Measuring and Testing Economic Laws of Production*

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Abstract

We search for the regularities observed in the production of goods. Despite the heterogeneity in sectoral growth rates, second order time polynomial seems to explain accurately enough the accumulation of production at every sector on different levels of aggregation. This is our first observation from Finnish economy with annual data, and we call it an *economic law of production*. Our second observation is that the measured acceleration at every sector deviates from zero statistically significantly. This is inconsistent with the assumption in neoclassical economics, that is, firms produce at their equilibrium flows of production. A different framework is thus needed for modeling firms' behavior. We test a Newtonian model for production against the neo-classical one, and our observation is that the former one works better with annual data at every tested industry in Finland.

Key words: Regularities of production, Industrial growth. JEL: C51, D21, D24.

1 Introduction

A literature of economic laws of production started at 1850s by [1] by introducing the concept of marginal productivity. Then [2] defined the law of decreasing marginal productivity, which is a cornerstone in neo-classical theory of production. The second state in finding regularities in production took place in the connection of estimating neo-classical production functions, see [3] and [4]. In [3], Cobb-Douglas -type of production function was introduced and estimated by aggregate level U.S. data of 1899-1922. In this estimation, the marginal productivity for labor was estimated as 3/4 and that of capital as 1/4; thus constant returns to scale were observed in aggregate production. However, the results in [3] have been questioned by [5] - [8]. In these articles it is shown that the observed success in about 80 years in estimating neo-classical production functions at aggregate and sectoral data has been based on a misunderstanding; the true estimated relation has been an accounting identity rather than a physical production function. Thus the whole research line of estimating production functions is in a new situation where new kind of frameworks are welcome. This is the topic of our research.

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Other kinds of laws in production have been obtained by using company level data in several countries. Refs. [9] and [10] report support for scaling laws for the distribution of firms' value added, growth of value added, sales, and employment by using Japanese and U. S. data. However, these 'laws of production' characterize statistical properties of the behavior of several firms, and not the properties of the possible aggregate level production functions as studied in [3] and others.

Here we introduce a fourth kind of empirical 'law of production'. We show that second order time polynomial is an accurate model for the accumulated production at industrial, sectoral, and aggregate level in Finnish economy with annual data. Our approach is analogous to kinematics in physics: "*Kinematics is the study of the geometry of motion; it deals with the mathematical description of motion in terms of position, velocity, and acceleration. Kinematics serves as a prelude to dynamics, which studies force as the cause of changes in motion*" ([11], p. 25).

We describe economic production analogously to the motion of an ideal particle — a body with no size and no internal structure. This ideal particle is represented by a point that measures the accumulated production; the point illustrates the position of an industry, sector, or the whole economy at a particular moment of time. This framework postulates that to understand why an economy grows we should measure the 'economic forces' acting upon industrial (sectoral) productions. We define the forces by transforming the neo-classical theory into a dynamic form analogous to Newtonian framework in physics, and we test this model against the static neo-classical one.

The study is organized as follows. The data is described in Section 2. The kinematics of production is defined in Section 3, and in Section 4 the empirical results of modeling accumulated production are shown. The two theories to be tested are presented in Section 5, and Section 6 shows the empirical results of testing the theories. Section 7 is a summary.

2 The data used in the study

We use annual sectoral production values at year 2000 prices measuring production volumes in the Finnish economy. The data contains the following 9 main sectors [12] at 1975-2002: 1 = A (Agriculture, forestry and hunting) + B (Fishing), 2 = C (Mining and quarrying) + D (Manufacturing) + E (Electricity, gas, and water supply), 3 = F (Construction), 4 = G (Trade, repair of motor vehicles and household goods) + H (Hotels and restaurants), 5 = I (Transport, storage and communication), 6 = J (Financial intermediation and insurance), 7 = K (Real estate and business activities), 8 = L (Administration, compulsory social security) + M (Education), 9 = N (Health and social work) + O (Other community, social and personal services) + P (Household service activities) − Financial intermediation services indirectly measured (FISIM). These sectors cover all Finnish production.

