Exotic hadrons in lattice QCD - examining the role of diquarks and the prediction of doubly heavy tetraquarks

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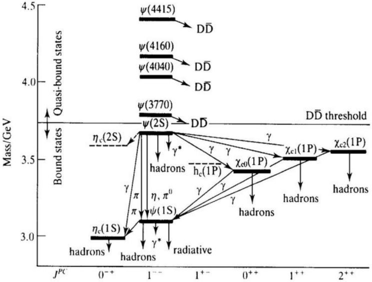
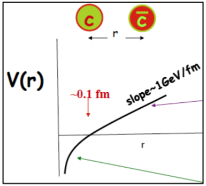
Heavy spectrum pre *B*-factories - A success story

Charmonium before B-factories

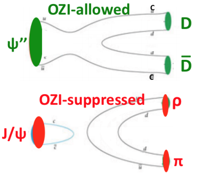


1980 – 2002 : no new charmonium states



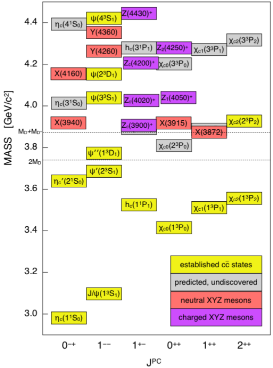
Before the advent of *B*-factories the study of heavy particles, in particular charmonia, can be seen as success story:

- predicted and measured masses agree
- potential model works well
- OZI-rule applies, no exceptions



Heavy spectrum after 2003 - a challenge to theory

*Mitchell, Olsen



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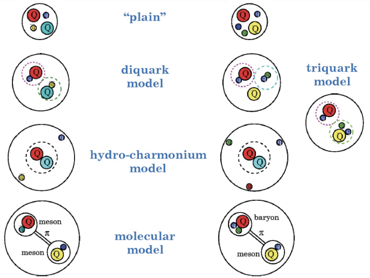


Newly discovered tetra- and pentaquarks are a challenge

- In 2003: $X(3872) \rightsquigarrow u\bar{d}c\bar{c}$ discovered at Belle
- Since then $\mathcal{O}(12)$ new heavy 4- and even 5-quark states observed
- None of them expected in quark models. Even worse: Predicted states not found.
- Many possible extensions of quark model thinkable.
- QCD origin, approximations often lead to contradictory statements

A new family of tetraquarks? - observation of T_{cc} at LHCb

Narrow state observed in $D^0 D^0 \pi^+$

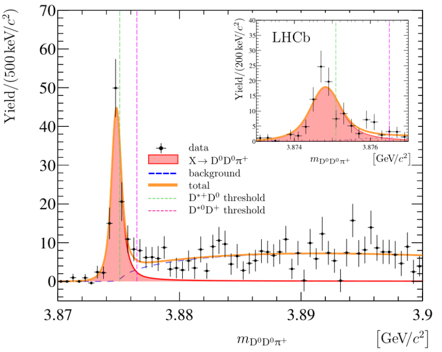
- Fitted to P-wave BW
- $\delta m = -273 \pm 61 \pm 5^{+11}_{-14} keV/c^2$ below $D^0 D^{*+}$ threshold
- $\Gamma = 410 \pm 165 \pm 43^{+18}_{-38} keV$

consistent with $cc\bar{u}\bar{d}$ tetraquark

- Possible family of states: $bc\bar{u}\bar{d}$, $bb\bar{u}\bar{d}$, $bb\bar{\ell}\bar{s}$, ...
- QN: $I(J^P) = 0(1^+)$
- Recent discussion in theory, both in pheno and lattice
  ⇝ predictions, binding mechanism

$B_{T_{cc}} = 0.3\text{MeV}$

⇝ LHCb-PAPER-2021-031



In the following:

- Non-time ordered review of discussion on the lattice
- Start with new work on diquarks as possible effective d.o.f's in QCD
- Followed by a status of current lattice doubly heavy tetraquark studies

The case for doubly heavy tetraquarks - Diquarks and $qq'\bar{Q}\bar{Q}'$ ($J^P = 1^+$)

Revisit ideas for stable multiquarks based on diquark d.o.f's

⤳Ader et al. ('82); Manohar, Wise ('93); ...

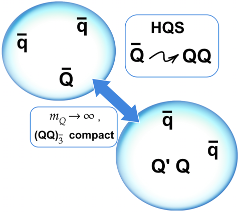
- Attractive $q - q$ interaction in "good" diquarks
- HQS ($Q \sim b$):
  - Anti-diquark acts like quark $[\bar{Q}\bar{Q}]_3 \leftrightarrow Q$
  - $[\bar{Q}\bar{Q}]_3^{m_Q \to \infty}$ becomes compact.
- Combine (HH)+(ll) diquarks into tetraquarks:

$$\boxed{\{qq'\}[\bar{Q}\bar{Q}'] = (qC\gamma_5 q')(\bar{Q}C\gamma_i\bar{Q}')}$$

- Renewed interest in pheno and lattice

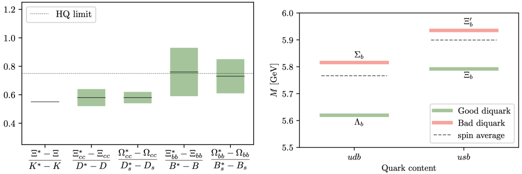
⤳Pheno: Karliner, Rosner ('17); Eichten, Quigg ('17); Czarnecki, Leng, Voloshin ('18); Mehen ('17); Maiani ('19)

⤳Lattice (et al.): AF ('16-'21); Bicudo, Wagner ('11-'19); Mathur ('18)



- PDG mesons/baryons provide constraints
- Deeply bound, prefer $\bar{b}\bar{b}$
  ⤳ closer to HQS
- Use diquark insights, binding deeper with
  ⤳ lighter good diquark
  ⤳ heavier bad diquark

Binding opportunity in model



Diquarks - attractive building blocks for exotic hadrons

Diquarks - an attractive concept

"The concept of diquarks is almost as old as the quark model, and actually predates QCD [1]"

⇝ Snowmass '20, [1] PR 155, 1601 (1967)

- Successful for low-lying baryons and exotic hadrons.
  - But, experimental evidence has been elusive.
- Well founded in QCD with many predictions. Light quarks:
  - special "good" $(\bar{3}_F, \bar{3}_c, J^P = 0^+)$ configuration
  - "good" diquarks experience attraction effect
  - large mass splitting in good, bad and not-even-bad
  - non-vanishing size or compact?
- For heavy quarks, with HQSS, diquarks can act as single antiquark $[QQ] \leftrightarrow \bar{Q}$.

⇝ opportunities for exotic hadrons, like $cc\bar{u}\bar{d}$ and $bb\bar{u}\bar{d}$.

good, bad and not-even

Diquark operator:

$$\boxed{D_\Gamma = q^c C \Gamma q'}$$

⇝ c, C =charge conjugation

⇝ Γ acts on Dirac space

J^P	C	F	Op: Γ
0^+	$\bar{3}$	$\bar{3}$	γ_5, $\gamma_0\gamma_5$
1^+	$\bar{3}$	6	γ_i, σ_{i0}
0^-	$\bar{3}$	6	$\mathbb{1}$, γ_0
1^-	$\bar{3}$	$\bar{3}$	$\gamma_i\gamma_5$, σ_{ij}

towards a clearer understanding and footing in QCD

Goal: Measure diquark properties in QCD non-perturbatively

- **spectrum:** *[diquark] mass differences are fundamental characteristics of QCD* `(Jaffe '05, arXiv:hep-ph/0409065)`
- **spatial correlations:** *study attraction and special status of the "good" diquark*
- **structure:** *estimate size and shape of the "good" diquark*

A gauge invariant probe - static quark as spectator

- A problem for the lattice is that diquarks are colored, i.e. not-gauge invariant.
  - Could fix a gauge, but then properties are gauge-dependent (masses, sizes,...)

