Statistical modeling of strategic behaviors: advances in microstructure.

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ABSTRACT

The goal of the paper is to show various results on agent-based financial market modeling[1–3] in the specific context of limit order book markets. I demonstrate how the interaction between heterogeneous market participants lead to specific limit order book markets characteristics, in particular the seemingly contradictory presence of correlations in the buy and sell market order flows with informational efficiency expressed by the absence of autocorrelations in the returns. By studying the effects of diversification among assets in this context, I provide an alternative explanation for the presence of long memory in these order flows. With the goal of provoking novel scientific debates on these important matters in finance, I then discuss the economic implications and predictions if one assumes that these plausible market mechanisms capture the main features for understanding how markets function.

Keywords: Agent-based model, complex systems, financial markets, stylized facts, limit order book, liquidity, diversification, business and management.

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Contents

1. INTRODUCTION.

Trading in the limit order book is a way to deal with market frictions such as transaction costs by placing limit orders whereas impatient traders place market orders for immediate execution. In this paper, I direct my agent-based study towards the limit order book which is receiving an intense research interest as reviewed in Bouchaud et al.[4] which reports on common facts between these markets. The growth in electronic trade execution systems has been indeed explosive and electronic trading is commonly based on a limit order book structure. Liquidity is the central issue of the limit order book trading. Limit orders can be seen as supplies of liquidity whereas market orders as demands of liquidity. The interaction between liquidity providers and informed traders demanding immediacy of execution is indeed the main ingredient of micro-structure studies on this topics and is a key ingredient of my present work. My approach aims also at bringing insights on the direct impact of trading strategies on speculative markets dynamics as opposed to the traditional zero-intelligence model studies.[5] I thus introduced an original agent based framework[3] for the limit order book with fully strategic traders' behaviors compatible with more general financial market models.[1]
The goal of the paper is to provide clear plausible explanations of seemingly contradictory empirical observations and a complete mathematical framework for understanding and controlling order book markets dynamics. In particular, by controlling the descriptive time scale, I show how market impact, linear by definition and the interaction between strategies in the limit order book market lead to the emergence of correlations in the order flows, yet with informational efficiency expressed by the absence of correlations in the returns. Even if information is Gaussian, the market order flows composed of informed traders is also determined by the volatility linked order placement in the book. The link between volatility and order placement in the model is also compatible with the observed relation between market volatility and the spread seen as the consequence of the minimal non-zero evaluation of risks and costs, or as the presence of a specialist fixing this spread.
An ambitious goal of my agent-based study is to provide a solid theory that integrates infrastructural elements with markets participants behaviors and their interactions, in order to produce a global systematic understanding of the features that dominate financial markets exchanges. For this purpose, I complete the framework for realistic modeling of modern markets by mathematical and statistical treatments of cross-markets dynamics.[2] These endogenous dynamics lead to more instabilities that are observed in the fluctuations of cross-assets correlations. Diversifications among assets[6] commonly sought to minimize risks are also responsible in this framework to longer memories in the order flows.

The article is organized as follows. I summarize in section 2 the complexity modeling approach for limit order book markets. In section 3, I remind the economic implications of the numerical experiments. In section 4, I complete the framework by studying the effects of diversifications in the cross-assets order book markets dynamics. I draw conclusions in the last section.

2. DEFINITION OF THE AGENT-BASED LIMIT ORDER BOOK STRUCTURE.

The model describes a limit order book where a single asset is traded by various types of agents. These traders populate the book with limit and market orders. Trading takes place at discrete time steps t. These periods may be interpreted on a wide range of time scales: from tick level to trading days or weeks. The trades in the model result from matching market orders against limit orders. The first group of agents is composed of n market neutral traders operating as liquidity providers: at each time period, each agent i of this group places limit orders (bid and ask) of one unit size at distance $\theta_i(t)$ ($\sim$ estimation for the volatility during [t,t+1]) from the price (mid-price which is the average of the best bid and best ask). Given a signal on expected price change ("new information") IID Gaussian noise $\epsilon_t \sim N(0, D^2)$ with $D=$ noise level, informed agents then send market orders to buy or sell limit orders that are at a distance below the expected market price. Prices then move up or down according to the direction of the trades. The model produces stochastic heterogeneity and sustains it through the updating of agents' strategies. Let us define in a more mathematical way the ingredients of the limit order dynamic model.

2.1. Strategic forces in the trading arena.

I describe first the adaptive decision-making rules at each time period. Liquidity providers' strategies are distances or thresholds $\theta_i(t)$ that determines their bid and ask orders. These positions results from their expectations of price movements and their risk aversion related for example to the uncertainty of execution and the waiting time where informed traders can take advantage of the liquidity they are providing. Each agent i places a bid and ask according to:

$$bid(i) = p_t - \theta_i(t)$$

$$ask(i) = p_t + \theta_i(t). \tag{1}$$

Given an information ϵ_t on the price change, which can have many potential origins (networks of friends, public news, private information) informed agents then send market orders to buy or sell limit orders that are at a distance below the expected market price. The market order flow is thus determined by the direction of the price change and the number of available limit orders in the book. Each liquidity provider's limit order will be likely to be executed or not according to:

- buy order likely to be executed if $\epsilon_t > \theta_i(t)$: $\phi_i(t) = +1$ (buy)
- sell order likely to be executed if $\epsilon_t < \theta_i(t)$: $\phi_i(t) = -1$ (sell)
- limit orders are canceled if not executed: $\phi_i(t) = 0$ (canceled)

So:

$$\phi_i(t) = 1_{\epsilon_t > \theta_i(t)} - 1_{\epsilon_t < -\theta_i(t)}. \tag{2}$$

In the model, orders placements and execution are very sensitive to the limit price, but not to the volume of the order. By construction, the time period is the typical cancellation time of limit orders' strategies responsible for the market activity at the studied time scale. One should be aware that the frequencies of execution and cancellation are comparable. The capabilities of the model demonstrated throughout the paper show that the typical cancellation time can be a relevant choice for determining the actors of the returns dynamics at a characteristic timescale. This quest and focus on the actors of the dynamics at a given time scale is indeed the strength and utility of the statistical approach when attacking problems with high number of interacting entities.

