Negative Energy Density, Repulsive Gravity, and Analog Models

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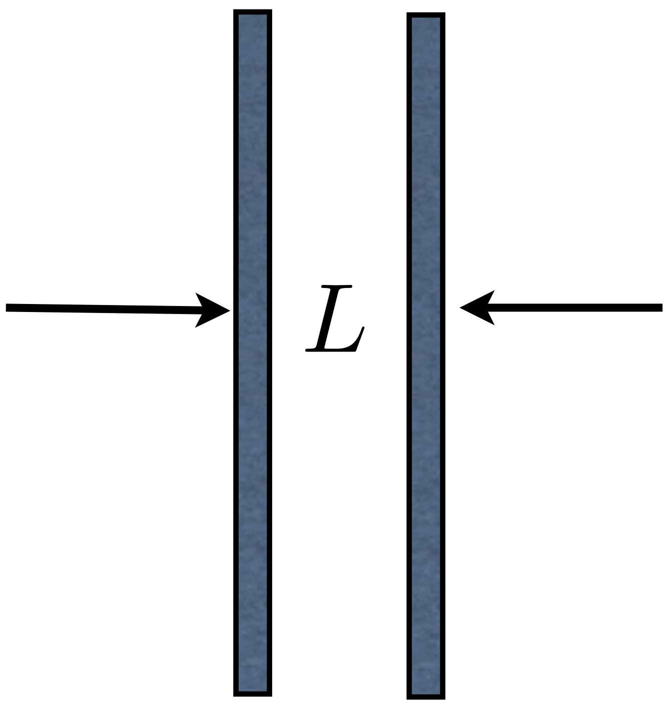
Source of the gravitational field:
stress energy tensor of matter,
including the energy density

Classical physics: energy density is positive and gravity is attractive

Quantum field theory: energy density can be negative - possibility of repulsive gravity

Negative energy as a subvacuum effect-suppression of usual vacuum fluctuations

Example: Casimir effect



$$\rho = -\frac{\hbar c \pi^2}{720 L^4}$$

Gravity determines what is zero energy.

Perfectly reflecting plates: constant density negative energy

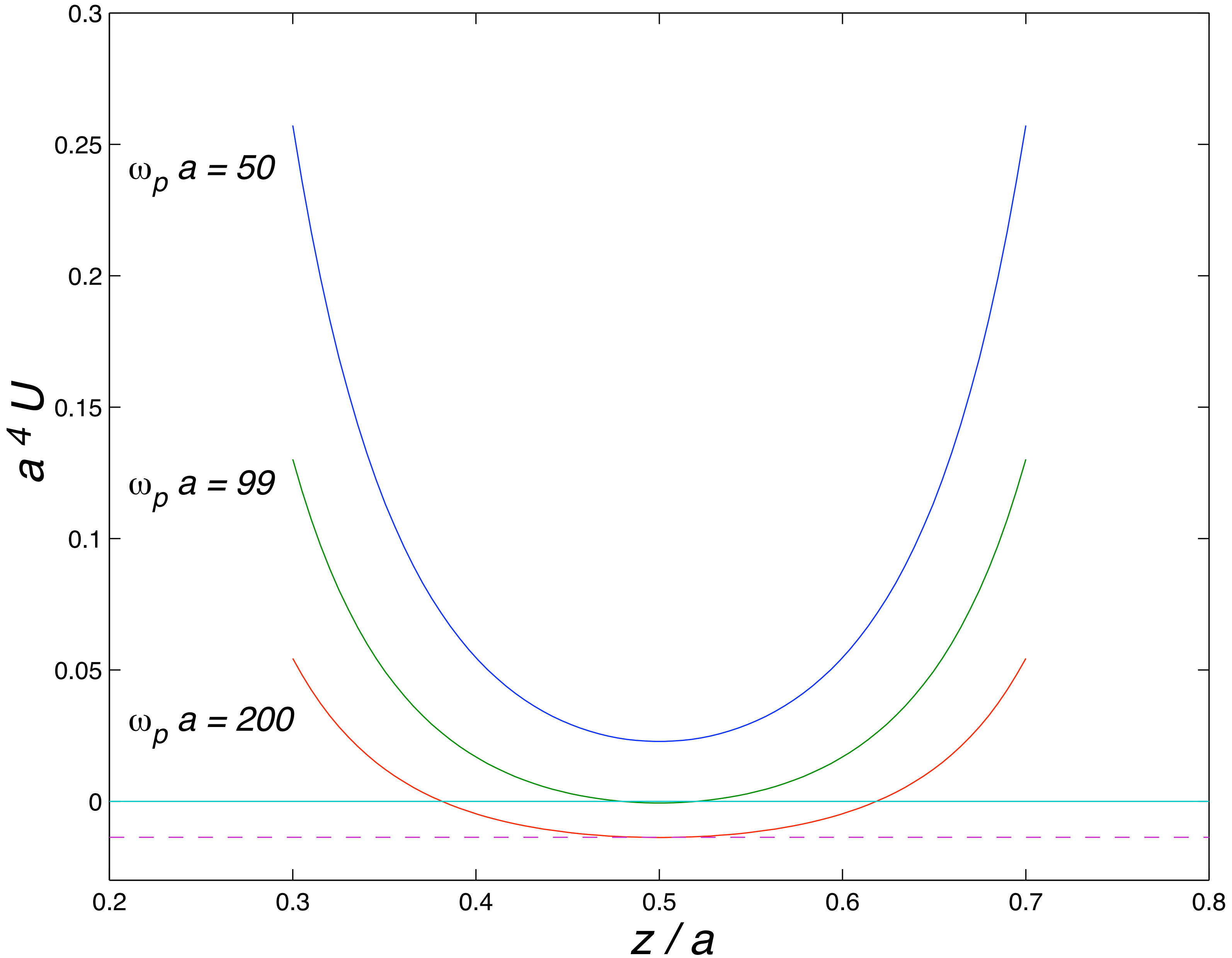
Casimir effect stress tensor: Brown & Maclay, DeWitt