$\rightsquigarrow$ `lattice and Dyson-Schwinger, see e.g. [15-20] in 2106.09080`

- **Alternative:** Static spectator quark Q ($m_Q \to \infty$) cancels in mass differences.
  - Diquark properties exposed in a gauge-invariant way.

$\rightsquigarrow$ `hep-lat/0510082, hep-lat/0509113, hep-lat/0609004, arxiv:1012.2353`

$$\boxed{C_\Gamma(t) \sim \exp\left[-t\left(m_{D_\Gamma} + m_Q + \mathcal{O}(m_Q^{-1})\right)\right]}$$

$\rightsquigarrow$ $t \to$ large, $m_Q \to$ large

- **Lattice correlator:** Diquark embedded in a static-light-light baryon

$$C_\Gamma(t) = \sum_{\vec{x}} \left\langle [D_\Gamma Q](\vec{x},t)\ [D_\Gamma Q]^\dagger(\vec{0},0)\right\rangle$$

$\rightsquigarrow$ static quark=Q and $D_\Gamma = q^c C\Gamma q$

$\rightsquigarrow$ flavor combinations ud, ℓs, ss'

$\rightsquigarrow$ static-light mesons $[\bar{Q}\Gamma q]$

setting up on the lattice - we recycle

- $n_f = 2+1$ full QCD, $32^3 \times 64$, $a = 0.090$fm, $a^{-1} = 2.194$GeV (PACS-CS gauges)
- $\boxed{m_\pi = 164, 299, 415, 575, 707\,\text{MeV}}$, $m_s \simeq m_s^{\text{phys}}$, propagators re-used from before
- Quenched gauge $a \simeq 0.1$fm, $\boxed{m_\pi^{\text{valence}} = 909\,\text{MeV}}$, to match `hep-lat/0509113`

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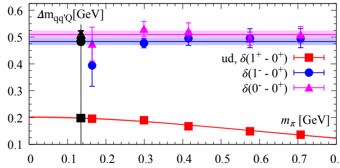
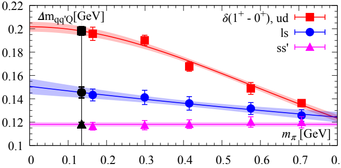
I. Diquark spectroscopy

Lattice spectroscopy - diquark-diquark differences

ud 0^+ versus 1^+, 0^- and 1^-



$(1^+ - 0^+)_{qq'}$ splitting



We consider differences of $qq'Q$ baryons:

$$\boxed{C_\Gamma^{qq'Q}(t) - C_{\gamma_5}^{qq'Q}(t)}$$

$\rightsquigarrow$ Q drops out

$\rightsquigarrow$ measures diquark-diquark mass difference

Bad-good diquark splitting:

- ○ Special status of good diquark observed
- ○ Good 0^+ *ud* diquark lies lowest in the spectrum
- ○ Bad 1^+ *ud* diquark 100-200 MeV above
- ○ 0^- and 1^- *ud* diquarks $\sim$ 0.5 GeV above
- ○ Pattern repeated in ℓs and ss'

$\Delta m_{qq'Q}(m_\pi)$ dependence:

- ○ Chiral limit: $\sim$ const
- ○ Heavy-quark limit: decreases $\sim 1/(m_{q_1} m_{q_2})$, with $m_\pi \sim (m_{q_1} + m_{q_2})$

$$\boxed{\delta(1^+ - 0^+)_{q_1 q_2} = A/\left[1 + \left(m_\pi/B\right)^{n\in 0,1,2}\right]}$$

Lattice spectroscopy - diquark-quark differences

We consider differences of a $qq'Q$ baryon and a light-static meson:

$$\boxed{C^{qq'Q}_{\Gamma=\gamma_5}(t) - C^{q'\bar{Q}}_{\gamma_5}(t)}$$

$\rightsquigarrow$ Q drops out

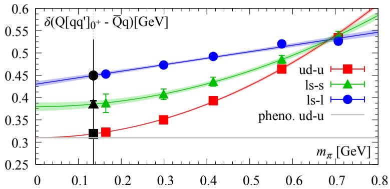
$\rightsquigarrow$ diquark-quark mass difference

$\Delta m_{qq'Q}(m_\pi)$ dependence:

- ○ Chiral vs. heavy-quark limiting behaviours, as before

$$\boxed{\delta(Q[q_1q_2]_{0^+} - \bar{Q}q_2) = C\,\left[1 + (m_\pi/D)^{n\in 0,1,2}\right]}$$

$Qqq' - \bar{Q}q'$ splittings



Diquark-quark splitting:

- ○ Established relative masses between a good diquark and an [anti]quark
- ○ May prove useful in identifying favourable tetra-, pentaquark channels
- ○ Omits possible distortions through additional light quarks, Pauli-blocking, spin-spin interactions ...

Diquark spectroscopy - phenomenological estimates

We want to compare our results with phenomenology

- ∘ Key resource: (Jaffe '05, arXiv:hep-ph/0409065), updated with PDG 2021 input
- ∘ For pheno estimates use charm and bottom hadron masses where leading $\mathcal{O}(1/m_Q)$ $(Q = c, b)$ can be cancelled

Four estimates considered:

- ∘ $\delta(1^+ - \ 0^+)_{ud}$: $\boxed{\frac{1}{3}\left(2M(\Sigma_Q^*) + M(\Sigma_Q)\right) - M(\Lambda_Q)}$
- ∘ $\delta(1^+ - 0^+)_{us}$: $\boxed{\frac{2}{3}\left(M(\Xi_Q^*) + M(\Sigma_Q) + M(\Omega_Q)\right) - M(\Xi_Q) - M(\Xi_Q')}$
- ∘ $\delta(Q[ud]_{0^+} - \bar{Q}u)$: $\boxed{M(\Lambda_Q) - \frac{1}{4}\left(M(P_{Qu}) + 3M(V_{Qu})\right)}$

⇝ P_{Qu}, V_{Qu} are the ground-state, heavy-light mesons

- ∘ $\delta(Q[us]_{0^+} - \bar{Q}s)$:

$$\boxed{M(\Xi_Q) + M(\Xi_Q') - \frac{1}{2}(M(\Sigma_Q) + M(\Omega_Q)) - \frac{1}{4}(M(P_{Qs}) + 3M(V_{Qs}))}$$

⇝ P_{Qs}, V_{Qs} are the ground-state, heavy-strange mesons

Diquark spectroscopy - comparing results

- We summarise the main spectroscopy results as:

All in [MeV]	$\delta E_{\rm lat}(m_\pi^{\rm phys})$	$\delta E_{\rm pheno}$	$\delta E_{\rm pheno}^{\rm bottom}$	$\delta E_{\rm pheno}^{\rm charm}$
$\delta(1^+ - 0^+)_{ud}$	198(4)	206(4)	206	210
$\delta(1^+ - 0^+)_{\ell s}$	145(5)	145(3)	145	148
$\delta(1^+ - 0^+)_{ss'}$	118(2)			
$\delta(Q[ud]_{0^+} - \bar{Q}u)$	319(1)	306(7)	306	313
$\delta(Q[\ell s]_{0^+} - \bar{Q}s)$	385(9)	397(1)	397	398
$\delta(Q[\ell s]_{0^+} - \bar{Q}\ell)$	450(6)			

$\rightsquigarrow$ updated pheno using PDG '21
$\rightsquigarrow$ use the bottom estimate for static
$\rightsquigarrow$ use charm-bottom difference as estimate for deviation from static
$\Rightarrow \lesssim \mathcal{O}(7)$MeV deviation

- Overall, very good agreement observed.

