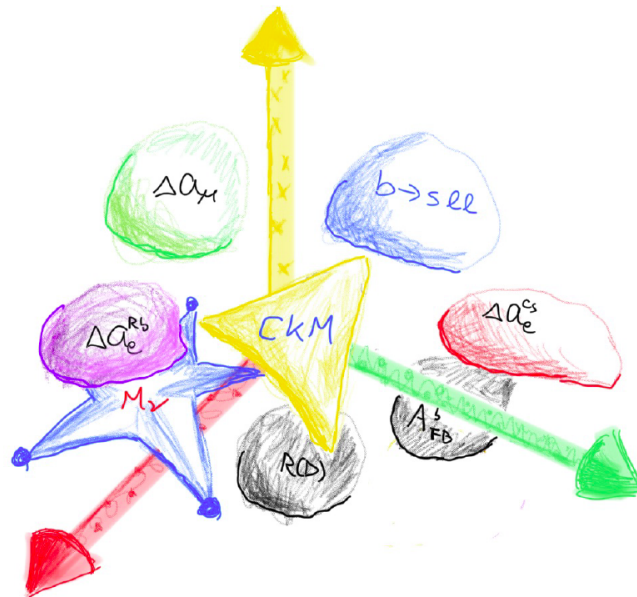


A colorful framework to accommodate the observed anomalies nowadays.



**We-Fu Chang**  
National Tsing Hua University  
IOPAS HEP seminar, Jan. 21, 2022

- Motivation–Anomalies
- The Model and its solutions
- Constraints and Numerical Study
- Phenomenological consequences
- Origin of the flavor pattern?
- Conclusion

See [JHEP 09 \(2021\) 043](#) for detailed ref.

# 1/6. Introduction

## Standard Model (SM)

= special relativity +

+ QM

QFT

particle content +

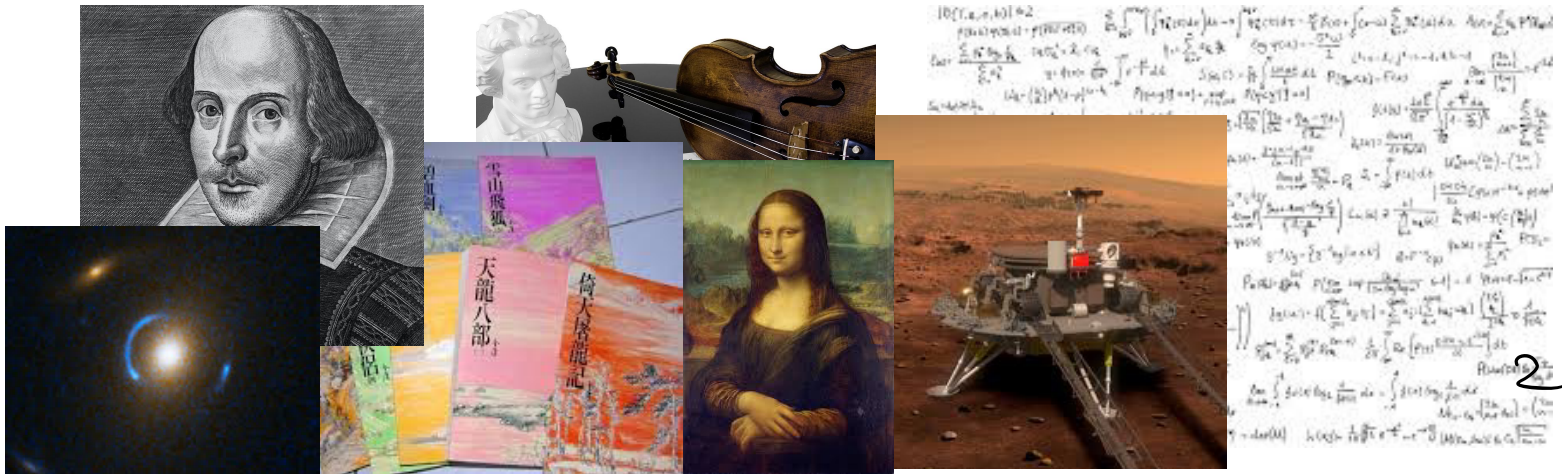
Quarks + Leptons  
+ Higgs

gauge symmetries  
and breaking

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

gluons  $W^\pm, Z^0, \gamma$

One of the highest intellectual achievements



A colorful framework to accommodate the observed anomalies in

# 1/6. Introduction

## Standard Model

$$\begin{array}{l}
 \simeq \text{special relativity} + \text{particle content} + \text{gauge symmetries and breaking} \\
 + \text{QM} \\
 \text{QFT} \qquad \text{Quarks + Leptons} \qquad SU(3)_C \times SU(2)_L \times U(1)_Y \\
 \qquad \qquad \qquad + \text{Higgs} \qquad \qquad \text{gluons} \qquad \qquad W^\pm, Z^0, \gamma
 \end{array}$$

## One of the highest intellectual achievements

$\Delta a_e$  : electron anomalous magnetic moment

$$\Delta a_e^{\text{exp}} = 1159652180.73 (28) \times 10^{-12}$$

PRA 83, 052122 (2011)

$$\Delta a_e^{\text{th}} = 1159652182.032 (720) \times 10^{-12}$$

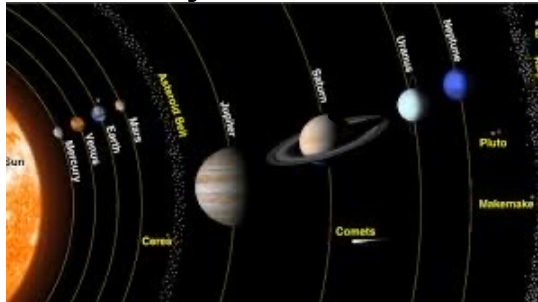
PRD 97, 036001 (2018)  $\alpha(\mathbb{R}_g \ 2016)$

$$\begin{array}{l}
 g = 1 \quad \text{in classical EM} \\
 g = 2 \quad \text{by Dirac Eq.} \\
 g = 2(1 + \Delta a)
 \end{array}$$



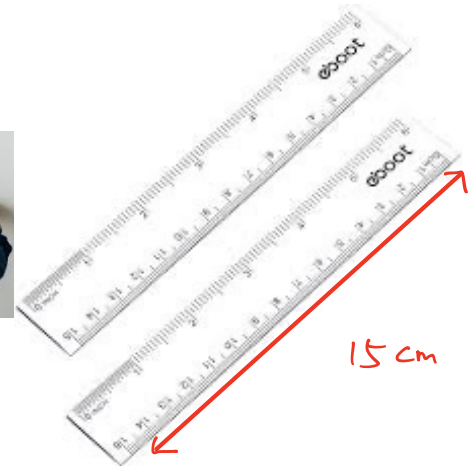
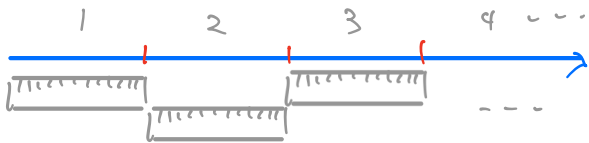
# 1/6. Introduction

What does  $10^{-12}$  accuracy mean ?



$$1AU = 1.5 \times 10^{11}m$$
$$10^{-12}AU \sim 15cm$$

Say, each measurement per second



$$1y \sim \pi \times 10^7 \text{ sec}$$

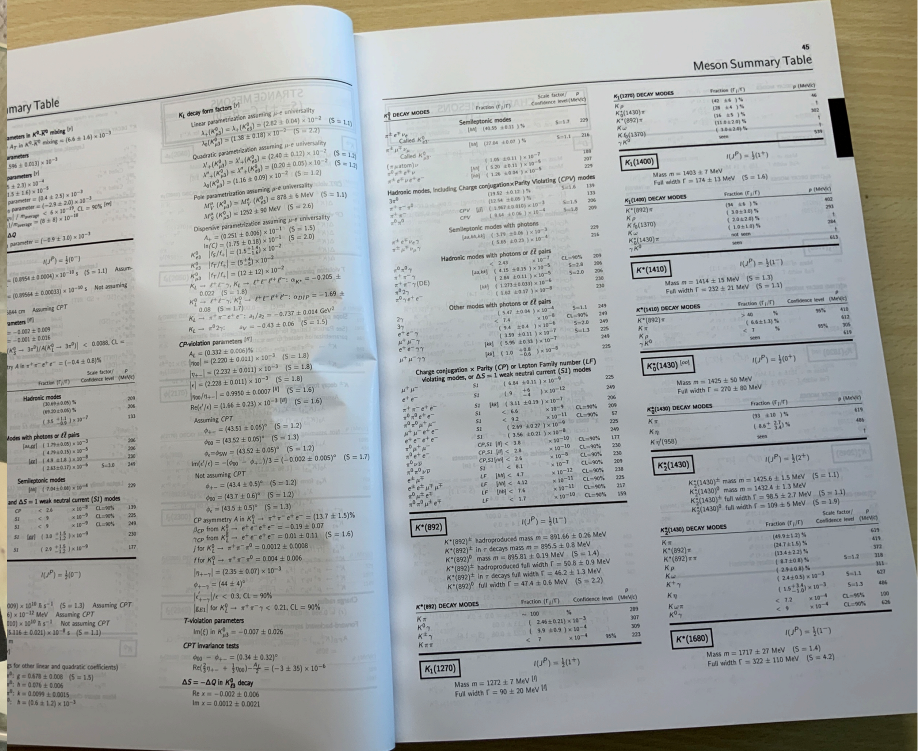
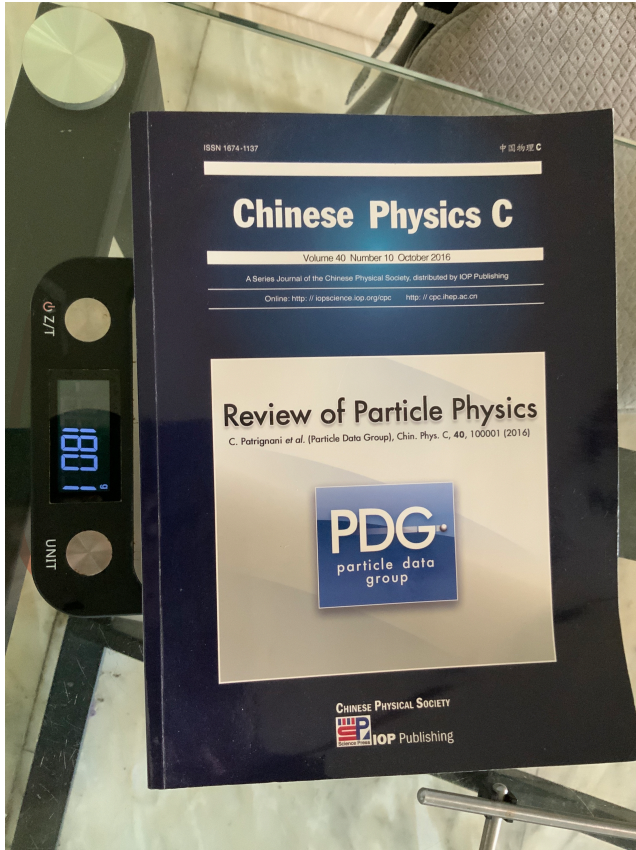
(w/o any rest)

You need to live for  $\sim 3 \times 10^4$  years to complete

this task, and **ONLY 1 measurement mistake is allowed!!**

2.5

Not only that



2.5

A colorful framework to accommodate the observed anomalies in

# 1/6. Introduction

However, SM is not the end of the story; it is incomplete.

SM=QM+SR

QFT

Quantum Gravity?  
Dark Energy ?

Dark Matter?

Why 3 generations?

+particle content

Quarks +Leptons  
+ Higgs

Neutrino mass?  
Antimatter? CPV?  
Flavor pattern?

+ gauge symmetries  
and breaking

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Non-perturbative

{ confinement  
proton mass }  $\Rightarrow$  { Models  
Lattice }



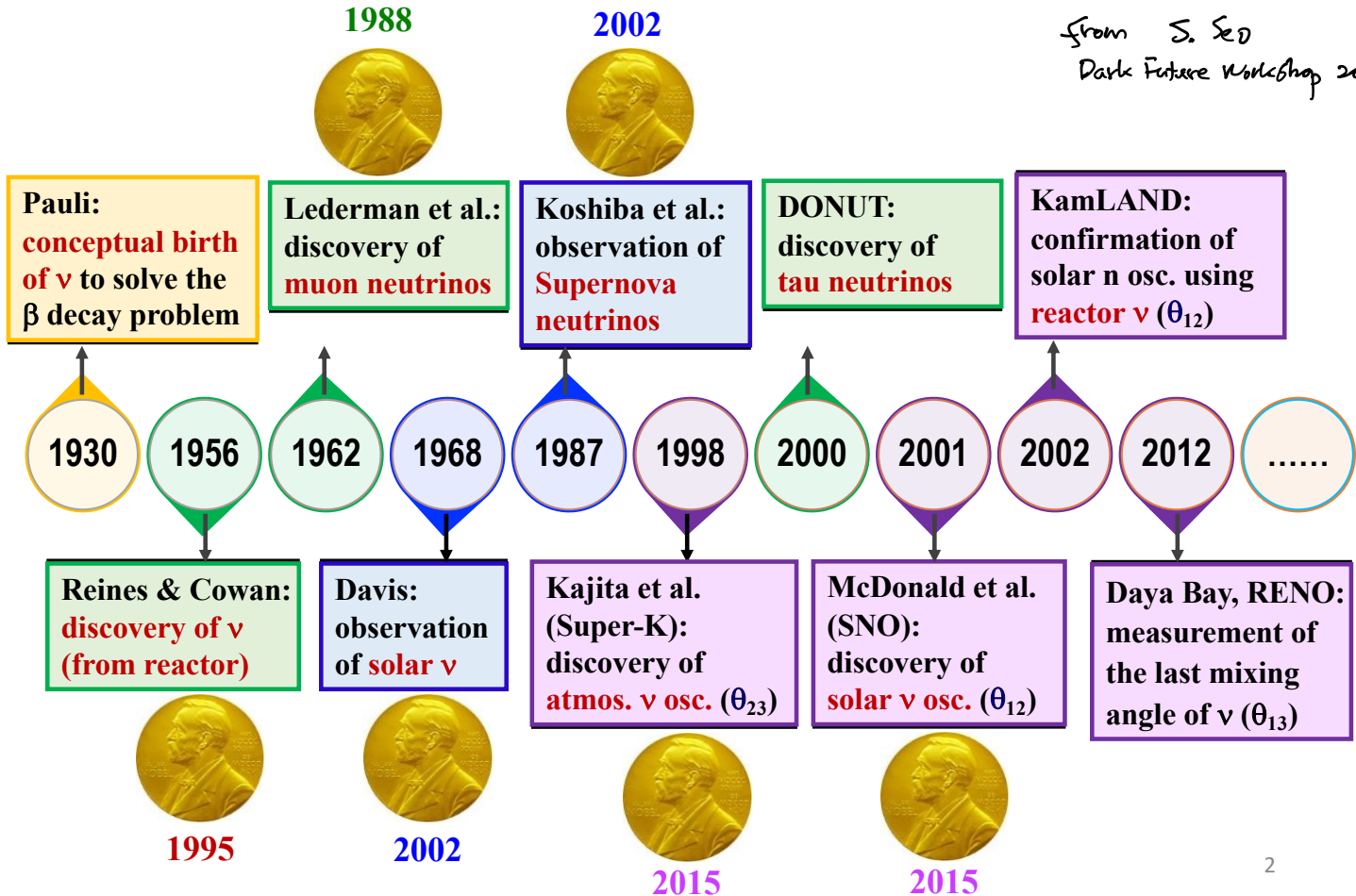
We might need some  
revolutionary ideas



the main playground for  
the traditional  
theoretical particle phys.