$$\langle T_{\mu\nu} \rangle = \rho \, \mathrm{diag}(1, -1, -1, 3)$$

Effects of finite reflectivity - can there be a large positive self energy? (Helfer & Lang)

Classical electrostatics: attractive forces but positive energy density

Result: energy density can be negative, but requires high reflectivity or large separations (Sopova & LF)



ω_p = plasma frequency

Non-classical quantum states

An example: A superposition of the vacuum and a two photon state

$$|\psi\rangle = \frac{1}{\sqrt{1+\varepsilon^2}}\left(|0\rangle + \varepsilon|2\rangle\right) \qquad |\varepsilon| \ll 1$$

$$\rho = \langle\psi| : T_{tt} : |\psi\rangle = 2\varepsilon \, \langle 0| : T_{tt} : |2\rangle + O(\varepsilon^2)$$

can be negative at a given spacetime point

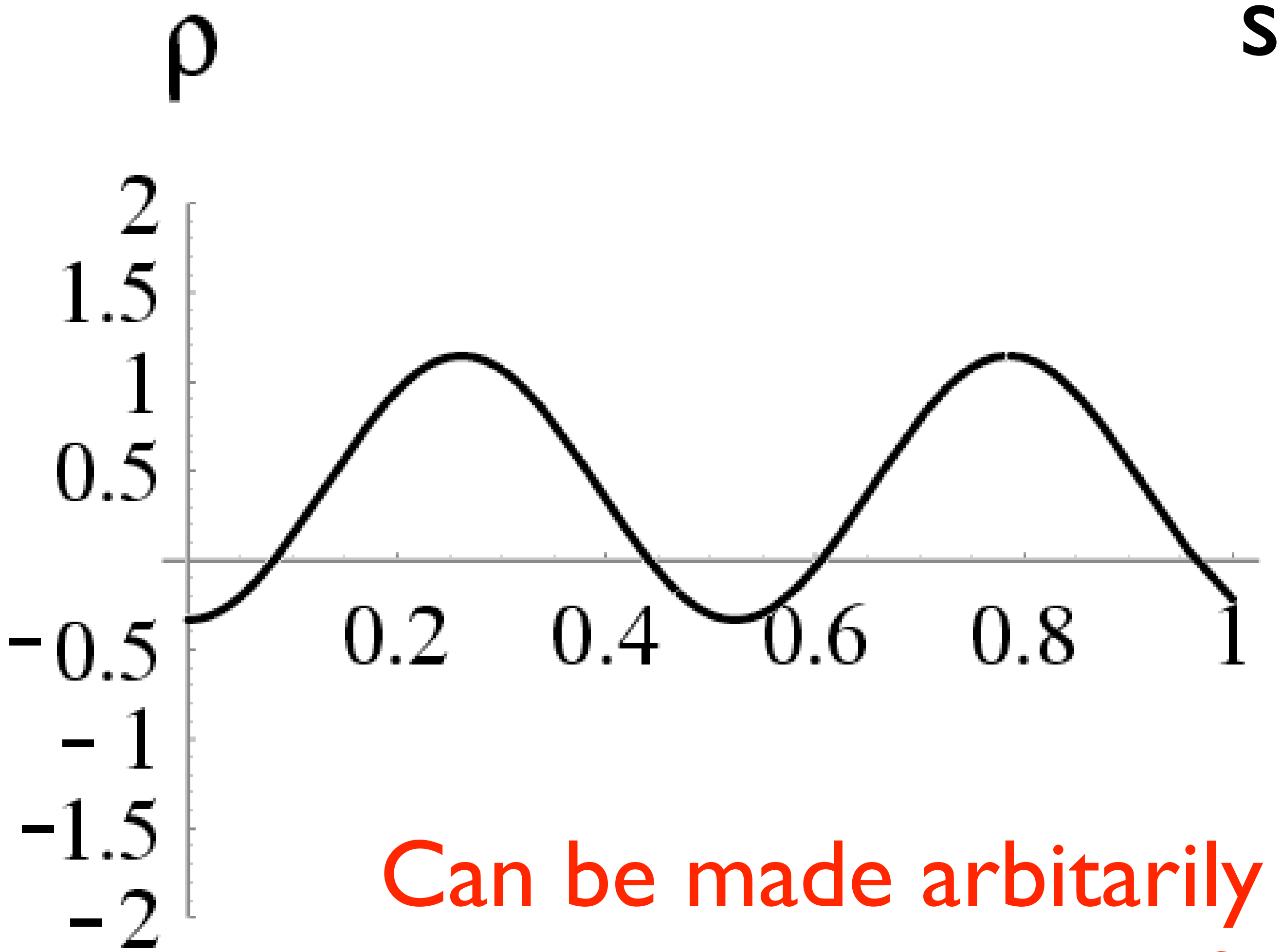
A quantum interference effect

Another example:

Negative energy density in a squeezed vacuum state (a superposition of all even particle number states):

$$\rho \propto \omega\, \sinh r [\sinh r - \cosh r\, \cos(2\omega\, t)]$$

squeeze parameter



Can be made arbitarily negative at a given point by increasing the frequency of the mode

Possible effects of negative energy:

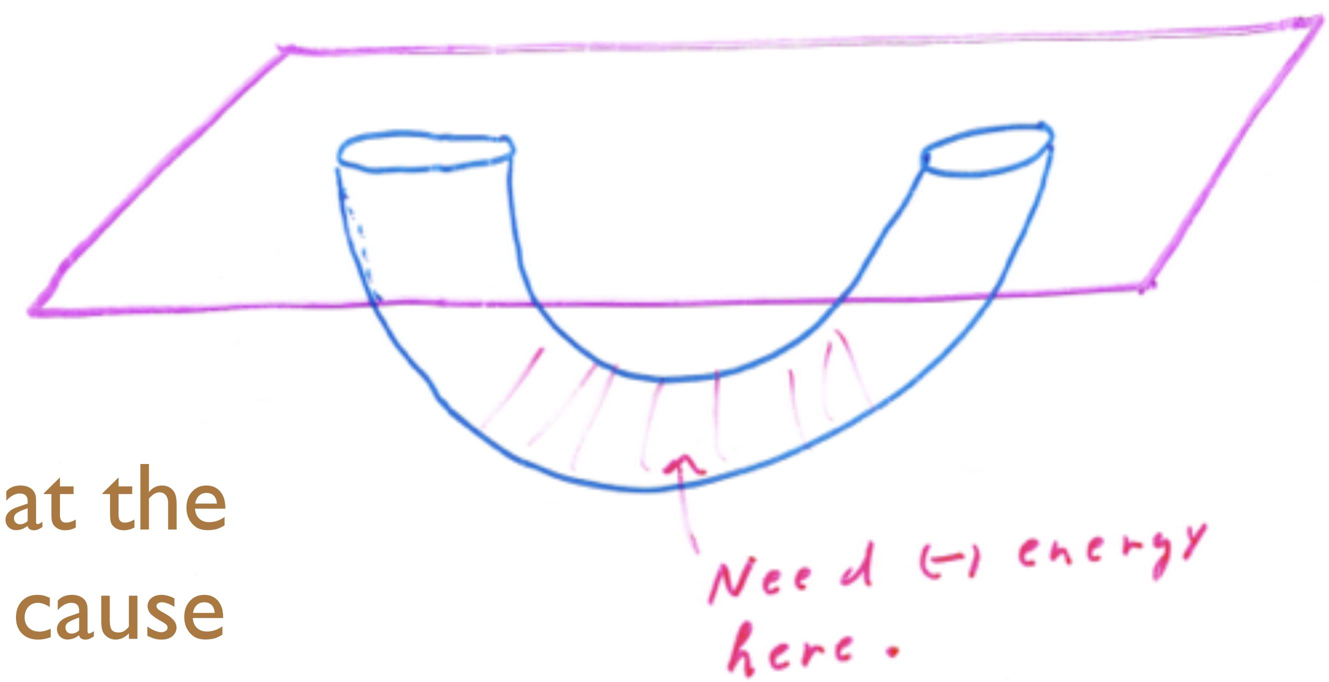
Repulsive gravity

Singularity avoidance

Violation of the weak energy conditions allows the singularity theorems to be evaded.

Traversable wormholes