In Finland, manufacturing is divided in 13 sectors: DA: Food products, beverages and tobacco, DB+DC: Textiles, textile products, leather and leather products, DD: Wood and wood products, DE: Pulp, paper and paper products, publishing and printing, DF: Refined petroleum products, coke and nuclear fuel, DG: Chemicals and chemical products, DH: Rubber and plastic products, DI: Other non-metallic mineral products, DJ: Basic metals and fabricated metal products, DK: Machinery and equipment, DL: Electrical and optical equipment, DM: Transport equipment, DN: Other manufacturing and recycling. These sectors cover the whole Finnish manufacturing, and we have annual data of these from years 1975-2008.

In empirical analysis, derivatives are approximated by difference quotients, e.g., by $\frac{\Delta Q_k(t)}{\Delta t}$ with $\Delta t = 1$ (*year*) is approximated $Q'_k(t)$.

3 Kinematics of production

Let time unit (t_0, t), where $t_0 < t$ are two time moments, be partitioned in intervals Δs (running time is denoted by s). The kinematics of production at sector i can then be defined as follows:

$$Q_i(t) = Q_i(t_0) + \int_{t_0}^{t} q_i(s)ds, \quad Q'_i(t) = q_i(t), \quad Q''_i(t) = q'_i(t),$$

where $Q_i(t_0)$ (*unit*) is the accumulated volume of production at sector i till time moment t_0, $Q'_i(t) = q_i(t)$ (*unit*/y) the momentous flow, and $Q''_i(t) = q'_i(t)$ (*unit*/y^2) the momentous acceleration of production at instant of time t; y is an arbitrary unit of time which can be a month, year, etc.[1] The measurement units of speed (*unit*/y) and acceleration (*unit*/y^2) become thus similar to those found in mechanics.

4 Empirical results for accumulated production

We estimate the following model for accumulated production Q_i (*unit*),

$$Q_i(t) = \int_{0}^{t} q_i(s)ds = a_i + b_i t + \frac{c_i}{2}t^2, \quad i = 1, 2, ..., \tag{1}$$

where t is time, q_i (*unit*/y) the flow of production at sector (industry) i, a_i, b_i, c_i are constants with units: *unit*, *unit*/y, *unit*/y^2, respectively, and $y = year$. These constants can be interpreted as follows: $q_i(t) = b_i + c_i t$, $q'(t) = c_i$; thus c_i measures the acceleration of production, see Section 3. The estimation results for model (1) for the economy level of production in Finland are in Table 1, those for the 9 main sectors are in Table 2, and those for the manufacturing industries are in Table 3; D-W is the Durbin-Watson statistic.

Sector	Constant (T-stat.)	*Time* (T-stat.)	$Time^2$ (T-stat.)	R^2	D-W
Aggregate	49758.54 (8.97)	60403.68 (63.47)	892.40 (26.21)	0.999	0.25

Table 1: Estimated model for accumulated aggregate production in Finland

Tables 1-3 show that acceleration of production has been statistically significantly negative in Sectors 1, 3 and in Industry DB+DC, and significantly positive in all other sectors and industries, and in the aggregate economy. The neoclassical assumption, that firms produce an equilibrium amount in a time unit, is thus rejected in every case. In other words, a statistically significant linear time trend exists in flows of production at all sectors and industries. The only statistically insignificant parameter estimate is $\hat{a}_6$ at Sector 6.

All the estimated models show a positive autocorrelation problem which implies that a cyclical term is missing from the models. However, we did not get rid of the autocorrelation problem by

[1] Measurement units are in brackets after the quantities.

