FLAVOUR PHYSICS AT THE NEUTRINO OSCILLATION FRONTIER

arXiv:1801.05656[hep]

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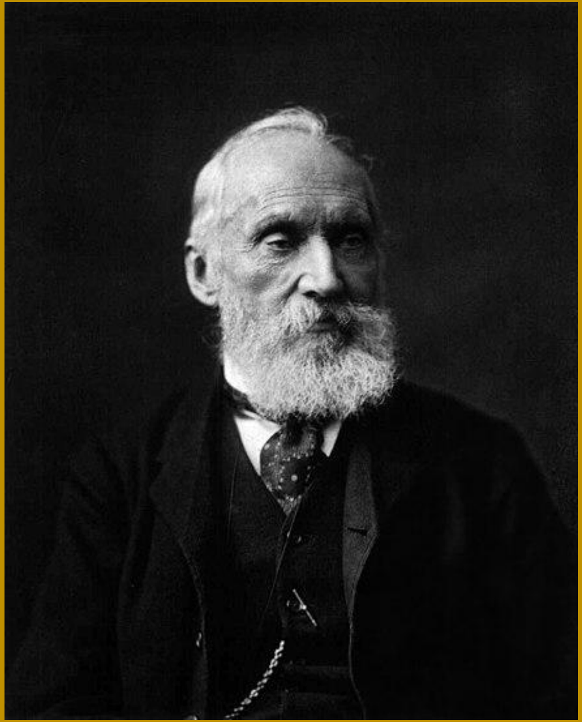
4th July 2018
@ Sinica

The beauty and clearness of dynamics theory, which asserts heat and light to be modes of motion, is at present obscured by two clouds.

-- Lord Kelvin 27th April 1900



YESTERDAY AND TODAY

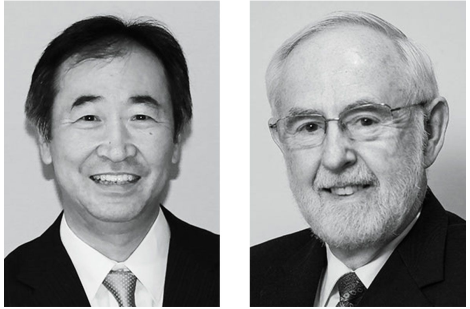
1. Beginning of 20th century, two clouds in Physics
   -- black body radiation & Michelson interference
   → quantum mechanics & relativity theory
   ** **PLUS**, the concept of symmetry is raised up.

2. Questions in today's physics (begging of 21th century)
   -- the neutrino oscillation (the massive neutrino)
   -- dark matter
   -- dark energy
   -- quantization of gravity… etc.

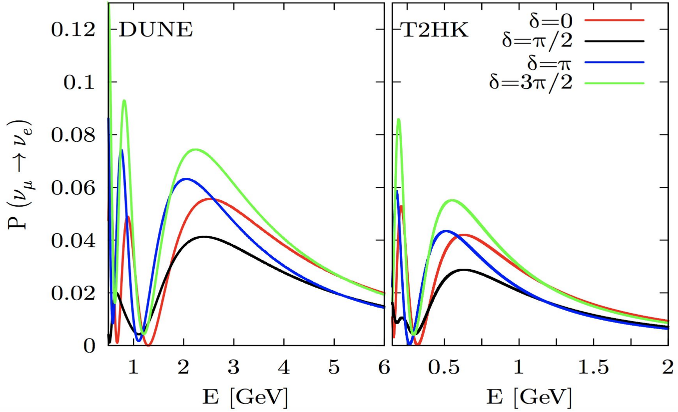
NEW LANGUAGE?
NEW CONCEPT?

NEUTRINO OSCILLATIONS

The neutrino oscillation, which is the phenomenon that some fraction of neutrinos change flavours in the quantity L(distance)/E(energy), have been confirmed since 1998. (Nobel Prize in physics, 2015)



Takaaki Kajita and Arthur B. McDonald



Standard neutrino oscillations are described by a CP phase, three rotations and two mass-squared differences(MSDs).

The Neutrino oscillation

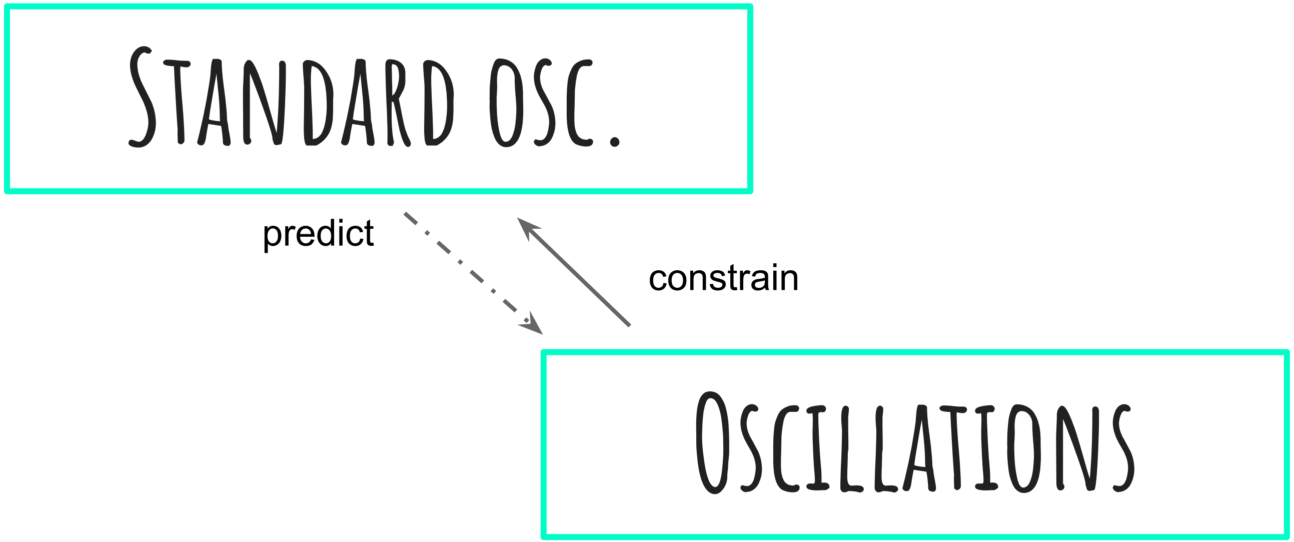
$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

@source/detector U_{PMNS} In Vacuum

$$U_{\mathrm{PMNS}} = U_{23}U_{13}U_{12}P,$$

$$= \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}e^{i\alpha_1} & 0 & 0\\ 0 & e^{i\alpha_2} & 0\\ 0 & 0 & 1\end{pmatrix}$$

NOT contribute to osc.


Standard osc.
predict
constrain
Oscillations

The Neutrino oscillation

S. F. King. 2015

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

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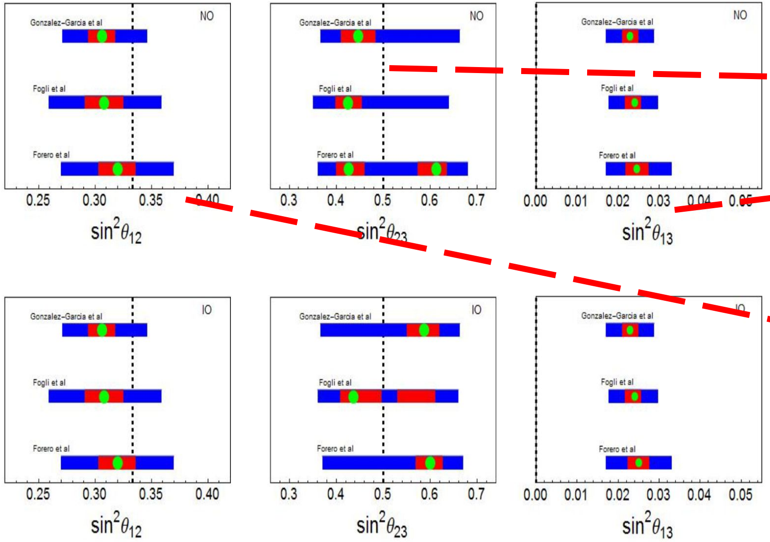
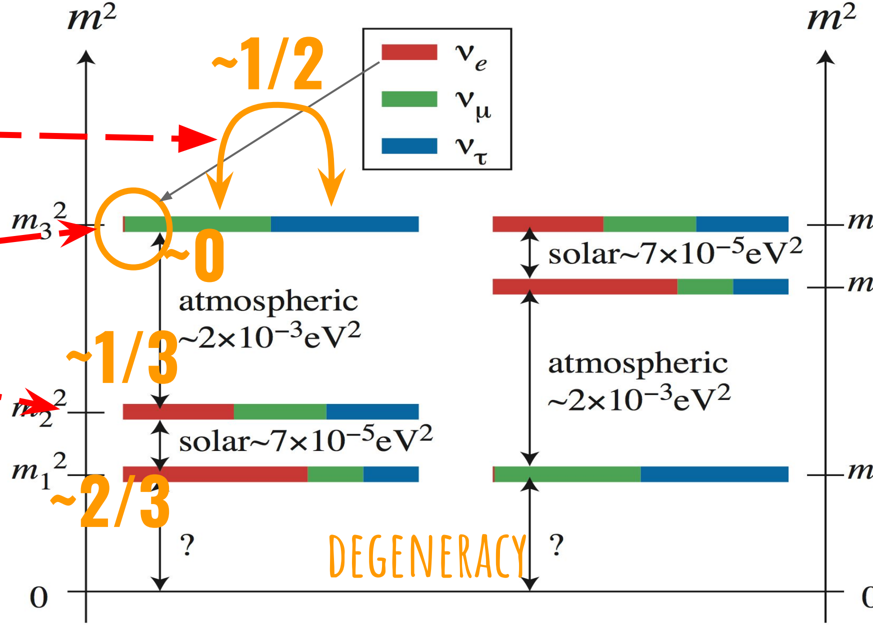
NOT CONTRIBUTE TO OSC.





The flavour symmetry at Leptonic sector

- The flavour symmetry **at the higher energy** (e.g. S_4, A_4, etc) explains neutrino mixing and simplifies the standard language for neutrino oscillations.
  e.g. Tri-bimaximal (TBM), which was a popular model

$$U_{\rm TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

based on A_4 or S_4

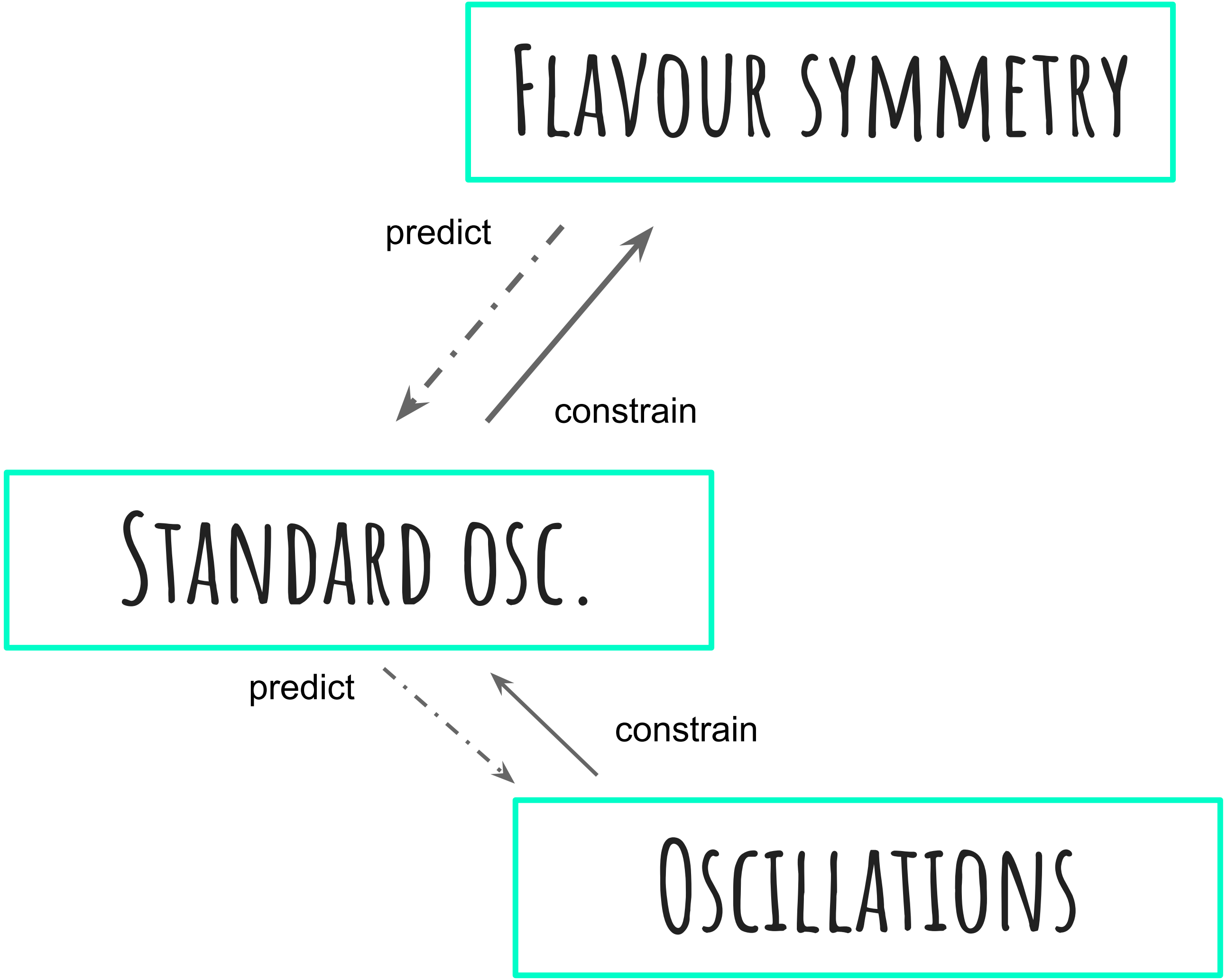
- TBM does not fit with the current θ_{13} measurement. NEED CORRECTIONs (s, a, r), by a flavon breaks A_4.etc….