Morris & Thorne



Negative energy is needed at the throat of the wormhole to cause light rays to defocus.

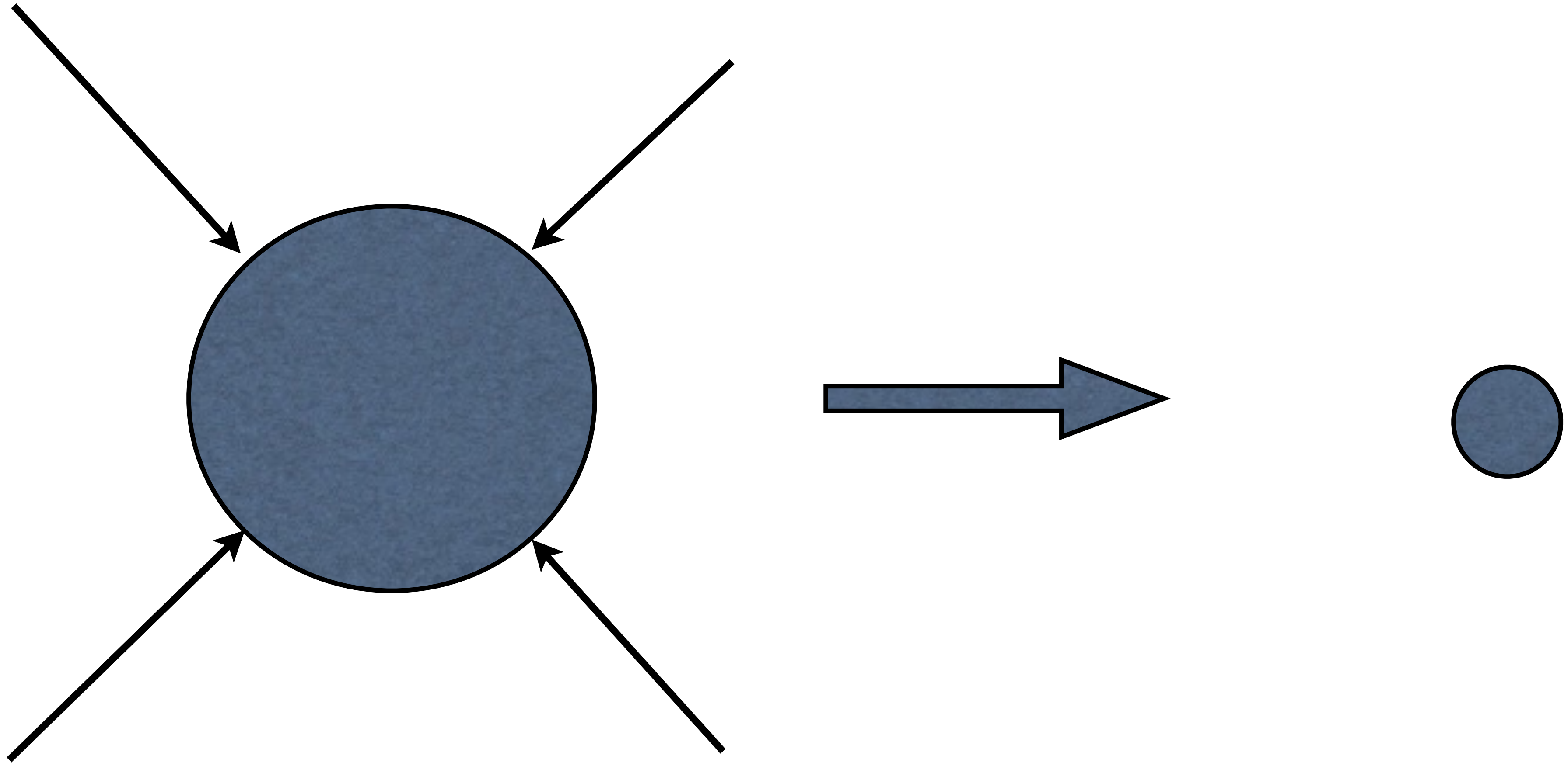
Faster than light travel - Alcubierre warp drive

