

# *Delineating QCD matter in extreme conditions*

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# Career

- 2008 : Ph.D., Kyoto University ( supervisor : Prof. Hideo Suganuma )

( **Hadron physics**; QCD sum rules for multi-quarks; holographic QCD)

2008 - 2011 : Posdoc, RIKEN-BNL research center, USA (host: Larry McLerran)

( **cold, dense QCD** in general; low dimensional condensed matter; baryons in large  $N_c$  )

2011 - 2013 : Posdoc, Bielefeld University, Germany (host: Aleksii Vuorinen)

( QCD in strong magnetic fields; ... )

2013 - 2015 : Research Associate, University of Illinois, Urbana-Champaign, USA

( **Neutron Stars**; 2-color QCD; ... ) (host: Gordon Baym)

2015 - Now : Associate professor, Central China Normal University

( Delineating neutron star phenomena; functional methods; ... )

# Achievements

- **29 (+1)** papers & 27 proceedings published (1307 cited in inspires, h-index 18)

most cited: *From hadrons to quarks in neutron stars: a review ('18)* : 205 times

*Quarkyonic Chiral Spirals ('10)* : 189 times

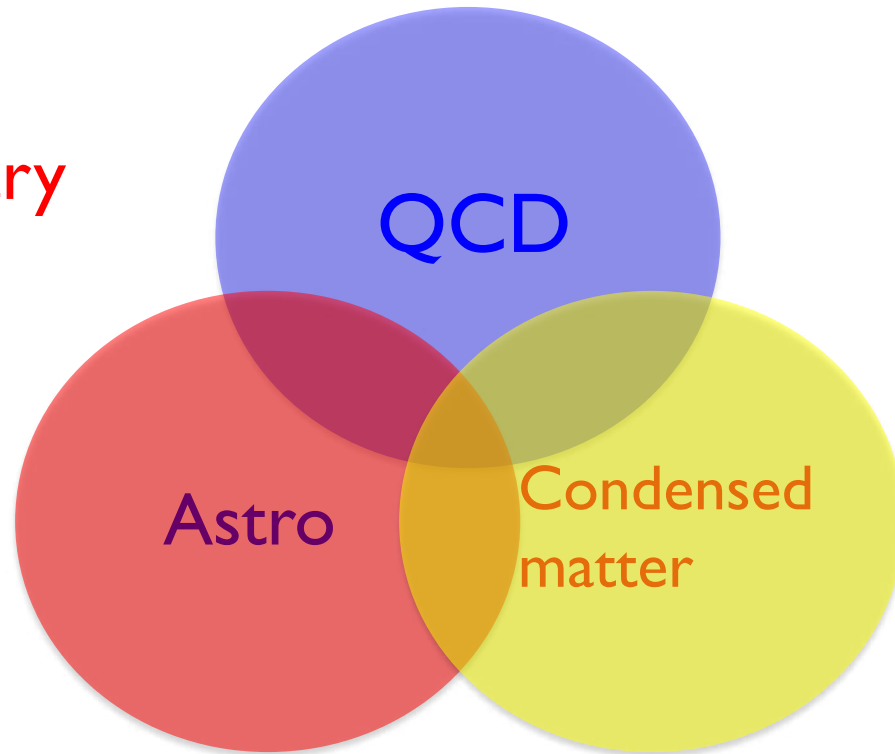
(will be presented today):

- **57** talks (**31** invited) in international conferences
- **44** seminars
- Honors: “*the most valued referee*” : Nucl. Phys. A ('13) & Phys. Lett. B ('14)  
*Special posdoc* (RIKEN, \$10,000 x 3-years)  
*NSFC* (2016: ~\$20,000, 2018-2021: ~ \$100,000 )

# Research interests

- **Hot & Dense QCD**
- QCD in magnetic fields
- Hadron spectroscopy
- Nuclear forces

Interdisciplinary

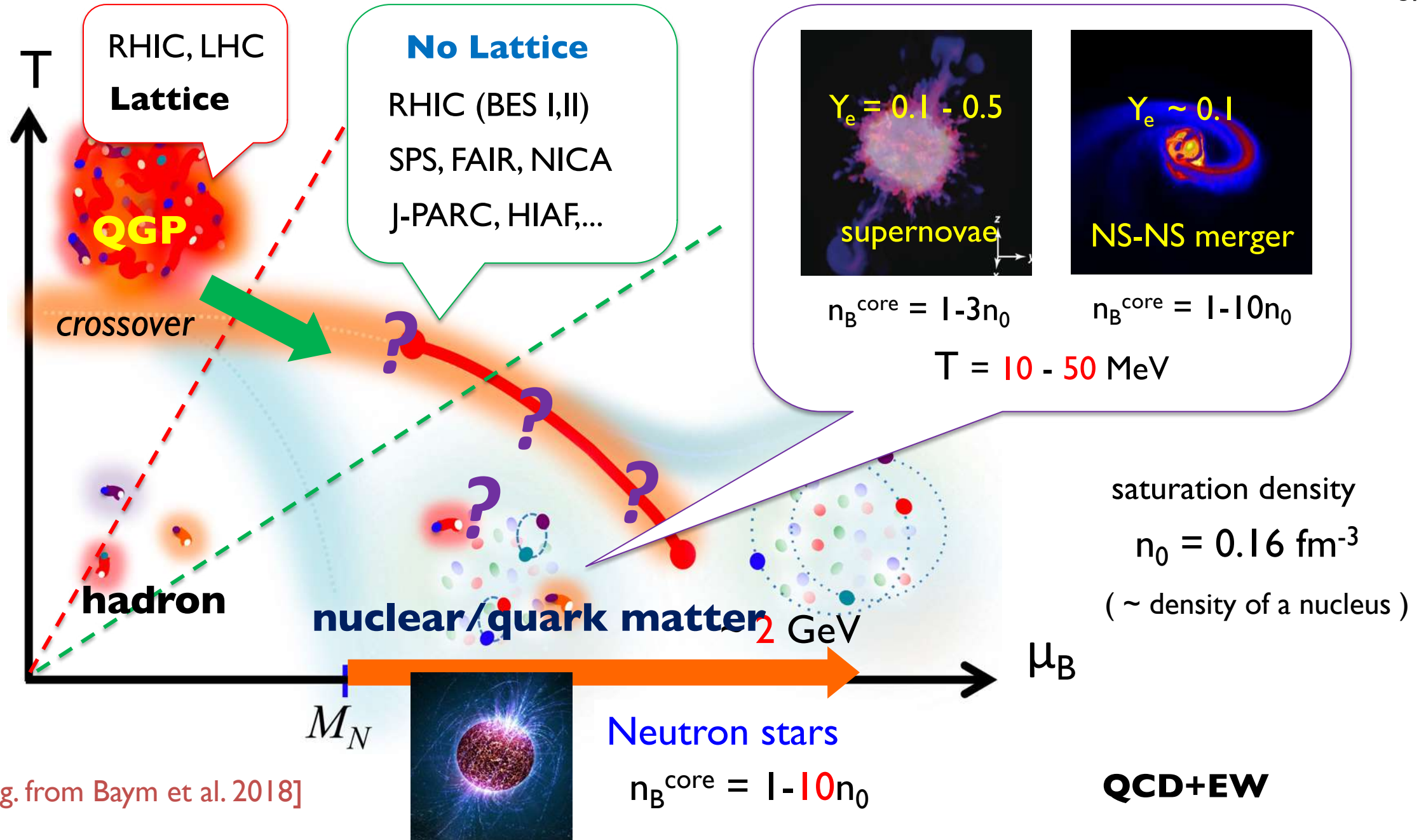


- **Neutron stars**
- Supernovae

- **Super-conductivity**
- Topological matter
- Low-dim. physics

# Contents

- 1, Dense QCD matter – overall sketch
- 2, 2-topics on dense QCD (achievements)
  - *Quarkyonic matter*
  - *Neutron star equations of state*
- 3, On-going works & future prospects



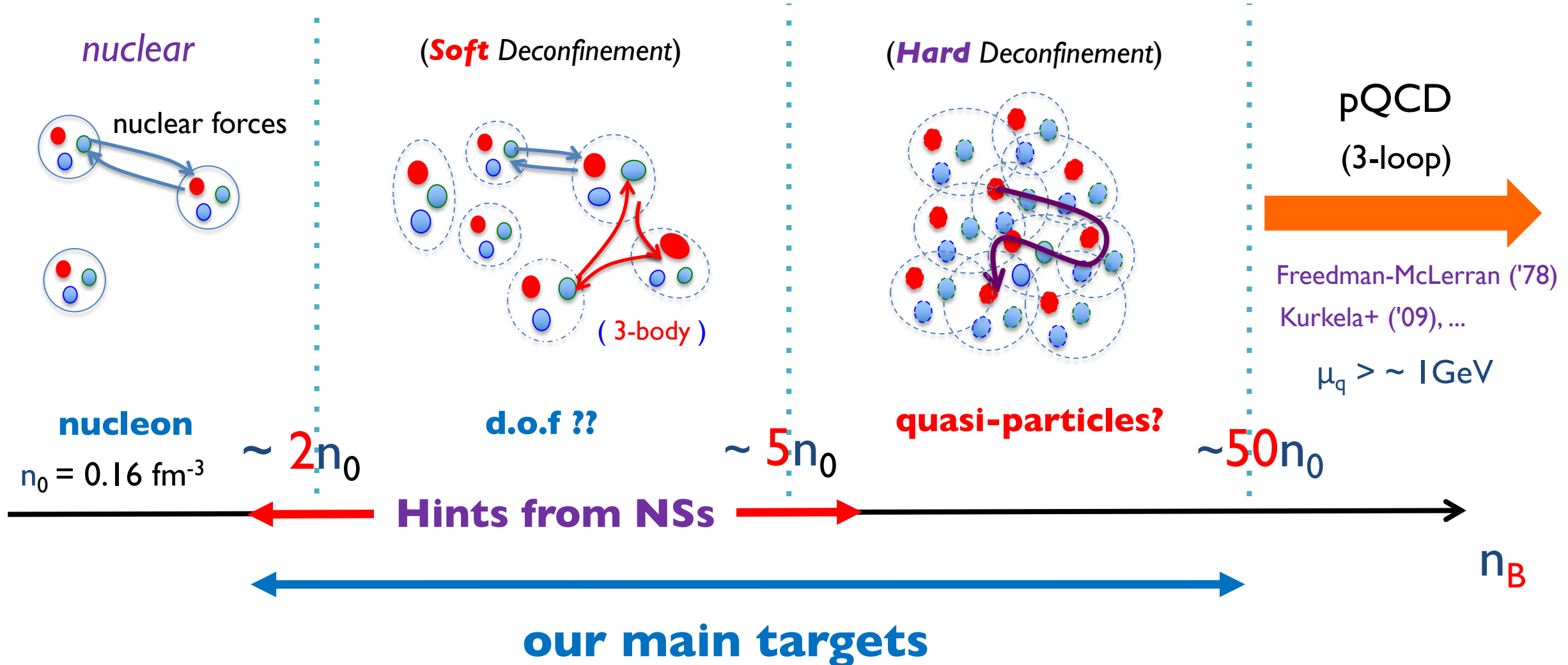
[Fig. from Baym et al. 2018]

# Dense QCD: my motivations

- 1, *Interdisciplinary* subjects : QCD x condensed matter x astrophysics
- 2, There remain *fundamental questions* & *paradoxes*
  - Rooms for new concepts & ideas
- 3, *Relevant pieces* to *complete* the QCD phase diagram
  - The long-standing problem; a lot of experimental & observational efforts
- 4, *Practically* important for *astrophysical* phenomena
  - e.g.)
    - Neutron star EoS: M-R relations & compositions
    - Hot EoS for supernovae & NS mergers
    - Reducing QCD uncertainties: beneficial for the beyond-SM physics

# Sketch of dense QCD matter

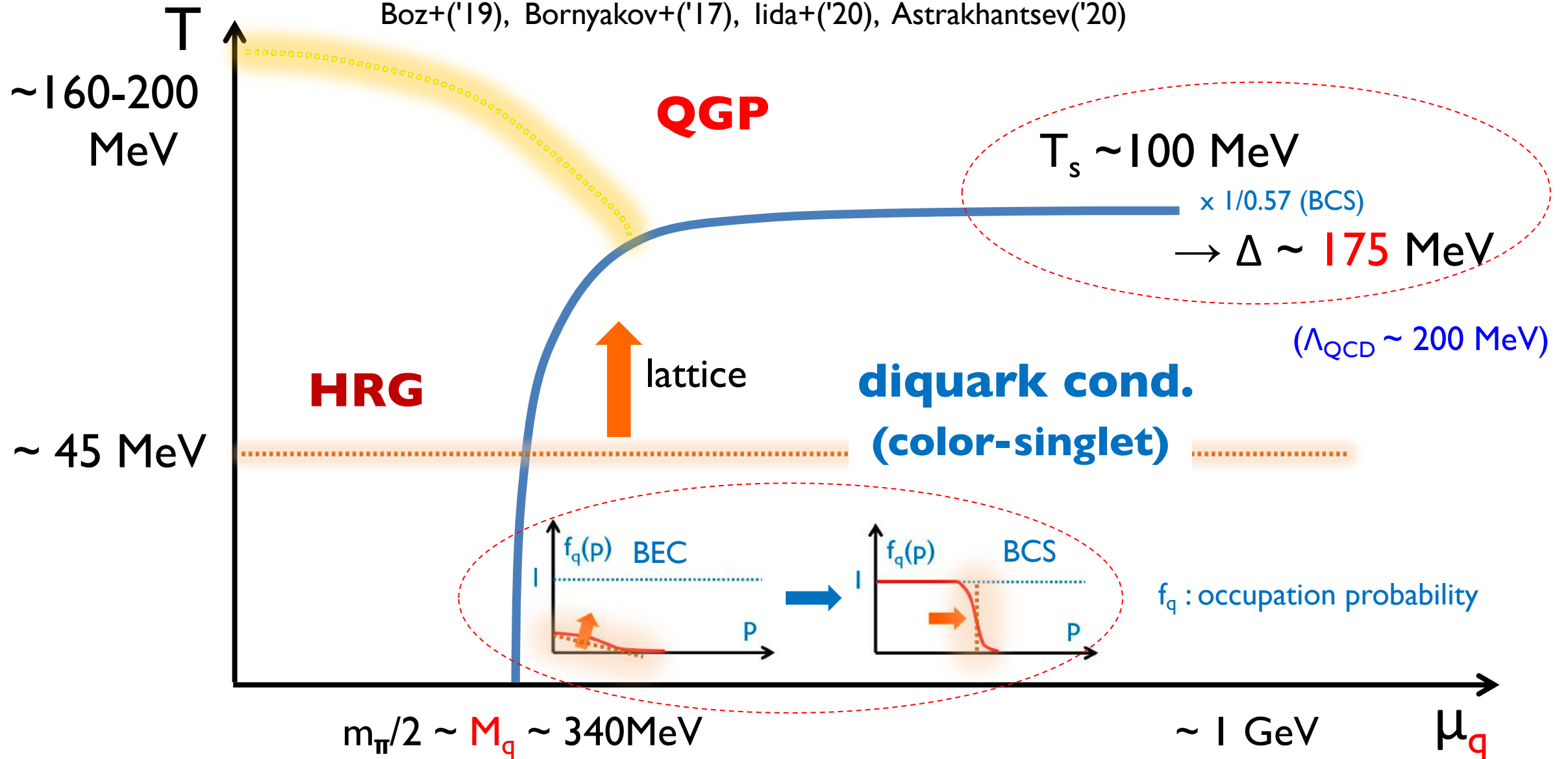
A sketch for  $N_c = 3$ : Masuda+('12), TK+('14), Fukushima+('20),...





