Spin polarization of particles in a thermal model

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References
e-Prints: arXiv:2112.02799 [hep-ph].
Publish paper: Phys. Rev. C 100, 054907 (2019); arXiv:1904.00002 [nucl-th].

ASIoP Seminar, March 4, 2022

Relativistic heavy ion collisions

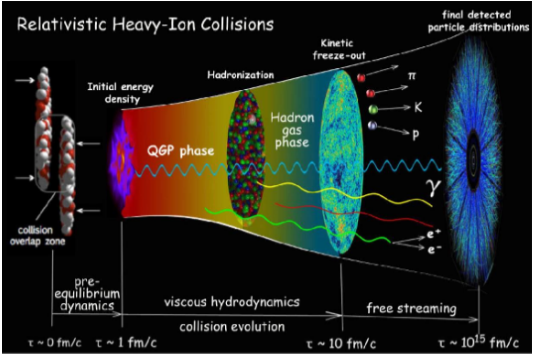


Figure: Dynamical evolution of a heavy ion collision [C. Shen, U. Heinz, Nucl. Phys. News 2015, 25, 6–11]

- Discover QGP.
- Study its properties such as, EOS, order of phase transition, transport coefficients etc.

Einstein-de Haas and Barnett Effects

- HIC can also provide opportunity to study consequences of spin-rotation coupling

Classical to quantum angular mometum transition

Barnett Effect

S. J. Barnett, Rev. Mod. Phys. 7, 129 (1935)

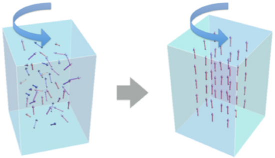


Figure: Mechanical rotation of an unmagnetized metallic object induces magnetization, an effective magnetic field emerges.

$$B_\Omega = \Omega/\gamma$$

Einstein-de Haas Effect

A. Einstein and W. de Haas, Deutsche Physikalische Gesellschaft, Verhandlungen 17, 152 (1915)

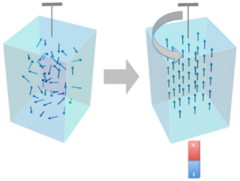


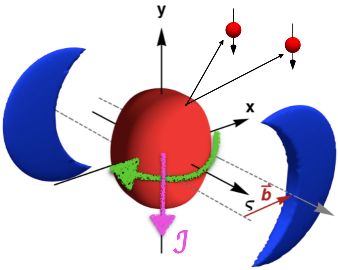
Figure: Application of magnetic field on an unmagnetized metallic object induces magnetization, body start rotating (mechanical angular momentum emerges)

Spin polarization of particles in heavy ion collisions

- Nuclei colliding at ultrarelativistic energies creates fireball of large orbital angular momentum $L_{init} \approx 10^5 \hbar$ (RHIC Au-Au 200 GeV, b=5 fm) [F. Becattini, F. Piccinini and J. Rizzo, Phys. Rev. C77, 024906 (2008)].

- Initially $J_{init} = L_{init}$, later some part of the angular momentum can be transferred from the orbital to the spin part $J_{final} = L_{final} + S_{final}$.

- This may induce spin polarization, similar to magnetomechanical Barnett effect [S. J. Barnett, Rev. Mod. Phys. 7, 129 (1935)].



Emerging particles are expected to be globally polarized with their spins on average pointing along the system angular momentum.

Global Λ polarization in RHIC experiment

In HIC experiments global polarization can be measured by the $\Lambda-$ hyperon weak decay into a proton and a negatively charged pion. [L. Adamczyk *et al.* (STAR), Nature 548 (2017) 62-65, arXiv:1701.06657 [nucl-ex]].

The average polarization $\bar{P}_H$ (where $H = \Lambda$ or $\bar{\Lambda}$) from $20 - 50\%$ central Au+Au collisions [L. Adamczyk *et al.* (STAR), Nature 548 (2017) 62-65, arXiv:1701.06657 [nucl-ex]].

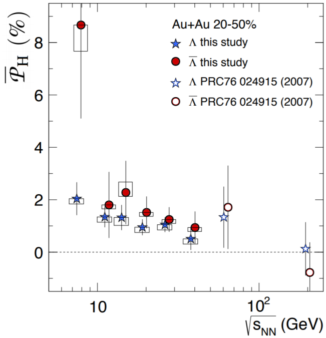


Figure: The average polarization versus collision energy

Initial Phenomenological prescription used to describe the data make use of hydrodynamic framework which deals with the spin polarization of particles at freeze-out.

[F. Becattini, I. Karpenko, M. Lisa, I. Upsal, S. Voloshin, Phys. Rev. C 95, 054902 (2017); F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013)]

Hydrodynamics=local thermodynamic equilibrium+conservation laws.

Ideal	Dissipative
$T^{\mu\nu}=\epsilon u^\mu u^\nu - P\Delta^{\mu\nu}$ $N^\mu = nu^\mu$ Unknowns: $\underbrace{\epsilon,\ P,\ n,\ u^\mu}_{=6}$	$T^{\mu\nu}=\epsilon u^\mu u^\nu - [P+\Pi]\Delta^{\mu\nu}+\pi^{\mu\nu}$ $N^\mu = nu^\mu+\nu^\mu$ Unknowns: $\underbrace{\epsilon,\ P,\ n,\ u^\mu,\ \Pi,\ \pi^{\mu\nu},\ \nu^\mu}_{=15}$
Equations: $\underbrace{\partial_\mu T^{\mu\nu}=0,\quad \partial_\mu N^\mu=0,\quad EoS}_{4+1+1=6}$	
Closed set of equations	9 additional equations are needed

