Diffusion in optimized networks

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Abstract

Diffusion is the process by which new products and practices are invented and successfully introduced into a society. Numerous studies on the diffusion of individual innovations have been done, many exhibiting common features such as the famous S-shaped diffusion curve. One basic question posed by the empirical study of innovation diffusion is why there is often a long lag time between an innovation's first appearance and the time when a substantial number of people have adopted it. Extensive amounts of theoretical and empirical literature have been devoted to explain this phenomenon and explore the mechanism behind it. New ideas, products, and innovations often take time to diffuse, and this fact is often attributed to the heterogeneity of human populations. In this paper, we optimize network for diffusion process by using evolutionary design method. And we show optimized network enable diffusion process occur at very low transmitting rate compared with the results on scale free network and random network. On optimized network the diffusion process goes to steady state more smoothly than other type of network. The aim of this paper is to understand how the structure of social networks determines the dynamics of various types of emergent properties occurring within various network structures.

Keywords: diffusion of innovations, diffusion threshold, evolutionary design

1. Introduction

For decades, social scientists, economists and physicists have been interested in the fundamental—and widespread—question of how infectious diseases, new technological practices, or the latest trends spread through society (Arthur, 1989). When a new technology appears, a society's members have the chance to become aware of the innovations of new technologies and incorporate it into their lives. The main study on diffusion modeling is based on the Bass model (Bass, 1969). The Bass diffusion model describes the process by which new products are adopted as an interaction between users and potential users. When an innovation is a good consumed by individuals, single consumers can decide whether or not to adopt it.

More specifically, the Bass model formalizes the aggregate level of penetration of a new product, emphasizing two processes: external influence via advertising and mass media and internal influence via word-of-mouth. The decision of a consumer is described as the probability of the consumer adopting the new product at a specific time, and it is assumed to depend on both external and internal influences. The Bass model displays a cumulative S curve of adopters: when the number of users of a new product is plotted against time, the resulting curve shows an S-shaped distribution—adoption proceeds slowly at first, accelerates as it spreads throughout the potential adopting population, and then slows down as the relevant population becomes saturated. The S-shape is a natural implication of the observation that adoption is usually an absorbing state. The fast growth of diffusion is generated by the interaction between early adopters and late adopters. The Bass model, however, does not specify the consumer decision-making process or how consumers communicate with and influence one other at the micro level. The Bass model assumes the population of consumers to be homogeneous—such diffusion models are referred to as aggregate models.

Rosenberg observed two dominant characteristics of the diffusion process: the overall slowness of the process, on one hand, and the wide variations in the rates of acceptance of different inventions on the other (Rosenberg, 1972). Empirical measurement and study have since confirmed his view. Why is diffusion sometimes slow? Why is it faster in some regions than others? Why do rates of diffusion differ among different types of innovations? What factors govern the wide variation in diffusion rates? Hall provides a comparative historical perspective on diffusion that looks at the broad economic, social, and institutional determinants (Hall, 2003). In the modern world, markets occasionally accept innovations very slowly, despite technological advances (Chakravorti, 2003).

Important processes that take place within social networks, such as the spreading of opinions and innovations, are influenced by the topological properties of those networks. Here, each node of the network represents a dynamical system. Individual systems are coupled according to the network topology. Thus, the topology of the network remains static while the states of the nodes change dynamically. Important processes studied within this framework include synchronization of the individual dynamical systems and contact processes, such as opinion formation and information diffusion. Studies like these have clarified that certain topological properties have strong impacts on the dynamics of and on networks (Ball, 2004).

The progress of study in complex network expands our knowledge about the property of existing social networks (Barabasi, 1999). Some social networks show that few agents have chances to communicate with many other agents; however, many agents are directly connected with a small number of other neighbors (colleague, friends, and family). This type of heterogeneity between agents suggests the difference of diffusion speed in each region over networks.

In recent years, motivated by applications to marketing, the result of diffusion study is not only used for predicting how well the new product or innovation will be adopted, but also is used for creating optimized network for diffusion. For example, many corporations make special portal site (or fun site) to make good communication network when they produce new products. Such web sites make word-of-mouth community, consisting of people who are adopters for same products regardless of their real-life location.