# Hints from **2-color QCD**: **lattice studies** **no sign problem!**

Boz+('19), Bornyakov+('17), Iida+('20), Astrakhantsev('20)

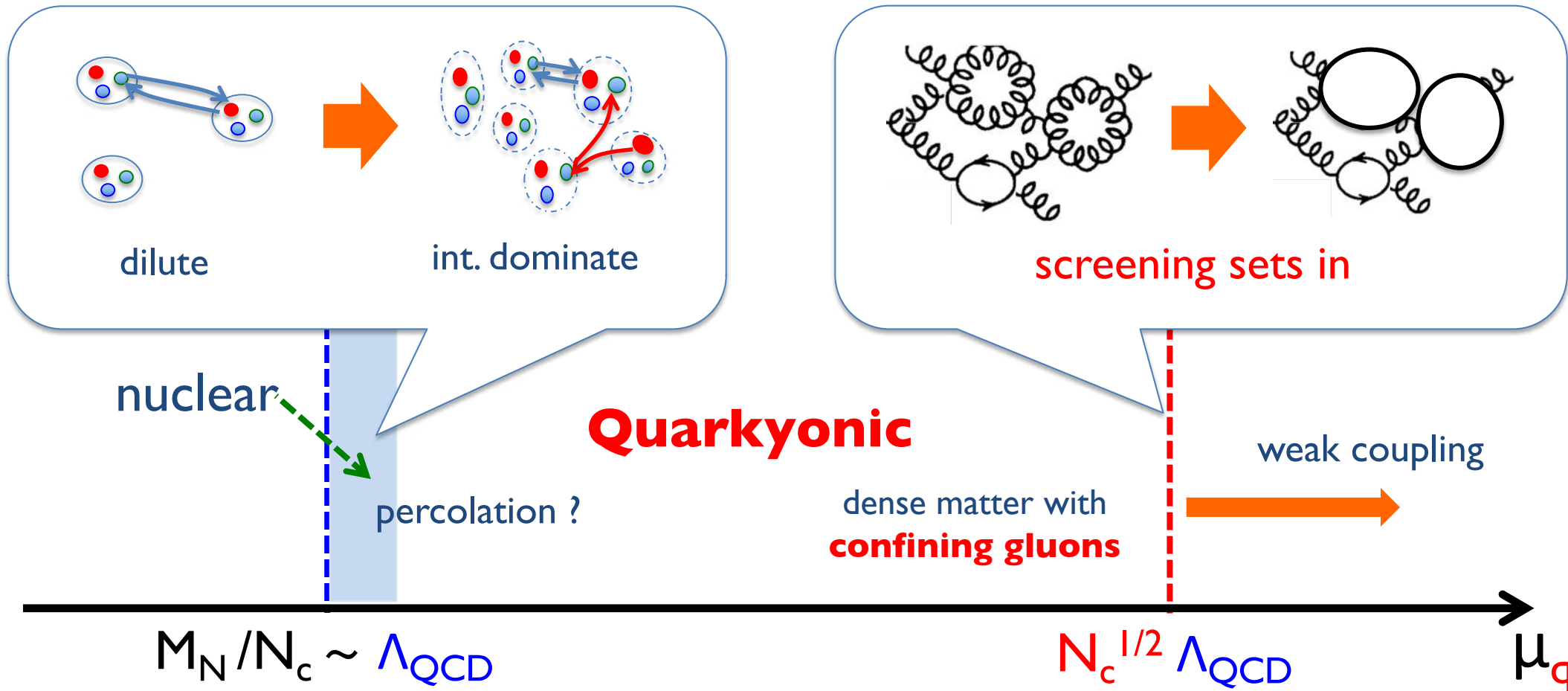


# Strong coupling at high density ?

The **interplay** between **gluons** & **dense quarks**

e.g.) **large  $N_c$** : [ McLerran-Pisarski '07 ]

$$N_c^2 \Lambda_{\text{QCD}}^2 \sim N_c \mu_q^2$$

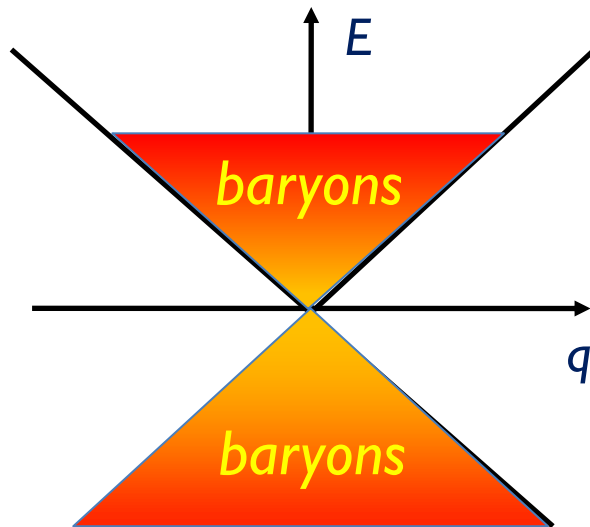


# baryon vs quark Fermi sea/surface

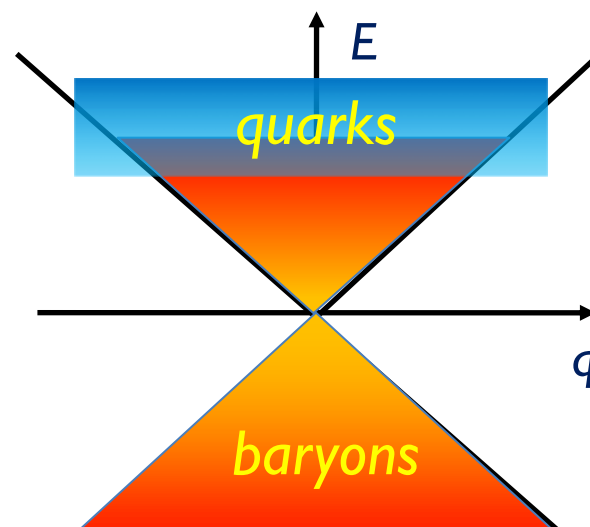
Dense matter with **confining gluons**

**effective d.o.f** ? (minimizing impacts of int.?)

use only colorless d.o.f.



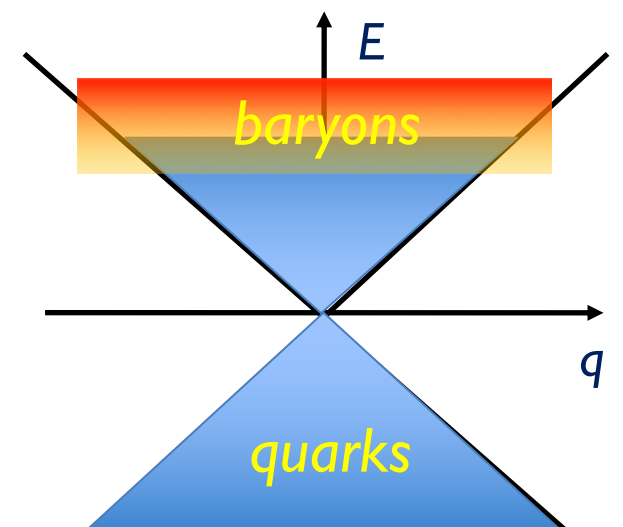
often claimed picture



transports  
thermal

bulk: EoS

McLerran-Pisarski ('07)



This question has **direct relevance** to *phenomenology* (e.g. neutron stars)

## *2-topics for dense matter*

- 1, Quarkyonic matter (theoretical topic)
- 2, Neutron star equations of state

# A solvable model for quarkyonic matter [TK '12]

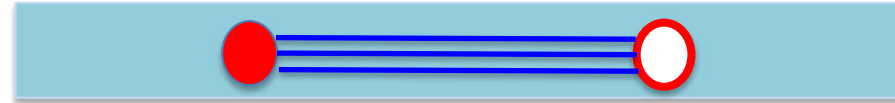
**QCD<sub>2</sub>**  
(axial gauge)

$$S_{\text{QCD}}^{2\text{D}} = \int_x \bar{\psi} (i\not{\partial} + \underline{\mu\gamma_0} + m) \psi + \int_{x,y} J_0^A(x) \underline{D_{00}^{AB}(x-y)} J_0^B(y)$$

[t Hooft '74]

**Confining** propagator :  $D_{00}^{AB}(x-y) = \delta_{AB} \times g_{2\text{D}}^2 |\vec{x} - \vec{y}|$

electric flux is squeezed in 1D



- The spectra in large  $N_c$ : mesons & baryons (solitons) [t Hooft '74, Witten '79, ...]
- At **finite density** ?

How does the **EoS** look like?

**Deconfined colors** after baryons overlap?

# A solvable model for quarkyonic matter [TK '12]

• Non-Abelian **bosonization**:  $S_{\text{QCD}}^{2\text{D}} = \int_x \bar{\psi} (i\cancel{\partial} + \underline{\mu}\gamma_0 + m) \psi + \int_{x,y} J_0^A(x) D_{00}^{AB}(x-y) J_0^B(y)$   
 [Novikov '82; Witten '84; ...]

*baryon num.* - *color* charge separation

$$h \sim e^{i\pi_A T_A}$$

$$S_{\text{QCD}}^{2\text{D}} = \underline{S_{U(1)}[\phi; \underline{\mu}]} + \underline{S_{k=1}^{\text{WZW}}[h]} + S_{\text{int}}[h] + \text{pert. of } O(m)$$

**free** theory **with**  $\mu$

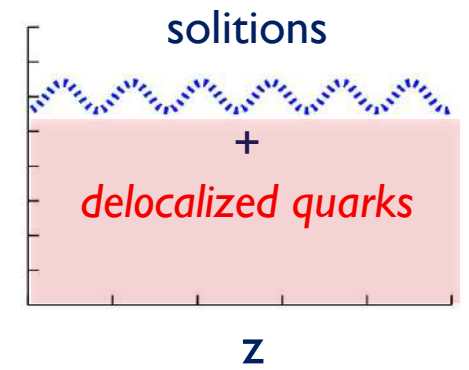
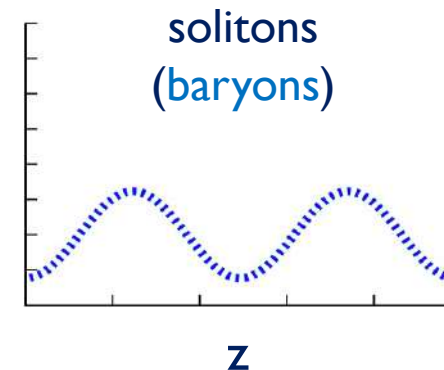
gauge dynamics ( **same as vac. !** )

## • Consequences:

- $P(\mu) =$  Free Fermi sea + Vac. int. energy (!)
- The colored objects remain confined (!)
- ChSB by *inhomogeneous chiral spirals*.

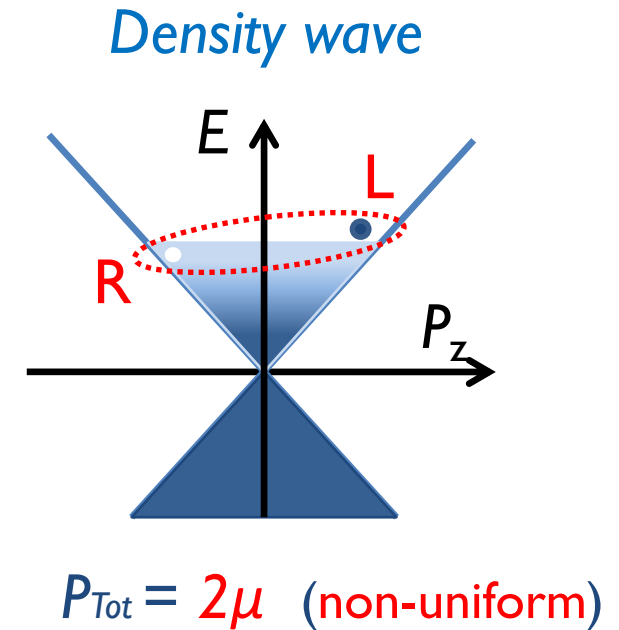
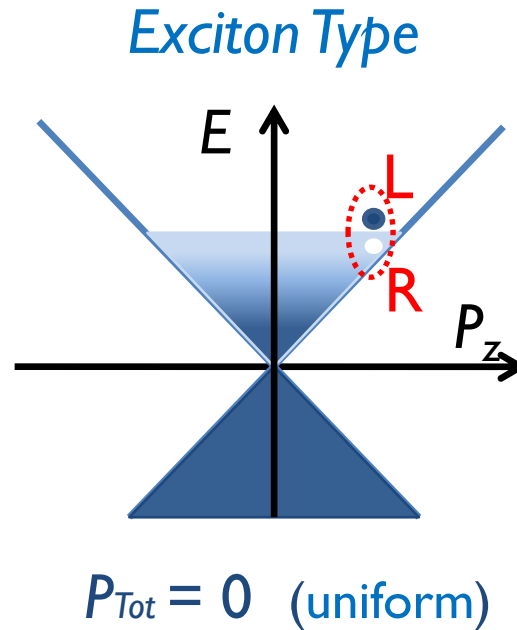
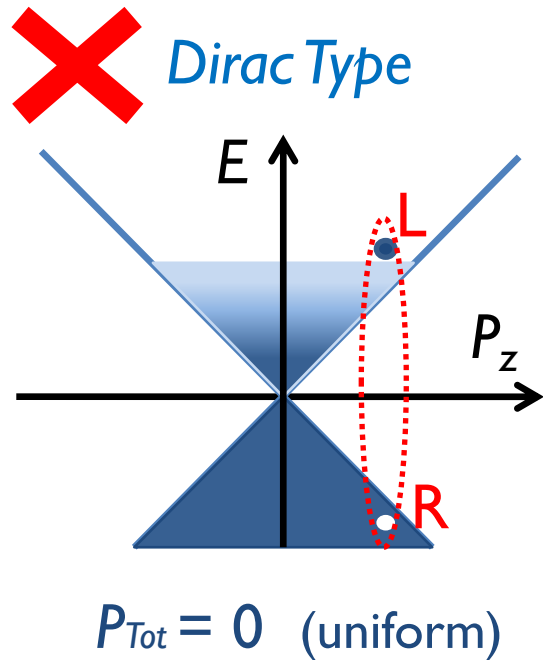
[see below]

