Backreaction Problems in Moving Atoms, Black Holes and the Early Universe via Generalized Fluctuation-Dissipation Relations

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- J.-T. Hsiang, B. L. Hu, and S. -Y. Lin, *Fluctuation-Dissipation and **Correlation-Propagation Relations** from the Nonequilibrium Dynamics of Detector-Quantum Field Systems* arXiv:1905.08596 (PRD)
- J.-T. Hsiang, B. L. Hu, S. -Y. Lin and K. Yamamoto, *Fluctuation-Dissipation and **Correlation-Propagation Relations** in 4D for Uniformly-Accelerated Detectors in a Quantum Field* arXiv:1906.00530 (PLB)

Outline

1. Backreaction vs Exact Soln and Test Field Approx.
2. Backreaction from an Open System's Perspective
3. Backreaction in Gravitation and Cosmology:

   3 Levels of theoretical structures: QFTCST,
   Semiclassial Gravity, Stochastic Gravity

4. Moving Atom, Radiating Black Hole, Early Universe:
   Backreaction a la Fluctuation-Dissipation Relations
5. Going deeper: N Detector-Quantum Field System
6. New relation: Correlation-Propagation Relations
7. Applications: Energy / Entropy, Quantum Information

A system interacting with a Quantum Field

System: Atoms / Detectors (with internal dof), q black hole (as atom), the universe (scale factor *a* obeys HO eq with – freq^2)

Quantum Field processes:

- vac polarization (e.g., trace anomaly), fluctuations (e.g., Casimir eff)
- **particle creation**

  from vacuum fluctuations parametrically amplified, e.g.,

  dynamical Casimir effect,

in the early universe (Planck time)

in spacetimes with event horizons:

  black holes: Hawking radiation, in de Sitter universe (static)

  uniformly accelerated detectors: Unruh radiation

Coherence and Fluctuations in the Interaction between Moving Atoms and a Quantum Field *

Experimental aspects
of RQI: Atoms & Optics

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In *Macroscopic Quantum Coherence*, edited by J. Swain and A. Widom
(World Scientific, Singapore, 1998) [quant-ph/9710061].

Based on work with J. Anglin, PRD53, 7003 (**1996**) | with D. Koks, PRD55, 4795 (**1997**)
N-Oscillators moving in a quantum field | non-uniform acceleration, nonMarkovian effects

ABSTRACT

Mesoscopic physics deals with three fundamental issues: quantum coherence, fluctuations and correlations. Here we analyze these issues for atom optics ,using a simplified model of an assembly of atoms (or detectors, which are particles with some internal degree of freedom) moving in arbitrary trajectories in a quantum field. Employing the influence functional formalism, we study the self-consistent effect of the field on the atoms, and their mutual interactions via coupling to the field. We derive the coupled Langevin equations for the atom assemblage and analyze the relation of dissipative dynamics of the atoms (detectors) with the correlation and fluctuations of the quantum field. This provides a useful theoretical framework for analyzing the coherent properties of atom-field systems.

Interaction between atoms (detectors) and a quantum field

(2nd part of my talk at RQI-N, 2010)

1. Models / Methods: Quantum field in terms of parametric oscillators
- QBM model of one or many oscillators in a common q. field
- Influence Functional Formalism: Dissipation and Noise kernels

2. Many atoms coupled through a quantum field
- Quantum Coherence, Fluctuations, Correlations: (RHA 96)
- Fluctuation-Dissipation / Correlation- Propagation relations:

  >> Hsiang et al, 2 papers, this talk and Hsiang's talk

3. Quantum Entanglement: Lin Chou Hu, PRD (2008), Lin & Hu, PRD (2009, 2010)

   Quantum Twins (RHK 97), Sec.VI of Lin, Chou & Hu, PRD91, 084063 (2015)
- Loriani et al Roura: Interference of Clocks: A Quantum Twin Paradox arXiv:1905.09102
- Autangle (Zhou et al)

4. Quantum Teleportation Lin, Chou & Hu,Phys. Rev. D91, 084063 (2015)

5. Applications:
- Atoms in a cavity, atom-array, ensemble; spin chain, strongly correlated systems
- Uniformly accelerated detector: backreaction / exact soln: Lin Hu 07
- Black hole fluctuations, entanglement and backreaction Hu Roura 07

Issue 2. Backreaction of field on system

- The next best thing to do, short of finding an exact solution.

- The simplest time-dependent perturbation theory (TDPT) results are valid only for a very short time span. Making general statements based on these calculations may be misleading. [compare with exact solutions]

- Best understood from an open system perspective

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Backreaction and the Unruh effect: New insights from exact solutions of uniformly accelerated detectors

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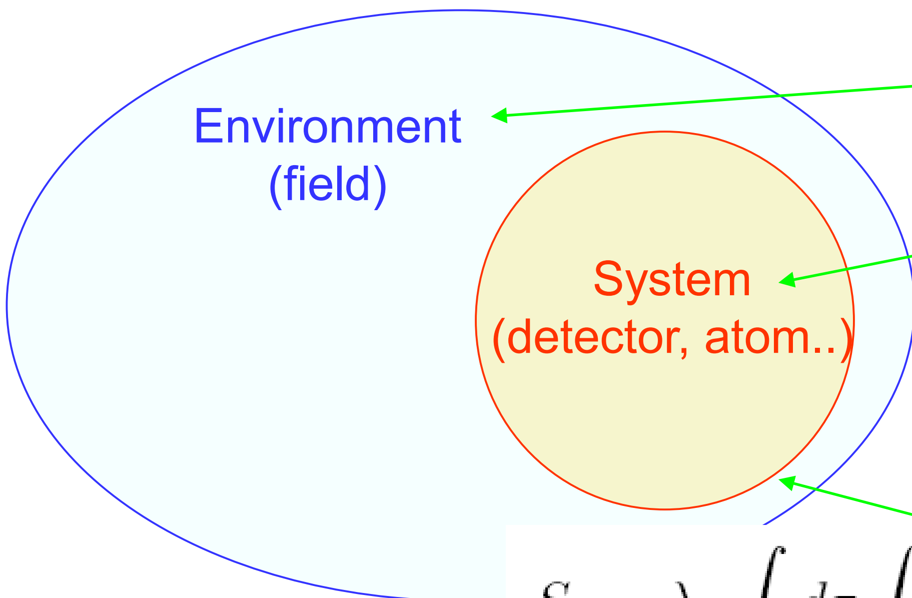
1) The detector no longer behaves like a perfect thermometer. From a calculation of the evolution of the reduced density matrix of the detector, we find that the transition probability from the initial ground state over an infinitely long duration of interaction derived from time-dependent perturbation theory is valid only in shart transient, corresponding to the Markovian regime.

2) The detector at late times never sees an exact Boltzmann distribution over the energy eigenstates of the free detector, thus in the non-Markovian regime covering a wider range of parameters the Unruh temperature cannot be identified inside the detector

Detector (Atom) - Field Model of BH info

II. The Model: A uniformly accelerated harmonic oscillator (Unruh-DeWitt detector) in (3+1)D initially in an excited state (pure state), coupled to a scalar field Minkowski vacuum (pure state) at t=0.

Follow what happens after t=0: Entanglement Dynamics



$$S_\Phi = -\int d^4x \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi$$

$$S_Q = \int d\tau \frac{m_0}{2} \left[(\partial_\tau Q)^2 - \Omega_0^2 Q^2 \right]$$

$$S_I = \lambda_0 \int d\tau \int d^4x Q(\tau) \Phi(x) \delta^4 \left(x^\mu - z^\mu(\tau) \right)$$

S. Y. Lin and B. L. Hu, *Class. Quant. Grav* 25:154004 *(2008)* [arXiv: 0710.0435]

B. L. Hu and S. Y. Lin, *PoSQG-Ph:019 (2007) (2008)* [arXiv:0712.3643]

Implications on BH information

Viewing the UD detector as the analog of a quantum black hole,
our results suggest:

1. All the initial information of the black hole will be encoded in its radiation (spontaneous emission in atom analog)
   -- consistent with the "no-hiding theorem" (Braunstein and Pati 2007),
   -- no information is hidden in the correlation between BH and the field.

2. Only in the ultraweak coupling limit can both the BH and the field restore most of its purity and quantum coherence.

3. In the non-Markovian regime (strong BH-field coupling), the area-eigenstate of BH could not form a good basis, and the final state of BH would be a complicated mixed state distributed widely from the ground state to the highly excited area-eigenstates.

4. Therefore, at late times BH could end up as a large remnant (with large expectation value of area operator) with all its initial information already leaked out and dispersed into the quantum field.

Issues 3: Stochasticity and nonMarkovianity

Nonequilibrium **Statistical Mechanics**: **Open Systems**

- Classical: **Projection Operator** Formalism Zwangzig-Mori (57,61) Grabert
- Quantum: **Influence Functional** Formalism Feynman-Vernon (63)

To see the **non-Markovianity** property: Begin with 2 coupled subsystems S1 and S2, each obeying a 2^{nd} order ODE. Solve the cpld system of ODEs self-consistently for each subsystem dynamics.

If only the dynamics of S1 is of interest, one can convert this system of equations into one **integral-differential equation** for just the dynamical variable of S1, but with **backreaction** from S2

The kernel is **time-nonlocal:** Because the dynamical time scales of S2 is different from that of S1.