# Milestones of $\nu$ History

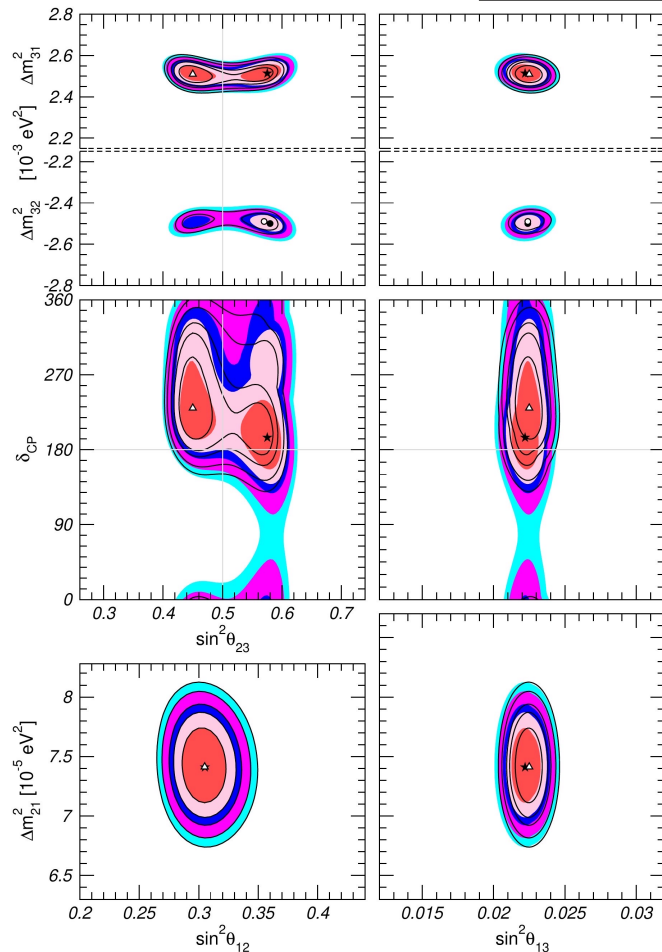
From S. Seo  
Dark Future Workshop 2021



NuFIT 5.1 (2021)

	Normal Ordering (best fit)				Inverted Ordering ( $\Delta\chi^2 = 2.6$ )					
	bfp $\pm 1\sigma$		$3\sigma$ range		bfp $\pm 1\sigma$		$3\sigma$ range			
	without SK atmospheric data	$\sin^2 \theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \rightarrow 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \rightarrow 0.343$	$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \rightarrow 35.86$	$33.45^{+0.77}_{-0.74}$
	$\theta_{23}/^\circ$	$0.573^{+0.018}_{-0.023}$	$0.405 \rightarrow 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \rightarrow 0.623$	$\theta_{23}/^\circ$	$49.2^{+1.0}_{-1.3}$	$39.5 \rightarrow 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \rightarrow 52.1$
	$\sin^2 \theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \rightarrow 0.02430$	$0.02238^{+0.00064}_{-0.00062}$	$0.02053 \rightarrow 0.02434$	$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \rightarrow 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \rightarrow 8.98$
	$\delta_{CP}/^\circ$	$194^{+52}_{-25}$	$105 \rightarrow 405$	$287^{+27}_{-32}$	$192 \rightarrow 361$	$\delta_{CP}/^\circ$	$194^{+52}_{-25}$	$105 \rightarrow 405$	$287^{+27}_{-32}$	$192 \rightarrow 361$
	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \rightarrow +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \rightarrow -2.413$
with SK atmospheric data	Normal Ordering (best fit)				Inverted Ordering ( $\Delta\chi^2 = 7.0$ )					
	bfp $\pm 1\sigma$		$3\sigma$ range		bfp $\pm 1\sigma$		$3\sigma$ range			
	$\sin^2 \theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \rightarrow 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \rightarrow 0.343$	$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	$31.27 \rightarrow 35.87$	$33.45^{+0.78}_{-0.75}$	$31.27 \rightarrow 35.87$
	$\theta_{23}/^\circ$	$0.450^{+0.019}_{-0.016}$	$0.408 \rightarrow 0.603$	$0.570^{+0.016}_{-0.022}$	$0.410 \rightarrow 0.613$	$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	$39.7 \rightarrow 50.9$	$49.0^{+0.9}_{-1.3}$	$39.8 \rightarrow 51.6$
	$\sin^2 \theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \rightarrow 0.02435$	$0.02241^{+0.00074}_{-0.00062}$	$0.02055 \rightarrow 0.02457$	$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	$8.25 \rightarrow 8.98$	$8.61^{+0.14}_{-0.12}$	$8.24 \rightarrow 9.02$
	$\delta_{CP}/^\circ$	$230^{+36}_{-25}$	$144 \rightarrow 350$	$278^{+22}_{-30}$	$194 \rightarrow 345$	$\delta_{CP}/^\circ$	$230^{+36}_{-25}$	$144 \rightarrow 350$	$278^{+22}_{-30}$	$194 \rightarrow 345$
	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.510^{+0.027}_{-0.027}$	$+2.430 \rightarrow +2.593$	$-2.490^{+0.026}_{-0.028}$	$-2.574 \rightarrow -2.410$

NuFIT 5.1 (2021)





# 1/6. Introduction

We have kept seeing many ambulances passing by.



# 1/6. Introduction

We have kept seeing many ambulances passing by.

Some wrecked . . . .



# 1/6. Introduction

We have kept seeing many ambulances passing by.

Some wrecked . . . .

Some keep going . . . .





# 1/6. Introduction

$b \rightarrow sll$

- $$R_K^{(*)} = \frac{\Gamma(B \rightarrow K^{(*)}\mu\mu)}{\Gamma(B \rightarrow K^{(*)}ee)}$$

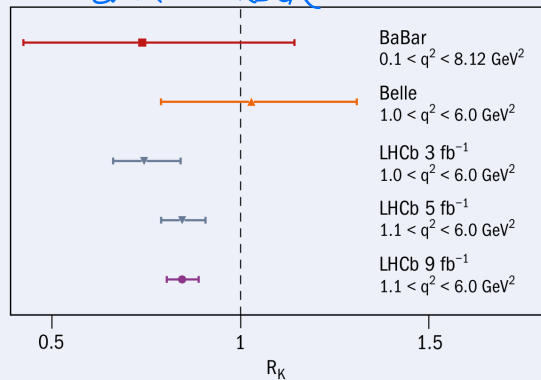
- $$R_K^{SM} = 1.0004_{-0.0007}^{+0.0008}$$

Geng

TABLE I. Key inputs used in

Observable	Value
	$(2.8_{-0.7}^{+0.8}) \times 10^{-9}$
$BR(B_s \rightarrow \mu^+\mu^-)$	$(2.9 \pm 0.7 \pm 0.2) \times 10^{-9}$
	$(3.09_{-0.43-0.11}^{+0.46+0.15}) \times 10^{-9}$
	$(2.842 \pm 0.333) \times 10^{-9}$
	$(3.63 \pm 0.13) \times 10^{-9}$
$R_K[1.1, 6]$	$0.846 \pm 0.044$
$R_K[1, 6]$	$1.03 \pm 0.28$
$R_{K^*}[0.045, 1.1]$	$0.660 \pm 0.113$
$R_{K^*}[1.1, 6]$	$0.685 \pm 0.122$
$R_{K^*}[0.045, 1.1]$	$0.52 \pm 0.365$
$R_{K^*}[1.1, 6]$	$0.96 \pm 0.463$

CENR COURIER



$$\mathcal{H}_{\text{eff}}^{b \rightarrow s\mu\mu} = -\frac{G_F}{\sqrt{2}} \tilde{V}_{tb} \tilde{V}_{ts}^* \frac{\alpha}{\pi} \sum_i C_i \mathcal{O}_i + H.c.$$

$$\mathcal{O}_9 = (\bar{s}\gamma^\alpha \hat{L}b) (\bar{\mu}\gamma_\alpha \mu), \quad \mathcal{O}_{10} = (\bar{s}\gamma^\alpha \hat{L}b) (\bar{\mu}\gamma_\alpha \gamma^5 \mu),$$

$$\mathcal{O}'_9 = (\bar{s}\gamma^\alpha \hat{R}b) (\bar{\mu}\gamma_\alpha \mu), \quad \mathcal{O}'_{10} = (\bar{s}\gamma^\alpha \hat{R}b) (\bar{\mu}\gamma_\alpha \gamma^5 \mu).$$

## Global fit:(after Moriond 2021)

Altmannshofer and Stangl, 2103. 13370

$$C_9 \simeq -0.73 \quad \text{or} \quad C_9 \simeq -C_{10} \simeq -0.39$$

$$\left( \text{theoretically clean modes only} \quad C_{10} \simeq 0.60, C_9 \simeq -C_{10} \simeq -0.35 \right)$$

Geng, Grinstein et al, 2103,12738

$$C_9 \simeq -0.82, \quad C_9 \simeq -C_{10} \simeq -0.40$$

$$C_{10} \simeq 0.65$$

# 1/6. Introduction

$$\Delta a_l$$

$\vec{\mu}$  (Magnetic moment)  $\sim I \vec{A}$  by EM

by Dimension-Analysis  $[\vec{\mu}] = \left(\frac{QL^2}{T}\right)_+$

and it is a pseudo-vector

$\sim$  spin

$$[q] = Q, [m] = M, [\vec{S}] = \hbar \vec{1} = \frac{ML^2}{T}$$

$$\Rightarrow \vec{\mu} \propto \frac{e}{m} \vec{S} \quad \vec{\mu} = \frac{ge\hbar}{2m} \frac{\vec{\sigma}}{2}$$

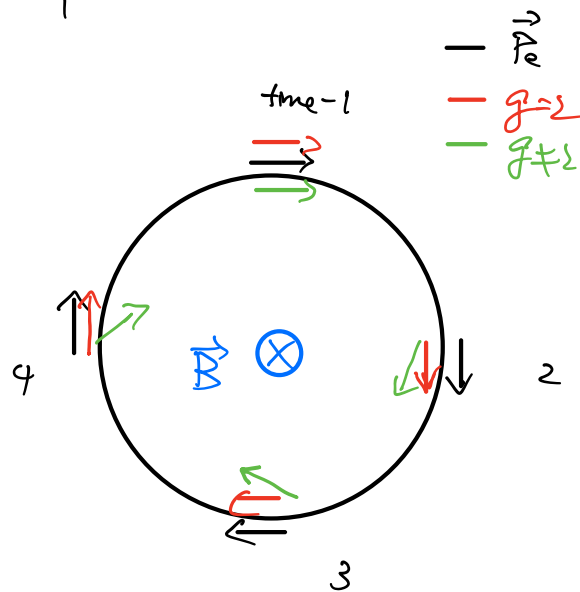
$g = 1$  in classical EM

Dirac Eq.  $\Rightarrow g = 2$  namely

(tree level)

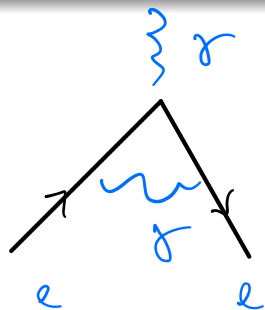
Quantum correction

$$g = 2(1 + \Delta a)$$



# 1/6. Introduction

$\Delta a_l$



$$\Delta a^{(1)} = \frac{\alpha}{2\pi} \sim 1.1617 \times 10^{-3}$$

$$\Delta a^{(2)} \sim O(10^{-7})$$



in terms of  $\mathcal{L} \supset \bar{L} \sigma^{\mu\nu} F_{\mu\nu} e_R H \Rightarrow \dim -6 \quad \frac{1}{\Lambda^2}$

chirality flip  $\Rightarrow \propto m_e$  (or  $M_F$  in the loop), another normalization  $\propto m_e^2$

extremely sensitive to  $\alpha \equiv \frac{e^2}{4\pi} \quad (m_e m_F)$

$$\Delta a_e^{Cs} = a_e^{exp} - a_e^{SM} \simeq (-8.7 \pm 3.6) \times 10^{-13}$$

$\alpha^{-1} = 137.035999046$  (27), recoil frequency of  $^{133}Cs$  in matter-wave interferometer 1812.04130

$\Delta a_e^{Rb} \simeq (+4.8 \pm 3.0) \times 10^{-13}$  recoil velocity of  $^{87}Rb$

$$\alpha^{-1} = 137.035999206$$
 (11)

Nature 588, 61

$\sim 5\sigma$  difference!!