II. Diquark structure

Diquarks - spatial correlations

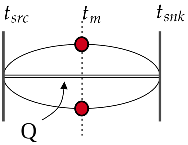
We access (good) diquark structure information through density-density correlations:

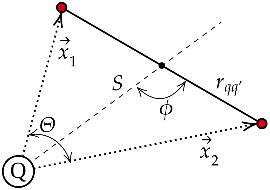
$$\boxed{C^{dd}_\Gamma(\vec{x}_1, \vec{x}_2, t) = \left\langle \mathcal{O}_\Gamma(\vec{0}, 2t)\ \rho(\vec{x}_1, t)\rho(\vec{x}_2, t)\ \mathcal{O}^\dagger_\Gamma(\vec{0}, 0) \right\rangle}$$

$\rightsquigarrow \mathcal{O}_\Gamma = q^c C\Gamma q$ and $\rho(\vec{x}, t) = \bar{q}(\vec{x}, t)\gamma_0 q(\vec{x}, t)$

$\rightsquigarrow t_m = (t_{snk} + t_{src})/2$ to minimize excited states

Main tool: Correlations between two light quarks' relative positions to the static quark





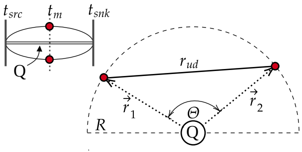
$\rightsquigarrow \vec{r}_{ud} = \vec{x}_2 - \vec{x}_1$ and $\vec{S} = (\vec{x}_1 - \vec{x}_2)/2$

$$\boxed{\rho_2(r_{ud}, S, \phi; \Gamma) = C^{dd}_\Gamma(\vec{x}_1, \vec{x}_2, t_m)}$$

Note, when S and r_{ud} fixed, distance between static quark Q and light quarks q, q' is

- Minimized for $\phi = \pi$, possible disruption due to Q is largest
- Maximized for $\phi = \pi/2$, possible disruption due to Q is smallest

Good diquark attraction



Setting $\phi = \pi/2$:

- $|\vec{x}_1| = |\vec{x}_2| = R$, use R, Θ:

$$\rho_2^{\perp}(R, \Theta) = \rho_2(r_{ud}, S, \pi/2)$$

- Attraction visible through increase in $\rho_2^{\perp}$ for small Θ at any fixed R

Two limiting cases for the two quarks:

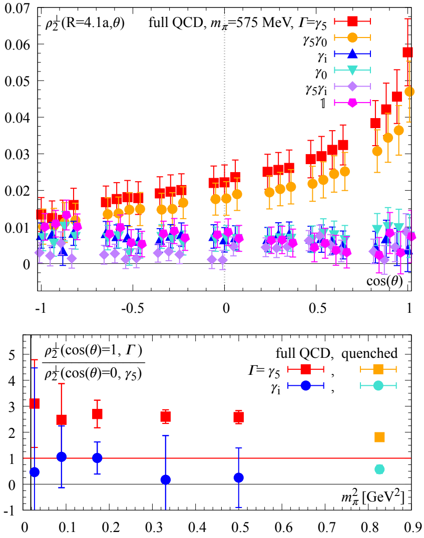
  - $\cos(\Theta) = 1$ on top of each other
  - $\cos(\Theta) = -1$ opposite each other

"Lift" as qualitative criterion:

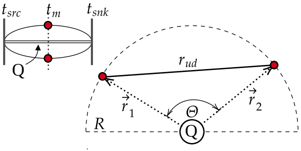
$$\frac{\rho_2^{\perp}(R, \Theta = 0, \Gamma)}{\rho_2^{\perp}(R, \Theta = \pi/2, \gamma_5)}$$

Increase observed in good diquark only

Spatial correlation over Θ



Good diquark size



- Distance between quarks:

  $$r_{ud} = R\sqrt{2\,(1-\cos(\Theta))}$$

  $\rightsquigarrow$ different visualisation

- $\rho_2^{\perp}(R, r_{ud}) \sim \exp(-r_{ud}/r_0)$

  $\rightsquigarrow$ "characteristic size" r_0

- Need to control:
  - interference from Q

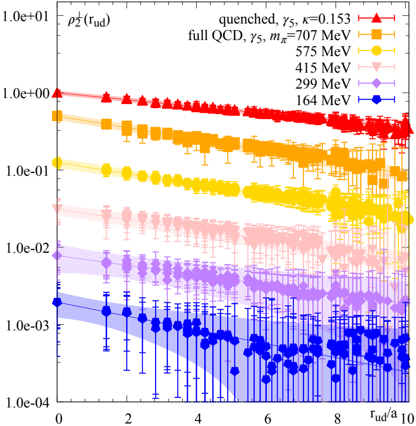
    $\rightsquigarrow$ we limit analysis to $r_{ud} < R$
  - periodicity effects

    $\rightsquigarrow$ in practice we find $L = 5r_0$

- Further checks:

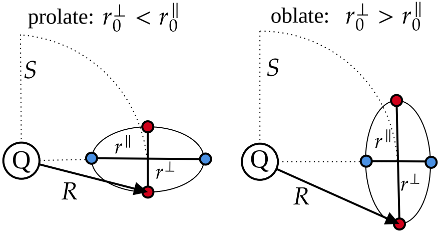
  $A(R, r_{ud} = 0) \sim \exp(-R/R_0)$

Spatial correlation over r_{ud}



- $r_{ud} = 0$ normalised, offset for each m_π
- all R shown simultaneously
- combined fits over $\forall R$ with shared r_0

Shape of good diquarks - studying oblateness



Tangential and radial spatial correlation decay

As opposed to before R $\neq$fixed:

- ○ $\phi = \pi$: radial correlation,
  size $\rightsquigarrow r_0^\parallel$
- ○ $\phi = \pi/2$: tangential correlation,
  size $\rightsquigarrow r_0^\perp$
- ○ $r_0^\perp / r_0^\parallel$ gives information on shape:
  $= 1$, spherical
  $\neq 1$, prolate/oblate

- Probe $J = 0$ nature of good diquark
- Diquark polarisation through static quark?

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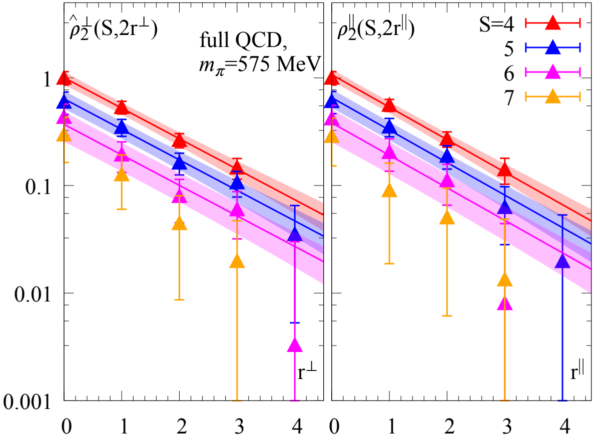
Oblateness - results



Goal:

- $r_0^{\perp}$, $r_0^{||}$ at fixed S

Technical issue:

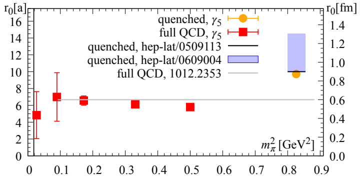
- ∘ ($||$) as before:
  $R = S$
- ∘ ($\perp$) different:
  $R = \sqrt{(r^{\perp})^2 + S^2}$

Solution:

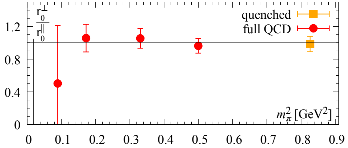
- ∘ Introduce "nuisance" paremeter R_0
- ∘ Adjusted in figure
- ∘ Parallel lines $\rightsquigarrow r_0^{\perp} = r_0^{||}$

Diquark structure - overview

Size dependence $r_0(m_\pi)$



Shape dependence $r_0^\perp/r_0^{||}(m_\pi)$



Good diquark size:

- Agreement w/ prev. quenched and dynamical
- Refinement through our results
- $r_0 \simeq \mathcal{O}(0.6)$fm weak m_π dependence

$\rightsquigarrow \sim r_{\text{meson, baryon}}$, arXiv:1604.02891

$r_0(m_\pi)$ dependence:

- $m_{q,q'} \uparrow$ should produce more compact object
- But, diquark attraction$\downarrow$ works opposite
- Former effect dominates at large m_π?
- But, in quenched diquarks definitely larger...

