Two-stock Portfolio Management with Periodic Trading

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Abstract

The fluctuation in the prices in a stock market can be separated into two time scales: a long term trend guided by financial principles and a short term trend governed by the trading mechanisms used. We proposed a mixed strategy for managing stock portfolios in which the long term trend is tracked by Markowitz's theory of mean-variance analysis, and the short term fluctuation in stock price is monitored by a trading threshold. This strategy is tested with the 24 stocks in the Hang Seng Index for the two years period from July 9 2007 to July 8 2009, which covers the financial Tsunami in 2008. In our strategy, the test is based on a trading period of two weeks (10 trading days). At the beginning of each trading period, a two-stock portfolio that has the largest Sharpe ratio among all of the possible combination of 24 chosen stocks from the Hang Seng Index is selected using mean variance analysis. On the day before the end of the trading period, we examine the price fluctuation of the chosen stocks to determine the trading strategy. A trading threshold is proposed to facilitate the trading decision so as to ensure that the updated portfolio still has the largest Sharpe ratio at the beginning of the next trading period. The yield of the portfolio based on this strategy is compared to the Hang Seng Index and the averaged price of the 24 stocks over the same period. The results show that this strategy of portfolio management yields a factor of 1.6 of the initial value, whereas the corresponding yield of the Hang Seng Index is a decrease in value by a factor of 0.8. Over the period of two years for the comparison, the investment using our portfolio management strategy maintains a positive return for a wide range of trading threshold. Our choice of a trading period of 10 days ensures that the effect of transaction fee is minimal. These positive results have also been obtained for different choice of trading period up to one month. Our strategy therefore allows higher flexibility in the trading scheme for investors of different trading habits. An important observation of our trading strategy is that it preserves the assets over the Tsunami in 2008, which is important to conservative investors who prefer protection in the worst situation.

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1. Introduction

Resource allocation in the financial portfolio management has been of continuous interest since the seminal work on the mean variance analysis by Markowitz [1, 2]. The mean variance analysis, which is based on the concept of diversification, suggests that the selection of two or more assets for investment can lower the risk involved in the investment of any individual asset, therefore providing a guideline in risk control in portfolio management. Markowitz's theory provides a simple and elegant solution for resource allocation, such as in the percentage of money invested in each constituent stock in the two-stock portfolios by specifying the investment frontier and the risk tolerable by the investor. In practice, however, one does not have a static picture of the mean nor the variance as they are time dependent. To handle this problem, pattern recognition [3,4], genetic algorithm[5,7], neural network[8], and fuzzy rule [9,10] are some of the approaches that have been applied in real application. In this paper, we introduce two time scales into the mean variance analysis. First of all, we assume that long term behavior provides a guidance to the trend of the stock in the near future. This point of view on the importance of long term behavior in resource allocation is very different from the point of view on time series forecasting, where the predictive power of a forecast relies heavily on an intelligent data-mining algorithm, applied not on the long or medium term data, but on the news and fluctuation of the market in the past few days. In order to accommodate the fluctuation in stock price in the short term, it will be desirable to incorporate these two different points of view, so that we have a general platform to construct a resource allocation algorithm, with the definition of the long time scale and short time scale given by the user. Recently, we have investigated a multi-agent system of stock traders, each making a two-stock portfolio using the mean-variance analysis [11]. The results of this work show that there exists portfolio with low risk and high return, in spite of the random nature of the stock price and the unknown mechanism between the price variations of individual stock. Indeed, in all the works on portfolio management involving stocks, a common goal is to pursue high return, low risk and consistent performance. Furthermore, we find that for the same set of data used in [11] that the short term fluctuation corresponds to a correlation of the stock price in about one to two days [12]. In this paper, we extend our previous works by considering both the long term and short term conditions for portfolio management with periodic trading. We perform the portfolio selection and trading over a fixed period (for example, trading can only take place at the end of a period of 10 days). This restriction can greatly reduce the effect of transaction fees on the performance of portfolio. This formulation allows more flexibility in the trading scheme for investors of different trading habits. The final result should produce an algorithm that avoid frequent trading, while providing a good guidance for selecting stock portfolios that yield good profit with low risks.

