# Euclidean path integral, entanglement entropy, and information loss paradox

Sasaki and DY, 1404.1565

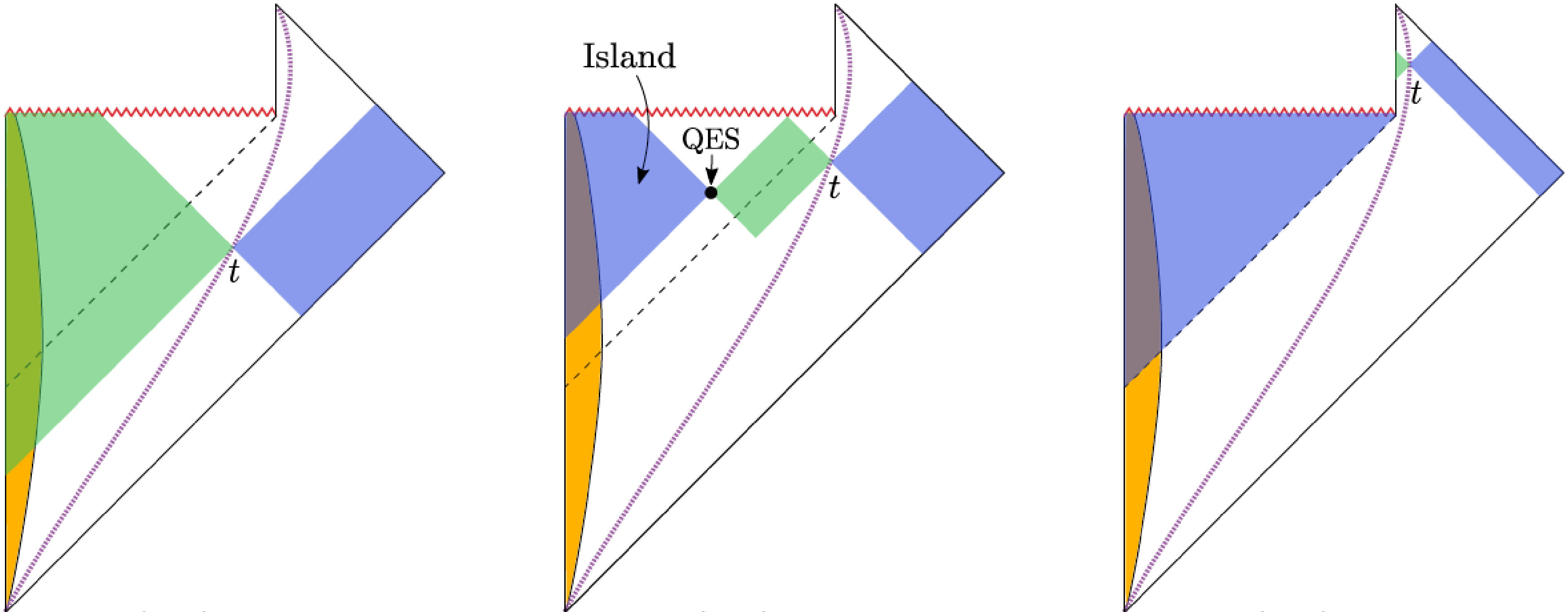
Chen, Sasaki and DY, 1806.03766

Chen, Sasaki and DY, 2005.07011

Brahma, Chen and DY, 2108.03593

Chen, Sasaki, DY and Yoon, 2111.01005

Dong-han Yeom 염동한
Department of Physics Education, Pusan National University 부산대학교 물리교육과
2021. 12. 24.

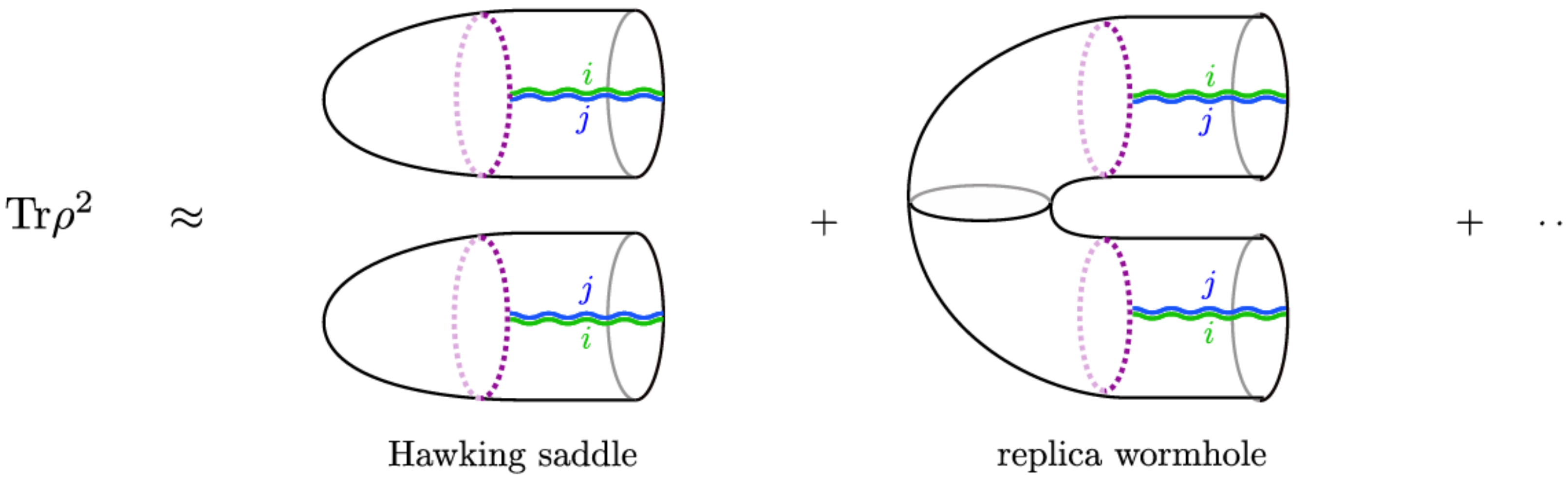



$t < t_{\text{Page}}$
(a)

$t > t_{\text{Page}}$
(b)

$t > t_{\text{Evap}}$
(c)

How can a black hole emit information
via **quantum extremal surfaces?**




The interpretation is very subtle because
this is a path-integral about **not** states **but** density matrices.

How can we embed this picture into the orthodox path–integral formulation?

# Let us follow the Euclidean path–integral approach.

## II. EUCLIDEAN QUANTUM GRAVITY

Black hole formation and evaporation can be thought of as a scattering process. One sends in particles and radiation from infinity and measures what comes back out to infinity. All measurements are made at infinity, where fields are weak and one never probes the strong field region in the middle. So one can't be sure a black hole forms, no matter how certain it might be in classical theory. I shall show that this possibility allows information to be preserved and to be returned to infinity.

I adopt the Euclidean approach [5], the only sane way to do quantum gravity nonperturbatively. One might think one should calculate the time evolution of the initial state by doing a path integral over all positive definite metrics that go between two surfaces that are a distance $T$ apart at infinity. One would then Wick rotate the time interval $T$ to the Lorentzian.

(Hawking, 2005)

$$|\boldsymbol{f}\rangle = \sum_j \boldsymbol{a_j}|\boldsymbol{f^j}\rangle$$

$$\langle \boldsymbol{f}|\boldsymbol{i}\rangle = \int_{\boldsymbol{i}\rightarrow \boldsymbol{f^j}} \boldsymbol{DgD\phi}\, \boldsymbol{e^{iS}}$$

path integral as a propagator

$$|\boldsymbol{f}\rangle = \sum_{j} \boldsymbol{a}_j |\boldsymbol{f}^j\rangle$$

$$\langle \boldsymbol{f}|\boldsymbol{i}\rangle = \int_{\boldsymbol{i}\to \boldsymbol{f}^j} \boldsymbol{D}\boldsymbol{g}\boldsymbol{D}\boldsymbol{\phi}\, \boldsymbol{e}^{-\boldsymbol{S}_{\boldsymbol{E}}}$$

Euclidean analytic continuation

$$|f\rangle = \sum_j a_j |f^j\rangle$$

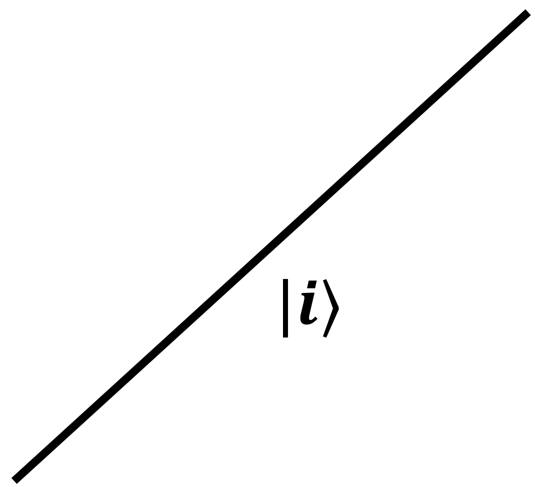
$$\langle f|i\rangle = \int_{i \to f^j} Dg D\phi \, e^{-S_E}$$