## baryon density distribution

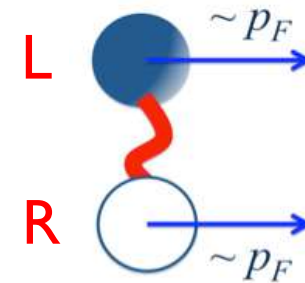


# Chiral symmetry in dense matter?

Candidates of chiral pairing (in general dimensions) [TK+ '09]

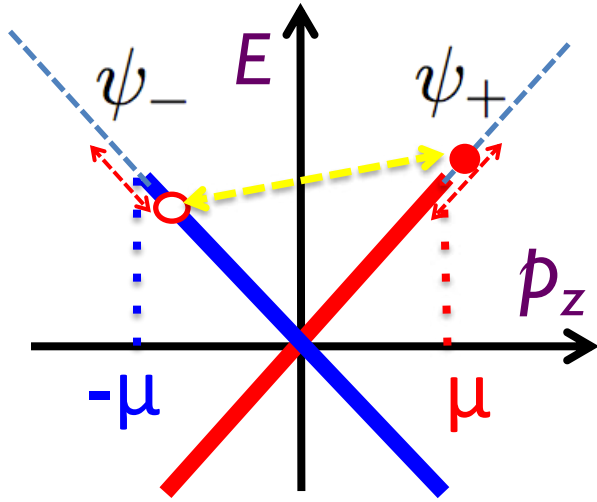


comoving *color-singlet* pairs of *particle-hole*



# Chiral spirals (2D-case)

[Thies+('00); Basar+('08); TK('11)..]



“ **moving directions** (z) ”  
(in any dim.)

$$\psi_{\pm} = \frac{1 \pm \gamma_0 \gamma_z}{2} \psi \quad (\gamma_5)_{2D}$$

Measure **momenta** from the **Fermi points** :

$$\begin{aligned} \psi_- &= e^{-i\mu z} \psi'_- \\ \psi_+ &= e^{i\mu z} \psi'_+ \end{aligned} \quad \text{then}$$

$$\begin{aligned} \bar{\psi}_{\pm} (i\partial + \mu\gamma_0) \psi_{\pm} &\longrightarrow \bar{\psi}'_{\pm} i\partial \psi'_{\pm} \\ J_{\mu}^A &\longrightarrow J'_{\mu}^A \end{aligned}$$

leading to the same gap equation as **in vacuum**:

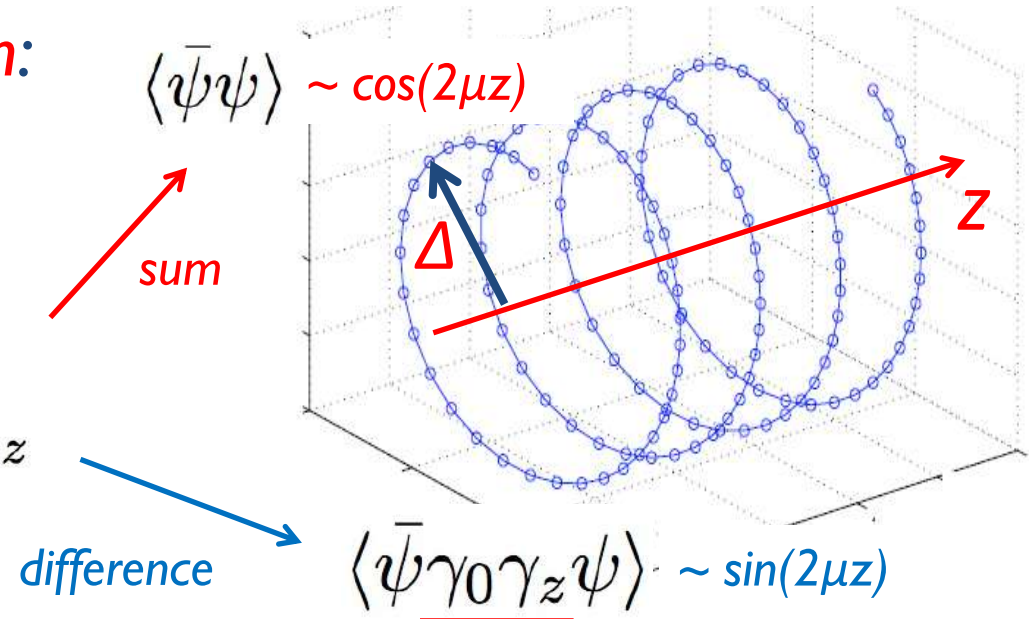
in "vacuum"

$$\langle \bar{\psi}'_+ \psi'_- \rangle = \langle \bar{\psi}'_- \psi'_+ \rangle = \Delta \quad \text{(known)}$$

in original bases

$$\langle \bar{\psi}_- \psi_+ \rangle = \Delta e^{2i\mu z}$$

$$\langle \bar{\psi}_+ \psi_- \rangle = \Delta e^{-2i\mu z}$$





# Chiral spirals: from 2D to 4D?

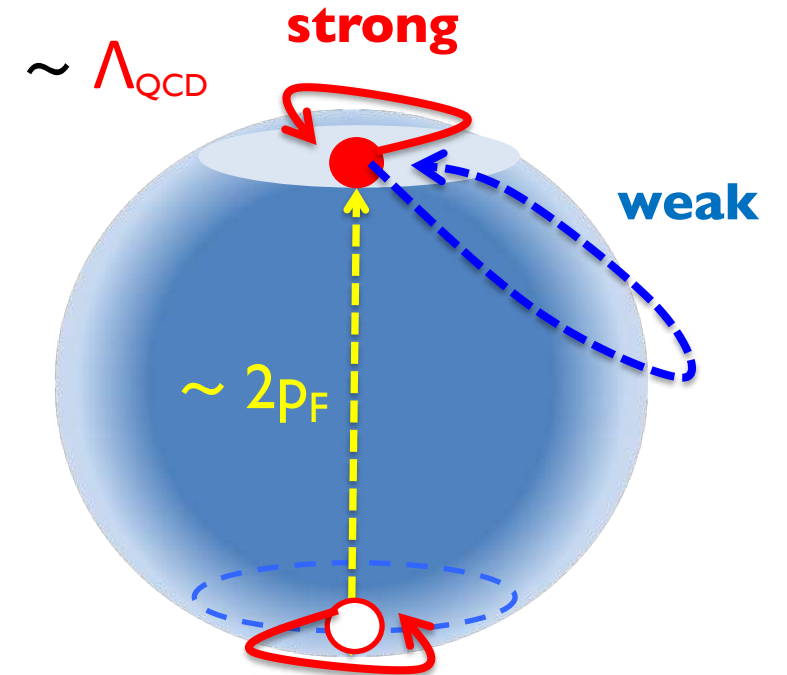
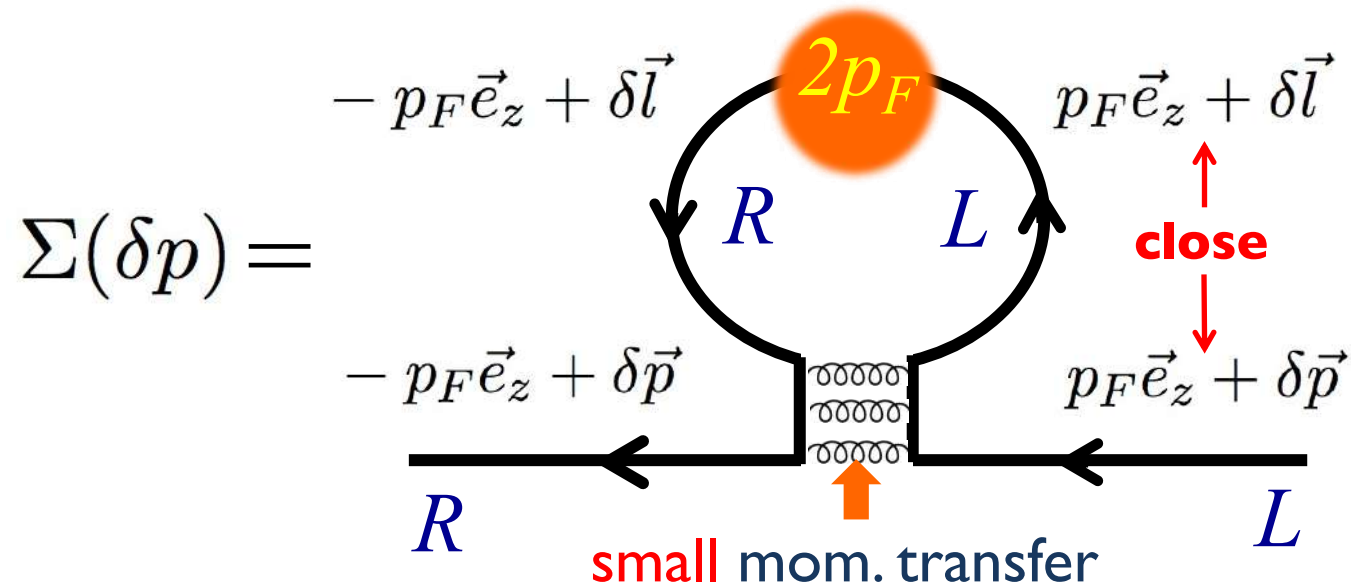
- A **confining** propagator for quark-antiquark (quark-hole) :

$$D_{\mu\nu} = C_F \times g_{\mu 0} g_{\nu 0} \times \frac{\sigma}{(\vec{p}^2)^2}$$

linear rising type, **Coulomb** gauge  
(ref: Gribov, Zwanziger)

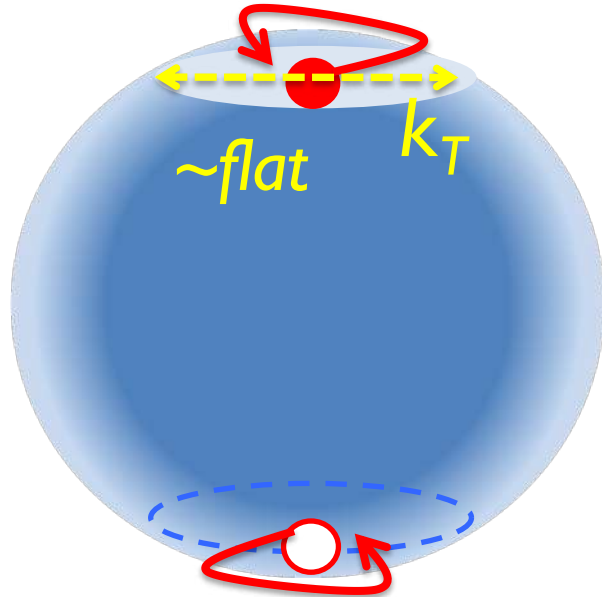
strong **IR** enhancement

- Schwinger-Dyson eq. for chiral spirals



# Dimensional reduction of integral eqs.

[TK-Hidaka-McLerran-Pisarski ('10)]



$$\Sigma(\delta p) = C \int \frac{d^2 k_L d^2 \vec{k}_T}{(2\pi)^4} \gamma_0 S(\delta k_L, \underline{\delta k}_T) \gamma_0 \frac{\sigma}{|\delta \vec{p} - \delta \vec{k}|^4}$$

quarks insensitive to  $k_T$

*factorization*

$$\int \frac{d^2 k_L}{(2\pi)^2} \gamma_0 S(\delta k_L, \underline{\vec{0}}_T) \gamma_0 \otimes \int \frac{d^2 \vec{k}_T}{2\pi} \frac{\sigma}{|\delta \vec{p} - \delta \vec{k}|^4}$$

2D propagator

2D confining propagator

Schwinger-Dyson eq.  
in **4-D** conf. model

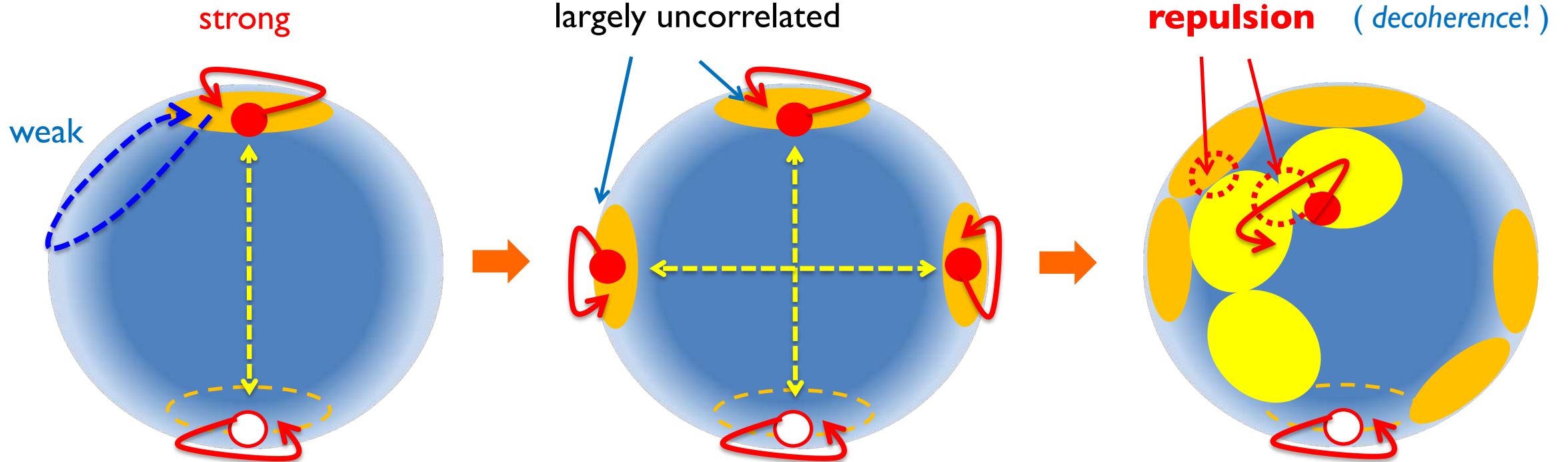


Schwinger-Dyson eq.  
in **2-D** QCD (axial gauge)