Thus **backreaction naturally introduces memory effects.**

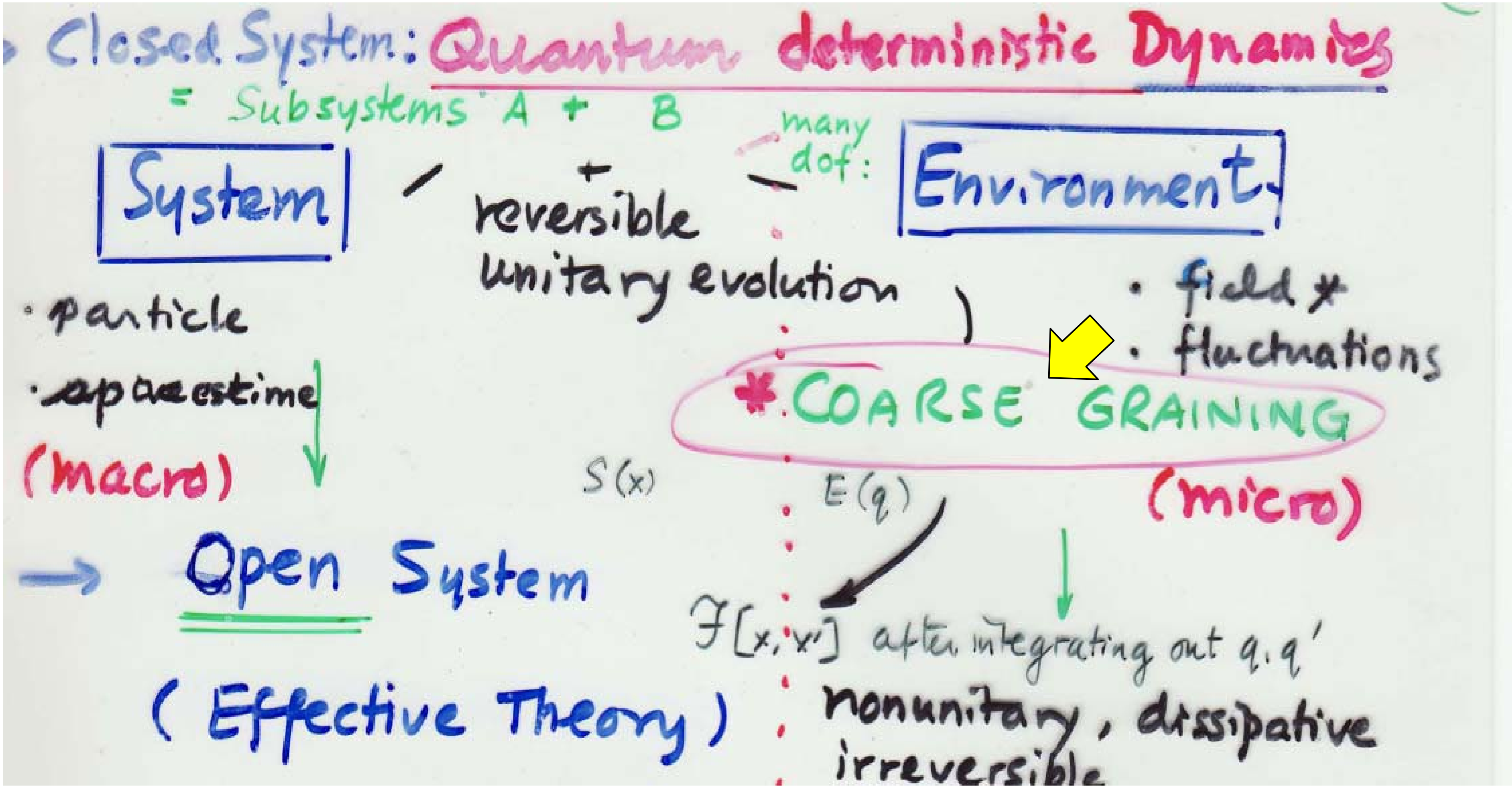
When S2 is assumed to have many more dof than S1, it is regarded as an **environment E to S**.

In an **open system approach**, one focuses on the dynamics of one chosen subsystem, coarse-grain (CG) the E and assign some distribution function for its fluctuations. The correlations of these stochastic variables are in general not a delta function: **Colored noise**.

The backreaction of CG - E (colored noise) on the system generates **nonlocal dissipative dynamics**

Noise (in E) and Dissipation (in S) are governed by a Fluctuation-Dissipation Relation.

"Opening up" a Quantum Closed System



nonMarkovianity: Back-action of environment on the open system always bring in memory: nonlocal kernels

Quantum Open System

• **Influence Functional** $\mathcal{F} = e^{iS_{IF}}$ / action
(Closed-time-path coarse-grained effective action)

S_{IF} : $\boxed{\mu + i\nu}$

Backreaction

$\boxed{\text{DISSIPATION}}$ kernel μ ← $\boxed{\text{NOISE}}$ / Fluctuation kernel $\nu(t,t')$ $\equiv \frac{1}{\hbar}\langle \xi(t)\xi(t')\rangle$

Decoherence

renormalization

UV divergences

Q

M maco — meso — μ ico

C

issues in Quantum Field Theory ◁ ▷ issues in N Eq. Stat. Mechanics

Quantum → Classical via (environment-induced) decoherence

Master Eqn., Fokker-Planck Eqn.
Langevin Eqn. $\boxed{\text{Classical Stochastic Dynamics}}$

e.g.

$$\underbrace{M \frac{d^2}{ds^2} X_c(s)}_{\text{CM of classical paths}} + 2M\int_0^s ds' \underbrace{\gamma(s-s')}_{\frac{d\gamma}{ds} = \mu \ \text{Dissipation}} \frac{d}{ds'} X_c(s') + \underbrace{M\omega_0^2}_{\Omega^2} X_c(s) = F_\xi(s) \quad \text{Noise}$$

ξ for linear cplg

Issues in an *Open System paradigm*

1. **Stochasticity**: Noise/ Fluctuations from coarse-grained environment: Colored noise
2. **Backreaction** of coarse-grained environment E on the system S: Memory
3. Nonunitarity: **Dissipative** (nonlocal) **Dynamics** of Opened system.
4. **NonMarkovianity**: different dynamics of S vs E

Quantum Brownian Motion (QBM) via the Influence Functional (IF) Method (Feynman Vernon 1963)

Open Quantum Systems:
System **S** (harmonic osc, Unruh-DeWitt detector, harmonic atom. Q black hole) + **Environment E** (quantum scalar field)

Osc with time-independent frequency: Caldeira-Leggett (83) Markov master eqn; Grabert Ingold Schramm (88) Unruh Zurek (89) Hu- Paz- Zhang (92) **nonMarkovian master equation for a general environment**

Osc with time-dependent frequency: Hu Matacz (94) → Quantum optics

Introduce a detector to probe the properties of a quantum field, obtain QFT results + NEq Stat Mech properties E.g., Hawking- Unruh effect derived from this approach

System (1HO) interacting *bilinearly* with an Environment (NHO): All Gaussian

$$
\begin{aligned}
S[x] &= \int_0^t ds \left[\frac{1}{2}M\dot{x}^2 - V(x)\right] \\
S_e[q_n] &= \int_0^t ds \sum_n \left[\frac{1}{2}m_n\dot{q}_n^2 - \frac{1}{2}m_n\omega_n^2 q_n^2\right] \\
S_{int}[x,\{q_n\}] &= \int_0^t ds \sum_n (-C_n x\, q_n)
\end{aligned}
$$

Closed-Time-Path /Schwinger-Keldysh/ in-in Effective Action

[Schwinger 61, Keldysh 63, Chou, Hao, Su, Yu 1981, Calzetta Hu 1987, 88 …]

$$
\begin{aligned}
e^{i\Gamma[x_+,x_-]} &= e^{iS[x_+]-iS[x_-]} \times \\
&\int_{CTP} \prod_n Dq_{n+} Dq_{n-} \left(e^{iS_e[\{q_{n+}\}]-iS_e[\{q_{n-}\}]} \right. \\
&\qquad \left. e^{iS_{int}[x_+,\{q_{n+}\}]-iS_{int}[x_-,\{q_{n-}\}]} \right) \\
&= e^{iS[x_+]-iS[x_-]+iS_{IF}[x_+,x_-]}
\end{aligned}
$$

where S IF is the Influence Action.

$$S_{IF}[x_+, x_-] = \sum_n \frac{1}{2} \int ds\, ds' \Big[x_+(s) G_{n++}(s,s') x_+(s') - x_+(s) G_{n+-}(s,s') x_-(s') - x_-(s) G_{n-+}(s,s') x_+(s') + x_-(s) G_{n--}(s,s') x_-(s')\Big]$$

where Gn are the Schwinger-Keldysh or closed time path (+, -) propagators:

$$\begin{aligned}
G_{n++}(s,s') &= -\eta_n(s-s')\,\mathrm{sgn}(s-s') + i\nu_n(s-s') \\
G_{n+-}(s,s') &= \eta_n(s-s') + i\nu_n(s-s') \\
G_{n-+}(s,s') &= -\eta_n(s-s') + i\nu_n(s-s') \\
G_{n--}(s,s') &= \eta_n(s-s')\,\mathrm{sgn}(s-s') + i\nu_n(s-s')
\end{aligned}$$

