Self-tuning scalar fields
The cosmological constant problem, Fab Four, and well-tempered cosmology

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Outline

The cosmological constant problem

- ❑The CCP
- ❑Weinberg's no-go theorem

Fab Four

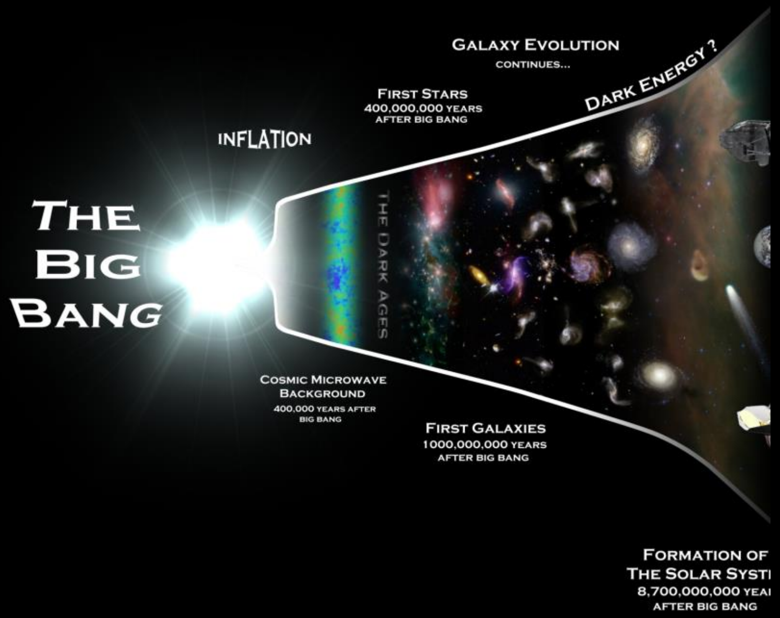
- ❑Horndeski theory
- ❑Fab Four

Well-tempered cosmology

- ❑Degeneracy, well-tempering
- ❑Well-tempered cosmology

Recent progress and future directions

The Cosmological Constant Problem

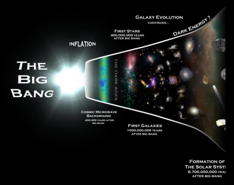


Our Universe

VS

Λ

The Cosmological Constant Problem



$$\lll \cdots \mathbf{50} \ll \cdots \lll$$

$$\Lambda$$

Our Universe

Steven Weinberg, *The cosmological constant problem*, Rev. Mod. Phys. 61 (1989) 1.
Antonio Padilla, *Lectures on the Cosmological Constant Problem*, arXiv:1502.05296.

Weinberg's no-go theorem

- A **no-fine tuning** theorem!
- Take metric g_{ab}, self-adjusting fields ϕ_i, assumptions:
   i. Dynamics, *Lagrangian* $\boldsymbol{L}(g_{ab}, \phi_i)$
   ii. Vacuum, *translation invariant*

$$g_{ab}, \phi_i = \text{constant}$$

- (ii) implies symmetry $M: x^a \to \left(M^{-1}\right)^a{}_b x^b$, M is a constant matrix:

$$g_{ab} \to g_{cd} M^c{}_a M^d{}_b$$

$$\boldsymbol{L}(g, \phi_i) \to \det(M)\, \boldsymbol{L}(g, \phi_i)$$

Weinberg's no-go theorem

- (ii) constant vacuum solution:

$$\delta_M \boldsymbol{L} = \frac{\partial \boldsymbol{L}}{\partial \phi_i} \delta_M \phi_i + \frac{\partial \boldsymbol{L}}{\partial g_{ab}} \delta_M g_{ab}$$

- (i) and (ii), vacuum equations (for illustration, both independent):

$$(\phi_i \text{ eqs.}) \frac{\partial \boldsymbol{L}}{\partial \phi_i} = 0 \quad \text{and} \quad (g_{ab} \text{ eq.}) \frac{\partial \boldsymbol{L}}{\partial g_{ab}} = 0$$