2.2. Price response to aggregate demand.

I now describe the dynamics of the model ruled by the evolution of price and updating of strategies.
The price is adjusted by the excess demand $Z_t = \sum \phi_i(t)$, which corresponds to the orders likely to be executed, through a price impact function g which depends on the total number of traded shares n:

$$r_t = \ln\left(\frac{p_{t+1}}{p_t}\right) = g(Z_t). \tag{3}$$

The empirical behavior[4] of this function indicates increased linearity and decreased slope while increasing n. I focus on the linear case defined as: $r_t = \frac{Z_t}{\lambda n}$ where $ng'(0) = \lambda^{-1}$. λ represents market depth at the studied time scale, the typical order imbalance needed to move the price by one point, normalized by total number of traded shares n and in this framework characterizes the order book depth. A market with increased market depth is more liquid. The current framework is inclusive (see Ghoulmié[7] for further discussions on market impact). By changing this function, one can explore the impact of mechanical execution on market learning dynamics.

2.3. Learning strategies for the fitness thresholds.

Initially, each liquidity provider agent has a trading rule given by the choice of a fitness threshold $\theta_i(0)$. At each time step, if the change in price is greater than the parameter δ, each agent i consider the price movement as significant and update with probability s according to the following defined rule: $\theta_i(t+1) = |r_t|$.
As a consequence of this rule, a fraction $s \in [0, 1]$ of these agents updates their strategies/thresholds using recent information. Because the limit order positions are directly related to the price changes, this updating rule implies that the limit orders are placed outside a spread 2δ. I introduced here the parameter δ which controls the spread, the difference between the best bid and best ask, and often fix it to 2δ. The reason of this choice for modeling the creation of a spread in the book is that agents' threshold are indeed an estimate of risk and it is thus wise to bound it at a non-zero minimal value.
Introducing IID random variables $u_i(t), i = 1...n, t \geq 0$ uniformly distributed on $[0, 1]$, which indicate whether agent i updates his threshold or not, we can write the learning rule as

$$\theta_i(t) = 1_{|r_{t-1}|>\delta}[1_{u_i(t)<s}|r_{t-1}| + 1_{u_i(t)\geq s}\theta_i(t-1)] + 1_{|r_{t-1}|\leq\delta}\theta_i(t-1). \tag{4}$$

Here ϵ_t represents randomness due to public news arrivals whereas the random variables $u_i(t)$ represent idiosyncratic sources of randomness. This way of updating can be seen as a stylized version of various estimators of volatility based on moving averages or squared returns. As a consequence of this mapping, a feedback loop is created between the volatility and the orders placement in the book.
The updating rule allows to differentiate between indistinguishable rational players. Indeed, given this probabilistic updating model, even if we start from an initially homogeneous population $\theta_i(0) = \theta_0$, heterogeneity develops into the population through the learning process which corresponds to changes in the strategies motivated by trading costs reduction. In this sense, the heterogeneity of agents strategies is endogenous in this model and, as we will see below, evolves through high fluctuations.

2.4. Features of the model.

Let us recall the main ingredients of the model defined above. At each time period :

• Informed agents receive a common signal $\epsilon_t \sim N(0, D^2)$.

• Each liquidity provider agent i uses a threshold $\theta_i(t)$ to set his bid and ask according to (1).

• if $|\epsilon_t| > \theta_i(t)$ informed agent considers the signal as significant and are likely to generate a market order $\phi_i(t)$ according to (2).

• The market price is affected by the excess demand and moves according to (3).

• Each liquidity provider agent adjusts, with probability s, her threshold according to (4).

With regard to some of the agent-based models considered in the literature, some important aspects are the following:

• Prices move through market fluctuations of supply and demand. Players can be "fundamentalist" and "chartist" traders. Price formation results from direct interaction of agents in the book.

• Information asymmetry: the model is based on the dynamics between informed traders and liquidity providers. Liquidity providers use market neutral strategies and differ in the way they process the information.

• Liquidity providers are localized in the book.

• Endogenous heterogeneity: heterogeneity of agents behavioral rules appears endogenously due to their learning strategies. There is also a "structural" heterogeneity[8–10] in the model between informed traders placing market orders and liquidity providers placing limit orders. This structural diversity is also compatible with a distinction between fundamentalists and chartists, between speculators with directional strategies and hedgers with market-neutral strategies.[11] The dynamics of this heterogeneity is responsible for the shape of the book.

• The spread is the consequence of the decision making under uncertainty. The order placement strategies are indeed based on an estimate of risk which is wise to set strictly greater than zero.

The model has tractable parameters: s describes the average updating frequency, D is the standard deviation of the news arrival process and λ is the market depth. Furthermore, as we will observe in the next section, if we require to interpret specifically the trading periods this will put a further restriction on the parameters, reducing the effective number of parameters. Nevertheless, the clear structure of the model generates time series of returns with interesting complex dynamics and with properties similar to those observed empirically.

Parameters	Monthly, Daily Dynamics	Intraday Dynamics
Number of agents n	$10^3 - 10^4$	$10^3 - 10^4$
Level of noise D	$10^{-3} - 10^{-2}$	$10^{-4} - 10^{-3}$
Updating frequency s	$10^{-1} - 10^{-2}$	$10^{-1} - 10^{-2}$
Market depth λ	$5 - 20$	$2 - 5$
spread 2δ	$\frac{10}{n\lambda}$	$\frac{10}{n\lambda}$

Table 1. Parameter values for the different time scales.

3. COMPUTER MODEL EXPERIMENTS.

The model is straightforward to simulate and the results can be reproduced in a robust way. I present some typical results that I obtained for all the simulated runs of the model when fixing a set of parameters. The simulations identify basic properties of the model and indicate the range of parameters that accord with the empirical data on asset returns. Table 1 recalls the various investigated ranges of the parameters for typical short term dynamics and long term dynamics.

Using the parameter ranges above, I perform an extensive simulation study of price behavior in this model. Figures 1, 2 and 3 illustrate typical sample paths obtained with different parameter values: they all generate series of returns with realistic ranges and realistic values of annualized volatility. For each series, the figures give the demand Z_t, the market activity $A(t) = |Z_t|$ which corresponds to the traded volume, the evolution of the price with a plot of the logarithmic price $\ln(p_t)$, the returns r_t, the annualized volatility, the histogram of returns both in linear and log scales, the ACF of returns C_r, and the ACF of absolute returns $C_{|r|}$.
We note that all the return series possess some regularities that match some of the empirical properties[3]:

• Excess volatility: the sample standard deviation of returns can be much larger than the standard deviation of the input noise representing news arrivals: $\hat{\sigma}(t) \gg D$. This is the consequence of orders' strategies that transform a signal representing news into binary trading at a fixed transaction volume. Paradoxically, it is the comparison of the news to estimates of the volatility that leads to an amplification of the volatility. In other words, even if agents are acting sensibly to minimize their risks, they are producing collectively more variability in the returns.
This result is observed for various values of T'.