For long times, consciously or unconsciously, people try to find optimal network topologies for diffusion of new information. As a result, in our life, there are many type networks such as hierarchical or flat. We are motivated by this demand to get good network for diffusion.

This paper investigates what kind of networks is good for diffusion. In many study about diffusion process, agents are represented by nodes (or vertices) and communications between nodes are represented by links (or edges). And any graph (or network) G can be represented by its adjacency matrix, A (G), which is a real symmetric matrix, $a_{i,j} = a_{j,i} = 1$, if vertices i and j are connected, or 0, if these two vertices are not connected. The spectrum of a graph is the set of eigenvalues of the graph's adjacency matrix. Adjacency matrix is often used as a good tool to manipulate and investigate the networks.

Wang suggests the maximum eigenvalue of the adjacency matrix is closely related to the spreading power of networks (Wang, 2003). The network with larger maximum eigenvalue helps spreading over the network. The spectrum of network, however, classifies network topology generally. We cannot get deterministic network topology of optimized network for diffusion.

In this paper, by using an evolutionary algorithm involving maximizing the maximum eigenvalue of the adjacency matrix of the network and minimization of average degree, they denotes low spreading threshold and network costs respectively, we design optimal networks which are suitable for diffusion of information. We evaluate the performance compared with Erdos and Reny (ER) model and Barabasi and Albert (BA) model.

The network on ER model is symmetric network, so there is no center and end node in the network. The distribution of number of links on each node (also called the degree) obeys a Poisson distribution. Consumers on ER model can communicate with others randomly over the network. The network on BA model is very asymmetric, and the degree distribution obeys power law. Consumers on BA model can communicate with other nodes via center node (node with many links). Both of them represent the property of the topology very well under several phenomena in our life.

The aim of this paper is to understand how the structure of social networks determines the dynamics of various types of emergent properties occurring within various network structures.

2. Diffusion model

Among research concerning dynamic diffusion processes, the diffusion of diseases has received the most attention (Colizza, 2006). The information diffusion process in social network can be viewed as like the viral propagation in which virus spreads on contact process between neighboring nodes (Dodds, 2004). Of course there are other effects in information diffusion, such as TV commercial or magazine advertising. While those are also important, in this paper we focus on the effects of contact process on network topology. In the study of viral propagation, two major diffusion models are accepted. These are SIS model and SIR model. In SIS model, the population is classified into two categories, which are susceptible (S) and infective (I). Susceptible person is changed into infective person by other infective person with the probability β**.** The informed person becomes susceptible again with the probability δ independently. In SIR model, the population is classified into three categories, which are susceptible (S), infected (I) and removed (R). The deference between SIS model and SIR model is infected people is removed with the probability δ in SIR model. The removed people do not communicate with other people after that.

At the beginning of the study about viral propagation, diffusion process is observed on homogeneous network model. Homogeneous model assumes that every individual can equally contact with all other individual. The SIS model on homogeneous network can be represented as:

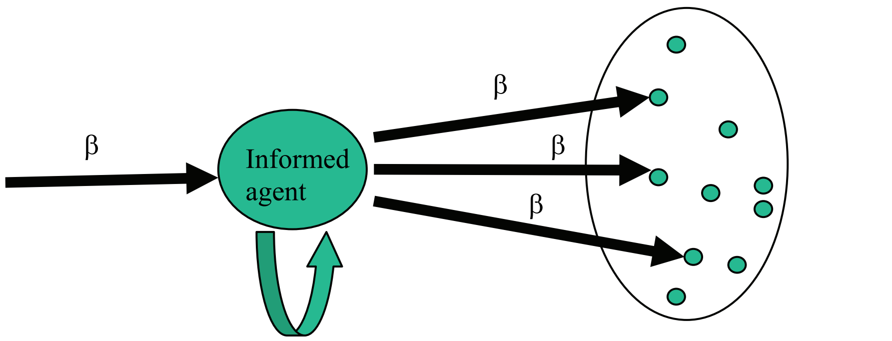
$$\frac{dS}{dt} = -\beta IS + \delta I$$
$$\frac{dI}{dt} = \beta IS - \delta I \quad (2.1)$$

and the SIR model can be represented as:

$$\frac{dS}{dt} = -\beta IS$$
$$\frac{dI}{dt} = \beta IS - \delta I \quad (2.2)$$
$$\frac{dR}{dt} = \delta I$$

The network with high link density and small variance between populations meets homogeneous assumption. However there are also many real networks which don't meet homogeneous model assumption, for example, the Internet, social network, metabolic network and so on. In many network, the number of contactable nodes of each node is restricted to some extent by the network regulation. Therefore we should consider the effect of the network topology.