9

# 1/6. Introduction

$$\Delta a_\mu$$



$$\sim \frac{\alpha}{2\pi} \sim \mathcal{O}(10^{-3})$$

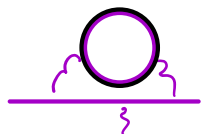
$$a_\mu^{QED} = 116584718.931 (104) \times 10^{-11}$$



$$\sim \frac{G m_\mu^2}{16\pi^2} \sim \mathcal{O}(10^{-9})$$

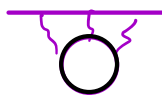
$$a_\mu^{EW} = (153.6 \pm 1.0) \times 10^{-11}$$

$a_\mu^{HVP}$ : Largest uncertainty



$$\sim \frac{\alpha m_\mu^2}{(16\pi^2) m_\pi^2} a_\mu^{QED} \sim \mathcal{O}(10^{-7})$$

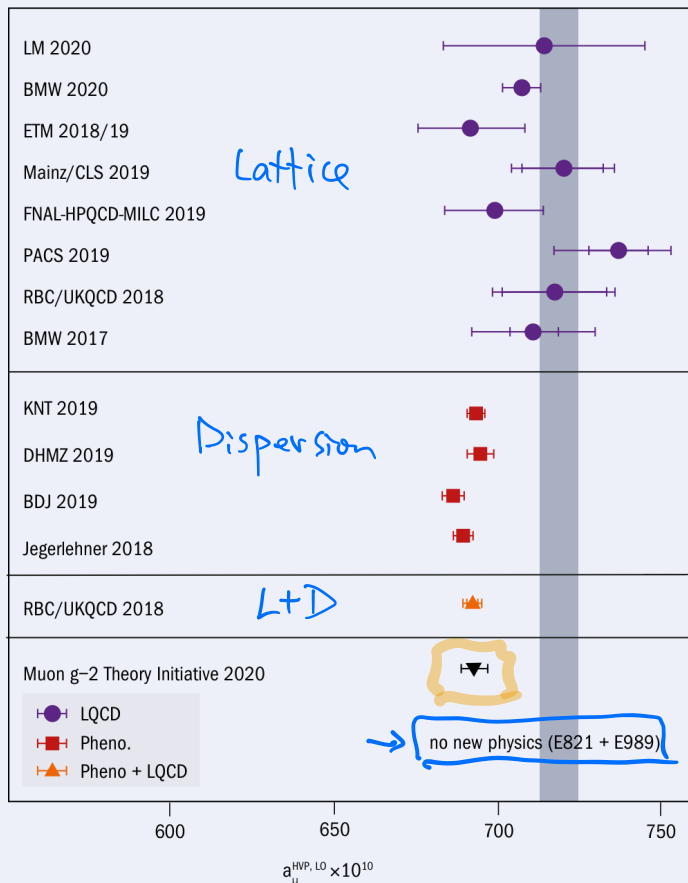
$$a_\mu^{HLBL} \times 10^{11} \sim \begin{cases} 90 \pm 20 & \text{pheno} \\ 80 \pm 35 & \text{Lattice} \end{cases}$$



$$\sim \frac{\alpha}{(16\pi^2)} a_\mu^{HVP} \sim \mathcal{O}(10^{-11})$$

$$\Delta a_\mu \simeq (25.1 \pm 5.9) \times 10^{-10}$$

CERN COURIER



$$\begin{aligned}
 & \bullet V_{CKM} \equiv V_L^u (V_L^d)^\dagger = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \sim 0.97 & \sim 0.23 & \sim 4 \times 10^{-3} \\ V_{cd} & V_{cs} & V_{cb} \\ \sim -0.123 & \sim 0.97 & \sim 4 \times 10^{-2} \\ V_{td} & V_{ts} & V_{tb} \\ \sim 8 \times 10^{-3} & \sim -4 \times 10^{-2} & \sim 0.99 \end{pmatrix} \quad |V_{us}| \sim |V_{cd}| \sim |\sin \theta_{\text{Cabbibo}}| \\
 & \bullet V_{CKM} V_{CKM}^\dagger = \mathbb{1}_{3 \times 3}
 \end{aligned}$$

$$\Rightarrow |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad \left( \text{since } |V_{ub}|^2 \sim 10^{-5}, \quad |V_{ud}|^2 + |V_{us}|^2 \simeq 1 \right)$$

- The present PDG value:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985 \pm 0.0005$$

From the 2nd neutrino conference(1974), Pennsylvania,  
(As already pointed out by Y Uchida at CLFV2016.)

- The solar neutrino talk by R.K. Ulrik

[AIP Conference Proceedings 22](#), 259 (1974); doi: 10.1063/1.2947415

The  $^{37}\text{Ar}$  production rates for the standard model and the low Z model are  $5.6 \pm 1.8$  SNU and  $1.4 \pm 0.35$  SNU, respectively. Taking Davis's result<sup>1</sup> without run 27 to be  $0.2 \pm 0.8$  SNU I find that the discrepancy between the experiment and the standard model to be  $2.7\sigma$  and while the discrepancy with the low Z model is  $1.4\sigma$ .

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- The Conference Summary by R.P. Feynman

I see no reason to include the solar neutrino problem in here because it is only 1-1/2 standard deviations off of some solar models and the principle is, you don't make a new theory because of 1-1/2 standard deviations.

[AIP Conference Proceedings 22](#), 299 (1974); doi: 10.1063/1.2947418





## 2/6. The Model and its solutions

- ①  $b \rightarrow s \ell \ell$  : NP connects both lepton/quark sectors
- ② Neutrino Majorana mass: Lepton# violation
- ③ CKM leakage : mixing between new fermion and the down ( or up) sector.
- ④ AMM: implies possible heavy fermion in the loop and the new DOFs must be charged.

What can it be?

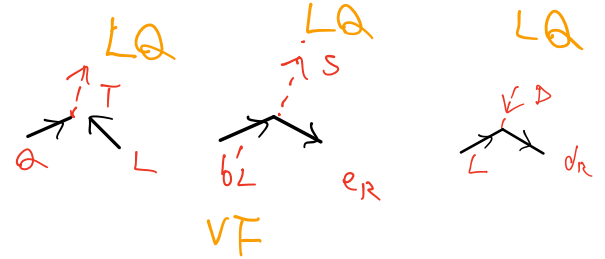
### A straightforward solution:

- ①  $b \rightarrow s \ell \ell$ : NP connects both lepton/quark sectors  $\Rightarrow$  Leptoquark
- ② Neutrino Majorana mass: Lepton# violation  $\Rightarrow$  LQ triplet + doublet
- ③ CKM leakage: mixing between new fermion and the down (or ~~up~~) sector.  $\Rightarrow$  vector  $b'$
- ④ AMM: implies possible heavy fermion in the loop  $\Rightarrow$   
and the new DOFs must be charged.  $\Rightarrow$  LQ doublet + singlet

# 2/6. The Model and its solutions

- 4 color states are employed in the model:

	New Fermion	New Scalar		
Fields	$b'_{L,R}$	$T = \begin{pmatrix} T^{\frac{2}{3}} \\ T^{-\frac{1}{3}} \\ T^{-\frac{4}{3}} \end{pmatrix}$	$D = \begin{pmatrix} D^{\frac{2}{3}} \\ D^{-\frac{1}{3}} \end{pmatrix}$	$S^{\frac{2}{3}}$
$SU(3)_c$	3	3	3	3
$SU(2)_L$	1	3	2	1
$U(1)_Y$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{6}$	$\frac{2}{3}$
lepton number	0	1	-1	-1
baryon number	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$



- Assuming 2-body decays:  $b' \rightarrow Wt, b\bar{\nu}, bH$ ,  $M_{b'} > 1.22 \text{ TeV}$

- LQ mass  $> 0.5 - 1.6 \text{ TeV}$  (see PDG)

ATLAS  
CMS } take  
 $M_{b'} = 1.5 \text{ TeV}$   
 $M_{LQ} \sim 1.10 \text{ TeV}$   
as ref. point

- Two possible scalar cubic interactions:

$$\begin{aligned} \mathcal{L} &\supset \mu_3 \{H, \tilde{D}\} \odot T + \mu_1 [H, D] S^{-\frac{2}{3}} + H.c. \\ &= \mu_3 \left[ H^+ D^{\frac{1}{3}} T^{-\frac{4}{3}} - \frac{1}{\sqrt{2}} \left( H^0 D^{\frac{1}{3}} - H^+ D^{-\frac{2}{3}} \right) T^{-\frac{1}{3}} - H^0 D^{-\frac{2}{3}} T^{\frac{2}{3}} \right] \\ &\quad - \mu_1 \frac{1}{\sqrt{2}} \left( H^0 D^{\frac{2}{3}} - H^+ D^{-\frac{1}{3}} \right) S^{-\frac{2}{3}} + H.c. \end{aligned}$$

- Assuming  $\mathcal{U}(1)_B$ , the most general Yukawa is:

$$\begin{aligned} \mathcal{L} \supset & -\tilde{\lambda}_T T^\dagger \cdot \{\bar{L}^c, Q\} - \tilde{\lambda}_D \bar{d}_R [L, D] - \tilde{\lambda}'_D \bar{b}'_R [L, D] - \tilde{\lambda}_S \bar{e}_R b'_L S^{-\frac{2}{3}} - \tilde{Y}'_d \bar{Q} b'_R H + H.c. \\ & = -\tilde{\lambda}_T \left[ \bar{\nu}^c u_L T^{-\frac{2}{3}} + (\bar{\nu}^c d_L + \bar{e}^c u_L) \frac{T^{\frac{1}{3}}}{\sqrt{2}} + \bar{e}^c d_L T^{\frac{4}{3}} \right] - \tilde{Y}'_d (\bar{u}_L H^+ + \bar{d}_L H^0) b'_R \\ & - \tilde{\lambda}_D \frac{\bar{d}_R}{\sqrt{2}} (\nu_L D^{-\frac{1}{3}} - e_L D^{\frac{2}{3}}) - \tilde{\lambda}'_D \frac{\bar{b}'_R}{\sqrt{2}} (\nu_L D^{-\frac{1}{3}} - e_L D^{\frac{2}{3}}) - \tilde{\lambda}_S \bar{e}_R b'_L S^{-\frac{2}{3}} + H.c. \end{aligned}$$

- In addition, new Dirac masses are allowed  $\mathcal{L} \supset M_1 \bar{b}'_R b'_L + M_2 \bar{d}_R b'_L + H.c.$

- down-type quark mass matrix becomes  $4 \times 4$  :

$$\mathcal{L} \supset -(\bar{d}_R, \bar{b}'_R) \mathcal{M}^d \begin{pmatrix} d_L \\ b'_L \end{pmatrix} + H.c., \quad \mathcal{M}^d = \begin{pmatrix} \frac{\tilde{Y}'_d v_0}{\sqrt{2}} & M_2 \\ \frac{\tilde{Y}'_d v_0}{\sqrt{2}} & M_1 \end{pmatrix}$$

- It can be diagonalized by  $4 \times 4$  biunitary rotation, and the CC int becomes

$$\mathcal{L} \supset \frac{g_2}{\sqrt{2}} (\bar{u}, \bar{c}, \bar{t}) \gamma^\alpha \hat{L} \tilde{V} \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix} W_\alpha^+ + H.c.$$

$$\tilde{V} = \begin{pmatrix} \tilde{V}_{ud} & \tilde{V}_{us} & \tilde{V}_{ub} & \tilde{V}_{ub'} \\ \tilde{V}_{cd} & \tilde{V}_{cs} & \tilde{V}_{cb} & \tilde{V}_{cb'} \\ \tilde{V}_{td} & \tilde{V}_{ts} & \tilde{V}_{tb} & \tilde{V}_{tb'} \end{pmatrix}$$

an unpleasant  
 $3 \times 4$  matrix!!

- Mathematical trick: consider an auxiliary 4x4 unitary matrix

$$\tilde{V}_4 \equiv \begin{pmatrix} U_L^u & 0 \\ 0 & 1 \end{pmatrix} \cdot (U_L^d)^\dagger = \begin{pmatrix} \tilde{V}_{ud} & \tilde{V}_{us} & \tilde{V}_{ub} & \tilde{V}_{ub'} \\ \tilde{V}_{cd} & \tilde{V}_{cs} & \tilde{V}_{cb} & \tilde{V}_{cb'} \\ \tilde{V}_{td} & \tilde{V}_{ts} & \tilde{V}_{tb} & \tilde{V}_{tb'} \\ (U_L^d)_{d4}^* & (U_L^d)_{s4}^* & (U_L^d)_{b4}^* & (U_L^d)_{b'4}^* \end{pmatrix}$$

- For simplicity, assume NO extra CPV  $\Rightarrow$   $\exists$  extra mixings between  $\begin{pmatrix} d \\ s \\ b \end{pmatrix} - b'$

$$(U_L^d)^\dagger = \begin{pmatrix} (U_{L3}^d)^\dagger & 0 \\ 0 & 1 \end{pmatrix} \cdot R_4, \text{ where } R_4 = \begin{pmatrix} c_1 & 0 & 0 & s_1 \\ -s_1 s_2 & c_2 & 0 & c_1 s_2 \\ -s_1 c_2 s_3 & -s_2 s_3 & c_3 & c_1 c_2 s_3 \\ -s_1 c_2 c_3 & -s_2 c_3 & -s_3 & c_1 c_2 c_3 \end{pmatrix}$$

- Finally, a delightful expression for  $\vec{V}_4$

$$\tilde{V}_4 = \begin{pmatrix} V_{CKM} & 0 \\ 0 & 1 \end{pmatrix} \cdot R_4$$

And

$$|\tilde{V}_{ud}|^2 + |\tilde{V}_{us}|^2 + |\tilde{V}_{ub}|^2 = 1 - |\tilde{V}_{ub'}|^2 \leq 1$$

leakage

From PDG:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = \underline{0.9985 \pm 0.0005}$$

and

$$\tilde{V}_{ub'} \simeq 0.9740s_1 + 0.2265c_1s_2 + 0.0036c_1c_2s_3e^{1.196i}$$

$d-b', s-b'$

we have

$$|s_1 + 0.233s_2| \simeq 0.039(7)$$

(note: only  $\theta_1, \theta_2$ )

Other predictions of CKM triangles :

$$\tilde{V}_{ud}\tilde{V}_{ub}^* + \tilde{V}_{cd}\tilde{V}_{cb}^* + \tilde{V}_{td}\tilde{V}_{tb}^* = -(U_L^d)_{d4}^*(U_L^d)_{b4}$$

$$|\tilde{V}_{cd}|^2 + |\tilde{V}_{cs}|^2 + |\tilde{V}_{cb}|^2 = 1 - |\tilde{V}_{cb'}|^2,$$

$$|\tilde{V}_{td}|^2 + |\tilde{V}_{ts}|^2 + |\tilde{V}_{tb}|^2 = 1 - |\tilde{V}_{tb'}|^2,$$

$$|\tilde{V}_{ud}|^2 + |\tilde{V}_{cd}|^2 + |\tilde{V}_{td}|^2 = 1 - |(U_L^d)_{d4}|^2,$$