- ϕ_i eqs. $\rightarrow \frac{\partial \boldsymbol{L}}{\partial g_{ab}} = \frac{g^{ab}}{2} \boldsymbol{L} \rightarrow$ $\boxed{\boldsymbol{L} = \sqrt{-g} \boldsymbol{V}(\phi_i)}$ BUT g_{ab} eq. $\rightarrow$ $\boxed{\boldsymbol{V}(\phi_i) = 0}$
- Interpretation: $\boldsymbol{V}(\phi_i)$ are VEV/mass scales, $\boldsymbol{V}(\phi_i) = 0$ is **fine tuning**.

Weinberg’s no-go-theorem

Constant vacuum fields $\rightarrow$ Fine tuning

Weinberg's no-go theorem

$$\phi(t) \quad \text{vs} \quad \Lambda$$

[8]For instance, we assumed that in the solution for flat space all fields are constant, but it might be that this solution preserves only some combination of translation and gauge invariance, in which case some gauge-noninvariant fields might vary with space-time position. (This is the case for the 3-form gauge field model discussed at the end of Sec. VII and in Sec. VIII.) Furthermore, it is possible that the foliation of field space, which allows us to replace the ψ_n with σ_a and ϕ, does not work throughout the whole of field space.

footnote 8, page 11

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Recent progress and future directions

Horndeski theory

- Most general scalar-tensor theory with second-order field equations;
- *Limits*: GR, Brans-Dicke theory, *f*(*R*), quintessence, kinetic braidings…
- Derived from Lovelock's theorem;
- About Gregory Horndeski: https://www.horndeskicontemporary.com/about

Horndeski's painting at Leiden, < *https://www.universiteitleiden.nl/binaries/content/gallery/ul2/main-images/horndeski-foto-bruno-van-wayenburg.jpg/horndeski-foto-bruno-van-wayenburg.jpg/d390xvar*>

PRL 108, 051101 (2012) PHYSICAL REVIEW LETTERS week ending 3 FEBRUARY 2012

General Second-Order Scalar-Tensor Theory and Self-Tuning

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Starting from the most general scalar-tensor theory with second-order field equations in four dimensions, we establish the unique action that will allow for the existence of a consistent self-tuning mechanism on Friedmann-Lemaître-Robertson-Walker backgrounds, and show how it can be understood as a combination of just four base Lagrangians with an intriguing geometric structure dependent on the Ricci scalar, the Einstein tensor, the double dual of the Riemann tensor, and the Gauss-Bonnet combination. Spacetime curvature can be screened from the net cosmological constant at any given moment because we allow the scalar field to break Poincaré invariance on the self-tuning vacua, thereby evading the Weinberg no-go theorem. We show how the four arbitrary functions of the scalar field combine in an elegant way opening up the possibility of obtaining nontrivial cosmological solutions.

DOI: 10.1103/PhysRevLett.108.051101 PACS numbers: 04.50.Kd, 98.80.Jk

Self tuning

- Theory admits Minkowski vacuum for any Λ;
- survives phase transitions $\Delta\Lambda$;
- permits *nontrivial cosmology.*

The Beatles, <https://www.masslive.com/resizer/-_GQoCduR6kVhaM3gN44q46H-Uo=/800x0/smart/arc-anglerfish-arc2-prod-advancelocal.s3.amazonaws.com/public/NJDUSZLU2BHADLZWGWXNQHTHAQ.JPG>

Fab Four

Scalar potentials that allow self-tuning within Horndeski:

$$\boldsymbol{L_J} = V_J(\phi)\boldsymbol{G^{ab}\nabla_a\phi\nabla_b\phi}$$

$$\boldsymbol{L_P} = V_P(\phi)\boldsymbol{P^{abcd}\nabla_a\phi\nabla_c\phi\nabla_b\nabla_d\phi}$$

$$\boldsymbol{L_G} = V_G(\phi)\boldsymbol{R}$$

$$\boldsymbol{L_R} = V_R(\phi)\hat{\mathcal{G}}$$

where G^{ab} is the Einstein tensor, P^{abcd} is the double dual of the Riemann tensor, and $\hat{\mathcal{G}} = R^{abcd}R_{abcd} - 4R^{ab}R_{ab} + R^2$ is the Gauss-Bonnet combination.

