# Key Parameters in Individual Donation Distribution

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**Abstract**

Individual donation distribution exhibits some power-law characters. From the primal donation model presented in our previous work, a simplified form of donation distribution is derived, in which two key parameters are involved. In this paper, we estimate these two parameters by using a specific estimating technology. Applying this estimation into two cases of donation for 2004 Indian Ocean Tsunami and for 2008 Wenchuan Earthquake, the meanings of the two parameters are inferred.

*Keywords:*
Individual donation, donation distribution, power law, KS statistics
*JEL:* D31,C13

## 1. Introduction

The power law distributions or Zipf distributions (P. Lévy., 1937; V. Pareto., 1986) are found in various natural and social economic systems (MEJ Newman., 2005), and they are now considered as an important property of these systems. This kind of scale-free law also governs many human individual behaviors which are mostly determined by his or her local external situation and inner psychology. For example, every individual always travel based on his or her own willing which has no tight connection with others, but the whole distribution of human travel distances has shown stable power law distribution (D. Brockmann et al., 2006). Every scholar determines what he or she should cite after a work respectively, but the citation distribution of many papers also has a scale free pattern (S. Redner., 1998). And more recently, two empirical studies have found that personal donation distribution has power-law character at large scale although the individual donation amount is determined by individual's situation (Q. Chen et al., 2009; F. Schweitzer et al., 2008). It is very interesting to explore how the large-scale patterns emerge from almost independent behaviors of human individuals.

In our previous work, we investigated a sample of individual donation for Wenchuan Earthquake in 2008 and found that the individuals' donations follow a power law distribution (Q. Chen et al., 2009). We also proposed a primal donation model to explain why the collective donation presents such a particular pattern. The basic idea of this model is that each donator would like to give a random share of his own wealth which follows a certain power law distribution overall. These primal donation amounts will vary from one person to another, and form a pattern that has approximate power law distribution in upper part but uniformed distribution in lower part. Then donators will adjust these primal donations to be practical on some constrain conditions like number preferences. Finally, the practical donations fit the actual ones well in entire pattern besides in some details.

We think that the data of individual donation can reveal some important information of the public at large, such as their wealth or desire to donate. As we have known, the amounts of wealth of the richest in a country or the world are always available from some magazines about fortune. But it needs arduous survey to obtain information about wealth of the masses. If the donation is related to personal wealth, the donation distribution would reflect the wealth distribution at a certain extent. Based on the realistic data and a rational model, we look deeply into the data and try to get key information out of them.

This paper simplified the primal-donation model and discussed the two key parameters which determine the

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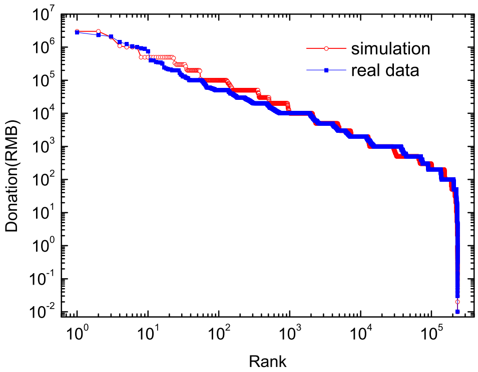


Figure 1: Given the value of parameters, we can use the primal-donation model proposed in Ref. (Q. Chen et al., 2009) to simulate the actual data (real data, the blue curve), and the estimation result (simulation, the red curve) is achieved. The corresponding parameters in the simplified model are: $\alpha = 2.136$ and $\beta = 232$.

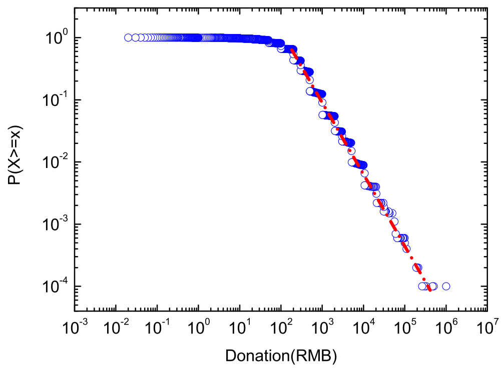


Figure 2: Liner fitting with the part intercepted from the upper tail of the curve presented in Figure 1. The new cumulative distribution is obtained (the open circle, blue), where the upper tail presents obvious power-law character. The dashed straight line is for comparison.

distribution of donation. The two parameters both have specific meanings: one is related to the degree of inequality in social wealth allocation, another is about donation amount of the kindest poorest individuals. In the following sections, we simplified the primal-donation model through removing some unimportant hypothesis. Then we selected two key parameters and discussed their meanings. Kolmogorov-Smirnov (KS) statistic approach (T. Hastie et al., 2005) is proposed to estimate the parameters. After that, we used two samples of data from *Chinese Red Cross Foundation* to estimate the parameters. At the end, some conclusions are offered.