η is used in HPZ92, called μ in later papers

The influence action S_{IF} can be written as

$$e^{iS_{IF}} = e^{-i\int_0^t ds \int_0^s ds' [\Delta x(s)\eta(s-s')\Sigma x(s')]} \; e^{-\frac{1}{2}\int_0^t ds \int_0^t ds' \, [\Delta x(s)\nu(s-s')\Delta x(s')]}$$

where $\Delta x(s) = x_+(s) - x_-(s)$ and $\Sigma x(s) = x_+(s) + x_-(s)$, and

$$\eta(s-s') = \sum_n \eta_n(s-s') = -\sum_n \frac{C_n^2}{2m_n\omega_n}\sin\omega_n(s-s')$$

Called μ in later papers

$$\nu(s-s') = \sum_n \nu_n(s-s') = \sum_n \frac{C_n^2}{2m_n\omega_n}\cos\omega_n(s-s')$$

Spectral density $\mathcal{I}(\omega)$ (later)

Feynman-Vernon Gaussian Integral Identity, Noise Kernel

Rewriting the imaginary part of S_{IF} as

$$
\begin{aligned}
& e^{-\frac{1}{2}\int \Delta x \nu \Delta x} \\
= \quad & N \int D\xi e^{-\frac{1}{2}\int \xi \nu^{-1} \xi} e^{-\frac{1}{2}\int \Delta x \nu \Delta x} \\
= \quad & N \int D\xi e^{-\frac{1}{2}\int (\xi - i\nu\Delta x)\nu^{-1}(\xi - i\nu\Delta x)} e^{-\frac{1}{2}\int \Delta x \nu \Delta x} \\
= \quad & N \int D\xi P[\xi] e^{i\int \xi \Delta x}
\end{aligned}
$$

where $P[\xi] = e^{-\frac{1}{2}\int \xi \nu^{-1} \xi}$ is the Gaussian probability density of the stochastic force ξ.

Due to this probability density one has the stochastic average $\langle \xi(s)\xi(s')\rangle_s = \nu(s - s')$ which is called the noise kernel.

Equation of Motion from the influence action

After this procedure the effective action

$$\begin{aligned}\Gamma[x_+, x_-] &= S[x_+] - S[x_-] \\ &\quad - \int_0^t ds \int_0^s ds' \Delta x(s) \eta(s-s') \Sigma x(s') \\ &\quad + \int_0^t ds \Delta x(s) \xi(s)\end{aligned}$$

called μ in later papers

The equation of motion for the particle is then given by

$$\left. \frac{\delta \Gamma[x_+, x_-]}{\delta x_+} \right|_{x_+ = x_- = x} = 0$$

Langevin Equation
Dissipation kernel

The equation of motion is a Langevin equation with the stochastic force $\xi(t)$,

$$M\ddot{x} + V'(x) + \int_0^t ds\,\eta(t-s)x(s) = \xi(t)$$

The integral term is related to dissipation as one can write

$$\eta(t) = \frac{d}{dt}\gamma(t) \Rightarrow \gamma(t) = \sum_n \frac{C_n^2}{2m_n\omega_n^2}\ cos\omega_n t$$

and we have

γ is called the damping kernel

$$M\ddot{x} + V'(x) + \int_0^t ds\,\gamma(t-s)\dot{x}(s) = \xi(t)$$

$\eta(s-s')$ is called the dissipation kernel. (called μ in later papers)

Issue 4: Fluctuation-Dissipation Relation

$$\nu(s) = \int_{-\infty}^{\infty} ds' \, K(s-s')\gamma(s')$$

in this simple case γ called damping kernel

$$K(s) = \int_0^{\infty} \frac{d\omega}{\pi} \omega \, \cos \omega s$$

Note the existence of FDR is a condition of self-consistency between the system dynamics with backaction from the environment.
This relation originates from the unitarity In the original closed system.

The coarse-grained environmental variables are now represented by noise and fluctuations. Their backreaction on the system imparts to it dissipative dynamics in the now opened system.

FDR called Optical theorem in particle physics (scattering)
Kramers Kronig relation in condensed matter physics
μ related to the dynamical susceptibility function

FDR for N detectors- Quantum Field System: Later.

Backreaction in Gravitation and Cosmology
-- Three levels of Theoretical Structure

FDR applied to Black hole & Cosmological backreaction problems

- Candelas Sciama (PRL1977) for a dynamic Kerr black hole emitting Hawking radiation.
- Mottola (PRD1986) for a static black hole (in a box) in quasi-equilibrium with its radiation via linear response theory

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- Hu & Sinha (1995): Dissipation of anisotropy in Bianchi Type I universe and fluctuations in particles creation.
- Campos & Verdaguer (1996) for weakly inhomogeneous cosmology

Black Holes

Backreaction of Hawking radiation on the black hole dynamics (since 80s)

- Quasi-stationary: Schwarzschild metric $m(t)$ due to Tmn of Φ (t) but system assumed to remain in quasi-equilibrium: Need to place the BH in a box or in AdS space. York et al (1983)

- Radiating BH: e.g., Bardeen (Vaidya metric)

FDR for static Black Hole (in a box)

Mottola showed that in some generalized Hartle-Hawking states a FDR exists between the expectation values of the commutator and anti-commutator of the energy-momentum tensor This FDR is similar to the standard thermal form found in linear response theory.

$$S_{abcd}(x,x') = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \coth\left(\frac{1}{2}\beta\omega\right) \tilde{D}_{abcd}(\mathbf{x},\mathbf{x}';\omega), \tag{3.5}$$

where S and D are the anticommutator and commutator functions of the energy-momentum tensor, respectively, and $\tilde{D}$ is the temporal Fourier transform of D. That is,

$$\begin{aligned} S_{abcd}(x,x') &= \langle\{\hat{T}_{ab}(x), \hat{T}_{cd}(x')\}\rangle_\beta \\ D_{abcd}(x,x') &= \langle[\hat{T}_{ab}(x), \hat{T}_{cd}(x')]\rangle_\beta. \end{aligned} \tag{3.6}$$

He also identifies the two-point function D as a dissipation kernel by relating it to the time rate of change of the energy density when the metric is slightly perturbed. Thus, Eq.(3.5) represents a *bona fide* FDR relating the fluctuations of a certain quantity (say, energy density) to the time rate of change of the very same quantity.

Linear Response Theory cannot provide self-consistent backreaction description

Two Critiques by Hu Raval Sinha (gr-qc/9901010)

1. About LRT: It is usually based on the assumption of a fixed, non-dynamical background (spacetime) and state (thermal) of the matter field(s). S-E cplg vanishingly weak.

- The spacetime and the state of matter should rather be determined in a self-consistent manner by their dynamics under mutual influence.

2. To Candelas and Sciama (1977) the fluctuation part represented by the noise kernel is amiss <= this is the centerpiece and major task of Stochastic Gravity 1994-96

Exact solutions (quantum gravity). Test Field Approximation (QFTCST*: Level 0), **Backreaction** (SCG: Level 1) w Fluctuations (StoGra: Level 2)+

Three Levels of Theoretical Structures

Einstein equation:

$$G_{\mu\nu}[g] = \kappa T_{\mu\nu}[g]$$

Semi-classical gravity (mean field):

$$G_{\mu\nu}[g] = \kappa \left(T_{\mu\nu}[g] + \langle T^{q}_{\mu\nu}[g] \rangle \right)$$

Stochastic gravity (including quantum fluctuations):

$$G_{\mu\nu}[g+h] = \kappa \left(T_{\mu\nu}[g+h] + \langle T^{q}_{\mu\nu}[g+h] \rangle + \xi_{\mu\nu}[g] \right)$$

to linear order in h, where $\xi_{\mu\nu}$ is the stochastic force induced by the quantum field fluctuations.

* Birrell & Davies, Parker & Toms: *QFTCST* (CUP 1982, 2009).

+ Hu & Verdaguer, *Semiclassical and Stochastic Gravity* (CUP 2020)

Backreaction of particle creation on the Dynamics of the early universe

Consider a massless conformal scalar quantum field in a Bianchi Type I universe. The normal modes satisfy:

$$\chi''_{\mathbf{k}}(\eta) + [\Omega^2_{\mathbf{k}}(\eta) + Q]\chi_{\mathbf{k}}(\eta) = 0.$$

s $\prime \equiv d/d\eta$ and

$$\Omega^2_k(\eta) \equiv \omega^2_k(t)a^2 = k^2 + m^2a^2$$

β_i. The line element of Bianchi Type I universe is thus

$$ds^2 = -dt^2 + \sum_{i=1}^{3} \ell_i^2(t)(dx^i)^2$$

where $\ell_i = a(t)e^{\beta_i}$ is the scale factor in the x_i direction.

$$q_{ij} = \beta'_{ij}$$

$$\frac{d}{d\eta}\left(\tilde{M}\frac{dq_{ij}}{d\eta}\right) + \mathcal{K}\frac{dq_{ij}}{d\eta} + kq_{ij} = c_{ij} + s_{ij}, \qquad (1.1.8)$$

$$\tilde{M} = \frac{1}{30(4\pi)^2}ln(\tilde{\mu}a) \qquad k = -\frac{a^2}{8\pi G_{\rm N}} + \frac{1}{90(4\pi)^2}\left[\left(\frac{a'}{a}\right)^2 + \left(\frac{a''}{a}\right)\right]$$

$$\mathcal{K}q_{ij} = \int d\eta_2 \int d\eta_1 f(\eta_2 - \eta_1)\frac{dq_{ij}}{d\eta_1},$$

$c_{ij} = \int J_{ij}(\eta)d\eta$ where Jij is an external source for switching on the anisotropy in the distant past. sij stochastic source - later

Rate of Particle production ~ Weyl curvature^2

With a real and causal equation of motion for $q_{ij} = \beta'_{ij}$ one can take the Fourier transform and identify from the dissipative term $i\omega\gamma q(\omega)$ (where $q \equiv q_{ij}q^{ij}$), i.e. the "resistance" component in a LCR circuit, the viscosity function $\gamma(\omega)$:

$$\gamma(\omega) = \frac{|\omega|^3}{60(4\pi)^2} \tag{1.1.5}$$

The damping of anisotropy going like ω^4 translates to a dependence on the quadrature of the second derivative of β_{ij}, which can be identified as the lowest order terms of the Weyl curvature tensor. This leads to the result that the rate of particle production in anisotropic or inhomogeneous cosmological spacetimes is proportional to the Weyl curvature-squared $C_{abcd}C^{abcd}$ of the background geometry.