$$\cong \sum_{i \to f^j} e^{-S_E^{\text{on-shell}}}$$

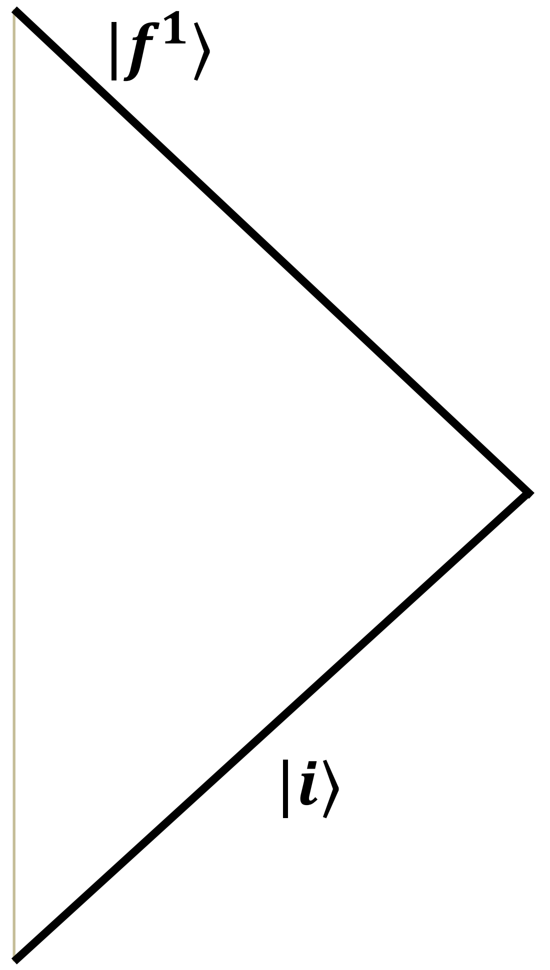
steepest-descent approximation

need to find/sum instantons

$$\langle f|i\rangle = \int_{i\to f^j} Dg D\phi \, e^{-S_E}$$



$$\langle f|i\rangle = \int_{i\to f^j} Dg D\phi \, e^{-S_E}$$



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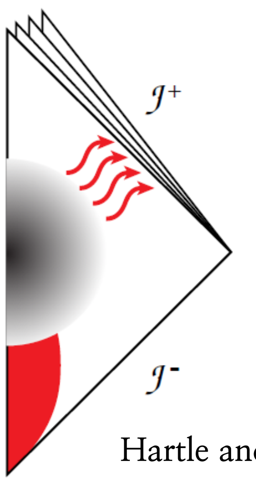
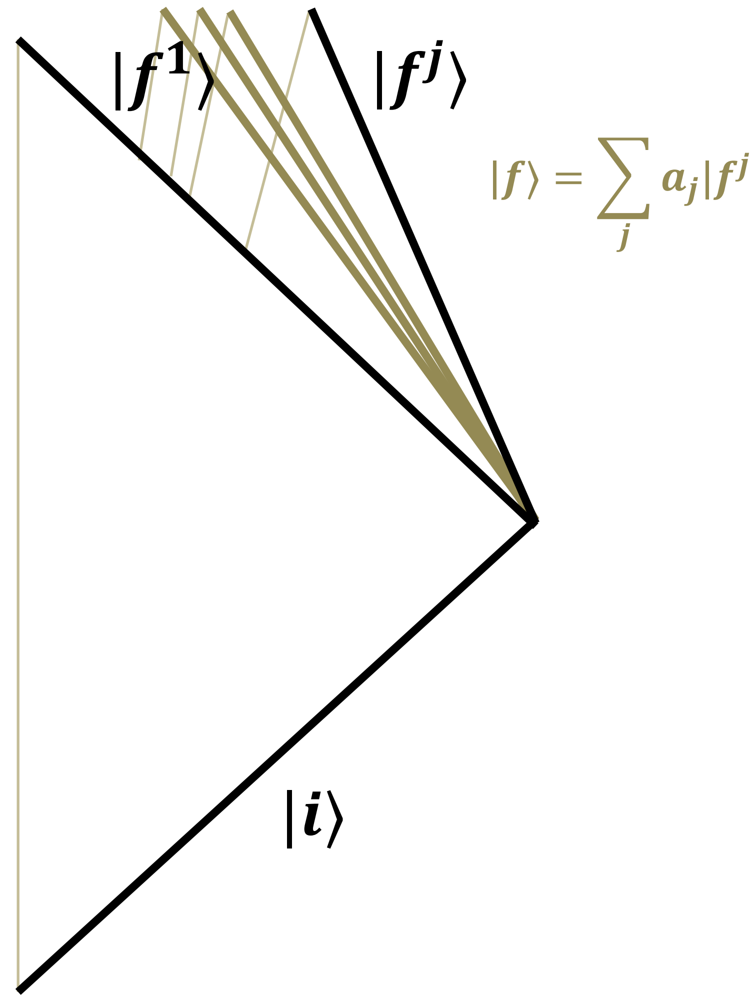
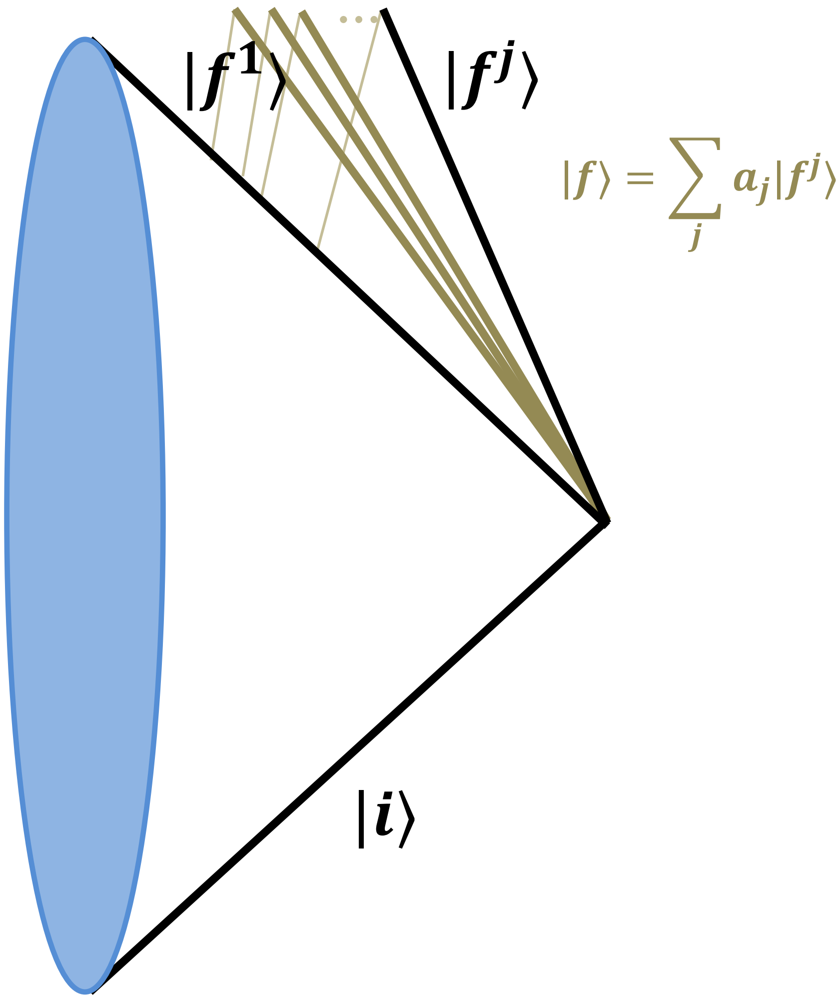