## 2. Simplified primal-donation model and two main parameters

In (Q. Chen et al., 2009), we assumed that the personal wealth follows a kind of distribution like $p(X = w) = \frac{\alpha-1}{\left(w_{\min}^{1-\alpha}-w_{\max}^{1-\alpha}\right)} w^{-\alpha}$, and each person wants to contribute a part $s_i$ of his or her wealth, while $s_i \in (0, \lambda)$ and $\lambda \leq 1$. Then the distribution of primal donation is given by

$$p(z) = \begin{cases} 0, & z \leq 0 \\ \frac{(\alpha-1)\left(w_{\min}^{-\alpha}-w_{\max}^{-\alpha}\right)}{\alpha\lambda\left(w_{\min}^{1-\alpha}-w_{\max}^{1-\alpha}\right)}, & 0 < z \leq \lambda w_{\min} \\ \frac{(\alpha-1)\left(\lambda^{\alpha} z^{-\alpha}-w_{\max}^{-\alpha}\right)}{\alpha\lambda\left(w_{\min}^{1-\alpha}-w_{\max}^{1-\alpha}\right)}, & \lambda w_{\min} < z \leq \lambda w_{\max} \\ 0, & z > \lambda w_{\max}. \end{cases} \tag{1}$$

For simplicity, we assume that the amount of the richest $w_{\max} \to \infty$, the function of donation distribution then turns into the following form:

$$p(z) = \begin{cases} 0, & z \leq 0 \\ \frac{(\alpha-1)}{\alpha}\beta^{-1}, & 0 < z \leq \beta \\ \frac{(\alpha-1)}{\alpha}\beta^{(\alpha-1)} z^{-\alpha}, & z > \beta. \end{cases} \tag{2}$$

where $\beta = \lambda w_{\min}$. Obviously, the density function of the distribution is totally determined by two key parameters: $\alpha$ and $\beta$. The value of parameter $\alpha$ determines the degree of decreasing slope in the lower part of the distribution and $\beta$ reflects the length of uniform distribution.

Of course, the two parameters both have certain economic meanings. As we know, $\alpha$ is derived from the power law exponent of personal wealth distribution, which determines the decaying speed of personal wealth. It also associates with the Gini coefficient (C.W. Gini., 1971) as $g = \frac{1}{(2\alpha-3)}$ which reflects the degree of inequality in social wealth allocation. $\beta$, defined as $\lambda w_{\min}$, is donation amount of the kindest poorest individuals. If either or both of the wealth of poorest $w_{\min}$ and the individual desire to donation $\lambda$ go up, $\beta$ will become bigger. The higher $\beta$ means higher average donation. So this parameter can reflect the enthusiasm of social donation. In order to find the exact form of donation distribution, the values of these two parameters need to be estimated from real data.

## 3. Parameter estimation

Using real data to determine parameters of a hypothetical function are called parameter estimation. Generally, when the form of the distribution function is supposed, the method of Maximum Likelihood Estimation can be used to calculate the values of parameters through maximizing the likelihood function. But here

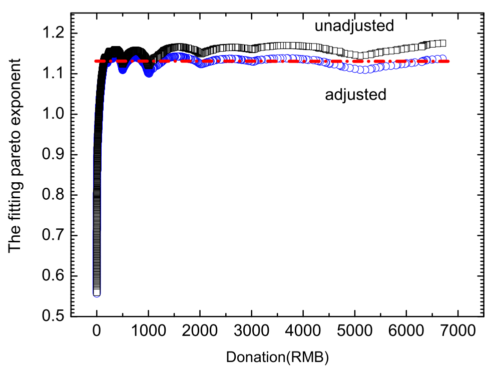


Figure 3: The result of fitting and estimating the data that greater than or equal some donation.

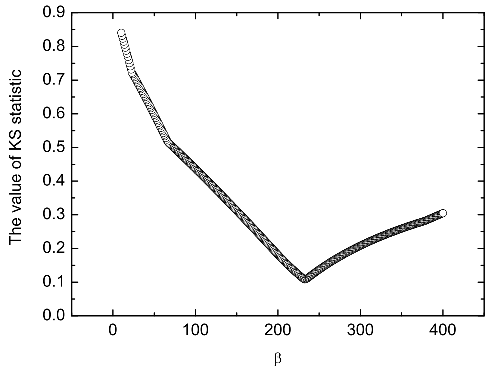


Figure 4: The value of KS statistic *vs.* $\beta$.

this method cannot be successfully applied to this case since the actual data does not follow exactly the distribution of primal donation as shown in Figure 1. In order to testify the validity of the proposed distribution function, we use a series of testing to make the following process of estimation.

Firstly, we estimate the power law exponent by the process of linear fitting. Considering that the final practical donation is adjusted from the primal donation, and some fewer donations are modified from previous bigger values. So we propose an approximate expression:

$$\overline{P}(X \geq x_i) = P(X \geq x_i) + \frac{[P(X \geq x_{i-1}) - P(X \geq x_i)]}{2} \quad (3)$$

Then we get a new cumulative distribution as shown in Figure 2, where the upper tail presents obviously a power-law character.