Time Machines

Can modify a traversable wormhole or warp drive to travel backwards in time

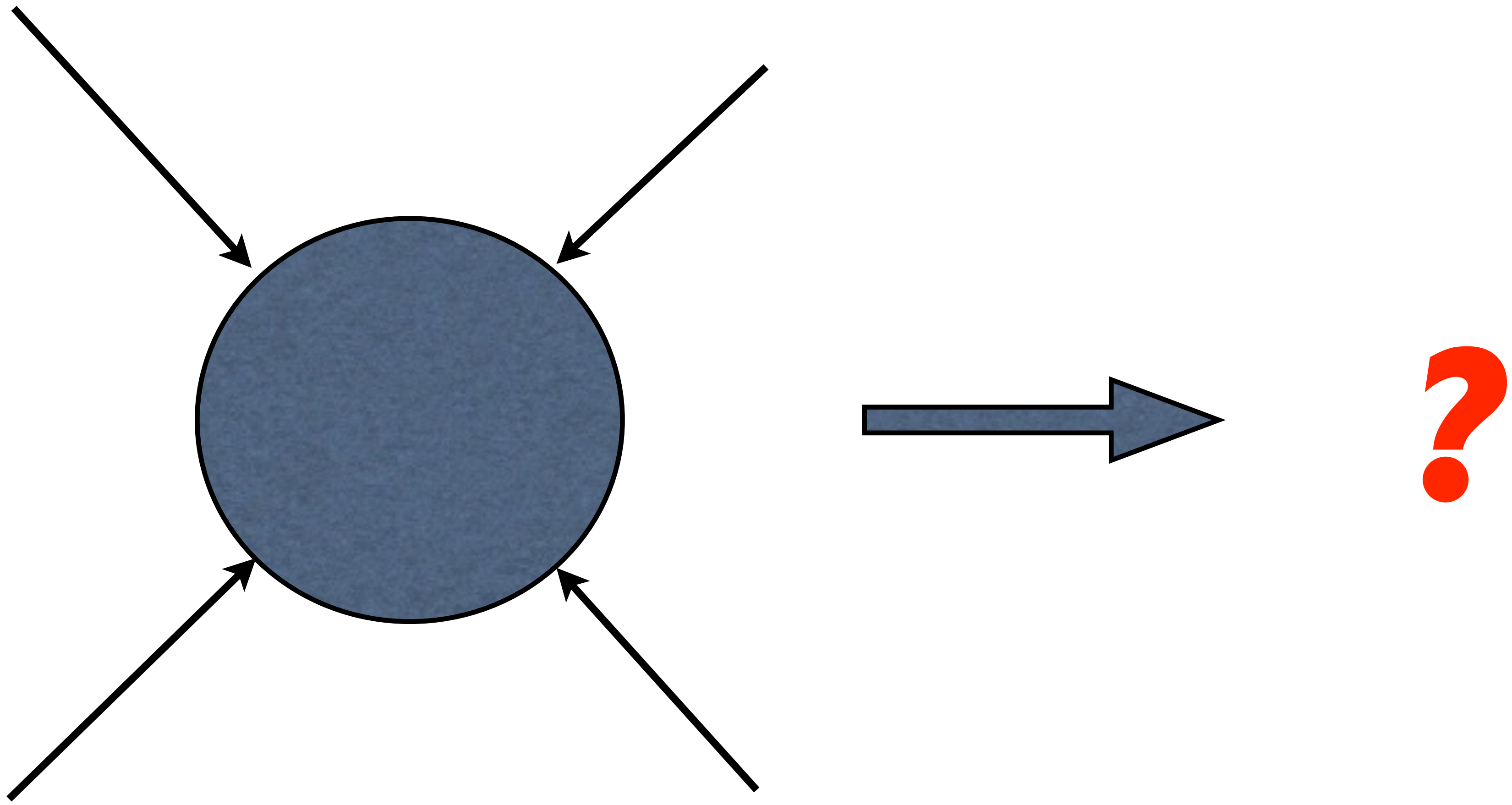
Hawking's Theorem: Negative energy is essential to build a time machine.