Sector	Constant (T-stat.)	*Time* (T-stat.)	$Time^2$ (T-stat.)	R^2	D-W
1	2528.10 (6.18)	4514.18 (64.39)	-15.33 (-6.11)	0.999	0.28
2	14478.27 (5.89)	10868.69 (25.77)	337.58 (22.37)	0.999	0.21
3	4127.73 (4.38)	7165.66 (44.32)	-14.93 (-2.58)	0.999	0.20
4	5525.35 (3.90)	8461.88 (30.98)	73.74 (6.71)	0.999	0.20
5	6951.82 (8.55)	3734.97 (26.80)	141.12 (28.30)	0.999	0.20
6	-46.33 (-0.07)	2585.29 (21.24)	30.71 (7.05)	0.998	0.18
7	7679.09 (11.58)	7203.11 (63.34)	220.42 (54.17)	0.999	0.26
8	5689.64 (13.78)	8314.94 (117.41)	57.26 (22.60)	0.999	0.18
9	2560.25 (3.20)	7641.61 (55.71)	57.65 (11.75)	0.999	0.18

Table 2: Estimated models for accumulated production in 9 main sectors

Industry	Constant (T-stat.)	*Time* (T-stat.)	$Time^2$ (T-stat.)	R^2	D-W
DA	1618.68 (7.30)	1097.23 (35.26)	19.19 (21.06)	0.999	0.11
DB+DC	818.77 (4.90)	1311.96 (55.98)	-15.74 (-22.93)	0.998	0.15
DD	986.50 (4.79)	511.80 (17.71)	15.96 (18.86)	0.998	0.11
DE	2452.06 (10.42)	3079.88 (93.32)	75.32 (77.92)	0.999	0.36
DF	746.00 (4.27)	66.44 (2.71)	9.60 (13.37)	0.992	0.22
DG	651.20 (8.43)	504.32 (46.58)	20.25 (63.85)	0.999	0.21
DH	293.25 (7.21)	295.96 (51.89)	11.68 (69.91)	0.999	0.36
DI	494.33 (3.57)	633.62 (32.59)	5.39 (9.46)	0.999	0.18
DJ	1794.50 (4.06)	557.44 (9.00)	50.81 (28.00)	0.998	0.18
DK	2180.18 (4.42)	1200.71 (17.35)	41.80 (20.62)	0.999	0.23
DL	14067.29 (3.22)	-3928.76 (-6.42)	215.76 (12.04)	0.947	0.14
DM	874.27 (9.95)	777.84 (63.15)	0.90 (2.51)	0.999	0.23
DN	330.50 (5.32)	445.68 (51.14)	3.50 (13.71)	0.999	0.20

Table 3: Estimated models for accumulated production in manufacturing

adding cyclical terms in the model, and so we report the results as such. The estimated model for the economy wide production is displayed in Figure 1, where the graph of the residual (the one fluctuating around 0) displays the autocorrelation problem.

5 Comparison of the two theories of production

5.1 Neo-classical theory

According to the neo-classical theory, the flow of production of a firm maximizes its profit. Let the profit Π_i ($\$/y$) of firm i in a perfectly competed industry be

$$\Pi_i = p_i q_i - C_i(q_i),$$

where p_i ($\$/unit$) is the price of the product of the firm, q_i ($unit/y$) the flow of production, and $C_i(q_i)$ ($\$/y$) the cost function. The assumption of profit maximization gives the following equation for the flow of production

$$\frac{\partial \Pi_i}{\partial q_i} = 0 \quad \Leftrightarrow \quad p_i = C_i'(q_i) \quad \Rightarrow \quad q_i = f_i(p_i), \quad f_i = C_i'^{-1}, \tag{2}$$

when $\partial^2 \Pi_i / \partial q_i^2 < 0$ is assumed. Without losing generality we can assume any function for f_i that represents all possible cost functions for the firm. We use Eq. (2) in testing the neo-classical theory. Industrial prices are approximated by price ratios $p_t q_t / p_0 q_t = p_t / p_0$, where industrial current value flows are divided by fixed price flows. The results are presented in Table 4.