• Mean-reverting volatility: the market price fluctuates endlessly and displays "stochastic volatility". The volatility, as measured by the moving average estimator $\hat{\sigma}(t)$, goes neither to zero nor to infinity and displays a mean-reverting behavior. This behavior is found in many empirical studies. GARCH models, on one hand, and stochastic volatility models, on the other, aim at reproducing this mean-reverting stochastic behavior of volatility.

• The simulated process generates a leptokurtic distribution of returns with (semi-)heavy tails, with a kurtosis around $\kappa \simeq 4 - 11$ ($\kappa = 4.5$ for the run presented in Figure 1) for daily returns and fatter tails in the returns $\kappa \simeq 11 - 50$ ($\kappa = 16.5$ for the run presented in Figure 2 and $\kappa = 35$ in Figure 3) for intraday dynamics including very short term timescales. This result confirms the role of $D_{eff} = D\lambda$ at describing both long term and short term dynamics and in having a unified framework to study financial market dynamics at various time scales. This result expresses a relation in the model between the level of diffusive behavior of the news, the market depth which describes the liquidity of the market and the trading period which can be interpreted in this framework as the typical cancellation time of agents' limit orders.

• The returns are uncorrelated: the sample autocorrelation function of the return exhibits an insignificant value (very similar to that of asset returns) at all lags, indicating the absence of linear serial dependence in the returns.

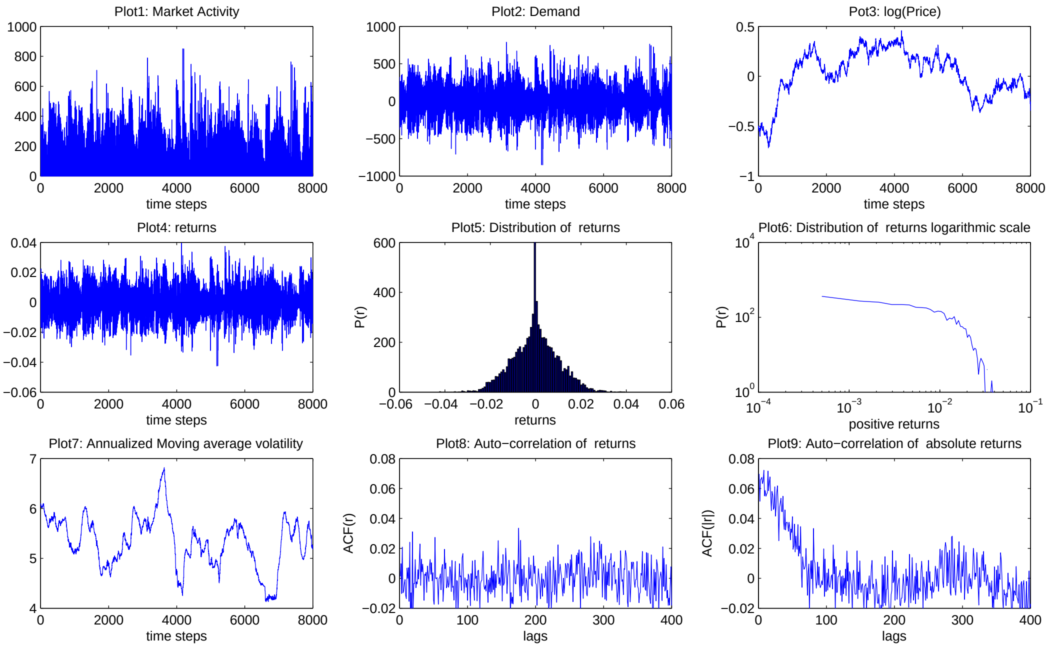

Figure 1. Daily prices and volatility behaviors generated numerically by the agent-based market model for $n = 2000$, $D = 0.001$, $\lambda = 10$, $s = 0.015$ and $\delta = \frac{5}{n\lambda}$. The simulated behaviors correspond to long term dynamics. One can observe in the presented pictures: the prices behavior in Plot3, the endogenous bursts of market activity in Plots 1, 2 and 4, the non-Gaussian distribution of the returns in Plots 5 and 6, the stochastic mean-reverting volatility in Plot7, the uncorrelation of the returns in Plot8 and the volatility clustering in Plot9.

• Diagnostics for volatility clustering : the autocorrelation function of absolute returns remains positive, and significantly above the autocorrelation of the returns, over many time lags, corresponding to persistence in the amplitude of returns over periods ranging from a few weeks to several months. This nonlinear dependence in the returns, shown in Figure 4, is an indication of volatility clustering. By construction of the model, these correlations are also in the market orders flows as observed empirically.

The computer controlled experiments confirm the role of D_{eff} as a control parameter for the descriptive time scale of the returns, and the role of $1/s$ as a time scale parameter that controls the dependence properties of the returns.

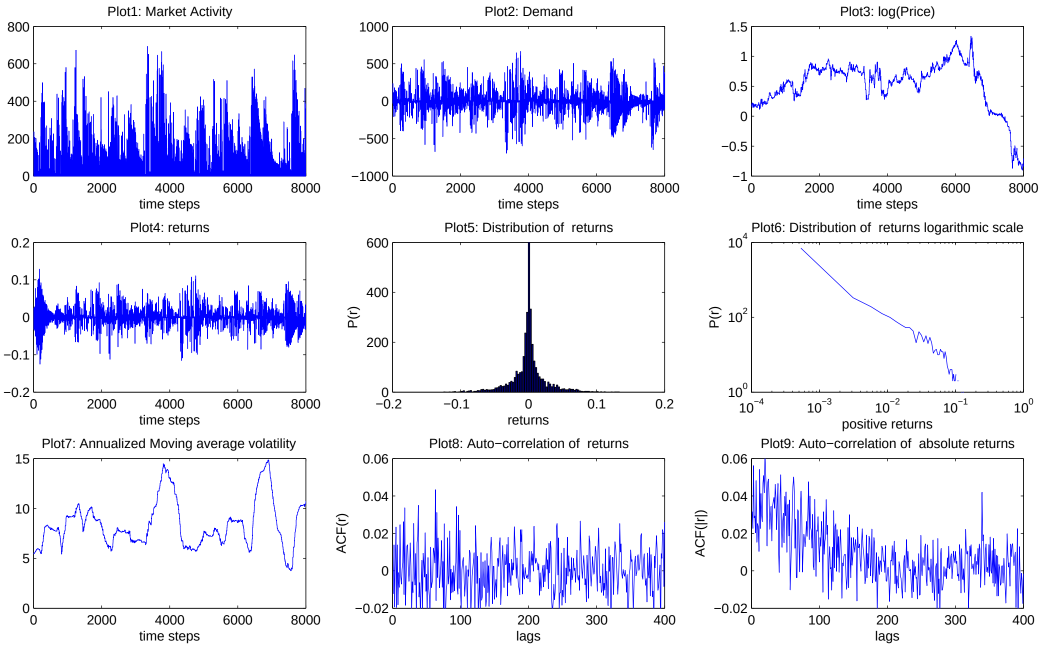

Figure 2. Intraday prices and volatility behaviors generated numerically by the agent-based market model for $n = 2000$, $D = 0.001$, $\lambda = 3$, $s = 0.015$ and $\delta = \frac{5}{n\lambda}$. In Plots 4 and 5, the distribution of returns has power law look. One note the fatter tails in the distribution of returns compared to the longer term dynamics. For the run presented, the kurtosis is 16.5 which is much higher than 4.5 for the run presented in Figure 1.

