

Hidden Higgs Portal Vector Matter for the Galactic Center
Gamma-Ray Excess from Two-Step Cascade Dark Matter
Annihilations, and Muon $g-2$

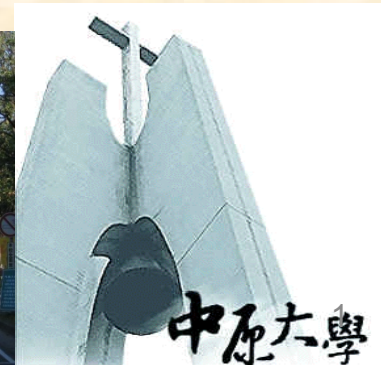
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Mini-Workshop on Dark Sector Phenomenology: Models, Satellites, and
Colliders

@ IoP, Academia Sinica

10/18/2018

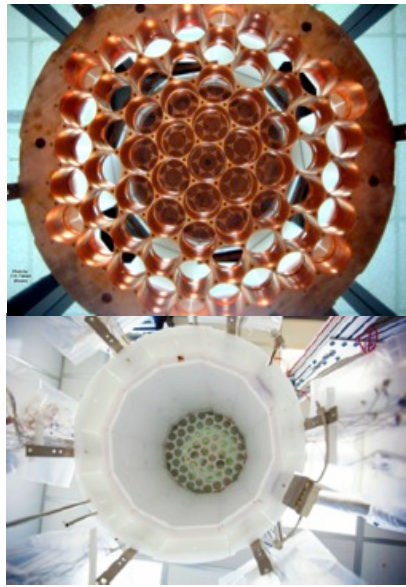
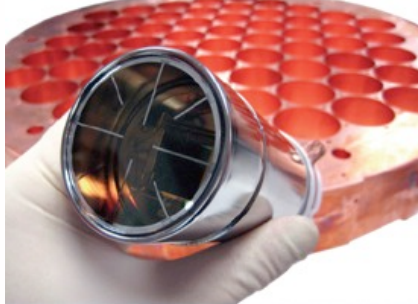


Outline

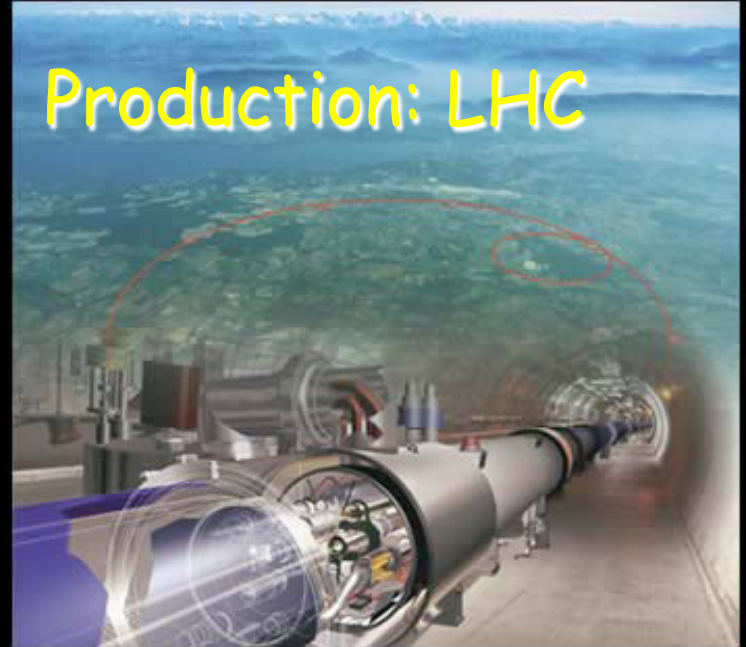
- ◆ Introduction :
- ◆ Motivation
- ◆ Model building:
- ◆ Theoretical and experimental constraints
- ◆ Summary

Four roads to Dark Matter

Direct (LUX)



Production: LHC



Indirect: Fermi



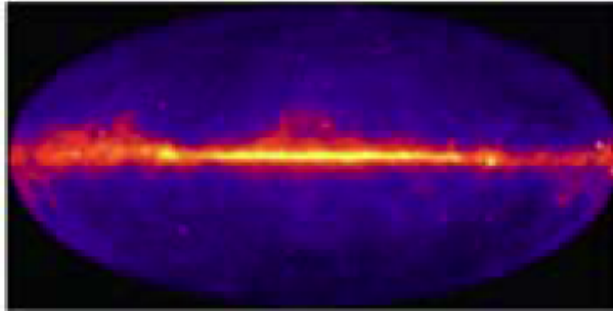
Gravitational observations:
Bullet cluster



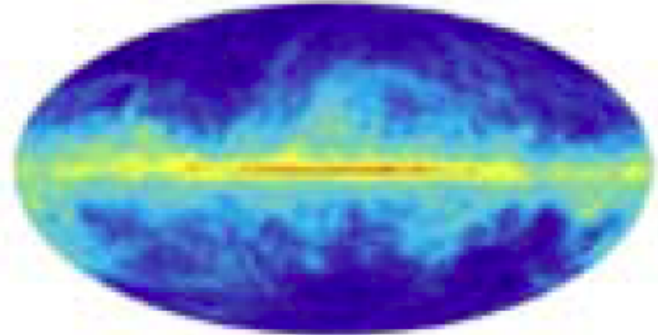
The possible gamma-ray excess surrounding the Galactic center suggested by Fermi-LAT observations has been interpreted as a variety of different phenomena such as

- (i) **a signal from WIMP dark matter annihilation,**
- (ii) gamma-ray emission from a population of millisecond pulsars,
- (iii) emission from cosmic rays injected in a sequence of burst-like events or continuum at the GC.

Fermi sky



Galactic diffuse



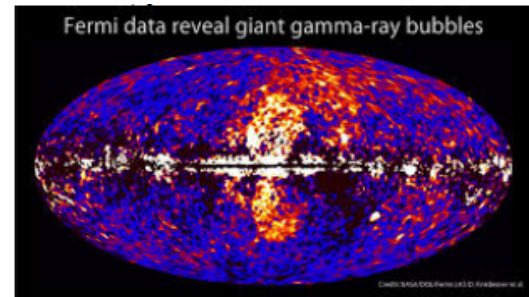
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Isotropic gamma-ray background



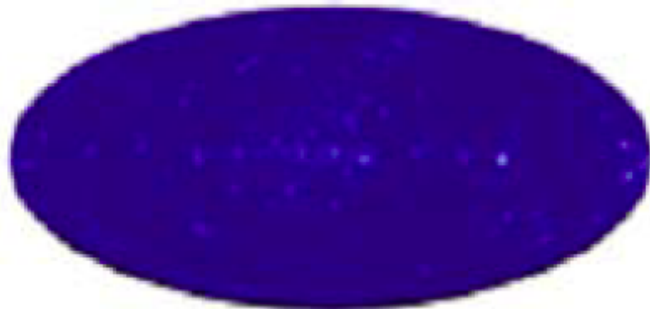
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Fermi bubbles



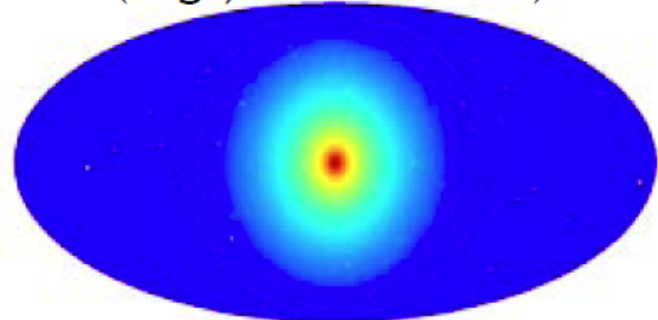
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point



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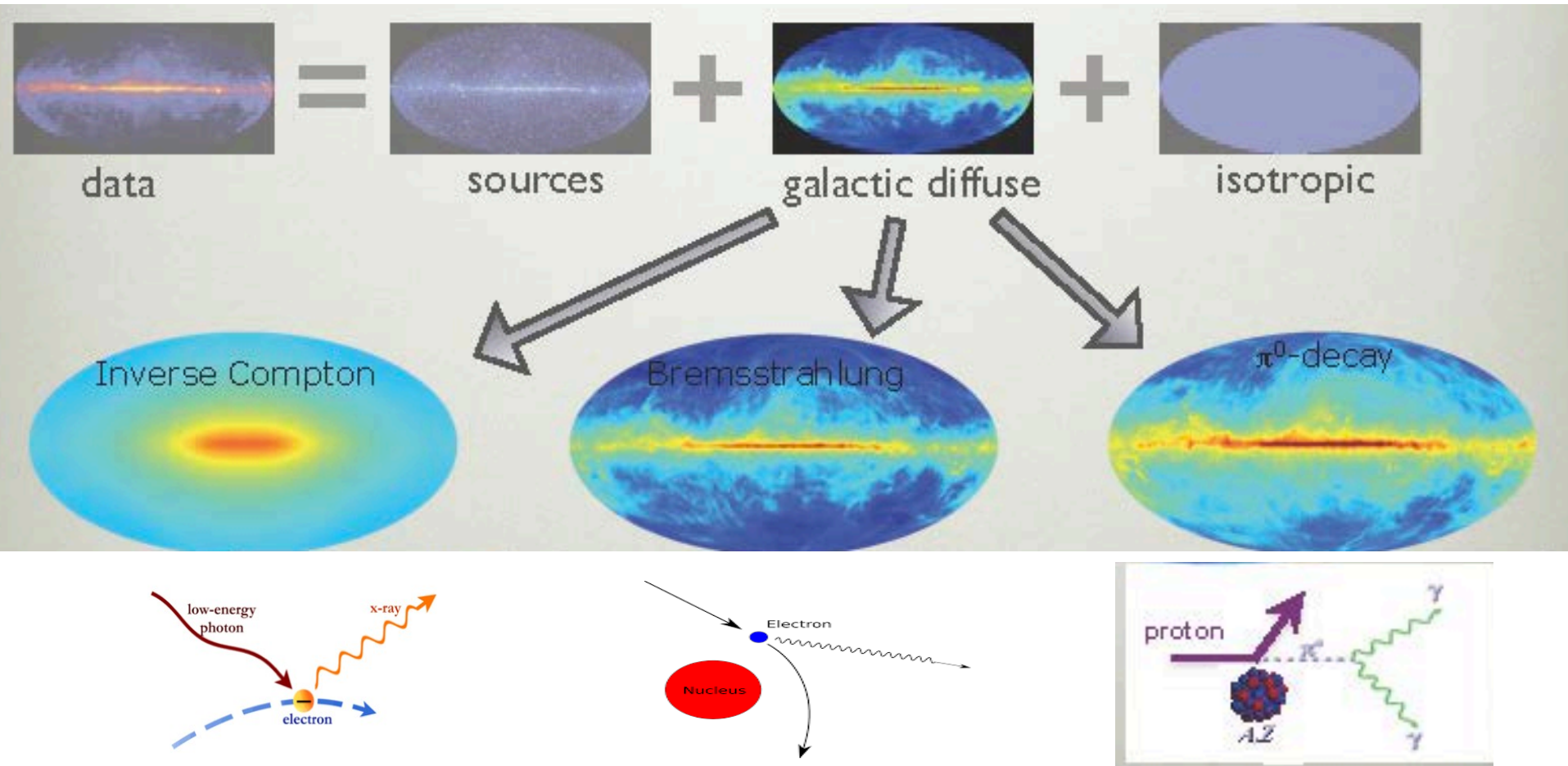
residual
(e.g., dark matter)