5.2 Newtonian theory

In [13] the profit function of firm i in a perfectly competed industry is assumed as

$$\Pi_i = p_i(t) q_i(t) - C_i(q_i(t), t),$$

where the quantities are as earlier but now a time-dependence is assumed in the flow of production, and in price and cost functions. The Newtonian theory of a firm can be stated as (see [13]),

$$q_i'(t) = \frac{1}{m_i} \frac{\partial \Pi_i}{\partial q_i}, \tag{3}$$

where $q_i'(t)$ $(unit/y^2)$ is the acceleration of production (see Section 3), $\partial \Pi_i / \partial q_i$ the force acting upon production, and positive constant m_i with unit $(y/unit)^2 \times \$$ measures the inertia ("mass") in the firm's adjustment of its flow of production; rigid technology, bottlenecks in the production process, etc. Neoclassical theory is a zero force situation in (3) at a fixed time moment: $\partial \Pi_i / \partial q_i = 0 \Rightarrow q_i'(t) = 0$.

Equation (3) is our first testable form for the Newtonian theory. To get another testable form of the theory, we assume the following time path for price

$$p_i(t) = a_{i0} + a_{i1} t + a_{i2} sin(b_i t). \tag{4}$$

A linear time trend exists in price if $a_{i1} \neq 0$, and the trigonometric term represents possible business cycle behavior with b_i as the frequency parameter. Empirical results for Eq. (4) for manufacturing industries are in Table 7 in Appendix. The results show that Eq. (4) works reasonably well in explaining the industrial prices; only in Industry DF no reasonable model is obtained. However, positive autocorrelation problem exists in the models, and it shows that the cyclical behavior of prices should be modeled in a more detailed way. Except DH, DI, and DM, all other industries show a statistically significant trigonometric term.

The cost function is assumed as

$$C_i(q_i(t), t) = c_{i0} + c_{i1} q_i(t) + \frac{1}{2} c_{i2} q_i^2(t) - c_{i3} t q_i(t),$$

where the last term represents possible decreasing costs with time ($c_{i3} \geq 0$) due to technical improvement. From these we get the marginal profit function as

$$\frac{\partial \Pi_i}{\partial q_i} = (a_{i0} - c_{i1}) - c_{i2} q_i(t) + (a_{i1} + c_{i3}) t + a_{i2} sin(b_i t).$$

The linear time trend in $\partial \Pi_i / \partial q_i$ may thus occur due to a positive time trend in price, or a negative time trend in costs. The cyclical behavior in $\partial \Pi_i / \partial q_i$ originates solely from price fluctuation. The differential equation in (3) becomes then

$$m_i q_i'(t) = z_{i0} - c_{i2} q_i(t) + z_{i1} t + a_{i2} sin(b_i t), \tag{5}$$

where $z_{i0} = a_{i0} - c_{i1}$, $z_{i1} = a_{i1} + c_{i3}$, and its solution is

$$\begin{aligned} q_i(t) &= \frac{z_{i0} c_{i2} - z_{i1} m_i}{c_{i2}^2} + \frac{a_{i2}}{b_i^2 m_i^2 + c_{i2}^2} \big(c_{i2} sin(b_i t) + b_i m_i cos(b_i t) \big) \\ &+ \frac{z_{i1}}{c_{i2}} t + C_{i1} e^{-\frac{c_{i2}}{m_i} t}, \end{aligned} \tag{6}$$

where C_{i1} is the constant of integration.

We compare empirically the neo-classical theory in (2) with the two forms of the Newtonian one in (3) and (6). Notice that the frequency parameter b_i and the exponential and linear time trend parameters, c_{i2}/m_i and z_{i1}/c_{i2}, may differ between industries.

6 Empirical results of testing the theories

For brevity, the testing is made only at the industry data which suits better for these micro level theories. Because time is abstracted in the neo-classical theory, and earlier on we observed a statistically significant linear time trend in production flows at all sectors and industries, we can conclude that as such the theory fails in describing the evolution of the industrial flows of production. However, we can test whether the price and the flow of production in an industry have a co-integration relation. In this testing we use Johansen's Unrestricted Co-integration Rank test, and these results are in Table 14 in Appendix. The results show that Industries DB+DC, DE, DI, DL, and DN have at most one co-integration relation at 0.05 level, while other industries show no such relation. Thus only in these industries the neo-classical theory has the possibility to work.