[ we know solutions: *chiral spirals* ]

# Interweaving chiral spirals

[TK-Pisarski-Tsvetlik ('10); TK-Fukushima-Hidaka-McLerran-Pisarski ('11)]



chiral spiral cond.  $\rightarrow$  energy **gain**

patch-patch int.  $\rightarrow$  energy **cost**

balance



a novel structure:

***interweaving CSs***

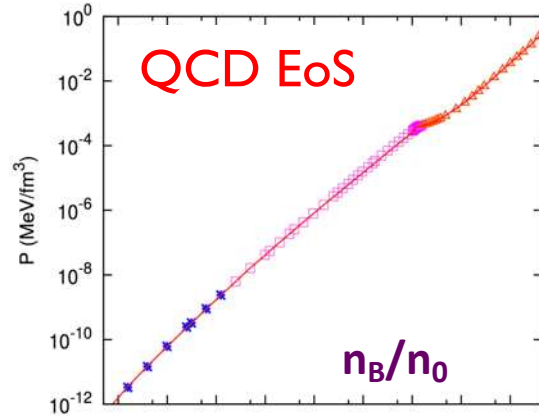
(energy gain: **comparable to the Color-super**)

## *2-topics for dense matter*

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# EoS & M-R relation

Einstein eq.:  $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$  ..... QCD EoS

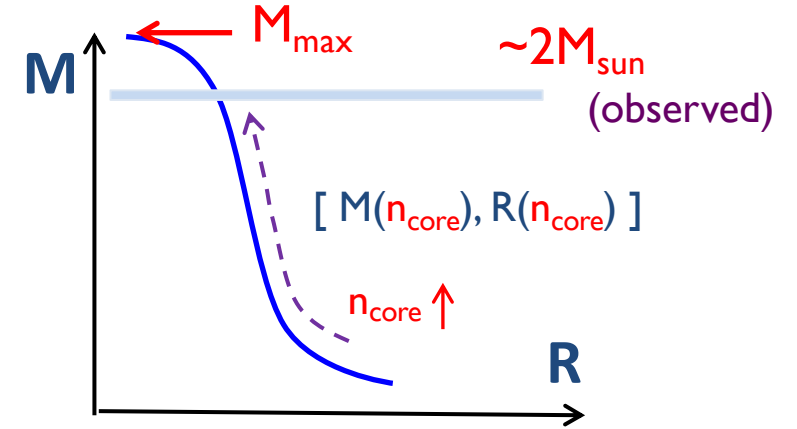


[for spherical NS → TOV eq.]



I-to-I correspondence

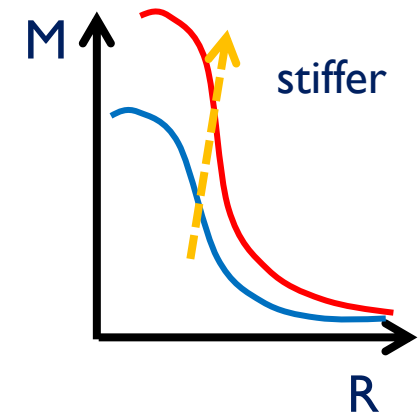
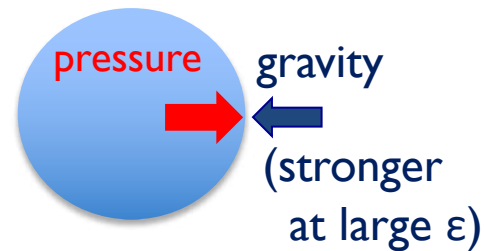
Lindblom (1992)

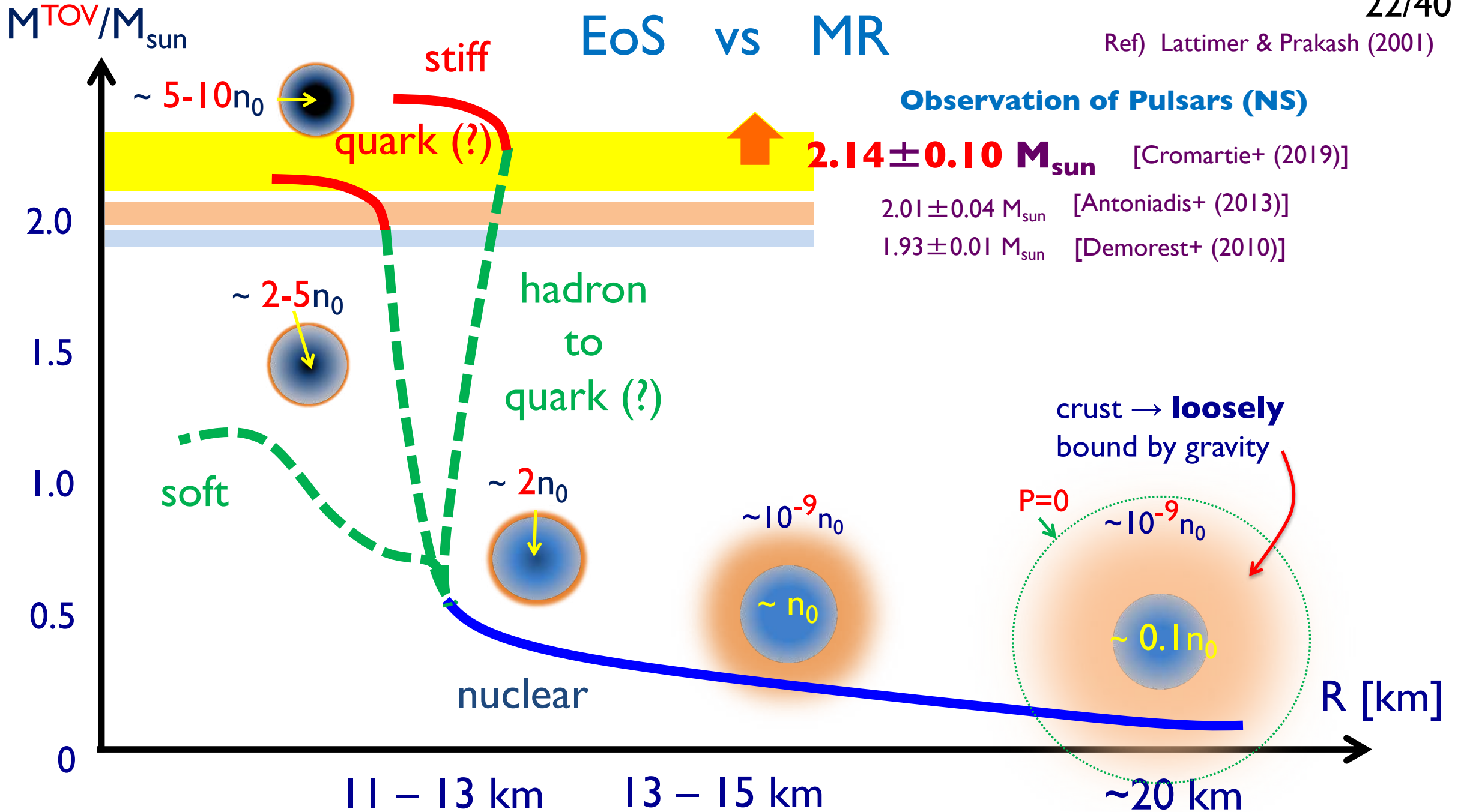


## Terminology (my convention)

1) **Stiff** EoS : P is large at given ε

2) **Soft-to-Stiff** EoS : Soft at  $n_B < 2n_0$  & Stiff at  $n_B > 5n_0$

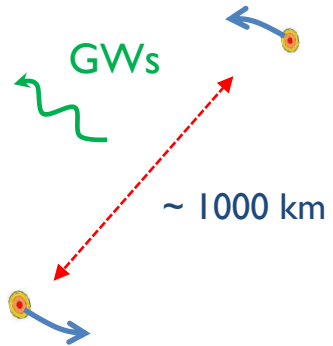




# NS-NS mergers [detectability **0.1~100 /year** ,aLIGO-Virgo-KAGRA]



## Early inspiral

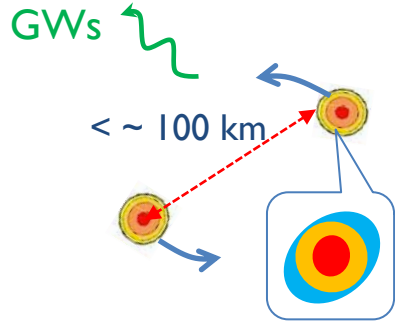


point particles

$M_1$  &  $M_2$   
spins (?)

## Tidally deformed

< 1 kHz



Finite size effect  
(tidal deformation)

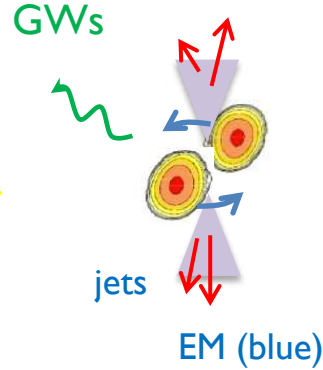
$\Lambda_{\text{obs}}$

$R_1$  &  $R_2$

(at  $\sim 1.4 M_{\text{sun}}$ )

## Merger

1 - 4 kHz



• oscillation freq. (GW)  $\sim 1 - 3$  kHz  $\rightarrow$

• EM signals (sGRB & kilonova)  $\rightarrow$

• life-time of merger  $\rightarrow$

$M/M_{\text{TOV}}^{\text{max}} > \sim 1.5$

too massive

$1.2 < M/M_{\text{TOV}}^{\text{max}} < 1.5$

short life

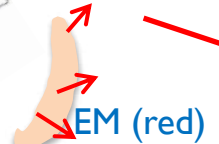
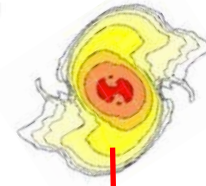
$M/M_{\text{TOV}}^{\text{max}} < 1.2$

long life

## BH

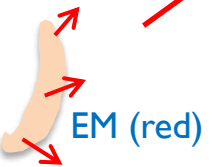
### HMNS

lifetime < 100 ms



**BH**

### SMNS



**NOT yet**

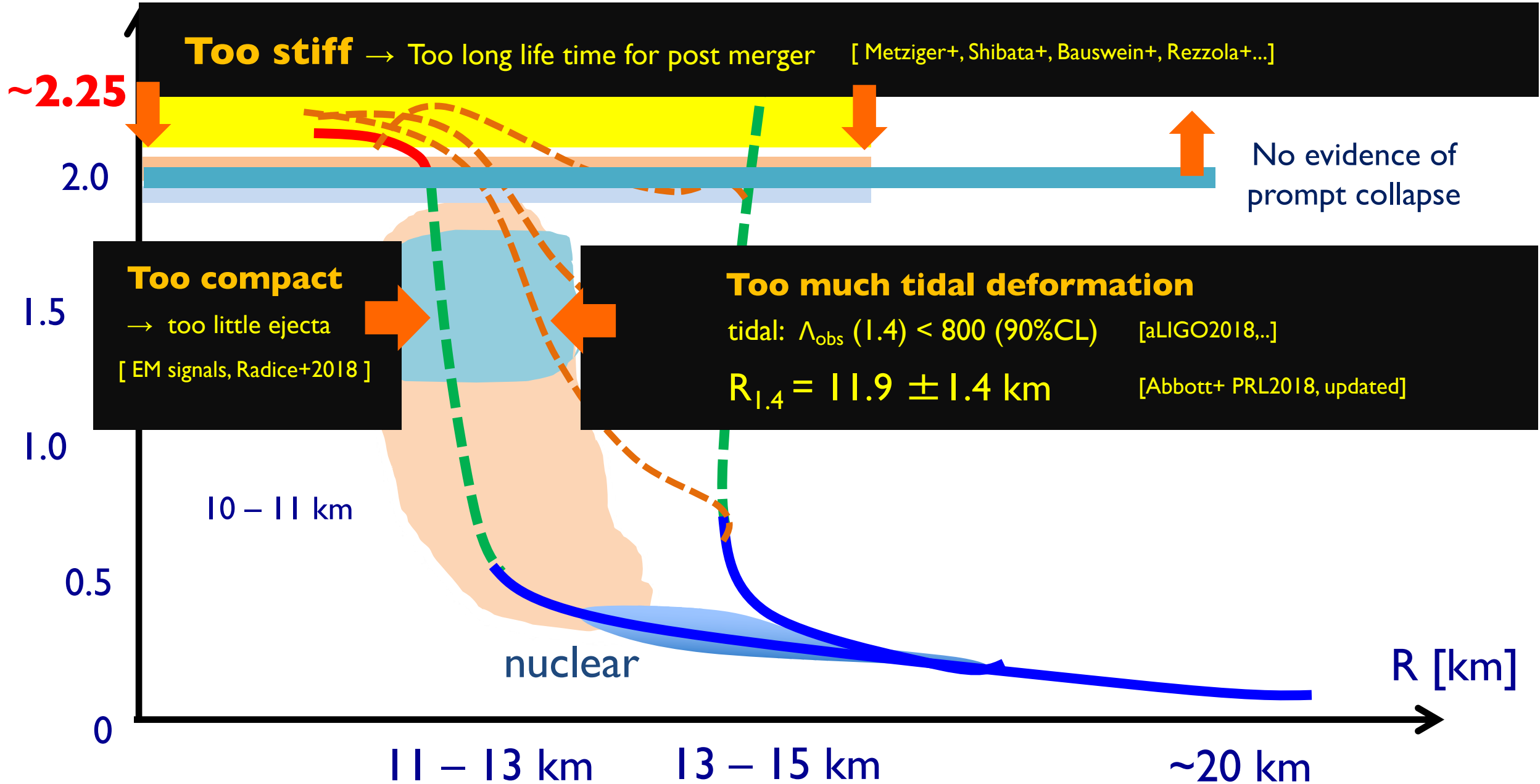
**R** (at  $> 1.4 M_{\text{sun}}$ )