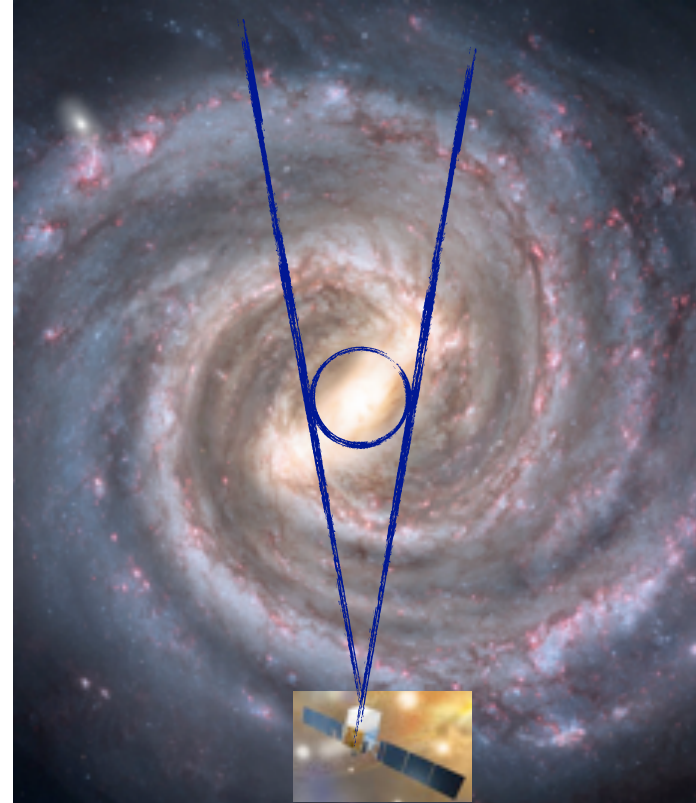
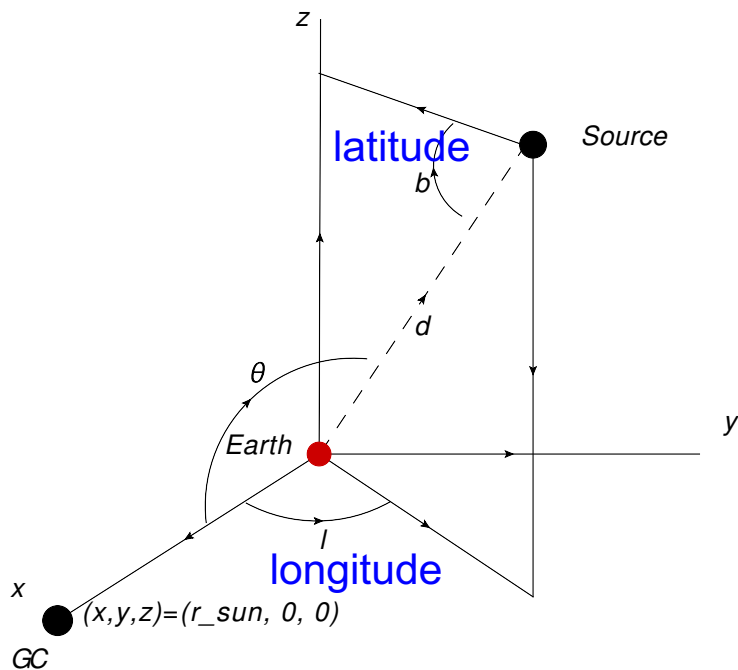


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Sources of Galactic Diffuse Emission (GDE)

1. Inverse Compton: CR electrons up-scattering low-energy photons
2. Neutral pion decays: CR protons inelastic collision with nuclei (gas)
3. Bremsstrahlung : CR electrons interacting with interstellar gas





The differential flux of gamma-ray

DM prompt γ -ray spectrum per annihilation

$$\frac{d\Phi_\gamma}{dE} = \frac{1}{8\pi m_X^2} \sum_f \langle \sigma v \rangle_f \left(\frac{dN_\gamma^f}{dE} \right)_X \frac{1}{\Delta\Omega} \underbrace{\int_{\Delta\Omega} \int_{\text{l.o.s.}} ds \rho^2(r(s, \psi)) d\Omega}_{\text{J-factor}}$$

depend on the observational region of interest (ROI) in a particular analysis

We will consider the galactic DM density distribution described by a generalized Navarro-Frenk-White (NFW) halo profile

$$\rho(r) = \rho_{\odot} \left(\frac{r}{r_{\odot}} \right)^{-\gamma} \left(\frac{1 + r/r_s}{1 + r_{\odot}/r_s} \right)^{\gamma-3}$$

where the scale radius $r_s=20$ kpc, r is the distance to the GC, $-\gamma$ is the inner log slope of the halo density near the GC, and ρ_{\odot} is the local DM density at $r_{\odot} = 8.5$ kpc, the radial distance of the Sun from the GC.

GCE results are sensitive to Galactic Diffuse Emission Models

1409.0042, systematics of 60 GDE models studied by Calore, Cholis and Weniger (CCW)

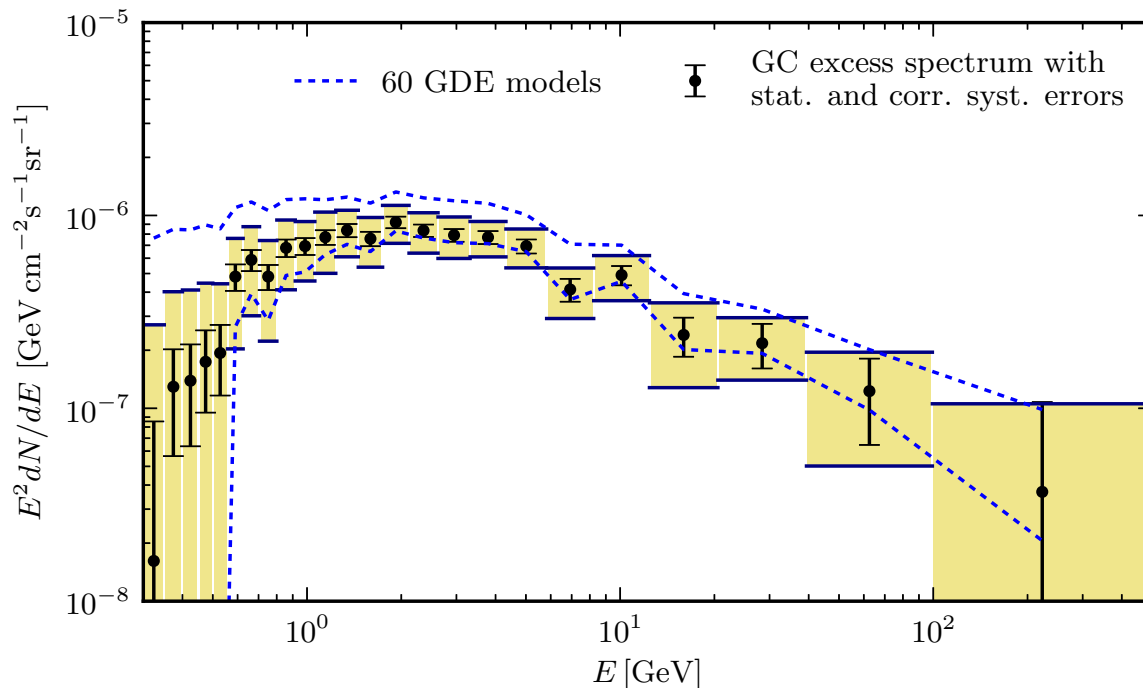
1411.2592, Agrawal, Batell, Fox, Harnik (use the CCW or preliminary Fermi GCE spectra)

1411.4647, Calore, Cholis, McCabe, Weniger (CCMW), follow up CCW's result

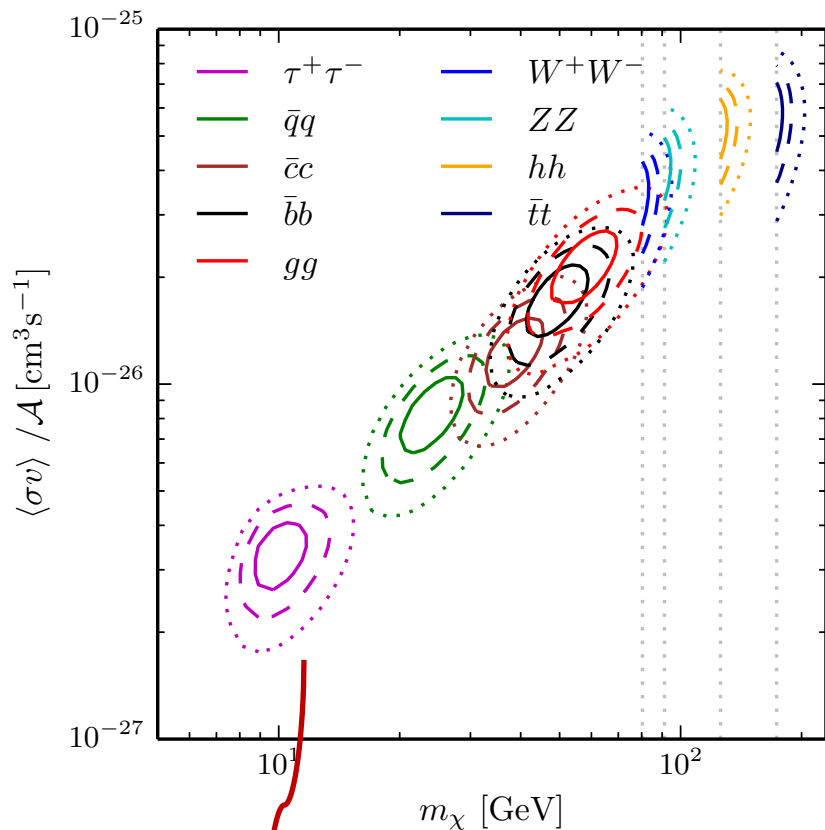
Use 60 GDE models and fit the gamma ray data (300 MeV to 500 GeV)

A Tale of Tails : the difference from earlier studies. The peak remains the same

ROI of $|\ell| < 20^\circ$ and $2^\circ < |b| < 20^\circ$



1409.0042



CCMW: Annihilation into gluons, $\bar{q}q, \bar{c}c, \bar{b}b, hh$ provides a good fit

Channel	$\langle\sigma v\rangle$ (10^{-26} cm ³ s ⁻¹)	m_χ (GeV)	χ^2_{\min}	p -value
$\bar{q}q$	$0.83^{+0.15}_{-0.13}$	$23.8^{+3.2}_{-2.6}$	26.7	0.22
$\bar{c}c$	$1.24^{+0.15}_{-0.15}$	$38.2^{+4.7}_{-3.9}$	23.6	0.37
$\bar{b}b$	$1.75^{+0.28}_{-0.26}$	$48.7^{+6.4}_{-5.2}$	23.9	0.35
$t\bar{t}$	$5.8^{+0.8}_{-0.8}$	$173.3^{+2.8}_{-0}$	43.9	0.003
gg	$2.16^{+0.35}_{-0.32}$	$57.5^{+7.5}_{-6.3}$	24.5	0.32
W^+W^-	$3.52^{+0.48}_{-0.48}$	$80.4^{+1.3}_{-0}$	36.7	0.026
ZZ	$4.12^{+0.55}_{-0.55}$	$91.2^{+1.53}_{-0}$	35.3	0.036
hh	$5.33^{+0.68}_{-0.68}$	$125.7^{+3.1}_{-0}$	29.5	0.13
$\tau^+\tau^-$	$0.337^{+0.047}_{-0.048}$	$9.96^{+1.05}_{-0.91}$	33.5	0.055
$[\mu^+\mu^-]$	$1.57^{+0.23}_{-0.23}$	$5.23^{+0.22}_{-0.27}$	43.9	0.0036] <small>ICS</small>