Energy of particles created = integrated braking power of anisotropy damping

Quantum Field Process   Spacetime Dynamics

equation) one can obtain the (spectral) power $P(\omega)$ dissipated by a velocity-dependent viscous force $\mathbf{F}$ acting on the background spacetime simply from $P(\omega) = \mathbf{F} \cdot \mathbf{v}$. The dissipated energy density $\rho(\omega)$ is obtained by integrating this '(spectral) braking power' $P(\omega)$ over all frequencies.

$$\rho_{dissipation} = \int_0^\infty \frac{d\omega}{2\pi} [\omega \beta_{ij}(\omega)^*][\gamma(\omega)\omega\beta_{ij}(\omega)]. \tag{1.1.6}$$

Alternatively, focusing on the matter field sector (the right-hand side of the SCE equation) one can calculate the energy density of particles created from the vacuum. The power spectrum of particle pairs created by a given anisotropy history is given by

$$\mathcal{P}(\omega) = \frac{1}{30\pi^2}\omega^4 Tr\beta^*(2\omega)\beta(2\omega) \tag{1.1.7}$$

Integrating over the full spectrum $\int_0^\infty d\omega(2\omega)\mathcal{P}(\omega)$ produces the total energy density of particle pairs created, which is seen to be precisely equal to the energy density dissipated in the dynamics of spacetime.

s_ij Colored Noise from Fluctuations of Q Field

with $s_{ij}(\eta) = \int d\eta' \xi_{ij}(\eta')$ where $\xi_{ij}(\eta)$ is a Gaussian type noise, which is completely characterized by its second moment

$$\langle \xi_{ij}(\eta) \rangle_\xi = 0$$
$$\langle \xi_{ij}(\eta_1)\xi_{kl}(\eta_2) \rangle_\xi = \nu_{ijkl}(\eta_1 - \eta_2). \qquad (1.1.9)$$

Here $\nu_{ijkl}(\eta_1 - \eta_2)$ is known as the noise kernel. It is the two-point time-correlation function of the external stochastic source $\xi_{ij}(\eta)$. Since this correlation function is non-local, this noise is colored. In the above the angled

The noise kernel ν for the spatial anisotropy is given by:

$$\nu(\eta) = \frac{1}{30(4\pi)^2} \int_0^{+\infty} \frac{d\omega}{2\pi} \pi\omega^4 \cos\omega\eta$$

It is related to the damping kernel γ by a Fluctuation-Dissipation Relation

$$\nu(\eta) = \int_0^\infty d\eta' K(\eta - \eta')\gamma(\eta')$$

where the fluctuation-dissipation kernel K(η) is given by

$$K(\eta) = \int_0^\infty \frac{d\omega}{\pi} \omega \cos\omega\eta.$$

5. FDR+CPR in N-Osc system + Q Field

Key points

- **Quantum Brownian Motion** 1 osc-scalar field bath: Langevin eq: Dissipation in the system is balanced by Fluctuations (noise) in the environment (scalar field)
- Fluctuation-Dissipation Relation in the (thermal) bath

- N coupled oscillators in a common scalar field bath
- **FDR between the oscillators and the bath** was shown (RHA): Diagonal components of a matrix relation,
- **Correlation-Propagation Relation**: Off-diag components
- Combined: Generalized FDR. (CPR unknown before)

- **Generalized FDR between detectors** shown recently (JT)

N uniformly-accelerated detectors (UAD) in the same quantum field

- FDR in field (known in LRT, note limitations)
- Correlation-Propagation Relation between two different detectors: [Raval Hu Anglin 1996]

- Latest work: [Details in J T Hsiang's talk following]

J.-T. Hsiang, B. L. Hu, and S. -Y. Lin, *Fluctuation-Dissipation and **Correlation-Propagation Relations** from the Nonequilibrium Dynamics of Detector-Quantum Field Systems* arXiv:1905.08596

J.-T. Hsiang, B. L. Hu, S. -Y. Lin and K. Yamamoto, *Fluctuation-Dissipation and **Correlation-Propagation Relations** in 4D for Uniformly-Accelerated Detectors in a Quantum Field (prepared for PLB)*

System: Charge, Mass, Atom, Black Hole, Universe
Environment: Quantum Field, Open Systems viewpt

Now let's use the N detector (atom) System- Q Field model to probe deeper into the relations

- between the system and its environment,

NEq dynamics, relaxation, FDR, energy balance

- between any two detectors mediated by E: CPR, nonMarkovianity, mutual influence

FDR + CPR = a matrix FDR

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Stochastic theory of accelerated detectors in a quantum field

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We analyze the statistical mechanical properties of n detectors in arbitrary states of motion interacting with one another via a quantum field. We use the open system concept and the influence functional method to calculate the influence of quantum fields on detectors in motion, and the mutual influence of detectors via fields. We discuss the difference between self and mutual impedance, advanced and retarded noise, and the relations between noise-correlations and dissipation-propagation. The mutual effects of detectors on one another can be studied from the Langevin equations derived from the influence functional, as it contains the back reaction of the field on the system self-consistently. We show the existence of general fluctuation-dissipation relations, and for trajectories without event horizons, correlation-propagation relations, which succinctly encapsulate these quantum statistical phenomena. These findings serve to clarify some existing confusions on the accelerated detector problem. The general methodology presented here could also serve as a platform to explore the quantum statistical properties of particles and fields, with practical applications in atomic and optical physics problems. [S0556-2821(96)03912-4]

A. Influence functional for N arbitrarily moving detectors

Consider N detectors $i=1,\ldots,N$ in $1+1$ dimensions with internal oscillator coordinates $Q_i(\tau_i)$, and trajectories $(x_i(\tau_i),t_i(\tau_i))$, τ_i being a parameter along the trajectory of detector i. In the following analysis, we do not need to assume that τ_i is the proper time, although this is, in most cases, a convenient choice. However, we will assume hereafter that the trajectories $(t_i(\tau_i),x_i(\tau_i))$ are smooth and that the parameters τ_i are chosen such that $t_i(\tau_i)$ is a strictly increasing function of τ_i .

The detectors are coupled to a massless scalar field $\phi(x,t)$ via the interaction action

$$S_{\mathrm{int}}=\sum_i e_i\int_{-\infty}^{t_i^{-1}(T)} d\tau_i s_i(\tau_i)\frac{dQ_i}{d\tau_i}\phi(x_i(\tau_i),t_i(\tau_i)). \qquad (2.1)$$

Here, T is a global Minkowski time coordinate which defines a spacelike hypersurface, e_i denotes the coupling constant of detector i to the field, $s_i(\tau_i)$ is the switching function for detector i (typically a step function), and t_i^{-1} is the inverse function of t_i. $t_i^{-1}(T)$ is, therefore, the value of τ_i at the point of intersection of the spacelike hypersurface defined by T with the trajectory of detector i. Note that the strictly increasing property of $t_i(\tau_i)$ implies that the inverse, if it exists, is unique.

The action of the system of detectors is

$$S_{\text{osc}} = \frac{1}{2}\sum_i \int_{-\infty}^{t_i^{-1}(T)} d\tau_i [(\partial_{\tau_i} Q_i)^2 - \Omega_i^2 Q_i^2]. \tag{2.2}$$

The scalar field action is given by

$$S_{\text{field}} = \frac{1}{2}\int_{-\infty}^{T} dt \int dx [(\partial_t \phi)^2 - (\partial_x \phi)^2] \tag{2.3}$$

and the complete action

$$S = S_{\text{field}} + S_{\text{osc}} + S_{\text{int}}. \tag{2.4}$$

2.4 Influence Kernel for Linear Coupling

We have established a correspondence between the parametric oscillators and a quantum field. This will facilitate the transcription of calculations for the former problem using the influence functional formalism to the latter. There is one final link to be added to complete the picture: How do we see that the influence kernel takes on the form (2.15)?