$r_0^\perp/r_0^{||}(m_\pi)$ dependence:

- Ratio $\simeq 1$ for all m_π
- Consistent w/ scalar, $J = 0$, shape
- No diquark polarisation through Q observed

Let's quickly revise - Diquarks on the lattice

Gauge invariant approach to diquarks in $n_f = 2+1$ lattice QCD

- ◦ Lattice setup with short chiral extrapolations, continuum limit still required

Diquark spectroscopy

- ◦ Special status of "good" diquark confirmed, attraction of 198(4)MeV over "bad"
- ◦ Chiral and flavor dependence modelled through simple Ansatz
- ◦ Very good agreement with phenomenological estimates

Diquark structure

- ◦ $q - q$ attraction in good diquark induces compact spatial correlation
- ◦ Good diquark size $r_0 \simeq \mathcal{O}(0.6)\mathrm{fm} \sim r_{\text{meson, baryon}}$, weakly m_π dependent
- ◦ Good diquark shape appears nearly spherical

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Doubly heavy tetraquarks in lattice QCD

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Confirm and predict doubly heavy tetraquarks non-perturbatively

Tetraquarks as ground states? What would their binding mechanism/properties be?

HQS-GDQ picture, consequences for $qq'\bar{Q}'\bar{Q}$ tetraquarks:

- ○ $J^P = 1^+$ *ground state* tetraquark below meson-meson threshold
- ○ Deeper binding with heavier quarks in the $\bar{Q}'\bar{Q}$ diquark
- ○ Deeper binding for lighter quarks in the qq' diquark

Ideal for lattice: Diquark dynamics and HQS could enable $J^P = 1^+$ *ground state doubly heavy tetraquarks* with flavor content $qq'\bar{Q}\bar{Q}'$.

Goal: $\Delta E = E_{\text{tetra}} - E_{\text{meson-meson}}$, e.g. in $bb\bar{u}\bar{d}$, $bb\bar{\ell}\bar{s}$ and others
$\Rightarrow$ Verify, quantify predictions of binding mechanism in mind.

Lattice point of view

- Hidden flavor $qQ\bar{q}'\bar{Q}$ are tetraquark candidates as excitations of $Q\bar{Q}'$.
  $\rightsquigarrow$ technical difficulty for lattice calculations, need to resolve many f.vol states.
  $\rightsquigarrow$ $qq'\bar{Q}\bar{Q}'$, i.e. ground state candidates would be better to handle.

In the following

- ○ Tetraquarks with two heavy (c, b) and two light (ℓ, s) quarks.
- ○ Lattice evidence for $bb\bar{u}\bar{d}$, $bb\bar{\ell}\bar{s}$.
- ○ Recent updates on systematics.
- ○ Survey of candidates status.

Lattice tetraquarks - 4 main approaches

1. Static quarks ($m_Q = \infty$)

Fitted potentials used to predict bound states and resonances.

- ○ Allows for potential formulation.
- ○ Ansatz fitted to lattice data.
- ○ Plug into Schrödinger Eq. for E_n.

$\rightsquigarrow b b \bar{u} \bar{d}$, Bicudo et al. ('17,'19)

3. Finite volume energy levels

Lattice energies equated to (un)observed states.

- ○ Operator matrix (GEVP) gives $\lambda_i \propto E_i$ $\Rightarrow$ Finite volume states.
- ○ Binding? Get $\Delta E = E_0 - E_{thresh}$.
- ○ Mechanism? Vary quark masses.

$\rightsquigarrow$AF et al. ('17,'18, '20), Hughes et al. ('17), Junnarkar et al. ('18), Leskovec et al. ('19), Mohanta et al. ('20)

2. HAL QCD method

Lattice potentials studied for scattering properties.

- ○ Expansion of energy dependent potential (systematics?).
- ○ Method under debate, best motivated for heavy systems.

$\rightsquigarrow$HAL QCD ('16,'18)

4. Scattering analysis

Lattice energies studied in terms of scattering phase shifts.

- ○ Excited state energies via GEVP.
- ○ Analyse fvol spectrum $\Rightarrow$ Resonant, bound, virtual bound, free.

$\rightsquigarrow$Hadron Spectrum Coll. ('18,'20)

Lattice tetraquarks - 4 step recipe

The main tool is to adopt a variational approach

Lattice GEVP gives access to finite volume energy states (masses, overlaps).

Beware: Operator overlaps do not necessarily connect to the naively expected structures. Be careful when equating lattice correlators with trial-wave functions.

Step I: Set up a basis of operators, here $J^P = 1^+$

Diquark-Antidiquark:

$$D = \left((q_a)^T (C\gamma_5) q'_b \right) \times \left[\bar{Q}_a (C\gamma_i)(\bar{Q}'_b)^T - a \leftrightarrow b \right]$$

Dimeson:

$$M = (\bar{b}_a \gamma_5 u_a)\,(\bar{b}_b \gamma_i d_b) - (\bar{b}_a \gamma_5 d_a)\,(\bar{b}_b \gamma_i u_b)$$

Step II: Solve the GEVP and fit the energies

$$F(t) = \begin{pmatrix} G_{DD}(t) & G_{DM}(t) \\ G_{MD}(t) & G_{MM}(t) \end{pmatrix}, \quad F(t)\nu = \lambda(t) F(t_0)\nu \,,$$

$$G_{\mathcal{O}_1\mathcal{O}_2} = \frac{C_{\mathcal{O}_1\mathcal{O}_2}(t)}{C_{PP}(t)C_{VV}(t)} \,,\ \lambda(t) = A e^{-\Delta E (t-t_0)} \,.$$

⇝ $\Delta E = E_{\text{tetra}} - E_{\text{thresh}}$ in case of binding correlator $(C_{\mathcal{O}_1\mathcal{O}_2}(t))/(C_{PP}(t)C_{VV}(t))$.

Most use these operators, but a larger basis has been worked out.
⇒ Need to be used by more groups.

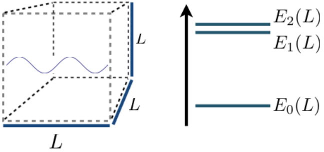
⇝ HadronSpectrum Coll. ('17)

Step III: Finite volume corrections

Large energy shifts are possible due to the finite lattice volume.