2. Investment Strategy

2.1 Mean-variance analysis for the long time scale

We first consider the resource allocation problem of a portfolio consisting of two stocks and cash. Let's denote the expected return $U(t)$ and variance $Var(t)$ by

$$U(t) = \frac{1}{Sample\ \ Size} \sum_{k=t-Sample\,Size+1}^{t} r(k) \tag{1}$$

$$Var(t) = \frac{1}{Sample\ Size - 1} \sum_{k=t-Sample\ Size+1}^{t} \left(r(k) - U(t)\right)^2 \tag{2}$$

where $r(t) = \dfrac{p(t) - p(t-1)}{p(t-1)}$ is the daily rate of return and $p(t)$ is the daily closing price of the stock. The sample size is chosen to be 50 days, which we consider to be sufficiently long so that the mean and variance are rather smooth function of time. In our study of a two-stock portfolio, the expected return and variance for stock pair (1,2) are given by

$$U_{12}(t,x) = U_1(t)x(t) + U_2(t)y(t) \tag{3}$$

$$Var_{12}(t,x) = Var_1(t)x^2(t) + Var_2(t)y^2(t) + 2Cov_{12}x(t)\cdot y(t) \tag{4}$$

where x and y are the fraction of the portfolio invested in stock 1 and in stock 2, respectively. Note that the constraint $x + y = 1$, with $x, y \in (0,1)$, implies that these quantities are function of *t* and *x* only. The covariance Cov_{12} of the two stocks is defined as

$$Cov_{12}(t) = \frac{1}{Sample\ Size} \sum_{k=t-Sample\ Size+1}^{t} \left(r_1(k) - U_1(t)\right)\left(r_2(k) - U_2(t)\right) \tag{5}$$

while the standard deviation of the two-stock portfolio is expressed as:

$$\sigma_{12}(t,x) = \sqrt{Var_{12}(t,x)} \tag{6}$$

To analyze this two-stock portfolio, we make use of a version of the Sharpe ratio defined as:

$$F_{12}(t,x) = U_{12}(t,x) / \sigma_{12}(t,x) \tag{7}$$

Note that this ratio is a function of *x*, so that we can find its maximum in the range of $x \in (0,1)$. In order to achieve maximum return per unit fluctuation or risk, we maximize

$F_{12}\left(t,x\right)$ with respect to x and denote this maximum value as $F_{12}^{*}\left(t\right)$ and the corresponding resource allocation at $\left(x(t),y(t)\right)=\left(x^{*},y^{*}=1-x^{*}\right)$. Note that this resource allocation of the portfolio refers to time t and stock pair (1,2). One may use exhaustive search with preset precision to obtain this value of $\left(x^{*},1-x^{*}\right)$ where the maximum of $F_{12}\left(t,x\right)$ occurs. In real application, one should use some efficient search algorithm to obtain this time dependent optimal resource allocation value $\left\{x^{*}_{ij}(t)\,|\,i=1,...N,\,j>i\right\}$ for all possible pair of stocks.

2.2 Short term trading criterion

With the selected portfolio according to the long term analysis, we introduce a trading criterion based on the short term condition. Since the averaged correlation length of the stock price is approximately 1.5 days in our previous analysis [12], we define the short term fluctuation of the portfolio as the price changes over two days: the trading day (*t*) and the day before the trading day (*t*-1). We then define the trading condition by examining the ratio $\rho\left(t\right)\equiv P(t)/P(t-1)$ where $P(t)$ is the price of the selected portfolio at time *t*. We then introduce a trading threshold θ to decide if a transaction is to be activated. If the ratio $\rho\left(t\right)$, exceeds the trading threshold, a transaction on the portfolio is activated. In our present model, the transaction is to convert the two-stock portfolio into cash. If $\rho\left(t\right)\leq\theta$, then there is no trading and we keep the original portfolio and wait for the end of next period. In this way, only portfolios with expected price higher than certain value can be traded. Setting low trading threshold reflects optimistic view to the stock market, while setting high trading threshold reflects conservative attitude.