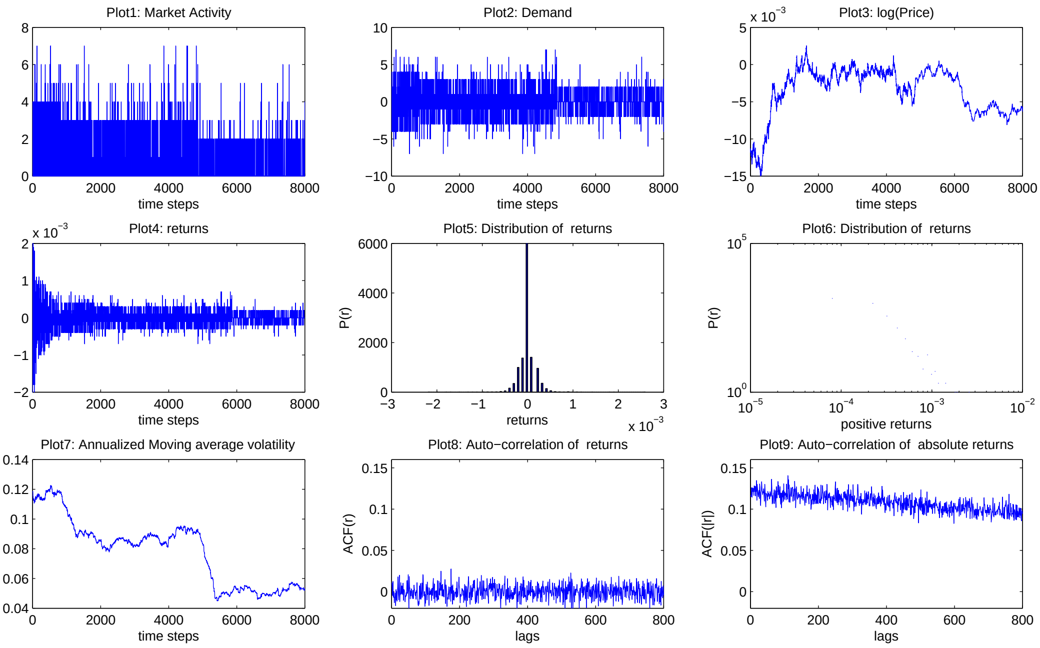

Figure 3. Very short term (1min) dynamics according to the model, generated numerically for $n = 10000$, $D = 0.0001$, $\lambda = 5$, $s = 0.015$ and $\delta = \frac{5}{n\lambda}$. The simulations can allow a tick by tick resolution. The distribution of returns in Plots 4 and 5 tends to be discrete which is consistent with ultra high frequency finance. The high occurrence of zeros, also observed with the model at longer term dynamics, are consistent with high frequency empirical finance. One can decrease the high number of zeros by simply adding a noise term with small amplitude for small effects in the price formation rule.

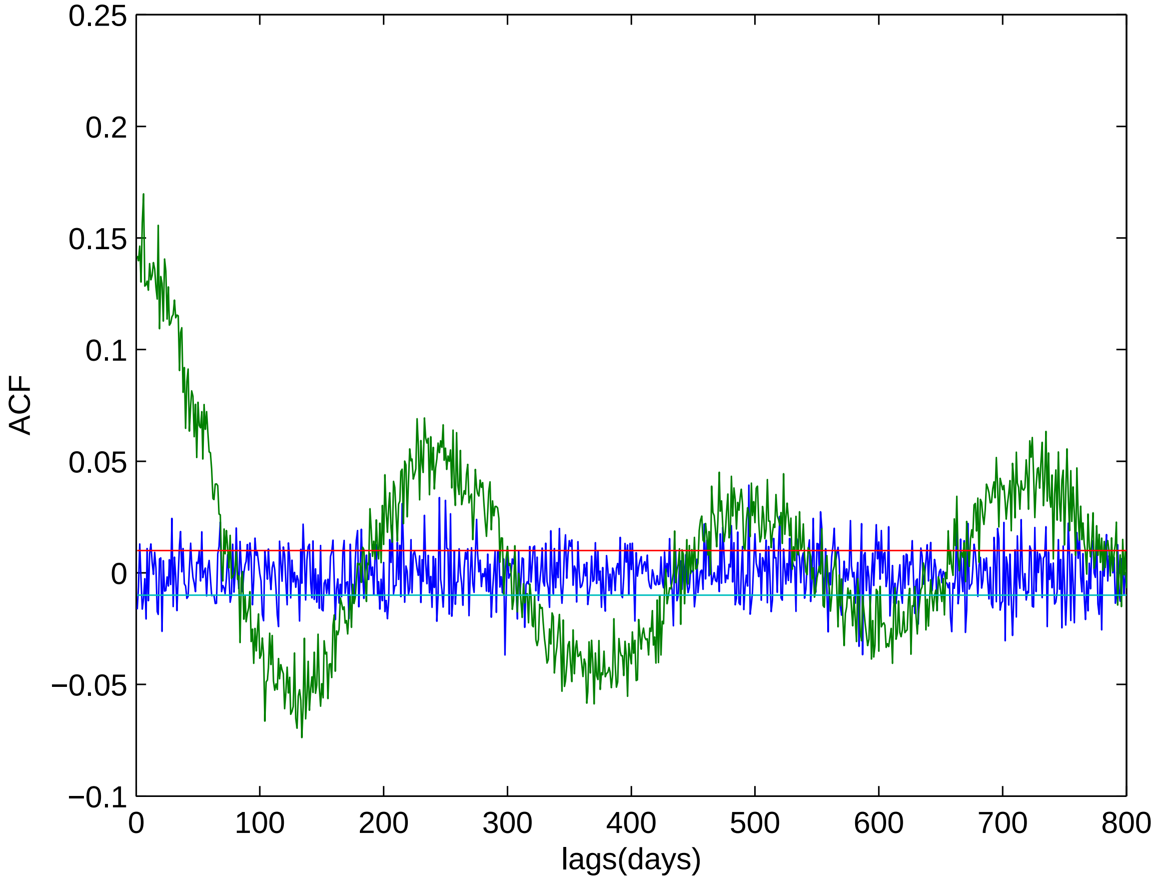

Figure 4. Volatility clustering phenomenon obtained for $n = 10000$, $D = 0.001$, $\lambda = 10$, $s = 0.015$ and $\delta = \frac{5}{n\lambda}$. Confidence intervals computed for 95% of a normal distribution are drawn which shows the uncorrelation of the returns. The autocorrelation function of the absolute returns remains positive for a period $1/s$ and then oscillates around zero before vanishing. These oscillations, that may link the adjustement frequency to some seasonalities observed in the data, are no longer observed when studying the multi-asset case[2] where the full long memory effect is obtained.