Fab Four: cosmology

The self-tuning vacuum (*Milne* slice of Minkowski):

$$H^2 = -k/a^2$$

Hamiltonian constraint:

$$H_J + H_P + H_G + H_R + \Lambda = -\rho_m$$

where

$$H_J = 3V_J(\phi)\dot{\phi}^2\left(3H^2 + k/a^2\right)$$

$$H_P = -3V_P(\phi)\dot{\phi}^3 H\left(5H^2 + 3k/a^2\right)$$

$$H_G = -6V_G(\phi)\left[\left(H^2 + k/a^2\right) + H\partial_t \ln\left(V_G(\phi)\right)\right]$$

$$H_R = -24\partial_t V_R(\phi) H\left(H^2 + k/a^2\right)$$

On-shell:

$$\boldsymbol{\mathcal{H}\left(\phi, \dot{\phi}, k \neq 0\right) + \Lambda = 0}$$

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Recent progress and future directions

Degeneracy

- $\boldsymbol{H = \text{constant}}$ (low energy vacuum), overconstrains the dynamical system

$$\dot{H} = \ddot{\phi} Z(\phi, \dot{\phi}, H) + Y(\phi, \dot{\phi}, H)$$
$$0 = \ddot{\phi} D(\phi, \dot{\phi}, H) + C(\phi, \dot{\phi}, H, \dot{H})$$

- In Fab Four,

$$D(\phi, \dot{\phi}, H) = C(\phi, \dot{\phi}, H) = 0$$

- Other **degeneracy**:

$$Z(\phi, \dot{\phi}, H) \propto D(\phi, \dot{\phi}, H) \quad \& \quad Y(\phi, \dot{\phi}, H) \propto C(\phi, \dot{\phi}, H, \dot{H})$$

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An IOP and SISSA journal

The well-tempered cosmological constant

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University of California, Berkeley, CA 94720, U.S.A.
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Astana, 010000 Kazakhstan

Well-tempered cosmology

- Use $\phi(t)$ to design a low energy vacuum state $\left(H(t) = h, \phi(t)\right)$ s.t.

$$3H^2 = \rho_\Lambda + \rho_\phi$$
$$2\dot{H} + 3H^2 = -P_\Lambda - P_\phi$$
$$\ddot{\phi} f + \dot{\phi} g + \phi k = j$$

Vacuum State : $\phi(t)$ vs $\mathbf{\Lambda}$

- Result: Screen *only* $\mathbf{\Lambda}$, with mass scales of order unity.
- Price: $K(X), G(X) \to q[K(X), G(X)]$

S. Appleby and E. V. Linder, *The Well-Tempered Cosmological Constant*, JCAP 07 (2018) 034 [1805.00470].

Well-tempered cosmology: An example

$$S_{\text{grav}}[g,\phi] = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \Lambda + \lambda^3 \phi - \frac{\lambda^3}{18h^2} \left(\frac{2\gamma}{\sqrt{X}} + \ln X \right) \partial^2 \phi \right]$$

On-shell ($H(t) = h$), dynamical equations

$$\ddot{\phi}(t) = 3h\dot{\phi}(t) \quad \rightarrow \quad \phi(t) = c_1 e^{3ht} + c_2$$

Hamiltonian constraint:

$$3h^2 M_{\text{Pl}}^2 = \Lambda - \frac{\sqrt{2}\gamma\lambda^3}{3h} - c_2\lambda^3$$

Dynamics in a well-tempered de Sitter model

$\rho_\Lambda/h^2 \sim 10^{10}$: Λ is ten(!) orders of magnitude > de Sitter vacuum