2.3 Trading period

In this work, we set each trading period to be 10 trading days. In each trading period, at most one transaction takes place. At the beginning of a trading period, one portfolio is selected from the stock pairs from the N constituent stocks of Hang Seng Index. Then, the short term trading criterion is applied to determine if a transaction is actually activated. If the trading does not occur due to the portfolio price ratio $\rho\left(t\right)$ is lower than the trading threshold θ, the investor then waits till next trading period. If trading occurs, the investor will sell the portfolio at the end of the trading period to keep the asset as cash.

3. Simulation Result

To perform numerical test of our theory, we select 24(=N) stocks that make up the Hang Seng Index. These stocks were all in the Hang Seng Index during the period between July 9, 2007 and July 8, 2009 (Table 1).

Table 1. The stocks whose real price data are used in this study

0001.HK	0002.HK	0003.HK	0004.HK	0005.HK	0006.HK	0011.HK	0012.HK
0013.HK	0016.HK	0019.HK	0023.HK	0066.HK	0101.HK	0144.HK	0267.HK
0291.HK	0293.HK	0330.HK	0494.HK	0762.HK	0883.HK	0941.HK	1199.HK

With these 24 stocks, we have a collection of M = 276 distinct pairs of stocks that can be the candidate of the optimum two-stock portfolio in the context of mean-variance analysis. The initial condition for the simulation is that the portfolio contains only cash. The transaction cost is 1 % of the stock price for each buying or selling. We numerically calculate the resource allocation of our money on optimum stock pair over the period from July 9, 2007 to July 8, 2009, which consists of 500 trading days. For a trading period of 10 days, the 500 trading days are then divided into 50 trading periods and there are at most 50 transactions. The term *return ratio*, defined by the ratio of the final asset to the initial asset, is used indicate the final asset in the following discussion.

By adopting the strategy introduced in section 2 with the trading threshold θ =1.01, the evolution of the asset is shown in Figure 1 in dark squares, as compared with the Hang Seng Index in open circles and the averaged stock price variation in dark triangles. At the end of the trading period, our strategy increases the asset by 60%, while Hang Seng Index drops 20% from the initial value.

From Fig.1, we see that before Sept 04, 2008, the return ratio, overcoming the 1% transaction fee, does not deviate much from the averaged stock price variation. The return is less than a portfolio that tracks the Hang Seng Index, since we tend to keep cash unless the stock market rises rather sharply. Nevertheless, the return is still impressive, making more than 15% increase in mid 2008 before the tsunami. The shape of the simulation curve looks similar to the Hang Seng Index, though generally with less fluctuation. This implies that the proposed strategy can predict the portfolio with above-average return at least in most of the "good" days. Indeed, if our strategy has no intelligence, for example, in the case of a random transaction, the transaction cost will incur great lost (each 10-day period when trading happens, one loses 1%, and over a period of 500 days, the loss can be really great). During the months of financial tsunami (September-October 2008), the stock price and Hang Seng Index nosedive while our portfolio strategy is more or less immune to the crash, because we mainly keep cash as determined by the short term trading criterion. From November 2008 onward, the return ratio is growing in pace with the stock price variation, indicating that during that rising period, the strategy can keep up with an early sign of a bull market and take profits by investing in a pair of good stocks, at a good combination provided by mean variance analysis with two time scales. As a result, the overall performance of this strategy in the "bad" days better than the stock price variation as well as the Hang Seng Index. In summary, our investment strategy provides good and stable return in good times, while in the crash period, it keeps cash due to a conservative decision mechanism using short term correction. Overall, it is a conservative strategy that can avoid penalty of transaction cost, avoid crash, and provide stable positive return in both the bull and the bear market.