Consider a particle detector following a trajectory $x^\mu(s)$ parametrized by s with internal coordinate Q coupled *linearly* to a quantum field ϕ with a coupling of the form $L_{int} = -eQ\phi(x(s))$. (The trajectory is denoted here simply by $x(s)$.) The *dissipation* and *noise* kernels μ and ν, are given respectively by [287]

Raval, Hu & Koks PRD 1997

$$\mu(s,s') = \frac{e^2}{2}G(x(s),x(s')) \equiv -i\frac{e^2}{2}\langle[\hat{\phi}(x(s)),\hat{\phi}(x(s'))]\rangle \tag{2.41}$$

$$\nu(s,s') = \frac{e^2}{2}G^{(1)}(x(s),x(s')) \equiv \frac{e^2}{2}\langle\{\hat{\phi}(x(s)),\hat{\phi}(x(s'))\}\rangle \tag{2.42}$$

where G and $G^{(1)}$ are the Schwinger and the Hadamard functions of the free field operator $\hat{\phi}$ evaluated for two points on the detector trajectory, $\langle\,\rangle$ denotes expectation value with respect to a vacuum state at some arbitrarily chosen initial time t_i, and $[\,,\,]$ and $\{\,,\,\}$ denote the commutator and anticommutator respectively. This result may be obtained either by integrating out the field degrees of freedom as in the Feynman-Vernon influence functional approach [40] or via manipulations of the coupled detector-field Heisenberg equations of motion in the canonical operator approach.

$\zeta \equiv \nu + i\mu$, called the *influence kernel*. For linear couplings, it follows from the above expressions that ζ is given by the Wightman function G^+: (This is def in RHA96, $e^2G_>$ in HHL19 $= i\,\zeta$)

$$\zeta(s,s') = e^2 G^+(x(s),x(s')) \equiv e^2 \langle \hat{\phi}(x(s)) \hat{\phi}(x(s')) \rangle . \tag{1.4}$$

The influence kernel thus admits the mode function representation (RHA96 picked sin and cos mode function following Hu Matacz PRD94)

$$\zeta(s,s') = e^2 \sum_k u_k(x(s)) u_k^*(x(s')), \tag{1.5}$$

the u_k's being the mode functions satisfying the field equations and defining the particular Fock space whose vacuum

Closed-Time-Path (CTP) coarse-grained (CG) effective action = FV influence action

$$S_{\mathrm{CG}} = \sum_i \int d\tau_i \left[m\, \dot{\Delta}_i^{(Q)}(\tau_i) \dot{\Sigma}_i^{(Q)}(\tau_i) - m\omega^2\, \Delta_i^{(Q)}(\tau_i) \Sigma_i^{(Q)}(\tau_i) \right]$$
$$+ \frac{1}{2} \int d^2x \sqrt{-g} \int d^2x' \sqrt{-g'}\, \Delta^{(j)}(x)\, G_R^{(\phi)}(x,x')\, \Sigma^{(j)}(x')$$
$$+ \frac{i}{2} \int d^2x \sqrt{-g} \int d^2x' \sqrt{-g'}\, \Delta^{(j)}(x)\, G_H^{(\phi)}(x,x')\, \Delta^{(j)}(x') \,.$$

$$\Delta^{(x)} = x^{(+)} - x^{(-)} \text{ and } \Sigma^{(x)} = (x^{(+)} + x^{(-)})/2$$

Retarded $$G_R^{(\phi)}(x,x') = i\,\theta(t-t')\ \mathrm{Tr}\Big(\big[\phi(x), \phi(x')\big]\, \rho_\phi \Big)$$

ρ_ϕ is the initial field state

Hadamard $$G_H^{(\phi)}(x,x') = \frac{1}{2}\ \mathrm{Tr}\Big(\big\{\phi(x, \phi(x')\big\}\, \rho_\phi \Big) \,,$$

Wightman function $$G_{>}^{(\phi)}(x,x') = i\,\langle \phi(x)\phi(x')\rangle = \frac{1}{2}\,G^{(\phi)}(x,x') + i\,G_{H}^{(\phi)}(x,x')\,.$$

The Wightman function of a massless scalar field in the Minkowski vacuum when expressed in terms of the Rindler coordinates (ξ,η) is given by:

$$G_{>}^{(\phi)}(x,x') = i\int_0^\infty \frac{d\kappa}{2\pi}\,\frac{1}{2\kappa}\left\{\left[\coth\frac{\pi\kappa}{\mathsf{a}}\,\cos\kappa(\Delta_\eta+\Delta_\xi) - i\,\sin\kappa(\Delta_\eta+\Delta_\xi)\right]\right.$$
$$\left. + \left[\coth\frac{\pi\kappa}{\mathsf{a}}\,\cos\kappa(\Delta_\eta-\Delta_\xi) - i\,\sin\kappa(\Delta_\eta-\Delta_\xi)\right]\right\}$$

$$\Delta_\eta = \eta - \eta' \text{ and } \Delta_\xi = \xi - \xi'$$

Unruh Temperature $T = \dfrac{\mathsf{a}}{2\pi}$

(generalized) Fluctuation-Dissipation (matrix) relation for the field:

$$\widetilde{G}_{H}^{(\phi)}(\kappa) = \coth\frac{\pi\kappa}{\mathsf{a}}\,\mathrm{Im}\,\widetilde{G}_{R}^{(\phi)}(\kappa)\,.$$

Diagonal terms: FDR
Off-diagonal terms: CPR: Correlation-Propagation Relation (RHA1996)

Uniformly-Accelerated Observer

UAO

$$\begin{cases} t = a^{-1} e^{a\xi} \sinh a\eta \\ x = a^{-1} e^{a\xi} \cosh a\eta \end{cases} \quad \frac{t}{x} = \tanh(a\eta)$$

Conformal transf. unimportant

$$-ds^2 = dt^2 - dx^2 = dU\,dV \;\longrightarrow\; -ds^2 = e^{2a\xi}\,du\,dv =$$

$$= e^{2a\xi}(d\eta^2 - d\xi^2) = d\tau^2 - dx^2$$

Minkowski

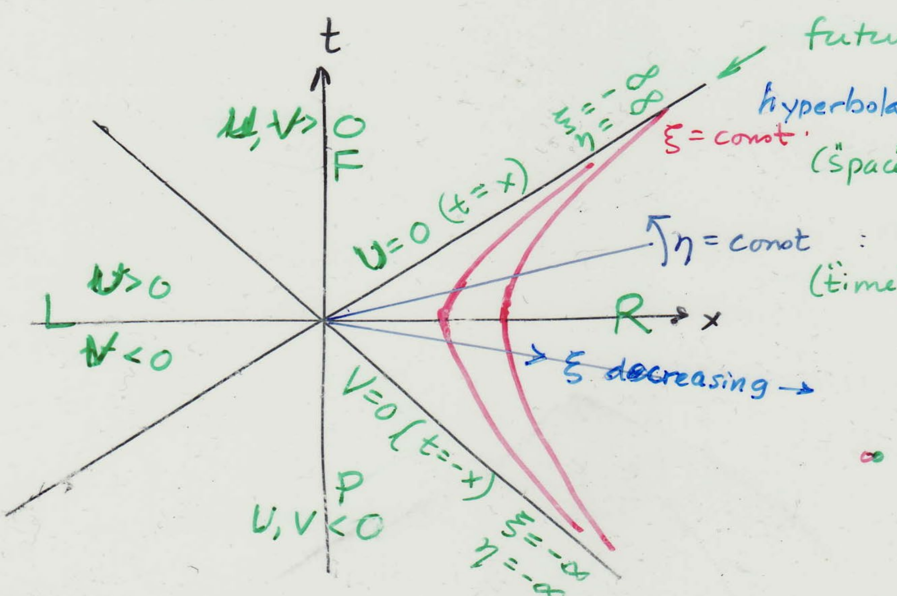
Light-cone $\begin{pmatrix} U \\ V \end{pmatrix} = t \mp x$

$$\begin{aligned} U &= -\tfrac{1}{a} e^{-au} \\ V &= \tfrac{1}{a} e^{av} \end{aligned} \;\longrightarrow\; \begin{pmatrix} u \\ v \end{pmatrix} = \tfrac{1}{2}(\eta \mp \xi)$$

exponential

Rindler

$$-\infty < \xi, \eta < +\infty$$



UAO:

hyperbola ("space") $x^2 - t^2 = \text{const} \quad \alpha^2 = \dfrac{e^{2a\xi}}{a^2}$

α^{-1}: proper acceleration

(e.g. MTW p.166)

accelerated observers' proper time $\tau = e^{a\xi}\eta$

$$\begin{cases} t = \alpha \sinh(\tau/\alpha) \\ x = \alpha \cosh(\tau/\alpha) \end{cases} \quad \begin{aligned} V &= a^{-1} e^{a\tau} \\ U &= -a^{-1} e^{-a\tau} \end{aligned}$$

Two detectors interacting via a common field

From GFDR of Q Field environment to GFDR between detectors: Need to solve for the system's Neq dynamics

- Fluctuations in Quantum Field: E

→ Stochastic component in system dynamics

- Quantum Radiation [Johnson & Hu, PRD2000]