Violations of the second law of thermodynamics



Shine negative energy on a black hole and reduce its horizon area.

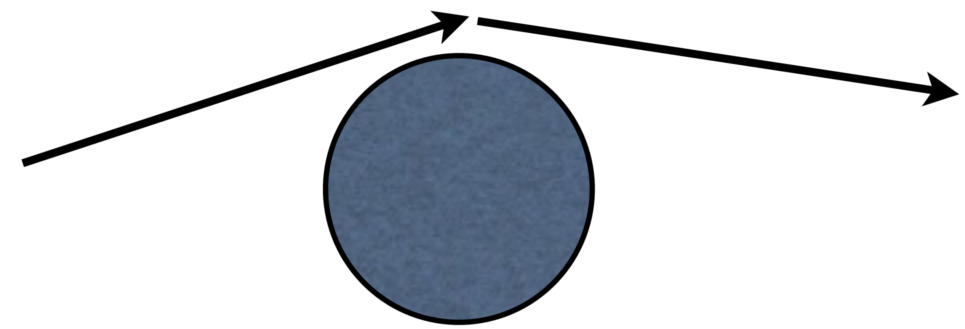
Violations of cosmic censorship: shine negative energy on an extreme black hole



Negative Energy and Superluminal Light Propagation

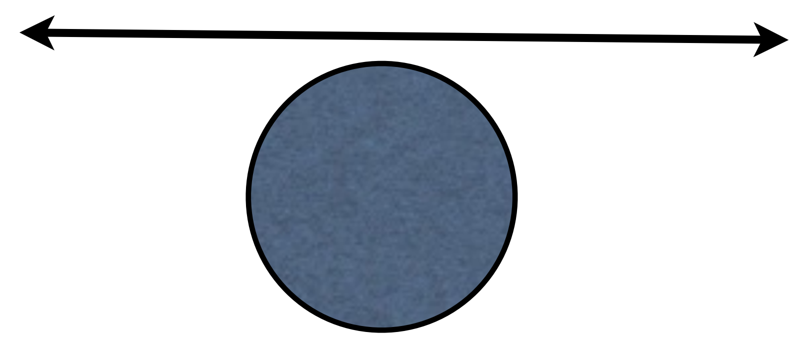
Effects on light rays in classical gravity:

Light deflection



Shapiro time delay

light slowed by the sun's gravitational field



Negative energy can lead to "Shapiro time advance", light travels faster than it would have in flat spacetime.

Local light speed is unchanged, but positive energy density increases the optical path length and negative energy density decreases it.

Constraints on negative energy in Minkowski spacetime (without boundaries):

1) Total energy is non-negative - positivity of the Hamiltonian

2) Averaged weak energy condition -

$$\int \langle T_{\mu\nu} \rangle \, u^\mu \, u^\nu \; d\tau \geq 0$$

u^μ= four velocity of an inertial observer

Neither of these conditions is strong enough to avoid negative energy problems; the positive energy could be very far from the negative energy.

3) Quantum inequalities -

$$\int \langle T_{\mu\nu} \rangle \, u^\mu \, u^\nu \, g(\tau, \tau_0) \, d\tau \geq -\frac{C}{\tau_0^d}$$

$g(\tau, \tau_0)$ = sampling function

C = positive constant

τ_0 = sampling time

d = spacetime dimension

Some Minkowski space examples with a Lorentzian sampling function:

Two dimensions (1+1)

$$\frac{\tau_0}{\pi}\int_{-\infty}^{\infty} \frac{\langle T_{\mu\nu}\rangle\, u^\mu\, u^\nu}{\tau^2+\tau_0^2}\, d\tau \geq -\frac{1}{48\pi\, \tau_0^2}$$

(Flanagan) Optimum bound

Four dimensions (3+1)

$$\frac{\tau_0}{\pi}\int_{-\infty}^{\infty} \frac{\langle T_{\mu\nu}\rangle\, u^\mu\, u^\nu}{\tau^2+\tau_0^2}\, d\tau \geq -\frac{3}{32\pi^2\, \tau_0^4}$$