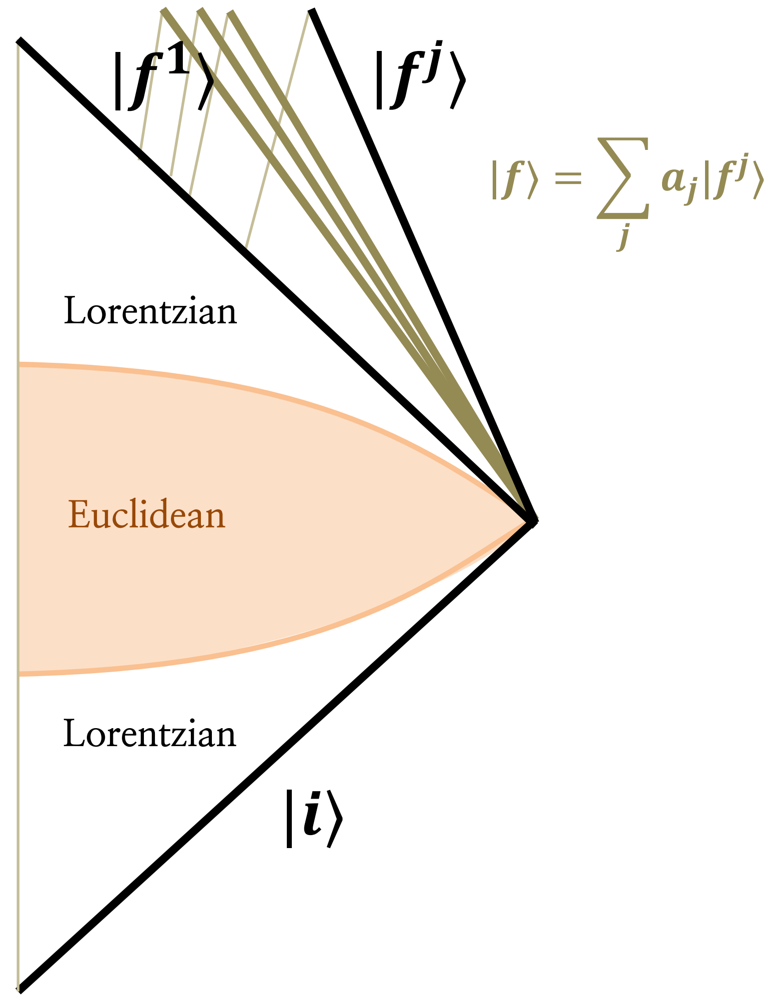

Hartle and Hertog, 2015



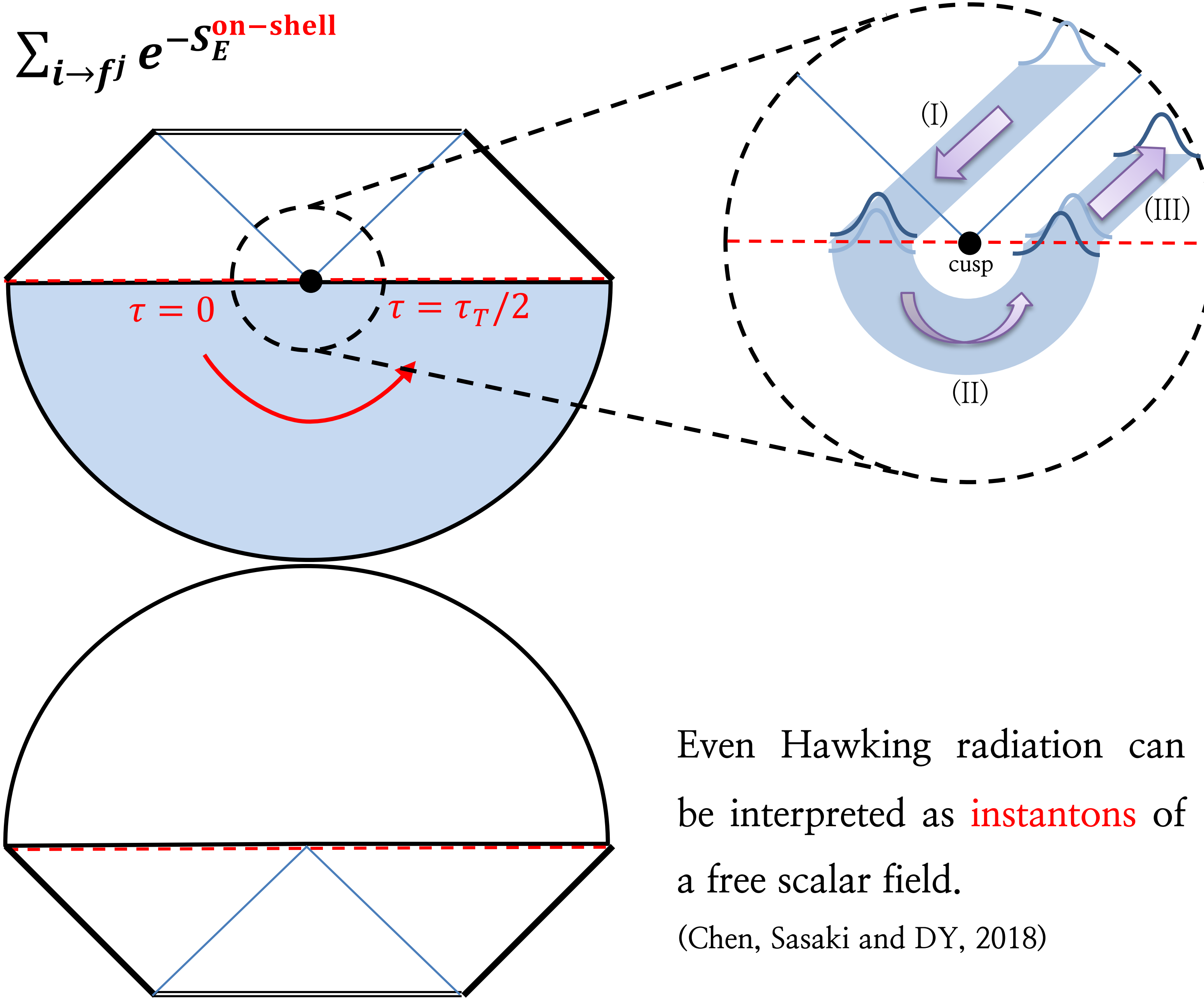
$$\langle f | i \rangle = \int_{i \to f^j} Dg D\phi \, e^{-S_E}$$



$$\langle f|i\rangle \cong \sum_{i\to f^j} e^{-S_E^{\text{on-shell}}}$$



$$\langle f|i\rangle \cong \sum_{i\to f^j} e^{-S_E^{\text{on-shell}}}$$




Even Hawking radiation can be interpreted as instantons of a free scalar field.

(Chen, Sasaki and DY, 2018)

After some computations, finally we can recover Hawking's result.

$$\Gamma \propto e^{-2B} \simeq e^{-8\pi M \delta M} \quad \text{if} \;\; \delta M \ll M$$

We further observe that there exist plenty of instantons with $\boldsymbol{\delta M/M \leq 1}$.

In the limit $\boldsymbol{\delta M = M}$, we obtain the trivial geometry,

where one can guarantee the existence of instantons.

Now, can we explain the Page curve
using the Euclidean path-integral approach?

Indeed, the computations of the Page curve are justified by the following two steps.

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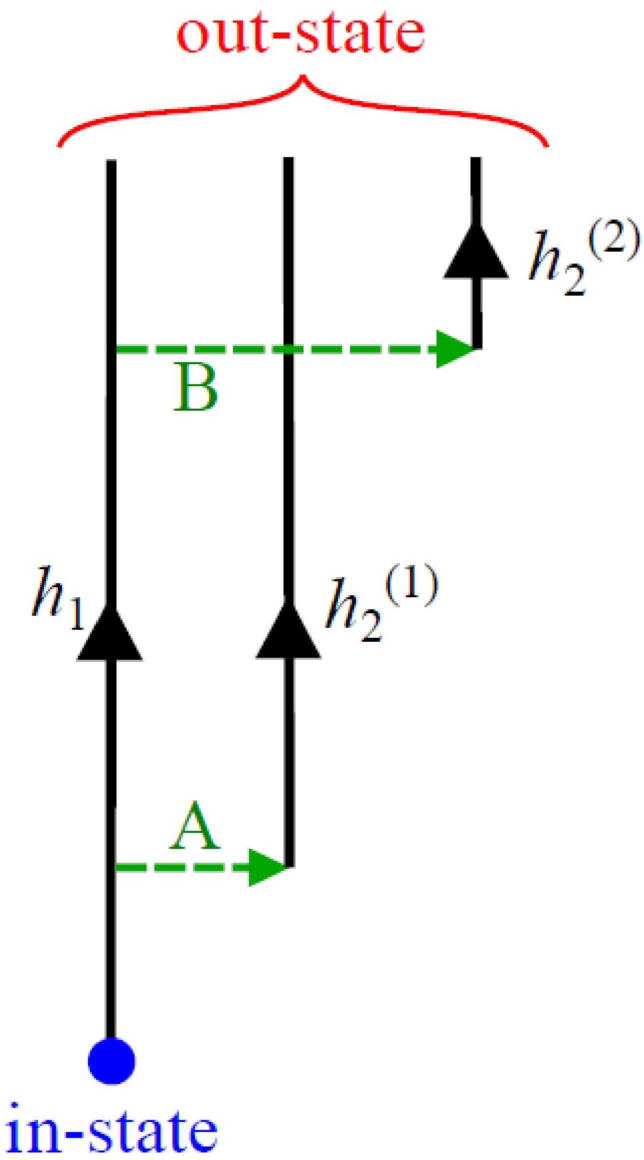
$$S \simeq p_1 S_1 + p_2 S_2$$

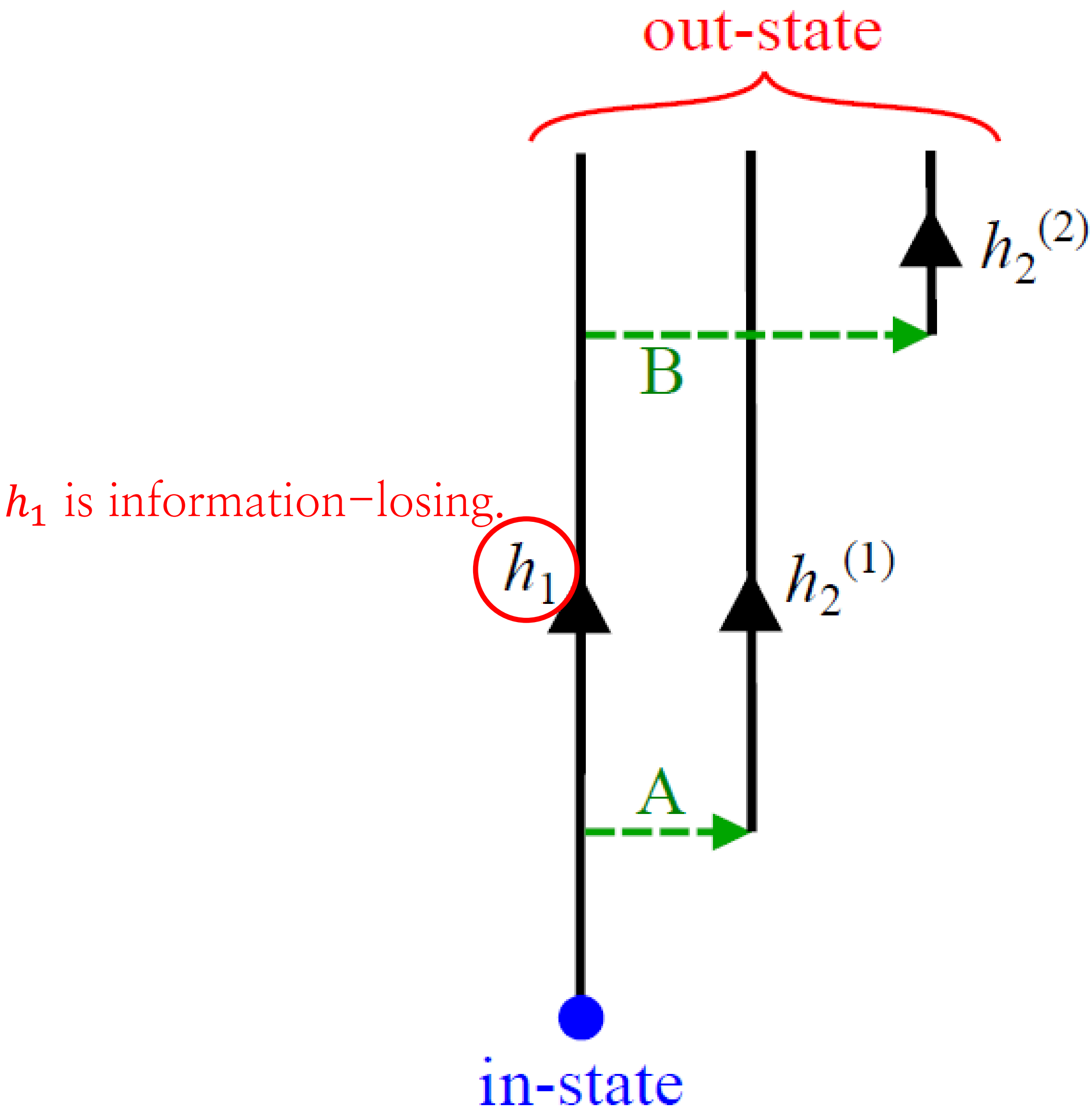
Indeed, the computations of the Page curve are justified by the following two steps.

Second, the probability of the information−preserving history is dominant at the late time.