Different models predict different relations between/among s,a,r ---> these relations are called ‘sum rules’.

very precise. $\sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s),$

NOT precise. $\sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a),$

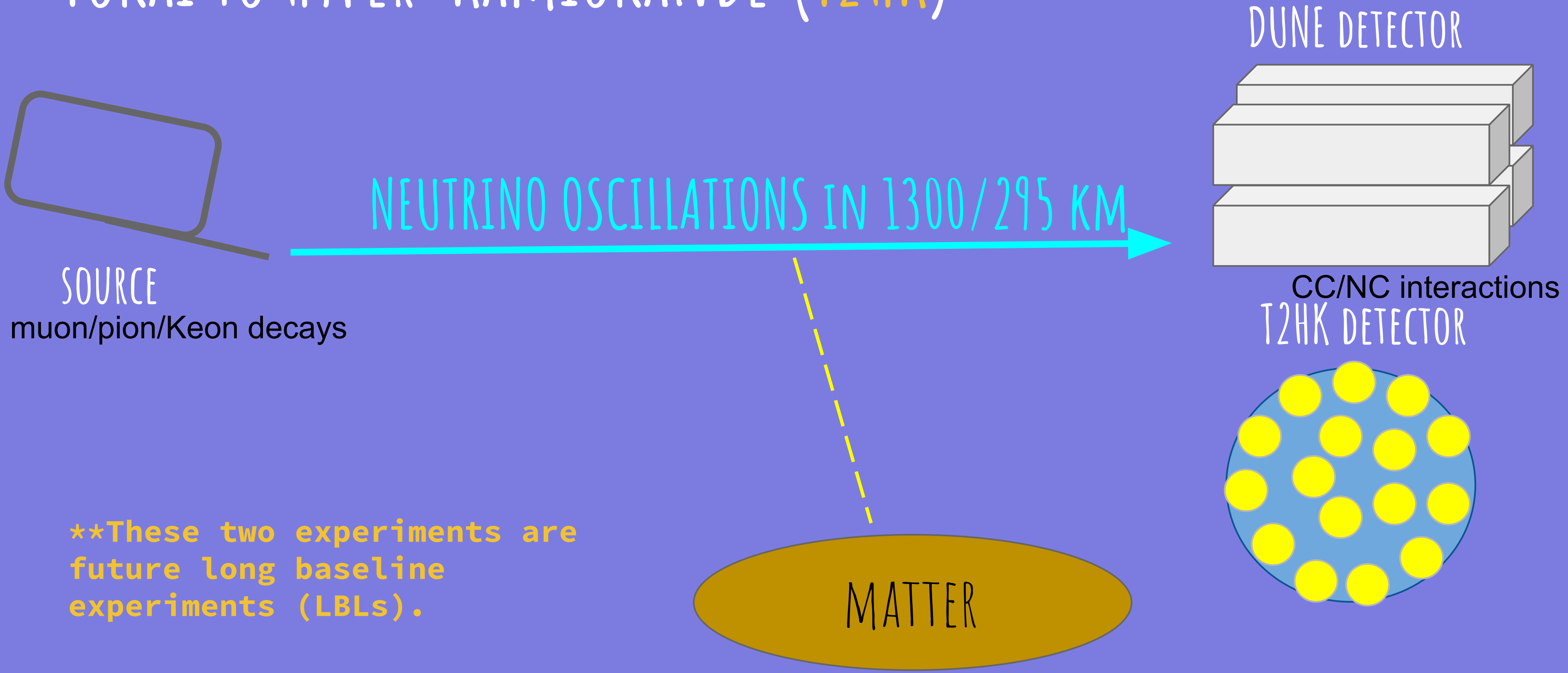
VERY precise. $\sin\theta_{13} = \frac{r}{\sqrt{2}},$



FLAVOUR SYMMETRY
predict
constrain
STANDARD OSC.
predict
constrain
OSCILLATIONS

Deep Underground Neutrino Experiment

Tokai to Hyper-Kamiokande (T2HK)



**These two experiments are future long baseline experiments (LBLs).

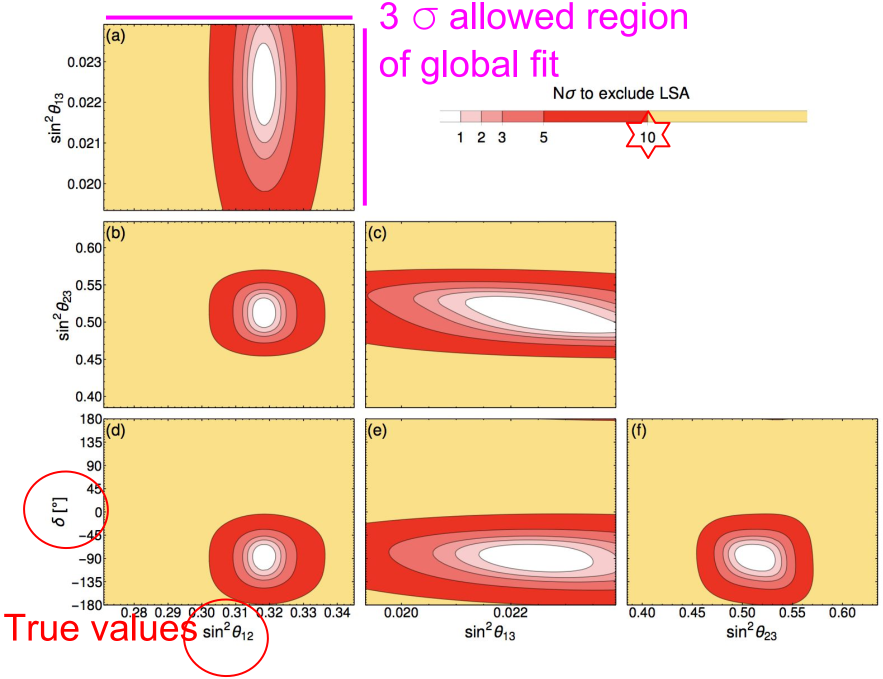
One flavour model in DUNE and T2HK

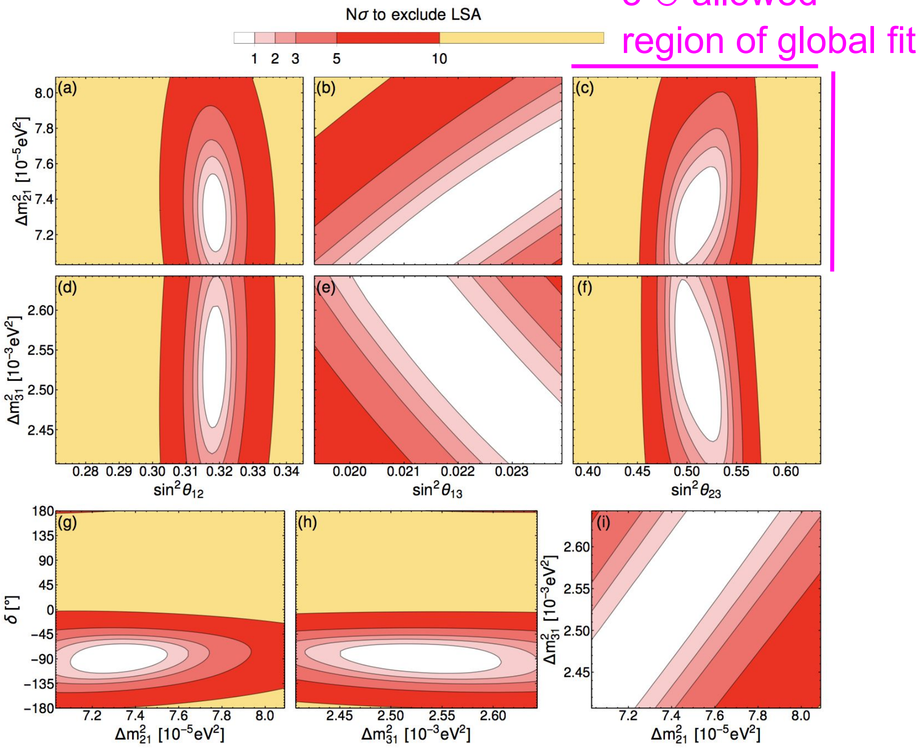
P. Ballet, et al. 2016

$$m^{\nu}_{\rm LSA} = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}$$

One of the most predictive models -- CSD3

ONLY three parameters. ---> Littlest Seesaw





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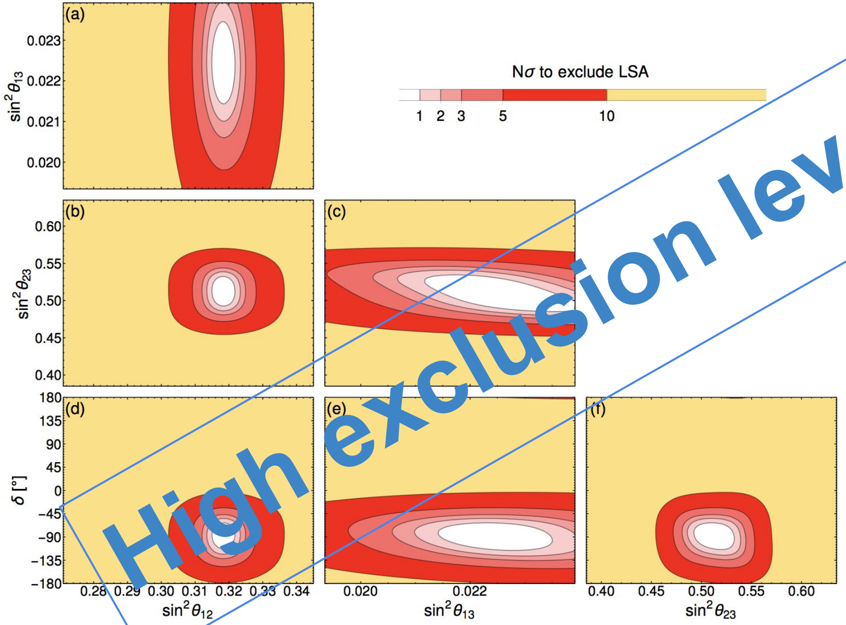
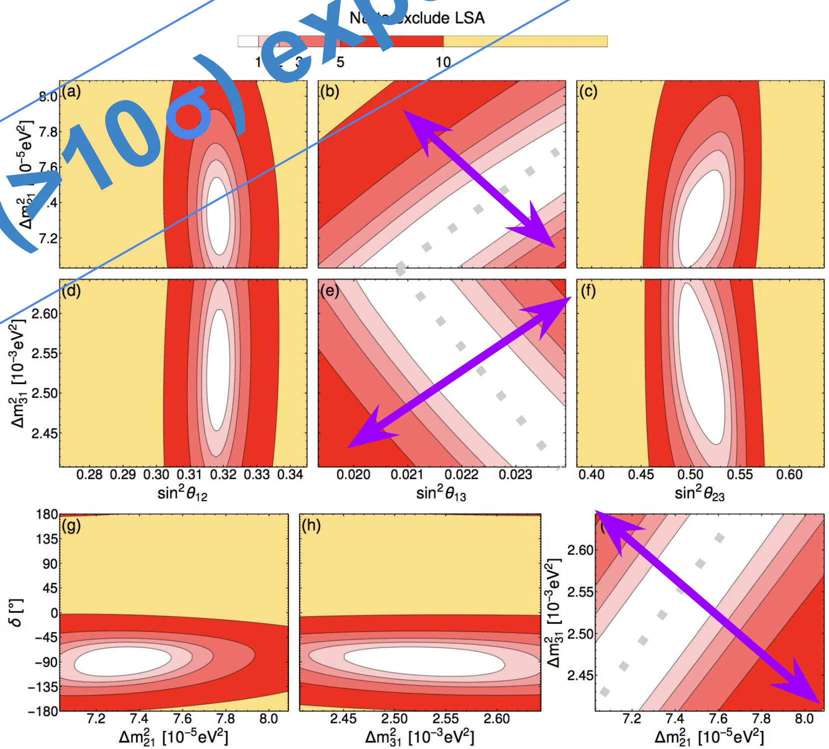




High exclusion level (>10σ) expected!!

BUT

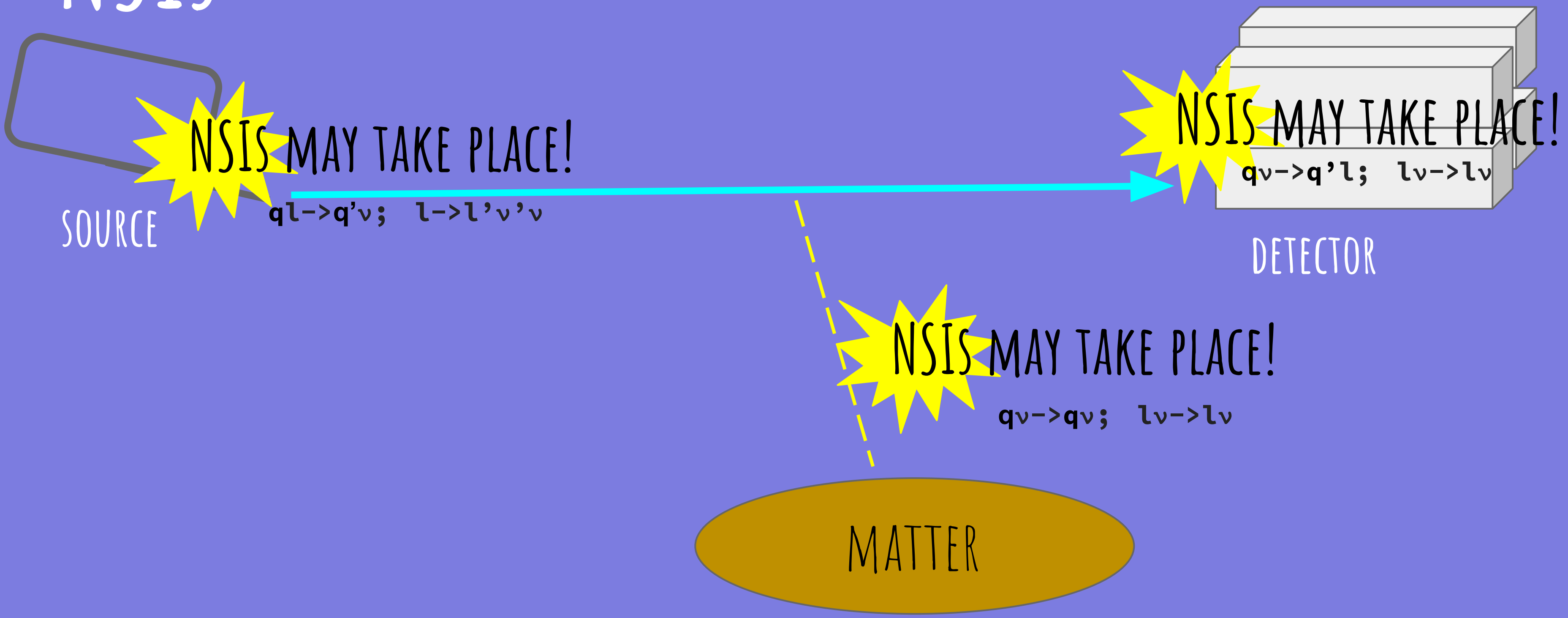
Even though we can do well in testing flavour models by measuring oscillation parameters, this does not lead us to say this is the final story.

I'VE BEEN THINKING...

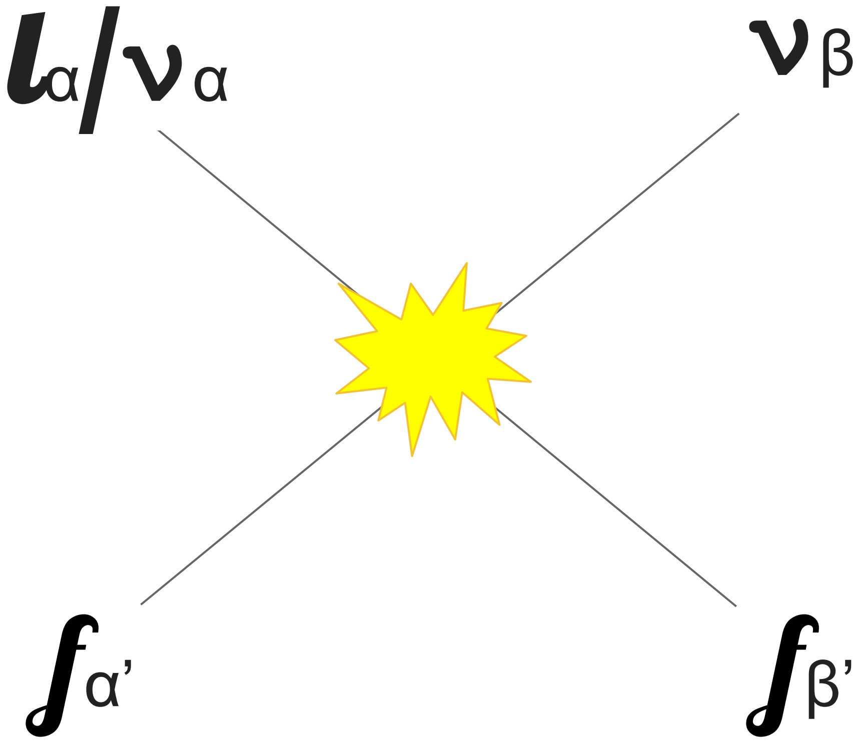
- Can we test the symmetry of flavour at the extreme (very high energy) in the attainable environment (upcoming experiments)?
- We cannot guarantee that any symmetry can be observed in the charged lepton sector. (charge assignment)
- We are entering the era of precision measurement for neutrino physics. (Superbeam, Neutrino Factory, LarTPC, etc….)

Nonstandard Interactions (NSIs), which are theoretically **flavour-dependent** interactions of neutrinos beyond the standard model, can be a new window.