The estimated neo-classical models for manufacturing industries are in Table 4. Because quantities p_i, p_i^2, $sin(p_i)$, and $exp(p_i)$ — where p_i is price at industry i — correlate at all industries roughly at rate 0.9 (in Table 8 in Appendix are the correlations in Industry DA), these variables cannot be used simultaneously in one model. Thus we chose the one of these that correlates most with the flow of production, and estimated the equation this way. The estimated models for the neo-classical theory show a reasonable rate of explanation at industries DB+DC, DE, DG, DH, DI, DJ, DL, and DN. However, the D-W statistic shows a high positive autocorrelation at every industry, and price negatively affects production at Industry DB+DC. In Figures 3 and 4, the two best ones of all the estimated neoclassical models are displayed. These models for Industries DG and DH show that autocorrelation in the residual (the graph circulating around 0) is a severe problem, and it shows that a cyclical term is missing in the model. Even though the models have a similar time trend as in the explained variable, the models are not accurate. We tested also whether inserting lagged prices would improve the models, even though they are not included in the theory as expressed in Eq. (2). However, lagged prices did not essentially improve any of the models.

The estimated Newtonian models of form (3) for manufacturing industries are in Table 5. We divided the industrial surplus by the flow of production to get the *'unit profit'* in an industry, and approximate marginal profitability by the unit profit. Only statistically significant parameters

Industry	Constant(T)	*price*(T)	*price*^{2}(T)	*sin*(*price*)(T)	R^2	D-W
DA	782.1(2.1)			1206.9(2.6)	0.18	0.08
DB+DC	1382.8(26.3)		-781.0(-11.9)		0.82	0.26
DD				1348.9(21.1)	0.25	0.19
DE				8887.4(46.8)	0.79	0.42
DF	394.5(10.3)				0.00	0.37
DG	405.7(5.2)		832.2(10.6)		0.78	0.41
DH	-146.0(-2.6)	1007.0(14.9)			0.87	0.78
DI	512.6(11.9)		470.8(7.9)		0.66	0.33
DJ			3025.5(27.2)		0.78	0.41
DK	986.1(3.3)		2547.0(6.1)		0.54	0.25
DL	28029.0(9.8)			-27963.0(-8.5)	0.69	0.15
DM	812.4(54.6)				0.00	0.76
DN	368.0(12.5)		242.7(7.0)		0.60	0.57

Table 4: Estimated neo-classical models for manufacturing industries

are reported in the table. Notice that the explained variable in (3) is acceleration, not flow of production as in Eq. (2) and (6). In the estimated Newtonian models of form (3), 7 out of

Industry	Constant (T-stat.)	unit profit (T-stat.)	R^2	D-W
DA				
DB+DC	-148.5 (-7.9)	784.0 (7.3)	0.63	1.59
DD				
DE		611.9 (3.2)	0.07	2.44
DF				
DG	-154.4 (-2.12)	525.4 (2.7)	0.18	1.69
DH				
DI	-121.7 (-3.4)	526.8 (4.0)	0.34	1.17
DJ				
DK	-754.9 (-5.1)	4037.4 (6.1)	0.54	1.81
DL	-979.1 (-3.4)	4283.1 (5.9)	0.53	1.07
DM				
DN	-118.9 (-3.0)	490.9 (3.2)	0.25	1.38

Table 5: Estimated Newtonian models (3) for manufacturing industries

13 industries show a statistically significant positive effect of unit profit on the acceleration of production. Thus the model gets some support from the data, but the rates of explanation are not impressive. We tested the Granger -causality between acceleration and unit profit at every industry and found that industries DD, DK, DN show a statistically significant causal relation from acceleration of production to unit profit, and only in industry DM a causal relation from unit profit to acceleration of production was observed. Thus we cannot find a clear causal relation between these quantities because they mostly move simultaneously. The best obtained model for Eq. (3) in Industry BD+BC is displayed in Figure 2.