Scenario I: Scattering state
The finite volume energy belongs to a scattering state, the corrections go as

$$E_{b,L} \sim E_{b,\infty} \cdot \left[1 + \frac{a}{L^3} + \mathcal{O}(\frac{1}{L^4})\right]$$



⇝ M. Hansen

Scenario II: Stable state
The corrections are exponentially suppressed with $\kappa = \sqrt{E_{b,\infty}^2 + p^2}$

$$E_{b,L} \sim E_{b,\infty} \cdot \left[1 + Ae^{-\kappa L}\right]$$

With a single volume available:

- In a bound state corrections are $\sim$ exp(binding momentum) ⇝ strong supp. $m_{\rm had}$ =heavy
- In a scattering state expect large deviation around threshold

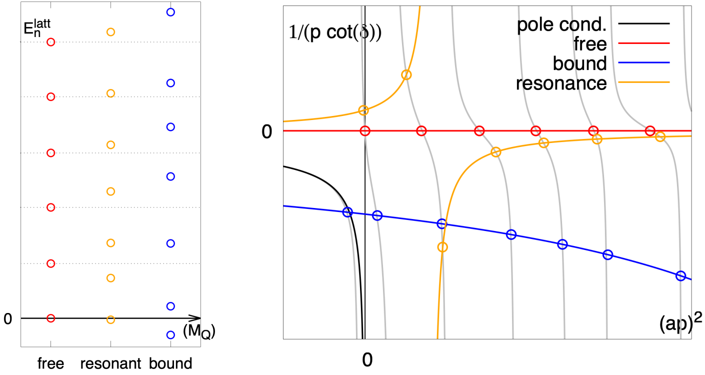
With multiple volumes available:

- Track mass dependence ⇝ decide bound/scatt. state
- Power law corrections might be too small to resolve

Step IV: Finite volume / Scattering analysis

Limitation: Small GEVP without f.vol analysis ok for deeply bound states.
Insufficient to tell apart free, resonant or virtual bd. states.

Extension: Connect energies to scattering phase shifts via finite volume quantisation conditions (Lüscher-formalism).



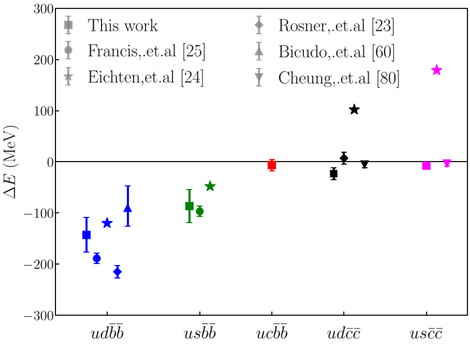
- ○ connect (many) f.vol states to scattering parameters (sketch: BW)
- ○ resonance: extra state(s) appear, lowest state close to threshold

What we know: A review of recent lattice studies

Anthony Francis, afrancis@nycu.edu.tw

What we know: Deeply bound $J^P = 1^+$ $bb\bar{u}\bar{d}$ and $bb\bar{\ell}\bar{s}$ tetraquarks

Community overview



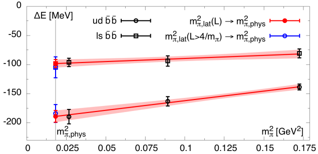
⇝Mathur et al. ('19)

Qualitative agreement with pheno

○ All three predictions met:
→ $J^P = 1^+$ bound ground state.
→ deeper binding with $m_Q \uparrow$.
→ deeper binding with $m_q \downarrow$.

○ $bb\bar{q}\bar{q}'$ are a focal point → All efforts observe deeply bound $bb\bar{u}\bar{d}$
.

AF et al. ('17)



· Junnarkar, Mathur, Padmanath ('18)
· Leskovec, Meinel, Plaumer, Wagner ('19)
· HadronSpectrum Coll. ('17)
· Mohanta, Basak ('20)
· Colquhoun, AF, Hudspith, Lewis, Maltman ('17, '18, '20)

Overview -possible doubly heavy tetraquark candidates

Surveying candidates

observed (>1 group)
no deep binding
observed (1 group)
not confirmed (>1 group)

channel	deeply bound
$J^P = 1^+$	$bb\bar{u}\bar{d}$ $bc\bar{u}\bar{d}$
	$bb\bar{\ell}\bar{s}$ $bc\bar{\ell}\bar{s}$
	$bs\bar{u}\bar{d}$ $cs\bar{u}\bar{d}$
	$bb\bar{u}\bar{c}$ $bb\bar{s}\bar{c}$
	$cc\bar{u}\bar{d}$ $cc\bar{\ell}\bar{s}$
	$bb\bar{b}\bar{b}$
$J^P = 0^+$	$bb\bar{u}\bar{u}$ $cc\bar{u}\bar{u}$
	$bb\bar{u}\bar{d}$ $bc\bar{u}\bar{d}$
	$bb\bar{\ell}\bar{s}$ $bc\bar{\ell}\bar{s}$
	$bb\bar{s}\bar{s}$ $cc\bar{s}\bar{s}$
	$bs\bar{u}\bar{d}$ $cs\bar{u}\bar{d}$
	$bb\bar{u}\bar{c}$ $bb\bar{s}\bar{c}$
	$bb\bar{c}\bar{c}$ $cc\bar{u}\bar{d}$
	$bb\bar{b}\bar{b}$

Deeply bound states

Focus: strong interaction stable

→ $bb\bar{u}\bar{d}$ and $bb\bar{\ell}\bar{s}$ in $J^P = 1^+$.
→ $cc\bar{q}\bar{q}'$ not deep.
→ $bc\bar{q}\bar{q}'$ not clear.
→ further candidates not observed.
→ none observed in $J^P = 0^+$.

⇝Bicudo et al. ('17), AF et al. ('17,'18, '20), HadSpec Coll. ('18), Hughes et al. ('17), Junnarkar et al. ('18), Leskovec et al. ('19), Mohanta et al. ('20)

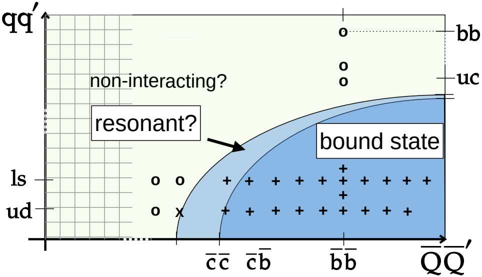
States above threshold, resonances?

→ $bb\bar{u}\bar{d}$ in $J^P = 1^+$ /w static quarks find a resonance just above threshold. ⇝Bicudo et al. ('19)
→ No results from other approaches.
→ What about $cs\bar{u}\bar{d}$?
⇝ under investigation Hudspith, AF et al.('20), HadSpec ('20)

Shallow binding?

○ $cc\bar{u}\bar{d}$ now observed by LHCb, robust lattice post-diction?
→ Work to remove current limitations.

A tunable system - binding diagram



○ Mapping out the flavor/mass binding diagram.
→ (Un-)binding transition?
→ Connecting resonance?

○ Surveying more J^{PC} candidates
→ Other binding mechanisms?
→ More exotica? ($cs\bar{u}\bar{d}$, $cc\bar{c}\bar{c}$,. . .)

Task: Establish the finite volume spectra and perform scattering analysis
→ What is the resonant/bound nature of the tetraquark candidates?

Recent lattice updates - including Lattice '21

Chiral limit

Majority of studies have performed extrapolations to m_{phys}.

Continuum limit

Few studies have taken (partial) continuum limits.