NSIs


NSIs MAY TAKE PLACE!
ql->q'ν; l->l'ν'ν
SOURCE
NSIs MAY TAKE PLACE!
qν->q'l; lν->lν
DETECTOR
NSIs MAY TAKE PLACE!
qν->qν; lν->lν
MATTER

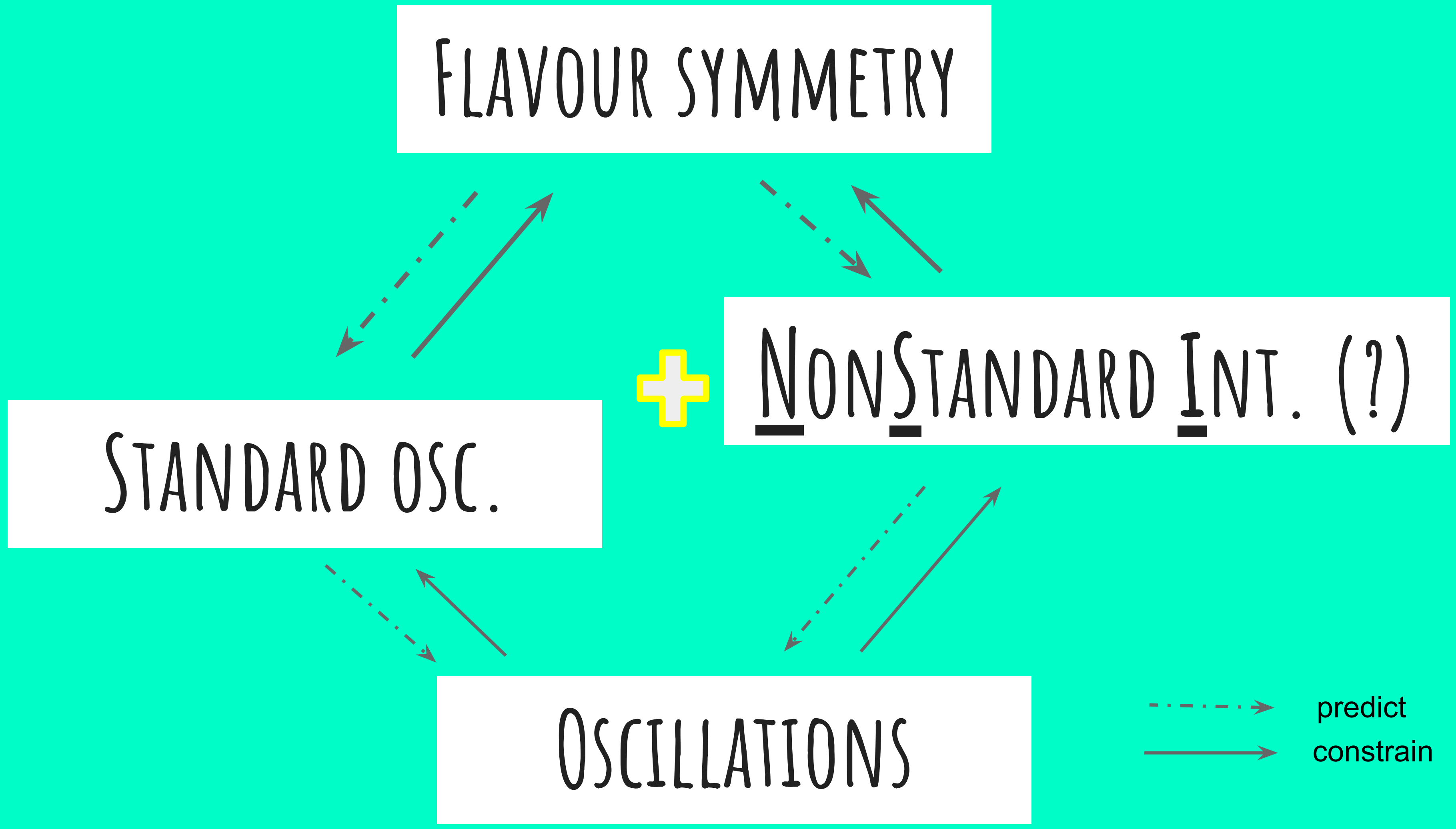
NSIs

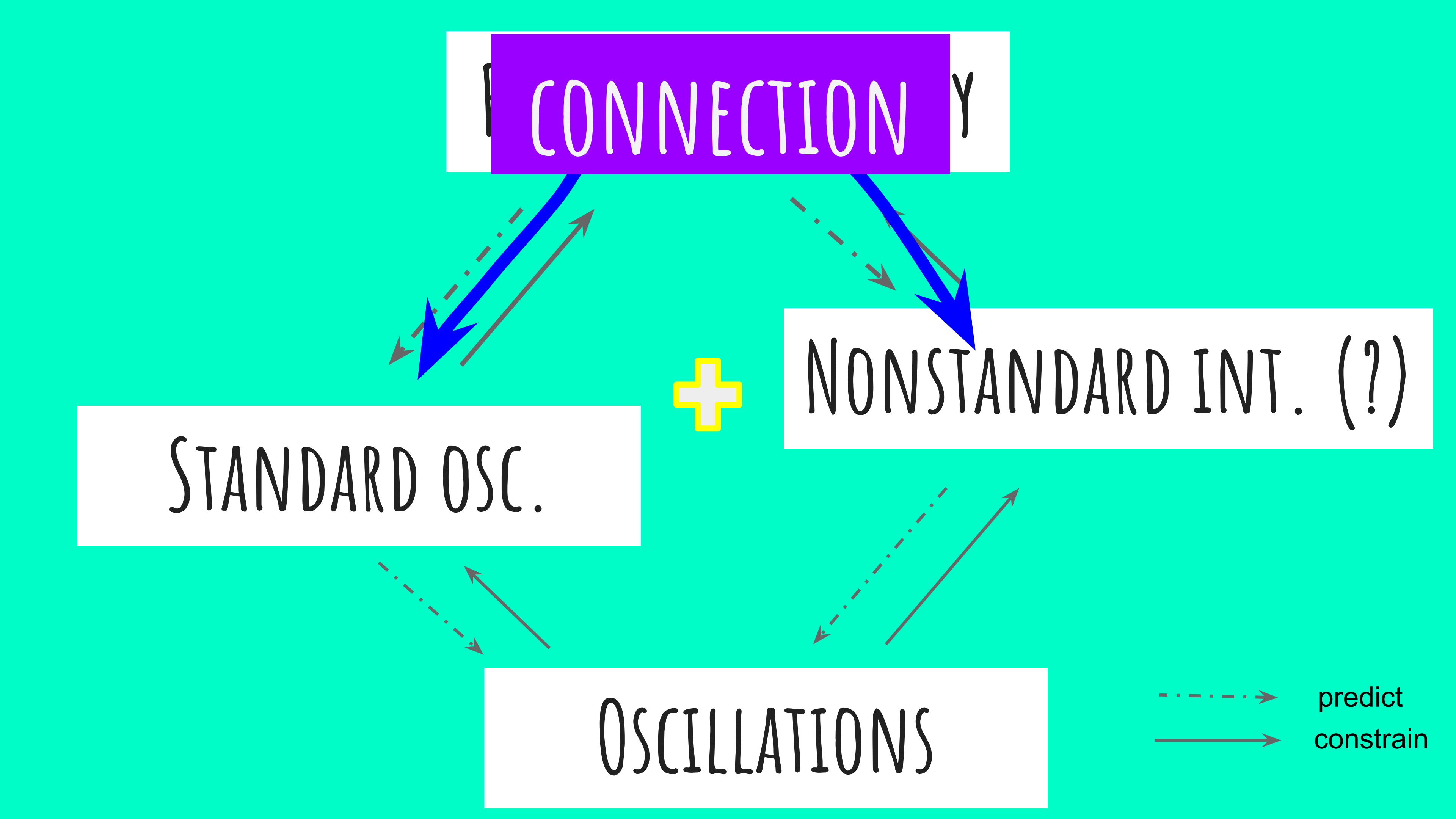


We will focus on those in matter, and describe them in a matrix in the Hamiltonian governing neutrino oscillations:

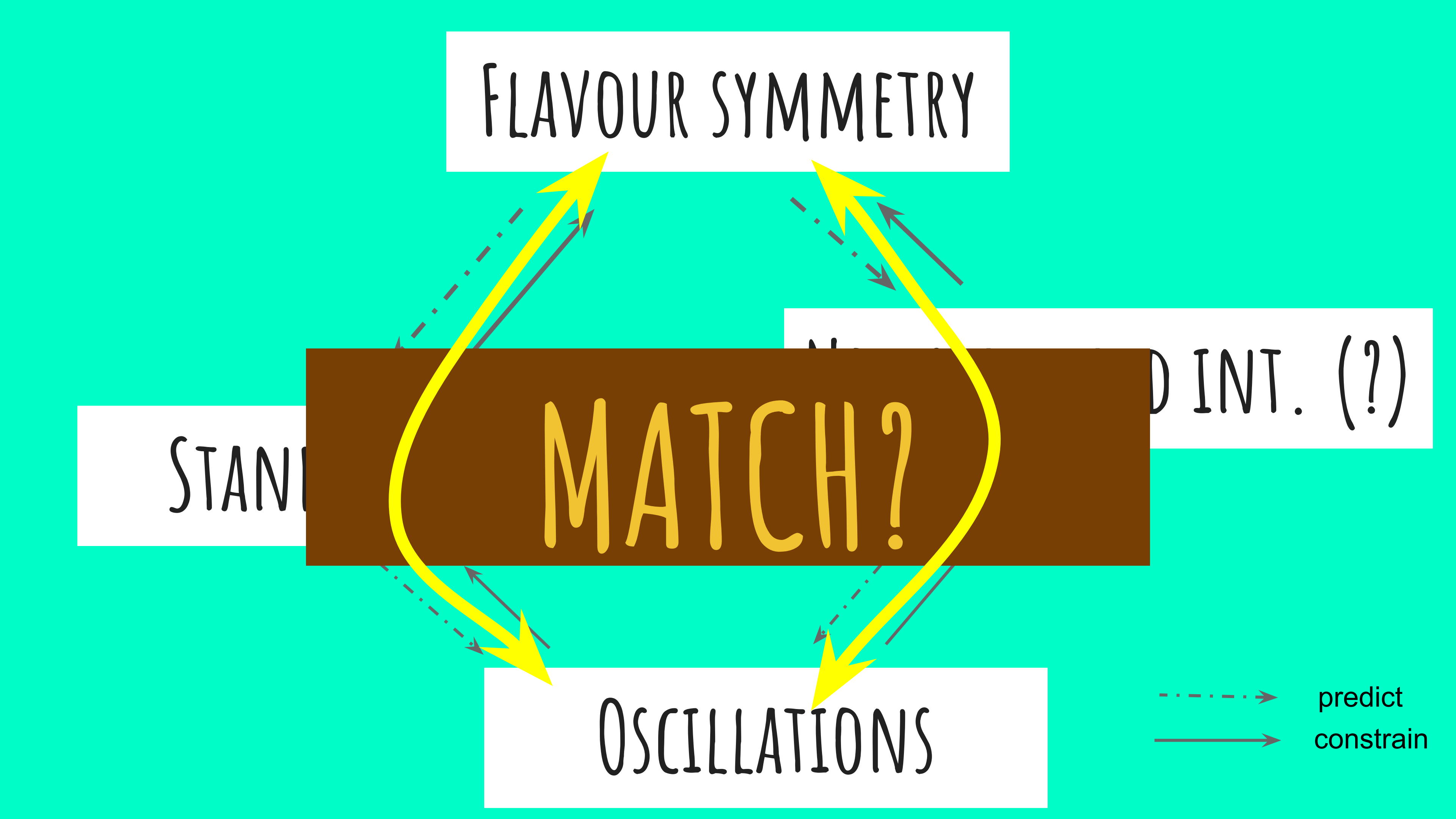
$$H = \frac{1}{2E}\left\{ U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} U^\dagger + A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + A \begin{pmatrix} \epsilon^{\rm m}_{ee} & \epsilon^{\rm m}_{e\mu} & \epsilon^{\rm m}_{e\tau} \\ \epsilon^{\rm m}_{\mu e} & \epsilon^{\rm m}_{\mu\mu} & \epsilon^{\rm m}_{\mu\tau} \\ \epsilon^{\rm m}_{\tau e} & \epsilon^{\rm m}_{\tau\mu} & \epsilon^{\rm m}_{\tau\tau} \end{pmatrix} \right\}$$

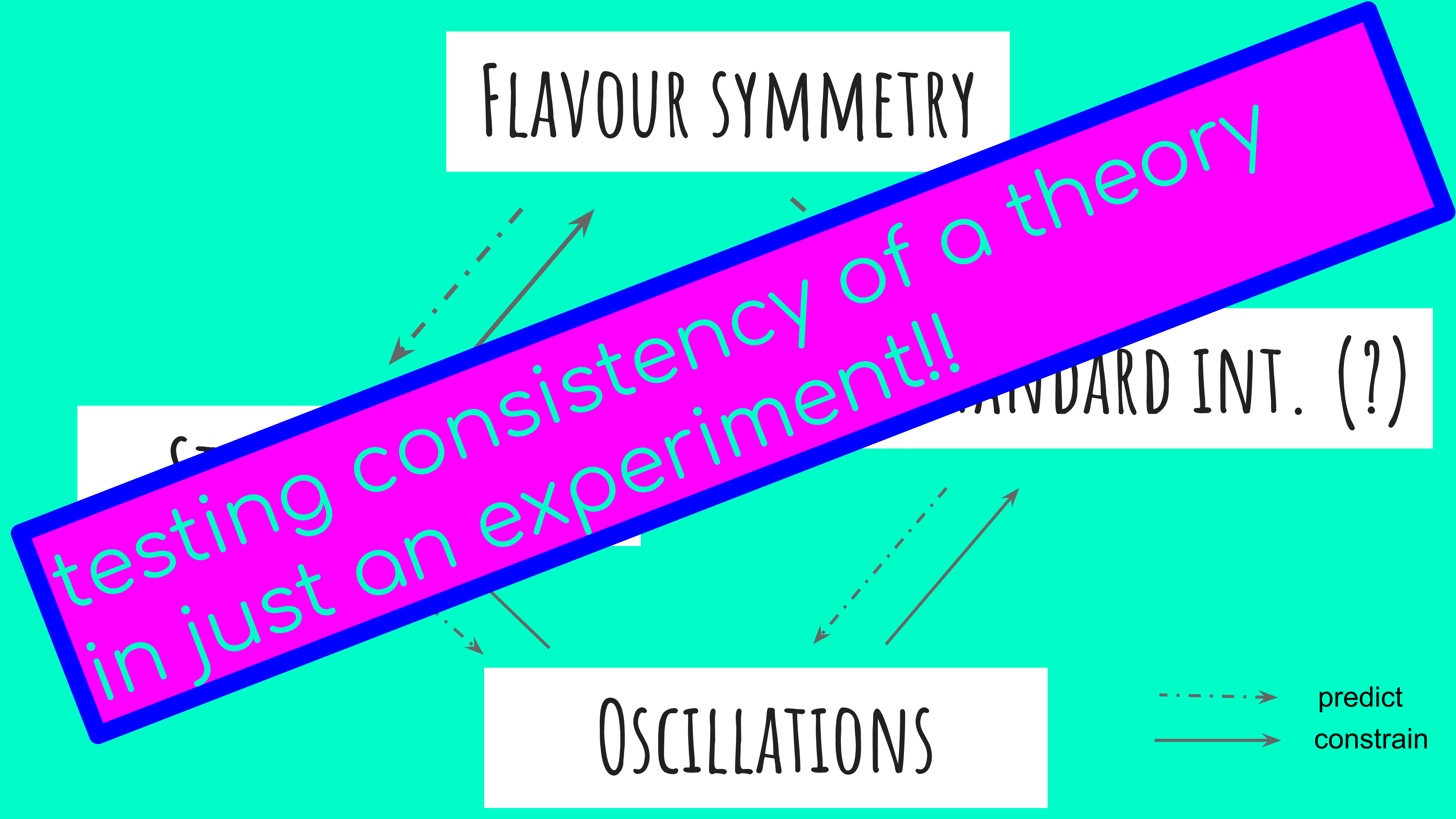
Oscillation in vacuum — Standard Matter effects — NSI Matter effects

