The Role of Symmetry in Strongly Coupled Universalities

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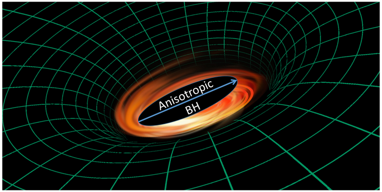
Outline

1. Introduction
2. Reduction of Symmetries
3. Phase Transitions
4. Universal Properties
5. Conclusions

Motivation I.

- New holographic theories(black holes) with a non-trivial Renormalization Group Flow.
- Strongly coupled anisotropic theories have significantly richer structure compared to isotropic ones.
  - → Non-local Observables, Transport coefficients, Diffusion, Brownian Heavy Quark Motion...
  - → **Characteristic Example:** Shear viscosity η over entropy density s: takes parametrically low values wrt degree of anisotropy $\frac{\eta}{s} < \frac{1}{4\pi}$. (Rebhan,Steineder 2011; D.G. 2012; Jain, Samanta Trivedy 2015;... D.G., Gursoy, Pedraza, 2018;...)
  - → Universalities are very rare!



(Anisotropy, Temperature)

- A generic holographic framework for observables has been developed.

Motivation II.

- Existence of strongly coupled anisotropic systems.
  - $\rightarrow$ Quark - Gluon Plasma.
  - $\rightarrow$ Anisotropic Materials.

    eg: (Pardo, Pickett 2009; Kobayashi, Suzumura, Piechon, Montambaux 2011; Fang, Fu 2015...)
- Strong (Magnetic) Fields in strongly coupled theories.

  eg: (Bali, Bruckmann, Endrodi, Fodor, Katz, Krieg et al. 2011)

Reminding-Example Slide: A Fixed Anisotropic Point

- The anisotropic hyperscaling violation metric in IR:

$$ds^2_{d+2} = u^{\frac{2\theta}{d}}\left(-u^{2z}\left(-f(u)dt^2 + dy_i^2\right) + u^2 dx_i^2 + \frac{du^2}{f(u)u^2}\right)$$

exhibits a critical exponent z and a hyperscaling violation exponent θ.

$$t \to \lambda^z t, \qquad y \to \lambda^z y, \qquad x \to \lambda x, \qquad u \to \frac{u}{\lambda}\,, \qquad ds \to \lambda^{\frac{\theta}{d}} ds\ .$$

- $z = 1$ and $\theta = 0$, is the AdS spacetime, dual to the conformal $\mathcal{N} = 4$ sYM theory.
- Thermodynamically it behaves as receiving contributions from a theory in $k - \theta$ dimensions with scaling z and from a $d - k$ dimensions conformal theory.

$$S \sim T^{\frac{k-\theta}{z} + d - k}$$

- Effective Space dimensionality for the dual theory!

(Chu, D.G., 2020)

Theories with Reduced Symmetry. A Pictorial Representation I:

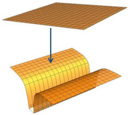
- The deformed theories with reduced symmetry.

Add D7-branes (heavy objects) extending along the x_1, x_2-directions which are homogeneously distributed along the x_3 with a uniform density.

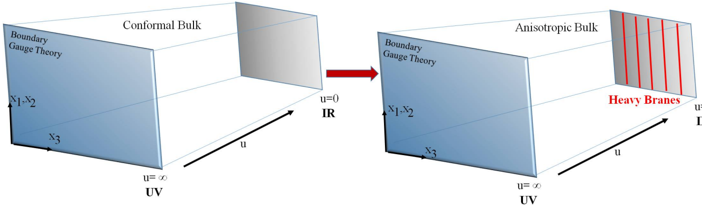
Spacetime (x_0, x_1, x_2, x_3)

AdS → D3

	x_0	x_1	x_2	x_3	u	S^5
D3	x	x	x	x		
D7	x	x	x			x

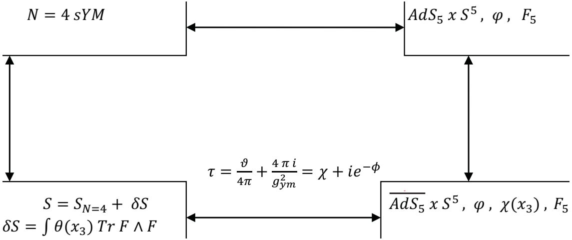


The space-time is modified:



Theories with Reduced Symmetry. A Pictorial Representation II:

- The deformed field theory with reduced symmetry.