Low p-value; is not excluded with 95% CL significance

This is for self-conjugate DM

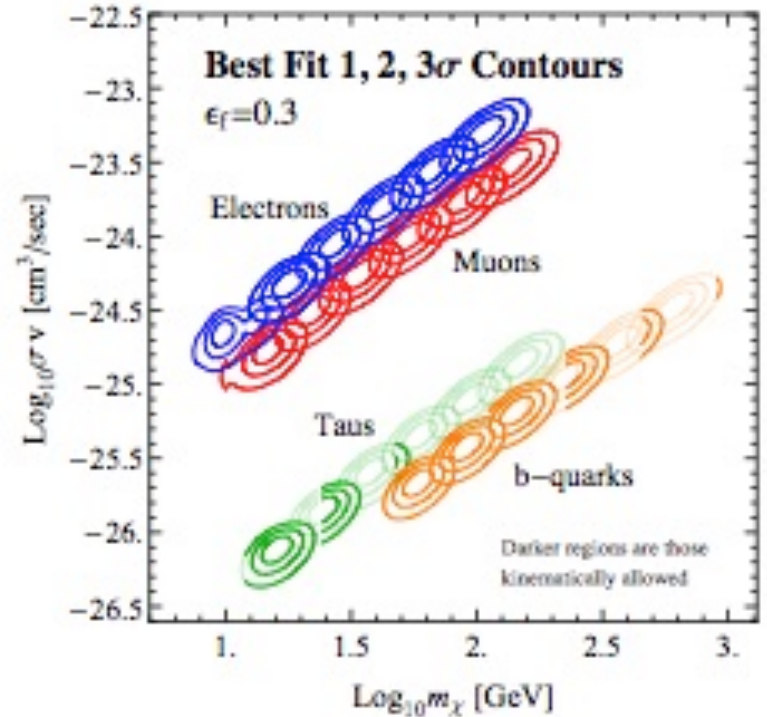
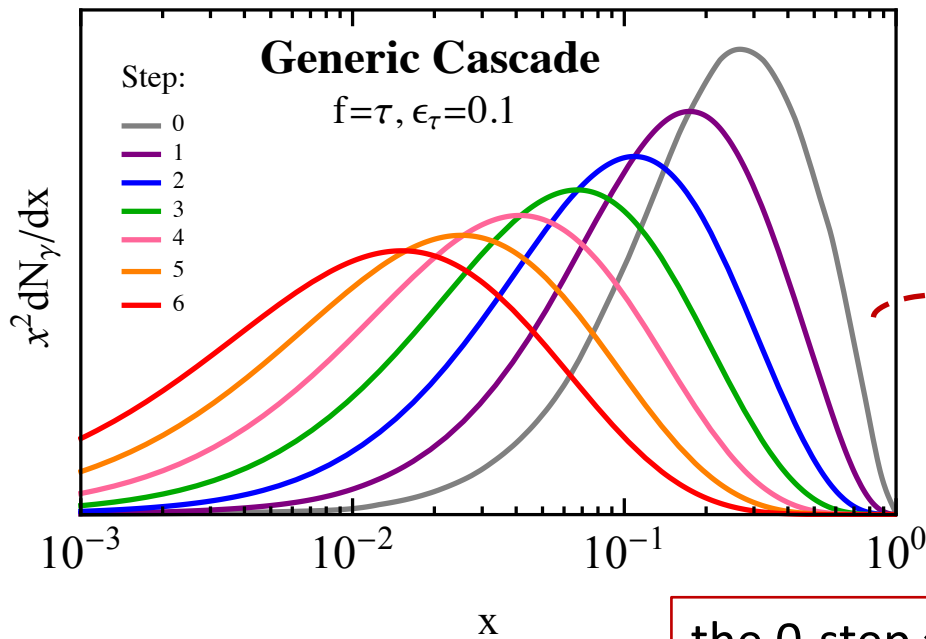
Idea

Multi-Step Cascade Annihilations of Dark Matter, Elor, Rodd, and Slatyer (2015)

$$\begin{aligned} \chi\chi &\rightarrow \phi_n\phi_n \rightarrow 2 \times \phi_{n-1}\phi_{n-1} \rightarrow \dots \\ &\rightarrow 2^{n-1} \times \phi_1\phi_1 \rightarrow 2^n \times f\bar{f}. \end{aligned}$$

1-6 steps for cascade annihilations

If the step is increased, the spectrum is broadened with a lower energy peak



the 0-step τ spectrum has a much larger contribution from leptonic and semi-leptonic $\tau^- \rightarrow \nu_\tau \ell \bar{\nu}_\ell, \nu_\tau d \bar{u}$

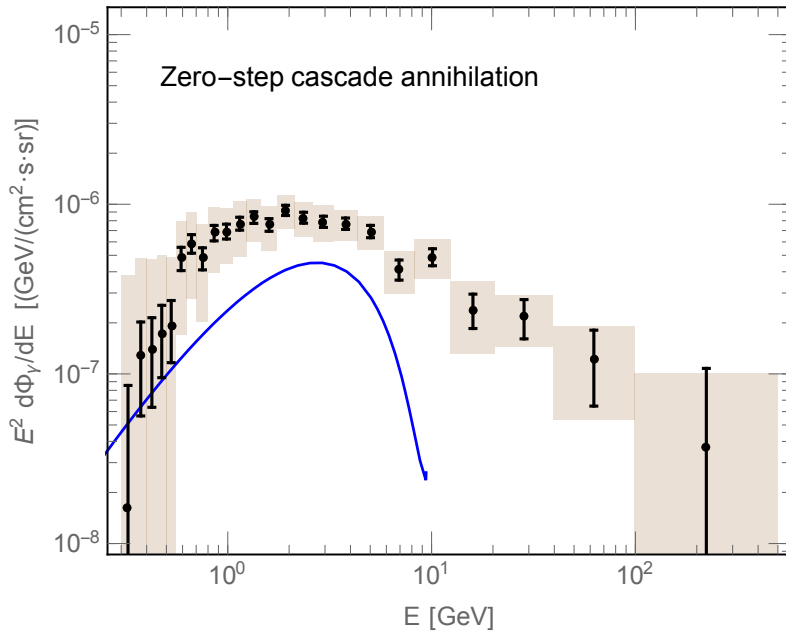
$$x = \frac{E}{m_{DM}}$$

$$\chi\chi \rightarrow \phi_n\phi_n \rightarrow 2 \times \phi_{n-1}\phi_{n-1} \rightarrow \dots$$

$$\rightarrow 2^{n-1} \times \phi_1\phi_1 \rightarrow 2^n \times f\bar{f}.$$

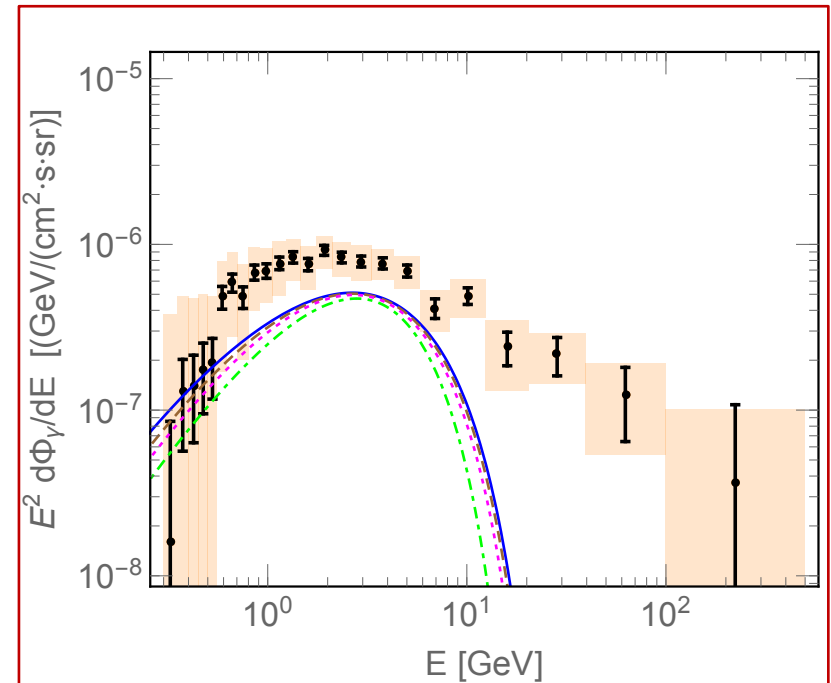
with $f \equiv \tau$

0-step, $n = 0$



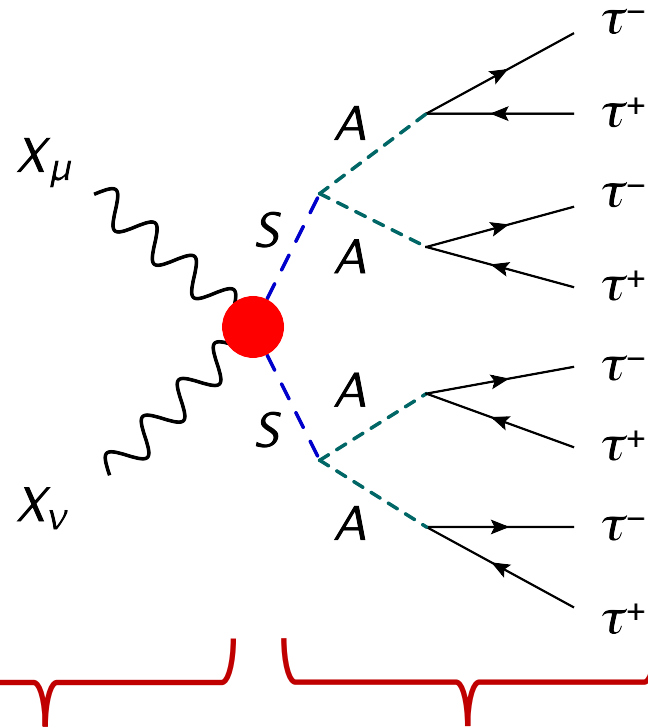
p-value of the best fit ~ 0.05

2-step, $n = 2$



p-value of the best fit: $0.12 \sim 0.22$

The model building for the galactic center gamma-ray emission



Vector dark matter

Leptophilic Next-to-Minimal 2HDM

2HDM: Two Higgs doublet model

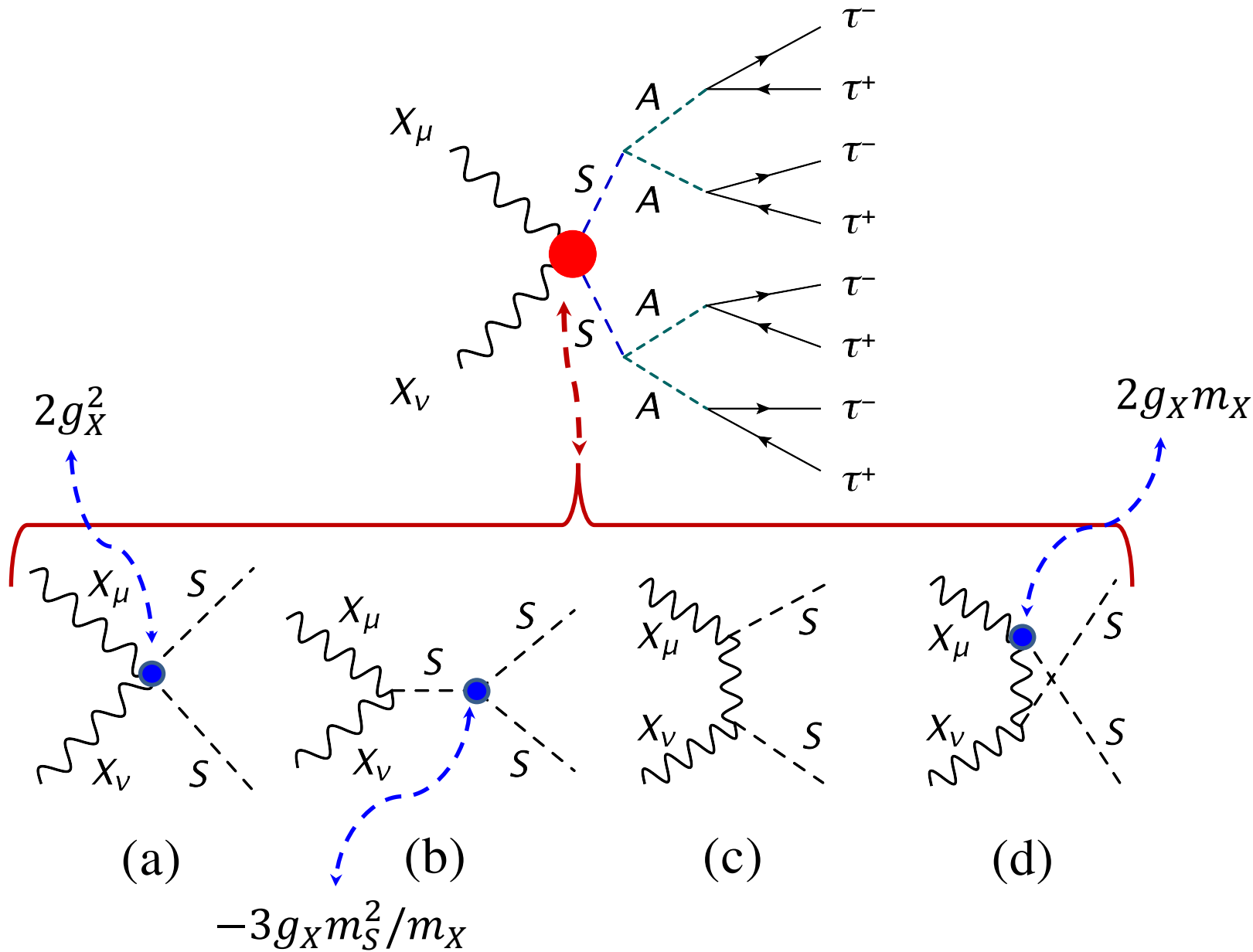
The relevant kinetic terms in the dark sector are

$$\mathcal{L}_{\text{DM}} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + (D_\mu\Phi_S)^\dagger(D^\mu\Phi_S)$$