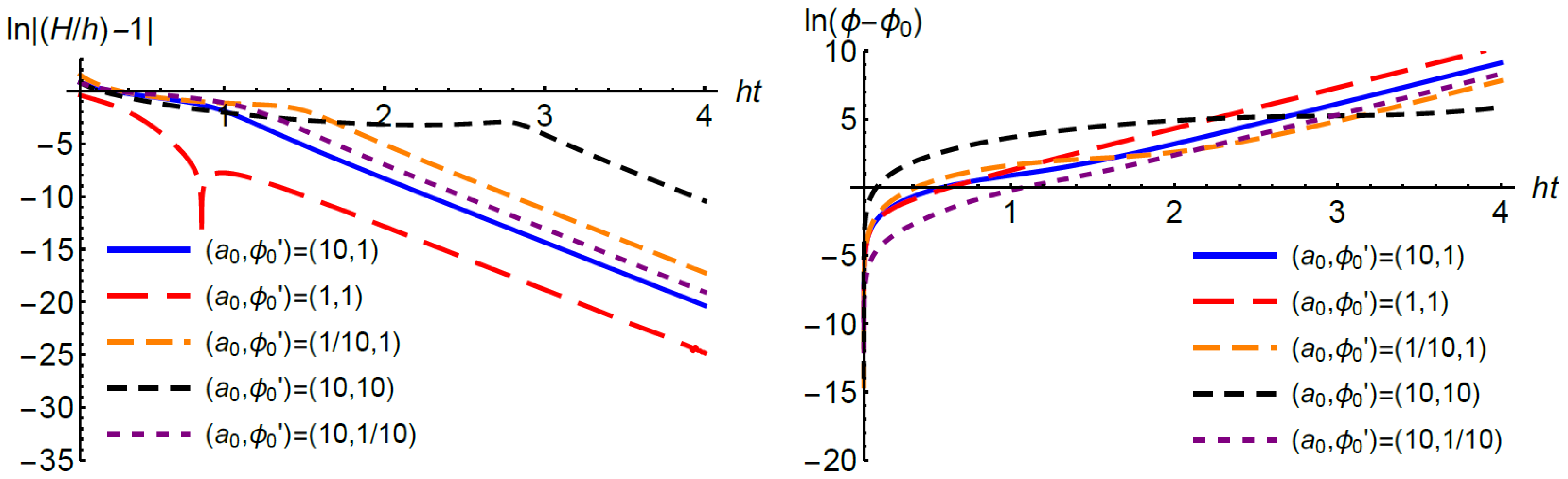

Figure 1. Hubble and scalar field evolution in a well-tempered model with $\rho_\Lambda/h^2 \sim 10^{10}$.

Dynamics in a well-tempered de Sitter model

$\rho_\Lambda / h^2 \sim 10^{10}$: Λ is ten(!) orders of magnitude > de Sitter vacuum