The estimated Newtonian models of form (6) for manufacturing industries are in Table 6.

The frequency parameters of the trigonometric functions in the industries are: $b_{DA} = b_{DB+DC} = b_{DD} = b_{DJ} = b_{DL} = 1/4$, $b_{DE} = b_{DG} = b_{DI} = b_{DK} = b_{DN} = 1/3$, $b_{DH} = b_{DM} = 1/2$, $b_{DF} = 1/12$, and the only industry where the exponential time parameter h deviates from 1 is DM, $h_{DM} = 1/4$. The correlations of the explaining variables in Tables 9-13 in Appendix show that in no equation explaining variables with a higher absolute correlation than 0.46 are used.

The obtained models are quite satisfactory for all other industries except DM. However, positive autocorrelation of residuals is still a problem, but not as big as with the neo-classical theory. The obtained models for every industry are better than those in the neo-classical theory, and the models for industries DG and DH are displayed in Figures 5, 6.

Industry	Constant	$Time$	$sin(b_i t)$	$cos(b_i t)$	$exp(h_i t)$	R^2	D-W
DA	1045.3(36.6)	41.1(28.1)	164.0(8.0)			0.96	1.11
DB+DC	1200.6(53.2)	-24.3(-20.8)	123.6(7.6)	-91.1(-5.5)		0.95	0.55
DD	467.0(15.8)	34.1(20.7)	149.4(6.9)	51.5(2.4)	-0.0(-3.9)	0.95	1.42
DE	2977.5(36.0)	151.1(32.1)		-175.3(-2.9)	-0.0(-2.6)	0.98	1.48
DF	264.3(4.3)	25.9(8.0)	-390.5(-3.7)		-0.0(-3.9)	0.69	1.27
DG	513.8(23.1)	38.8(32.7)		-49.8(-3.0)		0.98	1.12
DH	276.3(20.6)	24.0(34.0)		41.9(4.4)		0.97	1.16
DI	641.3(24.7)	10.1(7.0)	-60.9(-3.2)	-88.4(-4.8)	0.0(2.6)	0.86	0.82
DJ	498.0(6.0)	103.7(22.8)	271.1(4.5)		0.0(3.5)	0.96	0.99
DK	1315.2(13.8)	74.6(13.8)		-207.2(-3.0)	0.0(7.4)	0.94	1.03
DL	-3302.0(-7.4)	392.6(15.7)	2468.3(7.5)	1032.8(3.2)	0.0(7.2)	0.96	0.84
DM	800.7(58.4)		-43.3(-2.5)	65.3(3.8)	0.0(2.6)	0.43	1.20
DN	433.2(29.6)	7.4(9.5)		-51.1(-4.7)		0.83	0.93

Table 6: Estimated Newtonian models of form (6) for manufacturing industries

7 Conclusions

We evaluated empirically what kind of regularities exist in the production data of Finnish economy. Our first observation is that the second order time polynomial fits remarkably well in the accumulated production with annual data at all tested levels of aggregation. Second, a statistically significant linear time trend is observed at all sectors and industries. This questions the basic assumption in the neoclassical theory where time is abstracted from the analysis. We tested a Newtonian type of model for production against the neo-classical one, and observed that the former one works better in every industry. We used annual data from Finnish economy at 1975-2008. It would be interesting if similar tests were repeated with quarterly or monthly data, or with different countries and different levels of aggregation.