Finite volume

○ Initial volume scaling in one study. → **More work needed!**

Operator choice

○ One study uses non-local sinks, but local sources.
○ Two studies use a large basis in w-l approach. → **More work needed!**

Ground state systematics

○ The systematic due to the approach-from-below in w-l correlators is assessed through a box-sink construction. ⇝Hudspith, AF et al. ('20)
○ Corrections to energies ($\propto$ 25MeV) in w-l approach. → **Need careful re-evaluation!**

Structure properties

○ Study in potential approach. ⇝Wagner et al. ('21)
○ Studies using overlaps *caution required*. ⇝Mohanta,Basak('20); Wagner et al. ('21)

Deeper dive into recent updates: Structure properties

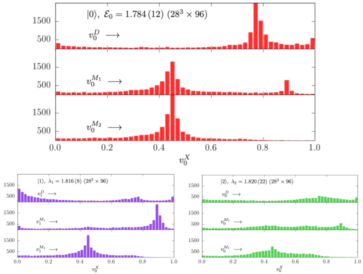
Structure properties - estimating overlaps from GEVPs

in principle: overlaps from GEVP give structure insight

- **Idea:** Overlaps give relative strengths of interpolating operator structures
- **Caveat:** Need well-defined operator structures.
  ⇝ Combining local sources with non-local sinks makes this ambiguous.
- *Possible solution:* Hermitian GEVP, e.g. via distillation approach

GEVP structure I

⇝Mohanta, Basak ('20)

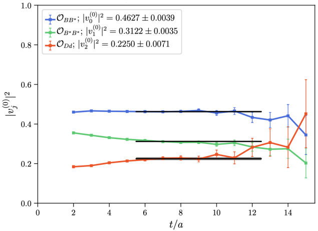


⇝NRQCD-HISQ, 3 × 3 GEVP, all local

- Diquark-type structure dominant

GEVP structure II

⇝Pflaumer ('21)



⇝local source, non-local sink

- Relative weights:

∼ 77% Dimeson vs. ∼ 23% Diquark-type

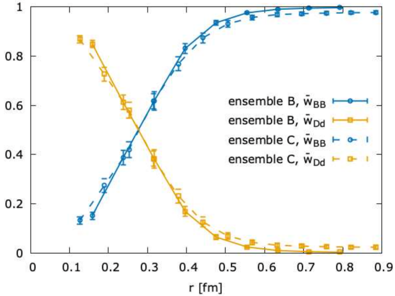
Structure properties - from the static potential

in principle: optimal trial states give structure insight

- **Idea:** Read off structure from weights of optimised trial states in Schrödinger Equation with lattice potential
- **Caveat:** Operator normalisation not trivial. Only clear connection when using static quarks. Potential needs to be interpolated
  $\rightsquigarrow$ Estimating systematics can be difficult.

Static potential structure

$\rightsquigarrow$Wagner ('21)



- $bb\bar{u}\bar{d}$ structure mixture
- Distance dependence:
  - $r \lesssim 0.2$fm: diquark-type dominance
  - $r \gtrsim 0.3$fm: dimeson dominance
- Relative weights:
  $\sim 60\%$ Dimeson vs. $\sim 40\%$ Diquark-type

The Full Program: A first lattice study of T_{CC}

A virtual bound state? - A lattice study of T_{CC} with unphysical quark masses

recall: performing the full finite volume analysis enables deeper insight

- **Idea:** Many lattice determined energy eigenstates are converted to scattering phase shifts via finite volume quantisation conditions.
- **Goal:** The extraction of the pole properties in the complex plane
- **Caveat:** The $E_B < 1$MeV of T_{CC} requires highly precise calculations at the physical point with many extra systematics under control (e.q. isospin breaking)
- *Possible solution:* Mapping of the pole trajectory with quark mass
- *Milestone:* The study of Padmanath, Prelovsek ('22) is a first step in this direction. They find a virtual bound state in T_{CC} at $m_\pi = 280$MeV.

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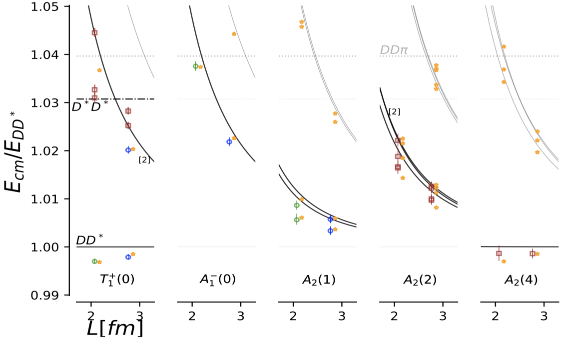
A virtual bound state? - A lattice study of T_{CC} with unphysical quark masses

Finite volume / scattering analysis - spectrum results

⇝Padmanat, Prelovsek ('22)



⇝distillation, only meson-meson operators used

- One lattice spacing $a = 0.086$ fm
- Two lattice volumes available, $\simeq 2$ fm and $\simeq 3$ fm
- One $m_\pi = 280$ MeV with 2 possible valence charm quark probes, one slightly below and one slightly above the physical charm quark mass.

A virtual bound state? - A lattice study of T_{CC} with unphysical quark masses

recall: performing the full finite volume analysis enables deeper insight

- **Idea:** Many lattice determined energy eigenstates are converted to scattering phase shifts via finite volume quantisation conditions.
- **Goal:** The extraction of the pole properties in the complex plane.
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Binding energy

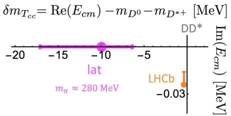

FIG. 3. The pole in the scattering amplitude related to T_{cc} in the complex energy plane: our lattice result at the heavier charm quark mass (magenta) and the LHCb result (orange).

A virtual bound state

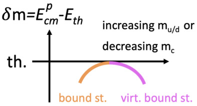

FIG. 4. Sketch of the binding energy for the (virtual) bound state dominated by the molecular component. It is based on a purely attractive potential $V(r)$ and partial wave $l = 0$ within quantum mechanics.

Towards new levels of precision: The open lattice initiative

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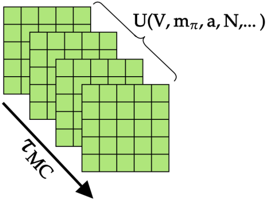
- A gauge field configuration is a single **snap-shot of the space-time** background on which the physics measurement is performed. A collection of snap-shots/ samples/ configurations is called an ensemble.
- Lattice simulations can be neatly separated into gauge field generation and observable calculation.



stabilised Wilson fermions (SWF) - an upgrade package

AF, Fritzsch, Lüscher, Rago ('19)

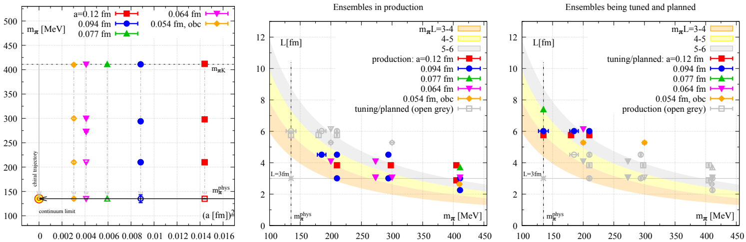
SMD = stochastic molecular dynamics ⇝ (algorithm between HMC and Langevin)

- Algorithmic improvements:
  - SMD decreases fluctuations and makes for a generally more stable run
  - SMD algorithm shows net gain in reduced autocorrelations at same cost
  - increase precision of internal numbers to quad
  - use supremum-norm to ensure minimum solve quality
- Fermion discretisation:
  - exponentiated Clover action
  - bound from below and guaranteed invertibility for Clover term
  - indication of (observable dependent) **scaling benefits** (see: further material)
- Combine with measures already deployed for the best, i.e. most stable, results.

SWF toolkit implemented in `openQCD-2.0`

OPEN LATtice initiative

Bringing together researchers from different institutes. Our aim is to generate state-of-the-art QCD gauge ensembles for physics applications and to share them with the community to strengthen open science.