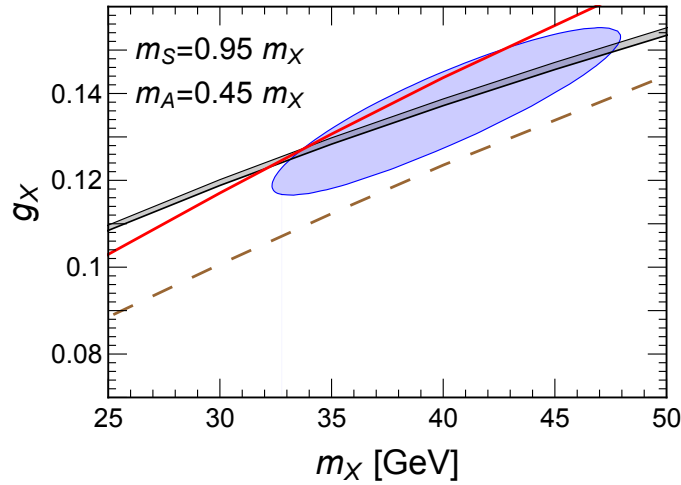
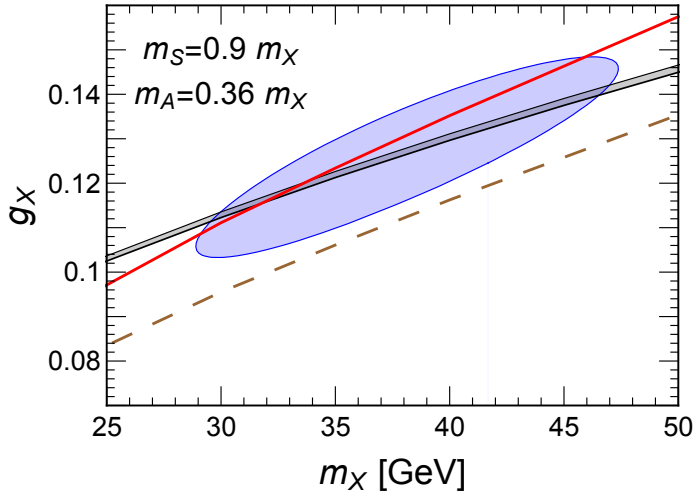
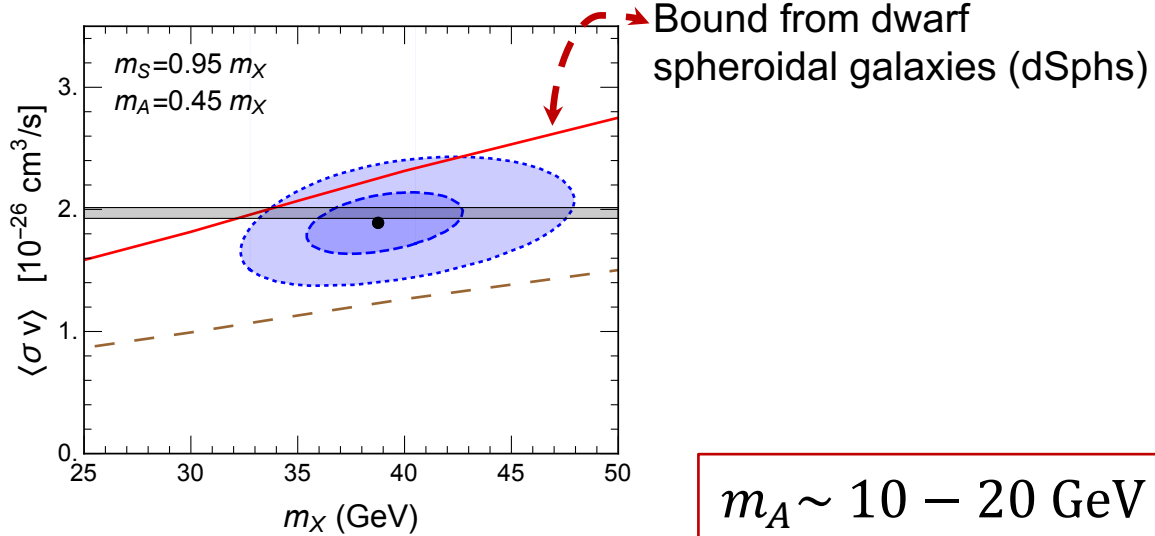
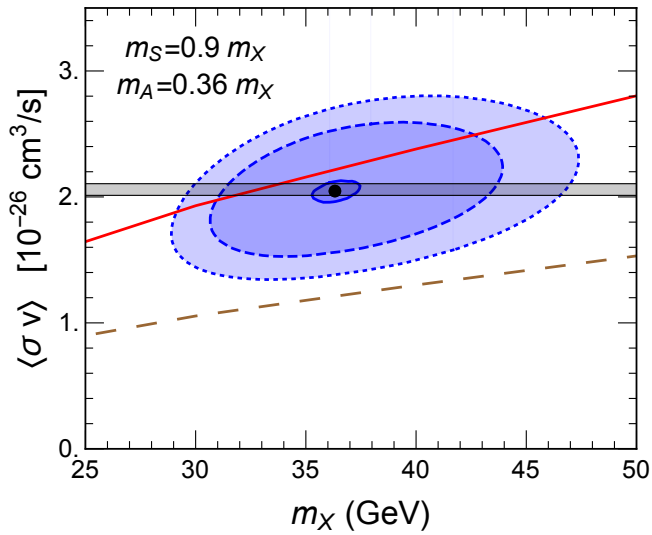
$$D_\mu\Phi_S = (\partial_\mu + ig_X Q_{\Phi_S} X_\mu)\Phi_S$$

$$X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$$

$U_{dm}(1)$ charge of Φ_S



Also relevant to dark matter relic density



Red line: 95% CL upper bound from combined gamma-ray data of 28 confirmed and 27 candidate dSphs, recently reported by the Fermi-LAT and DES Collaborations (2017)

grey region: conventional WIMP thermal relic density can be accounted for

The CP-conserving potential for the Higgs sector is described by

$$\begin{aligned} V = & m_{11}^2 |\Phi_1|^2 + m_{22}^2 |\Phi_2|^2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + h.c.) + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2} [(\Phi_1^\dagger \Phi_2)^2 + h.c.] \\ & + m_{33}^2 \Phi_S^\dagger \Phi_S + \frac{\lambda_6}{2} (\Phi_S^\dagger \Phi_S)^2 + \lambda_7 (\Phi_1^\dagger \Phi_1) (\Phi_S^\dagger \Phi_S) + \lambda_8 (\Phi_2^\dagger \Phi_2) (\Phi_S^\dagger \Phi_S) \end{aligned}$$

Z_2 symmetry: $\Phi_1 \rightarrow \Phi_1$, $\Phi_2 \rightarrow -\Phi_2$, and $\Phi_S \rightarrow \Phi_S$, under which the tree-level flavor changing neutral currents (FCNCs) are absent.

Φ_S is charged in the dark $U_{dm}(1)$ gauge group

After spontaneous symmetry breaking, a discrete Z_2' symmetry: $X_\mu \rightarrow -X_\mu$, $\Phi_S \rightarrow \Phi_S^*$, is still maintained

$$\begin{pmatrix} h_1^\pm \\ h_2^\pm \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix},$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} G^0 \\ A \end{pmatrix},$$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \cos \delta & -\sin \delta \\ 0 & \sin \delta & \cos \delta \end{pmatrix} \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix} \begin{pmatrix} H \\ h^0 \\ S \end{pmatrix}$$

α is the mixing angles of the neutral CP-even bosons (h_1, h_2) in the limit of $\delta, \theta \rightarrow 0$

β is the mixing angles of the charge bosons, and is related to the ratio of the two VEVs, $\tan \beta = v_2/v_1$.

Yukawa couplings of neutral type-X N2HDM Higgs

	$f = u, d$	l	
g_{hff}	c_α/s_β	$-s_\alpha/c_\beta$	Dominant in large $\tan \beta$
g_{Hff}	s_α/s_β	c_α/c_β	
g_{Sff}	$-(s_\alpha s_\theta + c_\alpha s_\delta)/s_\beta$	$(-c_\alpha s_\theta + s_\alpha s_\delta)/c_\beta$	
g_{Aff}	$\pm 1/t_\beta$	t_β	

Feature of L(N)2HDM with a the light CP-odd Higgs

😊 Consider the constraint on $\text{Br}(h \rightarrow AA)$

Current bound 0.2-0.4 dependent on m_A

In case of $g_{hAA} = 0$, we have ($M^2 \equiv m_{12}^2/(\sin \beta \cos \beta)$)

$$\begin{aligned}\sin(\beta - \alpha) &\simeq 1 - \frac{2}{\tan^2 \beta} \left(1 + \frac{m_h^2}{M^2} - \frac{2m_A^2}{M^2} \right) \\ \cos(\beta - \alpha) &\simeq \frac{2}{\tan \beta} \left(1 + \frac{m_h^2}{2M^2} - \frac{m_A^2}{M^2} \right).\end{aligned}$$

under the conditions of $t_\beta \gg 1$, $m_A^2/M^2 \ll 1$, $m_h^2/M^2 \ll 1$ and $s_{\beta-\alpha} \rightarrow 1$

The normalized Yukawa coupling of the SM Higgs to the lepton pair can thus be

$$g_{h\ell\ell} = -\frac{s_\alpha}{c_\beta} = s_{\beta-\alpha} - t_\beta c_{\beta-\alpha} \simeq -1 - \frac{m_h^2}{M^2} + 2\frac{m_A^2}{M^2}$$

In this case, the alignment limit, $s_{\beta-\alpha} \rightarrow 1$, reproduces the wrong-sign SM coupling $g_{h\ell\ell} \rightarrow -1$.