- $\Delta^{\mu\nu}=g^{\mu\nu}-u^\mu u^\nu$ and choice of Landau frame $T^{\mu\nu}u_\nu=\epsilon u^\mu$.
- For the simplest case, $\pi^{\mu\nu}=2\eta\sigma^{\mu\nu}$, $\Pi=\zeta\nabla^\mu u_\mu$ and $\nu^\mu=\kappa\nabla^\mu\xi$.
- Here, $\nabla^\mu=\Delta^{\mu\nu}\partial_\nu$ denotes the transverse gradient,

$\sigma^{\mu\nu}=\frac{1}{2}\left(\nabla^\mu u^\nu+\nabla^\nu u^\mu\right)-\frac{1}{3}\Delta^{\mu\nu}\left(\nabla^\lambda u_\lambda\right)$ is the shear flow tensor, while η, ζ and κ are the transport coefficients: namely coefficient of shear viscosity, bulk viscosity and charge or heat conductivity.

The main idea is to identify the spin polarization tensor $\omega_{\mu\nu}$ with thermal vorticity $\varpi^{\mu\nu}$ and then to obtain the results for spin polarization. Note that the relation that $\omega_{\mu\nu} = \varpi^{\mu\nu}$ hold true in global equibrium and proven in Ref. [F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013)]

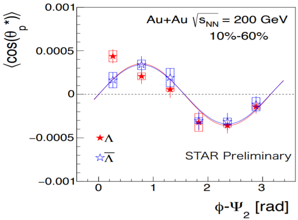
$$\omega^{\mu\nu} \quad \Leftrightarrow \quad \varpi^{\mu\nu} = -\frac{1}{2}\left(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu\right)$$

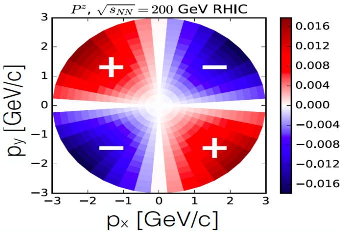
The Algorithm is:

1. Use hydrodynamic frameworks (ideal or viscous).
2. Find $\beta^\mu = u^\mu / T$ on the freeze-out hypersurface.
3. Calculate the thermal vorticity consider it as spin polarization tensor.
4. Make prediction about spin polarization using the modified Cooper Frye formula [F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013), R. Fang, L. Pang, Q. Wang, X. Wang Phys. Rev. C 94, 024904 (2016)].

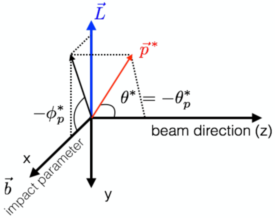
It describes the global polarization data. But there is a problem!

Problem with the thermal vorticity model





T. Niida, NPA 982 (2019) 511514 [1808.10482]; F. Becattini, I. Karpenko, PRL 120 (2018) no.1, 012302, [1707.07984]



$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_{\mathrm{H}}\mathbf{P_H}\cdot\mathbf{p}^*_p)$$

$$\langle\cos\theta^*_p\rangle = \int \frac{dN}{d\Omega^*}\cos\theta^*_p d\Omega^*$$

$$= \alpha_{\mathrm{H}} P_z \langle(\cos\theta^*_p)^2\rangle$$

$$\therefore\ P_z = \frac{\langle\cos\theta^*_p\rangle}{\alpha_{\mathrm{H}}\langle(\cos\theta^*_p)^2\rangle}$$

$$= \frac{3\langle\cos\theta^*_p\rangle}{\alpha_{\mathrm{H}}} \quad \text{(if perfect detector)}$$

α_{H}: hyperon decay parameter
θ_p^*: θ of daughter proton in Λ rest frame

Problem in explaining the Quadrupole structure! (Does not describe the local spin polarization)

Discovery of thermal shear

One of the reasons may be that in local equilibrium thermal vorticity is not same as spin polarization tensor.

F. Becattini el al. and S.Y.F. Liu et al. came out with idea of new hydrodynamic gradients that lead to local spin polarization.

[Phys. Lett. B 820 (2021) 136519, also arXiv:2103.14621 [nucl-th]]

[JHEP 07 (2021) 188, arXiv:2103.09200 [hep-ph]]

They suggested that thermal vorticity which is an antisymmetric combination of hydrodynamic gradients is the not the only thing that contributes to local spin polarization. There are also contribution from a symmetric hydrodynamic gradients defined in terms of shear stress tensor $\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu)$.

Thus in the local equilibrium spin polarization tensor is the combination of both the antisymmetric and symmetric hydrodynamic gradiets.

Discovery of thermal shear term

In case of Becattini et al, the spin polarization tensor can be written as
[Phys. Lett. B 820 (2021) 136519, also arXiv:2103.14621 [nucl-th]]

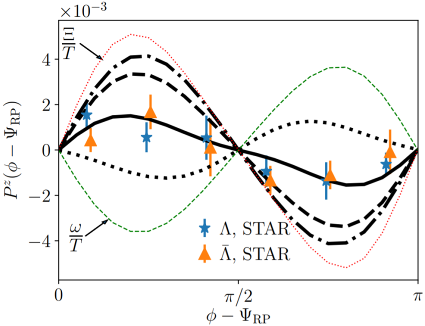
$$\omega^{\rho\sigma} = \varpi^{\rho\sigma} + 2\hat{t}^{\rho}\frac{p_{\lambda}}{E_p}\xi^{\lambda\sigma}$$

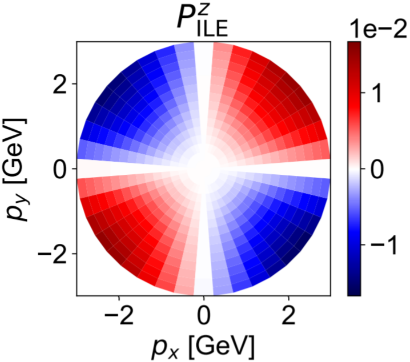
$\hat{t}^{\rho} = (1, 0, 0, 0)$ and $E_p = \sqrt{m^2 + |p|^2}$.
In case of S. Y. F. Liu et al the same can be written as [JHEP 07 (2021) 188, arXiv:2103.09200 [hep-ph]]

$$\omega^{\rho\sigma} = \varpi^{\rho\sigma} + u^{\rho}\frac{p_{\lambda}}{\bar{E}_p}\xi^{\lambda\sigma}$$

where $\bar{E}_p = u^{\mu}p_{\mu}$. Note that in rest frame of the fluid the thermal shear contribution in the two equations differs by a factor of 2.