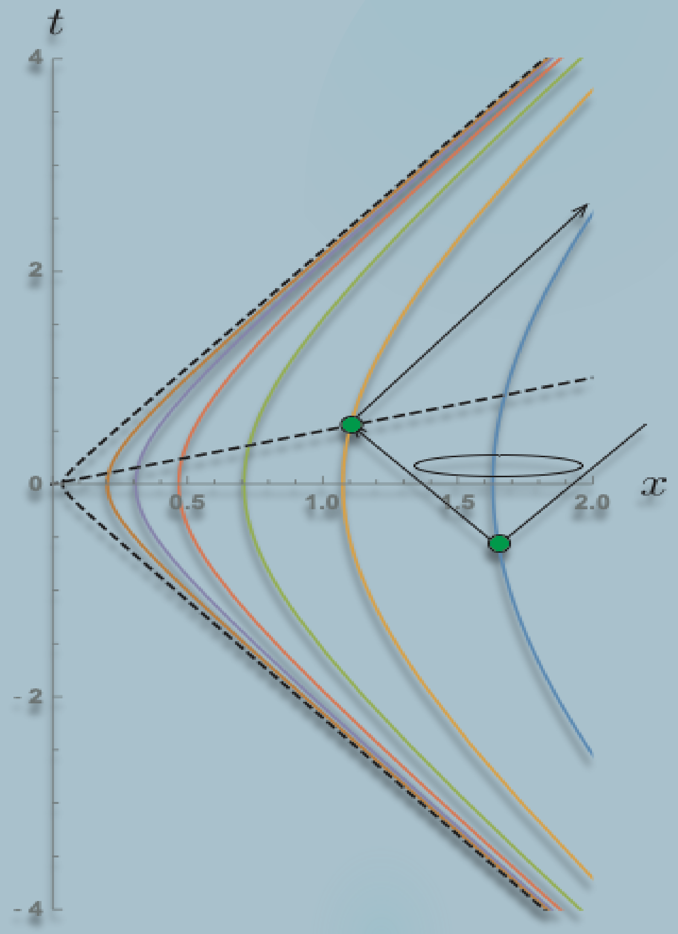
→ Reactive force (self-force) on system: S

→

- **NonMarkovianity** shows up in the:

Colored noise in E, correlated field fluctuations and (nonlocal) mutual influence in S

We examine these factors in
the Neq dynamics of the internal DoF,
the condition for reaching an equilibrium
the existence FDR-CPR among them



6. CPR

Influence Functional

$$\mathcal{F}[\{Q\};\{Q'\}] = \exp\left\{ -\frac{1}{\hbar}\left[\sum_{i,j=1}^{N} \int_{-\infty}^{t_i^{-1}(T)} \right.\right.$$

$$\times d\tau_i s_i(\tau_i) \int_{-\infty}^{t_j^{-1}(t_i(\tau_i))} d\tau_j' s_j(\tau_j')$$

$$\times \left(\frac{dQ_i}{d\tau_i} - \frac{dQ_i'}{d\tau_i} \right) \left(Z_{ij}(\tau_i, \tau_j') \frac{dQ_j}{d\tau_j'} \right.$$

$$\left.\left.\left. - Z_{ij}^{*}(\tau_i, \tau_j') \frac{dQ_j'}{d\tau_j'} \right) \right] \right\}, \qquad (2.14)$$

$$Z_{ij}(\tau_i, \tau_j') = \frac{2}{L} e_i e_j {\sum_{k,\sigma}}' \zeta_k(t_i(\tau_i), t_j(\tau_j')) u_k^{\sigma}(\tau_i) u_k^{\sigma}(\tau_j')$$

Two Inertial Detectors:

We now consider the case of two detectors moving on the inertial trajectories $x_1(\tau_1)=-x_0/2$, $x_2(\tau_2)=x_0/2$, and $t_1(\tau_1)=t_2(\tau_2)=\tau$, coupled to a scalar field initially in the Minkowski vacuum state, with coupling constants $e_{1,2}$. They are separated by a fixed coordinate distance x_0. As before, we will assume that both detectors have been forever switched on, i.e., $s_i(\tau)=1$, $i=1,2$.

$$Z_{11}(\tau,\tau')=\frac{e_1^2}{2\pi}\int_0^\infty \frac{dk}{k}\exp[-ik(\tau-\tau')],$$

$$Z_{22}(\tau,\tau')=\frac{e_2^2}{2\pi}\int_0^\infty \frac{dk}{k}\exp[-ik(\tau-\tau')],$$

$$Z_{12}(\tau,\tau')=Z_{21}(\tau,\tau')$$

$$=\frac{e_1 e_2}{2\pi}\int_0^\infty \frac{dk}{k}\exp[-ik(\tau-\tau')]\cos kx_0$$

In the above, the continuum limit in the mode sum is recovered through the replacement $\sum_k' \to \frac{L}{2\pi}\int_0^\infty dk$. We then obtain, after substituting for u_k^σ and ζ_k,

$$Z_{ij}(\tau_i, \tau_j') = \frac{e_i e_j}{2\pi}\int_0^\infty \frac{dk}{k} e^{-ik(t_i(\tau_i)-t_j(\tau_j'))}\cos k(x_i(\tau_i) - x_j(\tau_j')). \tag{2.16}$$

In this form, Z_{ij} is proportional to the two point function of the free scalar field in the Minkowski vacuum, evaluated for the two points lying on trajectories i and j of the detector system. It obeys the symmetry relation

$$Z_{ij}(\tau_i, \tau_j') = Z_{ji}^*(\tau_j', \tau_i) \tag{2.17}$$

Corresponding to (2.12), we may also split Z_{ij} into its real and imaginary parts. Thus we define

$$Z_{ij}(\tau_i, \tau_j') = \tilde{\nu}_{ij}(\tau_i, \tau_j') + i\tilde{\mu}_{ij}(\tau_i, \tau_j') \tag{2.18}$$

where (RHA96 picked sin and cos mode function following Hu Matacz PRD94)

$$\begin{aligned}\tilde{\nu}_{ij}(\tau_i, \tau_j') &= \frac{e_i e_j}{2\pi}\int_0^\infty \frac{dk}{k}\cos k(t_i(\tau_i) - t_j(\tau_j'))\cos k(x_i(\tau_i) - x_j(\tau_j')) \\ \tilde{\mu}_{ij}(\tau_i, \tau_j') &= -\frac{e_i e_j}{2\pi}\int_0^\infty \frac{dk}{k}\sin k(t_i(\tau_i) - t_j(\tau_j'))\cos k(x_i(\tau_i) - x_j(\tau_j')).\end{aligned} \tag{2.19}$$

$\tilde{\nu}$ and $\tilde{\mu}$ are proportional to the anticommutator and the commutator of the field in the Minkowski vacuum, respectively.

The coupled Langevin equations for the system are

$$\frac{d^2 Q_1}{d\tau^2}+\frac{e_1^2}{2}\frac{dQ_1}{d\tau}+\frac{e_1 e_2}{2}\frac{dQ_2}{d\tau}\Big|_{\tau-x_0}+\Omega_1^2 Q_1=\frac{d\eta_1}{d\tau}, \tag{3.27}$$

$$\frac{d^2 Q_2}{d\tau^2}+\frac{e_2^2}{2}\frac{dQ_2}{d\tau}+\frac{e_1 e_2}{2}\frac{dQ_1}{d\tau}\Big|_{\tau-x_0}+\Omega_2^2 Q_2=\frac{d\eta_2}{d\tau}, \tag{3.28}$$

where $\tau-x_0$ is the retarded time between the two trajectories, and

$$\eta_i(\tau_i)=e_i{\sum_{k,\sigma}}' \sqrt{\frac{2}{L}}\, u_k^\sigma(\tau_i)\xi_k^\sigma(t_i(\tau_i)).$$

$$\langle\{\eta_i(\tau),\eta_j(\tau')\}\rangle=\hbar\,\widetilde{\nu}_{ij}(\tau-\tau'). \tag{3.29}$$

$$\widetilde{Q}_2(\omega)=L_{22}(\omega)\,\widetilde{\eta}_2(\omega)+L_{21}(\omega)\,\widetilde{\eta}_1(\omega), \tag{3.33}$$

where L_{22} is the modified self-impedance of detector two because of the presence of detector one, and L_{21} is the mutual impedance:

$$L_{22}(\omega)=\chi_\omega^{(2)}(1-4\gamma_1\gamma_2 e^{-2i\omega x_0}\chi_\omega^{(1)}\chi_\omega^{(2)})^{-1},$$

$$L_{21}(\omega)=-2\sqrt{\gamma_1\gamma_2}e^{-i\omega x_0}\chi_\omega^{(2)}\chi_\omega^{(1)} \times(1-4\gamma_1\gamma_2 e^{-2i\omega x_0}\chi_\omega^{(1)}\chi_\omega^{(2)})^{-1}. \tag{3.34}$$

We obtain the Correlation Function

$$\langle\{Q_i(\omega),Q_j(\omega')\}\rangle=\sum_{\alpha=1}^{2}\sum_{\beta=1}^{2}L_{i\alpha}(\omega)L_{j\beta}(\omega') \times\langle\{\widetilde{\eta}_\alpha(\omega),\widetilde{\eta}_\beta(\omega')\}\rangle.$$

FDR for N osc with time-like trajectories

$$\tilde{\gamma}_{ii}(\tau_i,\tau_i') = -\frac{d}{d\tau_i}\tilde{\mu}_{ii}(\tau_i,\tau_i')$$

$$= \frac{e_i^2}{4}\left[\delta(v_i(\tau_i)-v_i(\tau_i'))\frac{dv_i}{d\tau_i}\right.$$

in adv. & ret. null coord (u, v)

$$\left. + \delta(u_i(\tau_i)-u_i(\tau_i'))\frac{du_i}{d\tau_i}\right]$$

$$= \tilde{\gamma}_{ii}^a(\tau_i,\tau_i') + \tilde{\gamma}_{ii}^r(\tau_i,\tau_i'),$$

ith detector on itself: radiation reaction, self force

The timelike property of the trajectories implies that $|dx_i/dt_i|<1$. Together with the fact that $t_i(\tau_i)$ are increasing functions of τ_i, this implies that $du_i/d\tau_i$ and $dv_i/d\tau_i$ are necessarily positive. It also implies that the functions $u_i(\tau_i)$ and $v_i(\tau_i)$ have unique inverses, if they exist. This can be

$$\tilde{\gamma}_{ii}(\tau_i,\tau_i') = \frac{e_i^2}{2}\delta(\tau_i-\tau_i'). \tag{3.7}$$

Thus we see that, for an arbitrary trajectory, the dissipation or radiation reaction kernel has the same form and is always local. This fact has been used in obtaining the dissipative term in the equations of motion for the accelerated detector and probe (4.1 and 4.7).