In this paper, we view information propagation as a dynamical birth-death process with self-recovery like SIS model, as shown in Fig. 1. An informed agent i propagate the information to another agent j in a single step with probability β, while at the same time an informed agent i may forget or lose interest with some probability δ. The ratio of the two factors $\lambda = \beta/\delta$ defines the relative diffusion rate of the information.



δ: forgetting rate β: Transmission probability to neighbors

Fig. 1 Information spread with forgetting

3. Evolutionary designed networks

In social systems that involve large numbers of interacting agents, emergent global stylized facts that arise from local interactions are a critical concept. The emergence of the Internet, for instance, marked the appearance of totally new forms of social and economic exchange (Sole, 2003). For instance, the choice of new added node to connect with makes global law like power law in the Internet (Balabashi, 1999). The dynamics on networks also develop well optimized topology. We can see the some results of network optimization in transport network, power grid network, the Internet and so on. These networks are often not static. We just see the snapshot of developing process. In this paper, we make optimized network as a result of accumulation of little improvement, which is produced by evolutionary design.

3.1 Definition of objective functions

We take two factors into consideration to design networks. One is maximum eigenvalue of adjacency matrix. This represents the network performance for diffusion process. The diffusion process will start at lower threshold (relative transmitting rate (β/δ)) on the network with small maximum eigenvalue. Another one is average degree. Average degree represents the costs to build the network topology. The following two subsections are devoted to these objective functions.

3.2.1 Eigenvalue of adjacency matrix

Any graph (network) G can be represented by its adjacency matrix A(G), which is a real symmetric matrix, $a_{i,j}=a_{j,i}=1$, if vertices i and j are connected, or 0, if these two vertices are not connected. We denote the probability that agent i aware the information at time t as $p_i(t)$. The column vector $p(t) = (p_1(t), p_2(t), \cdots, p_N(t))$ represents the awareness probabilities of the agent population. The transition of the awareness probabilities is now described as

$$p(t+1) = (\beta A + (1-\delta)I)\, p(t) \tag{3.1}$$

where I is a $N \times N$ identity matrix. The long-run behavior of the above system is determined the structure of the system matrix S=$\beta A + (1-\delta)I$. Wang etc (2003) also proved that the spectral of the system matrix S (the distribution of eigenvalue of S is closely related to the spectral of the adjacency matrix A), and we obtain

$$\lambda_i(S) = \beta\lambda_i(A) + (1-\delta),\ i = 1,2,\cdots N \tag{3.2}$$

where $\lambda_i(S)$ the i-th maximum eigenvalue of the system matrix S. The maximum eigenvalue $\lambda_i(S)$ is also called the principal eigenvalue of the system matrix. We denote the maximum eigenvalue of the system matrix S by $\lambda_1(S)$.

(i) $\lambda_1(S) < 1$, then p(t) converges to the zero vector
(ii) $\lambda_1(S) > 1$, then p(t) converges to infinity (3.3)

From the relation in (4.2), the conditions in (4.3) in terms of the principal eigenvalue of the system matrix S can be expressed using of the principal eigenvalue of adjacency matrix A:

(i) $\beta/\delta < 1/\lambda_1(A)$, then p(t) converges to the zero vector
(ii) $\beta/\delta > 1/\lambda_1(A)$, then p(t) converges to infinity (3.4)

From this, it is known that the diffusion process is governed by the threshold phenomenon, and therefore information spreads will start in the network only if

$$\beta/\delta > 1/\lambda_1(A) \tag{3.5}$$

Therefore, high infection rates lead to the possibility of positive diffusion, as do degree distributions with high variances. The rationale behind high-variance distribution is that there will be some hub nodes with highly informed, which may foster the diffusion, accelerating the transmission of information.

