{2d+1}$, then ϕ is an embedding from M ro $\mathbb{R}^{2d+1}$.*

One application of the Takens' Theorem is the following: for a time series $\{x_1, x_2, \cdots, x_n, \cdots\}$, if we can properly choose some embedding dimension m and a time delay τ to reconstruct its phase space

$$Y(t_i) = \{x(t_i), x(t_i + \tau, x(t_i + 2\tau), \cdots, x(t_i + (m-1)\tau\}, i = 1, 2, \cdots. \quad (1)$$

then we can recover some dynamical properties of equilibrium points and attractors.

2.2. Largest Lyapunov Exponent (LLE)

The Lyapunov exponents are related to the expanding or contracting nature of different directions for time series in phase space. Wolf et al. [8] suggested that LEs can quantify the initial sensitivity of time series.

Once applying Takens' theorem, a time series can be embedded in to a m-dimensional phase space, where m is an embedding dimension. There will be m Lyapunov exponents, say $\lambda_1 \geq \lambda_2 \geq \cdots \lambda_m$. Largest Lyapunov exponent (LLE) λ_1 reflects the chaotic degree of the time series. The larger the LLE, more sensitive the time series to initial values. LLEs help determine whether there is a chaotic uncertainty for financial time series.

Let $\|\cdot\|$ be a metric in the m-dimensional phase space. Time series (1) satisfies the equation $x_{n+1} = F(x_n)$, for all n. Then LLE satisfies the following equation.

$$\begin{aligned} \|\delta x_0\| e^{n\lambda_1} &= \|F^n(x_0 + \delta x_0 - F^n(x_0)\| \\ &\approx \|J(x_{n-1})J(x_{n-2})\cdots J(x_0)\delta x_0\| \end{aligned}$$

$J(x_0)$ is the Jacobian matrix at x_0 and δ represents a small value.

2.3. Wolf Algorithm

Wolf et al. [8] proposed a method to estimate the LLEs based on the evolutions of phase planes (two-dimensional phase spaces) and trajectories. This so-called Wolf Algorithm is used frequently for the studies of chaotic time series.

Let $\{x_1, x_2, x_3, \cdots, x_k, \cdots\}$ be a chaotic time series with embedding dimension m and time delay τ, then the reconstructed phase space Y is defined as follows.

$$Y(t_i) = \{x(t_i), x(t_i+\tau, x(t_i+2\tau), \cdots, x(t_i+(m-1)\tau\}, i = 1, 2, \cdots, N. \quad (2)$$

Choose the initial point $Y(t_0)$ whose distance with the nearest point $Y_0(t_0)$ is L_0. We track the evolutions of these two points until time t_1 with their distance greater than some value $\epsilon > 0$, $L'_0 = |Y(t_1) - Y_0(t_1)| > \epsilon$. Keep $Y(t_1)$, and find another point $Y_1(t_1)$ near $Y(t_1)$ such that $L_1 = |Y(t_1) - Y_1(t_1)| < \epsilon$. Continuing these processes until $Y(t)$ reaches the end of time series (with $i = N$); assume that the total number of evolutions is M, then the LLE is as follows (see FIgure 1).

$$\sigma = \frac{1}{t_M - t_0} \sum_{i=0}^{M} \ln \frac{L'_i}{L_i} \quad (3)$$

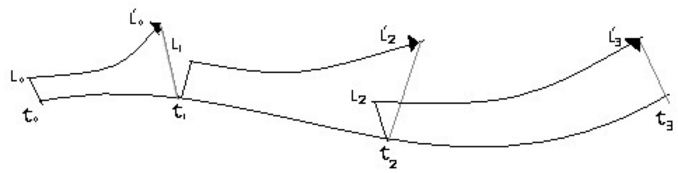

Figure 1: Wolf method : The LLE value of time series.

3. Chaotic Behaviors of the Exchange Rate Dynamics

The global financial crisis, is also known as the financial tsunami, had been outburst since the end of 2008. It was caused by domestic financial crisis (Subprime mortgage ruin) in the United States in 2007. This crisis is out of control later which led to a global financial storm in 2009.

In this section, we will explore the chaotic time series of the exchange rate by the LLE methodology within this period of time. We will investigate whether the chaotic characteristics exists for the time series generated by exchange rate of USD-TWD. The relationship between imports & exports and exchange rate will also be explored from LLE viewpoints.

3.1. Exchange Rate Dynamics in the Financial Crisis

We first investigate the exchange rate of USD-TWD from 2007 to 2010 (until July 30th). The United States is the most important partners of Taiwan international trading. USD-TWD is an influential factor for Taiwanese overall economic developments. There were 250, 251, 252, and 143 points for each year. There are 896 points totally (see Figure 2).