Hydrodynamic model predictions based on formula by Becattini et al. [arXiv:2103.14621]

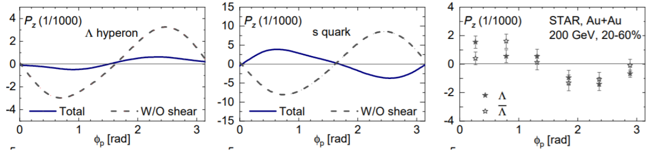




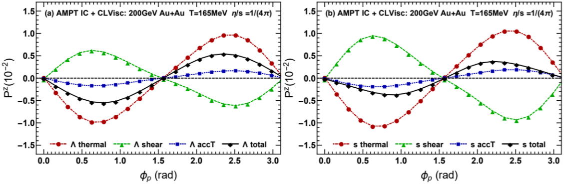
Sign changes, describe the data.

Temperature gradients terms of thermal vorticity and thermal shear not included.

Hydrodynamic model predictions based on Liu et al. Formula by Fu et al. describe the data only if the mass of $\Lambda-$ hyperon is replaced by strange quark mass.
[arXiv:2103:10403[hep-ph]]



In an anoither study using Hydrodynamic model based on Liu et al scenario does not describe the data [Cong Yi, Shi Pu, Di-Lun Yang, Phys.Rev.C 104 (2021) 6, 064901; arXiv: 2106.00238 [hep-ph]]



The reason might be that longitudinal spin polarization depend on various factors like, EoS, freeze-out temperature etc.

- None of the studies gives convincing results.
- Why not to give a try of the same using thermal model which have been in past used to describe the various hadronic yields, transverse momentum spectra, elliptic flow, HBT radii.

[W. Florkowski, AK, R. Ryblewski, Prog. Part. Nucl. Phys. 108, 103709 (2019)] [S. Bhadury, W. Florkowski, A. Jaiswal, AK, R. Ryblewski, arXiv:2002.03937 [hep-ph], arXiv:2008.10976 [nucl-th]]

Model Input

- In its standard fomulation, it uses only four parameters: Temperature T_f, Chemical Potential μ_{Bf}, proper time τ_f, and system size r_{max}.
- The two thermodynamic parameters T_f and μ_{Bf} are fitted from the ratios of hadronic abundances.
- For boost-invariant systems the ratios of hadron multiplicities at midrapidity, $dN/dy_p|y_p = 0$, are related to the ratios of densities, n_i, since

$$\left.\frac{dN_i/dy_p}{dN_j/dy_p}\right|_{y_p=0} = \frac{n_i}{n_j} = \frac{g_i \int d^3p f_i(p)}{g_j \int d^3p f_j(p)}$$

- $f_i(p) = \frac{1}{(2\pi)^3}\left(\exp\frac{(E_i(p)-\mu_B B_i)}{T} \pm 1\right)^{-1}$ is the distribution function, with energy, $E_i(p) = \sqrt{p^2 + m_i^2}$ and μ_B baryon chemical potential.

Fitted thermodynamic parameters	$T_f = 165 \pm 7 MeV$, $\mu_{Bf} = 41 \pm 5 MeV$
Ratios used for the fit	$\pi^-/\pi^+ = 1.00 \pm 0.02$

Table: Fitted parameters used to describe the PHENIX data ($\sqrt{s_{NN}} = 130$ GeV) [Acta Phys.Polon.B 33 (2002) 4235-4258].

Thermal Model

- The geometric ones, τ_f and r_{max} characterize the freeze-out hypersurface and hydrodynamic flow and can be obtained from the fit of experimentally observed transverse momentum spectra of particles from the Cooper-Frye Formula

$$\frac{dN}{d^2p_T dy_p} = \int \Delta\Sigma_\lambda p^\lambda f(\frac{p \cdot u}{T}, \mu_B)$$

- Freeze-out hypersurface is defined through the conditions $\tau_f = \sqrt{t^2 - x^2 - y^2 - z^2} = constt$ and $x^2 + y^2 \leq r_{max}^2$.
- The hydrodynamic flow is assumed to be Hubble-like form

$$u^\mu = \frac{x^\mu}{\tau_f} = \frac{t}{\tau_f}\left(1, \frac{x}{t}, \frac{y}{t}, \frac{z}{t}\right)$$

- In this case the integration over freeze-out hypersurface can be done and transverse momentum spectra will depend on τ_f and r_{max} for the given values of freeze-out temperature T_f and μ_{Bf} Therefore, we can fit the experimental data by choosing these parameters.