4. ORDER BOOK MARKETS CROSS-ASSETS DYNAMICS

4.1. Definition of the multi-asset model

I now extend the previous model to a two-asset case that can in fact be directly generalized to a higher number of assets. The model describes a market where two assets, with mid-prices denoted by $p_{1,t}$ and $p_{2,t}$ respectively, are traded by n liquidity providers agents at discrete time steps t and informed traders. Compared to the single asset case, at each time step, the model is updated according to the additional rule:

• Every informed agent receives a common signal $\epsilon_t \in N(0, D^2)$ for both assets.
• Each liquidity provider agent i has a strategy for the first asset, $\theta_{1,i}(t)$, and for the second asset, $\theta_{2,i}(t)$, and then places orders of respectively $\omega_{1,i}(t)$ and $\omega_{2,i}(t)$ units of assets. The weights $\omega_{1,i}(t)$ and $\omega_{2,i}(t)$ are defined positive and bounded by the following constrains:

$$\omega_{1,i}(t) + \omega_{2,i}(t) = 1. \tag{5}$$

• The asset prices $p_{1,t}$ and $p_{2,t}$ are affected by the excess demand and move according to

$$r_{1,t} = \ln\left(\frac{p_{1,t}}{p_{1,t-1}}\right) = g\left(\frac{\sum_i \omega_{1,i}\phi_{1,i}(t)}{n}\right), \tag{6}$$

$$r_{2,t} = \ln\left(\frac{p_{2,t}}{p_{2,t-1}}\right) = g\left(\frac{\sum_i \omega_{2,i}\phi_{2,i}(t)}{n}\right), \tag{7}$$

where $r_{1,t}$ and $r_{2,t}$ are the returns and g is the price impact function.
• Each agent updates, with probability s, its threshold $\theta_{1,i}(t)$ to $|r_{1,t}|$ and $\theta_{2,i}(t)$ to $|r_{2,t}|$.

• Each agent updates, with probability s, the weights according to

$$\omega_{1,i}(t) = \frac{1}{1 + e^{\beta . V_t}}, \tag{8}$$

$$\omega_{2,i}(t) = 1 - \omega_{1,i}(t), \tag{9}$$

where the difference between assets absolute returns $V_t = |r_{2,t}| - |r_{1,t}|$ is the fitness function that determines the adaptive asset allocation and β is the parameter that controls the intensity of choice between the two assets. The parameter β determines the rationality of agent's choice. For $\beta = 0$, agents keep the same equal weights on both assets independently of the fitness function, and the behavior of the returns is equivalent to the single asset market studied in the first section. For an infinite value of β, agents are fully rational and chose radically the most profitable asset, the asset with the highest absolute return.

4.2. Nonstationarity of assets returns

In this section I explore numerically the effects of the diversification on the complex behavior of asset returns. The analysis is similar to the multi-asset study[2] previously achieved and I recall the results obtained. In particular, by setting the other parameters in ranges similar to those required to get realistic dynamics in the single asset case, we study the role of the parameter β that characterizes the asset allocation. The price impact function has been chosen to be linear, $g(x) = x/\lambda$, with the same market depth λ for both assets. The behavior of the returns generated by the present model is significantly different from the single asset case. In particular, the returns, reported in Fig. 5 for $n = 1500$, $s = 0.015$, $D = 0.001$, $\lambda = 2$ and $\beta = 1000$, produce fluctuations that are larger than those of the the single asset model. This behavior is emphasized by the fatter tails observed in the probability distribution function and reported in Fig. 6 where a Gaussian distribution is also plotted for visual comparison.

Fig. 7 illustrates that additional differences from the single asset model are observed in the behavior of the annualized volatilities. It is seen that the volatilities of the two-asset model switch intermittently, in a sort of mean reverting behavior, from high to low values in an anti-correlated fashion. These sudden changes of regime

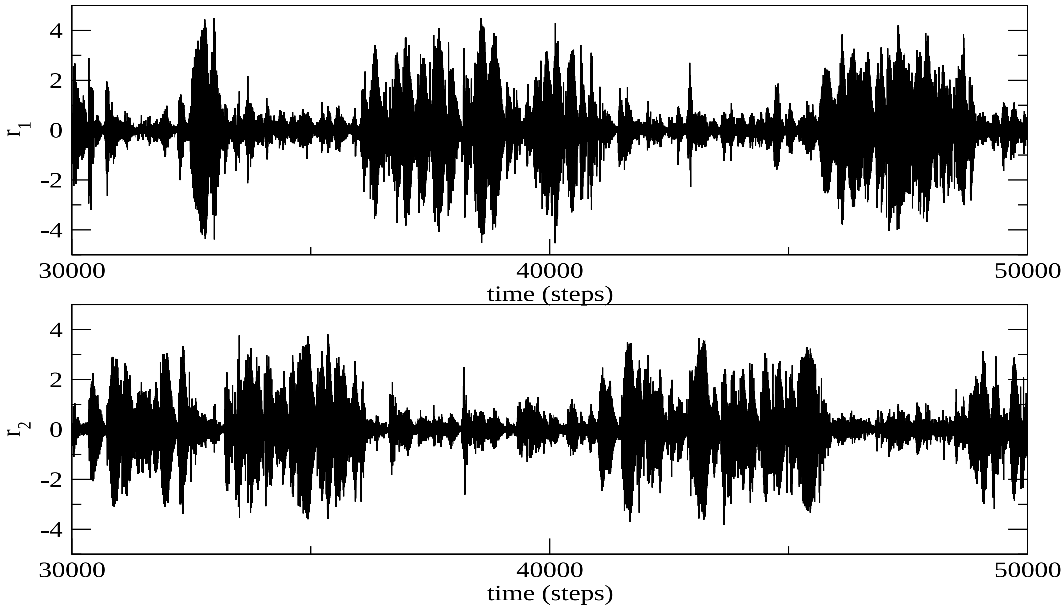

Figure 5. Time series plots of asset1 and asset2 returns, r_1 (top) and r_2 (bottom), generated numerically by the agent-based market model for $n = 1500$, $D = 0.001$, $\lambda = 2$, $s = 0.015$ and $\beta = 1000$. The time series exhibit regime-switching type of behaviors between a high and low volatility periods.

in the volatility are observed in financial data. Compared to the single asset model, this new result brings us closer to the empirical facts observed in financial data. Moreover, these higher fluctuations occur without structural changes in the market or as a result of changes in the fundamentals and are simply related to the diversification process: the agents having very similar trading strategies allocate intermittently more weight on one of the asset and, therefore, leading to higher fluctuations in the traded volume.