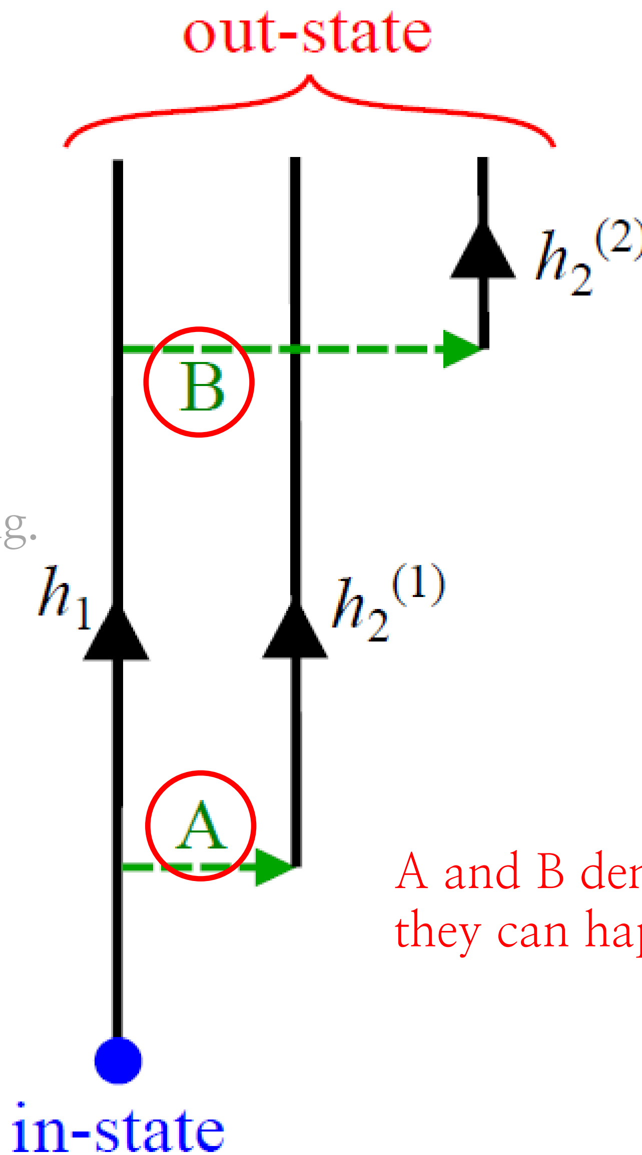
Then, one can reproduce the Page curve that we wanted.

$$S \simeq p_1 S_1 + p_2 S_2$$

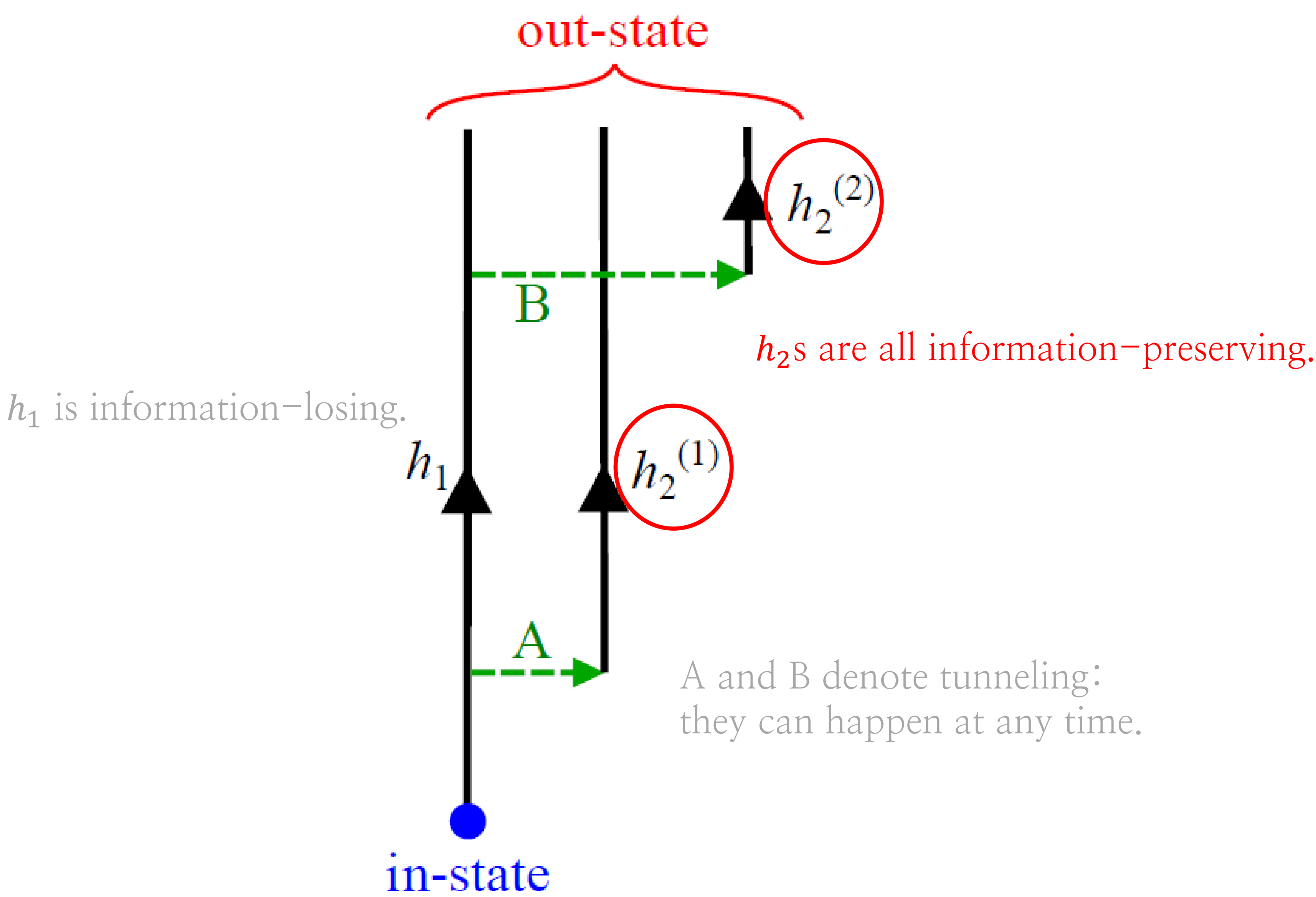

out-state
$h_2^{(2)}$
B
$h_1$
$h_2^{(1)}$
A
in-state

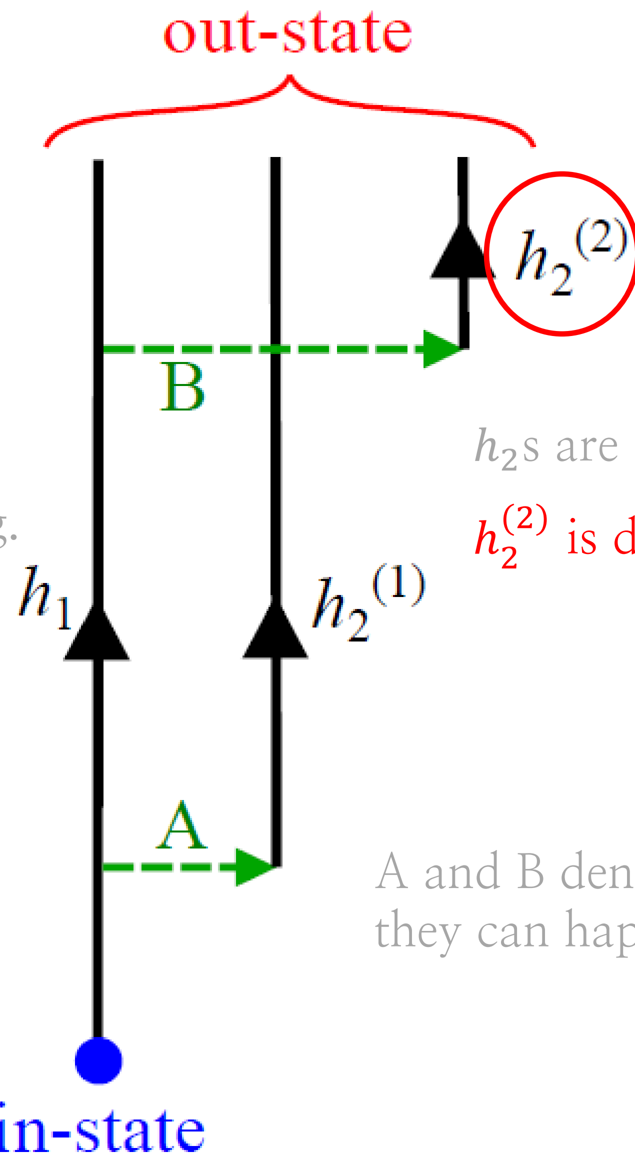

out-state
$h_2^{(2)}$
B
$h_1$ is information−losing.
$h_1$
$h_2^{(1)}$
A
in-state

$h_1$ is information−losing.




A and B denote tunneling:
they can happen at any time.


out-state
$h_2^{(2)}$
B
$h_2$s are all information−preserving.
$h_1$ is information−losing.
$h_1$
$h_2^{(1)}$
A
A and B denote tunneling:
they can happen at any time.
in-state




$h_2$s are all information−preserving.

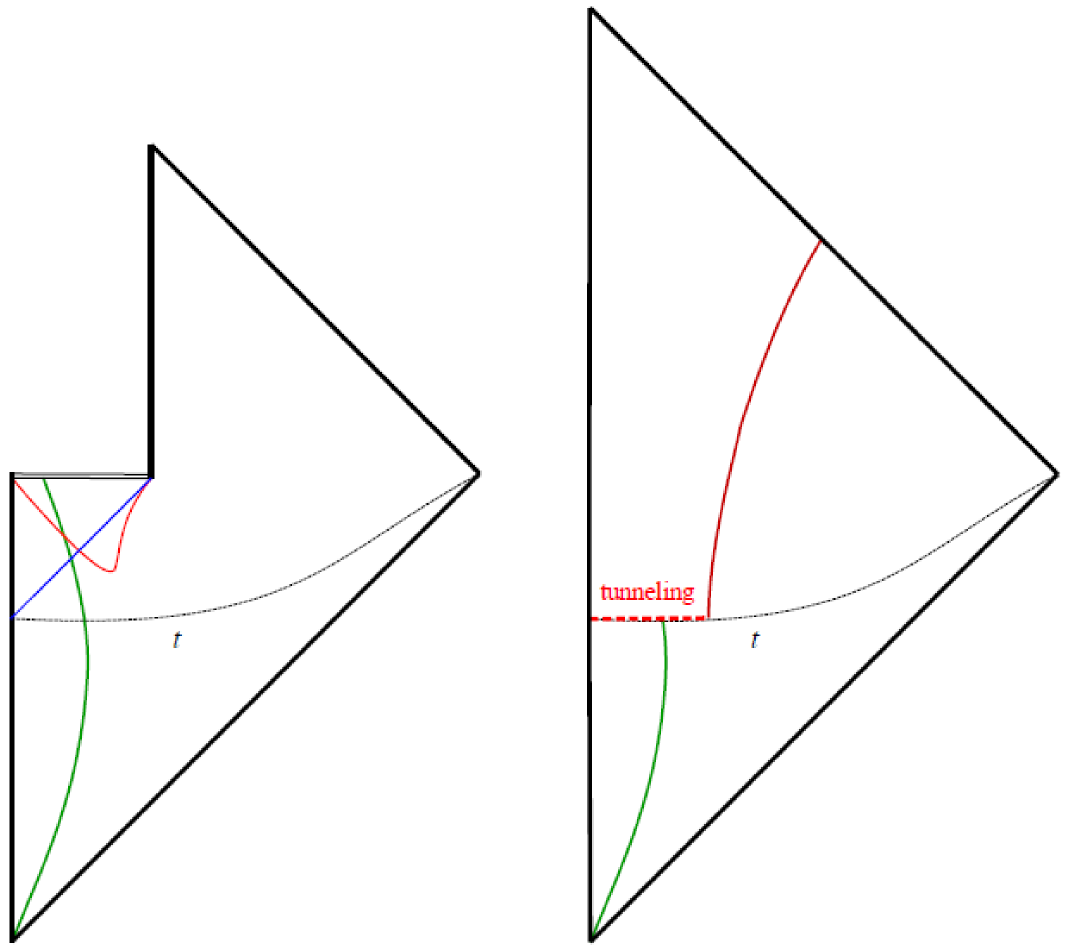
$h_2^{(2)}$ is dominated at late time.