3.1.2 Average degree of network

It is very simple question, "What is best network for diffusion process". The answer is "Complete graph", in which each node is connected to every other node. However it is not realistic to form complete graph as the size of network grows. It takes a lot of costs to make and keep links between nodes. The allocated resource to spend is usually limited. Therefore many realistic networks, for example, social network, railway network and neuron network redress balance between the performance and the number of link. In this paper we evaluate the cost of building network topology by average degree of network. We don't care geodetically distance between nodes, because we just want to study the relationship between network topology and the performance of diffusion process.

3.1.3 Weighted objective function

We evaluate each network by weighted objective function as follows,

$$E = \omega \frac{1}{\lambda_1(A)} + (1-\omega)\frac{<k>}{N-1} \tag{3.6}$$

where ω $(0 \leq \omega \leq 1)$ is a parameter controlling the linear combination of the inverse of $\lambda_1(A)$ and the average degree $\langle k \rangle$. The average degree $\langle k \rangle$ is normalized by $N - 1$.

The minimization of E makes the maximization of maximum eigenvalue, which is associated with diffusion threshold, of adjacency matrix of the network and the minimization of average degree, which denotes network costs.

3.2 Evolutionary network design method

Table 1 Parameter on genetic algorithm

Genetic algorithm model	Minimum Generation Gap model (Sato, 1996)
Initial population size	100
"Child" population size	100
Objective function	$E = \omega \frac{1}{\lambda_1(A)} + (1-\omega)\frac{<k>}{N-1}$

We apply genetic algorithm (MGG: Sato, 1996) to design optimized network. The genetic algorithm is metaheuristic algorithm. We use the adjacency matrix of the network as chromosome. The parameter on genetic algorithm is shown in Table 1.

In this way we design optimized network for diffusion varying parameter ω on Eq. (3.6).

4. Optimized network

We design optimized network by using genetic algorithm as mentioned in 3.2. The object of our design is to make the network which has large maximum eigenvalue with low network cost. The maximum eigenvalue of each optimized network is compared with random graph scale free networks, as shown in Fig. 2.

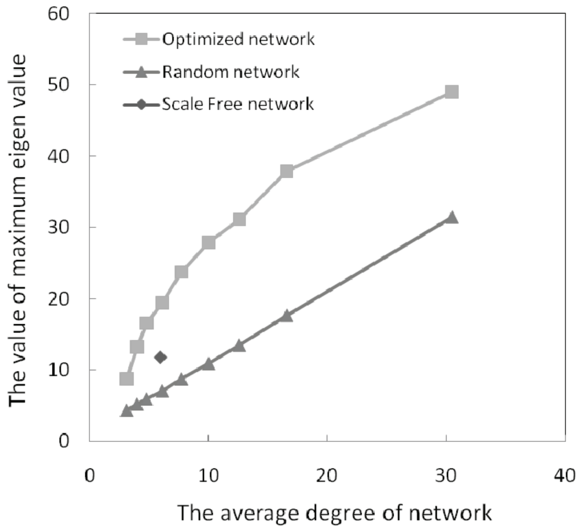


Fig. 2 The maximum eigenvalue of optimized network

We make optimized network which has 300 nodes varying parameter $\omega = (0.1, 0.2, \cdots, 0.9)$ in Eq. (3.6), from left. Each random network has same the number of average degree corresponding to optimized network. And a scale free network has approximately same the number of average degree (=5.96), corresponding to optimized network which has 6.11 average degree.

5. Diffusion simulation

We simulate diffusion process by the model which is explained in section 2, on optimized network, scale free network and random network. The point of this simulation is to show the effect of the deference of maximum eigenvalue on each network. We also show simulation results to confirm diffusion process will start at lower score (the ratio of transmitting rate β to forgetting rate δ) on the optimized network than the thresholds on other network topologies, in which maximum eigenvalue is smaller than optimized network.