😊 $h \rightarrow \tau\tau$ measurements, $|g_{h\tau\tau}| < 1.26$ at 2σ C.L., give $M^2 \geq 245 \text{ GeV}^2$ for $m_A \leq 20 \text{ GeV}$

Oblique parameters in EW precision measurements

Taking the limit $s_{\beta-\alpha} \rightarrow 1$, and keeping terms linear in $\sin\theta$ and $\sin\delta$, we obtain

$$\begin{aligned} S &\approx -\frac{1}{24\pi} \left(\frac{5}{3} + \frac{4(m_{H^\pm} - m_H)}{m_H} \right) \simeq -0.022 - 0.002 \times \frac{300 \text{ GeV}}{m_H} \frac{m_{H^\pm} - m_H}{10 \text{ GeV}}, \\ T &\approx \frac{1}{32\pi^2 \alpha_{\text{em}} v^2} m_H (m_{H^\pm} - m_H) \simeq 0.04 \times \frac{m_H}{300 \text{ GeV}} \frac{m_{H^\pm} - m_H}{10 \text{ GeV}}, \\ U &\approx \frac{1}{12\pi} \left(\frac{m_{H^\pm} - m_H}{m_H} \right) \simeq 0.001 \times \frac{300 \text{ GeV}}{m_H} \frac{m_{H^\pm} - m_H}{10 \text{ GeV}}, \end{aligned}$$

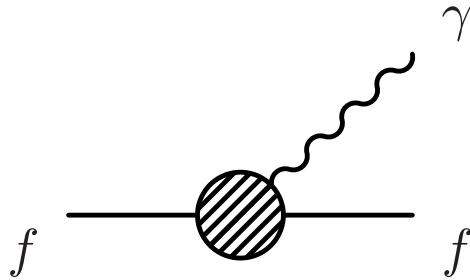
For $m_H \approx 300 \text{ GeV}$, $|m_{H^\pm} - m_H| \sim \mathcal{O}(10 \text{ GeV})$, the theoretical prediction is consistent with that from the data fit:

$$S = 0.05 \pm 0.10, \quad T = 0.08 \pm 0.12, \quad U = 0.02 \pm 0.10.$$

Muon $g-2$

$$\Delta H = -\vec{\mu} \cdot \vec{B},$$

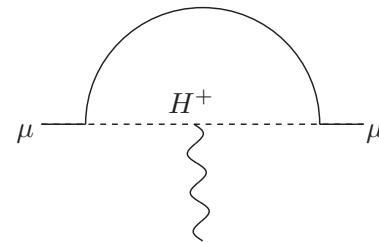
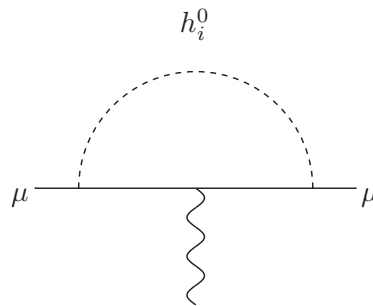
$$\mu = -\frac{e}{2m} \sigma = -g \frac{e}{2m} \vec{S}$$



$$i\mathcal{M} = -i\bar{u}_f e Q_f \left(F_1(q^2) \gamma^\mu + F_2(q^2) i \frac{\sigma^{\mu\nu} q_\nu}{2m_f} \right) u_f$$

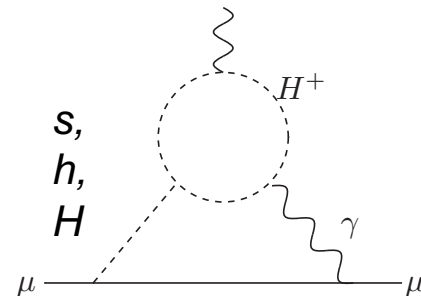
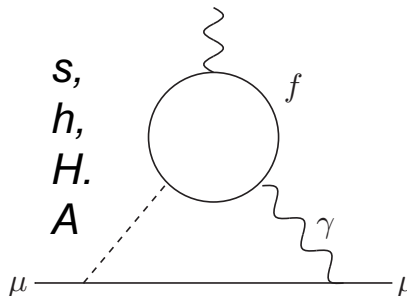
$$a_f = \frac{g_f - 2}{2} = F_2(0)$$

One-loop



Two-loop (Barr-Zee diagram)

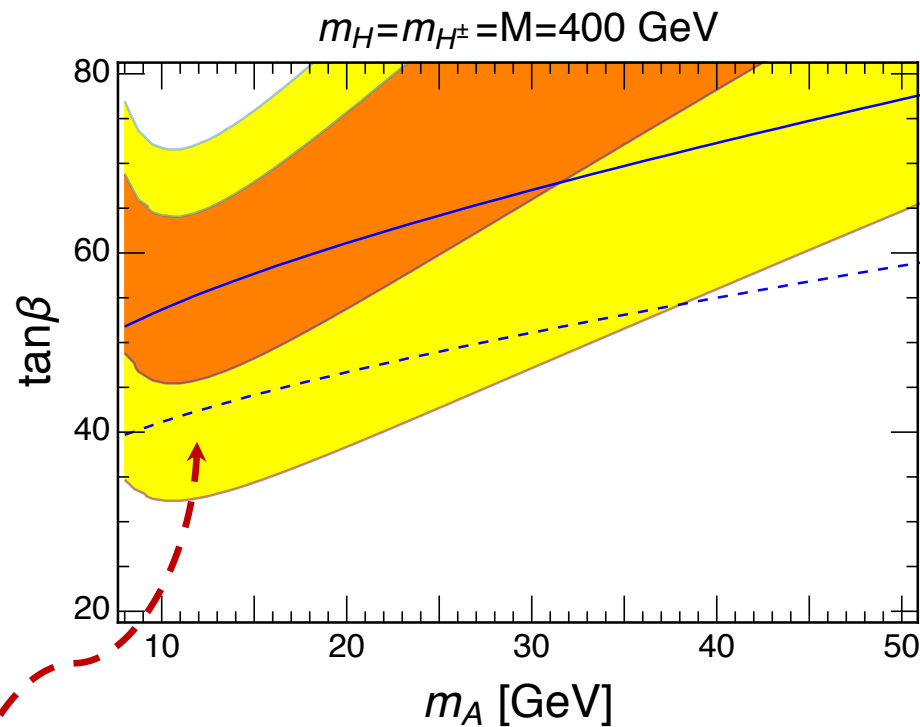
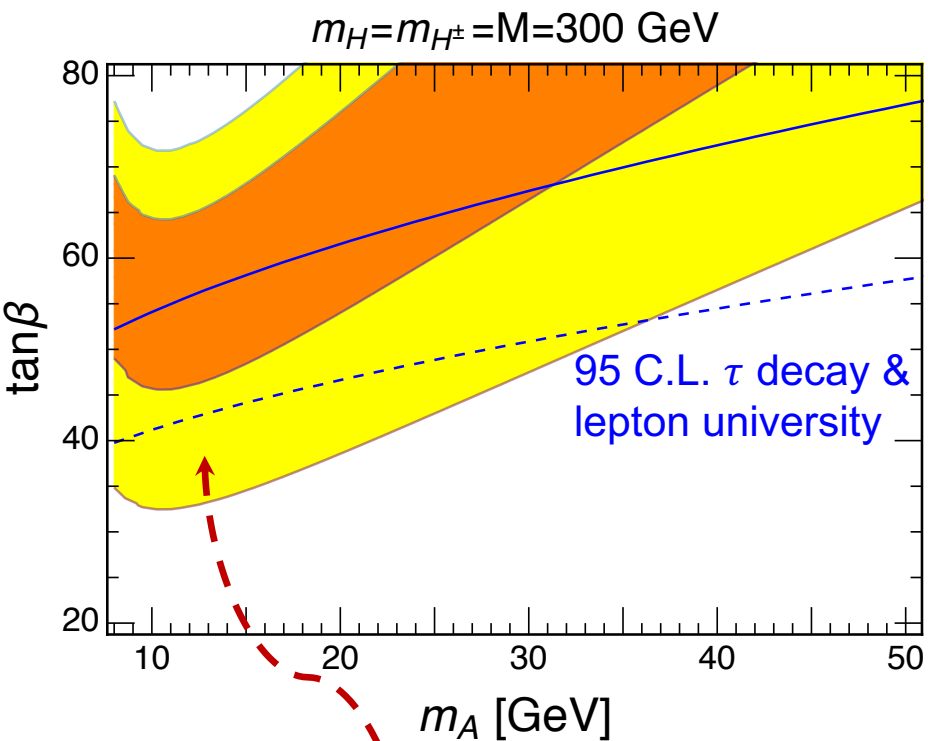
Contribution from the **light A boson** was pointed by Kingman Cheung, CH Chou, Otto, Kong (2001)



E821 collaboration (2004)

$$a_{\mu}^{\text{exp}} = 11\,659\,208.0 (5.4)(3.3) \times 10^{-10}$$

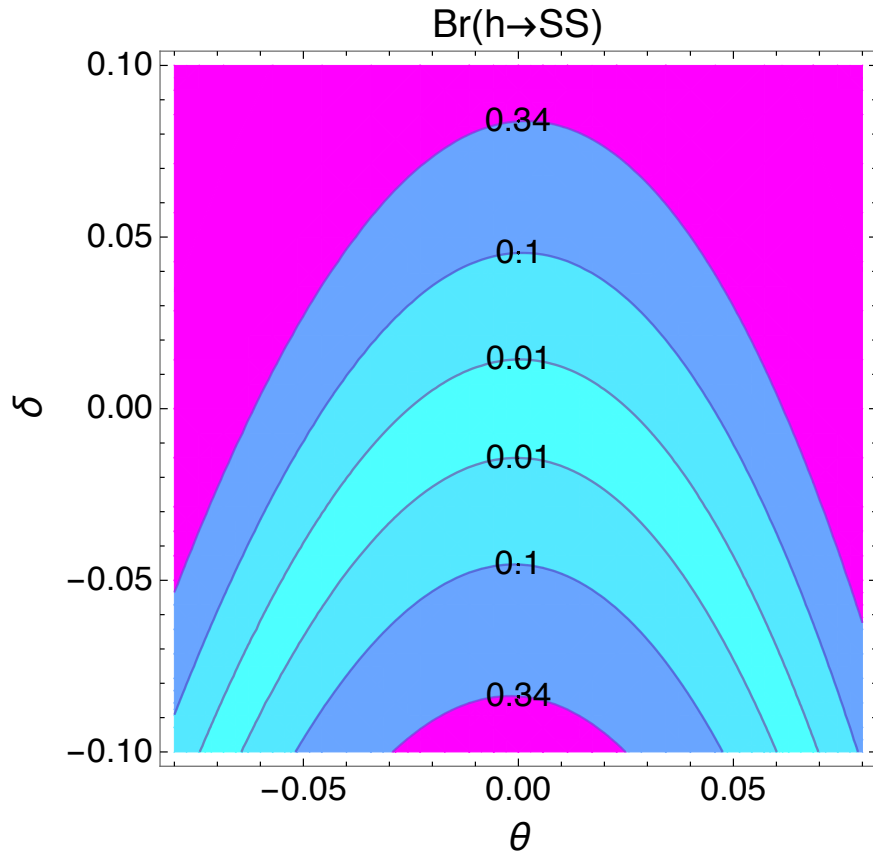
$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (26.1 \pm 8.0) \times 10^{-10}, \quad (\text{Hagiwara et. al.})$$



$m_A \sim 10 - 20 \text{ GeV}$ can explain muon $g-2$ anomaly at the 2σ level

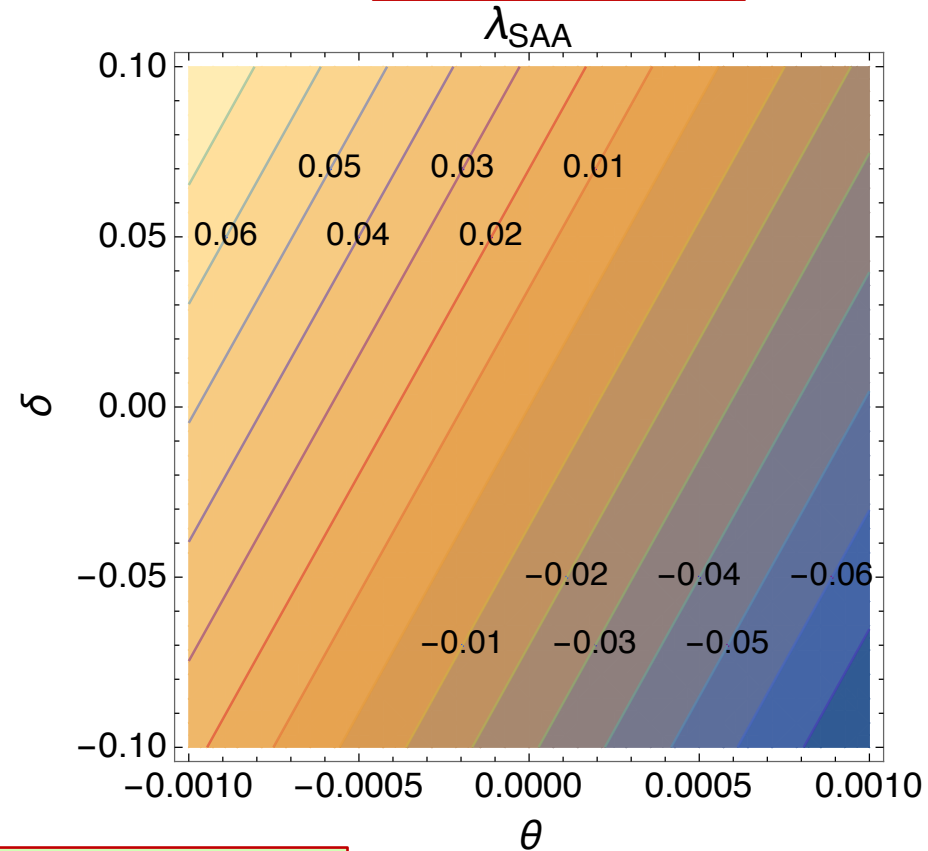
Determination of θ and δ

95% CL. limit: $\text{Br}(h \rightarrow \text{beyond SM}) < 34\%$



$\langle \sigma v \rangle_{XX \rightarrow AA} / \langle \sigma v \rangle_{XX \rightarrow SS} < 0.05$,