$h_1$ is information−losing.

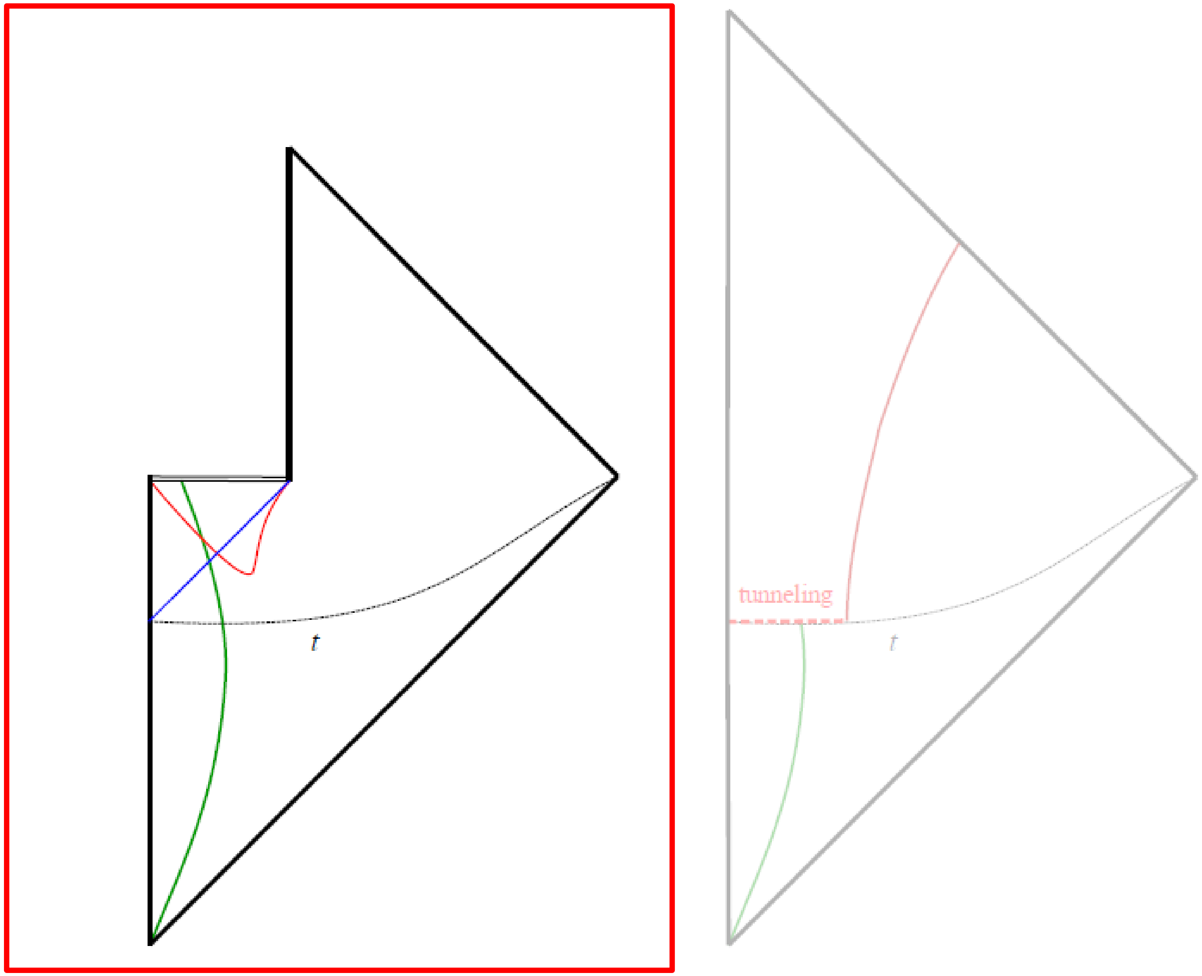
A and B denote tunneling:
they can happen at any time.

To summarize,
if we assume (1) multi-history condition
and (2) late-time dominance condition,
one can explain the Page curve.

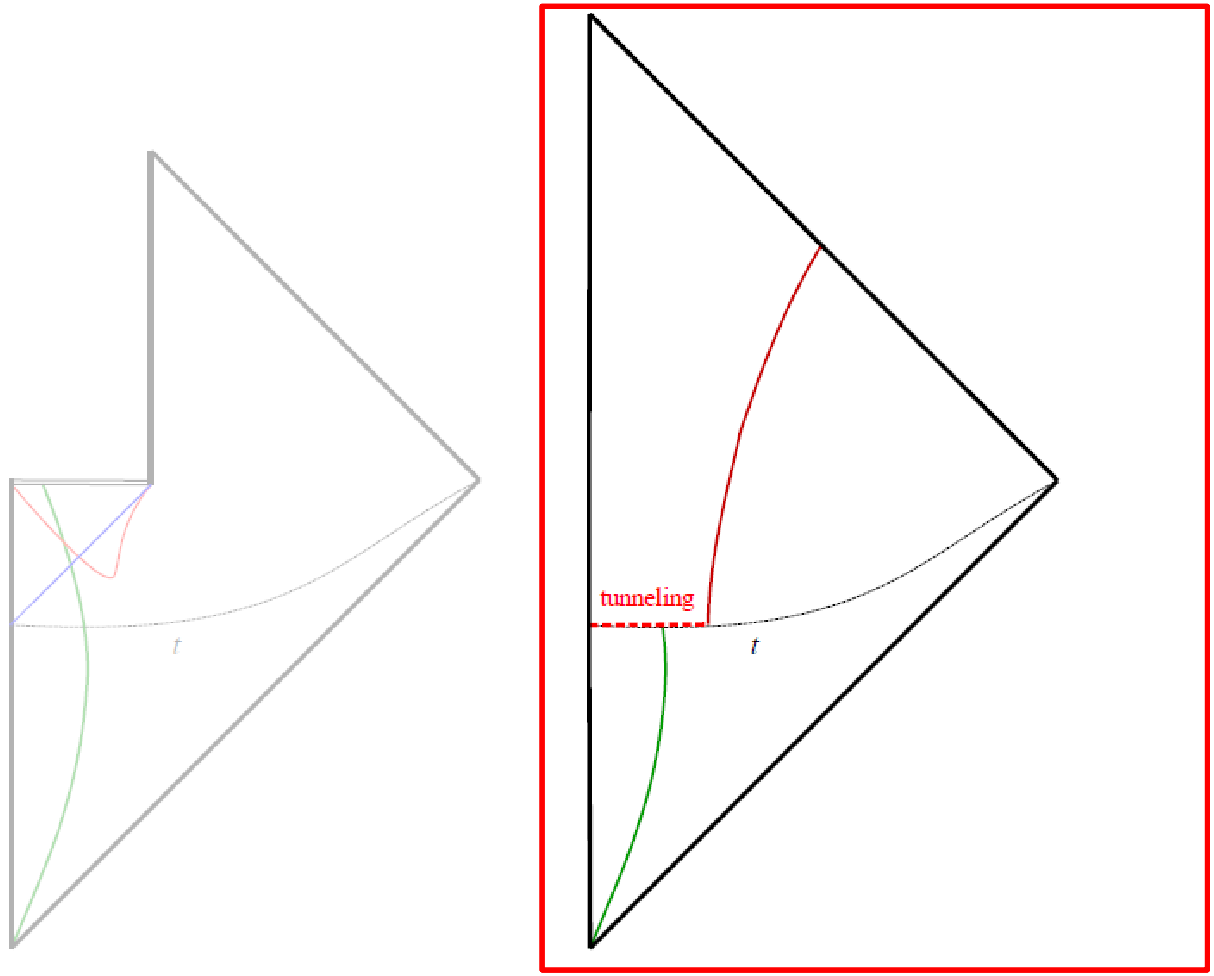
Can it be realized?