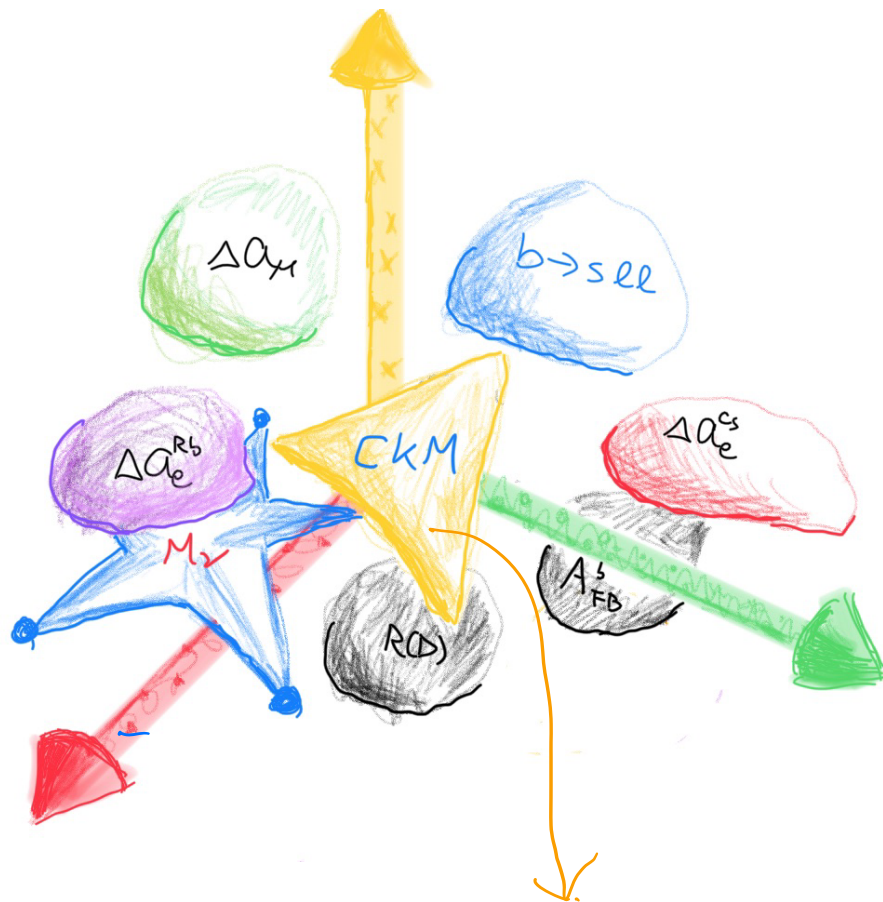
$$|\tilde{V}_{us}|^2 + |\tilde{V}_{cs}|^2 + |\tilde{V}_{ts}|^2 = 1 - |(U_L^d)_{s4}|^2,$$

$$|\tilde{V}_{ub}|^2 + |\tilde{V}_{cb}|^2 + |\tilde{V}_{tb}|^2 = 1 - |(U_L^d)_{b4}|^2,$$

In the mass basis,  $d_L$  and  $u_L$  have diff LH rotation.

$$\hookrightarrow -(X^T)_{ep} \left[ \bar{e}_e^c T^{\frac{4}{3}} + \bar{e}_e^c \frac{1}{\sqrt{2}} \right] d_{Lp} - (X^T)_{ep} \bar{V}_{pr}^{\dagger} \left[ \bar{e}_e^c T^{-\frac{2}{3}} + \bar{e}_e^c \frac{1}{\sqrt{2}} \right] u_{Lp}$$

using charged lepton/down quark as indices



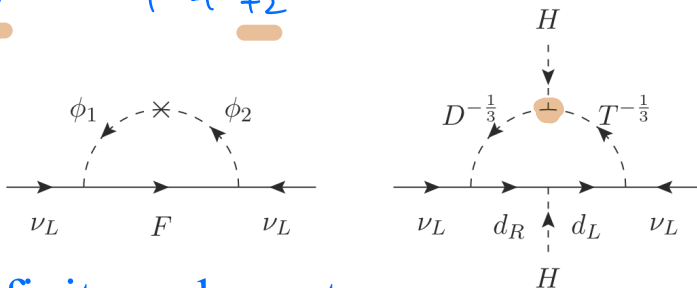
$$\begin{pmatrix} x & x & x & x \\ x & x & x & x \\ x & x & x & x \\ x & x & x & x \end{pmatrix} \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix}$$

Consider a general coupling:

$$\mathcal{L} \supset \lambda_{ij} \bar{F}_j \nu_{Li} \phi_1 + \kappa_{ij} \bar{F}_j \nu_i^c \phi_2 + H.c.$$

L-# :     - + | 0             - - + 2

$\mathcal{L}$  if  $\phi_1$  and  $\phi_2$  mix.



The 1-loop result is finite and exact:

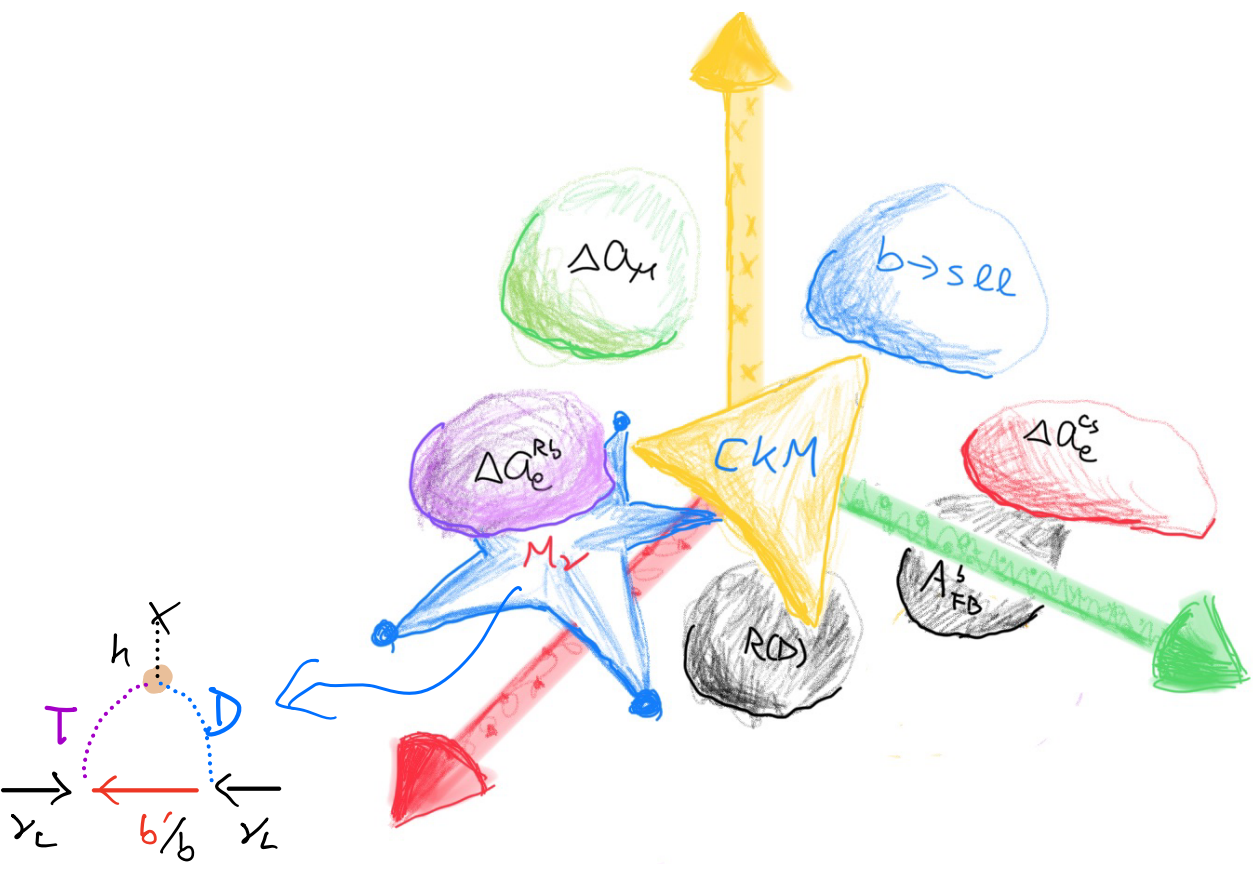
$$M_{ij}^\nu = \sum_k \frac{N_c^F m_k}{16\pi^2} s_\alpha c_\alpha (\kappa_{ik} \lambda_{jk} + \kappa_{jk} \lambda_{ik}) \left[ \frac{m_h^2}{m_h^2 - m_k^2} \ln \frac{m_h^2}{m_k^2} - \frac{m_l^2}{m_l^2 - m_k^2} \ln \frac{m_l^2}{m_k^2} \right]$$

$\alpha$ : the mixing angle between  $\phi_1$  and  $\phi_2$

In this model  $\phi_{1,2} \sim D, T$ , mixing  $\propto \mu_3$

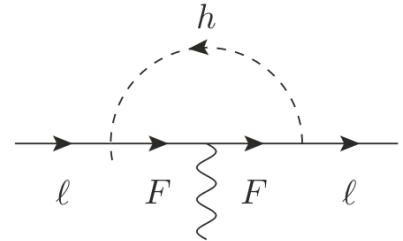
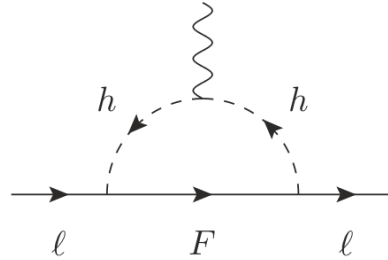
$$M_{ij}^\nu \simeq \sum_{k=d,s,b'} \frac{3m_k}{32\pi^2} (\lambda_{ik}^T \lambda_{jk}^D + \lambda_{jk}^T \lambda_{ik}^D) \frac{\mu_3 v_0}{M_D^2 - M_T^2} \ln \frac{M_T^2}{M_D^2}$$





## A general coupling

$$\mathcal{L} \supset \bar{F}(y_R^l \hat{R} + y_L^l \hat{L}) \ell h + H.c.$$



charge of F  
↓

$$\Delta a_l^h = \frac{-N_c^F (1 + Q_F) m_l^2}{8\pi^2} \int_0^1 dx x(1-x) \frac{x \frac{|y_L^l|^2 + |y_R^l|^2}{2} + \frac{m_F}{m_l} \Re[(y_R^l)^* y_L^l]}{x^2 m_l^2 + x(m_h^2 - m_l^2) + (1-x)m_F^2},$$

( $\Delta a_e^{cs} \Delta a_n < 0$ )

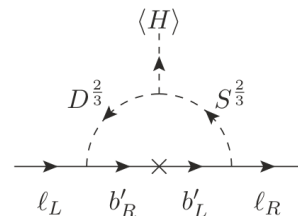
$$\Delta a_l^F = \frac{-N_c^F Q_F m_l^2}{8\pi^2} \int_0^1 dx x^2 \frac{(1-x) \frac{|y_L^l|^2 + |y_R^l|^2}{2} + \frac{m_F}{m_l} \Re[(y_R^l)^* y_L^l]}{x^2 m_l^2 + x(m_F^2 - m_l^2) + (1-x)m_h^2},$$

• If  $m_e \ll m_F$ ,

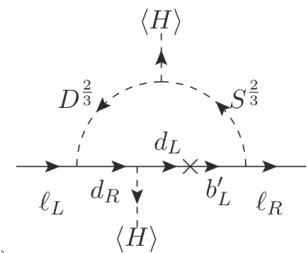
$$\Delta a_l = \Delta a_l^F + \Delta a_l^h \simeq -\frac{N_c^F \Re[(y_R^l)^* y_L^l]}{8\pi^2} \left(\frac{m_l}{m_F}\right) \mathcal{J}_{Q_F} \left(\frac{m_h^2}{m_F^2}\right)$$

and

$$\begin{aligned} \mathcal{J}_Q(\alpha) &= \int_0^1 dx \frac{x(1-x) + xQ}{x + (1-x)\alpha} \\ &= \frac{2Q(1-\alpha)(1-\alpha + \alpha \ln \alpha) + (1-\alpha^2 + 2\alpha \ln \alpha)}{2(1-\alpha)^3} \end{aligned}$$



(a)

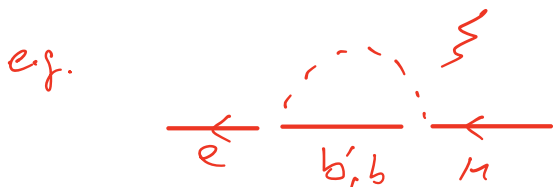


(b)

# 2/6. The Model and its solutions

$\Delta a_e$

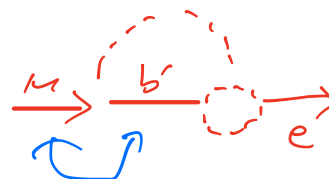
- NOTE: The diagrams also induce the FCNC  $L \rightarrow e \gamma$  transitions.



- A simple arrangement to avoid that,

$\begin{pmatrix} e \\ \mu \end{pmatrix}$  only couples to  $\begin{pmatrix} b \\ b' \end{pmatrix}$  or  $\begin{pmatrix} b' \\ b \end{pmatrix}$

$\Rightarrow b'$  Plays many roles in the model.