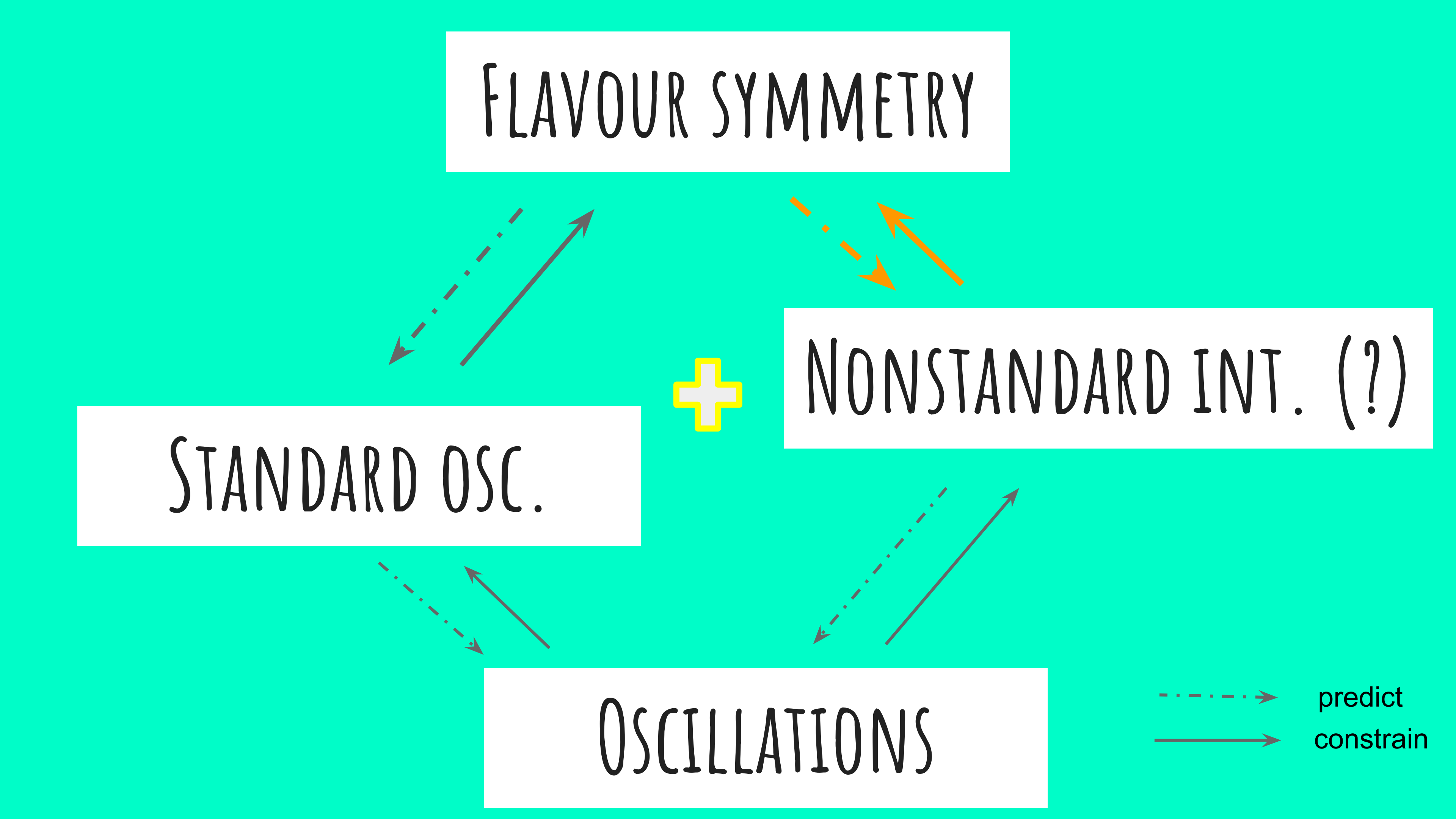

FLAVOUR SYMMETRY
STANDARD OSC.
+ NONSTANDARD INT. (?)
OSCILLATIONS
predict
constrain



CONNECTION
NONSTANDARD INT. (?)
STANDARD OSC.
OSCILLATIONS
predict
constrain


FLAVOUR SYMMETRY
MATCH?
OSCILLATIONS
predict
constrain


FLAVOUR SYMMETRY
DARD INT. (?)
OSCILLATIONS
predict
constrain
testing consistency of a theory
in just an experiment!!



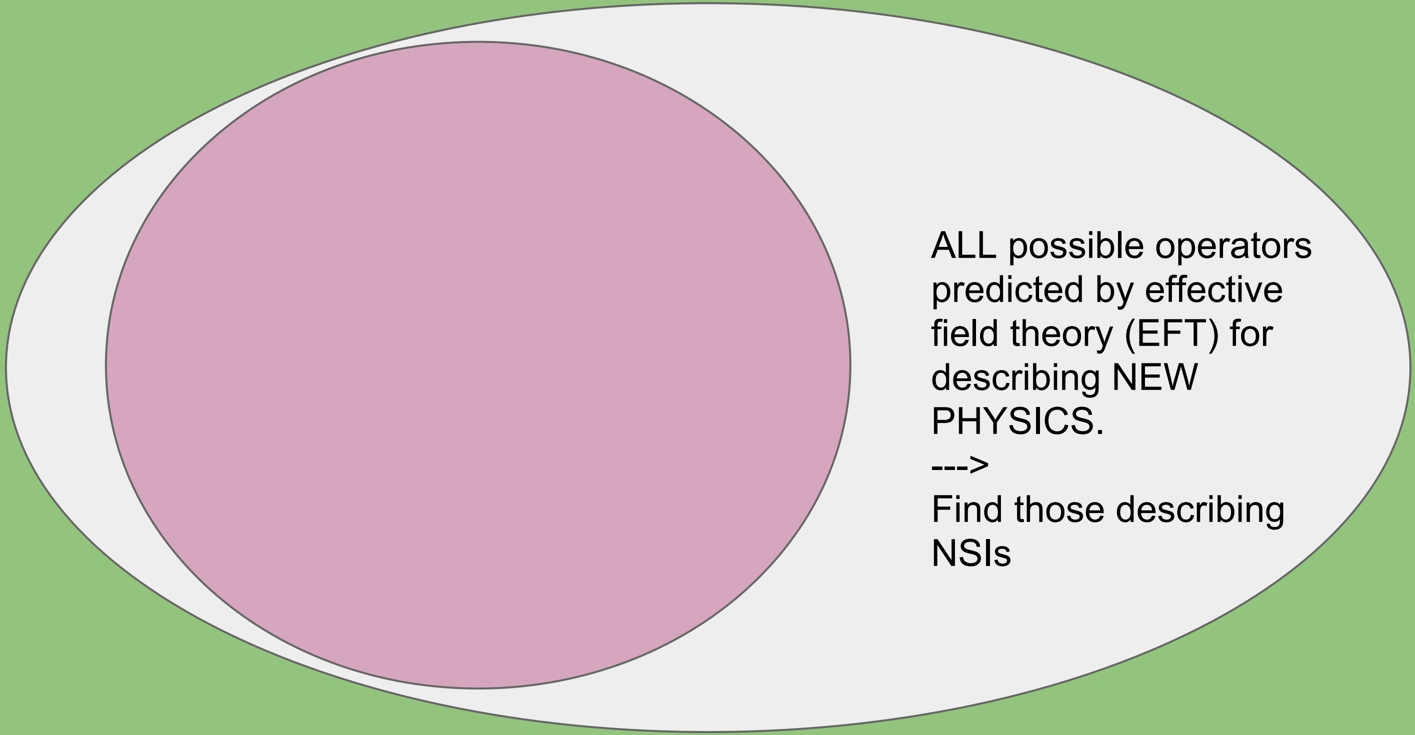
Flavour symmetry
Standard osc.
Nonstandard int. (?)
Oscillations
predict
constrain

Let's do it in a general way.

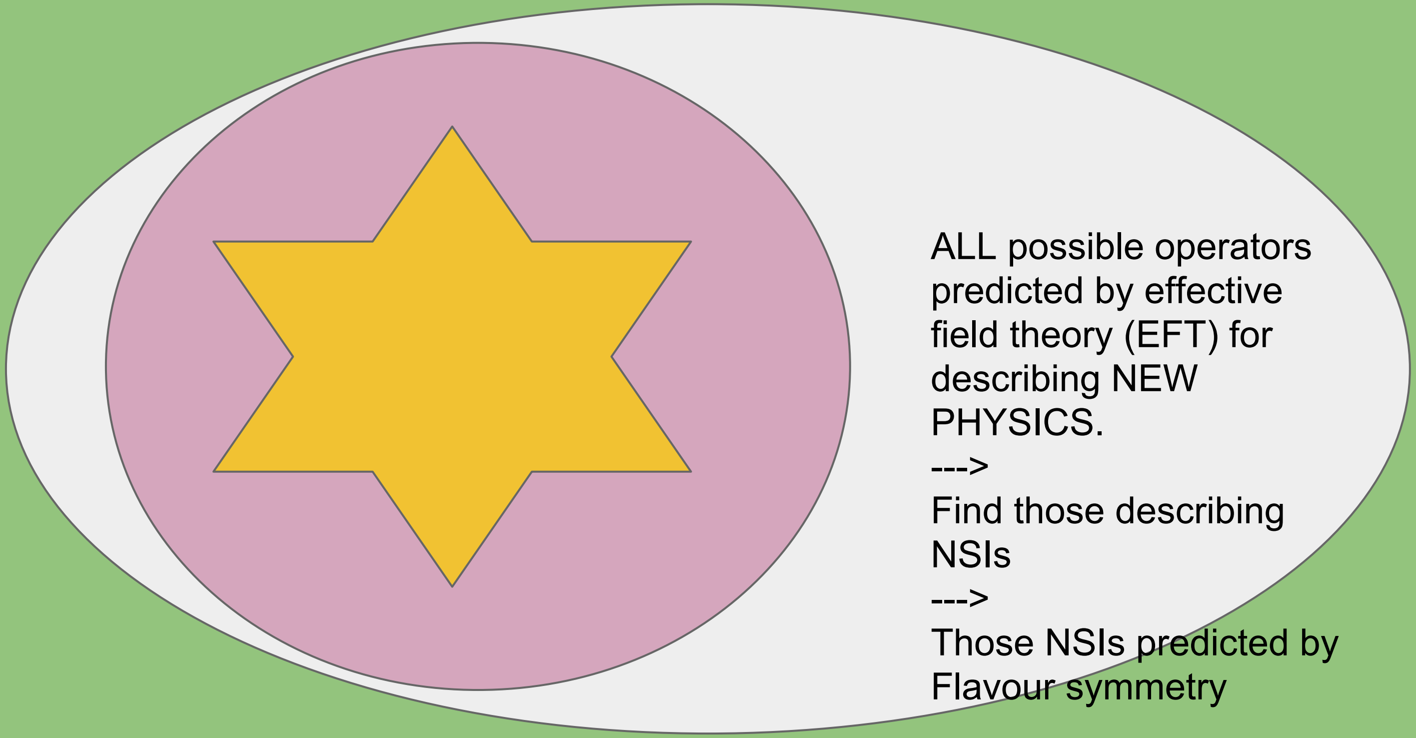
A general way

ALL possible operators predicted by effective field theory (EFT) for describing NEW PHYSICS.

A general way



A general way



EFT operators (D ≤ 8) for NSIs

Requirements:

1. Lorentz invariance and the SM gauge symmetry are satisfied around and above EW scale.
2. Lepton number and baryon number are conserved.
3. Involving 4 fermions and D-6 Higgs.
   In more details, 2 $SU(2)_L$ doublets L are needed for matter-effect NSIs, and at least 1 L is for NSIs at the source and detector.
4. NSIs are considered to avoid the strong constraints from 4-charge-fermion interactions.

EFT operators (D ≤ 8) for NSIs

$$\mathcal{L}_{\rm NSI} = 2\sqrt{2}G_F \sum_{p=1}^{12} c^p_{\alpha\beta\gamma\delta}\mathcal{O}^p_{\alpha\beta\gamma\delta} + \text{h.c.}$$

Label	Before EW breaking	After EW breaking	observation
$\mathcal{O}^1$	$\varepsilon_{ac}\varepsilon_{bd}(\overline{L_{a\alpha}}\gamma^\mu L_{b\beta})(\overline{L_{c\gamma}}\gamma_\mu L_{d\delta})$, $\varepsilon_{ac}\varepsilon_{bd}(\overline{L_{a\alpha}}\gamma^\mu L_{b\beta})(\overline{L_{c\gamma}}\gamma_\mu L_{d\delta})H^\dagger H$	$(\overline{\nu_{\alpha\rm L}}\gamma^\mu\nu_{\beta\rm L})(\overline{E_{\gamma\rm L}}\gamma_\mu E_{\delta\rm L})+(\overline{\nu_{\gamma\rm L}}\gamma^\mu\nu_{\delta\rm L})(\overline{E_{\alpha\rm L}}\gamma_\mu E_{\beta\rm L})$ $-(\overline{\nu_{\alpha\rm L}}\gamma^\mu\nu_{\delta\rm L})(\overline{E_{\gamma\rm L}}\gamma_\mu E_{\beta\rm L})-(\overline{\nu_{\gamma\rm L}}\gamma^\mu\nu_{\beta\rm L})(\overline{E_{\alpha\rm L}}\gamma_\mu E_{\delta\rm L})$	M
$\mathcal{O}^2$	$(\overline{L_\alpha}\tilde{H}\gamma^\mu\tilde{H}^\dagger L_\beta)(\overline{U_{\gamma\rm R}}\gamma_\mu U_{\delta\rm R})$	$(\overline{\nu_{\alpha\rm L}}\gamma^\mu\nu_{\beta\rm L})(\overline{U_{\gamma\rm R}}\gamma_\mu U_{\delta\rm R})$	M
$\mathcal{O}^3$	$(\overline{L_\alpha}\tilde{H}\gamma^\mu\tilde{H}^\dagger L_\beta)(\overline{D_{\gamma\rm R}}\gamma_\mu D_{\delta\rm R})$	$(\overline{\nu_{\alpha\rm L}}\gamma^\mu\nu_{\beta\rm L})(\overline{D_{\gamma\rm R}}\gamma_\mu D_{\delta\rm R})$	M
$\mathcal{O}^4$	$(\overline{L_\alpha}\tilde{H}\gamma^\mu\tilde{H}^\dagger L_\beta)(\overline{E_{\gamma\rm R}}\gamma_\mu E_{\delta\rm R})$	$(\overline{\nu_{\alpha\rm L}}\gamma^\mu\nu_{\beta\rm L})(\overline{E_{\gamma\rm R}}\gamma_\mu E_{\delta\rm R})$	M
$\mathcal{O}^5$	$(\overline{L_\alpha}\tilde{H}\gamma^\mu\tilde{H}^\dagger L_\beta)(\overline{Q_\gamma}\gamma_\mu Q_\delta)$	$(\overline{\nu_{\alpha\rm L}}\gamma^\mu\nu_{\beta\rm L})(\overline{U_{\gamma\rm L}}\gamma_\mu U_{\delta\rm L}+\overline{D_{\gamma\rm L}}\gamma_\mu D_{\delta\rm L})$	M
$\mathcal{O}^6$	$(\overline{L_\alpha}\tilde{H}\gamma^\mu\tilde{H}^\dagger L_\beta)(\overline{L_\gamma}\gamma_\mu L_\delta)$	$(\overline{\nu_{\alpha\rm L}}\gamma^\mu\nu_{\beta\rm L})(\overline{\nu_{\gamma\rm L}}\gamma_\mu\nu_{\delta\rm L}+\overline{E_{\gamma\rm L}}\gamma_\mu E_{\delta\rm L})$	M
$\mathcal{O}^7$	$(\overline{L_\alpha}\tilde{H}\gamma^\mu L_{b\beta})(\overline{Q_{b\gamma}}\gamma_\mu\tilde{H}^\dagger Q_\delta)$	$(\overline{\nu_{\alpha\rm L}}\gamma^\mu\nu_{\beta\rm L})(\overline{U_{\gamma\rm L}}\gamma_\mu U_{\delta\rm L})+(\overline{\nu_{\alpha\rm L}}\gamma^\mu E_{\beta\rm L})(\overline{D_{\gamma\rm L}}\gamma_\mu U_{\delta\rm L})$	S,M,D
$\mathcal{O}^8$	$\varepsilon_{bc}(\overline{L_\alpha}\tilde{H}\gamma^\mu L_{b\beta})(\overline{Q_\gamma}H\gamma_\mu Q_{c\delta})$	$(\overline{\nu_{\alpha\rm L}}\gamma^\mu\nu_{\beta\rm L})(\overline{D_{\gamma\rm L}}\gamma_\mu D_{\delta\rm L})-(\overline{\nu_{\alpha\rm L}}\gamma^\mu E_{\beta\rm L})(\overline{D_{\gamma\rm L}}\gamma_\mu U_{\delta\rm L})$	S,M,D
$\mathcal{O}^9$	$\varepsilon_{bc}(\overline{L_\alpha}\tilde{H}\gamma^\mu L_{b\beta})(\overline{Q_\gamma}H\gamma_\mu Q_{c\delta})$	$(\overline{\nu_{\alpha\rm L}}\gamma^\mu E_{\beta\rm L})(\overline{D_{\gamma\rm R}}\gamma_\mu U_{\delta\rm R})$	S,D
$\mathcal{O}^{10}$	$(\overline{L_\alpha}\tilde{H}\sigma^{\mu\nu}E_{\beta\rm R})(\overline{Q_\gamma}H\sigma_{\mu\nu}U_{\delta\rm R})$	$(\overline{\nu_{\alpha\rm L}}\sigma^{\mu\nu}E_{\beta\rm R})(\overline{D_{\gamma\rm L}}\sigma_{\mu\nu}U_{\delta\rm R})$	S,D
$\mathcal{O}^{11}$	$(\overline{L_\alpha}\tilde{H}E_{\beta\rm R})(\overline{D_{\gamma\rm R}}\tilde{H}^\dagger Q_\delta)$	$(\overline{\nu_{\alpha\rm L}}E_{\beta\rm R})(\overline{D_{\gamma\rm R}}U_{\delta\rm L})$	S,D
$\mathcal{O}^{12}$	$(\overline{L_\alpha}\tilde{H}E_{\beta\rm R})(\overline{Q_\gamma}HU_{\delta\rm R})$	$(\overline{\nu_{\alpha\rm L}}E_{\beta\rm R})(\overline{D_{\gamma\rm L}}U_{\delta\rm R})$	S,D