$M_{\text{eject}}$ , **R**

$M_{\text{TOV}}^{\text{max}}$

$M^{TOV}/M_{sun}$

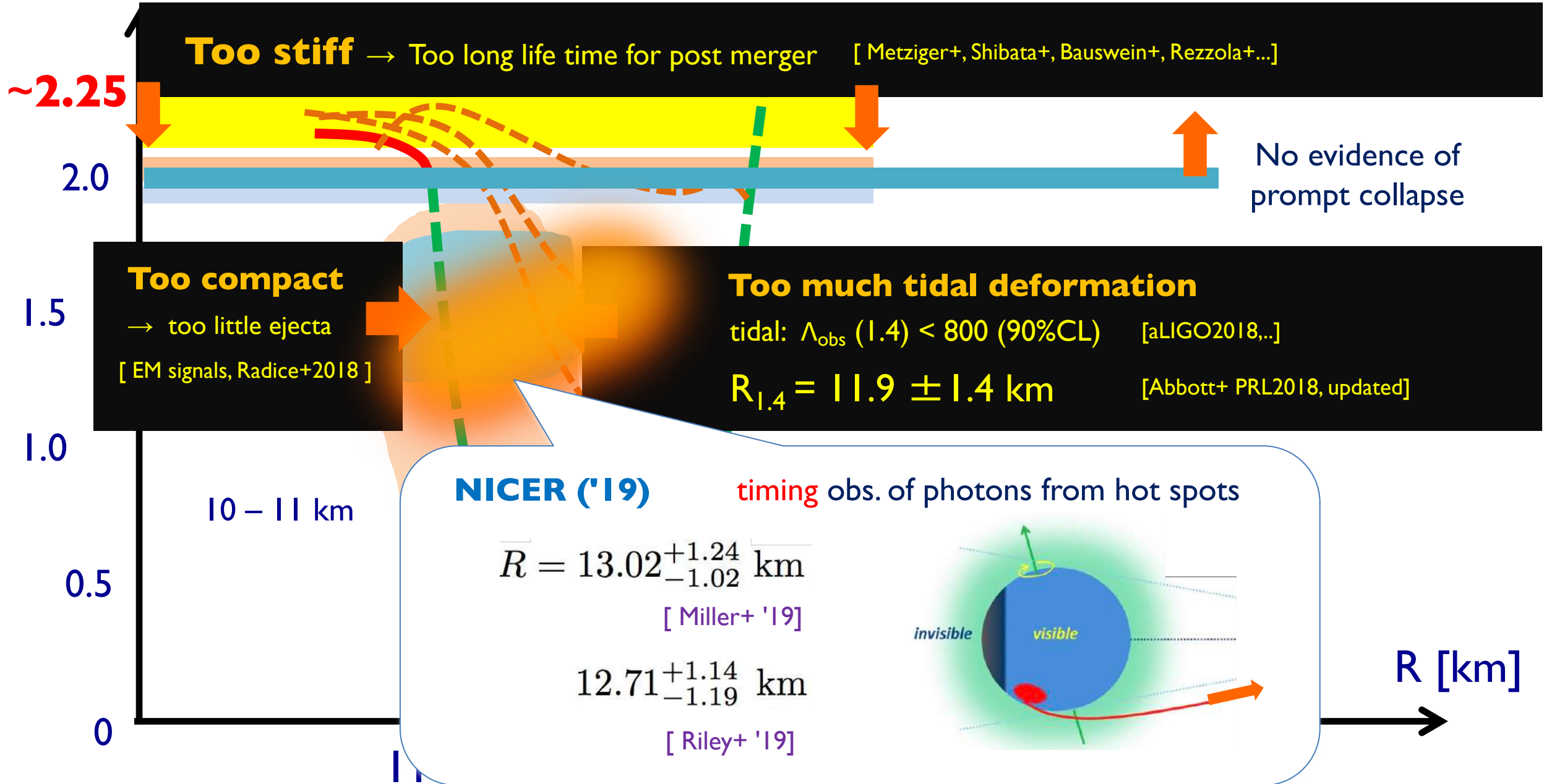
# Constraints (after GW170817 - GRB170817A - AT2017gfo)





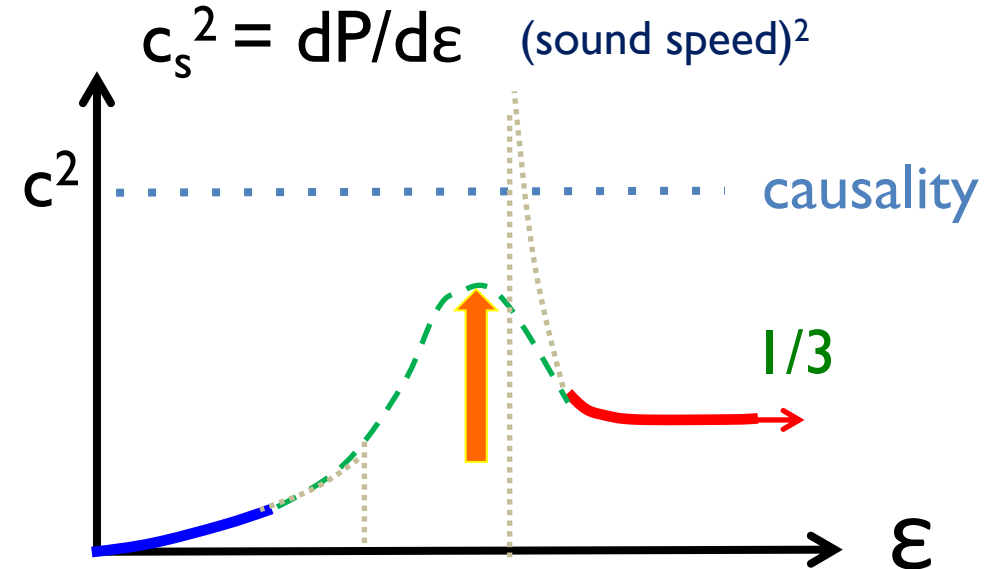
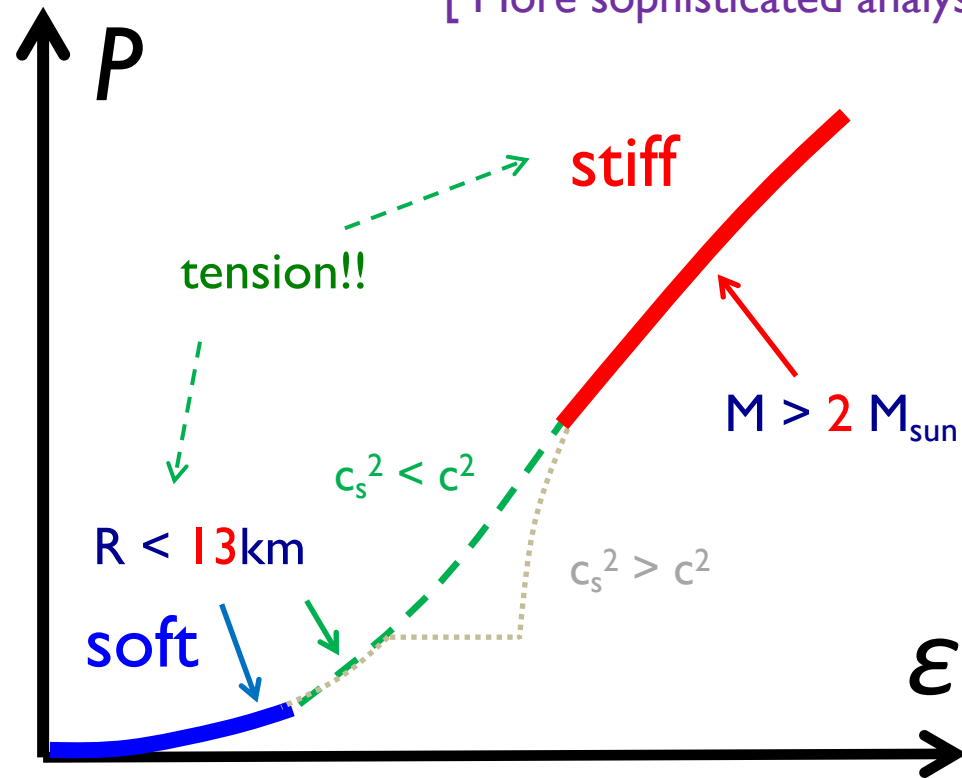
$M^{\text{TOV}}/M_{\text{sun}}$

# Constraints (after GW170817 - GRB170817A - AT2017gfo)

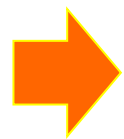


# Causality constraint

[ More sophisticated analyses: Han-Alford-Prakash '13, Bedaque-Steiner '15 ]



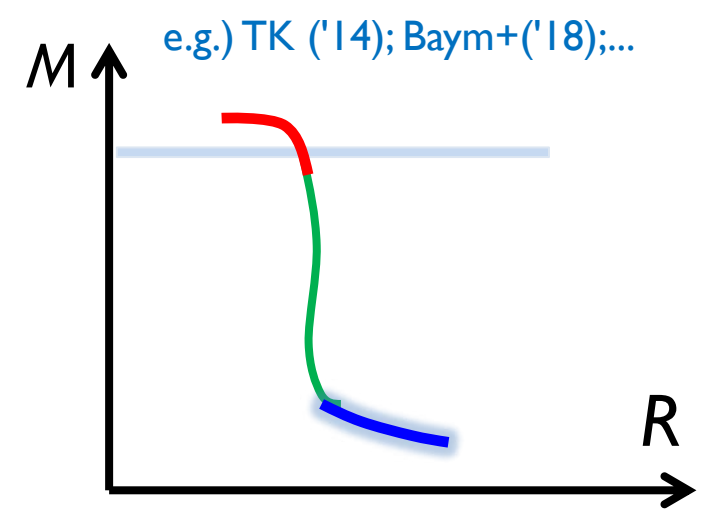
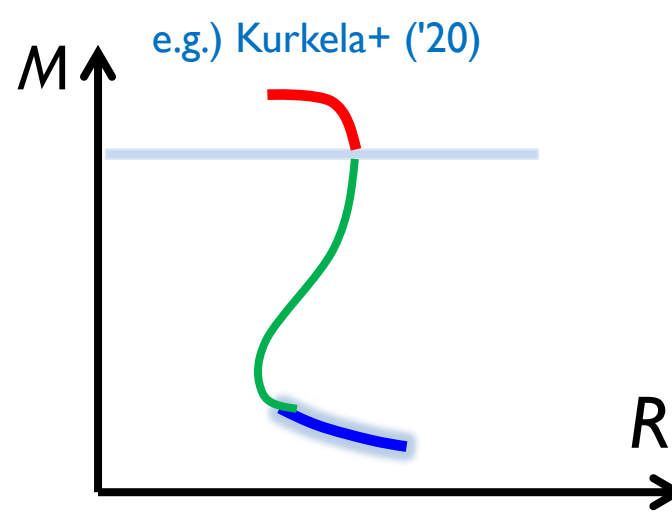
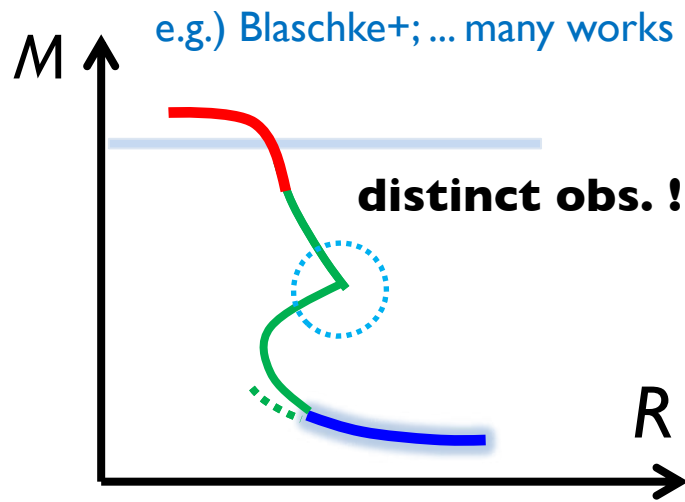
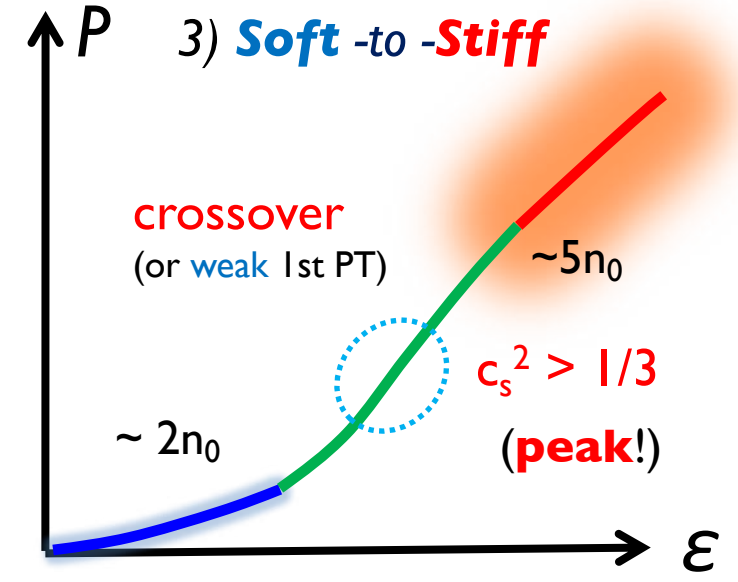
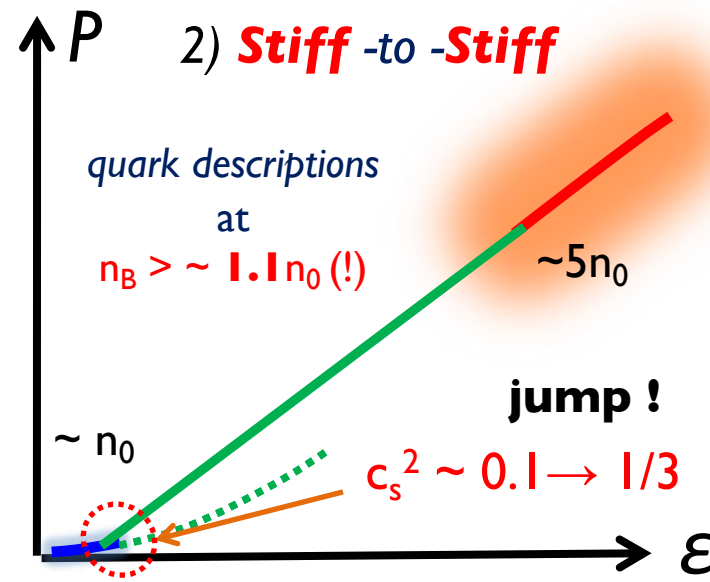
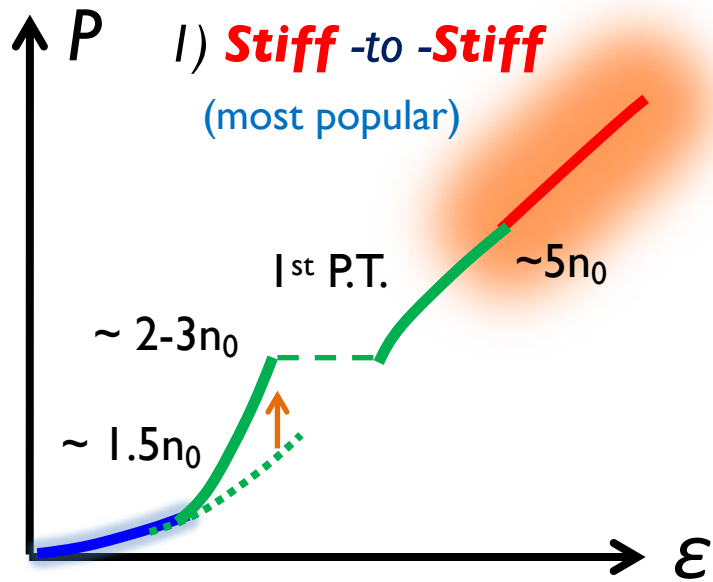
softer at low density & stiffer at high density