- The dual gravity side is the Lifshitz-like IIB Supergravity solutions

$$ds^2 = u^{2z}(dx_0^2 + dx_i^2) + u^2 dx_3^2 + \frac{du^2}{u^2} + ds_{S^5}^2 .$$

(Azeyanagi, Li, Takayanagi, 2009)

- Comment: $dC_8 \sim \star d\chi$ with the non-zero component $C_{x_0 x_1 x_2 S^5}$.

A Generalized Anisotropic Theory with External Fields

The generalized Einstein-Maxwell-Axion-Dilaton action with a potential and arbitrary couplings:

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi,\gamma,\sigma,\Delta) - \frac{1}{2}Z(\phi,\gamma)(\partial\chi)^2 - \frac{1}{4}Y(\phi,\lambda)F^2\right].$$

(D.G., Gursoy, Pedraza, 2022, in progress)

• For certain values of $(\sigma,\gamma,\lambda,\Delta)$ the action and the solution of eoms, are reduced of IIB supergravity.

(includes as a case: Mateos, Trancanelli, 2011)

• $Z(\phi,\gamma) = e^{\gamma\phi}$, $\quad Y(\phi,\lambda) = Y_0 e^{\lambda\phi}$.
• $V(\phi,\gamma,\sigma,\Delta)$: Polynomial form with Asymptotically AdS solution for small dilaton in the UV; and exponential form in the IR.

((Gubser, Nellore), Pufu, Rocha 2008a,b;Gursoy, Kiritsis, Nitti, 2007;...)

• Anisotropy: $\frac{\partial\chi}{\partial x_3} = \alpha$.
α: Uniform D7-brane charge density per unit length $\sim$ strength of Anisotropy.

Exponential Potential: Warm-up solution I

Consider a linear axion ansatz, and a magnetic field transverse to the axion deformation:

$$\chi = \alpha\, x_3\,, \qquad A_\mu = \{0, 0, -x_3 B/2, x_2 B/2, 0\}\,.$$

The solution in this case is

$$ds^2 = \tilde{L}^2 u^{\frac{2\theta}{3}} \left[-\frac{f(u)dt^2 + dx_1^2}{u^2} + \frac{c_2 dx_1^2}{u^{2z_2}} + \frac{c_3 dx_3^2}{u^{2z_3}} + \frac{du^2}{u^2 f(u)} \right],$$

$$\phi = c_\phi \log(\xi u)\,, \qquad f(u) = 1 - \left(\frac{u}{u_h}\right)^{2+z_2+z_3-\theta},$$

where

$$\theta = \frac{3\sigma(\lambda - 2\sigma)}{2\gamma^2 - 2\gamma(\lambda+\sigma) + \lambda^2 + 2\lambda\sigma - 2\sigma^2 + 2}\,, \quad z_2 = \frac{(\lambda - 2\sigma)(-\gamma + \lambda + \sigma)}{2\gamma^2 - 2\gamma(\lambda+\sigma) + \lambda^2 + 2\lambda\sigma - 2\sigma^2 + 2}$$

$$z_3 = \frac{\gamma(\lambda - 2\sigma)}{2\gamma^2 - 2\gamma(\lambda+\sigma) + \lambda^2 + 2\lambda\sigma - 2\sigma^2 + 2}\,.$$