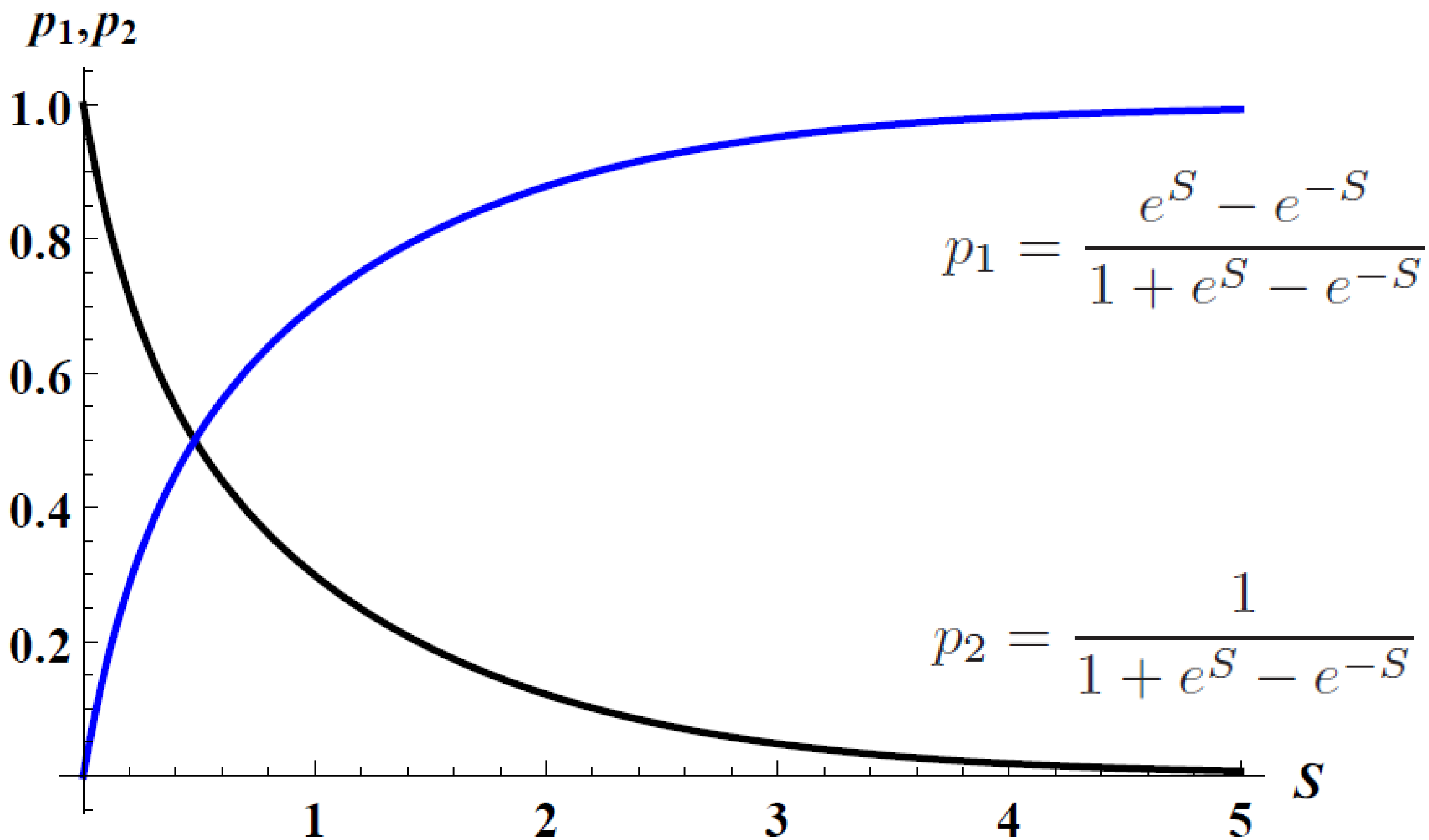

There exists a tunneling channel toward a trivial geometry thanks to the plenty of instantons.



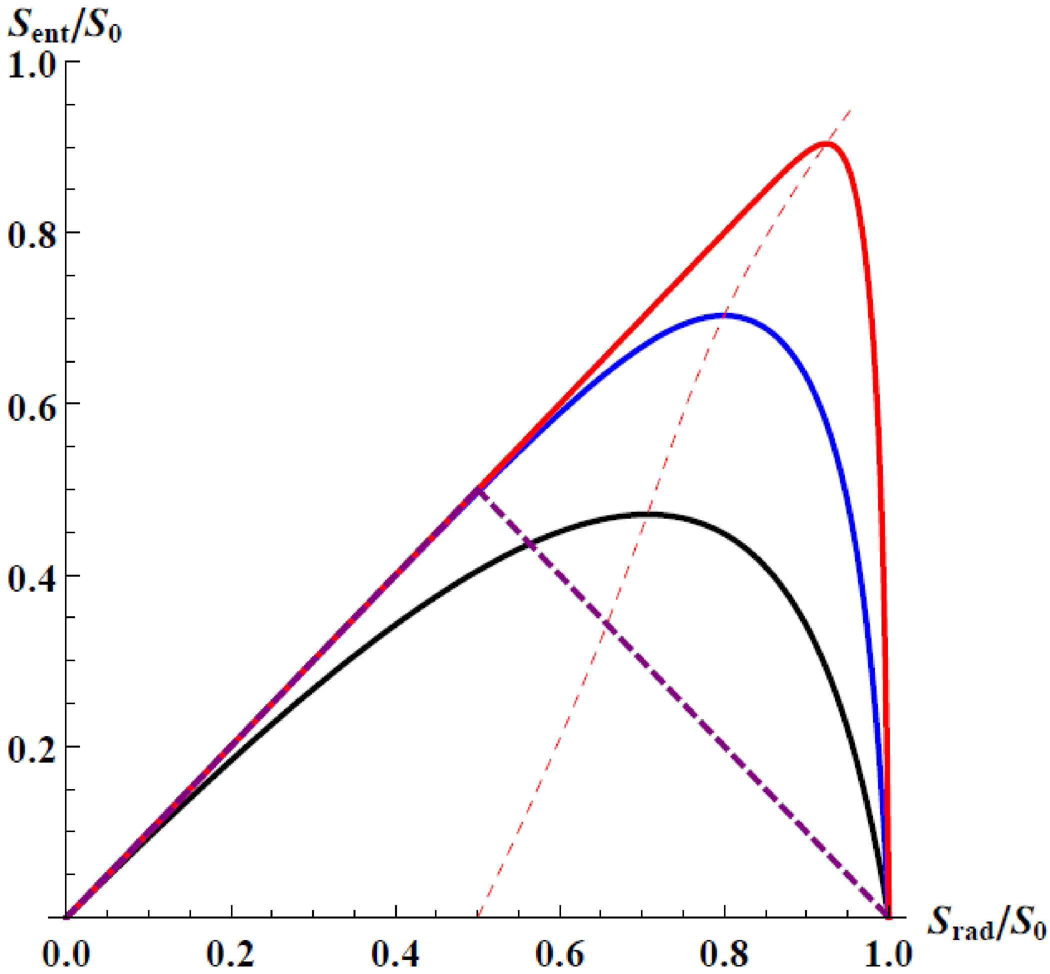

For the first history, we assume that the entanglement entropy monotonically increases.




However, since there is no interior for the second history, the entanglement entropy (between black hole and radiation) is zero.

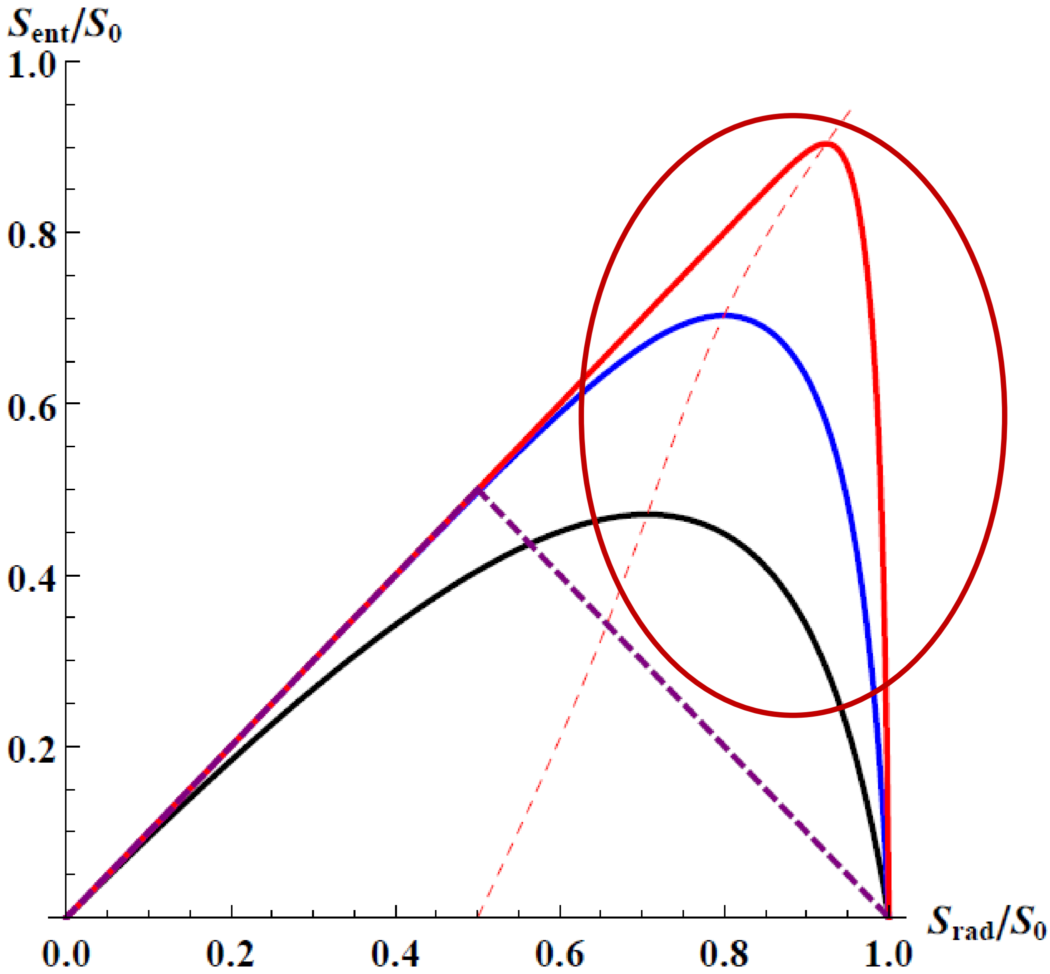



The probability to tunnel to a trivial geometry is dominated at the (very) late time (after some computations).




$$S \simeq p_1 S_1 + p_2 S_2$$

Therefore, one can **mimic** a Page curve,
while this is nothing to the with the quantum extremal surfaces,
but only relying on the Euclidean path integral.




$$S \simeq p_1 S_1 + p_2 S_2$$

One interesting point is that this allows a moment when the Boltzmann entropy is greater than its areal entropy. This might be a small remnant or a monster.

Is this sufficient?

Isn't it strange,
if the information loss paradox is resolved
without resolving a singularity?