Thermal Model

Values of parameters τ_f and r_{max} are listed by fitting transverse momentum spectra of the particles for different class of centralities [Acta Phys.Polon.B 33 (2002) 4235-4258]

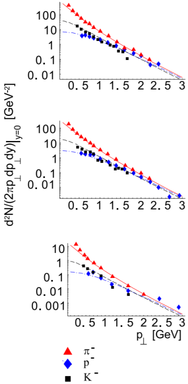


Figure: Model vs. experiment for the PHENIX data [PRL 88, 242301 (2002)] at three different centrality bins $c = 0-5\%$, $c = 15-30\%$, $c = 60-92\%$ (top to bottom) for pions, kaons, protons at $T_f = 165 MeV$, $\mu_{Bf} = 41 MeV$.

c %	τ_f [fm]	$r_{\max}$ [fm]
$0-15$	8.2	6.9
$15-30$	6.3	5.3
$60-92$	2.3	2.0

Table: Thermal model parameters used to describe the PHENIX data ($\sqrt{s_{NN}} = 130$ GeV)[PRL 88, 242301 (2002)]

Extended Thermal Model

In the realistic scenario we encounter the following situation in heavy ion collisions

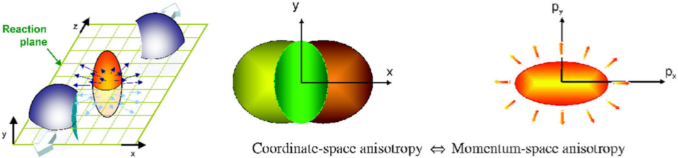


Figure: Initial coordinates space anisotropy get transsformed to the anisotropy in particle momenta distributions (flow) [Fig. taken from Ref. Pramana – J. Phys. (2021) 95:15]

One can perform the Fourier decomposition of the momentum space particle distributions in the $xy-$plane as follow,

$$\frac{dN}{d^2p_T dy_p} = \frac{dN}{2\pi p_T dp_T dy_p}[1 + 2\sum_{n=1}^{\infty} v_n \cos(n\phi_p)]$$

where, $v_n = \langle \cos(n\phi_p) \rangle$. At $y_p = 0$ the most dominant contribution comes from v_2. It is known as elliptic flow coefficient. To incorporate the phenomenon of elliptic flow thermal thermal model need to be extended.

Extended Thermal Model

- Thermal model need to be extended to include the elliptic flow phenomenon.
- This can be done by including the elliptic deformation of both: the emission region in the transverse plane and the transverse flow.
- Elliptic asymmetry in the transverse plane

$$\begin{aligned} x &= r_{\mathrm{max}}\sqrt{1-\epsilon}\cos\phi, \\ y &= r_{\mathrm{max}}\sqrt{1+\epsilon}\sin\phi. \end{aligned}$$

- ϕ is the azimuthal angle while r_{max} and ϵ are the model parameters.
- $\epsilon > 0$ indicates that the system formed in HICs is elongated in the y direction.
- Flow asymmetry is included by parametrizing the flow velocity as follows

$$u^\mu = \frac{1}{N}\left(t,\ x\sqrt{1+\delta},\ y\sqrt{1-\delta},\ z\right).$$

- δ characterizes anisotropy in the transverse flow.
- $\delta > 0$, indicates that there is a more flow in the reaction plane (elliptic flow).

$$N = \sqrt{\tau_f^2 - (x^2 - y^2)\,\delta}$$

- normalization contant, $\tau_f^2 = t^2 - x^2 - y^2 - z^2$ is the proper time characterizing freeze-out hypersurface.

Model parameters

- Now six parameters, T_f, μ_{Bf}, ϵ, δ, τ_f and r_{max}.
- The values of these parameters have been used before to describe the PHENIX data for transverse mometum spectra for three different centrality classes at the beam energy $\sqrt{s_{NN}} = 130$ GeV, freeze-out temperature $T_f = 0.165$ GeV and $\mu_{Bf} = 0.041$ GeV [A. Baran, PhD Thesis and J. Phys. G31 S1087-S1090 (2005)].

c %	ϵ	δ	τ_f [fm]	r_{max} [*fm*]
$0-15$	0.055	0.12	7.666	6.540
$15-30$	0.097	0.26	6.258	5.417
$30-60$	0.137	0.37	4.266	3.779

Decomposition of thermal vorticity and thermal shear in term of velocity gradient and temperature gradient terms

- Thermal vorticity is defined as

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu),$$

- The above equation can be rewritten as a sum of the two terms,

$$\varpi_{\mu\nu} = \underbrace{-\frac{1}{2T}\left(\partial_\mu u_\nu - \partial_\nu u_\mu\right)}_{\varpi^{I}_{\mu\nu}} + \underbrace{\frac{1}{2T^2}\left(u_\nu \partial_\mu T - u_\mu \partial_\nu T\right)}_{\varpi^{II}_{\mu\nu}}.$$

- Thermal shear tensor is defined as

$$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu),$$

$$\xi_{\mu\nu} = \underbrace{\frac{1}{2T}\left(\partial_\mu u_\nu + \partial_\nu u_\mu\right)}_{\xi^{I}_{\mu\nu}} \underbrace{-\frac{1}{2T^2}\left(u_\nu \partial_\mu T + u_\mu \partial_\nu T\right)}_{\xi^{II}_{\mu\nu}}.$$

Calculation of velocity and temperature gradient terms

- Velocity gradient terms $\varpi^{I}_{\mu\nu}$ and $\xi^{I}_{\mu\nu}$ can be directly obtained by the parametrized flow velocity as shown in one of the earlier slides.
- Temperature gradient term $\varpi^{II}_{\mu\nu}$ and $\xi^{II}_{\mu\nu}$ can be calculated by

$$\partial_\mu T^{\mu\nu}(x) = 0, \qquad T^{\mu\nu} = (e+p)u^\mu u^\nu - p g^{\mu\nu}, g^{00} = +1.$$

- From the above equation we can get the following two equations:

$$Du^\alpha = \frac{1}{T}\nabla^\alpha T,$$
$$DT = -T c_s^2 \partial_\alpha u^\alpha,$$

where, $D = u^\alpha \partial_\alpha$ and $\nabla^\alpha = \Delta^{\alpha\mu}\partial_\mu = \partial^\alpha - u^\alpha D$, with $\Delta^{\alpha\mu} = g^{\alpha\mu} - u^\alpha u^\mu$ and used that $e + p = sT$ and $s(T) = dp(T)/dT$. From the above two equations we can get

$$\boxed{\partial^\alpha T = T\left(Du^\alpha - c_s^2 u^\alpha \partial_\mu u^\mu\right)}$$

- Using the above expression the different components of temperature gradient terms $\varpi^{II}_{\mu\nu}$ and $\xi^{II}_{\mu\nu}$ can be obtained in terms of model parameters.