Exponential Potential: Warm-up solution II

Consider a linear axion ansatz, and a magnetic field parallel to the axion deformation:

$$\chi = \alpha\, x_3\,, \qquad A_\mu = \{0, -x_2 B/2, x_1 B/2, 0, 0\}\,,$$

The solution in this case is

$$ds^2 = \tilde{L}^2 u^{\frac{2\theta}{3}} \left[-\frac{f(u)dt^2}{u^2} + c_1 \left(\frac{dx_1^2 + dx_2^2}{u^{2z_1}} \right) + \frac{c_3 dx_3^2}{u^{2z_3}} + \frac{du^2}{u^2 f(u)} \right],$$

$$\phi = c_\phi \log(\xi u)\,, \qquad f(r) = 1 - \left(\frac{u}{u_h} \right)^{1+2z_1+z_3-\theta}$$

where

$$\theta = \frac{6\sigma(\gamma+\lambda-2\sigma)}{2\gamma^2+\lambda^2+2\lambda\sigma-5\sigma^2+4}\,, \qquad z_1 = \frac{(\lambda+\sigma)(\gamma+\lambda-2\sigma)}{2\gamma^2+\lambda^2+2\lambda\sigma-5\sigma^2+4}\,,$$

$$z_3 = \frac{2\gamma(\gamma+\lambda-2\sigma)}{2\gamma^2+\lambda^2+2\lambda\sigma-5\sigma^2+4}\,.$$

Theories with Phase Transitions and Non-trivial RG flows:

- How the Field Theory looks like?
  - ✓ 4d $SU(N)$ Strongly coupled anisotropic gauge theory.
  - ✓ Its dynamics are affected by a scalar operator $\mathcal{O}_\Delta$.
  - ✓ Anisotropy is introduced by another operator $\tilde{\mathcal{O}} \sim \theta(x_3) TrF \wedge F$ with a space dependent coupling.

- The gravity dual theory is an Einstein-Axion-Dilaton theory in 5 dimensions with a non-trivial potential.
  - ✓ A "backreacting" scalar field depending on spatial directions, the axion; and a non-trivial dilaton.
  - ✓ Solutions are non-trivial RG flows: Conformal fixed point in the UV $\Rightarrow$ Anisotropic (Hyperscaling Lifshitz-like) in IR.

- The vacuum state confines color and there exists a phase transition at finite T_c above which a deconfined plasma state arises.

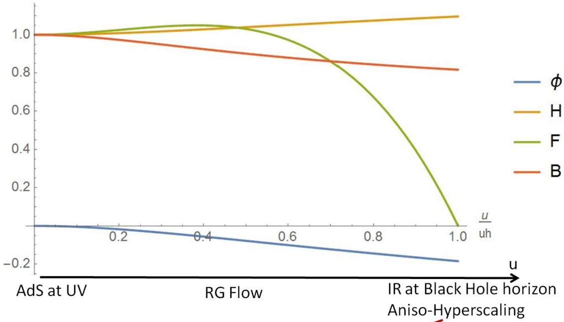
(D.G., Gursoy, Pedraza, 2018)

A Numerical Solution : The RG Flow

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}(u)\mathcal{B}(u)\, dt^2 + dx_1^2 + dx_2^2 + \mathcal{H}(u)dx_3^2 + \frac{du^2}{\mathcal{F}(u)}\right),$$

$$\chi = \alpha x_3\,, \qquad \phi = \phi(u)\,, \qquad \mathcal{F}(u_h) = 0\,, \qquad B = 0.$$

Solve perturbatively the equations of motion around the horizon of the black hole $u = u_h$. Use them as initial conditions to obtain the numerical full solution.



$$ds^2 = u^{-\frac{2\theta}{3}}\left(-u^{2z}\left(f(u)dt^2 + dx_{1,2}^2\right) + \tilde{\alpha}\, u^2 dx_3^2 + \frac{du^2}{f(u)u^2}\right)$$

Are the theories physical and stable?

$$\Downarrow$$
$$\Downarrow$$

✓ Energy Conditions Analysis: $T_{\mu\nu}N^\mu N^\nu \geq 0\ , \quad N^\mu N_\mu = 0\ .$

$$R_1^1 - R_0^0 \geq 0\ , \qquad R_3^3 - R_0^0 \geq 0\ , \qquad R_u^u - R_0^0 \geq 0\ .$$

AND

✓ Local Thermodynamical Stability Analysis

$$\Downarrow$$
$$\Downarrow$$

YES!