Generate **public** ensembles that

- exploit the SWF benefits
- enable a better controlled extrapolations
- controlled finite volume effects

Multi-stage plan:

- fix trajectory in a and m_π
- extend and establish infrastructure
- follow trajectory towards m_π^{phys}
- update shared data at each step

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Wrapping up

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Summary

Gauge invariant properties of diquarks on the lattice

- ∘ Special status of "good" diquark confirmed, attraction of 198(4)MeV over "bad"
- ∘ Very good agreement with phenomenological estimates
- ∘ $q - q$ attraction in good diquark induces compact spatial correlation
- ∘ Good diquark size $r_0 \simeq \mathcal{O}(0.6)\text{fm} \sim r_{\text{meson, baryon}}$, weakly m_π dependent
- ∘ Good diquark shape appears nearly spherical

Doubly heavy tetraquarks

- ∘ Lattice evidence for doubly heavy tetraquarks, esp. $bb\bar{u}\bar{d}$, $bb\bar{\ell}\bar{s}$
- ∘ Broad agreement with a description based on a diquark+HQS model
- ∘ Surveying potential candidates favors the $I(J^P) = 0(1^+)$ channel

Lattice status

- ∘ Lattice studies focussing on consolidating and estimating systemtatics
- ∘ First studies of tetraquark structure
- ∘ Operator bases need to be updated for robust structure predictions

Welcome T_{cc}, the first member of a new family of tetraquarks?

- ∘ Requires firm understanding of heavier candidates
- ∘ Lattice confirmation of $B_{T_{cc}} \lesssim 1\text{MeV}$ hard, but first results are exciting

Open lattice initative openlat1.gitlab.io

- ∘ Generate next level ensembles for new jump in precision lQCD calculations

謝謝

Thank you!

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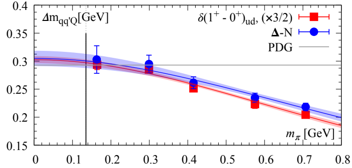
Further material

Δ-Nucleon mass difference

$[\Delta - N](m_\pi)$



Measured the mass difference of $\Delta - N$

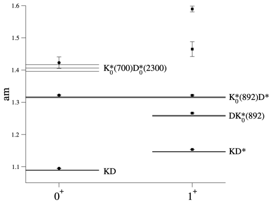
- Prediction: $\delta(\Delta - N) = 3/2 \times \delta(1^+ - 0^+)_{ud}$
- Same Ansatz as before
- Prediction holds well, even at fairly large m_π

Charm-strange $X(2900)$ - opportunity together with experiment

○ $X(2900)$, $cs\bar{u}\bar{d}$, is particularly interesting:
→ observed in experiment. *LHCb ('20)
→ within reach of lattice calculations.

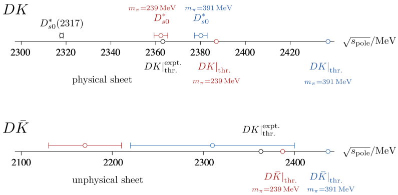
○ Two existing lattice studies fall just short of the interesting region:

Hudspith, Coulqhoun, AF, Lewis, Maltman ('20)



(b) $ud\bar{s}\bar{c}$

Hadron Spectrum Coll. ('20)



○ Close to D^*K^* threshold, but not enough operators to really probe.
○ Currently no indication $X(2900)$, a quotable **statement is premature**.

○ Focussed on DK and $D\bar{K}$ in the energy region $<$ 2500MeV.

Extension studies required and eagerly awaited.

All-heavy $cc\bar{c}\bar{c}$ - opportunity together with experiment

○ $cc\bar{c}\bar{c}$, is another interesting example:
→ observed in experiment. *LHCb ('20)
→ within reach of lattice calculations.

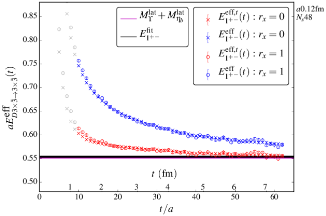
○ One existing lattice study in $bb\bar{b}\bar{b}$, focussed below threshold:

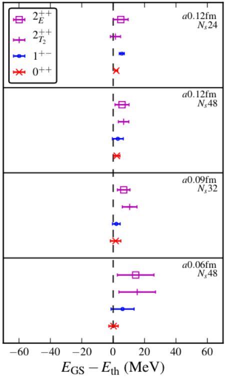
*Hughes et al. ('18)

Calculation using NRQCD in 0^{++}, 1^{+-} *and* 2^{++} *channels.*

⇒ No binding found.

Diquark-Antidiquark



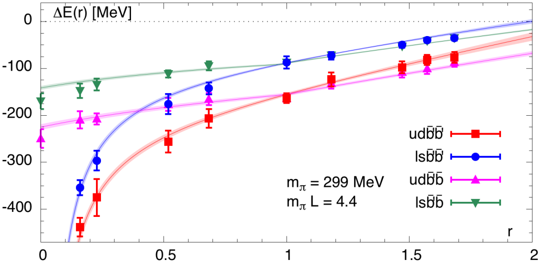


Extension study(?).

A tunable system - opportunity together with pheno

AF et al. ('18)

*5 parameter pheno-Ansatz in Appendix



- ◦ E.g. scans in $m_{b'}$ map out the heavy quark mass dependence.

◦ Away from physical masses the binding mechanism can be probed.
→ Mass dependence can be confronted with model predictions.
→ System can be tuned continuously from the bound to the resonant or non-interacting regimes.

→ Requires robust control of finite volume spectrum.

I. Deeper dive into recent updates: Ground state systematics

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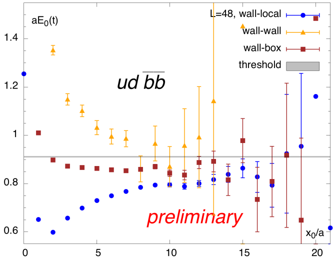
Consolidating results - the role of systematics



⇝Hudspith, AF et al. ('20)

Wall-local correlators approach the ground state from below.

$\Rightarrow$ Systematic **over estimate** of binding energies?

Update: Study that includes correlators using a wall-box approach to increase ground state overlap.

Box-sink construction

∘ Correlators made of Coulomb gauge fixed wall sources and local sinks are known to have exceptionally good signal-to-noise ratios. Properties:

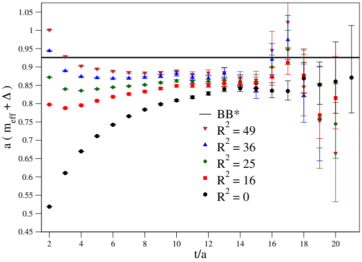
a. little resource requirements
b. benefit from mom. proj.
c. alternating sign in spec. decomp.
d. ground state reached from below!
e. GEVP is non-Hermitian
f. wall-wall correlators very noisy, but do not have problems *c*./*d*.

∘ Addressing *c*./*d*.

→ Sum the propagator over a sphere in R at the sink:

$$S^B(x,t) = \frac{1}{N}\sum_{r^2\leq R^2} S(x+r,t)$$

→ Tune R to reduce excited states



Use newly generated configurations

- ○ $n_f = 2 + 1$, $a[fm] = 0.089$
- ○ $L[fm] = 2.88 \rightarrow 4.32$
- ○ $m_\pi[MeV] = 192$

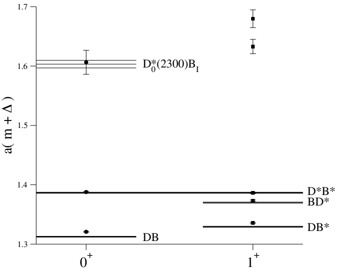
Improvement observed in $bb\bar{u}\bar{d}$, but visible shift in ground state energy!

The systematic due to the approach from below in w-l correlators can be assessed through the box construction:

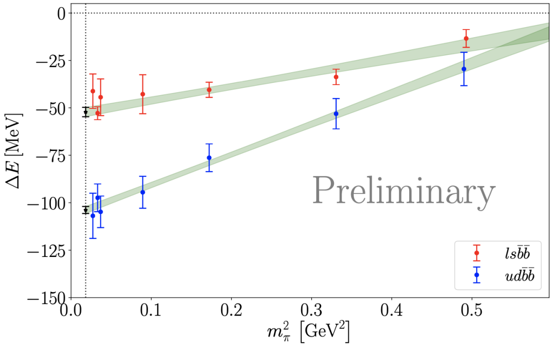
○ We find the corrections in the ground state energies to be significant.