I also perform an analysis of summary statistics for 5 consecutive periods of 10000 time steps for the time series generated by the multi-asset model and the single-asset model. These summary statistics are reported in Tabs. 2, 3 and 4 respectively. In the two-asset case we observe fluctuations of the kurtosis between 3 and 12, indicating nonstationarity of asset returns. In the single asset case, instead, this parameter is almost stationary. This result implies that the multi asset diversification of the proposed model promotes non-stationarity, an effective feature that often causes problems to policymakers. We must remark on the destabilizing role of this diversification in comparison with the benchmark single asset market case. This is an interesting case where diversification, which is usually seen as a way to reduce portfolio risk, increases market instability.

4.3. Nonlinear diagnostics

In the present section I estimate the autocorrelation function for the returns of the two assets as well as their absolute value. The results, reported in Fig. 8, show an absence of correlation in the returns time series (bottom). For the absolute value (top), the nonlinear diagnostics point out two different phases: the first, for small lags, is related to the presence of volatility clustering, a feature noticed also in the single asset model, the second persistent phase has not previously been observed. The first phase, analogously to that seen in the one asset model, corresponds to the clusters of volatility of length $1/s$ and can be described by an exponential decay with characteristic time scale of 70 periods. At large lags we instead observe a slower decay while the

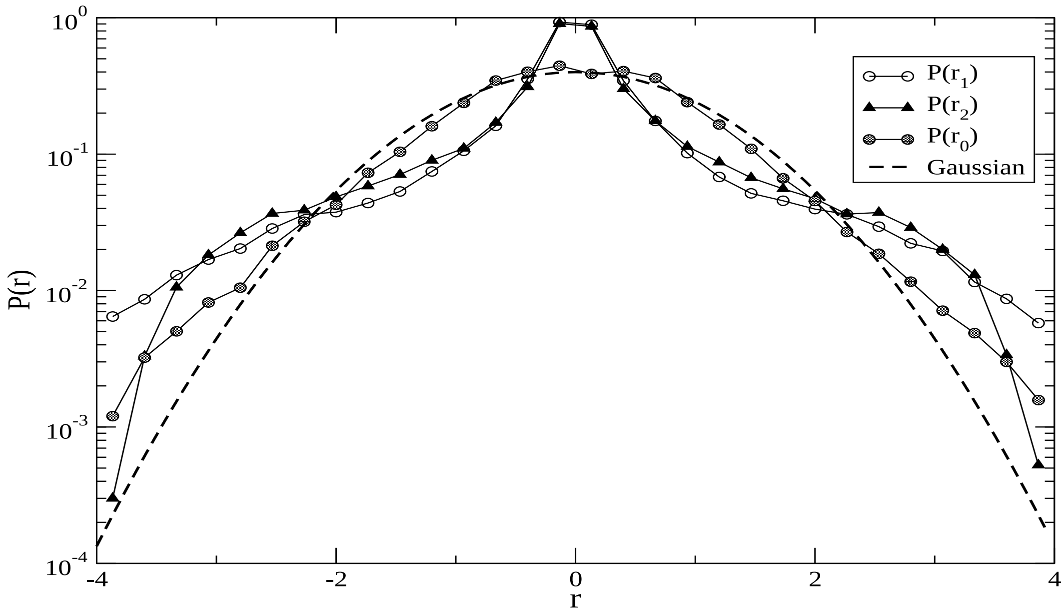

Figure 6. Empirical densities plots $P(r_1)$ and $P(r_2)$ for asset1 and asset2 time series returns, generated numerically by the agent-based market model for $n = 1500$, $D = 0.001$, $\lambda = 2$, $s = 0.015$ and $\beta = 1000$. These time series exhibit fatter tails than the Gaussian distribution. The Gaussian distribution case and the empirical density $P(r)$ for the returns generated by the single-asset model for $n = 1500$, $D = 0.001$, $\lambda = 10$ and $s = 0.015$ are also plotted for comparison.

Period	**mean**(10^{-4})	**std**(10^{-2})	**skew**(10^{-2})	**kurtosis**	**max**(10^{-2})	**min**(10^{-2})
1	1.1	1.3	24	11.1	10.19	-9.64
2	7.3	2.3	7.5	7.6	12	-12.53
3	0.6	4	-1.9	4	13.44	-13.38
4	3.9	3.1	1.9	6.5	12.87	-12.94
5	-2.4	2.8	0.5	5.7	12.29	-12.89

Table 2. Summary statistics for the returns of the first asset in the two-asset model. The estimation is obtained over 10000 observations.

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Period	**mean**(10^{-4})	**std**(10^{-2})	**skew**(10^{-2})	**kurtosis**	**max**(10^{-2})	**min**(10^{-2})
1	2.2	4.3	8.3	3.5	13.37	-13.63
2	6.8	3.6	4	4.3	12.85	-12.89
3	0.6	1.8	-7	11.6	12.71	-12.33
4	3.8	3.1	2.5	5.7	12.61	-11.89
5	-0.1	3.2	3	5.6	12.1	12.63

Table 3. Summary statistics for the returns of the second asset in the two-asset model. The estimation is obtained over 10000 observations.