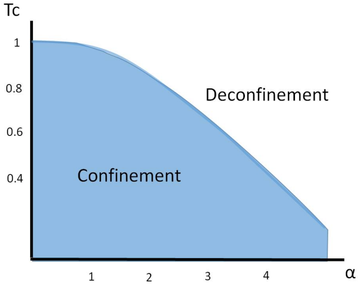
Phase Diagram

Properties of Heavy Quark Observables depend strongly on the Anisotropy.

(D.G. 2012; Chernicoff, Fernandez, Mateos, Trancanelli; Rebhan, Steineder 2012...)

Confinement/Deconfinement Phase Transitions?

- The Critical Temperature of the theories vs the anisotropy:



- Anisotropic strongly coupled systems have lower critical temperature.
- New phenomenon: **Inverse Anisotropic Catalysis**.

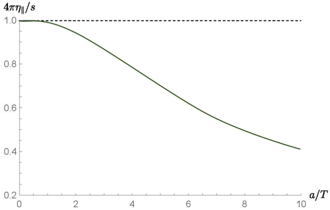
(DG, Gursoy, Pedraza 2018; related Aref'eva, Rannu 2018)

Universal Results: η/s in Theories with Broken Symmetry

Consider a finite T theory in the deconfined phase:

$$ds^2 = g_{tt}(u)dt^2 + g_{11}(u)(dx_1^2 + dx_2^2) + g_{33}(u)dx_3^2 + g_{uu}(u)du^2$$

- The anisotropic "shear viscosity" is obtained by the two-point function of the energy momentum tensor :



- The Ratio:

$$4\pi\frac{\eta_\parallel}{s} = \frac{g_{11}}{g_{33}}\bigg|_{u=u_h} \sim \left(\frac{T}{\alpha}\right)^p, \qquad p = 2 - \frac{2}{z} \sim (0,\infty)\ , \quad \alpha \gg T.$$

(Jain, Samanta Trivedy 2015; D.G., Gursoy, Pedraza, 2018)

- New Universalities?

$$4\pi\frac{\eta_\parallel}{s}\frac{\sigma_\perp}{\sigma_\parallel} \geq 1$$

(Inkof, Gouteraux, Kiselev, Kuppers, Link, Narozhny, Schmalian 2018, 2020; Rebhan, Steineder 2011)

Spinning Heavy Bound States

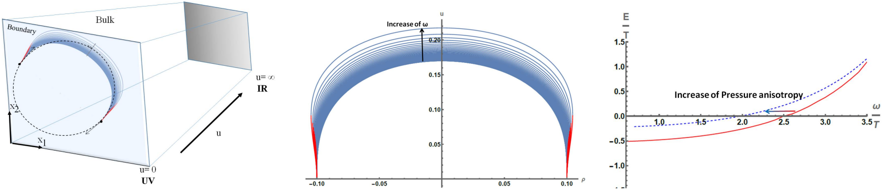
- A bound state spinning with angular frequency ω.

$$x_1 = \rho \cos\phi \ , \quad x_2 = \rho \sin\phi \ , \quad \phi = \omega t + [\theta(\sigma)] \ , \quad u = u(\sigma) \ , \quad \rho = \sigma \ .$$

The action reads

$$S_s = \int d\tau d\sigma \sqrt{-(g_{tt} + g_{x_1x_1}\rho^2\dot{\phi}^2)(g_{x_1x_1} + g_{uu}u'^2)}$$

- There are UV divergences: appropriate renormalization scheme needs to be applied.
- Worldsheet solutions of the equations of motion and the energy of the bound state:

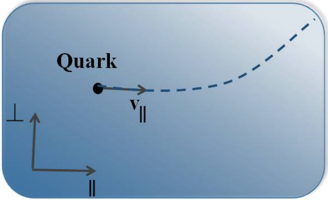


- Increase of anisotropy leads to easier dissociation of the state. Common behavior when rotational symmetry is broken.

