Scaling Properties of Foreign Exchange Returns

Jonathan A. Batten
Department of Finance
Hong Kong University of Science and Technology
Clear Water Bay, New Territories, Hong Kong
Email: jabatten@ust.hk
Office: +852-2358-8202
Fax: +852-2358-1749

Craig A. Ellis
School of Economics and Finance
University of Western Sydney
Locked Bag 1797
Penrith South DC, NSW 1797, Australia
Email: c.ellis@uws.edu.au
Fax: (61) 2-4626-6683

This study investigates the nature of financial asset returns, calculated as the inter-period change in the natural logarithm of the asset price, used as the basis for modelling key financial assets such as currencies and stocks and derivative securities, such as options that are based on their value. The paper has three key objectives: (1) investigate the independent and identically distributed (IID) assumption and the implications that arise for asset pricing in its application; (2) establish the links between the well-known Efficient Markets Hypothesis and the Random Walk Hypothesis (RWH) in the context of recent developments in the empirical asset pricing literature; and (3) consider the presence of non-linear dependence in financial markets, which remains a disputed anomaly in the finance and economics literature despite an extensive literature.

Specifically, we investigate volatility scaling and self-similarity in Brownian and fractional Brownian motion. The correct estimation of financial asset risk has important implications for investors using standard asset pricing models. Under the usual assumptions of independent and Gaussian distributed increments, traditional methods of estimating risk require an annualised risk coefficient, which is calculated by linear rescaling of the variance from shorter time intervals. Only when the returns series under observation are independent will rescaling provide correct estimates of the underlying level of investment risk. Dependence between increments in the returns series will instead lead the investor to underestimate, or overestimate, their exposure to risk. The higher the underlying levels of dependence, the greater the possibility of error in the estimation of risk.

An empirical example, using four legacy spot foreign exchange prices (DMK/USD, GBP/USD, SWF/USD and JPY/USD) enables estimation of the scaling relationships between the volatility of returns at different time intervals. The results (Table 1) provide evidence of dependence not found using traditional statistical techniques, with three of the four series tested having scale exponents (H) greater than 0.5 for all return intervals (H = 0.5 being the expected exponent value for independent increments). Imputed scale exponents between the intervals k = 1, 5, 22 and 252 are not similar, implying that the distribution of currency returns is not stable across intervals of different lengths.

While the exponent values of the currency return series are not significantly large enough to conclude in favour of statistical long-term dependence, the economic implication of the exponent values *are* significant. Using a simple Black-Scholes foreign currency option-pricing model, linearly rescaled volatility estimates are shown to misprice the option value by as much as 84.0% for out-of-the-money contracts. These results are significant since they demonstrate that small deviations from independence in asset returns can result in significant economic benefits or costs. Investors should therefore exercise caution when using short-term returns to estimate longer-term risk, so as to avoid underestimating their risk exposure.

In our discussion of the possible sources of option under-pricing, both conditional heteroskedasticity and underpricing bias by the Black-Scholes model have been considered. While statistically significant evidence of dependence in the conditional variance of short interval returns was found in all the currency series, this alone was unable to account for the very high levels of underpricing reported. General underpricing bias in the Black-Scholes model similarly implied a significantly lower level of underpricing than was found in the currency option prices. In conclusion, these results highlight the complex behaviour of financial asset returns and the inability of general pricing models to completely and accurately describe this behaviour.

Table 1. Imputed scale exponent (*H*) for estimation of *k* interval standard deviation from the standard deviation of *n* interval returns. [a]

Currency	Interval Length k =	Observed Standard Deviation	Imputed Scale Exponent (H)		
			n = 1	n = 5	n = 22
DMK/USD	1	0.007			
	5	0.015	0.495		
	22	0.032	0.507	0.521	
	252	0.133	0.534	0.550	0.567
SWF/USD	1	0.008			
	5	0.017	0.502		
	22	0.036	0.511	0.521	
	252	0.146	0.535	0.549	0.565
JPY/USD	1	0.007			
	5	0.015	0.499		
	22	0.034	0.521	0.546	
	252	0.142	0.546	0.565	0.576
GBP/USD	1	0.007			
	5	0.015	0.509		
	22	0.032	0.520	0.532	
	252	0.104	0.499	0.495	0.473

[a] This table shows imputed values for the scale exponent H. Using observed values of the standard deviation of k and n interval returns, the imputed scale exponent is the value of H for which rescaled values of the standard deviation of n interval returns exactly equal the standard deviation of k interval returns. Under the null hypothesis H_0, the value of the scale exponent should be H = 0.5. Rejection of the null hypothesis will imply that the currency series tested does not conform to a Gaussian random walk over the sample period. For example, the imputed value of H for estimating the annual return (k = 252) standard deviation from the standard deviation of daily returns (n = 1) for the JPY/USD is H = 0.546.