The fluctuation-dissipation relation now follows in a straightforward manner:

$$\tilde{\nu}_{ii}(\tau_i,\tau_i') = \int_{-\infty}^{\infty} ds K_i(\tau_i,s)\tilde{\gamma}_{ii}(s,\tau_i') \tag{3.8}$$

where

$$\begin{aligned} K_i(\tau_i,s) &= K_i^a(\tau_i,s)+K_i^r(\tau_i,s) \\ &= \int_0^\infty \frac{dk}{2\pi k}[\cos k(v_i(\tau_i)-v_i(s)) + \cos k(u_i(\tau_i)-u_i(s))]. \end{aligned} \tag{3.9}$$

ith detector on jth detector: Re Zij: Correlations of the field; Im Zij: propagation of radiation b/w two trajectories

3.2 Correlation-propagation relation

We now ask whether a similar relation holds between the real and imaginary parts of Z_{ij}, $i \neq j$. This would not be a fluctuation dissipation relation in the usual sense, as the real part of Z_{ij} describes correlations of the field between points on different trajectories rather than fluctuations, and its imaginary part describes the propagation of radiation between one detector and the other, rather than dissipation. We will call such relations "correlation-propagation" relations.

If points on different trajectories have space like separations, the relevant $\tilde{\gamma}_{ij}$ (defined as $-\frac{d\tilde{\mu}_{ij}}{d\tau_i}$) will vanish as a consequence of the vanishing of the commutator of a free field for points at spacelike separations. This is simply an expression of causality in the detector dynamics. However, the corresponding correlation $\tilde{\nu}_{ij}$ need not vanish, and hence there cannot be a general relation between these two kernels. Such a situation is realized most clearly, for example, in the case of two uniformly accelerating detectors, one in the right and the other in the left Rindler wedge. The trajectories, although individually timelike, are spacelike separated everywhere. The corresponding $\tilde{\gamma}_{12}$ and $\tilde{\gamma}_{21}$ will therefore vanish identically. However, $\tilde{\nu}_{12}$ and $\tilde{\nu}_{21}$ will remain non-zero, reflecting the highly correlated nature of the Minkowski vacuum state.

If the two pts on different trajectories are spacelike separated, e.g., one in L, the other in R: no CP relation.

CPR for Trajectories w/o horizons

Trajectories without horizons If, however, none of the detector trajectories possess past or future horizons (in Minkowski space this is true in particular for geodesic trajectories, but not *only* for geodesic trajectories), then each of them will lie completely within the causal future of the others. In that case, we can obtain correlation-propagation relations relating separately the advanced and retarded correlations to their "propagating" counterparts. These relations follow from the fluctuation dissipation relations along single trajectories derived above, essentially by a method of geometric construction : defining $\tilde{\gamma}^a_{ij} = -\frac{d\tilde{\mu}^a_{ij}}{d\tau_i}$ and similarly $\tilde{\gamma}^r_{ij}$, we have

$$\tilde{\gamma}^a_{ij}(\tau_i, \tau_j') = \frac{e_i e_j}{4}\delta(v_i(\tau_i) - v_j(\tau_j'))\frac{dv_i}{d\tau_i}. \tag{3.10}$$

Since the trajectory i does not possess horizons, the null coordinates u_i and v_i range from $-\infty$ to ∞. Thus the functions $v_i v_i^{-1}$ and $u_i u_i^{-1}$ are identity functions over the entire real line. Then we obtain, similar to equation (5.6),

$$\begin{aligned} \tilde{\gamma}^a_{ij}(\tau_i, \tau_j') &= \frac{e_i e_j}{4}\delta(\tau_i - v_i^{-1}(v_j(\tau_j'))) \\ &= \frac{e_j}{e_i}\tilde{\gamma}^a_{ii}(\tau_i, v_i^{-1}(v_j(\tau_j'))) \end{aligned} \tag{3.11}$$

and

$$\tilde{\gamma}^r_{ij}(\tau_i, \tau_j') = \frac{e_j}{e_i}\tilde{\gamma}^r_{ii}(\tau_i, u_i^{-1}(u_j(\tau_j'))). \tag{3.12}$$

The correlations $\tilde{\nu}_{ij}$ may be constructed from the noises $\tilde{\nu}_{ii}$ in an identical manner:

$$\begin{aligned}\tilde{\nu}^a_{ij}(\tau_i,\tau'_j) &= \frac{e_i e_j}{4\pi}\int_0^\infty \frac{dk}{k}\cos k(v_i(\tau_i) - v_i(v_i^{-1}v_j(\tau'_j))) \\ &= \frac{e_j}{e_i}\tilde{\nu}^a_{ii}(\tau_i, v_i^{-1}v_j(\tau'_j))\end{aligned} \tag{3.13}$$

where we have inserted the identity function $v_i v_i^{-1}$ in the first step. Also,

$$\tilde{\nu}^r_{ij}(\tau_i,\tau'_j) = \frac{e_j}{e_i}\tilde{\nu}^r_{ii}(\tau_i, u_i^{-1}u_j(\tau'_j)). \tag{3.14}$$

These two sets of constructions for the propagation and correlation kernels in terms of the dissipation and noise kernels enables us to write down the correlation-propagation relations simply by invoking the fluctuation-dissipation relations (5.8) as they separately apply to the advanced and retarded parts of the noise and dissipation along single trajectories:

$$\tilde{\nu}^{a,r}_{ij}(\tau_i,\tau'_j) = \int_{-\infty}^{\infty} ds K^{a,r}_i(\tau_i, s)\tilde{\gamma}^{a,r}_{ij}(s,\tau'_j), \tag{3.15}$$

Correlation-propagation relation

K^a_i and K^r_i being defined earlier (5.9). Since the quantities $\tilde{\gamma}_{ij}$ are really just δ-functions and the quantities $K^{a,r}_i$ are proportional to $\tilde{\nu}^{a,r}_{ii}$, these relations can be equivalently viewed as constructions of the correlations $\tilde{\nu}_{ij}$ from the noises $\tilde{\nu}_{ii}$.

The above relations hold for trajectories without event horizons.

Two moving detectors following arbitrary timelike trajectories $z_i(t)$

- We can construct the FDR/CPR of the field for two moving detectors following arbitrary timelike trajectories $z_i(t)$ as long as they do not possess the horizons.

- However, since in this case, such constructed two-point functions of the field in general are not time-translationally invariant, the corresponding generalized FDR can only be expressed in the form of a convolution integral in time.

> Further details about FDR-CPR in J T Hsiang's talk at RQI-Tainan

Stochastic dynamics

Variation of the influence action yields a Langevin Eq the internal DoF of the detector

$$m\ddot{Q}_i(\tau_i) + m\omega^2 Q_i(\tau_i) + \lambda^2 \sum_j \frac{d}{d\tau_i} \int d\tau_j \, G_R^{(\phi)}[z_i(\tau_i), z_j(\tau_j)]\dot{Q}_j(\tau_j) = -\lambda \, \zeta_i(\tau_i)$$

$$\langle\langle \zeta_i(\tau_i) \rangle\rangle = 0 \,, \qquad\qquad \langle\langle \zeta_i(\tau_i)\zeta_j(\tau_j) \rangle\rangle = G_H^{(\phi)}[z_i(\tau_i), z_j(\tau_j)] \,.$$

the noise force $\zeta_i(\tau_i)$ accounts for field fluctuations of the initial state at the location of the detector *i*,

the corresponding probability distribution is Gaussian

Q_i thus has a stochastic component, which is of quantum origin

Nonlocal potential, Self Force, Quantum Radiation, FDR, nonMarkovianity,

the nonlocal term $$-\lambda^2 \frac{d}{d\tau_i} \int d\tau_j \, G_R^{(\phi)}[z_i(\tau_i), z_j(\tau_j)] \dot{Q}_j(\tau_j)$$

(Lienard-Wiechert potential) depicts the stochastic motion of Q_j ;

Quantum radiation emitted by Q_j has two consequences:

local: reactive force (self force) which damps Q_i

nonlocal: influence from and influencing the other detectors. Memory, History dependent, non-Markovian

it is this quantum radiation – not classical radiation --which matches up with the field fluctuations to form the FDR (quantum to quantum)

Generalized FDR for the system dof

we obtain a generalized FDR for the internal DoF of the detectors (a matrix relation)

$$\widetilde{\mathbf{G}}_H^{(Q)}(\kappa) = \coth\frac{\pi\kappa}{\mathsf{a}}\,\mathrm{Im}\,\widetilde{\mathbf{G}}_R^{(Q)}(\kappa)$$

it looks similar to that of the field

$$\widetilde{\mathbf{G}}_H^{(\phi)}(\kappa) = \coth\frac{\pi\kappa}{\mathsf{a}}\,\mathrm{Im}\,\widetilde{\mathbf{G}}_R^{(\phi)}(\kappa)$$

it also looks similar to the one from LRT

$$\tilde{C}_{jk}(\omega) = \frac{\omega}{2}\coth\frac{\beta\omega}{2}\,\tilde{A}_{jk}(\omega)$$

But LRT is very restricted: Bath is nondynamical,

Our gFDR is obtained from the Neq dynamics of the system.