Sol-1 :

$$\Delta a_e \simeq 2.28 \times 10^{-5} \times [\lambda_{eb}^D \lambda_{eb}^S] \times \left( \frac{\mu_1}{\text{GeV}} \right) \times \mathcal{K}(b_D, b_S),$$

$$\Delta a_\mu \simeq 1.03 \times 10^{-10} \times [\lambda_{\mu b'}^D \lambda_{\mu b'}^S] \times \left( \frac{\mu_1}{\text{GeV}} \right) \left( \frac{1.5 \text{ TeV}}{M_{b'}} \right)^3 \times \mathcal{K}(\beta_D, \beta_S)$$

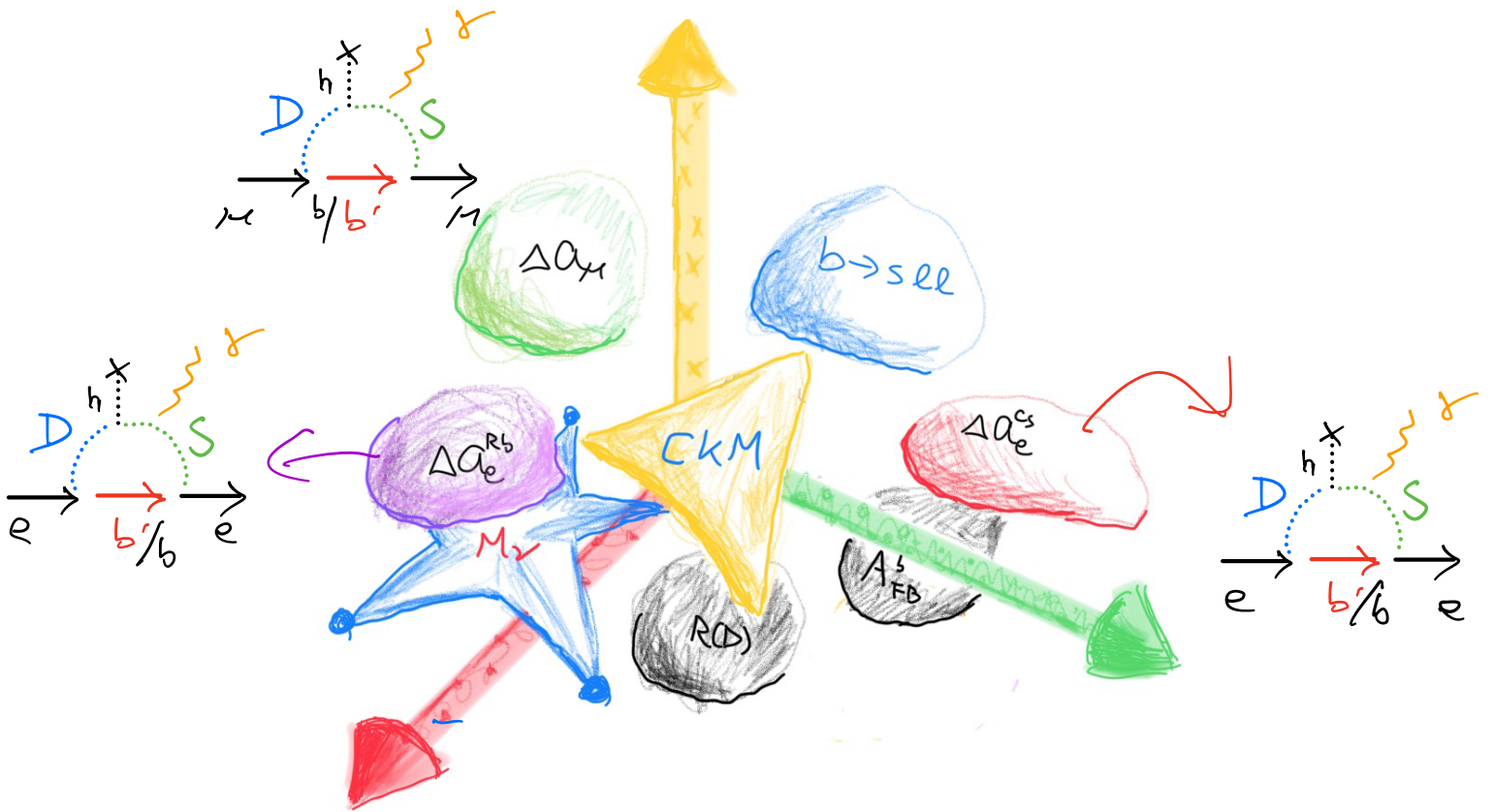
$(\text{CKM}, \Delta a_e, \Delta a_\mu, \text{FCNC})$   
 $\uparrow$

Sol-2 :

$$\Delta a_e \simeq 5.00 \times 10^{-13} \times [\lambda_{e b'}^D \lambda_{e b'}^S] \times \left( \frac{\mu_1}{\text{GeV}} \right) \left( \frac{1.5 \text{ TeV}}{M_{b'}} \right)^3 \times \mathcal{K}(\beta_D, \beta_S),$$

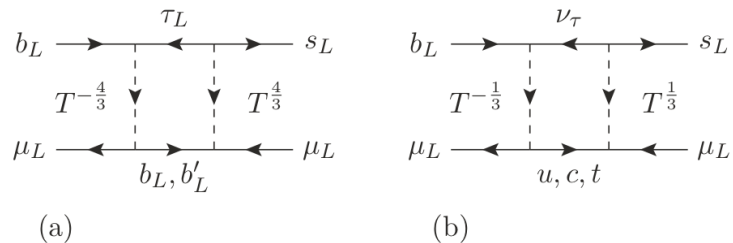
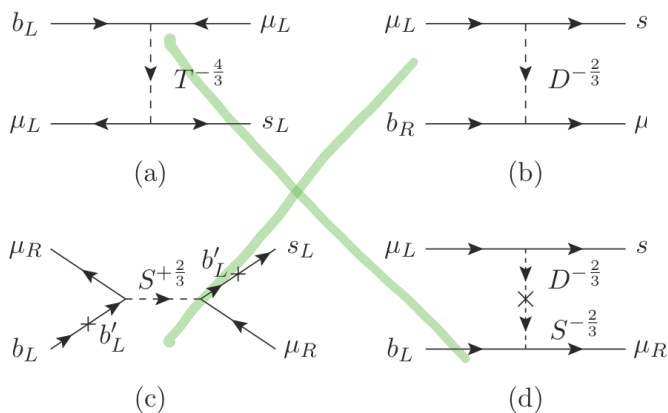
$$\Delta a_\mu \simeq 4.71 \times 10^{-3} \times [\lambda_{\mu b}^D \lambda_{\mu b}^S] \times \left( \frac{\mu_1}{\text{GeV}} \right) \times \mathcal{K}(b_D, b_S).$$

$$\mathcal{K}(a, b) \equiv \frac{\mathcal{J}_{-\frac{1}{3}}(a) - \mathcal{J}_{-\frac{1}{3}}(b)}{b - a}.$$



# 2/6. The Model and its solutions

$b \rightarrow s \ell \ell$



$$\mathcal{H}_{\text{eff}}^{b \rightarrow s \mu \mu} = -\frac{G_F}{\sqrt{2}} \tilde{V}_{tb} \tilde{V}_{ts}^* \frac{\alpha}{\pi} \sum_i C_i \mathcal{O}_i + H.c.$$

$$\mathcal{O}_9 = (\bar{s} \gamma^\alpha \hat{L} b) (\bar{\mu} \gamma_\alpha \mu), \quad \mathcal{O}_{10} = (\bar{s} \gamma^\alpha \hat{L} b) (\bar{\mu} \gamma_\alpha \gamma^5 \mu),$$

$$\mathcal{O}'_9 = (\bar{s} \gamma^\alpha \hat{R} b) (\bar{\mu} \gamma_\alpha \mu), \quad \mathcal{O}'_{10} = (\bar{s} \gamma^\alpha \hat{R} b) (\bar{\mu} \gamma_\alpha \gamma^5 \mu).$$

$$\mathcal{H}_{\text{eff}(a)}^{b \rightarrow s \mu \mu} \simeq -\frac{\lambda_{\tau b}^T (\lambda_{\tau s}^T)^*}{64\pi^2} \left( \frac{|\lambda_{\mu b'}^T|^2}{M^2} \mathcal{G}(\beta_T) + \frac{|\lambda_{\mu b}^T|^2}{m^2} \mathcal{G}(\beta_T) \right) (\bar{s} \gamma^\alpha \hat{L} b) (\bar{\mu} \gamma_\alpha \hat{L} \mu) + H.c.,$$

$$\mathcal{H}_{\text{eff}(b)}^{b \rightarrow s \mu \mu} \simeq \frac{\lambda_{\tau b}^T (\lambda_{\tau s}^T)^*}{64\pi^2} \frac{1}{4M_T^2} \left[ |\lambda_{\mu b'}^T|^2 (s_1^2 + s_2^2 + s_3^2) + |\lambda_{\mu b}^T|^2 \right] (\bar{s} \gamma^\alpha \hat{L} b) (\bar{\mu} \gamma_\alpha \hat{L} \mu) + H.c.$$

• The box diagram function:  $\mathcal{G}(x) = \left[ \frac{1}{1-x} + \frac{\ln x}{(1-x)^2} \right]$

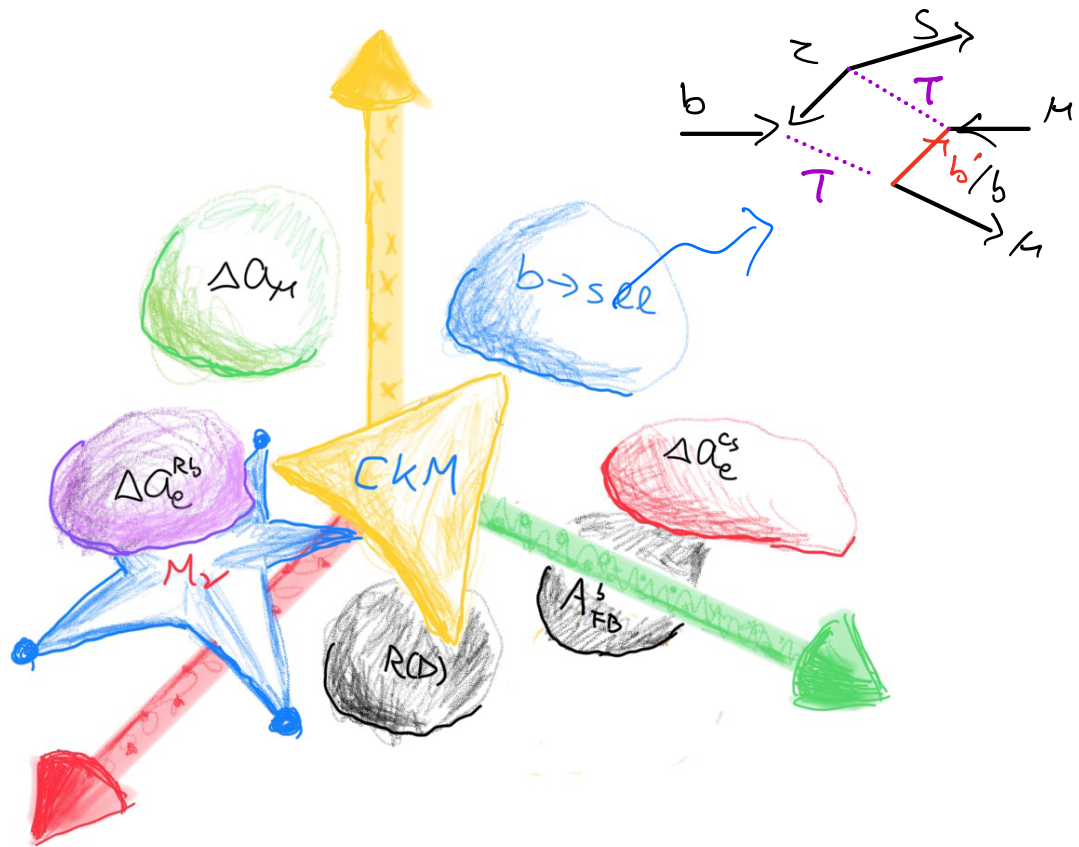
$$C_9 = -C_{10} \simeq \frac{\sqrt{2}}{128\pi\alpha} \frac{\lambda_{\tau b}^T (\lambda_{\tau s}^T)^*}{V_{tb} V_{ts}^* G_F M_T^2} \left[ |\lambda_{\mu b'}^T|^2 \beta'_T \mathcal{G}(\beta'_T) - \frac{5}{4} |\lambda_{\mu b}^T|^2 \right]$$

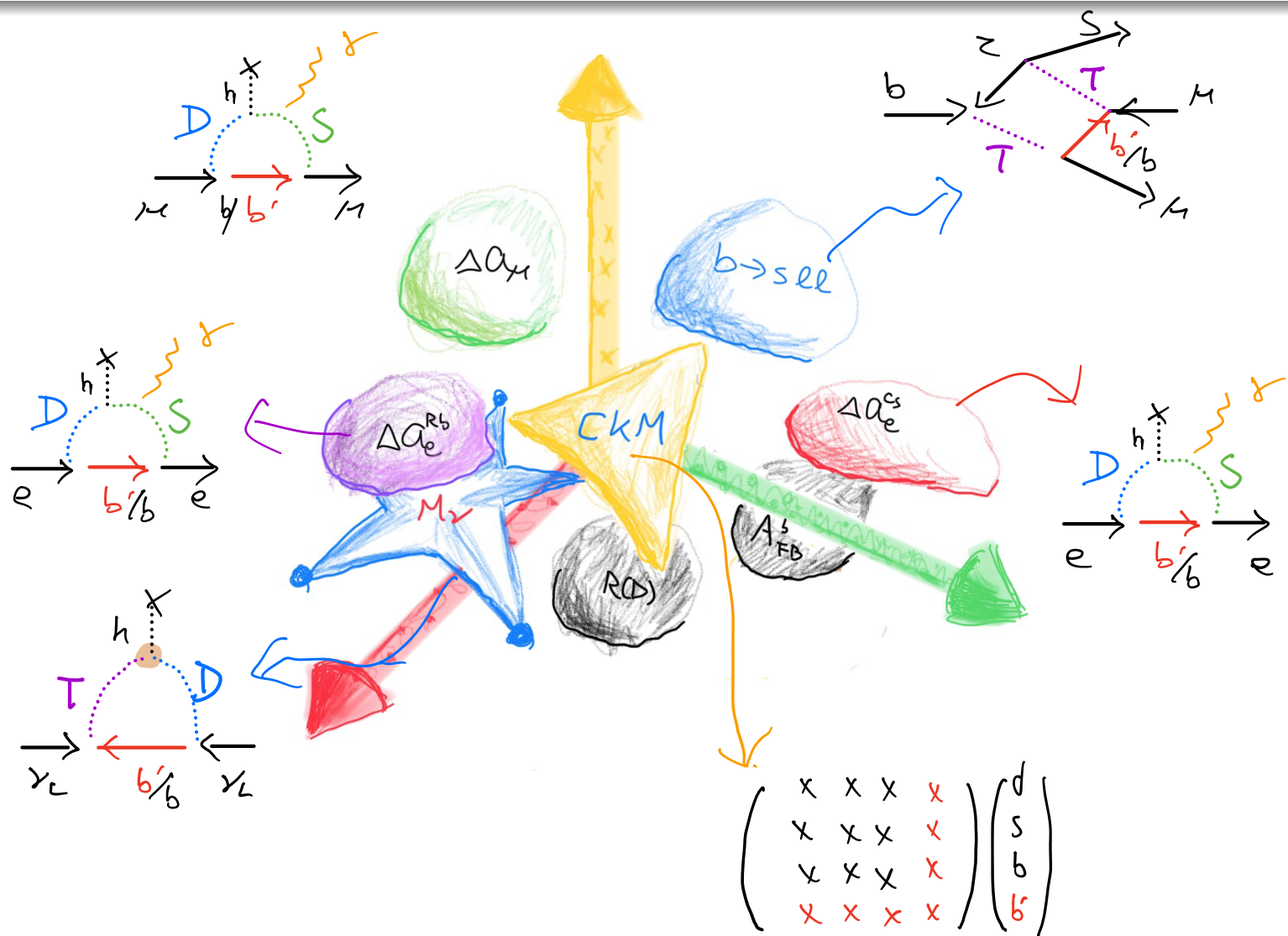
$$C'_9 = -C'_{10} = 0.$$

And we adopt the best fit values from  
Altmannshofer & Stangl 2103.13370

$$(C_9)^\mu = -(C_{10})^\mu \simeq -0.41 \pm 0.07,$$

$$(C_9)^e \simeq (C_{10})^e \simeq (C'_9)^e \simeq (C'_{10})^e \simeq 0$$







To accommodate the anomalies,

Anomaly	Requirement
$m_\nu$	$\mu_3 m_{b(\prime)} \lambda^D \lambda^T \simeq \mathcal{O}(10^{-5})(\text{GeV})^2$
$\Delta a_e^{Cs[Rb]}, \Delta a_\mu$ (Sol-1)	$\{\mu_1 \lambda_{eb}^D \lambda_{eb}^S, \mu_1 \lambda_{\mu b'}^D \lambda_{\mu b'}^S\} \simeq \{(49 \pm 20)[-27 \pm 17], (280 \pm 66)\} \text{GeV}$
$\Delta a_e^{Cs[Rb]}, \Delta a_\mu$ (Sol-2)	$\{\mu_1 \lambda_{eb'}^D \lambda_{eb'}^S, \mu_1 \lambda_{\mu b}^D \lambda_{\mu b}^S\} \simeq \{-(20.1 \pm 8.3)[+11.1 \pm 6.9], -(689 \pm 162)\} \text{GeV}$
$b \rightarrow sl^+ l^-$	$\lambda_{\tau b}^T (\lambda_{\tau s}^T)^* ( \lambda_{\mu b'}^T ^2 + 3.39  \lambda_{\mu b}^T ^2) \simeq -(1.07 \pm 0.18)$
Cabibbo angle anomaly	$ s_1 + 0.233 s_2  \simeq 0.039(7)$