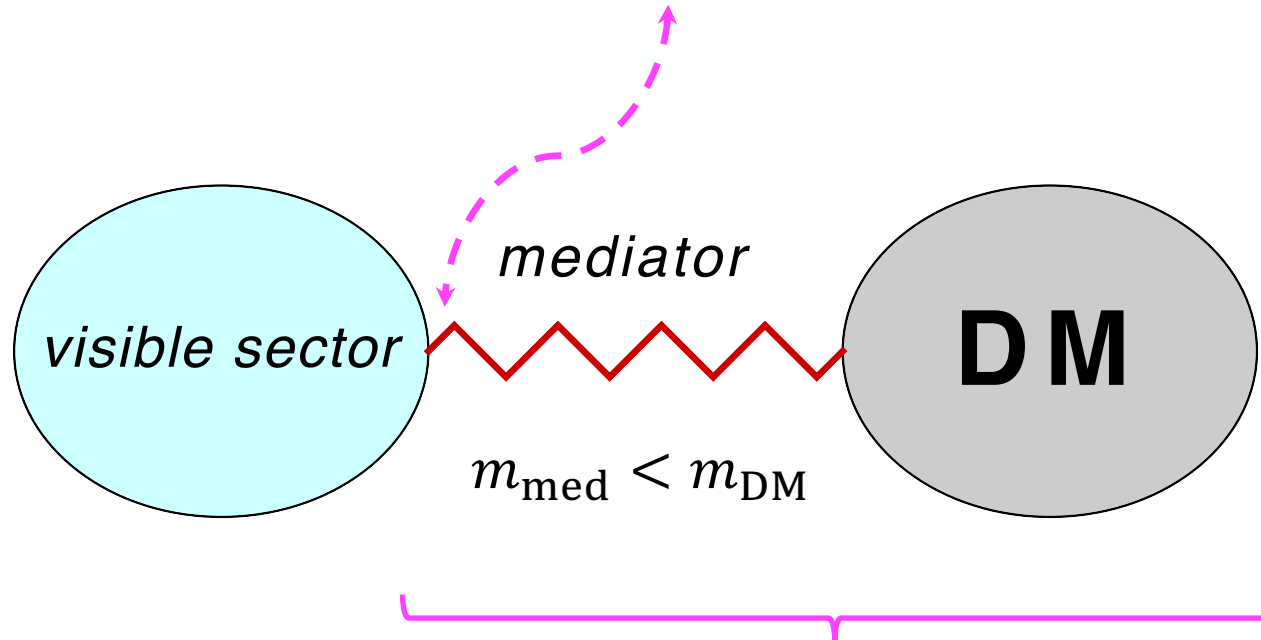
$\Rightarrow |\lambda_{SAA}| \lesssim 0.022$



$|\theta| \lesssim 0.001, \quad |\delta| \lesssim 0.088$

If taking $\delta = 0$, the constraint from the two-step cascade annihilation gives $|\theta| \leq 0.00043$

A very small coupling is allowed



Hidden sector
in which dark matter is secluded

Relic density: dependence on θ and δ

the same as the conventional WIMP scenario

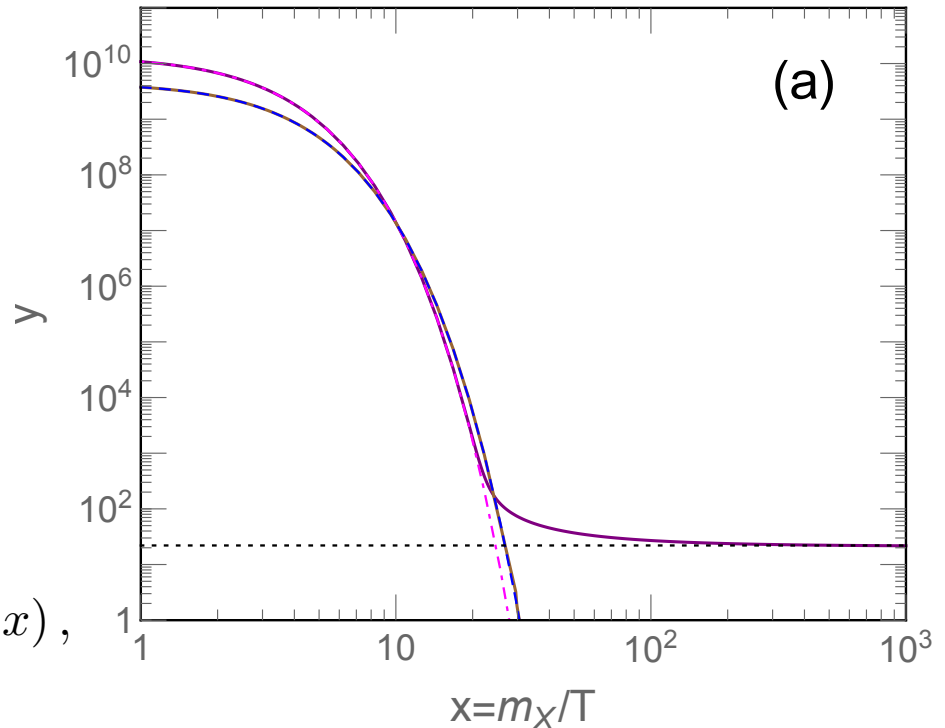
Normalized yields are

$$y_X(x) = \sqrt{\frac{\pi}{45G}} m_X g_*^{1/2} \langle \sigma v \rangle_{XX \rightarrow SS} Y_X(x),$$

$$y_S(x) = \sqrt{\frac{\pi}{45G}} m_X g_*^{1/2} \langle \sigma v \rangle_{XX \rightarrow SS} Y_S(x),$$

$$Y_i = \frac{n_i}{s}$$

$$(\theta, \delta) = (0.00043, 0)$$



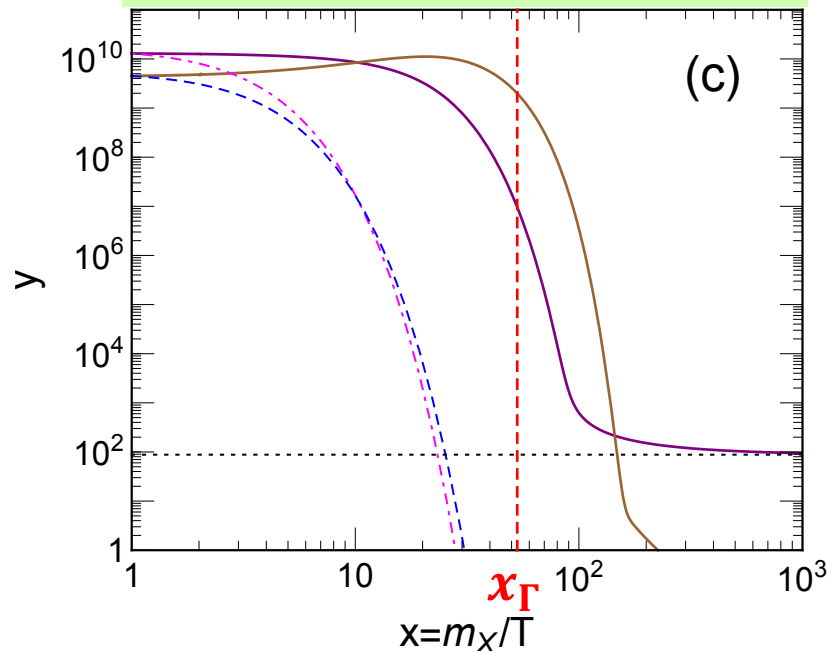
$$y_X^\infty = x_f \simeq 22$$

What happens for an even smaller mixing angle ?

Dark matter is well secluded in the hidden sector

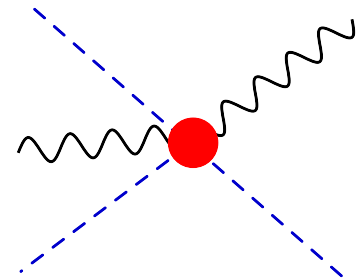
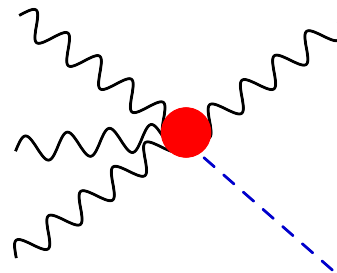
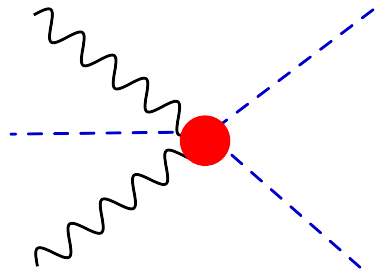
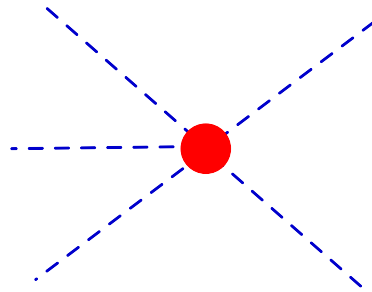
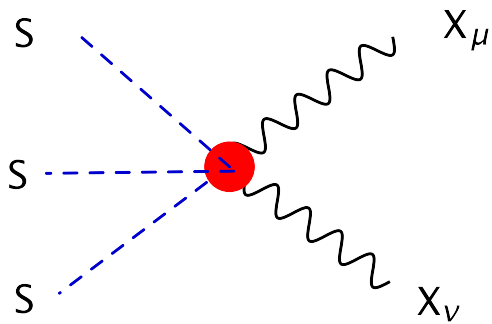
Ignoring the cannibal effects, the hidden sector may be out of equilibrium when $T \lesssim m_S$ (**co-decay dark matter**)

$$(\theta, \delta) = (1.2 \times 10^{-11}, 0)$$



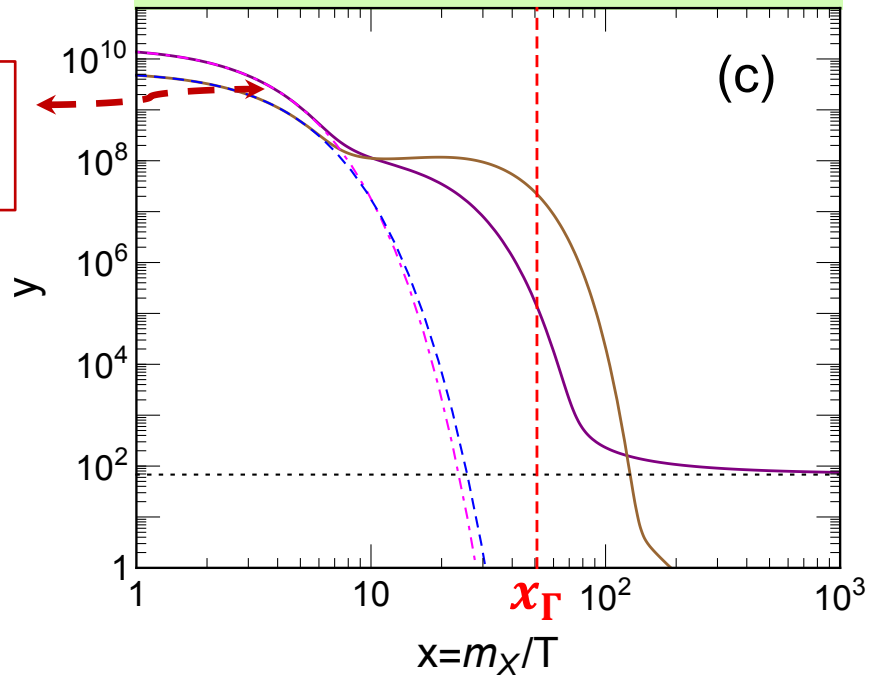
$$x_\Gamma = \frac{m_S}{T_S}$$

3 → 2 annihilation (cannibalism)



$$(\theta, \delta) = (1.2 \times 10^{-11}, 0)$$

3 → 2 annihilation
(cannibalism)