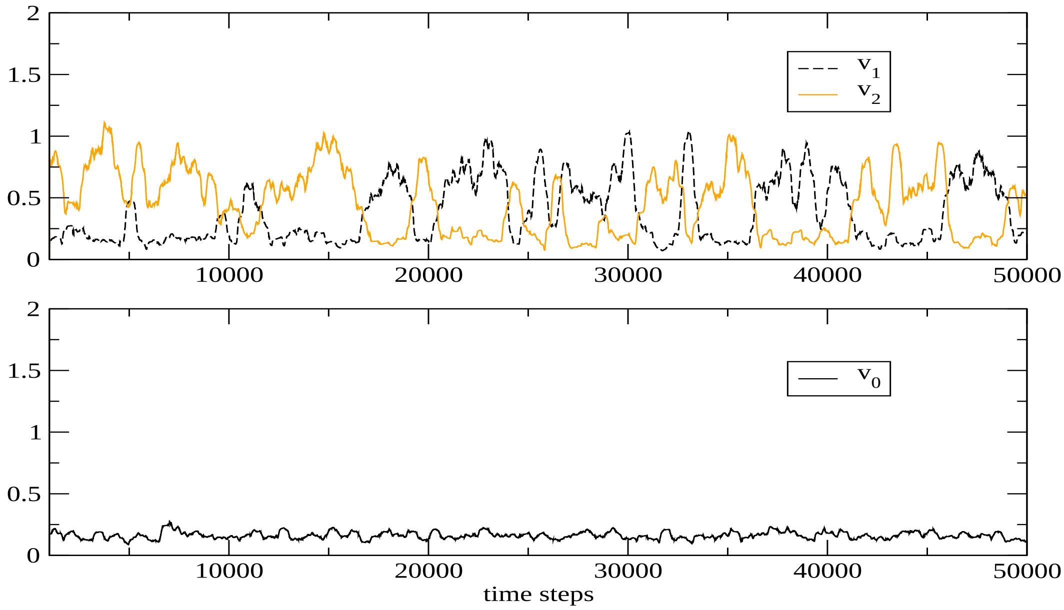

Figure 7. Top: Annualized volatilities v_1 and v_2 for the time series returns of the two assets generated numerically by the agent-based market model with $n = 1500$, $D = 0.001$, $\lambda = 2$, $s = 0.015$ and $\beta = 1000$. Bottom: Annualized volatility v_0 for the returns generated by the single-asset model for $n = 1500$, $D = 0.001$, $\lambda = 10$ and $s = 0.015$. The volatilities are calculated as the standard deviation of the returns on a moving window of 500 time steps multiplied by the annualization factor $\sqrt{250}$.

Period	**mean**(10^{-4})	**std**(10^{-2})	**skew**(10^{-2})	**kurtosis**	**max**(10^{-2})	**min**(10^{-2})
1	0.05	1.05	4.3	4.3	6.7	-5.3
2	0.5	1.04	4.2	4.2	5.13	-5.4
3	0.03	1.05	3.6	3.6	4.4	-5.2
4	2.7	1.05	3	4.7	5.3	-5.6
5	0.4	1.01	-5	4	5.3	-6.7

Table 4. Summary statistics for the returns in the single asset model. The estimation is obtained over 10000 observations.

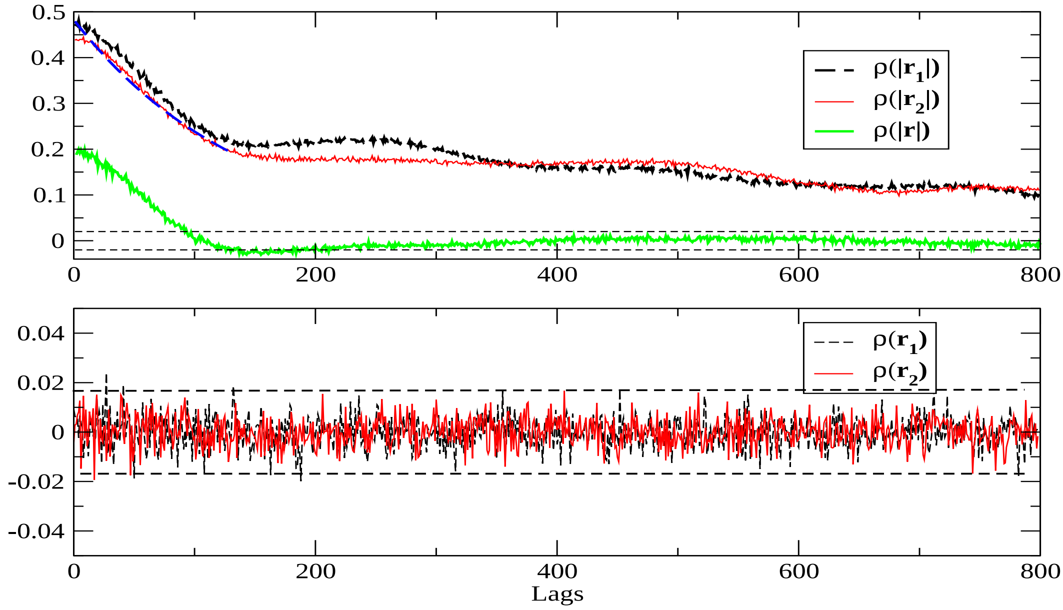

Figure 8. Top: Autocorrelation functions $\rho(|r_1|)$ and $\rho(|r_2|)$ of $|r_1|$ and $|r_2|$ generated numerically by the agent-based market model for $n = 1500$, $D = 0.001$, $\lambda = 2$, $s = 0.015$ and $\beta = 1000$. The volatility clustering phenomenon is reproduced: these functions remain significantly positive over a long period with an initial phase similar to the single asset case. An exponential fit (dashed line) of the initial decay has been plotted. We also plotted for comparison the autocorrelation function $\rho(|r|)$ for the absolute values of the returns generated by the single-asset model with $n = 1500$, $D = 0.001$, $\lambda = 10$ and $s = 0.015$. In the two-asset case, we observe at large lags a slower decay and the autocorrelations remain significantly positive. The persistence in the amplitude of the returns in the two-asset case is definitely closer to that observed in financial data than is the persistence generated in the one-asset case. Bottom: Corresponding autocorrelation functions $\rho(r_1)$ and $\rho(r_2)$ of r_1 and r_2. The multi-asset model generates an absence of autocorrelation in the returns.

autocorrelations remain significantly positive over a long period. I thus conclude that the fluctuations persist in the multi-asset case over a longer period than in the single asset case, Fig. 8. This behavior is definitely closer to that observed in financial time series than is the behavior exhibited by the single asset model.