• The minimal set of

$\lambda^T$  to address  $b \rightarrow sl$  &  $M_\nu$

$$\text{MinS}_T = \{\lambda_{\tau b}^T, \lambda_{\tau s}^T, \lambda_{\tau b'}^T, \lambda_{\mu b'}^T, \lambda_{\mu b}^T\}$$

• Need to check the exp. constraints

- ↳  $\mathcal{Z}\mathcal{Z} \ell\ell$  operators
- SM  $Z^0$  couplings
- $B_s - \bar{B}_s$  mixing
- $B \rightarrow K^{(*)} \nu \bar{\nu}$
- $\tau \rightarrow \mu \gamma, e \gamma$
- $b \rightarrow s \gamma$
- Neutrino data
- $0 \nu \beta\beta$  decay

- ZZll operators
- SM  $Z^0$  couplings
- $B_s - \bar{B}_s$  mixing
- $B \rightarrow K^{(*)} \nu \bar{\nu}$
- $Z \rightarrow \mu \gamma, e \gamma$
- $b \rightarrow s \gamma$
- Neutrino data
- $0 \nu \beta \beta$  decay

$$\left\{ \begin{array}{l} \lambda_{ed}^T \rightarrow B^+ \rightarrow \pi^+ e \mu, B^0 \rightarrow \bar{e} Z, \\ \quad \quad \quad \tau \rightarrow e k, \quad \mu-e \text{ conv.} \\ \lambda_{es}^T \rightarrow B^+ \rightarrow K^+ e \mu, \quad \mu-e \text{ conv.} \\ \lambda_{eb}^T \rightarrow \mu-e \text{ conv.} \quad b \rightarrow s ee \\ \lambda_{ed}^T \rightarrow K^+ \rightarrow \pi^+ \nu \bar{\nu} \\ \lambda_{es}^T \lambda_{ed}^T \rightarrow K-\bar{K} \text{ mixing} \\ \lambda_{edi}^T \rightarrow e \rightarrow e' \gamma \\ \lambda_{zd}^T \rightarrow K_L \rightarrow \mu \bar{\nu} \\ \lambda_{ms}^T \rightarrow \text{Kaon phys.} \end{array} \right.$$

• : Tree

• : Loop

$$\text{MinS}_T = \{ \lambda_{\tau b}^T, \lambda_{\tau s}^T, \lambda_{\tau b'}^T, \lambda_{\mu b'}^T, \lambda_{\mu b}^T \}$$

# 3/6. Constraints

## Low energy [qqll] effective operators

$(\bar{q}_k \gamma^\mu L_{ij})(\bar{l}_i \gamma_\mu L_{kj})$ $4M_F^2$	Wilson Coef.	Constraint	Model
$bb\mu\mu$	$2 \lambda_{ub}^T ^2$	211.1 [116]	1.06
$sbt\tau$	$2\lambda_{tb}^*(\lambda_{rs}^T)^*$	-	-0.14
$sbj\mu$	$0^a$	-	0
$sbj\tau$	$0$	-	0
$sbt\mu$	$2\lambda_{rb}^*(\lambda_{rs}^T)^*$	0.199 <sup>b</sup> [116]	0.11
$uut\mu$	$(\tilde{V}_{ub}(\lambda_{rb}^T)^* + \tilde{V}_{ub'}(\lambda_{rb'}^T)^* + \tilde{V}_{us}(\lambda_{rs}^T)^*) \times (\tilde{V}_{ub}^* \lambda_{ub}^T + \tilde{V}_{ub'}^* \lambda_{ub'}^T)$	0.13 [116]	0.0043
$uuj\mu$	$ \tilde{V}_{ub}(\lambda_{ub}^T)^* + \tilde{V}_{ub'}(\lambda_{ub'}^T)^* ^2$	1.03 [116]	0.017
$ucj\mu$	$(\tilde{V}_{ub}(\lambda_{ub}^T)^* + \tilde{V}_{ub'}(\lambda_{ub'}^T)^*) \times (\tilde{V}_{cb}^* \lambda_{ub}^T + \tilde{V}_{cb'}^* \lambda_{ub'}^T)$	0.11 <sup>c</sup> [116]	0*
$ccj\mu$	$ \tilde{V}_{cb}(\lambda_{cb}^T)^* + \tilde{V}_{cb'}(\lambda_{cb'}^T)^* ^2$	52.8 [116]	0*
$cc\tau\mu$	$(\tilde{V}_{cb}(\lambda_{cb}^T)^* + \tilde{V}_{cb'}(\lambda_{cb'}^T)^* + \tilde{V}_{cs}(\lambda_{cs}^T)^*) \times (\tilde{V}_{cb}^* \lambda_{cb}^T + \tilde{V}_{cb'}^* \lambda_{cb'}^T)$	211.1 [116]	0*
$tcj\mu$	$(\tilde{V}_{tb}(\lambda_{tb}^T)^* + \tilde{V}_{tb'}(\lambda_{tb'}^T)^*) \times (\tilde{V}_{cb}^* \lambda_{tb}^T + \tilde{V}_{cb'}^* \lambda_{tb'}^T)$	-	0*
$tc\tau\tau$	$(\tilde{V}_{tb}(\lambda_{tb}^T)^* + \tilde{V}_{tb'}(\lambda_{tb'}^T)^* + \tilde{V}_{ts}(\lambda_{ts}^T)^*) \times (\tilde{V}_{cb}^* \lambda_{tb}^T + \tilde{V}_{cb'}^* \lambda_{tb'}^T + \tilde{V}_{cs}^* \lambda_{ts}^T)$	-	-0.030
$tc\tau\mu$	$(\tilde{V}_{tb}(\lambda_{tb}^T)^* + \tilde{V}_{tb'}(\lambda_{tb'}^T)^* + \tilde{V}_{ts}(\lambda_{ts}^T)^*) \times (\tilde{V}_{cb}^* \lambda_{tb}^T + \tilde{V}_{cb'}^* \lambda_{tb'}^T)$	11.35 <sup>d</sup>	0*
$tcj\tau$	$(\tilde{V}_{tb}(\lambda_{tb}^T)^* + \tilde{V}_{tb'}(\lambda_{tb'}^T)^*) \times (\tilde{V}_{cb}^* \lambda_{tb}^T + \tilde{V}_{cb'}^* \lambda_{tb'}^T + \tilde{V}_{cs}^* \lambda_{ts}^T)$	11.35	0.02
$tuj\mu$	$(\tilde{V}_{ub}(\lambda_{ub}^T)^* + \tilde{V}_{ub'}(\lambda_{ub'}^T)^*) \times (\tilde{V}_{cb}^* \lambda_{ub}^T + \tilde{V}_{cb'}^* \lambda_{ub'}^T)$	-	0.09
$tut\tau$	$(\tilde{V}_{ub}(\lambda_{ub}^T)^* + \tilde{V}_{ub'}(\lambda_{ub'}^T)^* + \tilde{V}_{us}(\lambda_{rs}^T)^*) \times (\tilde{V}_{cb}^* \lambda_{ub}^T + \tilde{V}_{cb'}^* \lambda_{ub'}^T + \tilde{V}_{cs}^* \lambda_{rs}^T)$	-	-0.03
$tuj\tau$	$(\tilde{V}_{ub}(\lambda_{ub}^T)^* + \tilde{V}_{ub'}(\lambda_{ub'}^T)^* + \tilde{V}_{us}(\lambda_{rs}^T)^*) \times (\tilde{V}_{cb}^* \lambda_{ub}^T + \tilde{V}_{cb'}^* \lambda_{ub'}^T)$	11.35	-0.12
$tuj\tau$	$(\tilde{V}_{ub}(\lambda_{ub}^T)^* + \tilde{V}_{ub'}(\lambda_{ub'}^T)^*) \times (\tilde{V}_{cb}^* \lambda_{ub}^T + \tilde{V}_{cb'}^* \lambda_{ub'}^T + \tilde{V}_{cs}^* \lambda_{rs}^T)$	11.35	0.02

<sup>a</sup>There is no such effective operator at tree-level.

<sup>b</sup>We update this value by using the new data  $\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \tau^-) < 4.5 \times 10^{-5}$  [1].

<sup>c</sup>We update this value by using the new data  $\mathcal{B}(D^+ \rightarrow \pi^+ \mu^+ \mu^-) < 7.3 \times 10^{-8}$  [1].

<sup>d</sup>We obtain the limit by using the top quark decay width,  $\Gamma_t = 1.42\text{GeV}$ , and  $\mathcal{B}(t \rightarrow ql\tau) < 1.86 \times 10^{-5}$  [117].

$(\bar{d}_k \gamma^\mu L_{ij})(\bar{l}_i \gamma_\mu L_{kj})$ $4M_F^2$	Wilson Coef.	Constraint	Model
$suw\mu\mu$	0		0
$suw_\tau\mu$	$(\lambda_{rs}^T)^* (\tilde{V}_{ub}^* \lambda_{ub}^T + \tilde{V}_{ub'}^* \lambda_{ub'}^T)$	3.96	0.010
$suw_\tau\tau$	$(\lambda_{rs}^T)^* (\tilde{V}_{ub}^* \lambda_{ub}^T + \tilde{V}_{ub'}^* \lambda_{ub'}^T + \tilde{V}_{us}^* \lambda_{rs}^T)$	0.79	0.003
$suw_\mu\tau$	0		0
$scv\mu\mu$	0		0
$scv_\tau\mu$	$(\lambda_{rs}^T)^* (\tilde{V}_{cb}^* \lambda_{ub}^T + \tilde{V}_{cb'}^* \lambda_{ub'}^T)$	31.7	0*
$scv_\tau\tau$	$(\lambda_{rs}^T)^* (\tilde{V}_{cb}^* \lambda_{ub}^T + \tilde{V}_{cb'}^* \lambda_{ub'}^T + \tilde{V}_{cs}^* \lambda_{rs}^T)$	15.8	0.002
$scv_\mu\tau$	0		0
$buw\mu\mu$	$(\lambda_{rb}^T)^* (\tilde{V}_{ub}^* \lambda_{ub}^T + \tilde{V}_{ub'}^* \lambda_{ub'}^T)$	0.51	0.09
$buw_\tau\mu$	$(\lambda_{rb}^T)^* (\tilde{V}_{ub}^* \lambda_{ub}^T + \tilde{V}_{ub'}^* \lambda_{ub'}^T)$	0.51	-0.12
$buw_\tau\tau$	$(\lambda_{rb}^T)^* (\tilde{V}_{ub}^* \lambda_{ub}^T + \tilde{V}_{ub'}^* \lambda_{ub'}^T + \tilde{V}_{us}^* \lambda_{rs}^T)$	0.51	-0.03
$buw_\mu\tau$	$(\lambda_{rb}^T)^* (\tilde{V}_{ub}^* \lambda_{ub}^T + \tilde{V}_{ub'}^* \lambda_{ub'}^T + \tilde{V}_{us}^* \lambda_{rs}^T)$	0.51	0.02
$bcv_\mu\mu$	$(\lambda_{rb}^T)^* (\tilde{V}_{cb}^* \lambda_{ub}^T + \tilde{V}_{cb'}^* \lambda_{ub'}^T)$	5.41	0*
$bcv_\tau\mu$	$(\lambda_{rb}^T)^* (\tilde{V}_{cb}^* \lambda_{ub}^T + \tilde{V}_{cb'}^* \lambda_{ub'}^T)$	5.41	0*
$bcv_\tau\tau$	$(\lambda_{rb}^T)^* (\tilde{V}_{cb}^* \lambda_{ub}^T + \tilde{V}_{cb'}^* \lambda_{ub'}^T + \tilde{V}_{cs}^* \lambda_{rs}^T)$	5.41	-0.03 <sup>a</sup>
$bcv_\mu\tau$	$(\lambda_{rb}^T)^* (\tilde{V}_{cb}^* \lambda_{ub}^T + \tilde{V}_{cb'}^* \lambda_{ub'}^T + \tilde{V}_{cs}^* \lambda_{rs}^T)$	5.41	0.02

<sup>a</sup>This is the effective operator to address the  $R(D^{(*)})$  anomaly.

Carpentier & Davidson, EPJC70,1071

$b \rightarrow c \ell_i \nu$  ( $i = e, \mu$ ) universality better than  $\lesssim 1\%$  [180(01112)]

$\left[ \bar{b} \gamma_L^\alpha c \right] \left[ \bar{\nu}_L \gamma_{\alpha L} \ell \right]$      $\left[ \bar{b} \gamma_L^\alpha c \right] \left[ \bar{\nu}_L \gamma_{\alpha L} \mu \right]$

?

not SM CC

$\Rightarrow \tilde{V}_{cb}^* \lambda_{ub}^T + \tilde{V}_{cb'}^* \lambda_{ub'}^T = 0$

such that perfect  $\mu$ - $e$  univ.