Calculating of Spin Polarization of particles in a thermal model

- The mean spin polarization of particles is determined by calculating the average Pauli-Lubański (PL) vector $\langle \pi^{\star}_{\mu}(p) \rangle$ in the local rest frame of the particles.
- The average PL vector $\langle \pi_{\mu}(p) \rangle$ of particles having momentum p emitted from a given freeze-out hypersurface is given by the following formula

$$\langle \pi_{\mu}(p) \rangle = \frac{E_p \frac{d\Pi_{\mu}(p)}{d^3p}}{E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

$$E_p \frac{d\Pi_{\mu}(p)}{d^3p} = -\frac{\cosh(\xi)}{(2\pi)^3 m} \int e^{-\beta \cdot p} \, \Delta\Sigma_{\lambda} p^{\lambda} \, \tilde{\omega}_{\mu\beta} p^{\beta}.$$

$$E_p \frac{d\mathcal{N}(p)}{d^3p} = \frac{4\cosh(\xi)}{(2\pi)^3} \int e^{-\beta \cdot p} \, \Delta\Sigma_{\lambda} p^{\lambda} \, .$$

- We take formula by F. Becattini et al.

[F. Becattini, M. Buzzegoli, and A. Palermo, Phys. Lett. B 820 (2021) 136519, arXiv:2103.10917 [nucl-th]]

[F. Becattini, M. Buzzegoli, A. Palermo, G. Inghirami, and I. Karpenko, arXiv:2103.14621[nucl-th]]

$$\tilde{\omega}_{\mu\beta} p^{\beta} = \frac{1}{2} \epsilon_{\mu\beta\rho\sigma} p^{\beta} \left(\varpi^{\rho\sigma} + 2\hat{t}^{\rho} \frac{p_{\lambda}}{E_p} \xi^{\lambda\sigma} \right)$$

Calculating of Spin Polarization of particles in a thermal model

- Assuming that freeze-out takes place at a constant values of proper time,

$$\Delta\Sigma_\lambda \quad = \quad n_\lambda \, dxdy \, d\eta,$$

$$n^\lambda = \left(\sqrt{\tau_f^2 + x^2 + y^2}\cosh\eta, x, y, \sqrt{\tau_f^2 + x^2 + y^2}\sinh\eta\right)$$

- $\eta = \frac{1}{2}\ln\left[(t+z)/(t-z)\right] \; n^\lambda n_\lambda = \tau_f^2.$
- Parametrization of the particle four-momentum p^λ in terms of the transverse momentum $p_T = \sqrt{p_x^2 + p_y^2}$ and rapidity y_p,

$$p^\lambda \quad = \quad (E_p, p_x, p_y, p_z) = (m_T \cosh y_p, p_x, p_y, m_T \sinh y_p)\,,$$

- $m_T = \sqrt{m^2 + p_T^2}$ is the transverse mass and m is the particle mass.
- As the experimental measurements are done in the central rapidity region, we consider the case of $y_p = 0$ only. Furthermore, since we focus on the longitudinal spin polarization, we do not have to boost the four-vector $\langle\pi_\mu(p)\rangle$ to the particle rest frame, because $\langle\pi_z(p)\rangle$ is invariant under transverse boosts.

Tansverse momentum dependence of longitudinal component of the mean spin polarization three-vector of Λ-hyperon

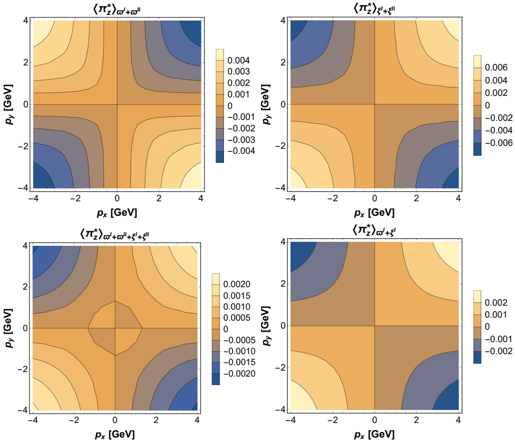


Figure: The longitudinal component of the mean spin polarization three-vector of Λ hyperon as a function of its transverse momentum for the centrality class c=0–15% with contribution from different terms of thermal vorticity and thermal shear tensor.

Tansverse momentum dependence of longitudinal component of the mean spin polarization three-vector of Λ-hyperon

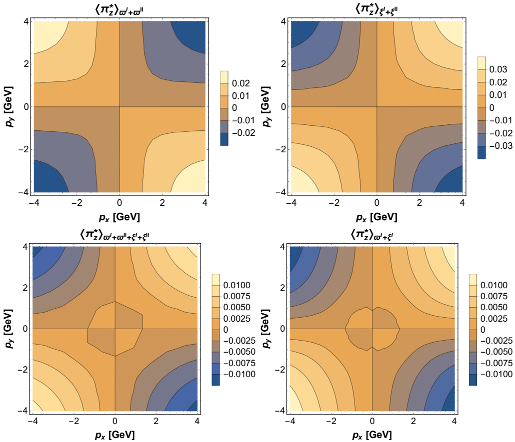


Figure: same as above but for the centrality class c=30–60%.