○ Throughout (on this single lattice) we observe a reduction of binding energies around 20 – 30MeV.

$bc\bar{u}\bar{d}$ binding strongly reduced



Consolidating results - re-evaluation with box-sinks



⇝Colquhoun, AF et al. ('21)

II. Deeper dive into recent updates: Operator choice and finite volumes

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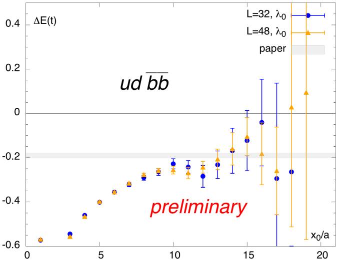
Finite volumes - first studies in $bb\bar{u}\bar{d}$

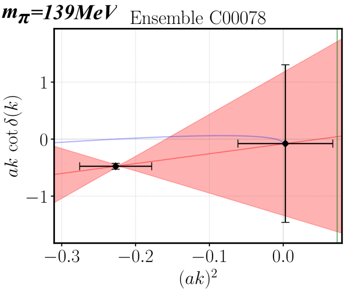
Volume scaling - varying L



New volumes for PACS-CS setup: $L = 48$, $m_\pi \simeq 164, 180, 200$MeV

*New work by Colquhoun, AF, Hudspith, Lewis, Maltman.

Lüscher formalism - varying P



First scattering analysis with DWF.

*Leskovec et al. ('19)

- ○ Good agreement left is a sign of stable scenario.

See e.g. Beane et al. ('17).

- ○ Similar signs in scattering analysis with 2 point ERE right.

Operator choices - larger bases and spatial structures

Local operators I

$$
\begin{aligned}
D(\Gamma_1,\Gamma_2) &= (\psi_a^T C\Gamma_1\phi_b)(\bar\theta_a C\Gamma_2\bar\omega_b^T),\\
E(\Gamma_1,\Gamma_2) &= (\psi_a^T C\Gamma_1\phi_b)(\bar\theta_a C\Gamma_2\bar\omega_b^T - \bar\theta_b C\Gamma_2\bar\omega_a^T),
\end{aligned}
$$

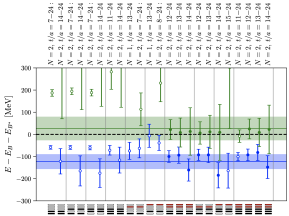
$$
M(\Gamma_1,\Gamma_2) = (\bar\theta\Gamma_1\psi)(\bar\omega\Gamma_2\phi), \qquad N(\Gamma_1,\Gamma_2) = (\bar\theta\Gamma_1\phi)(\bar\omega\Gamma_2\psi),
$$

$$
O(\Gamma_1,\Gamma_2) = (\bar\omega\Gamma_1\psi)(\bar\theta\Gamma_2\phi), \qquad P(\Gamma_1,\Gamma_2) = (\bar\omega\Gamma_1\phi)(\bar\theta\Gamma_2\psi).
$$

⇝Colquhoun, AF et al. ('21)

Non-local sinks*

*local sources, non-Hermitian GEVP, difficulty for structure interpretation



⇝Leskovec et al. ('19, '21)

Overall: Larger, varied, operator bases deployed!

Local operators II

Meson-Meson type

Diquark-Anti-diquark type

$[uC\gamma_5 d][\bar c C\gamma_i \bar b]$

$[uC\gamma_0\gamma_5 d][\bar c C\gamma_i\gamma_0 \bar b]$

$[uC\gamma_i d][\bar c C\gamma_5 \bar b]$

$\varepsilon_{ijk}[uC\gamma_j d][\bar c C\gamma_k \bar b]$

$[uC\gamma_i \bar b][dC\gamma_5 \bar c]$

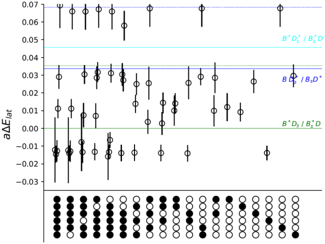
$[uC\gamma_5 \bar b][dC\gamma_i \bar c]$

$[u\bar b][dC\gamma_i\gamma_5 \bar c]$

$[u\bar c][dC\gamma_i\gamma_5 \bar b]$

$\varepsilon_{ijk}[uC\gamma_i \bar b][dC\gamma_k \bar c]$

$\bar b\bar c\ q_1 q_2\ 0(1^+)$

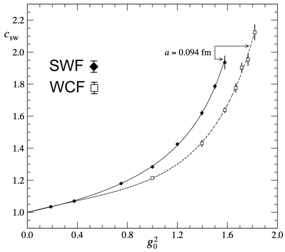


Basis dependence

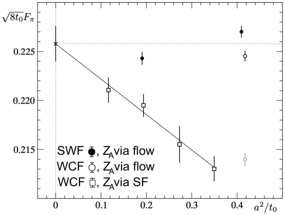
⇝Mathur et al. ('21)

First SWF calculations - relative scaling in a

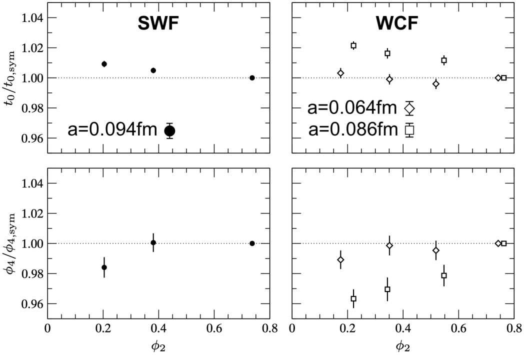
NP tuned c_{SW} (in SF)



f_π scaling in a



Chiral scaling at fixed a



In `arxiv/1911.04533` `(AF, P. Fritzsch, M. Lüscher, A. Rago)`:

- NP tuning of c_{SW} via SF, **extended to a=0.12fm**
- SWF: a=0.094fm ($m_\pi = 412, 293, 215$MeV),
  a=0.064fm ($m_\pi = 412$MeV)
  WCF: a=0.092fm ($m_\pi = 412$MeV)
- Z_A via Wilson flow
- overall, SWF show signs of some benefits ...

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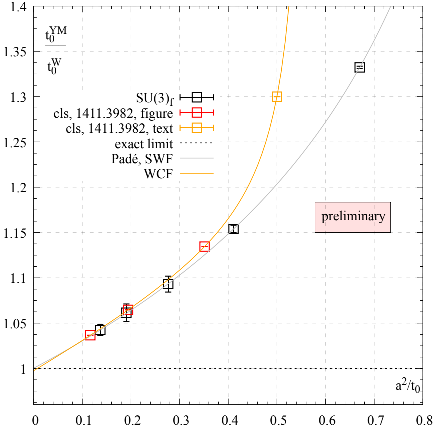
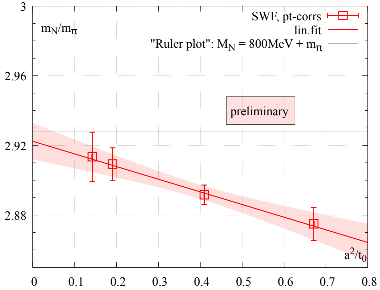
First SWF calculations - continuum extrapolations at the $SU(3)_F$ point

... we continue to see benefits at the $SU(3)_F$ point where $m_\pi = m_K = 412$MeV, or $\phi^4 = 1.115$, with TrM = fixed.

Relative effects in t_0 over a



Preliminary scaling of $m_{nucleon}/m_\pi$



- pt-pt corrs of the nucleon
- 2-state + 1-state fits to decide plateau
- "Ruler" pheno estimate, but here: $m_s \neq m_s^{\text{phys}}$