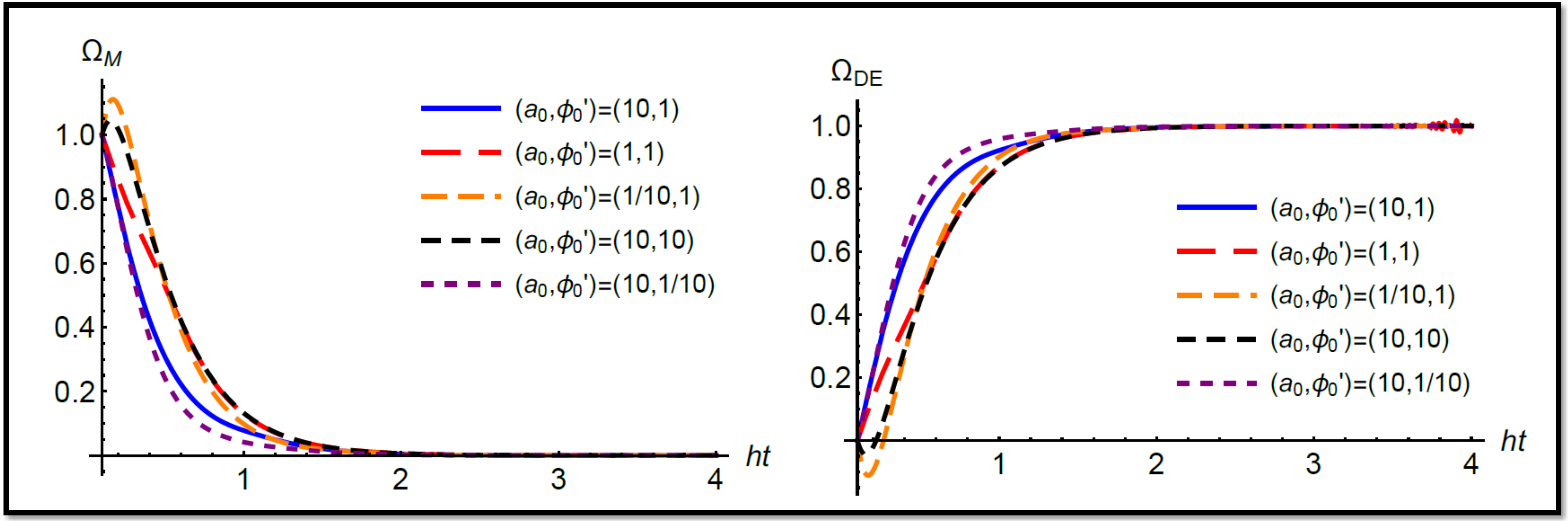

Figure 2. Matter and dark energy density parameters in well-tempered model with $\rho_\Lambda / h^2 \sim 10^{10}$.

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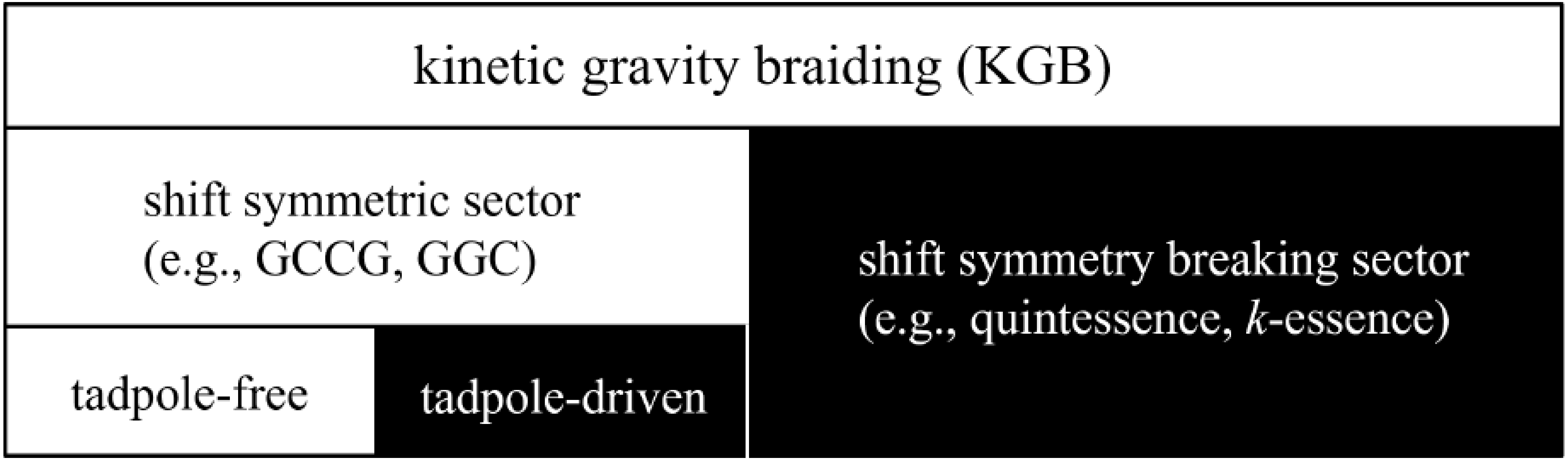
Recent progress and future directions