Experimental observables

- In experiment one observes the azimuthal angle dependence of the transverse momentum integrated longitudinal polarization.

- Momentum integrated polarization

$$\langle P_\mu \rangle = \frac{\int d^3p \langle \pi_\mu(p) \rangle E_p \frac{d\mathcal{N}(p)}{d^3p}}{\int d^3p E_p \frac{d\mathcal{N}(p)}{d^3p}} = \frac{\int d^3p E_p \frac{d\Pi_\mu(p)}{d^3p}}{\int d^3p E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

- Azimuthal angle dependence of Longitudinal component of spin polarization can be obtained by carrying out integration over the transverse momentum

$$\langle P(\phi_p) \rangle \equiv \frac{\int p_T dp_T E_p \frac{d\Pi^z(p)}{d^3p}}{\int d\phi_p p_T dp_T E_p \frac{d\mathcal{N}(p)}{d^3p}}.$$

Azimuthal angle dependence of Longitudinal component of spin polarization

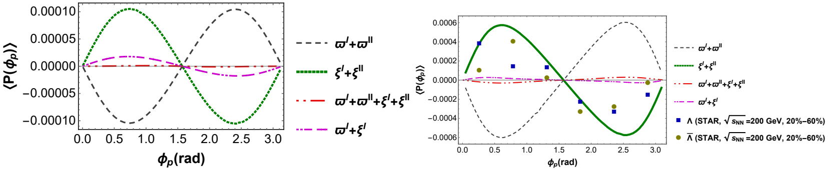


Figure: Azimuthal angle dependence of p_T-integrated (range p_T=0.5–6 GeV) longitudinal spin polarization for the centrality class c=0–15% and c=30–60%. For comparison we show the dependence of longitudinal spin polarization of Λ and $\bar{\Lambda}$ on azimuthal angle relative to second order event plane for the centrality class c=20–60% plotted using the STAR data at $\sqrt{s_{NN}}$ = 200 GeV [Phys.Rev.Lett. 123 (2019) no. 13, 132301, arXiv:1905.11917 [nucl-ex]].

Azimuthal angle dependence of Longitudinal component of spin polarization

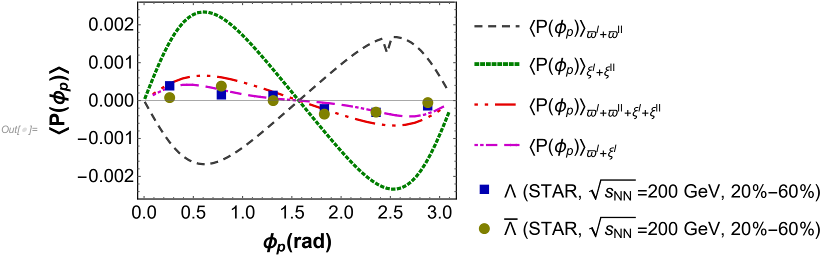


Figure: Azimuthal angle dependence of p_T-integrated (range p_T=0.5–6 GeV) longitudinal spin polarization for the centrality class c=30–60% when mass of lambda hyperon is replaced by strange quark.

Experimental observables

- $\langle \pi^z \rangle$ can be decomposed into the Fourier series, where only sine terms of even multiples of the azimuthal angle ϕ_p of the transverse momentum vector are non-vanishing [I. Karpenko, F. Becattini, Nucl. Phys. A 00 (2018) 1-4, arXiv:1811.00322 [nucl-th]] [I. Karpenko, arXiv:2101.04963v1 [nucl-th]]

$$\langle \pi^z \rangle \quad = \quad \frac{1}{2} \sum_{k=1}^{\infty} P_{2k}(p_T) \sin(2k\phi_p)$$

•Azimuthal harmonic (Polarization anisotropy)

$$\langle P_2 \rangle = \frac{\frac{1}{2\pi} \int p_T dp_T d\phi_p \sin(2\phi_p) \langle \pi^z(p) \rangle E_p \frac{d\mathcal{N}(p)}{d^3p}}{\int p_T dp_T d\phi_p E_p \frac{d\mathcal{N}(p)}{d^3p}} = \frac{\frac{1}{2\pi} \int d\phi_p p_T dp_T \sin(2\phi_p) E_p \frac{d\Pi^z(p)}{d^3p}}{\int d\phi_p p_T dp_T E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

- We have also looked at the polarization anisotropy treated as a function of p_T. In this case, we perform azimuthal integrals in both the numerator and denominator, keeping the transverse momentum fixed, namely

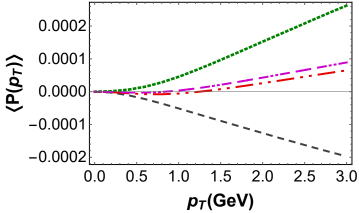
$$\langle P(p_T) \rangle \quad = \quad \frac{\frac{1}{2\pi} \int d\phi_p \sin(2\phi_p) E_p \frac{d\Pi_z(p)}{d^3p}}{\int d\phi_p E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

Azimuthal Harmonic

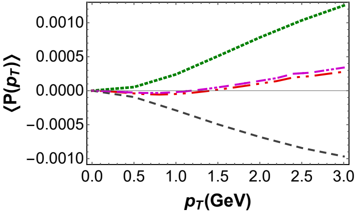
c %	$\langle P_2 \rangle_{\varpi^I+\varpi^{II}}$	$\langle P_2 \rangle_{\xi^I+\xi^{II}}$	$\langle P_2 \rangle_{\varpi^I+\xi^I}$	$\langle P_2 \rangle_{\varpi^I+\varpi^{II}+\xi^I+\xi^{II}}$
$0-15$	-0.000052	0.000052	8.8×10^{-6}	-3.9×10^{-7}
$15-30$	-0.000138	0.000136	7.5×10^{-6}	-1.8×10^{-6}
$30-60$	-0.000290	0.000277	0.0000104	-0.0000134

Table: n=2 harmonics of longitudial spin polarization due to various terms using the model parameters valid for different classes of collision centralities at $\sqrt{s_{NN}} = 130$ GeV. Temperature gradient contribution has been obtained using ideal equation of state ($c_s^2 = 1/3$). Integration of transverse momentum is carried out in the range p_T=0.5–6 GeV.