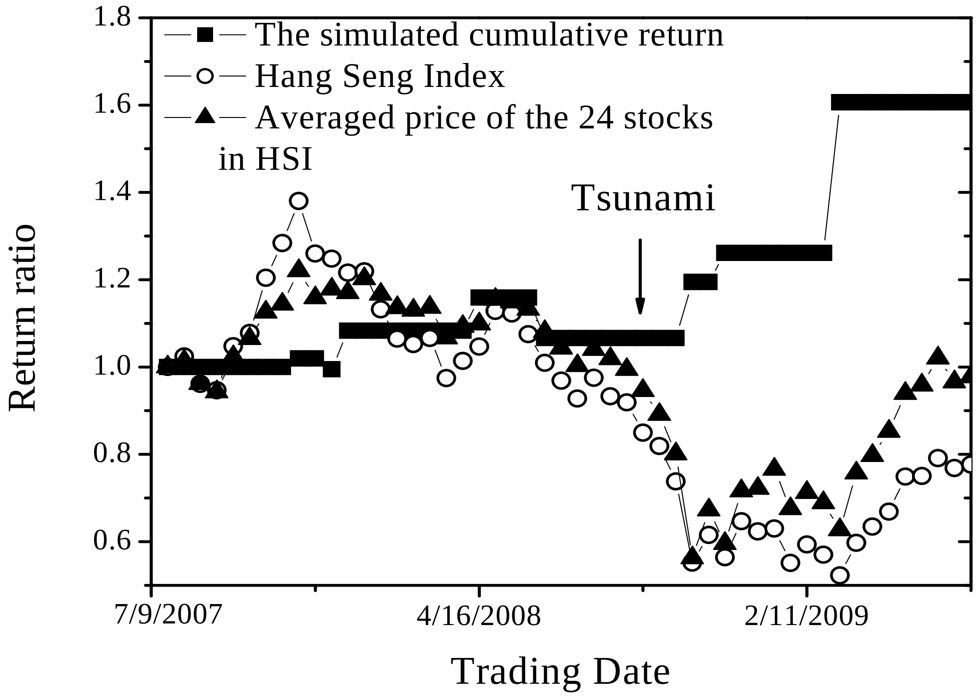

Fig.1 The return ratio by the proposed trading strategy in comparison with the stock average price variation and the Hang Seng Index over the same period.

To see more clearly the effect of the trading threshold in the short term trading criterion, we test the trading performance in the absence of transaction cost. This provides a fair comparison without the effect of transaction cost difference introduced by the trading frequency. The results are shown in Fig.2. The open circles represent the return ratio with the strategy with the trading threshold, and the open triangles represent the return ratio with the strategy without the trading threshold. In both cases, the transaction fee is zero. In this comparison, we see that even though the aggressive strategy without the trading threshold earns more than the conservative strategy with threshold at the beginning, the former loses quite a lot (more than 70% as referenced from the initial asset during the period) during the financial tsunami, whereas the latter preserves the assets as cash very well. In Fig.2, the solid line represents the return ratio with transaction cost and trading threshold (also shown as triangles in Fig.1). We notice that out of 50 possible transactions during the period of 500 days, only 8 transaction are actually activated. This implies that the trading threshold has been applied frequently to prevent unprofitable trading. This threshold definitely plays an important part in the decision process to make the strategy a more conservative and stable one. In Fig.2, we also show the return ratio with threshold $\theta = 1.01$ and a transaction fee of 1% for reference (solid line). We see that even with transaction cost, the good features of our strategy with threshold are maintained.