5.1 Network setting

In our simulation, the optimized networks by genetic algorithm ($\omega = 0.3, 0.4$) (which is mentioned in 4) is compared to other network topology which has similar average degree and same the number of nodes. The property of scale free network is that the degree distribution obeys the power law, which is observable in Internet AS level Topology. This scale free network is generated by Barabasi-Albert (BA) model. The main features with respect to how to make BA model are 1. Networks expand continuously by the addition of new nodes, 2. New nodes preferentially attach to sites that are already well connected. On the other hand random network is generated to have same average degree with optimized network.

We did simulation on three network topologies. The property of each network is shown in table 2. The each network has same a number of nodes (=300). And the network cost (the number of link) of optimized network and random network is lower than Scale free network.

Table 2. Table of network property

	Optimized network	Scale free network	Random network
The number of node	300	300	300
The number of link	716	894	716
The average distance	3.6	3.0	3.8
The number of average degree	4.8	6.0	4.8
The maximum eigenvalue of adjacency matrix	17	12	6

5.1 Simulation parameter setting

The maximum eigenvalue of adjacency matrix decides threshold for diffusion process. Therefore the ratio of transmission rate β to forgetting rate δ determines the information or products will diffuse or not and the fraction of informed nodes when diffusion process begin. The diffusion threshold τ is written by maximum eigenvalue of adjacency matrix $\lambda_1(A)$ as:

$$\tau = 1/(\lambda\_1(A)) \tag{5.1}$$

Therefore the diffusion process will start when the ratio of transmitting rate to forgetting rate becomes larger than threshold τ, as shown in Eq. (5.2).

$$\beta/\delta > 1/(\lambda\_1(A)) \tag{5.2}$$

We show diffusion threshold of each network, as shown in Table 3. $(< k >)/(< k\text{^}2 >)$ and $1/< k >$ in table 3 are diffusion process threshold of scale free network and random network, which are proposed by (Pastor-Satorras and Vespignani, 2002),(Kephart and White, 1991)

In each simulation we set transmission rate β as:

$$\beta = \text{Score} \times \delta/(\lambda\_1(A)) \tag{5.3}$$

where score is a scaling parameter to control relative transmitting rate. When the score equals 2, the information have two times effective transmitting rate compared with the information with the score equals 1. In this paper δ is fixed at 0.1 in all our simulation. We vary score parameter from 0.1 to 5 in each network.

Table 3 Comparison of diffusion process threshold

Network Topology	Threshold expression	Threshold transmitting rate (δ=0.1)	score $= \frac{\beta}{\delta} \div \frac{1}{\lambda_1(A_{optimized})}$
Optimized network	$\frac{1}{\lambda_1(A_{optimized})}$	**0.0060**	**1**
Scale free network	$\frac{<k>}{<k^2>}$	**0.0079**	**1.3**
	$\frac{1}{\lambda_1(A_{SF})}$	**0.0085**	**1.4**
Random network	$\frac{1}{<k>}$	**0.021**	**3.5**
	$\frac{1}{\lambda_1(A_{random})}$	**0.0168**	**2.8**

5.2 Simulation process

I. Selection of diffusion model

We select SIS model as information diffusion model. The population in network, which is denoted by nodes in network, is divided to susceptible and informed people.

II. Decision of the position and the number of initial infected node

The only one node which has maximum degree is selected as an initial infected node in all our simulation. We confirmed such node also has high eigenvector centrality (not included here).

III. Iteration diffusion process

Our simulation is done by iteration process. During each time step, informed nodes attempt to transmit each of its neighbors with probability β. In addition informed nodes return to susceptible node with the probability δ. The node's condition of a new informed node is updated at next time step. The simulation proceeds until diffusion process becomes steady state ($(dI(t))/dt \cong 0$).

5.3 Simulation results

Fig. 3 shows that diffusion process begins at lower threshold on optimized network, compared with other network topology. The maximum eigenvalue of adjacency matrix is seemed better indicator of diffusion thresholds than $\frac{<k>}{<k^2>}$ and $\frac{1}{<k>}$, which are proposed as diffusion thresholds of scale free network and random network respectively. At score =2 if fig.3, the optimized network has 8% fraction of informed nodes even though 2 or 0% on scale free network or random network respectively.