**severer constraint** on **P.T.** from the causality

(too radical 1<sup>st</sup> order P.T. is disfavored)

# Three possibilities in literatures



# The reasons we take the continuity as a *baseline*

Q) why **not** assuming the strong hadron-quark 1<sup>st</sup> order P.T.? (as in choice 1)

A) *nuclear forces*  $\sim$  *quark exchanges* (from meson exchange to hard core repulsion)

many-quark exchanges important at  $n_B > \sim 2n_0$

→ difficult to imagine **sharp interface** between quark-hadron matter

Q) why **not** switching to (almost free) quark matter at  $n_B \sim 1.1n_0$ ? (as in choice 2)

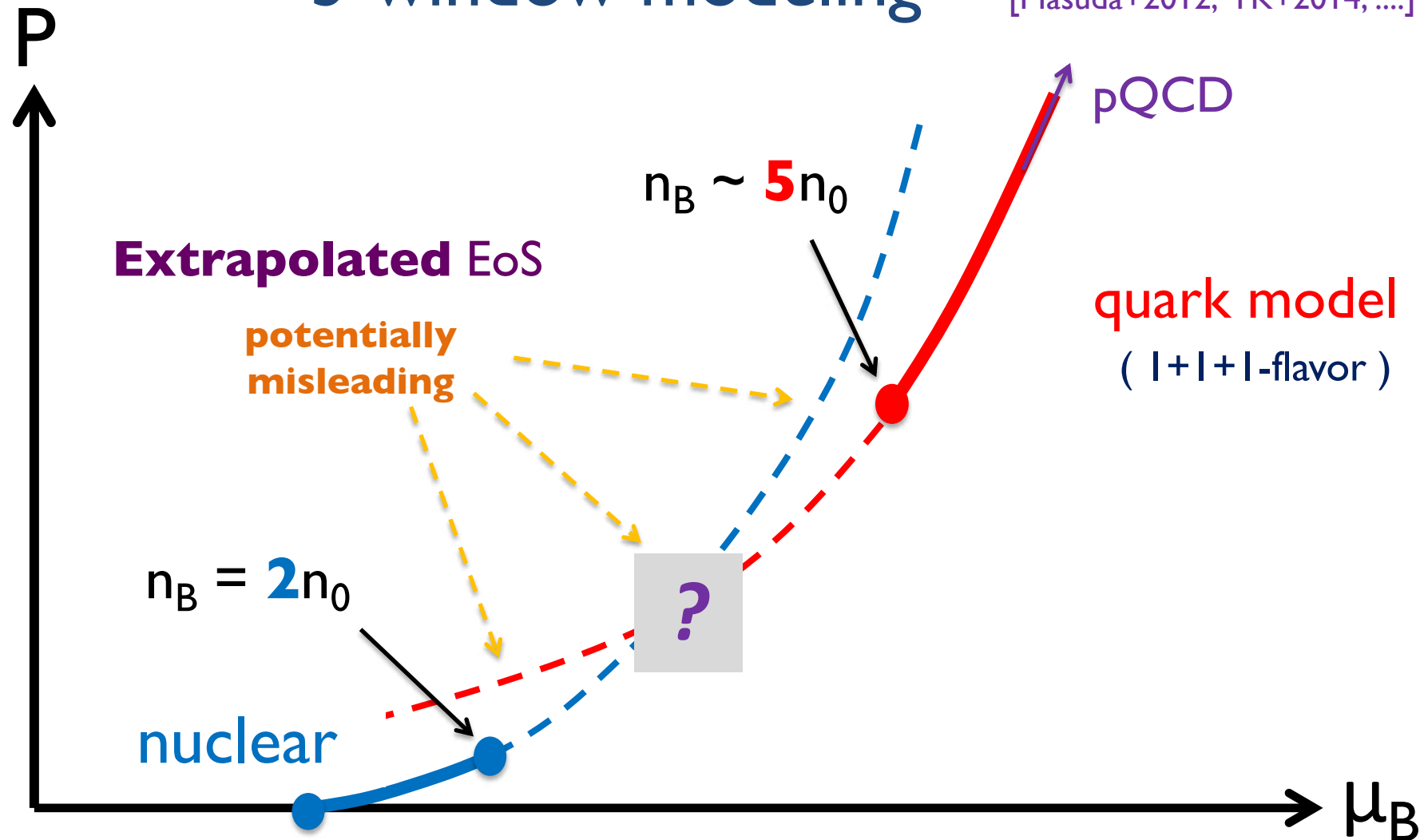
A) I don't know any models that allow a **positive jump** in  $c_s^2$ . (any P.T. doesn't allow either)

Also, nuclear cal. has problems *in precision* at  $\sim 1.1n_0$ , but not *in systematics*.

(nuclear cal. should be OK at low density)

# 3-window modeling

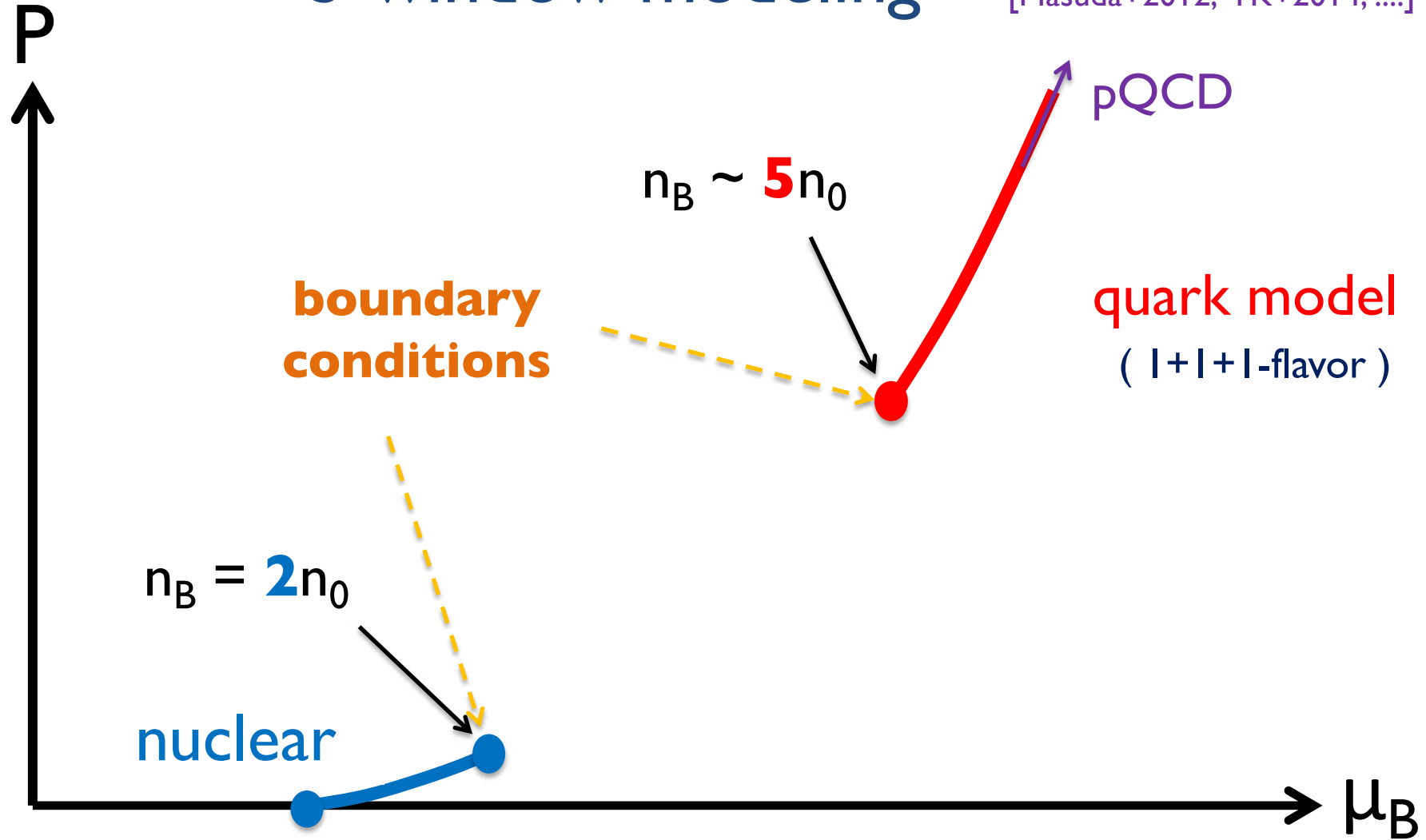
[Masuda+2012, TK+2014, ...]



[Akmal+1998, Togashi+2017,  
Hebeler+2017, Gandolfi+, ...]

# 3-window modeling

[Masuda+2012, TK+2014, ...]



$$n_B = 2n_0$$

nuclear

$$n_B \sim 5n_0$$

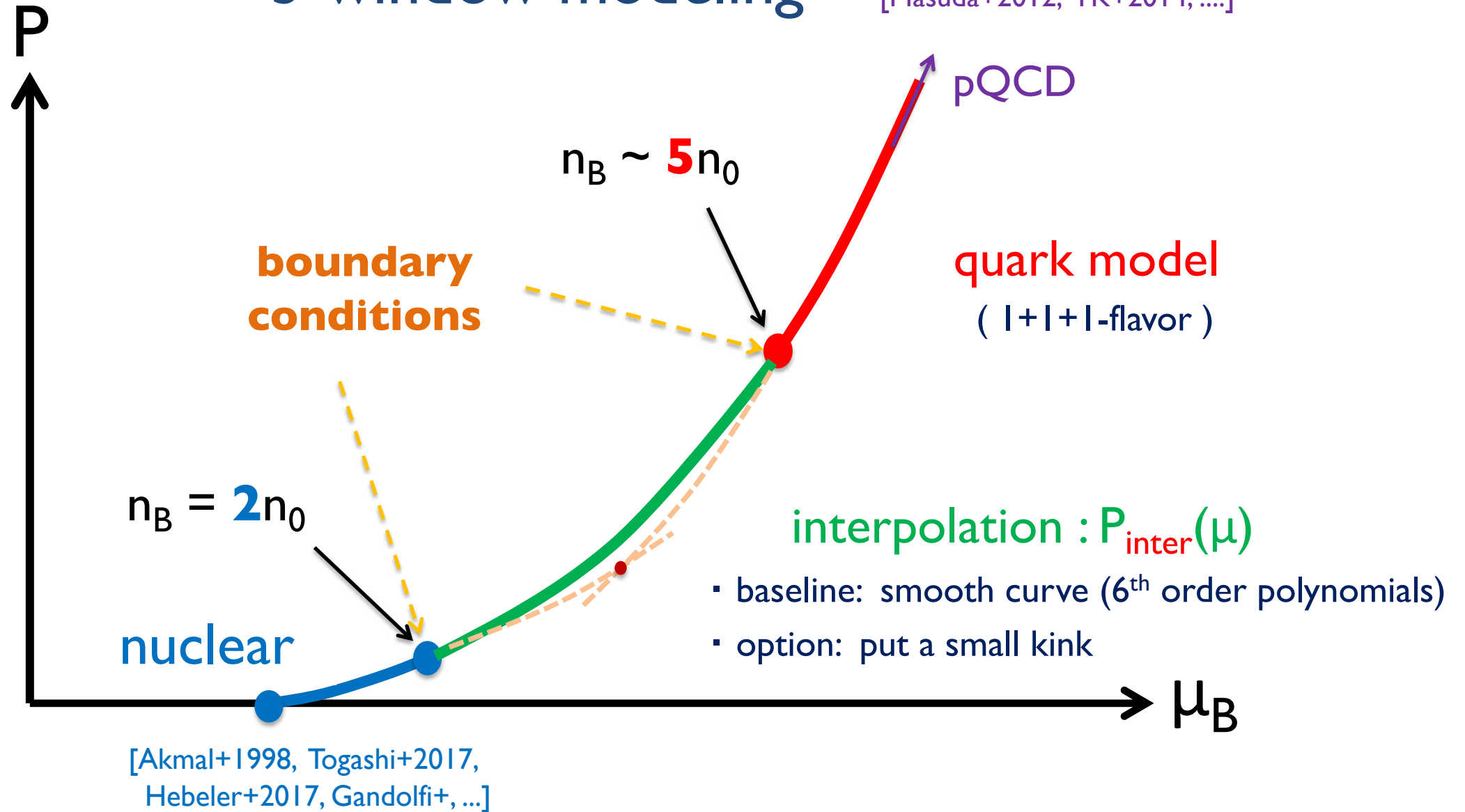
pQCD

quark model  
(1+1+1-flavor)

[Akmal+1998, Togashi+2017,  
Hebeler+2017, Gandolfi+, ...]