(LF&Roman, Fewster&Eveson)

In the limit that $\tau_0 \to \infty$, we recover the averaged weak energy condition as a special case.

$$\int \langle T_{\mu\nu} \rangle \, u^\mu \, u^\nu \; d\tau \geq 0$$

A negative energy density cannot last longer than about $\Delta t = |\rho|^{-1/4}$

(4D)

Physical implication:

The amount of negative energy that can be absorbed by a system in time t is less than $1/t$

In 4D, need the collecting area $< 1/t^2$

This is less than the quantum energy uncertainty on this time scale.

Maximally negative energy density as the lowest eigenvalue of an averaged stress tensor operator

Fewster & Teo, Dawson

Let $$\bar{T} = \int T_{\mu\nu} u^\mu u^\nu \, g(\tau, \tau_0) \, d\tau$$

The lowest eigenvalue of $\bar{T}$ is negative and is the optimum quantum inequality bound. The corresponding eigenstate is a squeezed vacuum state.

In many cases, the mean number of particles in the state of maximally negative averaged energy density is of order one, so the vacuum + two particle state is a good approximation.

Korolov & LF

Estimate of the typical magnitudes of the gravitational effects of negative energy density:

$$\delta T_{\mu\nu} \sim \frac{1}{\tau^4} \qquad \tau = \text{characteristic time scale}$$

Ricci tensor

$$\delta R_{\mu\nu} \sim \frac{\ell_P^2}{\tau^4} \qquad \ell_P = \text{Planck length}$$

metric perturbation

$$\delta h \sim \left(\frac{\ell_P}{\tau}\right)^2$$

Very small on macroscopic scales

Negative energy or related subvacuum effects might still be observable.

Some proposed laboratory experiments:

1) Effects on magnetic moments of spin systems

LF, Grove & Ottewill

Basic idea: vacuum fluctuations cause de-alignment of spins and their suppression can cause a temporary re-alignment, or increase in magnetization.

Very small effect.

2) Effects on atomic decay rates   LF & Roman

Basic idea: vacuum fluctuations are essential or radiative decay of excite states, and their suppression decreases the decay rate

3) Effects on the speed of light in a nonlinear material   DeLorenci & LF

Basic idea: A background electric field in a nonlinear material can change the speed of a probe pulse, analogous to the effect of a gravitational field.

Consider a situation where both fields are polarized in the z-direction, but propagating in the x-direction.

Electric field: $$E_i = \delta_{iz}\, E = \delta_{iz}\, E(t,x)$$

Polarization: $$P_z = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \cdots$$

linear term

non-linear terms

where $$\chi^{(1)} = \chi^{(1)}_{zz}\,, \quad \chi^{(2)} = \chi^{(2)}_{zzz}\,, \quad \chi^{(3)} = \chi^{(3)}_{zzzz}$$

Material can be anisotropic, but only z-components of the susceptibility tensors contribute here.

Let $E = E_0 + E_1$ ← probe field

lower frequency background field ↗

Assume that the material is approximately dispersionless over the frequency range defined by these two frequencies, and $|E_1| < |E_0|$

Also consider the case where $\chi_2 = 0$, but $\chi_3 > 0$, the Kerr effect.

Equation for $E_1(t,x)$

$$\frac{\partial^2 E_1}{\partial x^2} - \frac{1}{v^2}(1+3\epsilon_2)\frac{\partial^2 E_1}{\partial t^2} = 0$$

where $$v = \frac{c}{\sqrt{1+\chi^{(1)}}}$$ Wave speed in the linear approximation

$$\epsilon_2 = \frac{\chi^{(3)}}{1+\chi^{(1)}} E_0^2(t,x)$$

Wave speed in the presence of the background field $$u = \frac{v}{1+3\epsilon_2} \approx v\left(1 - \frac{3}{2}\epsilon_2\right)$$