MATTER

SOURCE , DETECTOR

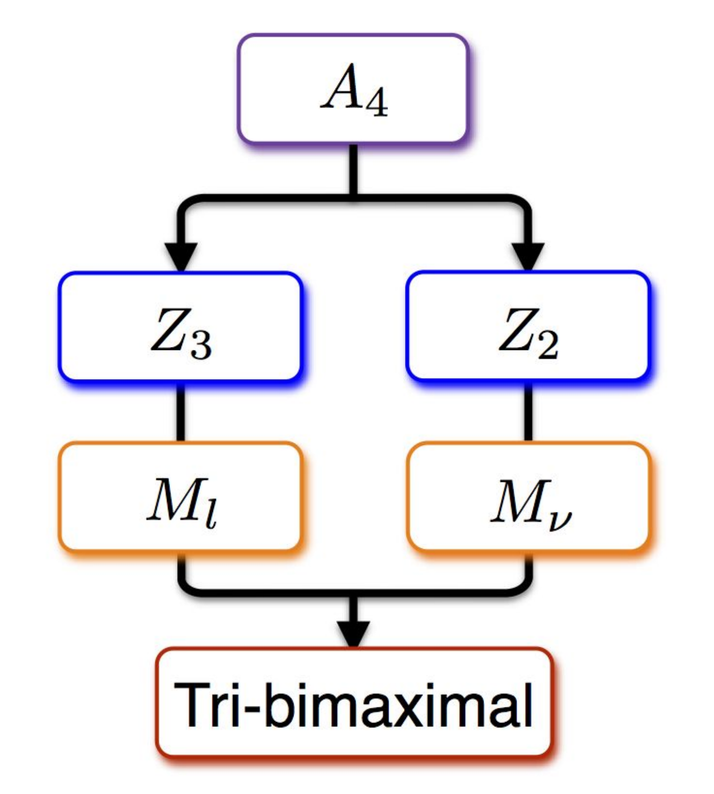
NSIs structures are predicted according to the considered NSI operator.

We study all possible structure of NSIs over these operators.

In this talk, we focus on matter effect NSIs ($\mathcal{O}^{1\text{-}8}$).

We consider A_4 and $A_4 \rightarrow Z_2$

- Model 1: A_4 $(1, 1', 1'', 3)$ is conserved in the whole lepton sector.
- Model 2: A_4 is broken by a scalar 'Flavon' ($\chi=(1\ 1\ 1)^T$); Z_2 (Z_3) is preserved in the neutrino (charged-lepton) sector.
- Both models can explain the current oscillation measurements.
- A lepton doublet L is always assigned a triplet of A_4 (3).
- For the other fermions, we scan over all possibilities $(1, 1', 1'', 3)$.
- interact with 1st-generation quarks or charged leptons in matter.



EFT under A_4 Flavour symmetries

e.g. (the same procedure is applied for the others)

$$(\overline{L}L)_{\mathbf{1}}(\overline{L}L)_{\mathbf{1}}\,,\ (\overline{L}L)_{\mathbf{1}'}(\overline{L}L)_{\mathbf{1}''}\,,\ (\overline{L}L)_{\mathbf{3}_{\rm S}}(\overline{L}L)_{\mathbf{3}_{\rm S}}\,,\ (\overline{L}L)_{\mathbf{3}_{\rm A}}(\overline{L}L)_{\mathbf{3}_{\rm A}}\,,\ (\overline{L}L)_{\mathbf{3}_{\rm S}}(\overline{L}L)_{\mathbf{3}_{\rm A}}$$

Predicts vanishing NSIs

$$c^1_{\mu\mu11} = c^1_{\tau\tau11}\,,\ c^1_{ee11} = c^1_{\alpha\beta11} = 0 \text{ for } \alpha \neq \beta$$

$$\mathcal{L}_{\rm NSI} = 2\sqrt{2}G_F \sum_{p=1}^{12} c^p_{\alpha\beta\gamma\delta}\mathcal{O}^p_{\alpha\beta\gamma\delta} + \text{h.c.}$$

$$\mathbb{T}'_{12} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \propto 2\mathbb{T}_{11} - \mathbb{T}_{12}$$

EFT OPERATORS UNDER FLAVOUR SYMMETRIES

All possible flavour dependences

All possible assignments **All possible operators**

	Representations	A_4-invariant operators	NSI textures
$\mathcal{O}^1$	$L \sim \mathbf{3}$	$(\overline{L}L)_{\mathbf{1}}(\overline{L}L)_{\mathbf{1}}$, $(\overline{L}L)_{\mathbf{1}'}(\overline{L}L)_{\mathbf{1}''}$, $(\overline{L}L)_{\mathbf{3}_\mathrm{S}}(\overline{L}L)_{\mathbf{3}_\mathrm{S}}$, $(\overline{L}L)_{\mathbf{3}_\mathrm{A}}(\overline{L}L)_{\mathbf{3}_\mathrm{A}}$	$2\mathbb{T}_{11} - \mathbb{T}_{12}$
$\mathcal{O}^{2-8}$	$L \sim \mathbf{3}, F \sim \mathbf{1}, \mathbf{1}', \mathbf{1}'', \mathbf{3}$	$(\overline{L}L)_{\mathbf{1}}(\overline{F}F)_{\mathbf{1}}$	$\mathbb{T}_{11}$
	$L \sim \mathbf{3}, F \sim \mathbf{3}$	$(\overline{L}L)_{\mathbf{3}_\mathrm{S}}(\overline{F}F)_{\mathbf{3}_\mathrm{S}}$	$\mathbb{T}_{12}$
		$(\overline{L}L)_{\mathbf{3}_\mathrm{A}}(\overline{F}F)_{\mathbf{3}_\mathrm{S}}$	$\mathbb{T}_{13}$

	Representations	Z_2-invariant operators	NSI textures
$\chi\mathcal{O}^1$	$\chi \sim \mathbf{3}, L \sim \mathbf{3}$	$\chi\big((\overline{L}L)_{\mathbf{3}_\mathrm{S}}(\overline{L}L)_{\mathbf{1},\mathbf{1}',\mathbf{1}''}\big)_{\mathbf{3}}$, $\chi\big((\overline{L}L)_{\mathbf{3}_\mathrm{S}}(\overline{L}L)_{\mathbf{3}_\mathrm{S}}\big)_{\mathbf{3}_\mathrm{S}}$, $\chi\big((\overline{L}L)_{\mathbf{3}_\mathrm{A}}(\overline{L}L)_{\mathbf{3}_\mathrm{A}}\big)_{\mathbf{3}_\mathrm{S}}$	$\frac{1}{3}(2\mathbb{T}_{11} - \mathbb{T}_{12} + 2\mathbb{T}_{21} + 2\mathbb{T}_{23})$
		$\chi\big((\overline{L}L)_{\mathbf{3}_\mathrm{A}}(\overline{L}L)_{\mathbf{1},\mathbf{1}',\mathbf{1}''}\big)_{\mathbf{3}}$, $\chi\big((\overline{L}L)_{\mathbf{3}_\mathrm{S}}(\overline{L}L)_{\mathbf{3}_\mathrm{A}}\big)_{\mathbf{3}_\mathrm{S}}$	$\mathbb{T}_{13}$
$\chi\mathcal{O}^{2-8}$	$\chi \sim \mathbf{3}, L \sim \mathbf{3}, F \sim \mathbf{1}, \mathbf{1}', \mathbf{1}'', \mathbf{3}$	$\chi(\overline{L}L)_{\mathbf{3}_\mathrm{S}}(\overline{F}F)_{\mathbf{1}}$	$\mathbb{T}_{12} + \mathbb{T}_{22}$
		$\chi(\overline{L}L)_{\mathbf{3}_\mathrm{A}}(\overline{F}F)_{\mathbf{1}}$	$\mathbb{T}_{13} + \mathbb{T}_{23}$
	$\chi \sim \mathbf{3}, L \sim \mathbf{3}, F \sim \mathbf{3}$	$\chi\big((\overline{L}L)_{\mathbf{3}_\mathrm{S}}(\overline{F}F)_{\mathbf{3}_\mathrm{S}}\big)_{\mathbf{3}_\mathrm{S}}$	$2\mathbb{T}_{12} - \mathbb{T}_{22}$
		$\chi\big((\overline{L}L)_{\mathbf{3}_\mathrm{A}}(\overline{F}F)_{\mathbf{3}_\mathrm{S}}\big)_{\mathbf{3}_\mathrm{S}}$	$2\mathbb{T}_{13} - \mathbb{T}_{23}$
		$\chi\big((\overline{L}L)_{\mathbf{3}_\mathrm{S}}(\overline{F}F)_{\mathbf{3}_\mathrm{S}}\big)_{\mathbf{3}_\mathrm{A}}$	$\mathbb{T}_{32}$
		$\chi\big((\overline{L}L)_{\mathbf{3}_\mathrm{A}}(\overline{F}F)_{\mathbf{3}_\mathrm{S}}\big)_{\mathbf{3}_\mathrm{A}}$	$\mathbb{T}_{33}$

$$\mathbb{T}_{11} \equiv \mathbb{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbb{T}_{12} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \mathbb{T}_{13} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

diagonal

NOT diagonal

$$\mathbb{T}_{21} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad \mathbb{T}_{22} = \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ -1 & 2 & 0 \end{pmatrix}, \quad \mathbb{T}_{23} = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\mathbb{T}_{31} = \begin{pmatrix} 0 & -i & i \\ i & 0 & -i \\ -i & i & 0 \end{pmatrix}, \quad \mathbb{T}_{32} = \begin{pmatrix} 0 & i & -i \\ -i & 0 & -2i \\ i & 2i & 0 \end{pmatrix}, \quad \mathbb{T}_{33} = \begin{pmatrix} 0 & i & i \\ -i & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}.$$

Matter effect NSIs under A_4 and $A_4 \to Z_2$

- NSIs in Matter are described by a matrix in the Hamiltonian under flavour basis.
- Under the flavour symmetry A4 or Z2, this matrix can be expanded by 'textures' ($\mathbb{T}_{mn}$).
- The textures preserving A4 forbid the flavour transition entries.

$$H = \frac{1}{2E}\left\{U\begin{pmatrix}0&0&0\\0&\Delta m^2_{21}&0\\0&0&\Delta m^2_{31}\end{pmatrix}U^\dagger + A\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix} + A\begin{pmatrix}\epsilon^{\mathrm{m}}_{ee}&\epsilon^{\mathrm{m}}_{e\mu}&\epsilon^{\mathrm{m}}_{e\tau}\\\epsilon^{\mathrm{m}}_{\mu e}&\epsilon^{\mathrm{m}}_{\mu\mu}&\epsilon^{\mathrm{m}}_{\mu\tau}\\\epsilon^{\mathrm{m}}_{\tau e}&\epsilon^{\mathrm{m}}_{\tau\mu}&\epsilon^{\mathrm{m}}_{\tau\tau}\end{pmatrix}\right\}$$

Matter NSI effects

$$\epsilon \equiv \begin{pmatrix}\epsilon_{ee}&\epsilon_{e\mu}&\epsilon_{e\tau}\\\epsilon_{\mu e}&\epsilon_{\mu\mu}&\epsilon_{\mu\tau}\\\epsilon_{\tau e}&\epsilon_{\tau\mu}&\epsilon_{\tau\tau}\end{pmatrix} \equiv \begin{pmatrix}\epsilon_{ee}&|\epsilon_{e\mu}|e^{i\phi_{e\mu}}&|\epsilon_{e\tau}|e^{i\phi_{e\tau}}\\|\epsilon_{\mu e}|e^{-i\phi_{e\mu}}&\epsilon_{\mu\mu}&|\epsilon_{\mu\tau}|e^{i\phi_{\mu\tau}}\\|\epsilon_{e\tau}|e^{-i\phi_{e\tau}}&|\epsilon_{\mu\tau}|e^{-i\phi_{\mu\tau}}&\epsilon_{\tau\tau}\end{pmatrix} = \sum_{m,n=1,2,3}\alpha_{mn}\mathbb{T}_{mn}/N_{mn},$$

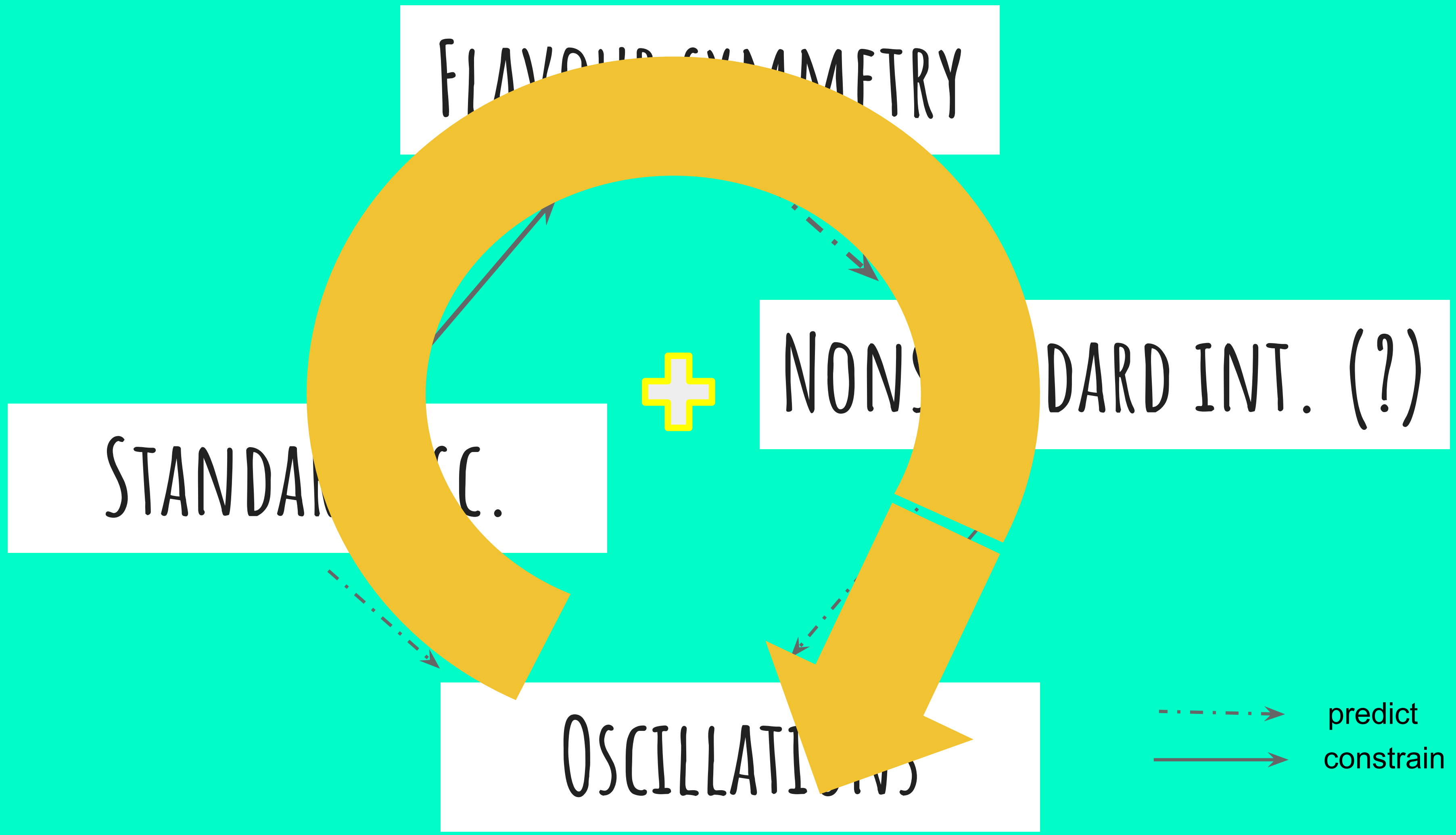
$$\mathbb{T}_{11} \equiv \mathbb{1} = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}, \quad \mathbb{T}_{12} = \begin{pmatrix}2&0&0\\0&-1&0\\0&0&-1\end{pmatrix}, \quad \mathbb{T}_{13} = \begin{pmatrix}0&0&0\\0&1&0\\0&0&-1\end{pmatrix}$$