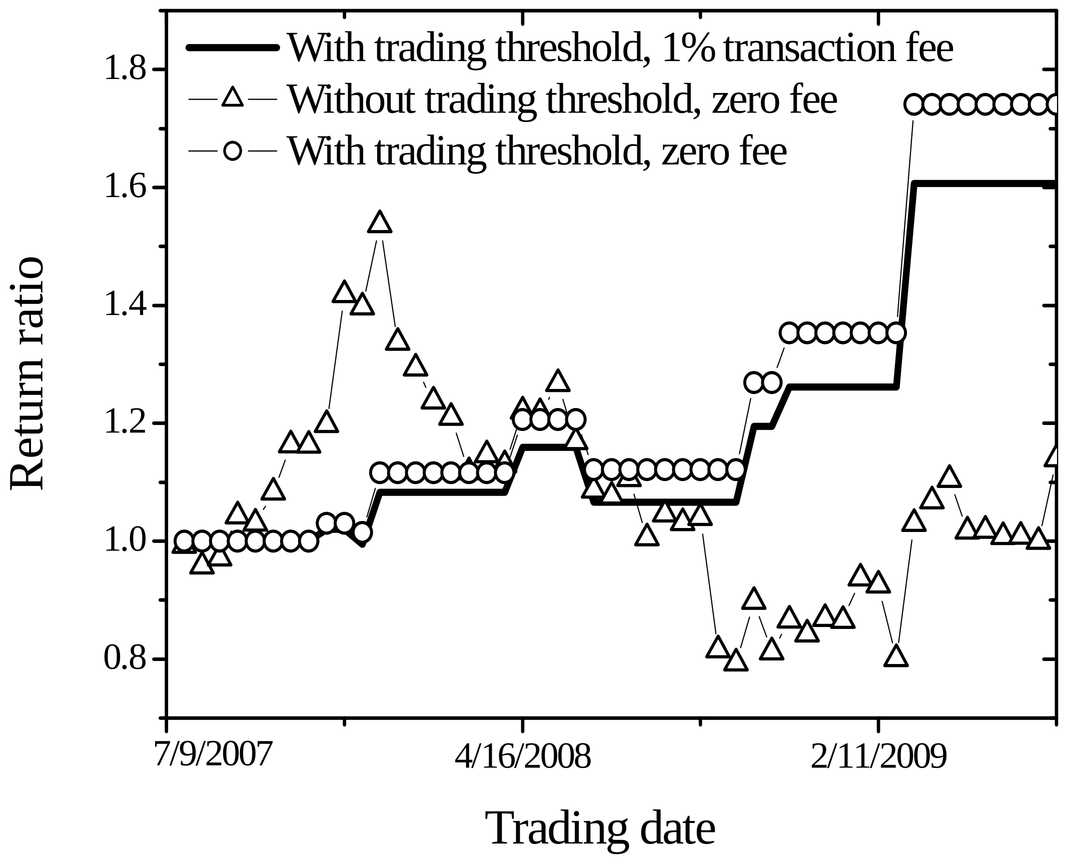

Fig.2 The return comparison between the trading strategy with trading threshold $\theta = 1.01$ (open circles) and without (open triangles) trading threshold, both without transaction cost. The solid line represents the simulated return ratio with transaction cost shown in Fig.1 for comparison.

Now we discuss the effect the threshold on the probability of trading. We show in Fig.3 (in open circles) the result of the threshold dependent return ratio at the end the period of 500 days.From Fig.3, we see that the return ratio increases with the trading threshold, but cuts off when the threshold factor is larger than 1.025, when this threshold value prevents any trading. The probability of the trading, defined as the number of days when trading actually happens divided by the total number of possible trading days, is shown on the right axis of Fig.3 with solid line. This probability of trading has a negative correlation with the return ratio. This confirms that a high return ratio can be designed by increasing the threshold factor and reducing the trading probability in the theoretical framework of mean-variance analysis. We suggest that investors initially set a high trading threshold, such as 1.05, and then gradually decrease the trading threshold until the trading is actually activated. According to Fig. 3, the process will provide the investors with high return ratio around the peek shown in the solid line.