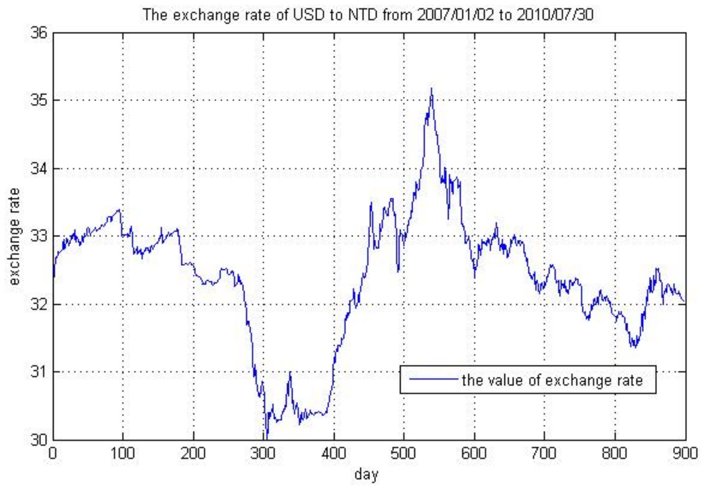

Figure 2: The values of exchange rate from 2007 to 2010(days)

LLE values for this time series is shown in Table 1 and Table 2 also later. In fact, they are the "reciprocals" of the original LLEs. The purpose of adopting reciprocals is to reduce the gap between the exchange rate and the LLE values. On the other hand, the range of the original LLE values were small for this time series. The range of these values are the decimal values between -1 and 1. Table 1 is the reciprocals of LLEs.

Table 1: LLE for the daily exchange rate from 2007 to 2010.)

Time Period	2007-2010	2007	2008	2009	2010
(Rec.) LLE Value	22.3376	22.474	22.632	22.555	22.1627

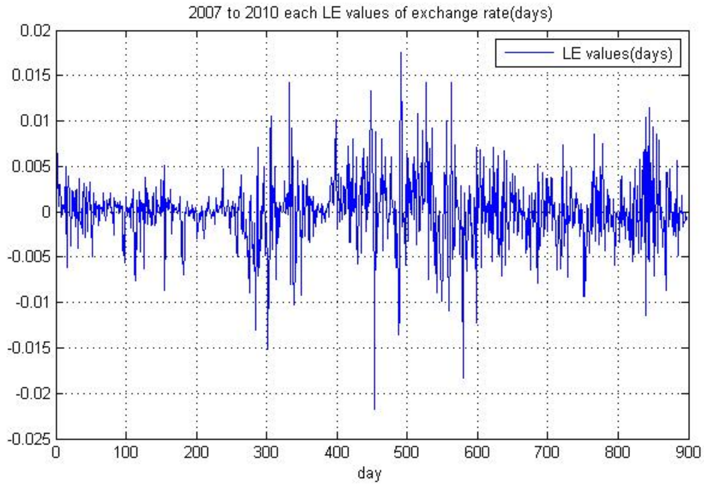

Figure 3: The LE values of exchange rate from 2007 to 2010(days)

LLEs for exchange rate of USD-NTD are positive, which shows the characteristics of chaotic time series in the market of exchange rate within this period of time.

3.2. Analysis of Exchange Rate Dynamics From Imports and Exports Information.

We investigate the relationship between exchange rate and export & import. This analysis is based on monthly-average data provided by the Central Bank and the DGBAS (The Directorate General of Budget, Accounting and Statistics) of Taiwan (see Figure 4). Accordingly, the lowest point (the 16th) of USD-TWD exchange rate was on April 2008, and the highest point (the 27th) was on March 2009. On the other hand, the highest point (the 20th) of the imports was on August 2008, while the lowest point(the 25th) of the imports was on January 2009. The lowest point (the 25th) of the exports was on January 2009 while the highest point of (the 41th) the imports was on May 2009.

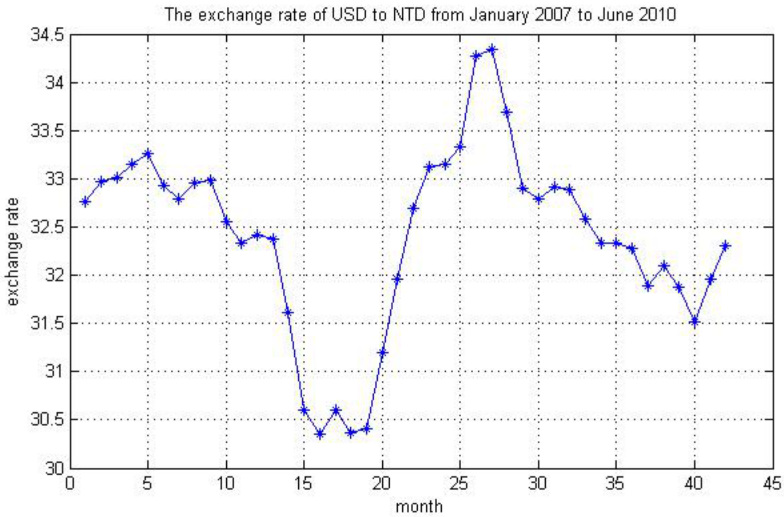

Figure 4: The monthly-average exchange rate of USD-TWD from 2007 to 2010.