4.4. Robustness and implications of the results

A parametric analysis of the two-asset model confirms the robustness of the results presented in the previous two sections: this is a desirable property for practitioners when building market models. Varying the updating frequency, s, affects the initial exponential decay of the autocorrelation function of the volatilities. The characteristic time of this decay is, indeed, roughly proportional to $1/s$. When increasing significantly the number of agents, n, we have to decrease the parameter D_{eff} in order to get an effective impact of the asset allocation on market moves and reproduce the results reported in the previous subsection. When increasing the intensity of choice, β, the diversification is more effective and the results are more pronounced: for $\beta = 10000$, the fluctuations in the kurtosis of the returns is higher as are the fluctuations in V_t. The autocorrelation function of the instantaneous volatility, instead, remains unchanged against variations of β for both assets, see Fig. 9. The limit case with infinite β exhibits two distinct behaviors for the volatilities of the two assets as a result of the radical choice: returns similar to the single asset case for the most often chosen asset and returns with

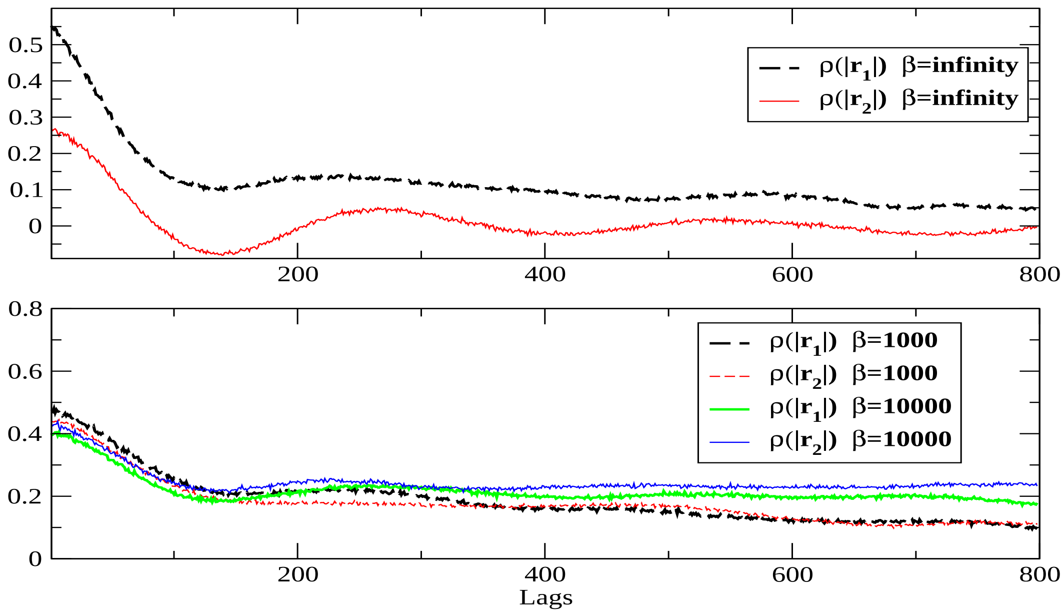

Figure 9. Top: Autocorrelation functions $\rho(|r_1|)$ and $\rho(|r_2|)$ of $|r_1|$ and $|r_2|$, absolute values of asset1 and asset2 time series returns generated numerically by the agent-based market model for $n = 1500$, $D = 0.001$, $\lambda = 2$, $s = 0.015$ and an infinite value of β. This limit case, which is the radical choice case, exhibit two distinct behaviors for the volatilities of the two assets returns. Bottom: Autocorrelation functions $\rho(|r_1|)$ and $\rho(|r_2|)$ of $|r_1|$ and $|r_2|$, absolute values of the two assets time series returns generated numerically by the agent-based market model for $n = 1500$, $D = 0.001$, $\lambda = 2$, $s = 0.015$, $\beta = 1000$ and $\beta = 10000$. When varying the intensity of choice, the model is robust in generating higher persistence in the fluctuations compared to the single asset case.

lower amplitude but with higher variations and persistence in the volatility for the other. The autocorrelation function for this particular case is reported in Fig. 9 where it is evident that the volatility behaves similarly to the single asset case which is plotted alongside.

By using a comprehensive and dynamic approach in the asset allocation decision this agent-based model also offers a plausible mechanism for generating correlations among returns of different assets. The model is successful in reproducing an observed phenomenon in speculative markets where traders crowd or flock from one volatile security to another. Moreover, I computed the correlation coefficient between assets returns for 5 consecutive periods of 10000 time steps generated by the multi-asset model. This correlation parameter fluctuates between 0.3 and 0.7 and we conclude that the correlations are unstable. This instability emerges endogenously as a result of trading strategies and not from the input signal that can be interpreted as the fundamentals of these assets. I note the relationship between volatilities and correlations. The correlation parameter is indeed positively correlated to the difference between volatilities which determines the asset allocation choice and its impact on market movements. This agent-based approach can thus lead to a better appreciation of the relationships among many asset classes, and this should improve the financial decision-making process in limit order book markets.

5. DISCUSSIONS AND CONCLUSIONS

Part of my research consists in building solid theoretical market models that are able to explain the origins of observed statistical regularities in terms of economic behavior of market participants and their interactions: this is a challenging aggregation problem where I explain the behavior of macro-variables such as prices and trading volumes, starting from micro-variables, i.e. individual agents behavior. In the context of limit order book markets that characterizes the modern electronic functioning of financial markets, I showed how in a mean field Ising style market model with original adaptive modeling of agents behavior, the interaction between heterogeneous trading strategies and market impact lead to the major empirical price properties such as excessive fluctuations in assets returns and to characteristics specific to these markets regarding the profile of the book. By using a Boltzmann-Gibbs rule for the dynamic asset allocation in the model, I reconcile seemingly contradictory notions in finance, the informational efficiency characterized by the absence of autocorrelation in assets returns and the long memory in market orders buy and sell flows.

What is novel in this work is to gather in a unified framework and in the specific context of limit order book markets many makets dynamics mechanisms and observed statistical behaviors. I remind the readers that in the present work I have studied with very few hypothesis and a sensible use of parameters, limit order book dynamics for all timescales, and specific characteristics of order book markets dynamics along with cross-market dynamics. The reason for that is first to show that taking into account cross-markets dynamics is the key step to capture all the empirical facts rather than trying ingeniously to create single asset models that matches perfectly all the stylized facts or to just focus on the role of market impact. In light of my results, studying the nonlinear role of market impact in limit order book dynamics without attempting to consider the effects of cross-markets dynamics is incomplete. Due to the comprehensive structure of the model, the results obtained through computer controlled experiments can be explained and traced back analytically to agents' adaptive behavior. Because we are dealing with a general framework, I believe one can draw bold conclusions regarding risks that arise from my works. Indeed, even if individually agents are acting in a sensible way regarding their risks, by being risk averse and by diversifying, the collective consequences of their trades are in both cases dramatic increases of market instabilities. This is a call for policymakers for seeking novel and global approaches regarding risk taking decisions. Agent based modeling allows also to study more sophisticated dynamics and learn more about financial markets, however because we can understand here many empirical properties with the presented models, I believe it is fair to draw at this point far reaching conclusions and question many research directions and policies taken in this field.

List of Figures

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