A4

$$\mathbb{T}_{21} = \begin{pmatrix}0&1&1\\1&0&1\\1&1&0\end{pmatrix}, \quad \mathbb{T}_{22} = \begin{pmatrix}0&-1&-1\\-1&0&2\\-1&2&0\end{pmatrix}, \quad \mathbb{T}_{23} = \begin{pmatrix}0&-1&1\\-1&0&0\\1&0&0\end{pmatrix},$$

$$\mathbb{T}_{31} = \begin{pmatrix}0&-i&i\\i&0&-i\\-i&i&0\end{pmatrix}, \quad \mathbb{T}_{32} = \begin{pmatrix}0&i&-i\\-i&0&-2i\\i&2i&0\end{pmatrix}, \quad \mathbb{T}_{33} = \begin{pmatrix}0&i&i\\-i&0&0\\-i&0&0\end{pmatrix}.$$

Z2



predict
constrain

With global-fit results

$\tilde{\epsilon}_{ee}(\equiv \epsilon_{ee} - \epsilon_{\mu\mu})$	$3\alpha_{12}/\sqrt{6} - \alpha_{13}/\sqrt{2}$
$\tilde{\epsilon}_{\tau\tau}(\equiv \epsilon_{\tau\tau} - \epsilon_{\mu\mu})$	$-2\alpha_{13}/\sqrt{2}$
$\epsilon_{e\mu}$	$\alpha_{21}/\sqrt{6} - \alpha_{22}/\sqrt{12} - \alpha_{23}/2 + i\left(-\alpha_{31}/\sqrt{6} + \alpha_{32}/\sqrt{12} + \alpha_{33}/2\right)$
$\epsilon_{e\tau}$	$\alpha_{21}/\sqrt{6} - \alpha_{22}/\sqrt{12} + \alpha_{23}/2 + i\left(\alpha_{31}/\sqrt{6} - \alpha_{32}/\sqrt{12} + \alpha_{33}/2\right)$
$\epsilon_{\mu\tau}$	$\alpha_{21}/\sqrt{6} + 2\alpha_{22}/\sqrt{12} + i\left(-\alpha_{31}/\sqrt{6} - \alpha_{32}/\sqrt{12}\right)$

Global fit data:

- Include solar and atmospheric neutrino data.
- Assume all element of NSI matrix are real.

M.C. Gonzalez-Garcia, et al, 2013.

Interact with u/d quarks only

Assume Gaussian

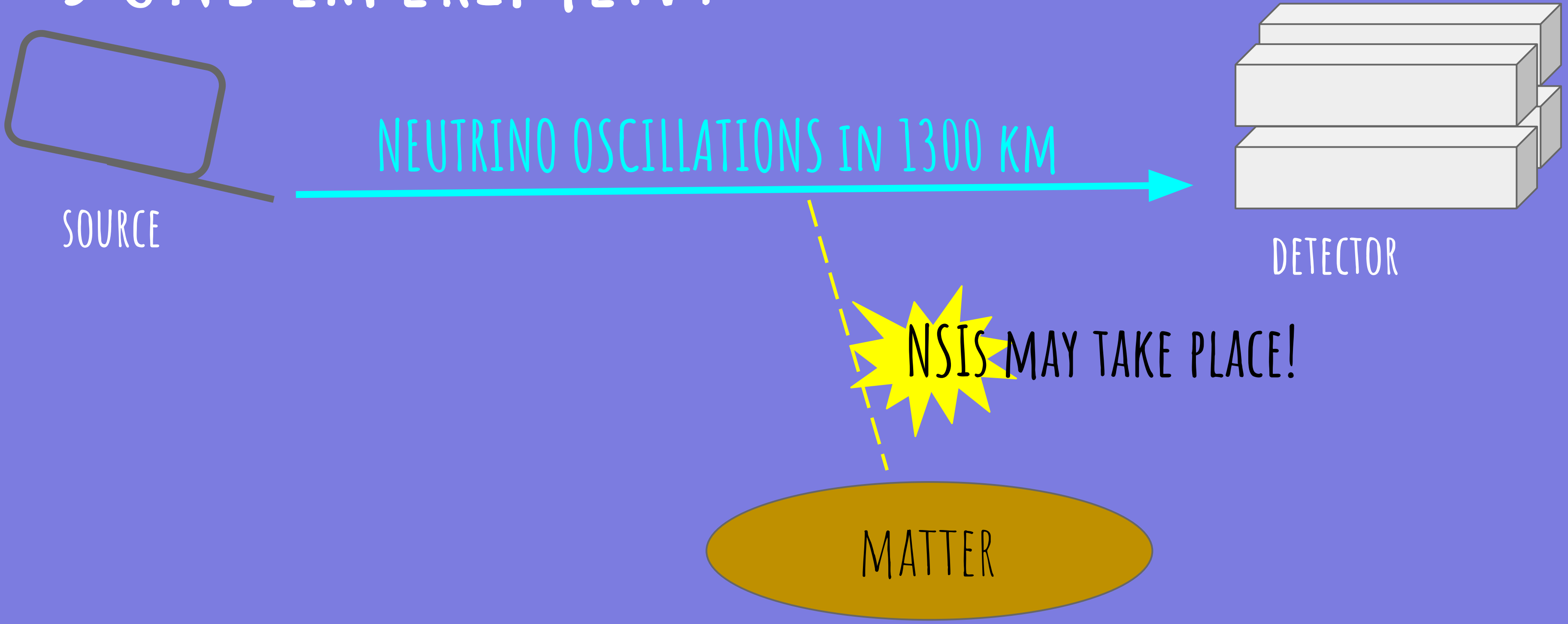
1σ bounds of global fit results			
$\tilde{\epsilon}^u_{ee}$	$[0.188, 0.376]$	$\tilde{\epsilon}^d_{ee}$	$[0.203, 0.384]$
$\tilde{\epsilon}^u_{\tau\tau}$	$[-0.003, 0.012]$	$\tilde{\epsilon}^d_{\tau\tau}$	$[-0.003, 0.012]$
$\epsilon^u_{e\mu}$	$[-0.046, 0.002]$	$\epsilon^d_{e\mu}$	$[-0.048, 0]$
$\epsilon^u_{e\tau}$	$[-0.038, 0.065]$	$\epsilon^d_{e\tau}$	$[-0.036, 0.066]$
$\epsilon^u_{\mu\tau}$	$[-0.004, 0.003]$	$\epsilon^d_{\mu\tau}$	$[-0.004, 0.003]$

1σ bounds by global fit results			
α^u_{12}	$[0.089, 0.247]$	α^d_{12}	$[0.099, 0.26]$
α^u_{13}	$[-0.003, 0.007]$	α^d_{13}	$[-0.003, 0.007]$
α^u_{21}	$[-0.045, 0.049]$	α^d_{21}	$[-0.045, 0.047]$
α^u_{22}	$[-0.037, 0.03]$	α^d_{22}	$[-0.035, 0.0302]$
α^u_{23}	$[-0.019, 0.096]$	α^d_{23}	$[-0.0154, 0.096]$

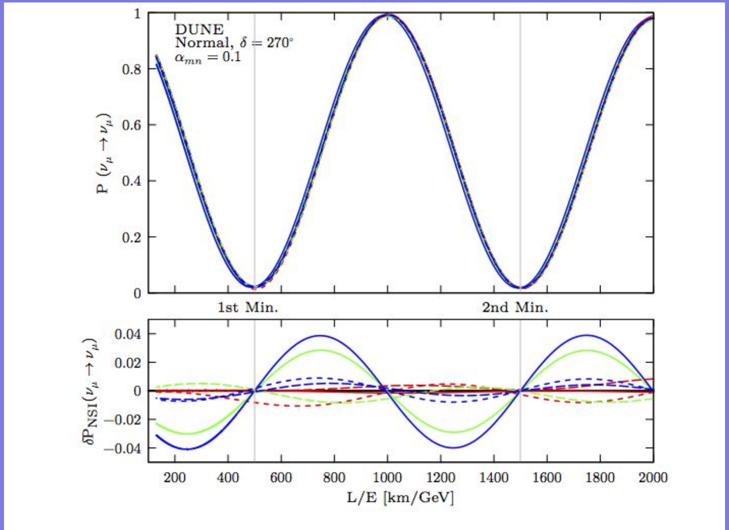
weakest

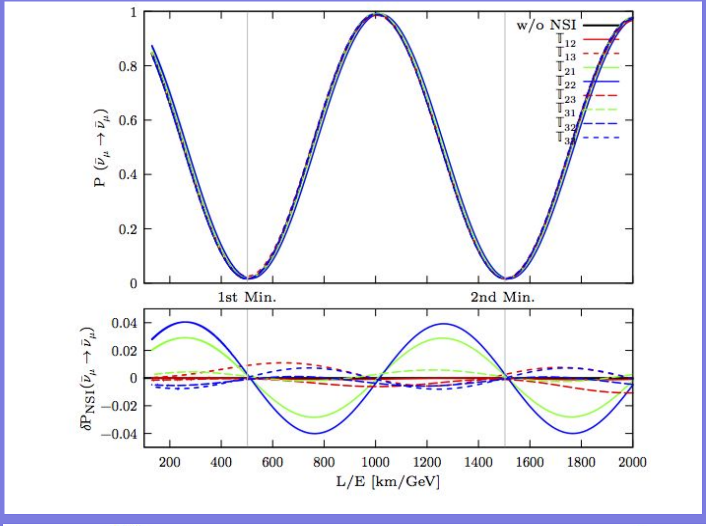
strongest

DUNE EXPERIMENT

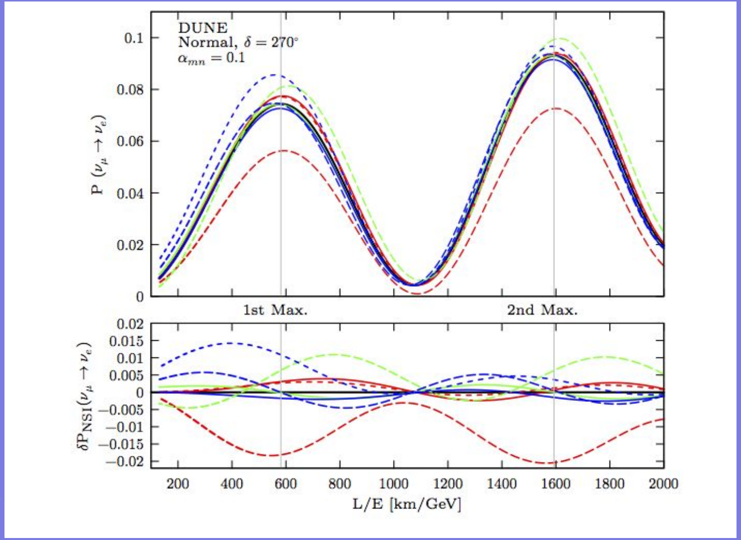


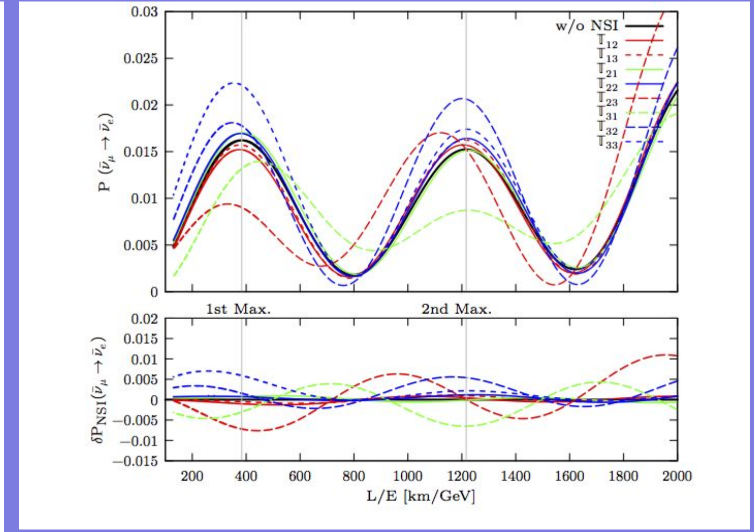
@DUNE





measuring the size



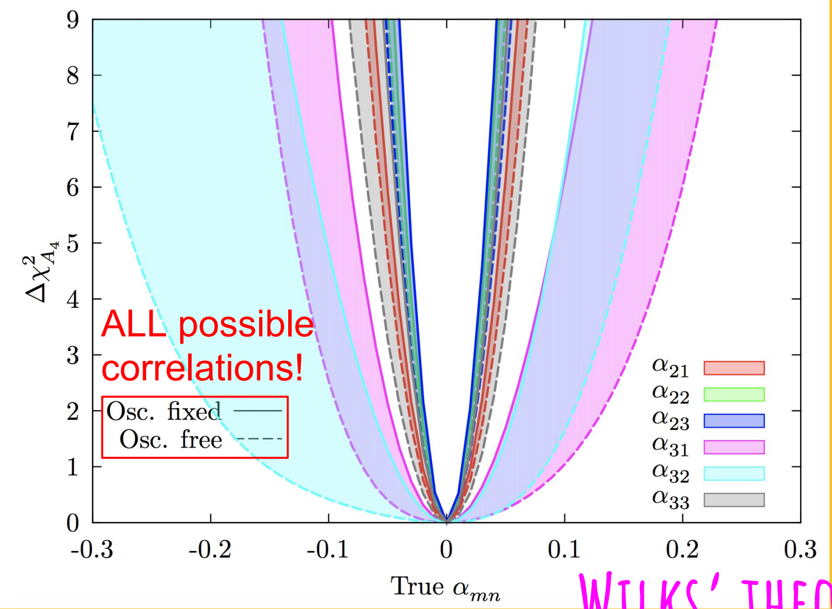


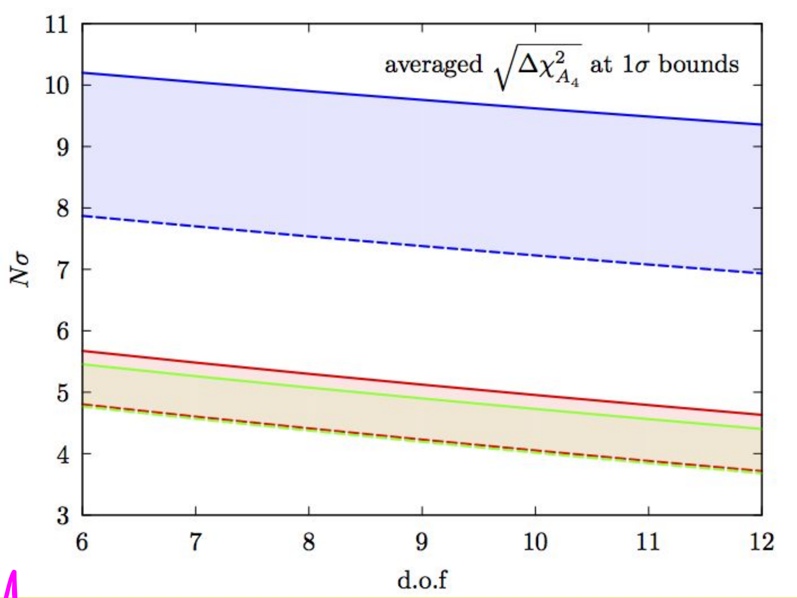
verifying textures

Testing A4 @DUNE with GLoBES

$$\Delta\chi^2_{A_4} \equiv \chi^2\big|_{\alpha_{2n}=\alpha_{3n}=0} - \chi^2_{b.f.}$$

	Osc. Para.	$\alpha_{12},\ \alpha_{13}$	$\alpha_{2n},\ \alpha_{3n}$
true values	fix them at B.F.	fix them at 0	change one; fix the other at 0
tested values	all fixed or free	allow them varying	fix all at 0





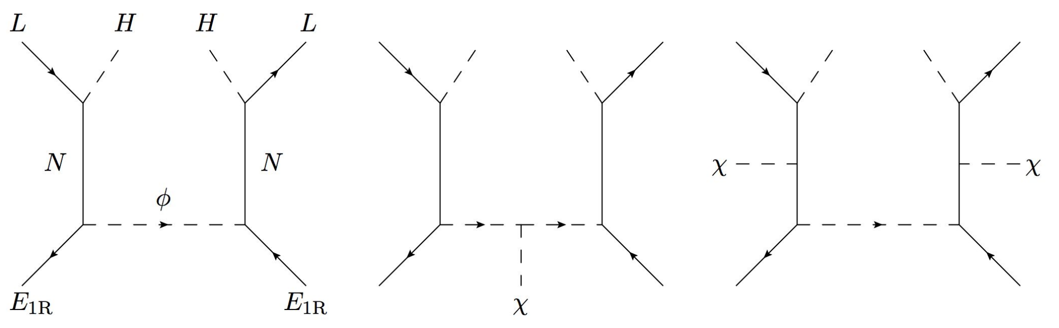
WILKS' THEOREM

Chi-squared value is defined in a Gaussian distribution with the dimension that is the difference between those of two models.