If the part we intercept from the upper tail is different, the value of parameter $\alpha$ given by linear regression will be different. Figure 3 shows the result of fitting and estimating the data that greater than or equal some donation. It can be seen that the values of parameter estimated with adjusted data are more stable and smooth. The average of slope of the regression line is about $\hat{k} = 1.135$, corresponding to $\hat{\alpha} = 2.135$. The error between this estimated value and the actual one ($\alpha = 2.136$) is smaller than 0.5%.

To estimate $\beta$, we employ Kolmogorov-Smirnov (KS) statistic, whose definition is as follow: $D = \max_{x \in R} |F(x) - P(x)|$, where $F(x)$ denotes the empirical CDF and $P(x)$ is hypothesized CDF. Because KS value concerns about the difference between two cumulative distribution functions, some local fluctuations on the probability distribution function will not produce lasting influence. With the simulated donation data, we can get empirical CDF which is essentially similar to the cumulative distribution in Figure 2.

Given that $\hat{\alpha} = 2.135$, the values of $\beta$ and KS are presented in Figure 4. This figure shows that KS changes with $\beta$ and its minimum appears at the point of $\beta = 232$, which is exactly the same as the initial given value. After the preceding procedures, we finally get the estimated values of those two parameters $\alpha$ and $\beta$ successfully, while we also get the distribution function from Equation (2).

## 4. Application and comparison

The data we used for estimating these two parameters come from *Chinese Red Cross Foundation*, who records the donation it has received. In this section, we select two samples of data, the donation for 2008 Wenchuan Earthquake and for 2004 Indian Ocean Tsunami for application and comparison. The cumulative distributions of donation are plotted in Figure 5, from which it can be seen that both present power law in the upper part, which are similar to the form that have been specified in our previous works. With the method described in previous section, we use adjusted Pareto's plot to estimate the parameter of Pareto exponent. Through calculating the average of relatively stable region in Figure 6, we get the results that the average of slope of the regression line are $\hat{k} = 1.18$ and $\hat{k} = 1.13$ for the two cases respectively, corresponding to $\hat{\alpha} = 2.18$ and $\hat{\alpha} = 2.13$. The estimated parameters of the data of the donation for 2008 Wenchuan Earthquake are consistent with our previous work.

Now we estimate the donation of the kindest poorest people. The best fit of KS statistics yields $\hat{\beta} = 166$

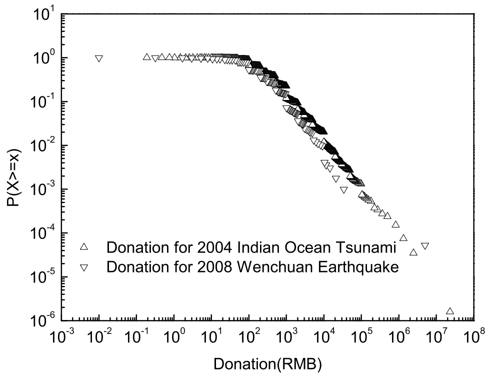


Figure 5: Cumulative donation distributions of two cases in log-log coordinates.

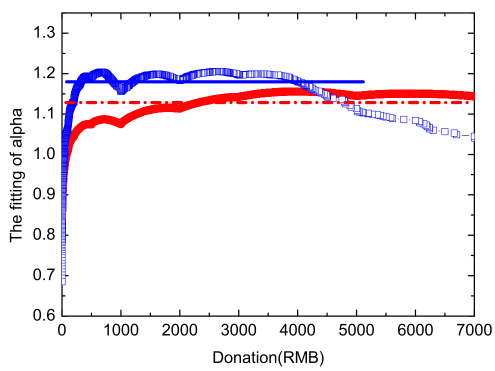


Figure 6: The adjusted result of fitting and estimating the data that greater than or equal some donation.

and $\hat{\beta} = 279$ for the two cases respectively as the corresponding $\hat{\alpha}$s are given. We can see that $\alpha$ shows a small shift, but $\beta$ varies widely in these two cases. The bigger $\alpha$ implies bigger gap between the Chinese poor and rich. The increase in $\alpha$ means that there was a remarkable exacerbation in wealth inequality during the given period. The change of $\beta$ tells us that the Chinese mainland's enthusiasm to donation for 2008 Wenchuan Earthquake was higher than for 2004 Indian Ocean Tsunami. Supposing the wealth held by the Chinese poorest remained stable, we can conclude that the degree of individual desire to donate in case of 2008 was about 2 times as in case of 2004. We anticipate that more estimations and comparisons of these two parameters can be carried out for different cases.

## 5. Conclusion

This paper simplified the model of primal donation in Ref. (Q. Chen et al., 2009), indicating that two parameters determine the donation distribution, and discussed their economic meanings. Then we introduced one estimating method and revealed some meaningful information. In particular, we found the degree of inequality of wealth distribution is very serious, and it still becomes worsen during the years of 2004-2008. As the wealth status and donation willingness of people have not been reported by other statistical approaches, with this model we can estimate some core economic indexes that may be helpful to propose and implement relevant economic policies.

## 6. Acknowledgement


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