# 3/6. Constraints

NC int. / A<sub>FB</sub>

- In the interaction basis  $(g^{SM} = T_3 - \theta \sin^2 \theta_w)$ ,

$$\mathcal{L} \supset \frac{g_2}{c_W} \left[ g_L^{SM} \sum_{i=1}^3 \bar{d}_{Li} \gamma^\alpha d_{Li} + g_R^{SM} \sum_{i=1}^3 \bar{d}_{Ri} \gamma^\alpha d_{Ri} + g_R^{SM} (\bar{b}'_L \gamma^\alpha b'_L + \bar{b}'_R \gamma^\alpha b'_R) \right] Z_\alpha$$

- In SM  $g_R^b = g_L^b + \frac{1}{2}$

$$\mathcal{L} \supset \frac{g_2}{c_W} \left[ g_L^{SM} \sum_{i=1}^4 \bar{d}_{Li} \gamma^\alpha d_{Li} + g_R^{SM} \sum_{i=1}^4 \bar{d}_{Ri} \gamma^\alpha d_{Ri} + \frac{1}{2} (\bar{b}'_L \gamma^\alpha b'_L) \right] Z_\alpha$$

- Then, in the mass basis,

$Z \not\sim b$

$$\mathcal{L} \supset \frac{g_2}{c_W} \left[ \sum_{\alpha=s,d,b,b'} \bar{d}_\alpha \gamma^\alpha (g_L^{SM} \hat{L} + g_R^{SM} \hat{R}) d_\alpha \right] Z_\mu + \frac{g_2}{2c_W} \sum_{\alpha,\beta=s,d,b,b'} \kappa_{\alpha\beta} [(\bar{d}_\alpha \gamma^\alpha \hat{L} d_\beta)] Z_\alpha, \quad \kappa_{\alpha\beta} \equiv (U_L^d)_{\alpha 4} [(U_L^d)_{\beta 4}]^*$$

In our parameterization, the FCNC couplings

$$\kappa_{sd} = \kappa_{ds} = s_1 s_2 c_2 c_3^2, \quad \kappa_{sb} = \kappa_{bs} = s_2 s_3 c_3, \quad \kappa_{bd} = \kappa_{db} = s_1 s_3 c_2 c_3$$

- To avoid Tree-level FCNC mediated by  $Z^0$ , ONLY 1  $\theta \neq 0$

- For that particular  $\theta_F$

$$g_{d_F,R}^{SM} \Rightarrow g_{d_i,R}^{SM}, \quad g_{d_F,L}^{SM} \Rightarrow g_{d_i,L}^{SM} + \frac{1}{2} |(U_L^d)_{4d_F}|^2$$

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# 3/6. Constraints

NC int. /  $A_{FB}$

$$A_F, A_{FB} \propto \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2}, \quad \Gamma \propto g_L^2 + g_R^2$$

$$g_L^{SM} \sim -0.1423$$

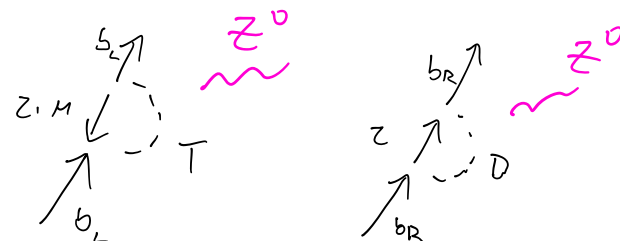
$$g_R^{SM} \sim 0.077$$

- Long standing LEP puzzle:  $A_{FB}^b \sim -2.3\delta$ , requires  $\theta_3 \neq 0, \Leftrightarrow \text{CKM}$  (b-b')

Only  $\theta_1 \neq 0$  is viable to satisfy all Exp. limits

$$|s_1| \simeq 0.039(7)$$

negligible  $\delta A_{FB}^d$



## • How about loop?

In  $\overline{\text{MS}}$  and on-shell renormalization

$$\delta g_L^b \simeq \frac{|\lambda_{\tau b}^T|^2 + |\lambda_{\mu b}^T|^2}{64\pi^2} \left[ \left( -1 + \frac{5}{3}s_W^2 \right) \frac{1}{9\beta_Z} - s_W^2 \frac{2 \ln \beta_Z + \frac{1}{3} + i\pi/2}{3\beta_Z} \right],$$

$$\delta g_R^b \simeq \frac{|\lambda_{\tau b}^D|^2}{64\pi^2} \left[ \left( -\frac{1}{3}s_W^2 \right) \frac{1}{9\beta_Z} + s_W^2 \frac{2 \ln \beta_Z + \frac{1}{3} + i\pi/2}{3\beta_Z} \right],$$

- $|\delta g_{L,R}^b| \sim \mathcal{O}(10^{-5}) |\lambda^{\tau, D}|^2$ , not helpful to  $A_{FB}^b$

- Wait for future Z-pole exp. (FCC-ee, ILC, CEPC....)

# 3/6. Constraints

Neutrino data

- For  $M_\nu$ ,  $\Delta a_e$  more parameters are needed  $\left( \text{MinS}_T = \{\lambda_{\tau b}^T, \lambda_{\tau s}^T, \lambda_{\tau b'}^T, \lambda_{\mu b'}^T, \lambda_{\mu b}^T\} \right)$ 
  - for  $m_\nu$
  - for  $\Delta a_e, \Delta a_\mu$

$\lambda_{e b'}^D \lambda_{e b'}^T$

$$\text{MinS} = \text{MinS}_T \cup \{ \lambda_{e b'}^D, \lambda_{\tau b'}^D, \lambda_{\mu b}^D, \lambda_{\tau b}^D, \lambda_{e b'}^S, \lambda_{\mu b}^S \} .$$

$$\mathcal{M}^\nu \simeq N^\nu \begin{pmatrix} 0 & \lambda_{e b'}^D \lambda_{\mu b'}^T & \lambda_{e b'}^D \lambda_{\tau b'}^T \\ \lambda_{e b'}^D \lambda_{\mu b'}^T & 2\rho_b \lambda_{\mu b}^D \lambda_{\mu b}^T & \rho_b (\lambda_{\mu b}^D \lambda_{\tau b}^T + \lambda_{\tau b}^D \lambda_{\mu b}^T) + \lambda_{\mu b'}^D \lambda_{\tau b'}^T \\ \lambda_{e b'}^D \lambda_{\tau b'}^T & \rho_b (\lambda_{\mu b}^D \lambda_{\tau b}^T + \lambda_{\tau b}^D \lambda_{\mu b}^T) + \lambda_{\mu b'}^D \lambda_{\tau b'}^T & 2\rho_b \lambda_{\tau b}^D \lambda_{\tau b}^T + 2\lambda_{\tau b'}^D \lambda_{\tau b'}^T \end{pmatrix}$$

- $M_{\nu ee} = 0$ , only NH is allowed

$$\rho_b = m_b / M_{b'} \text{ and } N^\nu = \frac{3\mu_3 v_0 M_{b'}}{32\pi^2 M_{LQ}^2}$$

- $\mathcal{M}^\nu \simeq \begin{pmatrix} 0 & 0.90792 & 0.13812 \\ 0.90792 & -2.4923 & -2.7643 \\ 0.13812 & -2.7643 & -1.9353 \end{pmatrix} \times 10^{-2} \text{eV}$ 
 $\theta_{12} \simeq 33.0^\circ, \quad \theta_{23} \simeq 48.7^\circ, \quad \theta_{13} \simeq 8.6^\circ, \quad \delta_{CP} = 0^\circ,$ 
 $\Delta m_{21}^2 \sim 7.47 \times 10^{-5} \text{eV}^2, \quad \Delta m_{31}^2 \sim 2.53 \times 10^{-3} \text{eV}^2.$

(Just one out of  $\infty$  example)

v.s. 1 $\sigma$  best fit (w. SK atm) NuFit

$$\theta_{12} \in (32.7 - 34.21)^\circ, \quad \theta_{23} \in (48.0 - 50.1)^\circ, \quad \theta_{13} \in (8.45 - 8.69)^\circ, \quad \delta_{CP} \in (173 - 224)^\circ,$$


$$\Delta m_{21}^2 \in (7.22 - 7.63) \times 10^{-5} \text{eV}^2, \quad \Delta m_{31}^2 \in (2.489 - 2.543) \times 10^{-3} \text{eV}^2.$$

 The neutrino mass pattern ( $\sim 1\sigma$ )

$\Delta a_e^{cs}$  [ $\Delta a_e^{Rb}$ ],  $\Delta a_\mu$ , the Cabibbo angle,  $b \rightarrow sll$  anomalies

can be accommodated by the following viable model parameters:

$$\begin{aligned}
 M_{LQ} &\simeq 1.0 \text{ TeV}, & M_{b'} &= 1.5 \text{ TeV}, & \mu_1 &= 2.3[-0.82] \text{ TeV}, & \mu_3 &\simeq 0.5 \text{ keV}, \\
 \mu_1 \lambda_{eb'}^S \lambda_{eb'}^D &= -12[4] \text{ GeV}, & \mu_1 \lambda_{\mu b}^S \lambda_{\mu b}^D &= -690 \text{ GeV}, \\
 \theta_2 = \theta_3 &= 0, & \sin \theta_1 &= 0.039, \\
 \lambda_{\mu b'}^T &\simeq -3.3, & \lambda_{\tau b'}^T &\simeq -0.51, & \lambda_{\mu b}^T &\simeq -0.72, & \lambda_{\tau b}^T &\simeq 0.93, & \lambda_{\tau s}^T &\simeq -0.08, \\
 \lambda_{eb'}^D &\simeq -0.002, & \lambda_{\mu b}^D &\simeq 3.4, & \lambda_{\tau b'}^D &\simeq 0.008, & \lambda_{\tau b}^D &\simeq -0.58,
 \end{aligned}$$

 Note that  $\mathcal{O}\left(\frac{\mu_3}{M_{LQ}}\right) \sim 10^{-9} \rightsquigarrow$  global L#.



# 4/6. Phenomenological consequences

- $B^+ \rightarrow K^+ \mu^+ \tau^-$ ,  $B_s - \bar{B}_s$  mixing,  $D^{\pm} \rightarrow \pi^{\pm} \mu^{\pm} \mu^{\mp}$ ,  $\tau \rightarrow \mu \gamma$

stringently set the parameter space boundaries.

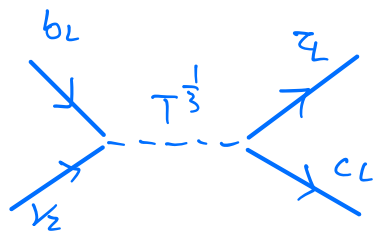
$b \rightarrow s \tau \tau$  at tree-level

$$\mathcal{H}_{\text{eff}}^{b \rightarrow s \tau \tau} \simeq -\frac{G_F}{\sqrt{2}} \tilde{V}_{tb} \tilde{V}_{ts}^* \frac{\alpha}{\pi} C^{bs\tau\tau} [\bar{s} \gamma^\alpha \hat{L} b] [\bar{\tau} \gamma_\alpha (1 - \gamma^5) \tau] + H.c.,$$

$$C^{bs\tau\tau} \simeq \frac{\sqrt{2} \pi}{4\alpha} \frac{\lambda_{\tau b}^T (\lambda_{\tau s}^T)^*}{\tilde{V}_{tb} \tilde{V}_{ts}^* G_F M_T^2} = 23.2 \times \left( \frac{\text{TeV}}{M_T} \right)^2, \quad (\text{c.f. } C_{SM}^{bs\tau\tau} \simeq -4.3)$$

$$\Rightarrow \mathcal{B}_r(B_s \rightarrow \tau^+ \tau^-) \simeq 10^{-5} \quad \text{still } \ll \sim \mathcal{O}(10^{-3}) \quad \begin{array}{l} B_s \rightarrow \tau^+ \tau^- \text{ LHCb [1703.02508]} \\ B^+ \rightarrow K^+ \tau^+ \tau^- \text{ BaBar [1605.09637]} \end{array}$$

$RC(D^{(*)})$



$RC(D^{(*)})$  needs  $C_{\tau L} \simeq 0.08$  [1904.09311]

but  $C_{\tau L}^{NP} \lesssim 0.01$  in this model  
(with real parameters)

$$\mathcal{H}_{\text{eff}}^{CC} \supset - \left[ \frac{\lambda_{\tau s}^T (\lambda_{\tau b}^T)^* \tilde{V}_{cs} + \lambda_{\tau b}^T (\lambda_{\tau b'}^T)^* \tilde{V}_{cb'} + |\lambda_{\tau b}^T|^2 \tilde{V}_{cb}}{4M_T^2} \right] (\bar{c} \gamma^\alpha \hat{L} b) (\bar{\tau} \gamma_\alpha \hat{L} \nu_\tau) + H.c.$$

For the benchmark point,

$$\mathcal{B}(T^{-\frac{1}{3}} \rightarrow b\nu_\mu) \simeq 18.9\%, \quad \mathcal{B}(T^{-\frac{1}{3}} \rightarrow b\nu_\tau) \simeq 30.9\%, \quad \mathcal{B}(T^{-\frac{1}{3}} \rightarrow s\nu_\tau) \simeq 2.1 \times 10^{-3},$$

$$\mathcal{B}(T^{-\frac{1}{3}} \rightarrow \tau t) \simeq 30.8\%, \quad \mathcal{B}(T^{-\frac{1}{3}} \rightarrow \mu t) \simeq 18.9\%, \quad \mathcal{B}(T^{-\frac{1}{3}} \rightarrow \tau c) \simeq 2.0 \times 10^{-3}.$$

$$\mathcal{B}(T^{\frac{2}{3}} \rightarrow t\nu_\mu) \simeq 37.9\%, \quad \mathcal{B}(T^{\frac{2}{3}} \rightarrow t\nu_\tau) \simeq 61.7\%, \quad \mathcal{B}(T^{\frac{2}{3}} \rightarrow c\nu_\tau) \simeq 0.4\%,$$

$$\mathcal{B}(T^{-\frac{4}{3}} \rightarrow b\mu^-) \simeq 37.9\%, \quad \mathcal{B}(T^{-\frac{4}{3}} \rightarrow b\tau^-) \simeq 61.7\%, \quad \mathcal{B}(T^{-\frac{4}{3}} \rightarrow s\tau^-) \simeq 0.4\%.$$

$$\mathcal{B}(D^{-\frac{1}{3}} \rightarrow b\bar{\nu}_\mu) \simeq 97.1\%, \quad \mathcal{B}(D^{-\frac{1}{3}} \rightarrow b\bar{\nu}_\tau) \simeq 2.9\%,$$

$$\mathcal{B}(D^{\frac{2}{3}} \rightarrow b\mu^+) \simeq 97.1\%, \quad \mathcal{B}(D^{\frac{2}{3}} \rightarrow b\tau^+) \simeq 2.9\%,$$

$$\mathcal{B}(S^{\frac{2}{3}} \rightarrow b\mu^+) \simeq 100\%,$$

$$\mathcal{B}(b' \rightarrow uW^-) \simeq 1.2\%, \quad \mathcal{B}(b' \rightarrow cW^-) \simeq 6 \times 10^{-4}, \quad \mathcal{B}(b' \rightarrow tW^-) \simeq 9 \times 10^{-7},$$

$$\mathcal{B}(b' \rightarrow \bar{\nu}T^{-\frac{1}{3}}) \simeq 19.7\%, \quad \mathcal{B}(b' \rightarrow \mu^+T^{-\frac{4}{3}}) \simeq 38.5\%,$$

$$\mathcal{B}(b' \rightarrow \tau^+T^{-\frac{4}{3}}) \simeq 0.9\%, \quad \mathcal{B}(b' \rightarrow eS^{\frac{2}{3}}) \simeq 39.7\%,$$

Very diff. from the simple assumptions implemented  
in searching for LQ,  $b'$ .