The final state of the system in general is NOT a canonical Gibbs state unless the sys-bath coupling is vanishingly small

The off-diagonal terms are in general nonzero even there is no direct interaction among the sys constituents

Generalized FDR = FDR + CPR

generalized FDR is a matrix equation containing two sets of relations

diagonal terms: conventional fluctuation-dissipation relations — connecting local frictional force and noise force for each detector

off-diagonal terms: correlation-propagation relations — relating the correlation and non-Markovian influence between detectors;

They are not present if

(1) there is no direct interaction between detectors and

(2) the bath (QF) does not play any dynamical role

7. FDR used for Energy Balance and Information Flow

- The generalized FDR guarantees the energy balance between the detectors, and the balance between the detector and the quantum field.
- Use these relations to analyze quantum information issues for different observers in Rindler, black hole spacetimes and inflationary universes. Begin with mutual information.

Fluctuations, Correlations (here) → Entanglement, Teleportation (next)

- Energy/entropy, kt ln 2, Landauer relation etc.

Thank You!

Rindler spacetime: Uniformly accelerated detectors (UAD)

N uniformly accelerating Unruh-DeWitt detectors in an ambient scalar field in 1+1 Minkowski space,

The external degree of freedom Q_i of each detector is described by

$$t = \frac{e^{\mathsf{a}\xi}}{\mathsf{a}} \sinh \mathsf{a}\eta \,, \qquad z = \frac{e^{\mathsf{a}\xi}}{\mathsf{a}} \cosh \mathsf{a}\eta \,,$$

Its proper acceleration is $\alpha = \mathsf{a}\, e^{-\mathsf{a}\xi}$

the field ϕ is a massless scalar field, initially prepared in global Minkowski vacuum.

the internal DoF of the detector is modeled by a SHO.

its coupling with the field is turned on* at $t = 0$

The interaction action takes the form (λi is called e_i in RHA)

$$\dot{Q} = \frac{dQ}{d\tau}$$

$$S_{\rm INT}[Q_i, \phi] = \int d^2x \sqrt{-g}\; j_i(x)\phi(x)\,, \quad j_i(x) = \lambda_i \int d\tau_i\; \dot{Q}_i(\tau_i)\, \frac{\delta^{(2)}[x^\mu - z_i^\mu(\tau_i)]}{\sqrt{-g}}\,,$$

the initial state of the internal DoF can be arbitrary

the field looks thermal to the detector

the internal DoF of the detector is not in equilibrium with the field, we need to follow its nonequilibrium evolution to find out whether at late times they settle into equilibrium

J.-T. Hsiang, B. L. Hu, and S. -Y. Lin, *Fluctuation-Dissipation and* ***Correlation-Propagation Relations*** *from the Nonequilibrium Dynamics of Detector-Quantum Field Systems* arXiv:1905.08596 (PRD)]

Black Hole Information Loss Paradox (Hawking, 1976)

Black hole radiation is in a mixed (thermal) state.

If the black hole continues to evaporate until it disappears completely,

a precisely known initial pure state (BH + field vacuum, a state with specified classical quantities – mass, angular momentum and charge plus full coherence information in the quantum field) will evolve to

a mixed state (nothing? + thermal radiation, a state permitting only probability assignments to various alternatives): Breakdown of unitarity?

- Related to the **End-state issue**. Could the information
  - be retained by a stable BH **remnant**?
  - be stored in a naked **singularity**
  - be transferred to a **baby universe**?
  - leak out with the Hawking radiation? Where does the information reside?

A. In the correlation between the field and the BH? ← most particle physicists' opinion (but countered by recent QI result of Braunstein and Pati, PRL07)

B. Transferred to the field completely, appearing to be mixed by approximately local measurement? (Wilczek 93)

- Information stored in field correlations, flows from lower order to higher order (Hu, Erice 95). Can in principle be retrieved, but very difficult. Apparent information `loss' due to the limitation in measurement by a local observer.

Atom analog of quantum black hole

Area of BH ~ action variable (e.g. energy) of atom
Hawking radiation of BH ~ spontaneous emission of atom

- Quantized horizon area, "area-eigenstates" (Bekenstein 1997)
- Einstein A and B coefficients of BH (Bekenstein and Meisels 1976)
- Spectroscopy of BH (Bekenstein and Mukhanov 1995)

- Assume the combined (BH atom + quantum field) system is unitary. Information gets carried away thru the Hawking radiation. But how so?

An analog: Quantum harmonic oscillator (~ BH) in a quantum field

2D: Anglin, Laflamme, Zurek and Paz 1995,
Lin and Hu, Class. Quant. Grav. 25:154004 (2008)

- Working on a harmonic oscillator + quantum field model we obtained an exact solution (so it includes full backreaction), and thus can treat strong coupling and non-Markovian behavior never attainable before.
- To follow the information trail, we calculate the entanglement dynamics between the two sectors and see how coherence in each sector evolves.

Implications on BH information

Viewing the UD detector as the analog of a quantum black hole,
our results suggest:

1. All the initial information of the black hole will be encoded in its radiation (spontaneous emission in atom analog)
   -- consistent with the "no-hiding theorem" (Braunstein and Pati 2007),
   -- no information is hidden in the correlation between BH and the field.

2. Only in the ultraweak coupling limit can both the BH and the field restore most of its purity and quantum coherence.

3. In the non-Markovian regime (strong BH-field coupling), the area-eigenstate of BH could not form a good basis, and the final state of BH would be a complicated mixed state distributed widely from the ground state to the highly excited area-eigenstates.

4. Therefore, at late times BH could end up as a large remnant (with large expectation value of area operator) with all its initial information already leaked out and dispersed into the quantum field.

Quantum Twin Paradox [Lin, Behunin and Hu, work in progress]

We expect...

- Information of the initial state of the detectors will be dispersed into the field.
- Initial entanglement between the detectors, if any, would disappear (sudden death) if the traveling time of Bob is long enough.
- During the acceleration and deceleration stages, Bob will experience something similar to the Unruh effect.
- If Bob and Alice are sitting close enough to each other in the final stage, then the field induced quantum entanglement would take over, namely, entanglement will be created (revived) by mutual influences mediated by the field.
- **No time reversal symmetry in the history of the system.**
- Finite time acceleration. No event horizon
- Time dilation of Bob should be considered.
- Mutual influence due to the deceleration of Bob does not have an impact on Alice.

FDR description of Backreaction of Quantum Processes in Black holes and the Early Universe

Black Holes

Backreaction of Hawking radiation on the black hole dynamics (since 80s)

- Quasi-stationary: Schwarzschild metric m(t) due to Tmn of Φ (t) but system assumed to remain in quasi-equilibrium: Need to place the BH in a box or in AdS space. York et al (1983)

- Radiating BH: e.g., Bardeen (Vaidya metric)

FDR for Black Hole (Candelas & Sciama 1977)

As a starting point they considered the *classical* relation, due to Hartle and Hawking [47], between energy flux transmitted across the horizon of a perturbed black hole and the shear [3]:

$$\frac{d^2E}{dtd\Omega} = \frac{M^2}{\pi} \mid \sigma(2M) \mid^2, \tag{4.4}$$

where $\sigma(2M)$ is the perturbed shear of the null congruence which generates the future horizon.

In turn, the dissipation of horizon area with respect to the advanced null coordinate is related to the energy flux across the horizon, and the above equation becomes (see, for example [48])

$$\frac{dA}{dv} = 4M \int_H \mid \sigma \mid^2 dA, \tag{4.5}$$

the integral being performed over the horizon.

The classical formula above immediately suggests a fluctuation-dissipation description: the dissipation in area is related linearly to the squared absolute value of the shear amplitude. This description is even more relevant when the gravitational perturbations are quantized. Then the integrand of the right-hand-side of Eq.(4.5) is $\langle\sigma^*\sigma\rangle$, the expectation value being taken with respect to an appropriate quantum state. Candelas and Sciama choose this state to be the Unruh vacuum, arguing that it is the vacuum which approximates best a flux of radiation from the hole at large radii.

Substitution of this quantity in (4.5), the left-hand-side of that equation now represents the dissipation in area due to the Hawking flux of gravitational radiation, and the right-hand-side comes from pure quantum fluctuations of gravitons (as opposed to semiclassical fluctuations of gravitational perturbations, which are induced by the presence of quantum matter).

Critiques: by Hu,Raval Sinha 1999

Does not relate dissipation of a certain quantity (horizon area) to the fluctuations of the same quantity.

- Need to compute the two-point function of the area, a four-point function of the graviton field, related to a two-point function of the stress tensor, called Noise Kernel
- Noise Kernel, the centerpiece of stochastic gravity theory (Hu & Verdaguer et al) enters into the FDR description of backreaction