Figure 5 shows the total trades of Taiwanese economy declined completely in January 2009. Such declination was more than the half of the average value in 2009. It was also the weakest international trading in a single month for the past 40 years.

We implement the Wolf algorithm with delay-time $\tau = 1$. According to the Takens' theorem, the embedding dimension of the phase space was 3 ($m = 3$). Moreover, when the delay-time is $\tau = 1$ and the embedding dimension varies among $m = 3, 4, 5, 6$, the discrepancy in those calculated values of LLEs was slight. Thus LLEs will remained unaffected by the embedding dimension.

These LLE values in Table 2 are also the reciprocals of the original LLEs. They remain stable within a small range when the financial crisis broke out.

According to Table 2, reciprocals of LLEs for Exports, Imports and exchange rate are smallest in 2009, which implies original LLEs of these values are largest among all these years.

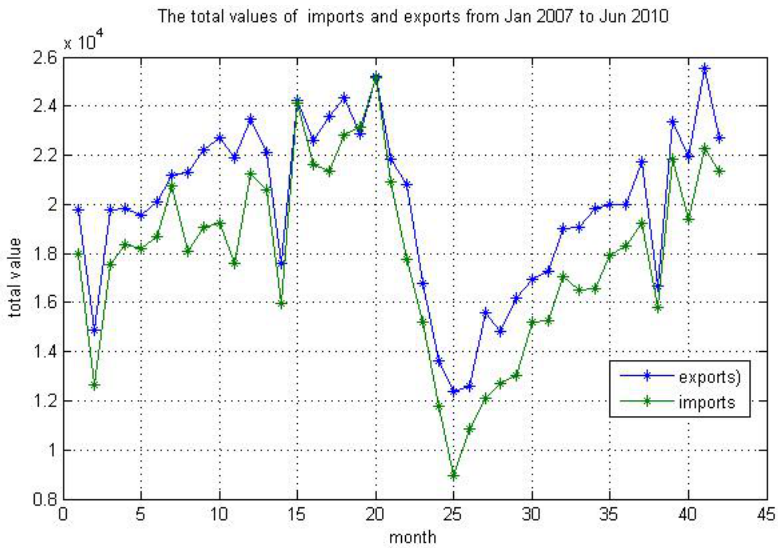

Figure 5: The total values of imports and exports from 2007 to 2010.

Table 2: (Rec.)LLEs for the exports, imports, and exchange rate from 2007 to 2010

Time Period	2007-2010	2007	2008	2009	2010
Expoer	14712.46	14958.40	12159.01	11011.86	15409.05
Import	13579.70	13558.83	10934.34	9058.58	14049.56
Exchange Rate	22.55	22.56	22.69	22.76	22.25

4. Future Research

For the future research, we will analyze some control measures activated by the Central Bank to the exchange rate market. Stabilization of the exchange rate market affected by controlling fluctuating exchange rate is another research direction. We need to identify more factors that affect the exchange rate such as the international oil prices, international markets, and the overall national productivity.

We also explore the relationship between the fluctuations of exchange rate and the monetary policies established by the Central Bank. Comparing with the values calculated by LLE methodology, we will analyze some control measures activated by the Central Bank to the exchange rate market.

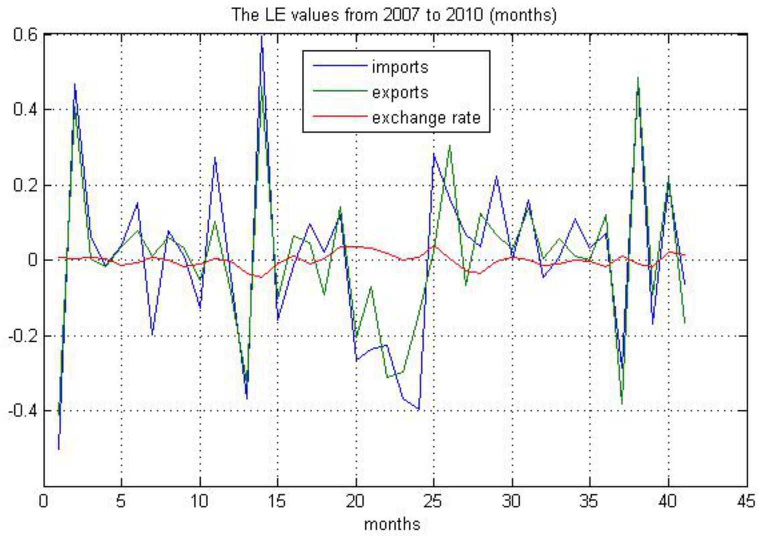

Figure 6: The LE values of imports, exports, and exchange rate from 2007 to 2010(months)

5. Conclusions

The time series of exchange rate, imports, and exports are composed of chaotic dynamics since they have positive Lyapunov exponents. LLE can show the basic features of chaotic dynamics and distinguish short-term chaotic behavior of a fixed period of time. USD-TWD examples can demonstrate that the LLE methodology can be used in the study of financial behaviors.

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