Transverse mometum dependence of polarization anisotropy



(a)



(b)

Figure: Transverse-momentum dependence of $n = 2$ harmonic of longitudinal spin polarization. for the centrality class c=0-15% and c=30-60% with temperature gradient terms evaluated by taking $c_s^2 = 1/3$.

Summary

1. We presented the study of tansverse momentum dependence of the longitudinal component of spin polarization of Λ-hyperons in a thermal model that includes the contribution from both the thermal vorticity and thermal shear tensor.
2. Our results suggests that the sign of quadrupole structure of longitudinal polarization can be changed by including the thermal shear effects.
3. Analysis of azimuthal angle dependence of the longitudinal component of spin polarization vector suggest that contributions of thermal vorticity and thermal shear are of opposite sign and nearly identically cancel each other leading to the disagreement with the data if mass of the particle is taken to be equal to $\Lambda-$ hyperon mass.
4. Experimental data for the azimuthal angle dependence of the longitudinal component of spin polarization vector can be described by replacing the $\Lambda-$ hyperon mass to strange quark mass.
5. We have also presented the results for the polarization anisotropy (n=2 harmonic of longitudinal spin polarization) and its transverse momentum dependence.

Current work and future plan

- Recently a new variable known as helicity polarization defined as the local spin polarization projected to the momentum direction of polarized hadrons has been proposed that can be used to reveal local parity violation in hot QCD matter in relativistic heavy ion collisions.

$$P_H(p) = \hat{\mathbf{p}} \cdot \langle \pi(p) \rangle$$

- Helicity polarization in a thermal model [With Prof. Di-Lun Yang]
- Spin alignment of vector meson [Planned to work with Prof. Di-Lun Yang]

THANK YOU FOR YOUR ATTENTION

Connection between spin polarization and thermal vorticity

The density operator [D. Zubarev, Nonequilibrium Statistical Thermodynamics (Springer, 1974); F. Becattini, Phys. Rev. Lett. 108, 244502 (2012)],

$$\hat{\rho}(t) \sim \exp\left[-\int d^3\Sigma_\mu(x)\left(\hat{T}^{\mu\nu}(x)b_\nu(x) - \frac{1}{2}\hat{J}^{\mu,\alpha\beta}(x)\omega_{\alpha\beta}(x) - \hat{N}^\mu(x)\xi(x)\right)\right].$$

$d^3\Sigma_\mu$ is an element of a space-like, three-dimensional hypersurface Σ_μ. We can take it as, $d^3\Sigma_\mu = (dV, 0, 0, 0)$. The operators $\hat{T}^{\mu\nu}(x)$, $\hat{J}^{\mu,\alpha\beta}(x)$ and $\hat{N}^\mu(x)$ are the energy-momentum, angular momentum and charge operators respectively.

In global thermodynamic equilibrium the operator $\hat{\rho}(t)$ should be independent of time.

$$\partial_\mu\left(\hat{T}^{\mu\nu}(x)b_\nu(x) - \frac{1}{2}\hat{J}^{\mu,\alpha\beta}(x)\omega_{\alpha\beta}(x) - \hat{N}^\mu(x)\xi(x)\right)$$
$$= \hat{T}^{\mu\nu}(x)\left(\partial_\mu b_\nu(x)\right) - \frac{1}{2}\hat{J}^{\mu,\alpha\beta}(x)\left(\partial_\mu\omega_{\alpha\beta}(x)\right) - \hat{N}^\mu(x)\partial_\mu\xi(x) = 0.$$

From above equation we can conclude that $\omega_{\alpha\beta} = \omega^0_{\alpha\beta}$, $\xi = \xi^0$, But

For asymmetric energy momentum tensor we must have, $\partial_\mu b_\nu$=0, $\Rightarrow b_\nu = b^0_\nu$.

For symmetric energy momentum tensor, $\partial_\mu b_\nu + \partial_\nu b_\mu = 0$, $\Rightarrow b_\nu = b^0_\nu + \delta\omega^0_{\nu\rho}x^\rho$.

Global equilibrium; particle with spin

Total angular momentum

$$\hat{J}^{\mu,\alpha\beta}(x) = \hat{L}^{\mu,\alpha\beta}(x) + \hat{S}^{\mu,\alpha\beta}(x).$$

Using above equation, we can write two cases discussed above can be expressed by a single form of the density operator

$$\hat{\rho}_{\rm EQ} \quad = \quad \exp\left[-\int d^3\Sigma_\mu(x)\left(\hat{T}^{\mu\nu}(x)\beta_\nu(x) - \frac{1}{2}\hat{S}^{\mu,\alpha\beta}(x)\omega^0_{\alpha\beta} - \hat{N}^\mu(x)\xi^0\right)\right].$$

For asymmetric energy-momentum tensor $\beta_\mu(x) = b^0_\mu + \omega^0_{\mu\gamma}x^\gamma$.
$\beta_\mu(x)$ is a Killing vector, $\omega_{\mu\gamma} = \omega^0_{\mu\gamma} = \varpi_{\mu\nu}$.