Highlights of recent progress

- (2012) Fab Four derivation, cosmology [1106.2000, 1112.4866, 1208.3373]
- …
- (2018) The Well-Tempered Cosmological Constant [1805.00470]
- (2019) WTC broadened to Horndeski sectors with $c = 1$ [1812.05480]
- (2020) Black hole in a FF-type self tuning model [1912.09199]
- (2020) WTC Horndeski Variations [2009.01720], WTC Fugue in B Flat [2009.01723]
- (2020) An expansion of well tempered gravity [2012.03965]
- (2021) **Self tuning KGB** [2101.00965, **RCB**]
- (2021) **WTC extended to teleparallel gravity** [**2107.08762**, **2108.02500**, **RCB**, Jackson Levi Said, Maria Caruana, Stephen Appleby]

Highlights of recent progress

- (2021) Fab Four generalized to AdS vacuum [2111.11448]
- (2022) WTC Scales, exploring relation between the mass scales [2201.02211]
- (2022) A minimal self tuning model to solve the CCP [2201.09016]
- (2022) **Nondegenerate self tuning** [*to appear*, Stephen Appleby, **RCB**]
- (20xx) …



Shift symmetry: $\phi \to \phi + C$ for constant C
Well tempering applies to the shaded sectors; trivial scalar/FF-type self tuning the all

Self-tuning KGB

(2101.00965, RCB, JCAP2021)

- The tadpole $\boldsymbol{S \sim \lambda^3 \phi}$
- No-tempering theorem
- Self tuning VS $\boldsymbol{\Lambda}$ VS Laplace condition

Recent progress: teleparallel gravity extensions

- **TEGR**: Teleparallel Equivalent of GR
- **Teledeski**: ***Tele***parallel Analogue of Horn***deski*** gravity

$$\mathcal{L}_{\text{Tele}} := G_{\text{Tele}}\big(\underbrace{\phi, X}_{\phi(t)}, \underbrace{T, T_{\text{ax}}, T_{\text{vec}}}_{T_{abc}(t)}, \underbrace{I_2, J_1, J_3, J_5, J_6, J_8, J_{10}}_{\phi - T \text{ couplings}}\big)$$

- **Well-tempered Teledeski models**
  - ❑2107.08762, **RCB**, Jackson Levi Said, Maria Caruana, Stephen Appleby, JCAP2021
  - ❑2108.02500, **RCB**, Jackson Levi Said, Maria Caruana, Stephen Appleby, CQG2021

Old Fab Four, <https://www.teahub.io/photos/full/23-231788_fab-four-1968-large-wallpaper-beatles-old.jpg>

Fab Four: The long and winding road

Scalar potentials that allow self-tuning within Horndeski
[**2111.11448**, Edmund **Copeland**, Sukhraj **Ghataore**, Florian **Niedermann**, Antonio **Padilla**]:

FF2012 (Milne slice)

$$L_J = V_J(\phi) G^{ab} \nabla_a \phi \nabla_b \phi$$

$$L_P = V_P(\phi) P^{abcd} \nabla_a \phi \nabla_c \phi \nabla_b \nabla_d \phi$$

$$L_G = V_G(\phi) R$$

$$L_R = V_R(\phi) \hat{\mathcal{G}}$$

FF2021 (AdS vacuum, curvature $-q^2$)

$$L_J = V_J(\phi)(G^{ab} \boldsymbol{- 3q^2 g^{ab}}) \nabla_a \phi \nabla_b \phi$$

$$L_P = V_P(\phi)\left(P^{abcd} + \boldsymbol{2q^2 g^{a[c} g^{d]b}}\right) \nabla_a \phi \nabla_c \phi \nabla_b \nabla_d \phi$$

$$L_G = V_G(\phi)\left(R + \boldsymbol{12q^2}\right)$$

$$L_R = V_R(\phi)\left(\hat{\mathcal{G}} - \boldsymbol{24q^4}\right)$$

where G^{ab} is the Einstein tensor, P^{abcd} is the double dual of the Riemann tensor, and $\hat{\mathcal{G}} = R^{abcd} R_{abcd} - 4R^{ab} R_{ab} + R^2$ is the Gauss-Bonnet combination.