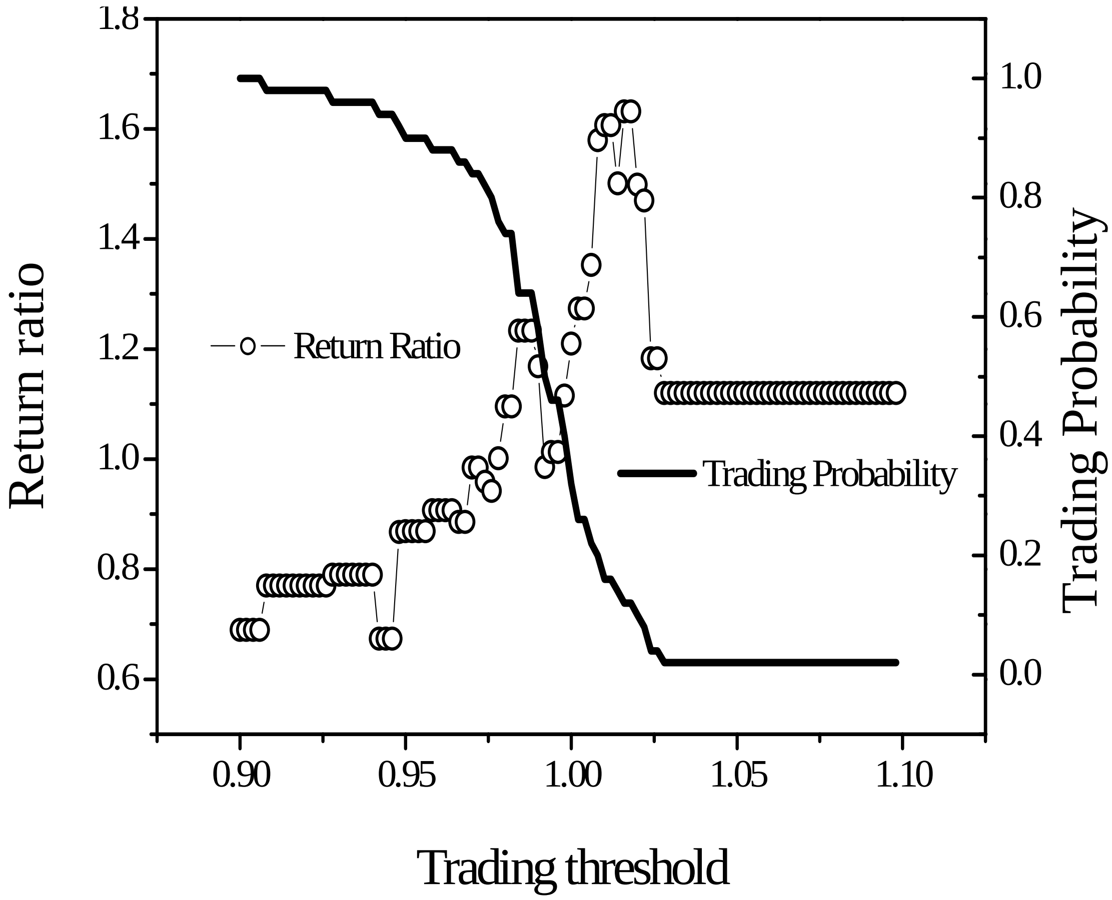

Fig.3 The return ratio by the proposed strategy as a function of the trading threshold (open circles). The trading probability is shown on the right axis (solid line).

4. Conclusion

We proposed a rather conservative strategy of investment using the time dependent mean-variance analysis on a two-stock portfolio. The time dependence covers both the long term aspect of the pair of stocks, as well as the short term stock price change. The long term aspect is determined by computing the "Sharpe ratio" at time *t*, while the short term stock price return provides a risk control. By setting a critical value for the trading threshold, we can avoid loss caused by the transaction fees. Numerical simulation of this trading strategy with real data on a set of blue chips in the Hang Seng Index indicates good return on bull market and small loss on bear market. The overall performance of our strategy beats the performance of the chosen set of stocks without short term control as well as the Hang Seng Index. This strategy should therefore be suitable for conservative investors.

Acknowledgements

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