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Appendix

Industry	Constant (T)	*Time* (T)	$sin(b_i t)$ (T)	R^2	D-W
DA	0.8 (18.8)	0.0 (5.7)	-0.2 (-7.1)	0.74	0.23
DB+DC	0.5 (25.0)	0.0 (19.0)	-0.1 (-4.5)	0.93	0.32
DD	0.7 (16.4)	0.0 (6.0)	-0.1 (-4.7)	0.67	1.03
DE	0.4 (9.8)	0.0 (9.2)	0.1 (1.9)	0.73	0.42
DF					
DG	0.6 (19.7)	0.0 (14.4)	-0.1 (-3.4)	0.90	0.47
DH	0.46 (23.8)	0.0 (21.5)		0.94	0.60
DI	0.4 (16.8)	0.0 (15.4)		0.88	0.30
DJ	0.6 (32.8)	0.0 (21.1)	-0.1 (-4.1)	0.94	1.19
DK	0.5 (28.4)	0.0 (21.5)	-0.1 (-6.9)	0.95	0.82
DL	0.9 (14.7)	-0.0 (-11.9)	1.1 (12.7)	0.87	0.53
DM	0.5 (14.9)	0.0 (16.7)		0.90	0.80
DN	0.6 (19.2)	0.0 (11.5)	-0.0 (-2.3)	0.84	0.23

Table 7: Estimated models for industrial prices

	pa	pa^2	$sin(pa)$	$exp(pa)$
pa	1.00	0.99	0.99	0.99
pa^2		1.00	0.96	0.99
$sin(pa)$			1.00	0.96
$exp(pa)$			1.00	

Table 8: Correlations of variables

	t	$sin(t/4)$	$cos(t/4)$	$exp(t)$
t	1.00	-0.08	0.12	0.42
$sin(t/4)$		1.00	0.05	0.29
$cos(t/4)$			1.00	-0.14
$exp(t)$				1.00

Table 9: Correlations of variables

	$exp(t/4)$	$sin(t/2)$	$cos(t/2)$
$exp(t/4)$	1.00	-0.04	-0.32
$sin(t/2)$		1.00	0.05
$cos(t/2)$			1.00

Table 10: Correlations of variables

	t	$sin(t/3)$	$cos(t/3)$	$exp(t)$
t	1.00	-0.28	-0.28	0.42
$sin(t/3)$		1.00	0.10	-0.37
$cos(t/3)$			1.00	-0.04
$exp(t)$				1.00

Table 11: Correlations of variables

	t	$sin(t/12)$	$exp(t)$
t	1.00	0.45	0.42
$sin(t/12)$		1.00	-0.24
$exp(t)$			1.00

Table 12: Correlations of variables

	t	$sin(t/2)$	$cos(t/2)$	$exp(t)$
t	1.00	-0.08	-0.18	0.42
$sin(t/2)$		1.00	0.05	-0.21
$cos(t/2)$			1.00	-0.27
$exp(t)$				1.00

Table 13: Correlations of variables

Industry	prob. for no relation	prob. for at most 1 relation
DA	0.56	0.74
DB+DC	0.06	0.02*
DD	0.29	0.20
DE	0.24	0.03*
DF	0.12	0.07
DG	0.64	0.45
DH	0.30	0.25
DI	0.34	0.03*
DJ	0.88	0.94
DK	0.75	0.53
DL	0.06	0.01*
DM	0.07	0.31
DN	0.02*	0.01*