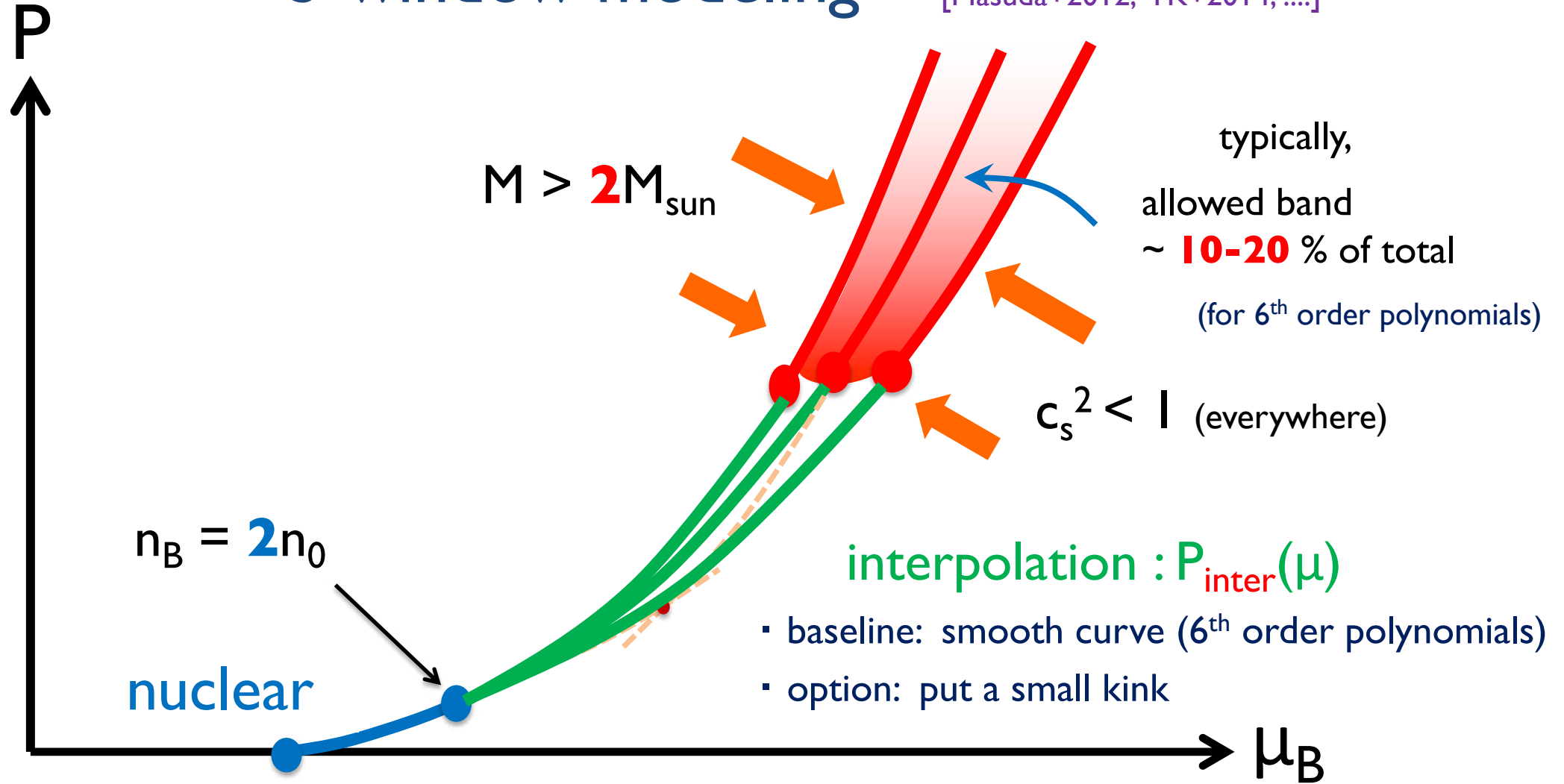
# 3-window modeling

[Masuda+2012, TK+2014, ...]



# 3-window modeling

[Masuda+2012, TK+2014, ...]



[Akmal+1998, Togashi+2017, Hebeler+2017, Gandolfi+, ...]



# A quark model for $n_B > \sim 5n_0$ ( $\sim 1\text{fm}^{-3}$ )

A guide : *Hadron-Quark continuity* : eff. Hamiltonian continuously evolves from hadron physics

**"3-window"** [Manohar-Georgi 1983, Weinberg 2010,...]

$Q < \sim 0.2 \text{ GeV}$

very long-range ( $> 1\text{fm}$ )

confinement

$0.2 \text{ GeV} < Q < 1-2 \text{ GeV}$

constituent quarks + OGE  
(quasi-particles)

**chiral SB & color-mag. int.**  
**& baryon-baryon. int.**

$\sim 2 \text{ GeV} < Q$

short range

pQCD

A template)

chiral

color-mag.

nB-nB int.

$$\mathcal{H} = \mathcal{H}_{\text{NJL}} - \underline{H} \sum_A (q\Gamma_A q)(\bar{q}\Gamma_A \bar{q}) + \underline{g_V} (\bar{q}\gamma_0 q)^2$$

solve within **MF**

+ color- & charge- neutrality  
+  $\beta$ -equilibrium

[Masuda+2015, TK+2014, Blaschke+...]



*color-superconductivity* & *smooth chiral restoration*

# Color-magnetic interaction play **many** roles

## 1) **Coupling** $\propto$ **velocity** $\sim$ $p/E$

become important in **relativistic regime** & **high density**

## 2) **Pairing**: strongly channel dependent

hadron mass ordering: N- $\Delta$ , etc. [DeRujula+ (1975), Isgur-Karl (1978), ...]

color-super-conductivity

[Alford, Wilczek, Rajagopal, Schafer, ... 1998-]

## 3) **Baryon-Baryon int.**: **short-range** correlation

( **Pauli + color-mag.** )

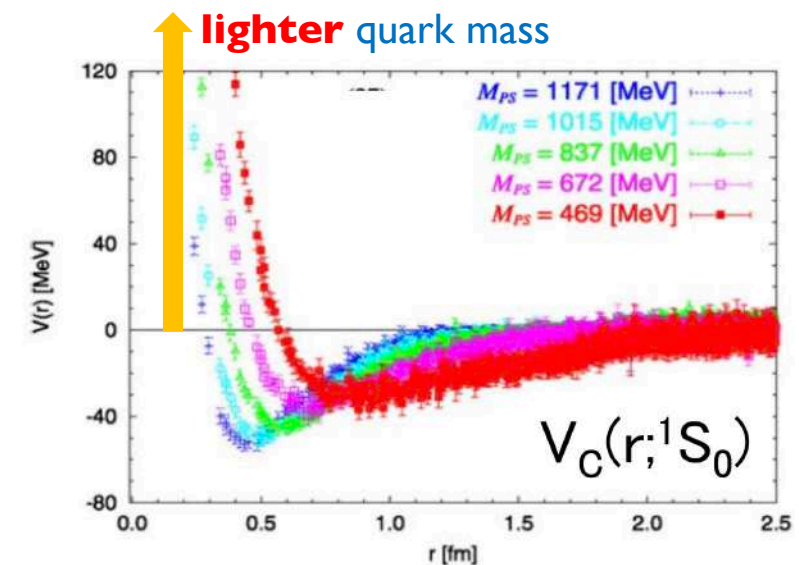
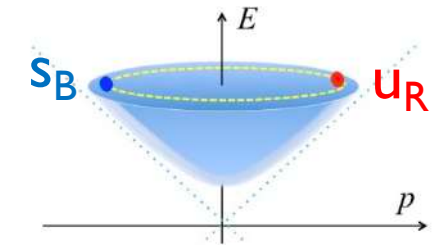
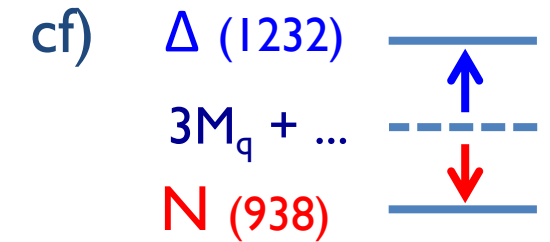
[Oka-Yazaki (1980), ...]

channel dep.  $\rightarrow$  **non-universal** hard core (some are **attractive!**)

mass dep.  $\rightarrow$  **stronger** hard core in **relativistic** quarks

$\rightarrow$  **consistent with the lattice QCD**

[HAL-collaboration]