# The Wheeler–DeWitt equation

$$\hat{\mathcal{H}}\Psi = 0$$

quantum Hamiltonian constraint

wave function of field space

Let us study the quantum gravitational wave function inside a black hole.

$$ds^2 = -N^2(t)dt^2 + e^{2X(t)}dR^2 + r_s^2 e^{2(Y(t)-X(t))}d\Omega_2^2$$

One way to present a metric inside a black hole

$$X(t) = \log \tan \frac{t}{2}$$

$$Y(t) = \log \frac{1}{2} \sin t$$

A classical solution inside a Schwarzschild black hole.

$$ds^2 = -N^2(t)dt^2 + e^{2X(t)}dR^2 + r_s^2 e^{2(Y(t)-X(t))}d\Omega_2^2$$

One way to present a metric inside a black hole

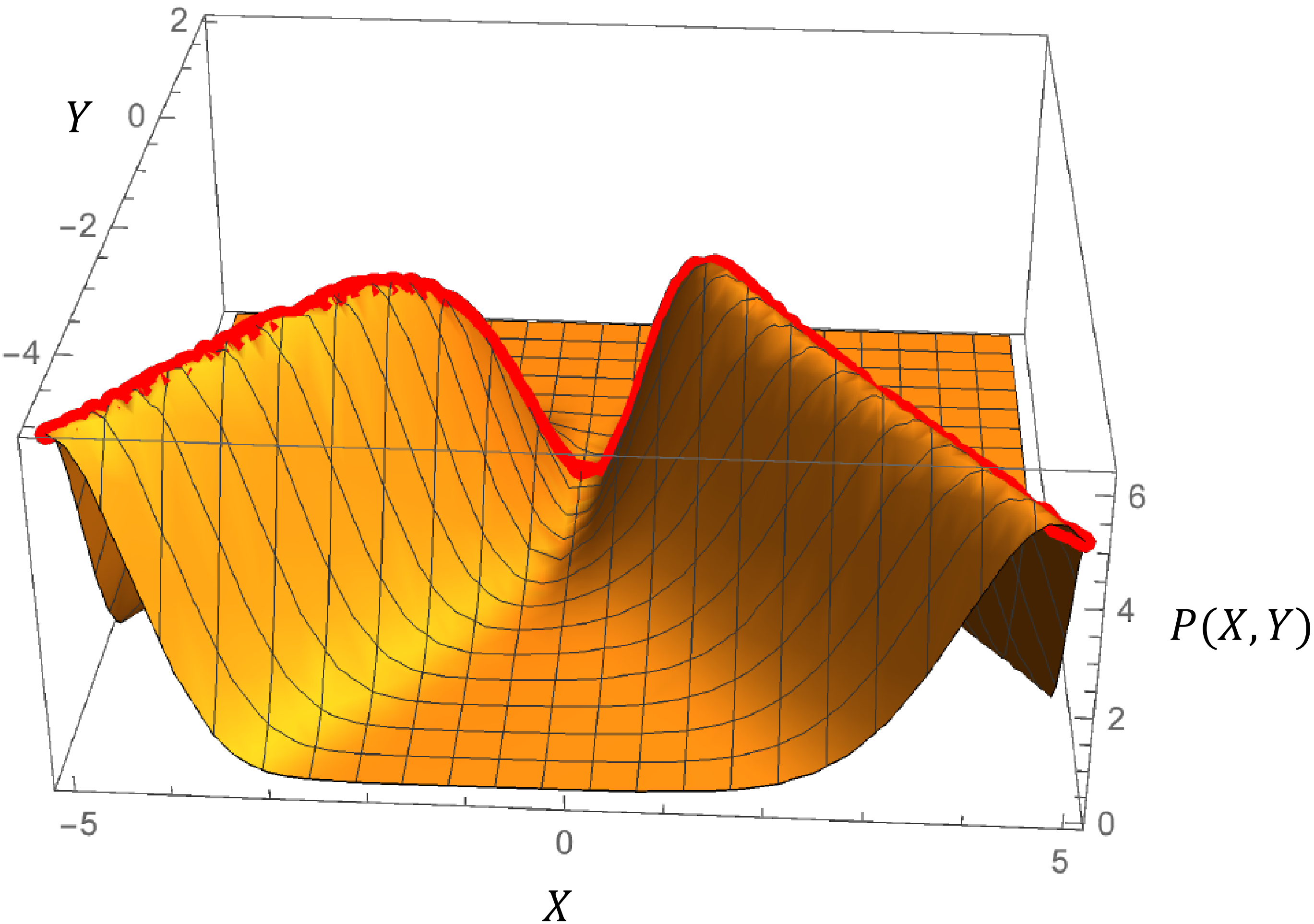
$$\left(\frac{\partial^2}{\partial X^2} - \frac{\partial^2}{\partial Y^2} + 4r_s^2 e^{2Y}\right)\Psi[X,Y] = 0$$

The Wheeler−DeWitt equation presented by $X$ and $Y$.
This was also known previously,
e.g., gr−qc/9411070, hep−th/0107250, etc.

$$\Psi[X,Y] = \int_{-\infty}^{\infty} f(k)\, e^{-ikX} I_{ik}(2r_s e^{Y}) dk$$

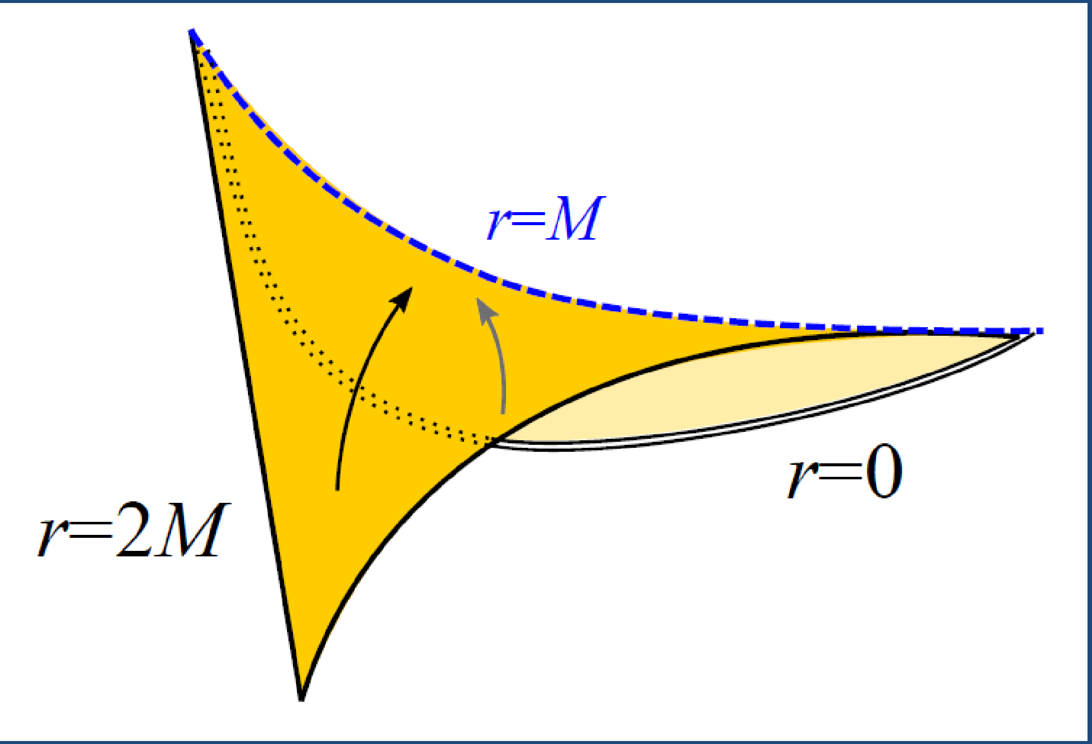
$$f(k) = \frac{2Ae^{-\sigma^2 k^2/2}}{\Gamma(-ik) r_s^{ik}}$$

We will impose the boundary condition such that
the wave function as a (Gaussian) peak at the event horizon,
because it is reasonable to assume that
the solution is classical at the horizon.

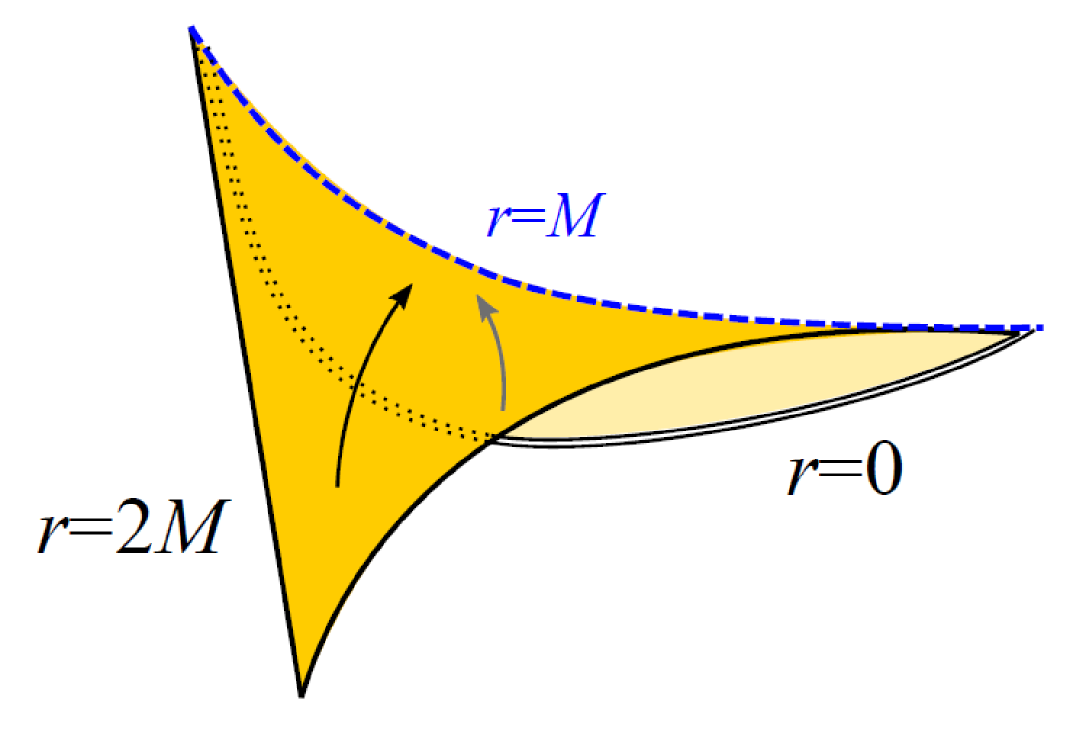



This steepest−descent coincides well with the classical trajectory.

$$Y = -\log(e^{X} + e^{-X})$$




Annihilation to nothing.