Z2 Symmetry with one UV complet model

$$\mathcal{L} = (D_\mu\phi)^\dagger(D^\mu\phi) - (M_\phi^2)_{\alpha\beta}\phi_\alpha^*\phi_\beta + \overline{N}i\partial\!\!\!/ N - M_{N\alpha\beta}\overline{N_{\alpha R}}N_{\beta L} - \kappa_{\alpha\beta\gamma}\overline{E_{\alpha R}}N_{\beta L}\phi_\gamma^* - y_{\alpha\beta}\overline{L_\alpha}\tilde{H}N_{\beta R} + \text{h.c.}\,,$$

$$\epsilon^e = \frac{1}{8G_F^2}(yM_N^{-1}\kappa_e)(M_\phi^2)^{-1}(yM_N^{-1}\kappa_e)^\dagger$$



(a) (b) (c)

$$\frac{\mu_N}{v_\chi}\left[\frac{2}{3}g_S\big(\chi(\overline{N_L}N_R)_{\mathbf{3}_S}\big)_{\mathbf{1}} - \frac{2}{\sqrt{3}}g_A\big(\chi(\overline{N_L}N_R)_{\mathbf{3}_A}\big)_{\mathbf{1}}\right] + \text{h.c.}$$

$$\frac{\mu_\phi^2}{v_\chi}\left[\frac{2}{3}f_S\Big(\chi(\phi^*\phi)_{\mathbf{3}_S}\Big)_{\mathbf{1}} - \frac{2}{\sqrt{3}}f_A\Big(\chi(\phi^*\phi)_{\mathbf{3}_A}\Big)_{\mathbf{1}}\right]$$

NSI matrix is preditected:

$$\begin{pmatrix} -x & x+y-z-iw & x+y+z+iw \\ x-z+iw & z & y-iw \\ x+z-iw & y+iw & -z \end{pmatrix}$$

Z_2 Symmetry with global-fit results

$$\begin{pmatrix} -x & x+y-z-iw & x+y+z+iw \\ x-z+iw & z & y-iw \\ x+z-iw & y+iw & -z \end{pmatrix}$$

compatitive

Global Fit		Global Fit		DUNE sensitivity	
w^u	– (x3)	w^d	– (x3)	w	$[-0.013, 0.025]$
x^u	$[-0.034, 0.013]$	x^d	$[-0.035, 0.012]$	x	$[-0.1, 0.1]$
y^u	$[-0.004, 0.003]$	y^d	$[-0.004, 0.003]$	y	$[-0.01, 0.01]$
z^u	$[-0.002, 0.005]$	z^d	$[-0.002, 0.005]$	z	$[-0.007, 0.017]$

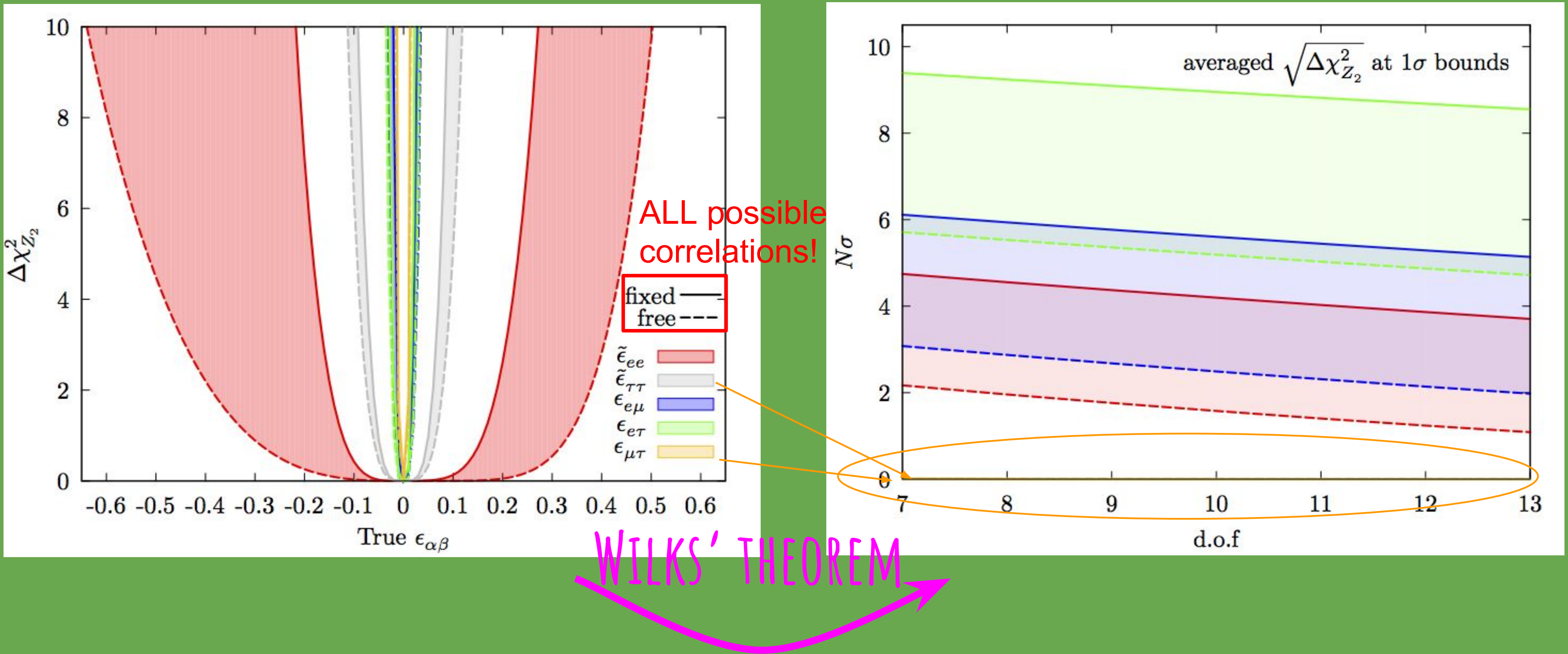
Predicts the NSI matrix:

$$\epsilon = \begin{pmatrix} 0 & x & x \\ x & x & 0 \\ x & 0 & x \end{pmatrix}$$

$$\epsilon_{e\mu} = \epsilon_{e\tau} = -\tilde{\epsilon}_{ee}\,,$$
$$\epsilon_{\mu\tau} = \tilde{\epsilon}_{\tau\tau} = 0\,.$$

Z_2 Symmetry @DUNE with GLoBES

$$\Delta\chi^2_{Z_2} \equiv \chi^2\big|_x - \chi^2_{b.f.}$$



CONCLUSIONS

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CONCLUSIONS

- We have proposed an idea to test the consistency of the flavour symmetry by measuring both oscillation and NSI parameters.
- The flavour symmetry predicts the flavour dependence of NSIs.
- We see the high exclusion level for DUNE to test A4 and Z2 symmetry.
- We should not waste these tiny but influential information for flavour symmetries.

Next...

- Test more symmetry models for NSIs.
- With sum rules for oscillation parameters (s,a,r).
- Other phenomenologies,
  e.g. non-unitarity of U_{PMNS} (TEXONO?).
- Including collider data.
- … …


Thank you for your attention!!!!!!!!!!!!!!!!!!!!!!!!!!

BACKUPS

The leptonic flavour symmetry

- The flavour symmetry at the higher energy (e.g. A4, Z2...etc) explains neutrino mixing and simplifies the standard language for the oscillations.
- Under a certain symmetry, the pattern of neutrino mixing and MSDs is imposed by **flavons**, which are too heavy to be detected in the prediction.

e.g. Trimaximal (TM),

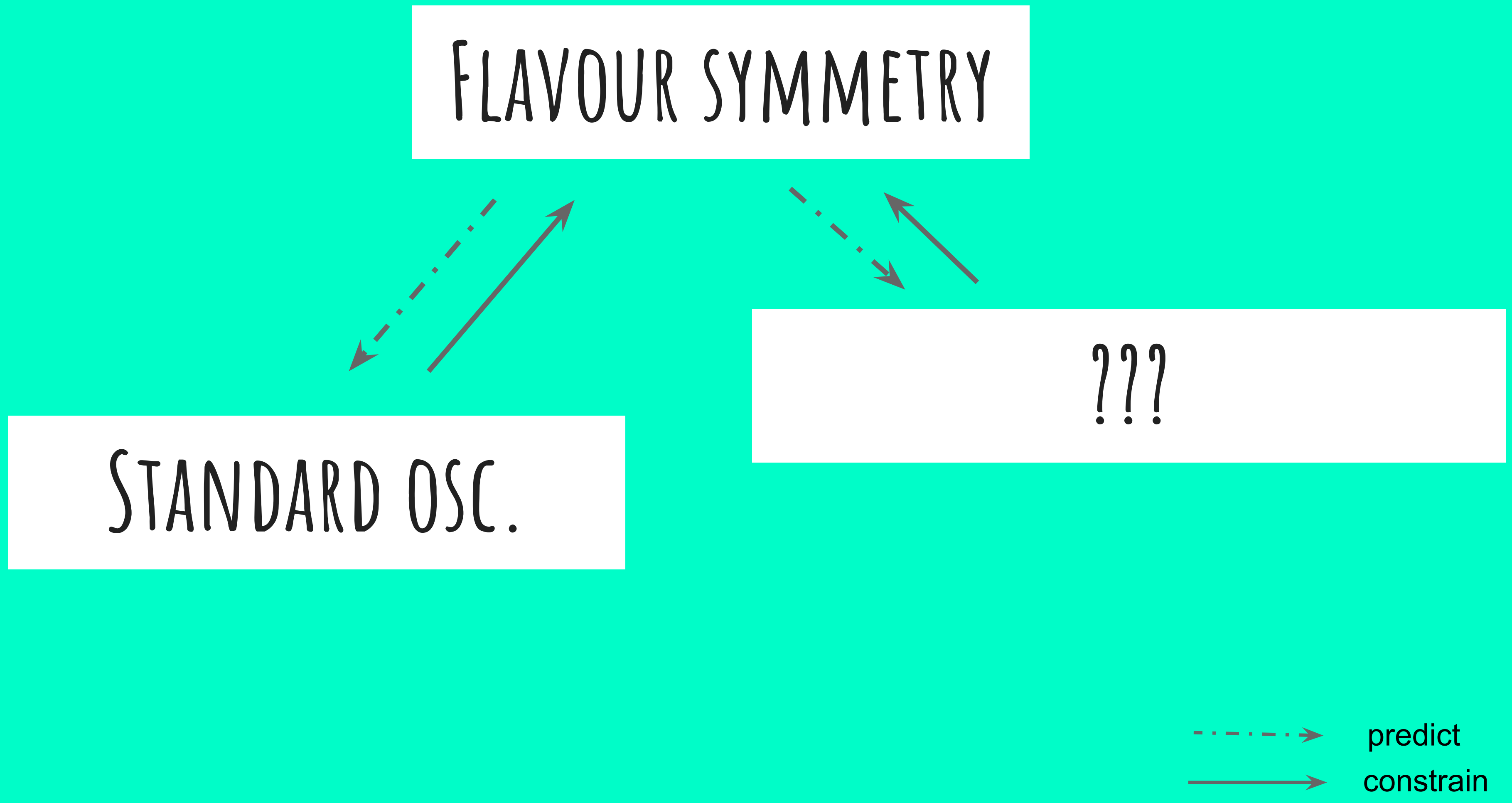
$$U_{\rm TM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{r}{\sqrt{2}} e^{-i\delta} \\ \frac{-1}{\sqrt{6}}(1-a+re^{-i\delta}) & \frac{1}{\sqrt{3}}(1-a-\frac{1}{2}re^{-i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+a-re^{-i\delta}) & \frac{-1}{\sqrt{3}}(1+a+\frac{1}{2}re^{-i\delta}) & \frac{r}{\sqrt{2}}(1-a) \end{pmatrix} P,$$

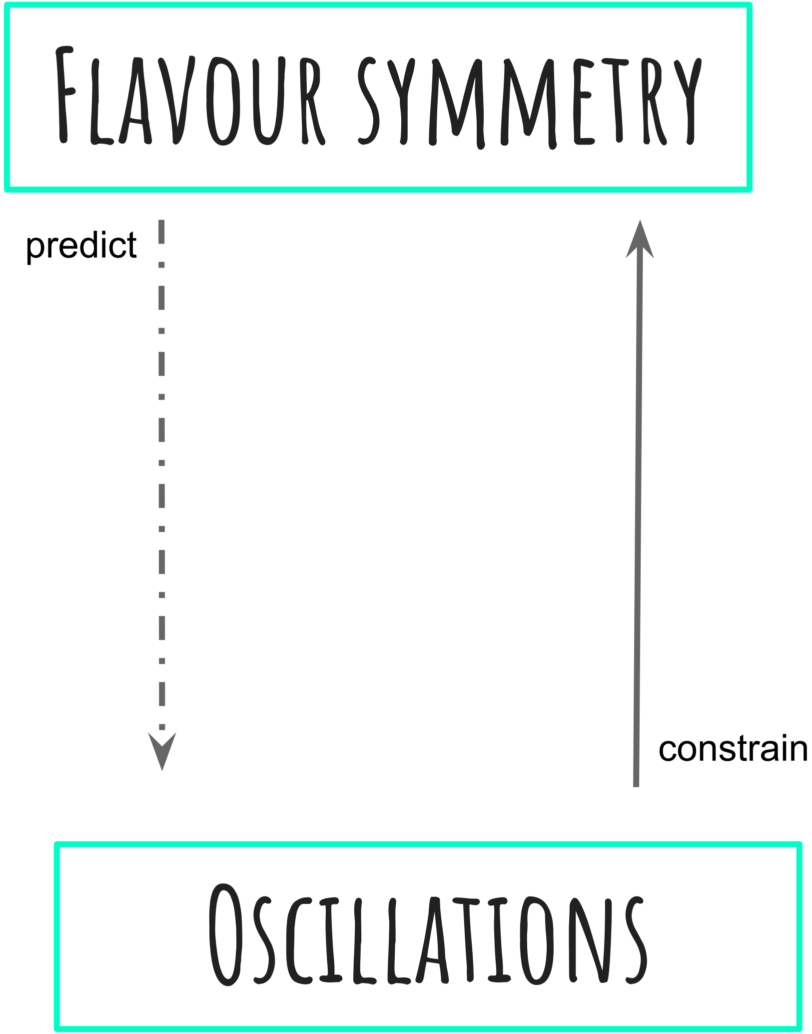
$$a = r\cos\delta\,, \qquad \mathrm{TM}_1$$

$$a = -\frac{1}{2} r\cos\delta\,, \qquad \mathrm{TM}_2\,,$$

I've been thinking...