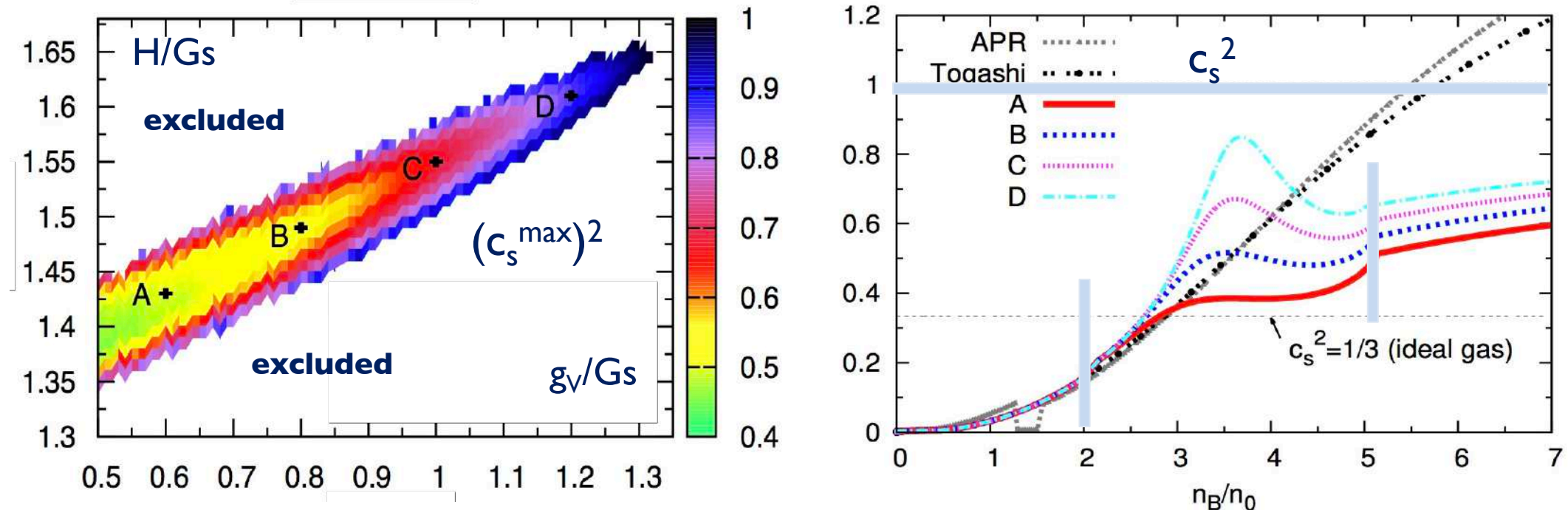


# An exercise: survey for $(g_V, H)_{@5n_0}$ [Baym+ '19]

**Step 1)** Prepare **realistic** nuclear EoS for **0.5-2n<sub>0</sub>**

[e.g. Akmal+1998, **Togashi+2017**, ChEFT, ...] **→ radius constraint OK**

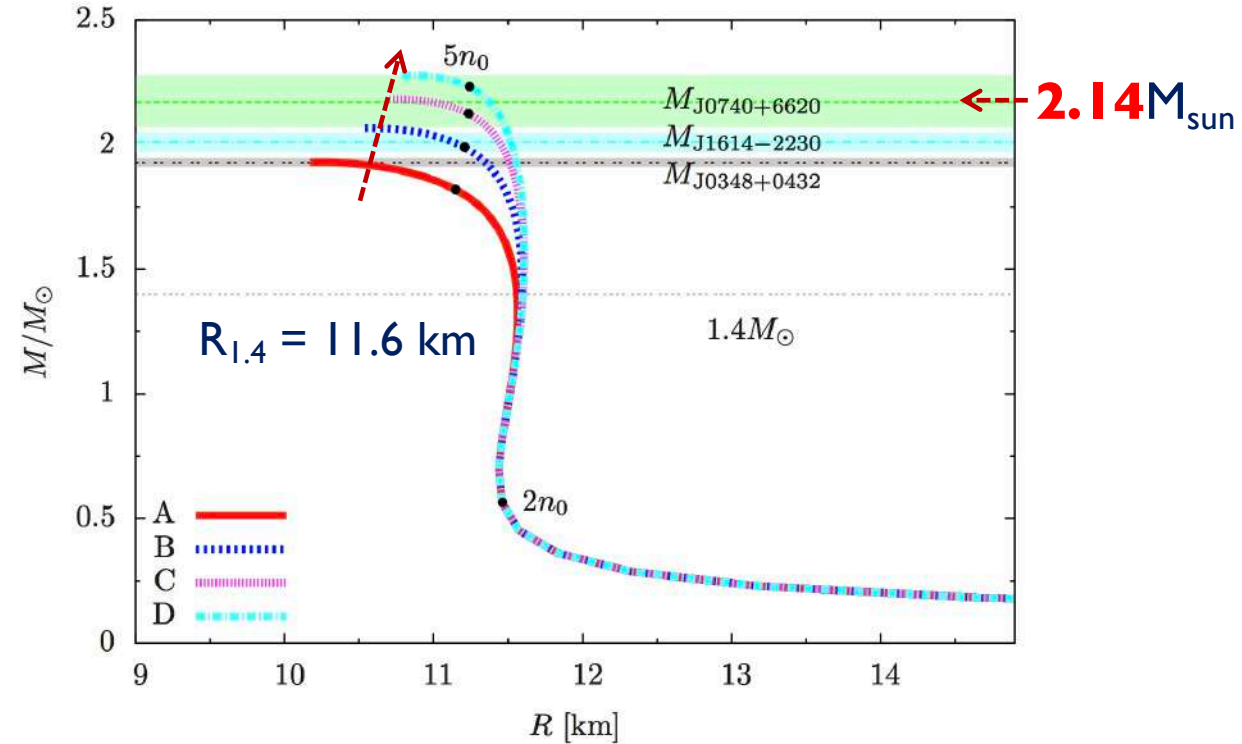
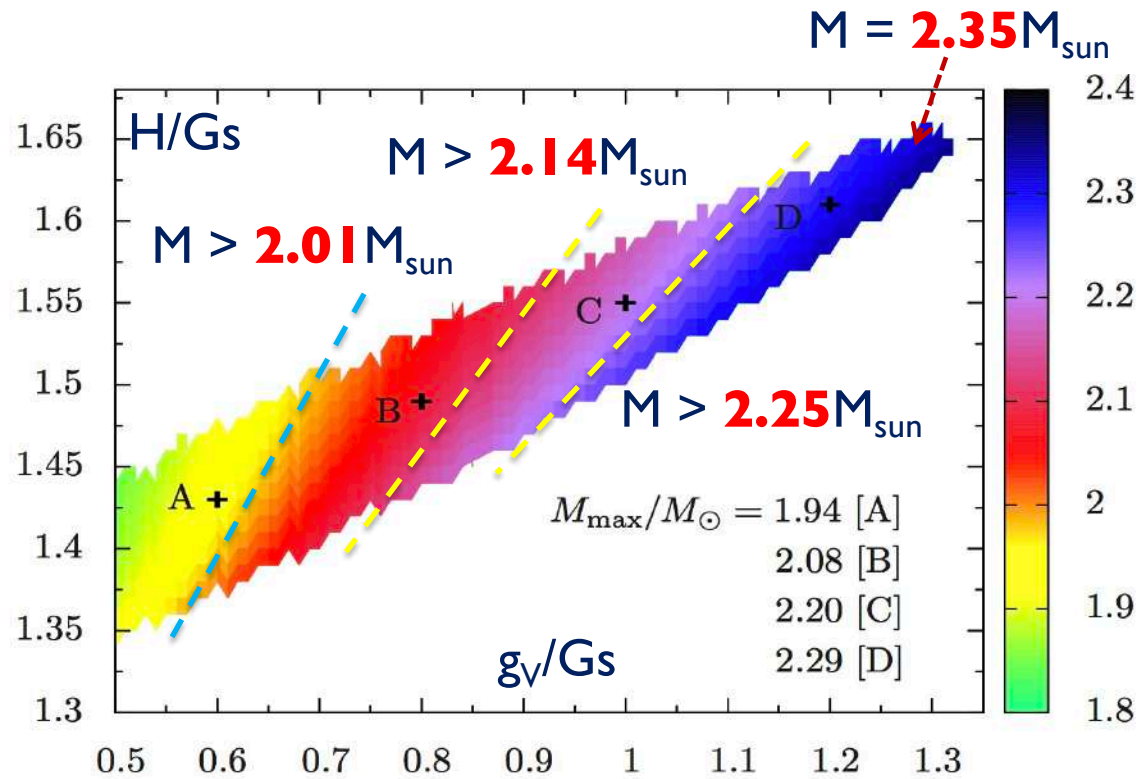
**Step 2)** Survey the range of  $(g_V, H)$  compatible with **causality & stability**



# An exercise: survey for $(g_v, H)_{@5n0}$

[Baym+ 2019]

**Step3)** Impose  $M > 2M_{\text{sun}}$



bottom line:  $(g_v, H)_{@5n0} \sim (G_s)_{@vac}$

On-going works & future prospects

# Next 10 years ? *NS-physics*

## More experimental programs to come:

[ (2<sup>nd</sup>) aLIGO-aVirgo (2015-), KAGRA(2019-), IndIGO (2023-); (3<sup>rd</sup>) Einstein-Telescope (2030s), ...]

- more events (1-100/year) of NS-NS (BH) mergers → **statistical** analyses

### & *if we are lucky:*

- an event happens at **not far** distance → significant **post-merger GWs** at **1-4 kHz**
- *asymmetric* binaries → more *head-on* collisions

**core** heated up; reaching  $T > 20$  MeV at  $n_B > 2n_0$       *thermal EoS* ( $Y_e \sim 0.1$ )

### & *if we are very lucky: (a few event per century)*

- **galactic** supernovae (like SNI987A): GWs & **neutrinos**      *thermal EoS* ( $Y_e \sim 0.4$ )  
(detector has improved by  $> 10^4$  x Kamiokande)

Need: **EoS** catalog & **corresponding GW** templates

# EoS projects I

I) General purpose EoS:  $P(\mu_B, \mu_L, T)$  [with Baym-Furusawa-Hatsuda-Togashi]

with *explicit* descriptions for  $2-5n_0$ , including:

- *matter composition* (strangeness, lepton fractions, ...)
- *thermal excitations* (d.o.f. near the Fermi surface)
- *microphysics* (based on hadron physics & QCD)

Testing several continuity scenarios **one by one, systematically** :

1) nuclear-**2SC**;

[TK-Hou-Okafor-Togashi, to appear]

2) hypernuclear-**CFL**;

[work in progress]

3) nuclear-**quarkyonic**

[work in progress]

# EoS projects 2

**my role:** responsible for *EoS tables*; clarification of *microphysics*

- 2) *NS-merger simulations* (RIKEN-iTHEMS collaboration)  
[ with Y. Huang, S. Nagataki, L. Biotti, K. Takami, H. Sotani, Hatsuda, Togashi ]
- 3) *Machine learning* for global analyses (Illinois-RIKEN collaboration)  
[ with N. Yunes, J. Noronha-Hostler, Baym, Hatsuda, Togashi ]
- 4) Construct *cyber-infra structure of EoS* in *full QCD phase diagram*  
[ with C. Ratti, V. Dexheimer, J. Noronha-Hostler, J. Noronha, and N. Yunes ]  
also connections to *low energy heavy-ion experiments*

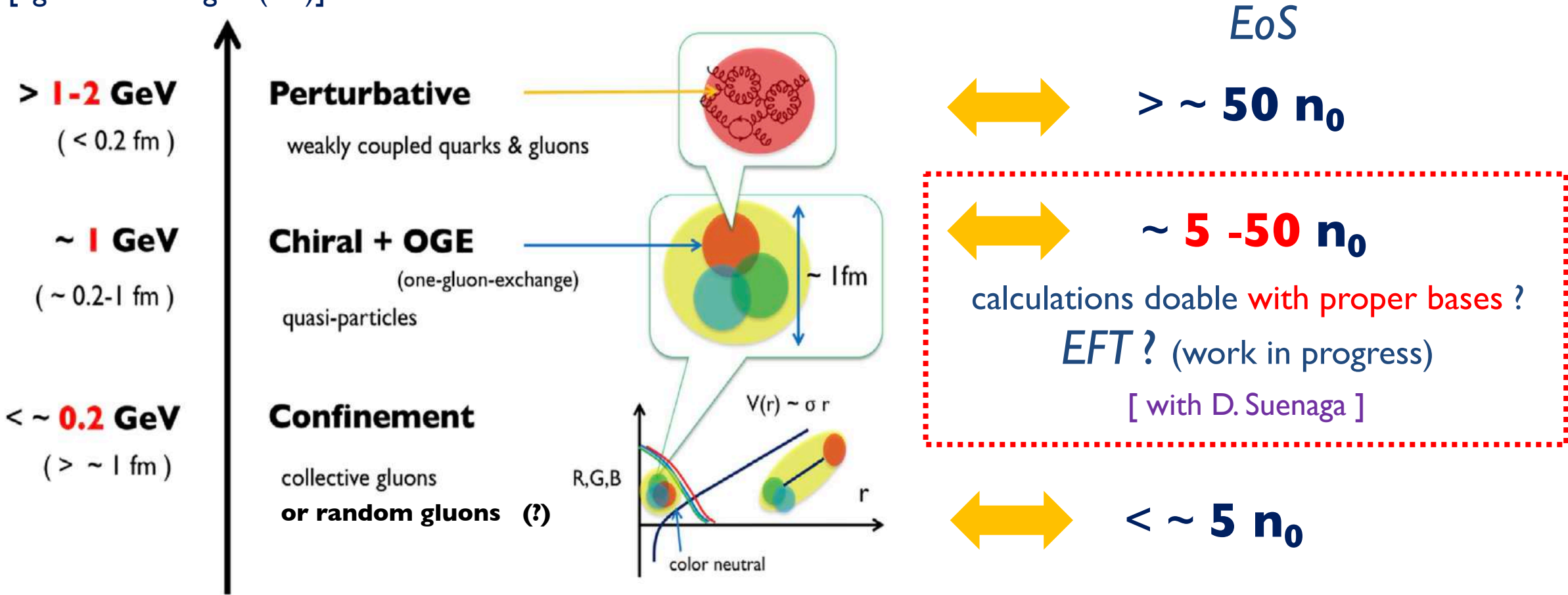


# Next 5 years ? *beyond pQCD*

*quasi-particles* → the **residual** interactions get **under control**

e.g.) **a** hadron [Manohar-Georgi('83), Weinberg('10)]

[fig. from Suenaga+ ('19)]

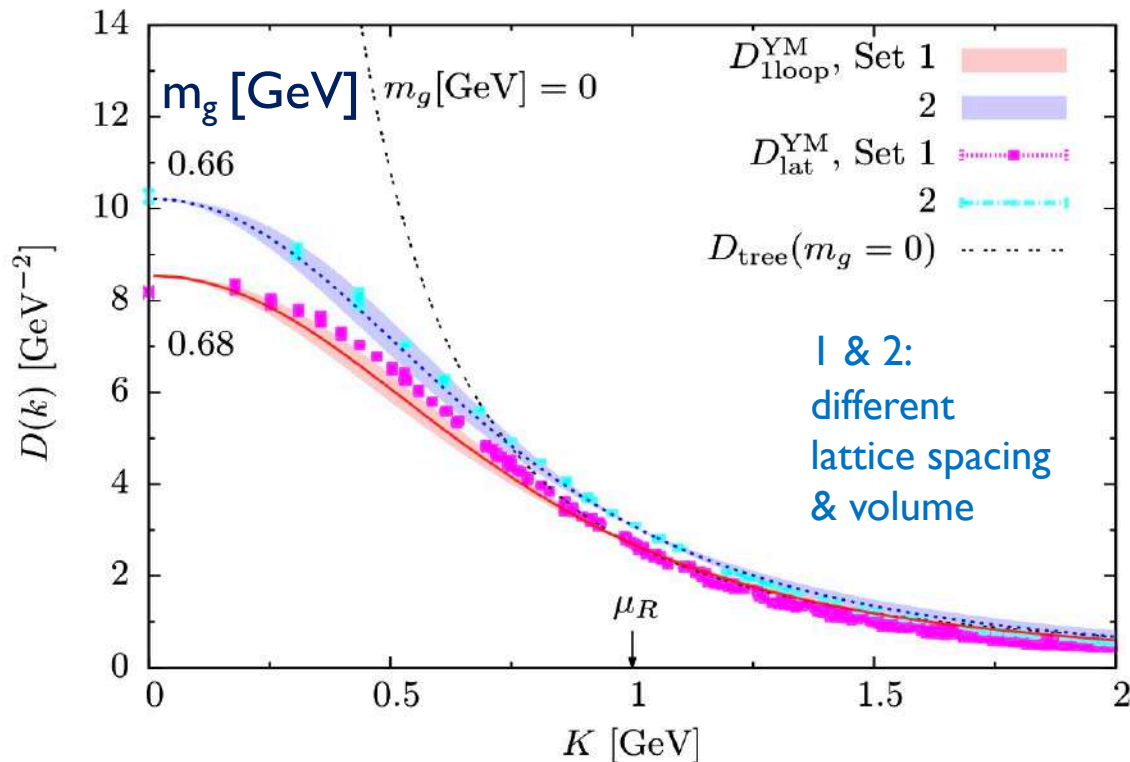


# quasi-gluons & perturbative stability (Landau gauge)

Gribov ('78): found gauge copies in QCD  $\rightarrow$  negative FP-det. (!)

suggested a **condition** to keep FP-det. **positive** ; effective mass  $m_g \sim 600$  MeV  
for gluon **excitations**

comparisons with  $p$ YM data (Boz+'19)



**error bands** for theory side

variations:  $\alpha_s = \mathbf{1-3}$  (!)  $\rightarrow$  only  $< \sim \mathbf{10\%}$  corrections!

at **small  $k$**

$$D_{\text{tree}}^{-1}(k) \sim \underline{k^2} \ll m_g^2$$

$$\Pi(k) \sim \underline{\alpha_s} \times \underline{k^2}$$

better **systematics** in loop corrections  
(once  $m_g$  is properly chosen)

## Next 5 years ? *QCD-like theories*

Numerical simulations of  $N_c=3$  QCD at finite density remain difficult problems

For *2-color* QCD, *isospin-QCD*, *QCD in magnetic fields*:

*lattice simulations are doable*; very useful **lab. to test theories !**

On-going work: **2-color** gluon propagators **in medium** [ *quasi-particle domain* ]

1-loop: Suenaga-TK ('19); RG-improvement (work in progress)

2nd step: calculations of **2-color** EoS, and compare it with lattice

3rd step: calculations of **3-color** EoS [ *quasi-particle domain* ]

# Summary

The physics of dense QCD is a long-standing topic, but still there are *revolutionary discoveries* to refresh our understandings.

## ***collaborative works are essential***

- The NS physics will not be closed by astrophysics community alone, even if everything could be measured;
- The QCD phase diagram will not be closed by QCD community alone, even if the sign problem on the lattice were solved;
- from hadron spectroscopy to nuclear forces to quark matter:  
unified descriptions are called for. (on the right track)