We plot the fraction of informed nodes at each simulation step, which is average value of 100 same experiments, as shown in fig.4. In the both case (score=2, 3.9), the speed of diffusion process on optimized network is faster than other. Therefore, the optimized network has the properties, which are low diffusion threshold and fast diffusion speed.

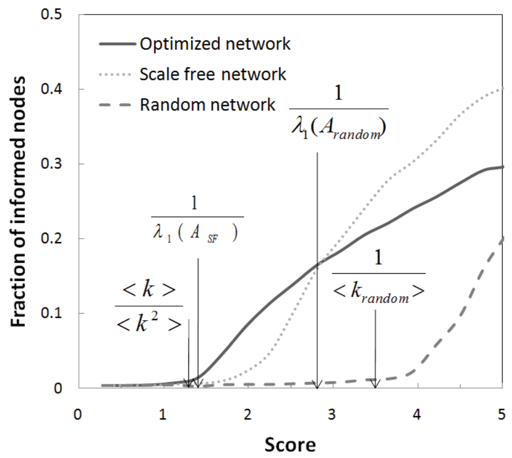
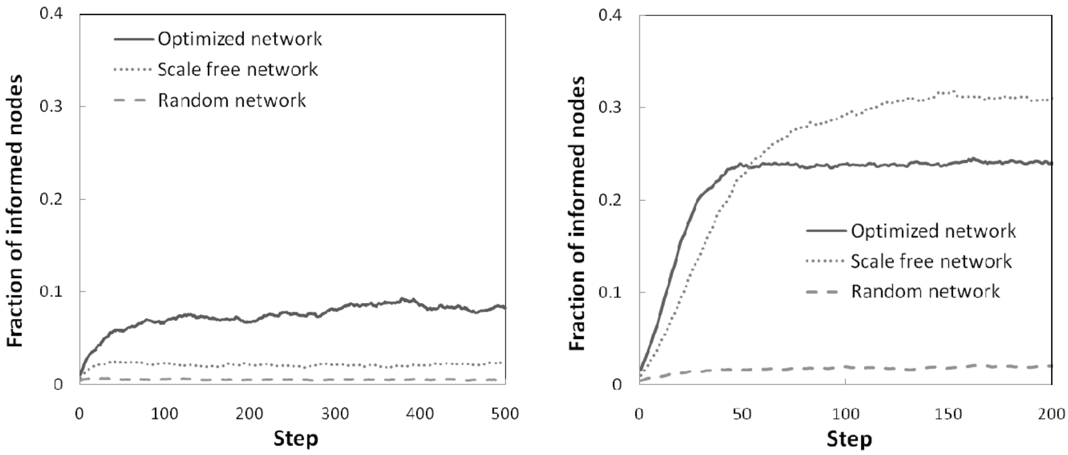


Fig. 3 Early start of diffusion process on optimized network

We vary transmitting rate β in each network (δ is fixed at 0.1), and plot fraction of infected nodes at steady state versus score (log scale) on each network topology. The score is calculated by Score $= \beta/\delta \times \lambda_1(A_{optimized})$, where $A_{optimized}$ denotes adjacency matrix of optimized network, which is designed by evolutionary design. $\frac{<k>}{<k^2>}$ and $\frac{1}{<k>}$ are proposed diffusion thresholds of scale free network and random network (Pastor-Satorras and Vesignani , 2002) (Kephart and White, 1991).



(a) Score=2 (b) Score=3.9

Fig. 4 The change of fraction of informed nodes in simulation

6. Conclusion

Suppose that a number of consumers mutually transmit information about newly launched information or products through a social network in which they are involved. Diffusion processes based on the epidemiological models describe information spread in terms of transmissibility and forgetting at the individual level. These are extended diffusion models that explicitly include decisions influenced by social situations and word-of-mouth processes. In this paper we study about the impact of network topology in diffusion process on SIS model. In sis model nodes of network are once informed, can randomly forget, however after forgetting they become susceptible once again. We show the method to design optimized network for diffusion process by evolutionary design. We show optimized network enables diffusion process to rise at lower threshold compared with scale free network and random network which have lower and about same network cost.

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