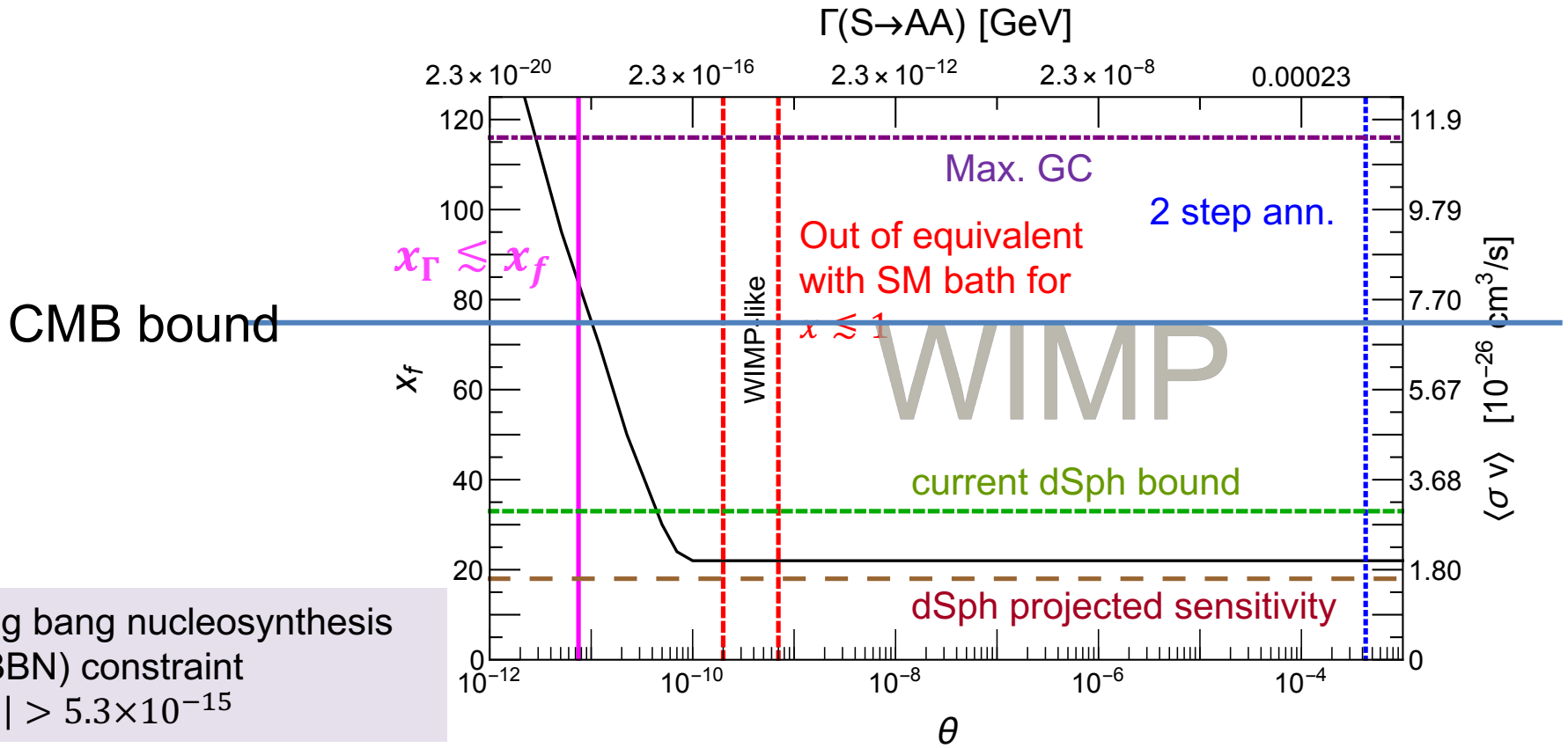
$$y_X^\infty = x_f \approx 70$$

$$\Omega_{\text{DM}} h^2 \simeq \frac{1.04 \times 10^9 \text{ GeV}^{-1}}{J \sqrt{8\pi g_*} M_{\text{pl}}} = 0.1198 \pm 0.0026$$

$$J = \int_{x_f}^{\infty} \frac{\langle \sigma v \rangle_{XX \rightarrow SS}}{x^2} dx \approx \frac{\langle \sigma v \rangle_{XX \rightarrow SS}^{(0)}}{x_f}$$

That is $\langle \sigma v \rangle_{XX \rightarrow SS}^{(0)} \propto x_f$

It hints that $\langle \sigma v \rangle_{XX \rightarrow SS}^{(0)}$ will be 3 times larger than the conventional WIMP case



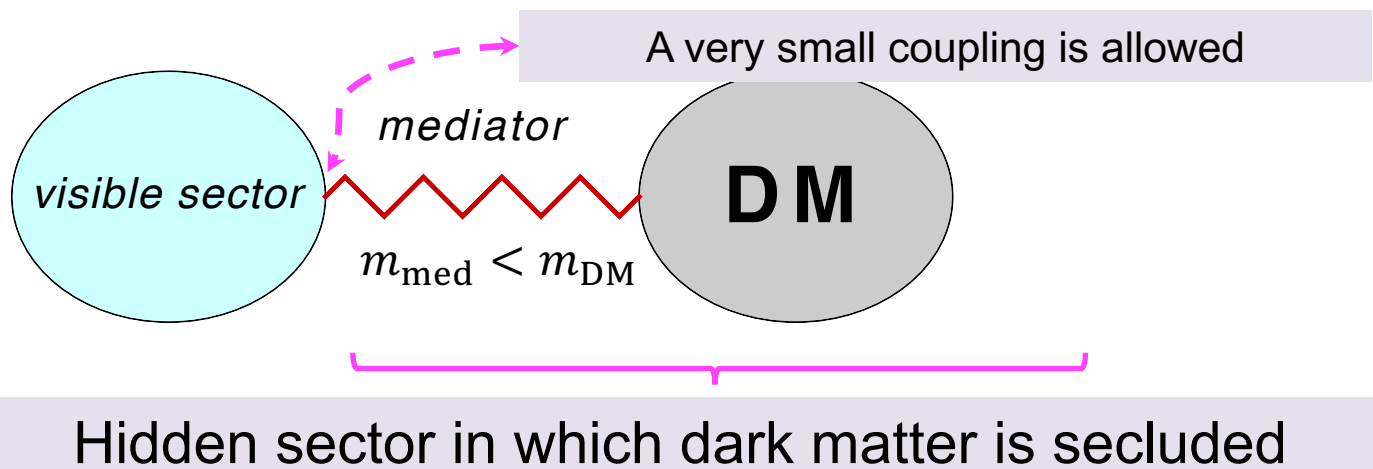
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Summary

1. 2-step cascade DM annihilation can well account for GC gamma-ray emission.
2. We have modeled a LN2HDM portal vector model
3. This model exhibits $m_A \sim 10 - 20$ GeV that can explain muon g-2 anomaly at the 2σ level.
4. The mechanism resulting in the **cannibally co-decaying** vector dark matter can explain the GC gamma-ray emission, the relic density simultaneously, and other constraints.



The Yukawa Sectors

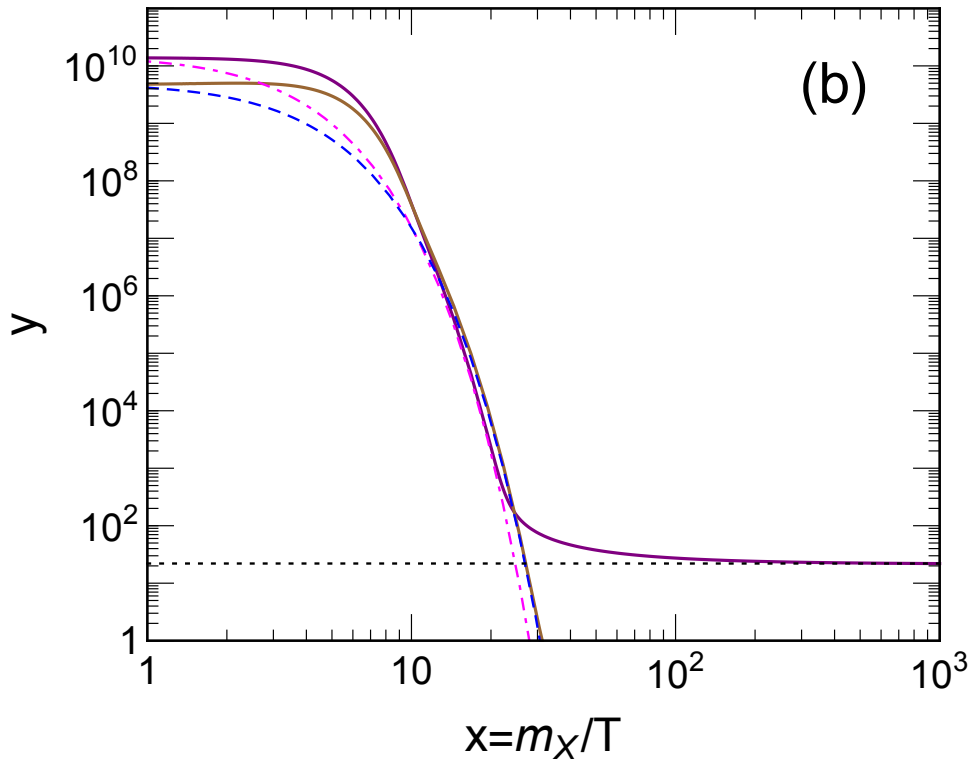
$$\mathcal{L}_{\text{Yukawa}} = -\bar{Q}_L y_u \tilde{\Phi}_2 u_R - \bar{Q}_L y_d \Phi_2 d_R - \bar{L}_L y_\ell \Phi_1 \ell_R + h.c.$$

The type-X Yukawa interactions are imposed a Z2 symmetry only to the right-handed quarks

Yukawa couplings of neutral type-X N2HDM Higgs

	$f = u, d$	l	
g_{hff}	c_α/s_β	$-s_\alpha/c_\beta$	Dominant in large $\tan\beta$
g_{Hff}	s_α/s_β	c_α/c_β	
g_{Sff}	$-(s_\alpha s_\theta + c_\alpha s_\delta)/s_\beta$	$(-c_\alpha s_\theta + s_\alpha s_\delta)/c_\beta$	
g_{Aff}	$\pm 1/t_\beta$	t_β	

TABLE I. The tree level Yukawa couplings of the neutral type-X N2HDM Higgs bosons, keeping terms linear in $\sin\theta$ and $\sin\delta$, with respect to that of the SM Higgs.



$$(\theta, \delta) = (2.0 \times 10^{-10}, 0)$$

$$y_X^\infty = x_f \simeq 22$$

$$\Omega_{\text{DM}} h^2 \simeq \frac{1.04 \times 10^9 \text{ GeV}^{-1}}{J \sqrt{8\pi g_*} M_{\text{pl}}} = 0.1198 \pm 0.0026$$