(D.G., 2022, in progress)

Langevin Dynamics and Brownian Motion

Consider a heavy quark ($M \gg T$) moving in a strongly coupled plasma.



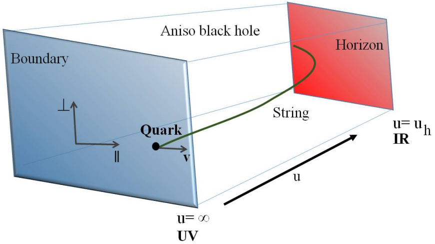
The Macroscopic Langevin equation:

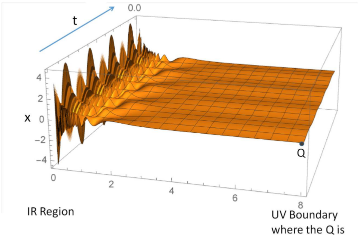
$$\dot{p}_i(t) = -\eta_D p_i(t) + \xi_i(t) ,$$

p: the momentum of the particle, η_D: the friction coefficient, ξ: the random force.

$$\langle \xi_{\parallel,\perp}(t) \rangle = 0 , \qquad \langle \xi_{\parallel,\perp}(t) \xi_{\parallel,\perp}(t') \rangle = \kappa_{\parallel,\perp} \delta(t-t') , \qquad \left\langle p^2_{\parallel,\perp} \right\rangle = 2\kappa_{\parallel,\perp} \mathcal{T}$$

Langevin Dynamics and Brownian Motion





$$\frac{\kappa_{\|}}{\kappa_{\perp}} = \left.\frac{\left(g_{00}\, g_{\|\|}\right)'}{g_{\perp\perp}\, g_{\|\|}\left(\frac{g_{00}}{g_{\|\|}}\right)'}\right|_{u=u_{wh}}, \qquad \left\langle p^2_{\|,\perp} \right\rangle \sim \kappa_{\|,\perp}\, \mathcal{T}$$

A Universal Inequality for Isotropic Theory:

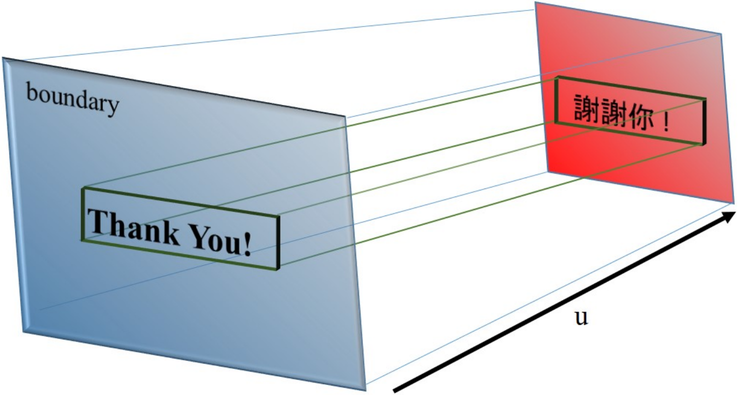
$\kappa_{\|} \geq \kappa_{\perp}$ for any isotropic strongly coupled plasma!

Can be inverted in the anisotropic theories: $\kappa_{\|} \geq < \kappa_{\perp}$.

(Gursoy, Kiritsis, Mazzanti, Nitti, 2010; D.G, Soltanpanahi, 2013a,b; D.G. 2018)

Conclusions

- ✓ Strongly coupled theories with a conformal UV fixed point and an IR fixed point with reduced symmetries.
- ✓ Observation: In strongly coupled theories many phenomena are more sensitive to the presence of the anisotropy than the source that triggers it.
- ✓ The phase transitions occur at lower critical Temperature as the anisotropy is increased = Inverse Anisotropic Catalysis!
- ✓ Several Universal Isotropic relations are anisotropically violated. Look for new Universalities!


boundary
Thank You!
謝謝你！
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