- Test the symmetry of flavour at the extreme (very high energy) in the attainable environment (upcoming experiments) in an indirect way?


Flavour symmetry
Standard osc.
???
predict
constrain


Flavour symmetry
predict
constrain
Oscillations

Trust Higgs Mechanism before detecting it

Higgs vacuum energy: unknown til detecting it

$$\frac{W^+/W^- \text{ Mass}}{Z \text{ Mass}} = \frac{g^H_W \cdot \mathbf{v}}{g^H_Z \cdot \mathbf{v}}$$

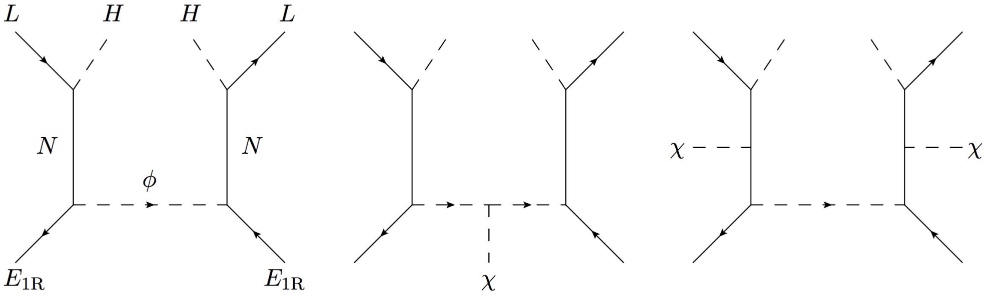
measurement

Two g’s (interaction strengths) can be predicted by $SU(2)_L$ X $U(1)_Y$ gauge theory.

UV COMPLETION AND A4->Z2

$$\mathcal{L} = (D_\mu\phi)^\dagger(D^\mu\phi) - (M_\phi^2)_{\alpha\beta}\phi_\alpha^*\phi_\beta + \overline{N}i\partial\!\!\!/ N - M_{N\alpha\beta}\overline{N_{\alpha R}}N_{\beta L}$$
$$-\kappa_{\alpha\beta\gamma}\overline{E_{\alpha R}}N_{\beta L}\phi_\gamma^* - y_{\alpha\beta}\overline{L_\alpha}\tilde{H}N_{\beta R} + \text{h.c.}\,,$$

- We introduce one sterile neutrino and one charged scaler.
- The main constraint is by the nonunitarity, which allows NSIs at the level of 0.01.
- The NSI matrix is simplified with 4 parameters (x,y,z,w).



(a) (b) (c)

$$\begin{pmatrix} y & x-z-iw & x+z+iw \\ x-z+iw & x+z & y-iw \\ x+z-iw & y+iw & x-z \end{pmatrix}$$

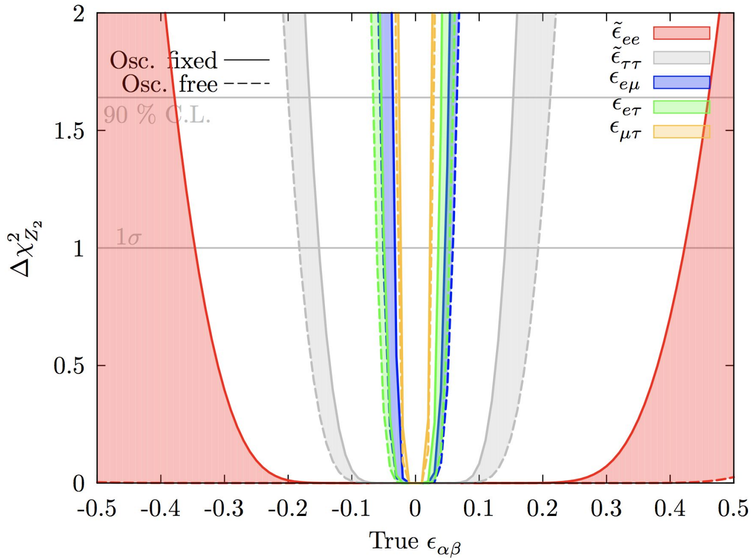
Testing Z2 @DUNE

- DUNE can measure x, y, z with the precision competitive to global fit results. In addition, it can measure the parameter for image texture with high precision.
- By global fit result, we find strongest constraint on y, z, w, and are suggested the NSI matrix with x parameter. Two sum rules are also raised up.
- DUNE can test these two sum rules with high exclusion level, except the ee component.

Global Fit		Global Fit		DUNE sensitivity	
w^u	$-$	w^d	$-$	w	$[-0.013, 0.025]$
x^u	$[-0.034, 0.013]$	x^d	$[-0.035, 0.012]$	x	$[-0.1, 0.1]$
y^u	$[-0.004, 0.003]$	y^d	$[-0.004, 0.003]$	y	$[-0.01, 0.01]$
z^u	$[-0.002, 0.005]$	z^d	$[-0.002, 0.005]$	z	$[-0.007, 0.017]$

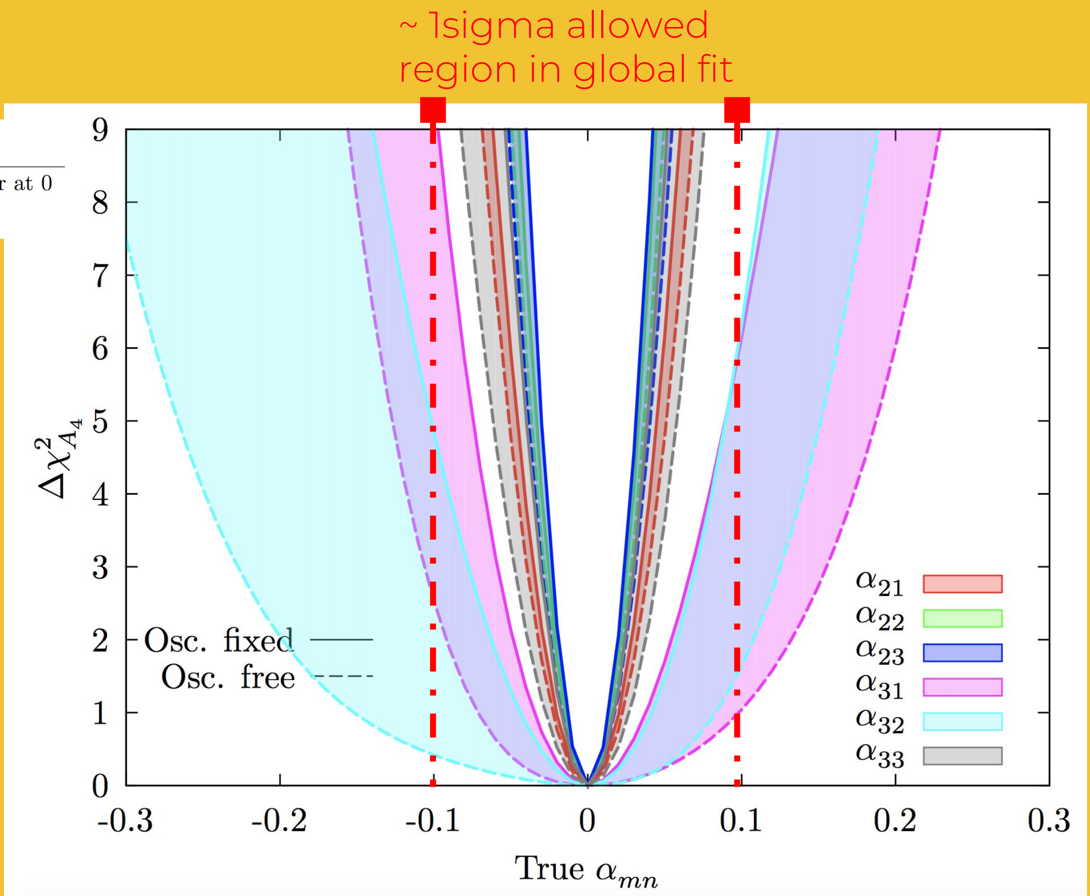
$$\epsilon = \begin{pmatrix} 0 & x & x \\ x & x & 0 \\ x & 0 & x \end{pmatrix} .$$

$$\epsilon_{e\mu} = \epsilon_{e\tau} = -\tilde{\epsilon}_{ee} \, ,$$
$$\epsilon_{\mu\tau} = \tilde{\epsilon}_{\tau\tau} = 0 \, .$$



Testing A4 @DUNE

	Osc. Para.	$\alpha_{12},\ \alpha_{13}$	$\alpha_{2n},\ \alpha_{3n}$
true values	fix them at B.F.	fix them at 0	change one; fix the other at 0
tested values	all fixed or free	allow them varying	fix all at 0



- High exclusion level for A4 symmetry is seen. It can reach ~3sigma level around the 1sigma boundary of Global Fit results.
- Including atmospheric neutrino data is the key.
- Given a symmetry model, the result should locate in the band.

Testing A4 @DUNE

1. We cover all possible correlations with oscillation parameters, and consider possible degree of freedom. The results can been seen as a general case.
2. We assume Wilks' theorem is applicable for high-sensitivity cases.
3. High exclusion level for A4 symmetry is seen.
4. α_{23} has better sensitivity at the right panel, because its 1σ allowed region is wider in GF results.

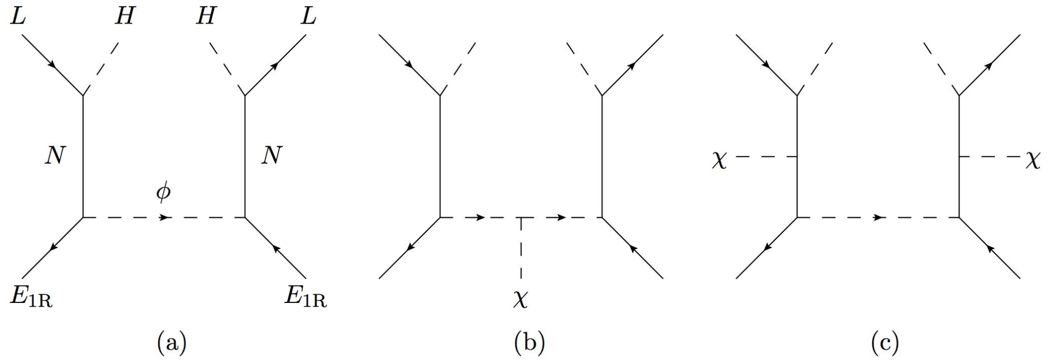
Testing Z2 @DUNE

1. We cover all possible correlations with oscillation parameters, and consider possible degree of freedom. The results can been seen as a general case.
2. We assume Wilks' theorem is applicable for high-sensitivity cases.
3. High exclusion level for A4 symmetry is seen.
4. α_{23} has better sensitivity at the right panel, because its 1σ allowed region is wider in GF results.

Z_2 Symmetry with one UV completion

$$\mathcal{L} = (D_\mu\phi)^\dagger(D^\mu\phi) - (M_\phi^2)_{\alpha\beta}\phi_\alpha^*\phi_\beta + \overline{N}i\partial\!\!\!/ N - M_{N\alpha\beta}\overline{N_{\alpha\mathrm{R}}}N_{\beta\mathrm{L}} - \kappa_{\alpha\beta\gamma}\overline{E_{\alpha\mathrm{R}}}N_{\beta\mathrm{L}}\phi_\gamma^* - y_{\alpha\beta}\overline{L_\alpha}\tilde{H}N_{\beta\mathrm{R}} + \mathrm{h.c.}\,,$$

$$\epsilon^e = \frac{1}{8G_F^2}(yM_N^{-1}\kappa_e)(M_\phi^2)^{-1}(yM_N^{-1}\kappa_e)^\dagger$$



$$\frac{\mu_\phi^2}{v_\chi}\left[\frac{2}{3}f_\mathrm{S}\Big(\chi(\phi^*\phi)_{\mathbf{3}_\mathrm{S}}\Big)_{\mathbf{1}} - \frac{2}{\sqrt{3}}f_\mathrm{A}\Big(\chi(\phi^*\phi)_{\mathbf{3}_\mathrm{A}}\Big)_{\mathbf{1}}\right]$$

$$\frac{\mu_N}{v_\chi}\left[\frac{2}{3}g_\mathrm{S}\big(\chi(\overline{N_\mathrm{L}}N_\mathrm{R})_{\mathbf{3}_\mathrm{S}}\big)_{\mathbf{1}} - \frac{2}{\sqrt{3}}g_\mathrm{A}\big(\chi(\overline{N_\mathrm{L}}N_\mathrm{R})_{\mathbf{3}_\mathrm{A}}\big)_{\mathbf{1}}\right] + \mathrm{h.c.}$$

$$\epsilon^e = \alpha_0\{[-(2+|g_\mathrm{S}|^2+|g_\mathrm{A}|^2)(|g_\mathrm{S}|^2+|g_\mathrm{A}|^2) + 4\mathrm{Re}(g_\mathrm{S}^2+g_\mathrm{A}^2) + 4[\mathrm{Im}(g_\mathrm{S}^*g_\mathrm{A})]^2]\mathbb{T}_1 - 2\mathrm{Re}(g_\mathrm{S})\mathbb{T}_2 - 2\mathrm{Re}(g_\mathrm{A})\mathbb{T}_3 - 2\mathrm{Im}(g_\mathrm{S}^*g_\mathrm{A})\mathbb{T}_4\}\ .$$

Z_2 Symmetry in global fits

$$\epsilon^e = \alpha_0 \{ [-(2+|g_S|^2+|g_A|^2)(|g_S|^2+|g_A|^2) + 4\mathrm{Re}(g_S^2+g_A^2) + 4[\mathrm{Im}(g_S^* g_A)]^2] \mathbb{T}_1 - 2\mathrm{Re}(g_S)\mathbb{T}_2 - 2\mathrm{Re}(g_A)\mathbb{T}_3 - 2\mathrm{Im}(g_S^* g_A)\mathbb{T}_4 \} \, .$$

$$\frac{\mu_N}{v_\chi}\left[\frac{2}{3} g_S \left(\chi(\overline{N_L} N_R)_{\mathbf{3}_S}\right)_{\mathbf{1}} - \frac{2}{\sqrt{3}} g_A \left(\chi(\overline{N_L} N_R)_{\mathbf{3}_A}\right)_{\mathbf{1}}\right] + \mathrm{h.c.}$$

$$\frac{\mu_\phi^2}{v_\chi}\left[\frac{2}{3} f_S \left(\chi(\phi^*\phi)_{\mathbf{3}_S}\right)_{\mathbf{1}} - \frac{2}{\sqrt{3}} f_A \left(\chi(\phi^*\phi)_{\mathbf{3}_A}\right)_{\mathbf{1}}\right]$$

$$\begin{pmatrix} -x & x+y-z-iw & x+y+z+iw \\ x-z+iw & z & y-iw \\ x+z-iw & y+iw & -z \end{pmatrix}$$

$$x \equiv \alpha_2,\; y \equiv -\frac{\alpha_1}{3} + \frac{2\alpha_2}{3\sqrt{2}},\; z \equiv \frac{\alpha_3}{\sqrt{3}} \text{ and } w \equiv \frac{\alpha_{31}}{\sqrt{6}}$$

$$\epsilon = \begin{pmatrix} 0 & x & x \\ x & x & 0 \\ x & 0 & x \end{pmatrix}$$

Global Fit		Global Fit		DUNE sensitivity	
w^u	– **x3**	w^d	– **x3**	w	$[-0.013, 0.025]$
x^u	$[-0.034, 0.013]$	x^d	$[-0.035, 0.012]$	x	$[-0.1, 0.1]$
y^u	$[-0.004, 0.003]$	y^d	$[-0.004, 0.003]$	y	$[-0.01, 0.01]$
z^u	$[-0.002, 0.005]$	z^d	$[-0.002, 0.005]$	z	$[-0.007, 0.017]$

$$\epsilon_{e\mu} = \epsilon_{e\tau} = -\tilde{\epsilon}_{ee}\,, \qquad \epsilon_{\mu\tau} = \tilde{\epsilon}_{\tau\tau} = 0\,.$$