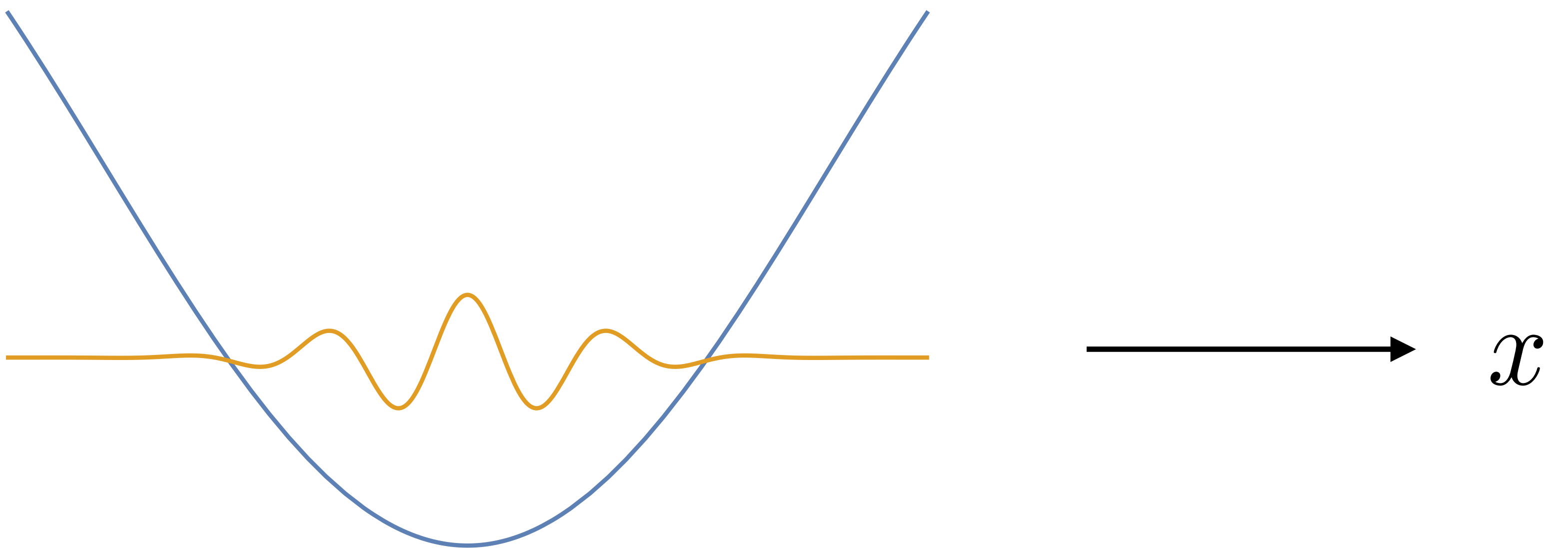
Let the background field represent a non-classical photon state and set $E_0^2 \rightarrow \langle E_0^2(x,t) \rangle$

Regions where $\langle E_0^2(x,t) \rangle < 0$ lead to "superluminal" pulses in that $u > v$

Here the subvacuum effect is that the pulse travels faster than it would in the absence of the background field.

Given that $\langle E_0^2(x,t) \rangle < 0$ is a transient effect, can we have an effect which is large enough to observe?

One possibility:
Let the probe pulse propagate with the $\langle E_0^2(x,t)\rangle < 0$ region.



The modified speed of the pulse leads to a phase shift $\Delta\varphi \propto \langle E_0^2\rangle\,\Delta x$ compared to a pulse in a $\langle E_0^2\rangle = 0$ region.

$\Delta x =$ travel distance

Estimate of the magnitude of the effect:

Let $\chi^{(3)} = 3 \times 10^{-19} \mathrm{m}^2 \mathrm{V}^{-2}$ (e.g., Si or Ge)

For the background field in a squeezed vacuum state with $\lambda = 1 \mu\mathrm{m}$

$$|\langle {E_0}^2 \rangle| \approx 1 (\mu\mathrm{m})^{-4} \approx (6 \times 10^4 \mathrm{V m}^{-1})^2$$

$$\Delta\varphi \approx \left(\frac{\chi^{(3)} \langle {E_0}^2 \rangle}{10^{-9}} \right) \left(\frac{z}{10\mathrm{m}} \right) \left(\frac{0.1 \mu\mathrm{m}}{\lambda_p} \right)$$

λ_p ↑ characteristic wavelength of the probe pulse

Can this be observed?

Summary

1) Quantum field theory allows states in which subvacuum effects occur. Examples include negative energy density. These are quantum interference effects.

2) Negative energy density leads to repulsive gravity effects, including superluminal propagation.

3) Negative energy density and other subvacuum effects are limited by quantum inequalities.

4) These effects may still be large enough to be observable, especially in analog models.

5) One example is a model of superluminal light propagation in a nonlinear dielectric.