For symmetric energy-momentum tensor $\beta_\mu(x) = b^0_\mu + (\delta\omega^0_{\mu\gamma} + \omega^0_{\mu\gamma})x^\gamma$.
$\beta_\mu(x)$ is again a Killing vector, $\omega_{\mu\gamma} = \omega^0_{\mu\gamma} \neq \varpi_{\mu\nu}(= \delta\omega^0_{\mu\gamma} + \omega^0_{\mu\gamma})$.

Go Back

Local thermodynamic equilibrium; particle with spin

We define the statistical operator for local equilibrium by the same form as

$$\hat{\rho}_{\mathrm{eq}} = \exp\left[-\int d^3\Sigma_\mu(x)\left(\hat{T}^{\mu\nu}(x)\beta_\nu(x) - \frac{1}{2}\hat{S}^{\mu,\alpha\beta}(x)\omega_{\alpha\beta}(x) - \hat{N}^\mu(x)\xi(x)\right)\right].$$

We allow for arbitrary form of $\beta_\mu(x)$ [not a killing vector] and $\xi = \xi(x)$ and two cases for $\omega_{\mu\nu}$.

$\omega_{\mu\nu} = \varpi_{\mu\nu}$.

local equilibrium

$\omega_{\mu\nu} \neq \varpi_{\mu\nu}$.

extended local equilibrium

Global and local equilibrium – spinless particles

Boltzmann equation

$$p^{\mu}\partial_{\mu}f(x,p) = C[f(x,p)]$$

1. Satisfied exactly for free streaming.

2. Satisfied in global equilibrium:

via some constraint equations on the hydrodynamic parameters (μ, T, u_{μ}) used to specify the form of $feq(x,p)$.

3. Does not vanish in the local thermodynamic equilibrium:

constraints on the hydrodynamic parameters are found by adding further assumptions (for instance momentum moments, $\int dPp_{\mu}\ldots f(x,p)$, yielding the conservation laws for energy and momentum etc)

For free streamimg $= 0$

Global equilbrium $= 0$

Local equailibrium $= 0$

Global equilibrium and the Killing equation

The equilibrium distribution function has the form

$$f_{\mathrm{eq}}(x,p) \sim \exp\left[\xi(x) - \beta_\mu(x) p^\mu\right] \qquad \text{with} \qquad \beta_\mu = u_\mu(x)/T(x) \qquad \xi = \mu(x)/T(x).$$

It satisfies the LHS of Boltzmann equation *i.e.* $p^\mu \partial_\mu f_{\mathrm{eq}}(x,p) = 0$ via,

$$p^\mu \partial_\mu \xi + p^\mu p^\nu \partial_\mu \beta_\nu = p^\mu \partial_\mu \xi + \frac{1}{2} p^\mu p^\nu (\partial_\mu \beta_\nu + \partial_\nu \beta_\mu) = 0$$

One arrives at constraints

$$\partial_\mu \xi = 0$$

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0 \quad \text{Killing equation}$$

giving

$$\xi = \text{constant}$$

$$\beta_\mu(x) = \beta^0_\mu + \omega^0_{\mu\nu} x^\nu \qquad \text{with} \qquad \beta^0_\mu = \text{const}, \quad \omega^0_{\mu\nu} = -\omega^0_{\nu\mu} = \text{constant}.$$

Thermal vorticity is given by

$$\varpi_{\mu\nu} = -\frac{1}{2} (\partial_\mu \beta_\nu - \partial_\nu \beta_\mu) \equiv \omega^0_{\mu\nu}.$$

Motivation

Nuclei colliding at ultrarelativistic energies creates fireball of large orbital angular momentum $L_{init} \approx 10^5\hbar$ (RHIC Au-Au 200 GeV, b=5 fm) [F. Becattini, F. Piccinini and J. Rizzo, Phys. Rev. C77, 024906 (2008)].

Initially $J_{init} = L_{init}$, later some part of the angular momentum can be transferred from the orbital to the spin part $J_{final} = L_{final} + S_{final}$.

This may induce spin polarization, similar to magnetomechanical Barnett effect [S. J. Barnett, Rev. Mod. Phys. 7, 129 (1935)].

Emerging particles are expected to be globally polarized with their spins on average pointing along the system angular momentum.

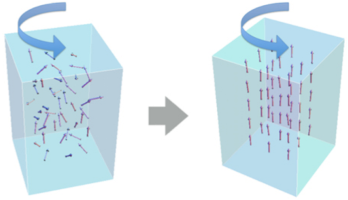


Figure: Mechanical rotation of an unmagnetized metallic object induces magnetization, an effective magnetic field emerges.

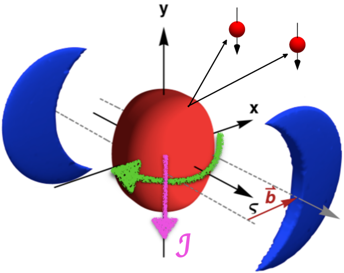
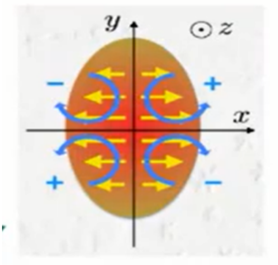


Figure: Polarization of particles in non-central heavy ion collision

Anisotropic expansion give rise to local vorticities in diffrent quadrant



[S.Voloshin, EPJ Web Conf. 171, 07002 (2018)]

[F. Becattini, I. Karpenko, PRL 120 (2018) no.1, 012302, [1707.07984]]

Spin polarization should have quadrupole structure in terms of transverse momentum.