(cf. 100%  $b' \rightarrow Wt, bZ, bH$ )

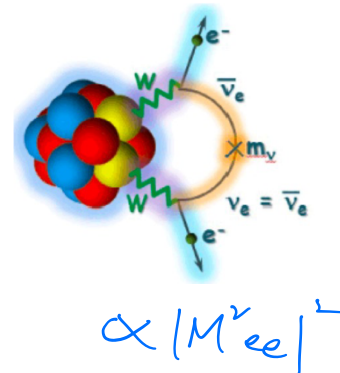
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# 4/6. Phenomenological consequences

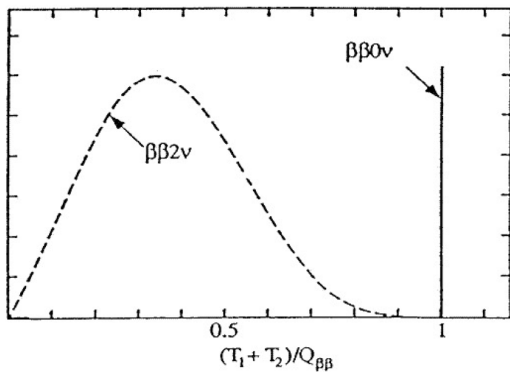
$0\beta\beta$  - decay

Table 2  $T_{1/2}^{0\nu}$  and  $\langle m_{\beta\beta} \rangle$  limits (90% CL) from the most recent measurements, sorted by mass number

Isotope	$T_{1/2}^{0\nu}$ ( $\times 10^{25}$ years)	$\langle m_{\beta\beta} \rangle$ (eV)	Experiment	Reference
$^{48}\text{Ca}$	$> 5.8 \times 10^{-3}$	$< 3.5\text{--}22$	ELEGANT-IV	159
$^{76}\text{Ge}$	$> 8.0$	$< 0.12\text{--}0.26$	GERDA	160
	$> 1.9$	$< 0.24\text{--}0.52$	MAJORANA DEMONSTRATOR	161
$^{82}\text{Se}$	$> 3.6 \times 10^{-2}$	$< 0.89\text{--}2.43$	NEMO-3	162
$^{96}\text{Zr}$	$> 9.2 \times 10^{-4}$	$< 7.2\text{--}19.5$	NEMO-3	163
$^{100}\text{Mo}$	$> 1.1 \times 10^{-1}$	$< 0.33\text{--}0.62$	NEMO-3	164
$^{116}\text{Cd}$	$> 2.2 \times 10^{-2}$	$< 1.0\text{--}1.7$	Aurora	165
$^{128}\text{Te}$	$> 1.1 \times 10^{-2}$	NE	C. Arnaboldi et al.	166
$^{130}\text{Te}$	$> 1.5$	$< 0.11\text{--}0.52$	CUORE	126
$^{136}\text{Xe}$	$> 10.7$	$< 0.061\text{--}0.165$	KamLAND-Zen	167
	$> 1.8$	$< 0.15\text{--}0.40$	EXO-200	168
$^{150}\text{Nd}$	$> 2.0 \times 10^{-3}$	$< 1.6\text{--}5.3$	NEMO-3	169



Annu. Rev. Nucl. Part. Sci. 2019.69:219-251.



$$Q_{\beta\beta} \equiv M(A, Z) - M(A, Z+2)$$

Due to  $\mu$ - $e$  conv. even with  $\lambda_{T_{e5}} \neq 0$

$$|M_{ee}^V| \lesssim 3 \times 10^{-4} \text{ eV}$$

# 5/6. Origin of the flavor pattern?

- We need a very special flavor pattern *(usually call for some nontrivial flavor symmetry)*

- In 4D QFT, the Higgs Yukawa after SSB

$$y \bar{\Psi}_{1L} \Psi_{2R} H \rightarrow \frac{y v_0}{\sqrt{2}} \bar{\Psi}_{1L} \Psi_{2R} \Rightarrow m \propto y$$

- In 5D, with extra-dimension  $z$

$$\Psi_{1L}^{5D} = \Psi_{1L}^{4D}(X^\mu) \phi_1(z) \quad \Psi_{2R}^{5D} = \Psi_{2R}^{4D}(X^\mu) \phi_2(z)$$

$$H^{5D} = H^{4D}(X^\mu) f(z), \quad \mathcal{L}_{5D} \sim y_{5D} \bar{\Psi}_{1L}^{5D} \Psi_{2R}^{5D} H^{5D}$$

- Integrating out the 5-th dim.

$$\mathcal{L}_{4D} = \int dz \mathcal{L}_{5D} \propto \bar{\Psi}_{1L}^{4D} \Psi_{2R}^{4D} H^{4D} \int dz \phi_1^*(z) \phi_2(z) f(z)$$

$$\Rightarrow y_{4D} \sim y_{5D} \times \int dz \phi_1^*(z) \phi_2(z) f(z)$$

# 5/6. Origin of the flavor pattern?

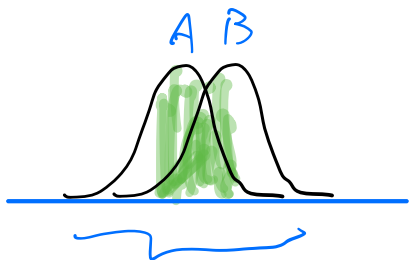
- One toy model : split fermion (Arkani-Hamed, 9903417)

All  $\phi_i(z) \sim$  Gaussian with universal

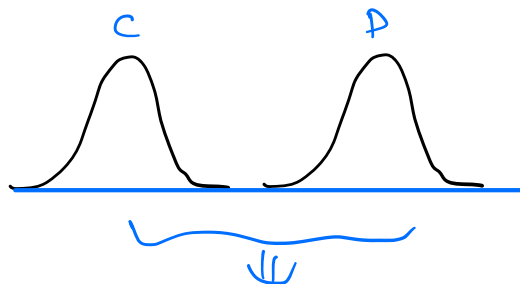
$\sigma_{SF}$  , cluster

$$y_{4D} \sim \exp \left[ -\frac{\Delta z^2}{2\sigma_{SF}^2} \right],$$

$\Delta z$	1	2	5
exp	$\sim 0.6$	$\sim 0.13$	$10^{-11}$



vs



The effective  $y_{4D}^{AB} \sim O(1)$

$y_{4D}^{CD} \ll \ll 1$

# 5/6. Origin of the flavor pattern?

## One example SF configuration

$$\{N_T, N_D, N_S\} = \{4.5, 6.2, 5.5\}.$$

and

$$\begin{aligned} \{z_{\tau L}, z_{\mu L}, z_{eL}, z_{\mu R}, z_{eR}\} &= \{0, -0.49, 7.75[7.80], -4.99[-4.72], -0.89[-0.97]\}, \\ \{z_{dL}, z_{sL}, z_{bL}, z_{b'_L}, z_{bR}, z_{b'_R}\} &= \{-5.27[-5.53], 2.86[2.90], -2.14[-2.13], \\ &\quad -1.90[-1.95], -2.06[-2.09], 3.66[3.74]\}, \end{aligned}$$

which is able to reproduce every parameter within  $\sim 50\%$

$$\begin{aligned} |\lambda_{\mu b'}^T| &= 1.66[1.56], & |\lambda_{\tau b'}^T| &= 0.74[0.68], & |\lambda_{\mu b}^T| &= 1.15[1.17], \\ |\lambda_{\tau b}^T| &= 0.46[0.46], & |\lambda_{\tau s}^T| &= 7.5[6.8] \times 10^{-2}, \\ |\lambda_{eb'}^D| &= 1.5[1.6] \times 10^{-3}, & |\lambda_{\mu b}^D| &= 1.78[1.71], & |\lambda_{\tau b'}^D| &= 7.5[5.8] \times 10^{-3}, & |\lambda_{\tau b}^D| &= 0.74[0.69], \\ |\lambda_{eb'}^S| &= 3.31[3.42], & |\lambda_{\mu b}^S| &= 9.4[19.2] \times 10^{-2}. \end{aligned}$$

$$\begin{aligned} |\lambda_{eb}^D| &= 7.5[3.6] \times 10^{-21}, & |\lambda_{\mu b'}^D| &= 1.1[0.8] \times 10^{-3}, \\ |\lambda_{ed}^T| &= 7.0[0.13] \times 10^{-37}, & |\lambda_{es}^T| &= 2.9[2.8] \times 10^{-5}, & |\lambda_{eb}^T| &= 2.6[1.8] \times 10^{-21}, \\ |\lambda_{eb'}^T| &= 2.7[1.1] \times 10^{-20}, & |\lambda_{\mu d}^T| &= 4.8[1.4] \times 10^{-5}, & |\lambda_{\tau d}^T| &= 4.2[1.0] \times 10^{-6}, \end{aligned}$$

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## 5/6. Origin of the flavor pattern?

In addition to the hierarchical Yukawa, the nearly conserved  $L\#$ , or

$$\mathcal{O}\left(\frac{\mu_3}{M_{LQ}}\right) \sim 10^{-9}$$

can be easily arranged in this 5D model.

Recall that,  $\mu_3 \sim 0.5 \text{ KeV}$ ,  $M_{LQ}, M_{b'}, \mu_1 \sim \mathcal{O}(\text{TeV})$

$$\mathcal{L} \supset \mu_3 \{H, \tilde{D}\}^{\otimes T} + \mu_1 [H, D] S^{-\frac{L}{3}}$$

$\Rightarrow$  **D&T** have different orbiting parties  
(such that nearly orthogonal to each other)

**D&S** have the same ...

Overlapping  $\sim \mathcal{O}(1)$

e.g. T: (+,-), D, S: (-,+) on the

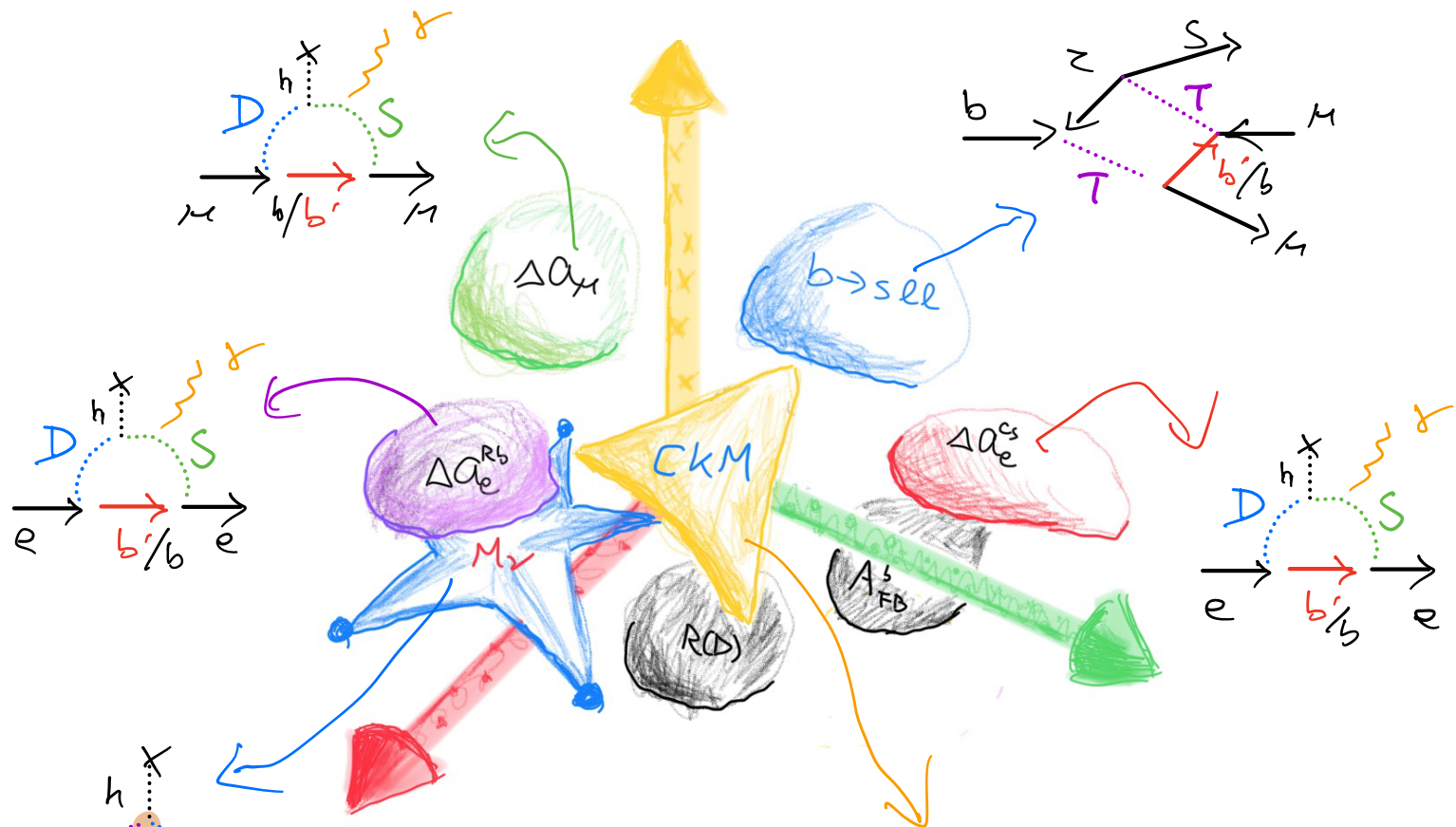
$$\frac{S_1}{Z_2 \times Z_2}$$

orbifold

## 6/6. Conclusion

- Neutrino mass,  $b \rightarrow sll$ ,  $\Delta a_\mu$ ,  $\Delta a_e^{Cs[Rb]}$ , and  $CKM$  anomalies can be accommodated in SM+ 3 scalar leptoquarks  $(3, 3, -1/3)$ ,  $(3, 2, 1/6)$ ,  $(3, 1, 2/3)$  +vector  $b'$  with  $U(1)_B$ .
- Viable benchmark point with minimal set of real parameters.
- Solid prediction: normal hierarchy with  $\mathcal{M}_{ee}^\nu \lesssim 3 \times 10^{-4}$  eV.
- Nontrivial leptoquark/ $b'$  decay branching ratios.
- Nearly conserved global Lepton number
- Split fermion is one possible origin of the flavor pattern





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 \end{pmatrix}
 \begin{pmatrix}
 d \\
 s \\
 b \\
 b'
 \end{pmatrix}$$