Table 14: Results of co-integration relations between p and q

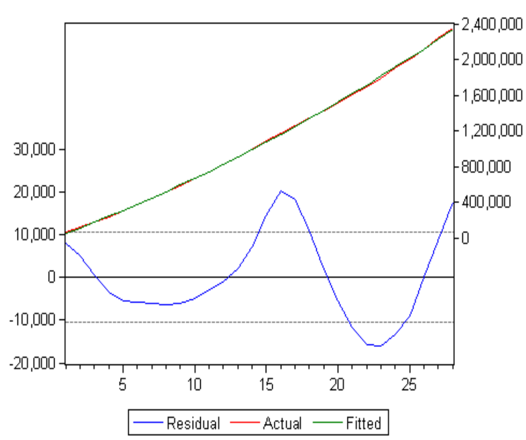

Figure 1: Estimated model for aggregate accumulated production

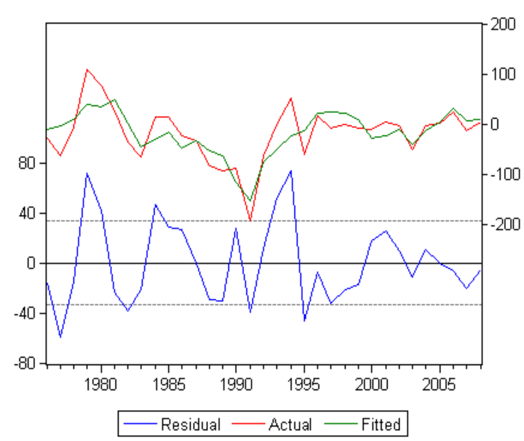

Figure 2: Estimated Newtonian model (3) for Industry DB+DC

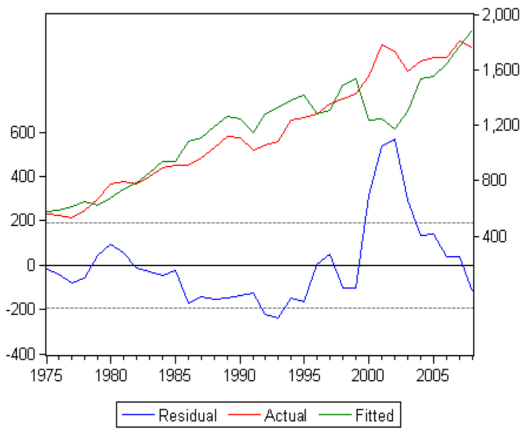

Figure 3: Neo-classical model for DG

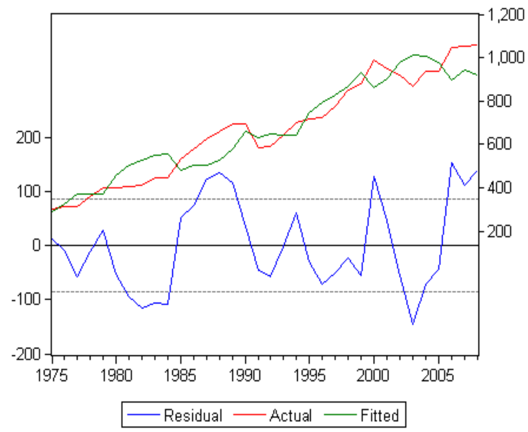

Figure 4: Neo-classical model for DH

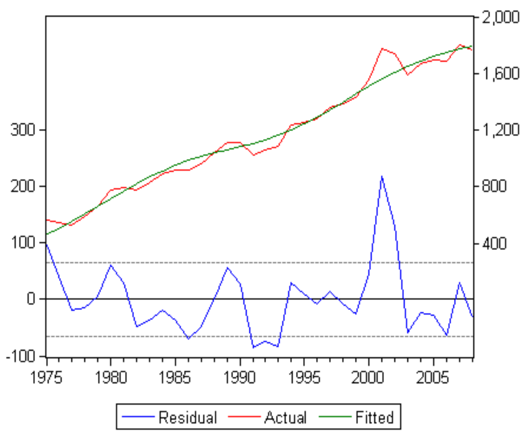

Figure 5: Newtonian model for DG

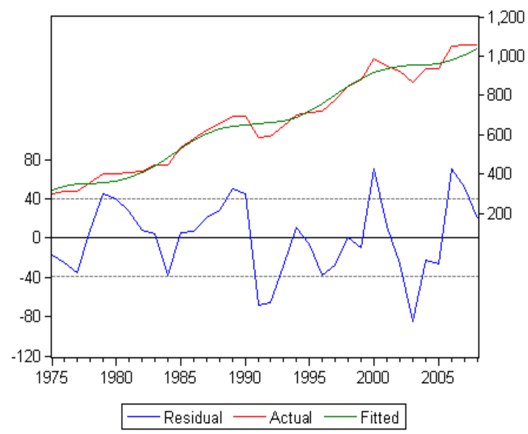

Figure 6: Newtonian model for DH