Beyond degeneracy

- Degeneracy allows an ansatz $H = \text{constant}$ (e.g., low-energy vacuum) to be an "exact" solution to the system of equations;
- Price: Special ϕ and g_{ab} couplings in the action;

$$S_G[g_{ab}, \phi] = \int d^4x \sqrt{-g} \left(\frac{M_{\text{Pl}}^2 R}{2} - \Lambda - \lambda^3 \phi + \epsilon X + \frac{\epsilon^2}{\kappa^3} X \partial^2 \phi \right)$$

If $\kappa = \lambda$, degeneracy is satisfied, **but**, "Isn't this another kind of fine tuning?"

- **Q**: "Can we break degeneracy and retain its desirable features?"

Nondegenerate self tuning

- Minimal self tuning to solve the CCP [2201.09016, Khan & Taylor]

$$S_G[g_{ab},\phi] = \int d^4x\sqrt{-g}\left(\mathrm{EH} - \Lambda - \boldsymbol{\lambda^3\phi} + X - \frac{\sqrt{2X}}{3h}\partial^2\phi\right)$$

- Tadpole cosmology [*to appear*, Stephen Appleby & **RCB**]

$$S_G[g_{ab},\phi] = \int d^4x\sqrt{-g}\left(\mathrm{EH} - \Lambda - \lambda^3\phi + \epsilon X + \frac{\boldsymbol{\epsilon^2}}{\boldsymbol{\kappa^3}}X\partial^2\phi + \frac{\boldsymbol{m^2\phi^2}}{\boldsymbol{2}} + \boldsymbol{M^{2-n}\phi^n R}\right)$$

Tadpole cosmology

Canonical scalar + Tadpole:

$$S_G[g_{ab}, \phi] = \int d^4x \sqrt{-g} \left(\frac{M_{\rm Pl}^2}{2} R - \Lambda - \lambda^3 \phi + \epsilon X \right)$$

Field equations:

$$-2M_{\rm Pl}^2 \dot{H} = \epsilon \dot{\phi}^2$$

$$0 = \epsilon \ddot{\phi} + 3\epsilon H \dot{\phi} + \lambda^3$$

- If $\lambda = 0$, the standard $\left(3M_{\rm Pl}^2 H^2 = \Lambda, \phi = \text{constant}\right)$ is a solution;
- If $\lambda \neq 0$, the fields H and ϕ must be time dependent.

Model	Section	Asymptotic State	Dynamical $\not\Lambda$	Ghost Free
Unequal Mass Terms $\kappa \neq \lambda$	IV A	Minkowski	✓	✓
Linear Coupling $M\phi R$	IV B	Minkowski	✓	✓
Nonlinear Coupling $\beta^{2-n}\phi^n R$	IV B	Minkowski	✓	✓
Including Matter $\rho \neq 0$	IV C	Minkowski	✓	✓
Scalar Field Mass $m_\phi^2\phi^2$	IV D	de Sitter or Unstable	✓	✓

Tadpole cosmology
Self tuning without degeneracy

Outlook

- ✓ CCP challenges our understanding of the Universe;
- ✓ What not to do? No-go theorem;
- ✓ Degeneracy:
  - ✓ Fab Four
  - ✓ Well Tempered Cosmology
- ✓ Timeline of recent progress
- ✓ Self tuning beyond degeneracy
- ❑ Fab Four departures? WT beyond flat FLRW? Observational constraints? Laplace instability? …

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Jackson Levi Said and Maria Caruana

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University of the Philippines

To the Institute of Physics,

Academia Sinica

"The effort to understand the Universe is one of the very few things that lifts human life a little above the level of farce, and gives it some of the grace of tragedy."

- SW