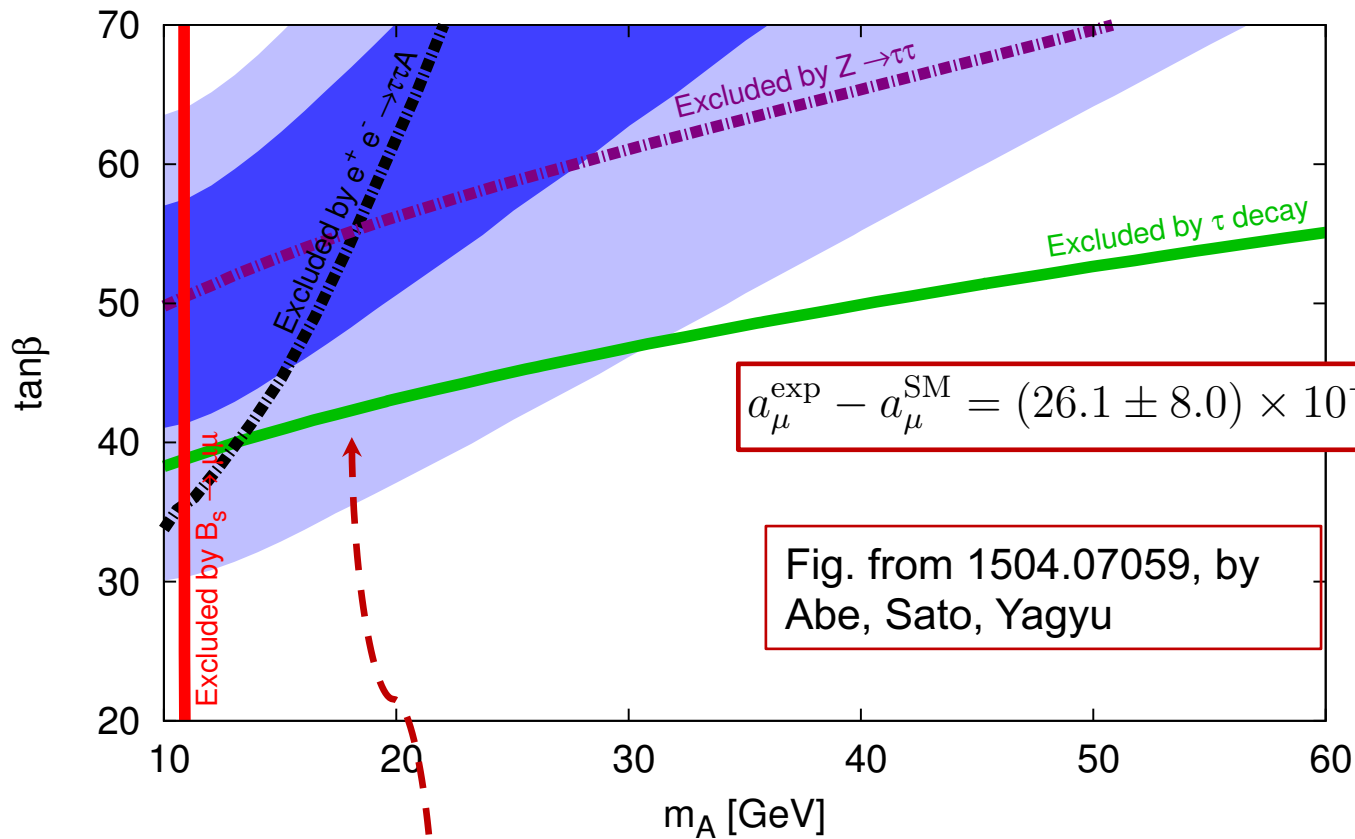
$$J = \int_{x_f}^{\infty} \frac{\langle \sigma v \rangle_{XX \rightarrow SS}}{x^2} dx \simeq \frac{\langle \sigma v \rangle_{XX \rightarrow SS}^{(0)}}{x_f}$$

That is $\langle \sigma v \rangle_{XX \rightarrow SS}^0 \propto x_f$

E821 collaboration (2004)

$$a_{\mu}^{\text{exp}} = 11\,659\,208.0 (5.4)(3.3) \times 10^{-10}$$

$$m_{H^0} = m_{H^+} = 300 \text{ GeV}$$

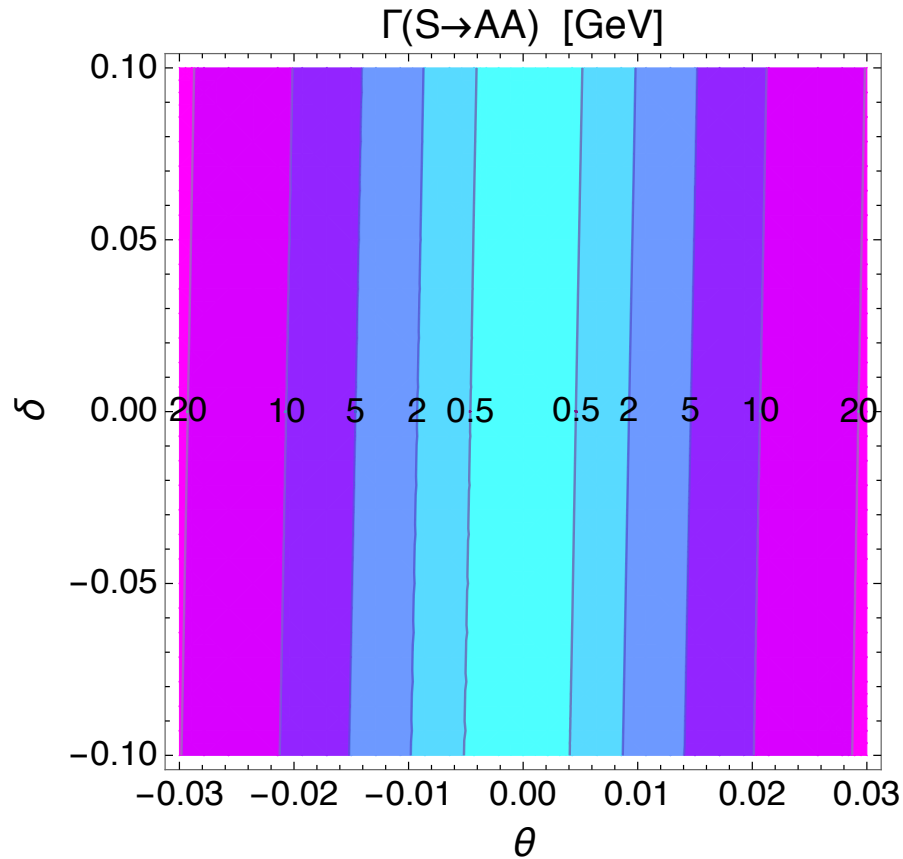


$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (26.1 \pm 8.0) \times 10^{-10}, \quad (\text{Hagiwara et. al.})$$

Fig. from 1504.07059, by
Abe, Sato, Yagyu

$m_A \sim 10 - 20 \text{ GeV}$ can explain muon g-2 anomaly at the 2σ level

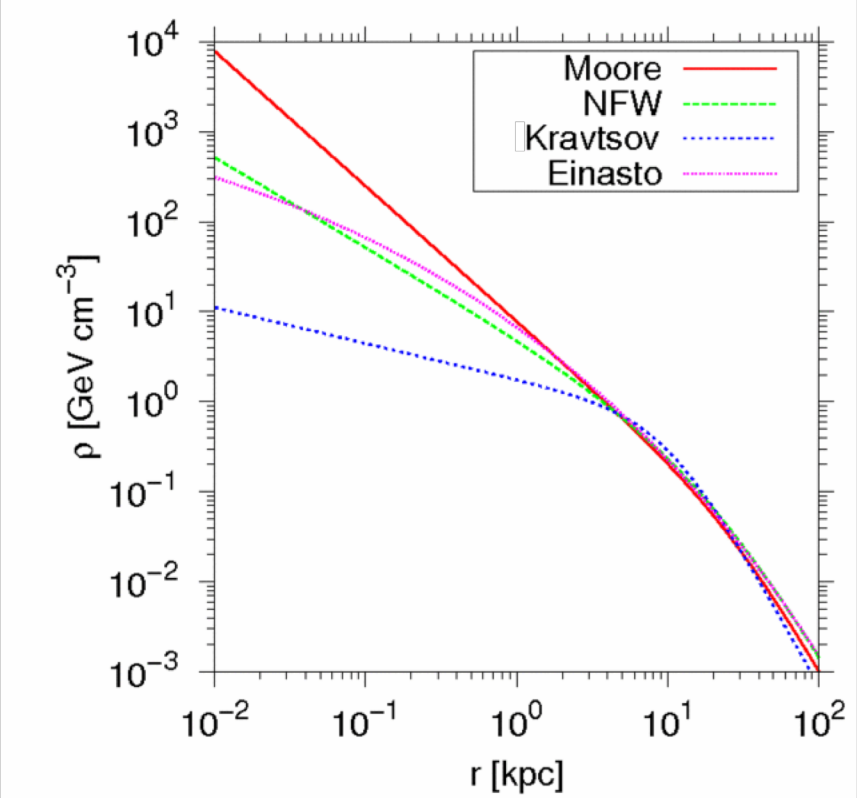
The width of S is narrow



$$|\theta| \lesssim 0.001, \quad |\delta| \lesssim 0.088$$



$$\Gamma_S/m_S \lesssim 6.7 \times 10^{-4} \text{ for } |\theta| \lesssim 0.001$$



Two-Higgs Doublet Model

$$\begin{aligned}
 V(\Phi_1, \Phi_2) = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.}) \\
 & + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\
 & + \left[\frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \lambda_6 (\Phi_1^\dagger \Phi_1) (\Phi_1^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + \text{H.c.} \right]
 \end{aligned}$$

discrete symmetry is imposed: $\Phi_1 \rightarrow \Phi_1, \Phi_2 \rightarrow -\Phi_2$, which forbids λ_6 and λ_7 but may be softly broken by the mixing term m_{12}^2 .

$$\begin{aligned}
 \Phi_1 &= \begin{pmatrix} G^+ \cos \beta - H^+ \sin \beta \\ \frac{1}{\sqrt{2}} [v \cos \beta - h \sin \alpha + H \cos \alpha + i(G^0 \cos \beta - A \sin \beta)] \end{pmatrix} \\
 \Phi_2 &= \begin{pmatrix} G^+ \sin \beta + H^+ \cos \beta \\ \frac{1}{\sqrt{2}} [v \sin \beta + h \cos \alpha + H \sin \alpha + i(G^0 \sin \beta + A \cos \beta)] \end{pmatrix}
 \end{aligned}$$

Yukawa coupling

$$-\mathcal{L}_Y = \begin{cases} \overline{Q'_L} Y^u \tilde{\Phi}_2 u'_R + \overline{Q'_L} Y^d \Phi_2 d'_R + \overline{L'_L} Y^\ell \Phi_2 \ell'_R + \text{H.c.} & \text{(Type I)} \\ \overline{Q'_L} Y^u \tilde{\Phi}_2 u'_R + \overline{Q'_L} Y^d \Phi_1 d'_R + \overline{L'_L} Y^\ell \Phi_1 \ell'_R + \text{H.c.} & \text{(Type II)} \\ \overline{Q'_L} Y^u \tilde{\Phi}_2 u'_R + \overline{Q'_L} Y^d \Phi_2 d'_R + \overline{L'_L} Y^\ell \Phi_1 \ell'_R + \text{H.c.} & \text{(Type X)} \\ \overline{Q'_L} Y^u \tilde{\Phi}_2 u'_R + \overline{Q'_L} Y^d \Phi_1 d'_R + \overline{L'_L} Y^\ell \Phi_2 \ell'_R + \text{H.c.} & \text{(Type Y)} \end{cases}$$

	Φ_1	Φ_2	u_{iR}	d_{iR}	ℓ_{iR}	Q_{iL}, L_{iL}	ξ_u	ξ_d	ξ_ℓ
Type I	+	-	-	-	-	+	$\cot \beta$	$\cot \beta$	$\cot \beta$
Type II	+	-	-	+	+	+	$\cot \beta$	$-\tan \beta$	$-\tan \beta$
Type X (lepton-specific)	+	-	-	-	+	+	$\cot \beta$	$\cot \beta$	$-\tan \beta$
Type Y (flipped)	+	-	-	+	-	+	$\cot \beta$	$-\tan \beta$	$\cot \beta$

$$g_{hff} = -\frac{m_f}{v} (s_{\beta-\alpha} + \xi_f c_{\beta-\alpha})$$

$$g_{Hff} = -\frac{m_f}{v} (c_{\beta-\alpha} - \xi_f s_{\beta-\alpha})$$

$$g_{Aff} = i \operatorname{sgn}(Q_f) \frac{m_f}{v} \xi_f$$