Also, a possible realization of the DeWitt boundary condition.

**No Future in Black Holes**

Malcolm J. Perry*
*School of Physics and Astronomy, Queen Mary University of London, Mile End Road, London E1 4NS, UK*†
(Dated: June 18, 2021)

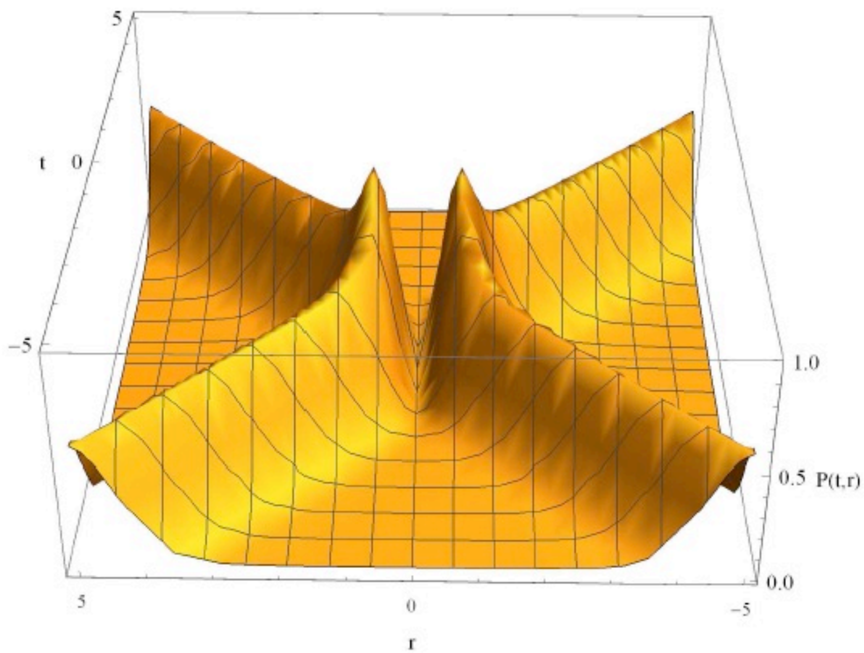
The black hole information paradox has been with us for some time. We outline the nature of the paradox. We then propose a resolution based on an examination of the properties of quantum gravity under circumstances that give rise to a classical singularity. We show that the gravitational wavefunction vanishes as one gets close to the classical singularity. This results in a future boundary condition inside the black hole that allows for quantum information to be recovered in the evaporation process.

This might be more generic than we expected.

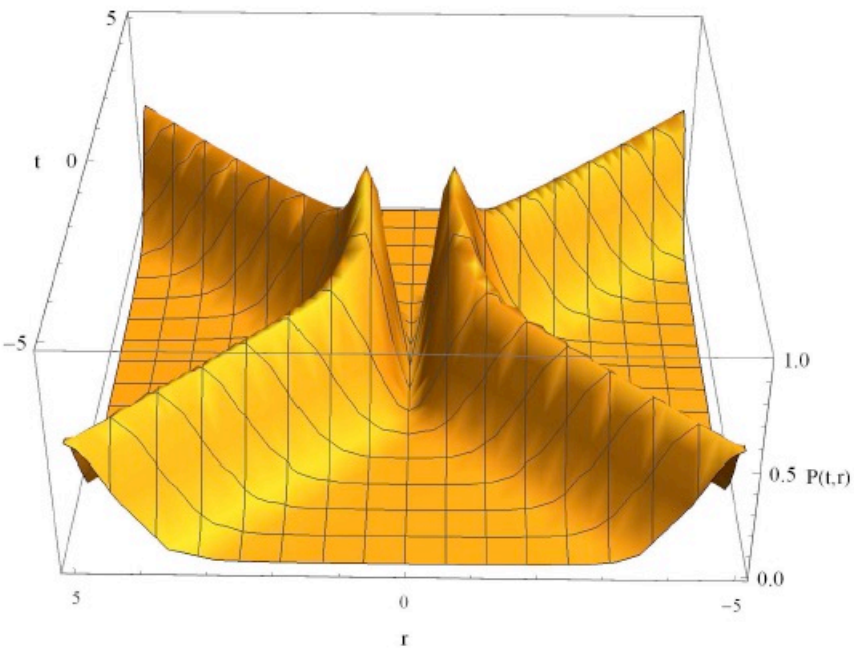
Can we extend to dynamical cases?

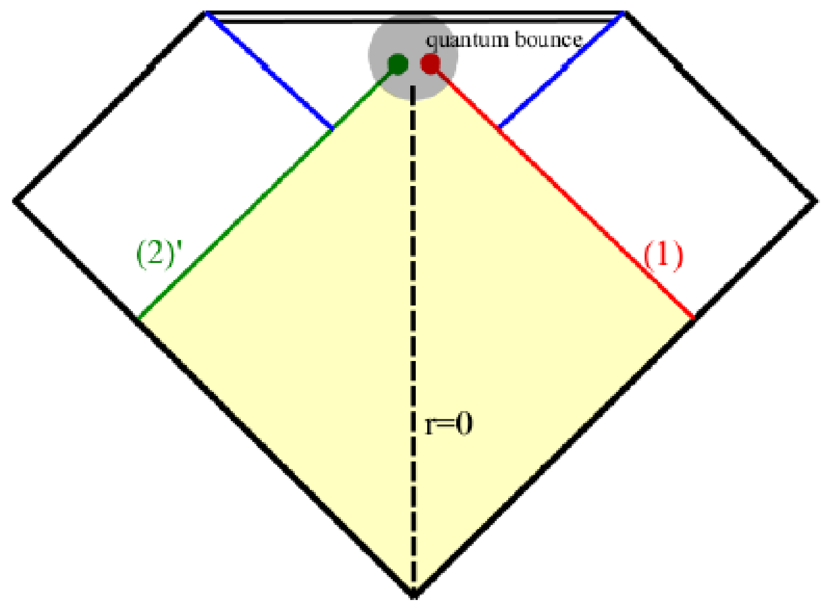
$$S = \int d\tau \, (p_u \dot{u} + p_v \dot{v} - n p_u p_v)$$

$$\Psi(t, r) = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{dp}{\sqrt{p}} \, \phi(p) \, e^{-ipt} \, \sin rp$$

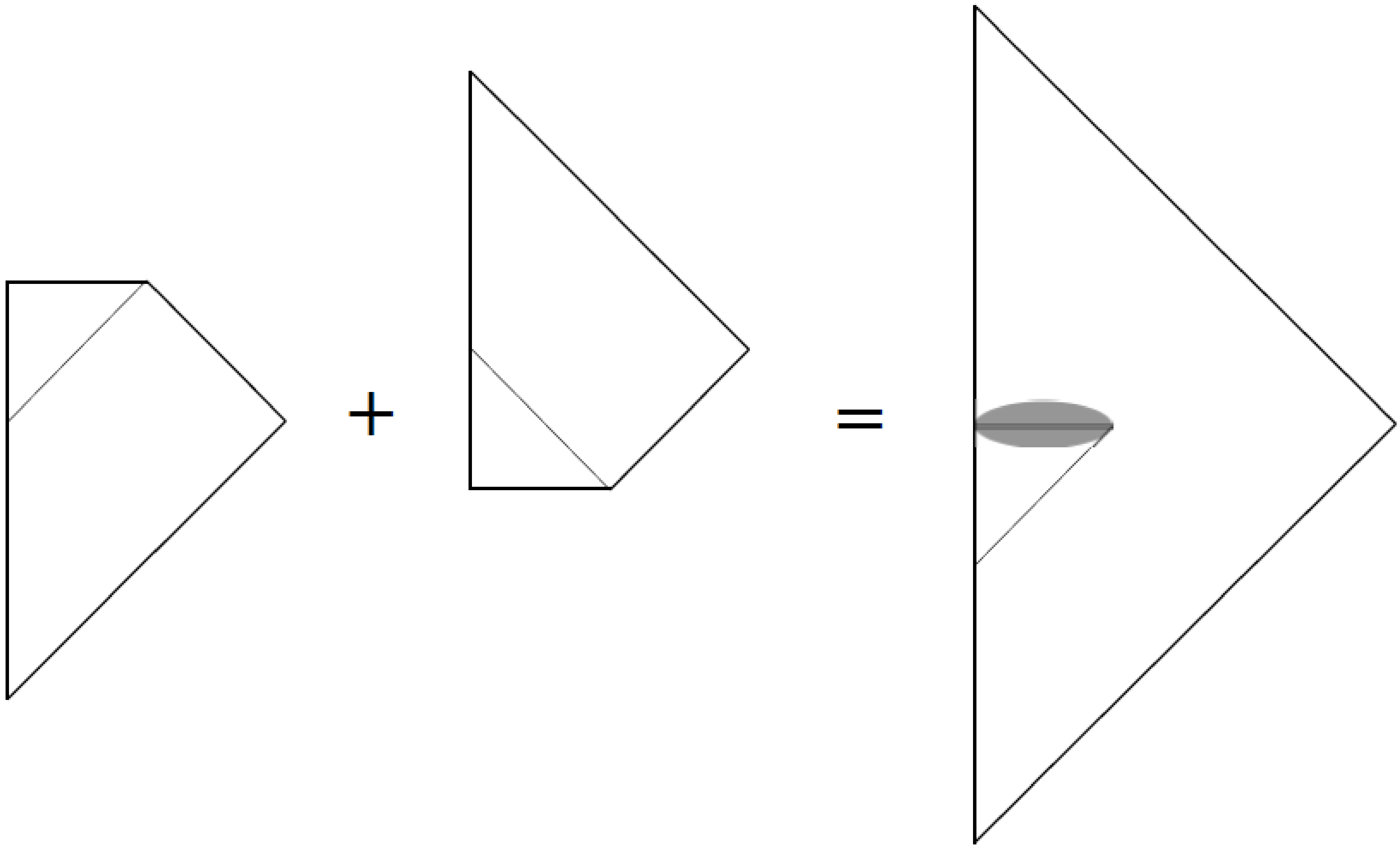



Hajicek–Kiefer model:
quantized thin–shell collapse model

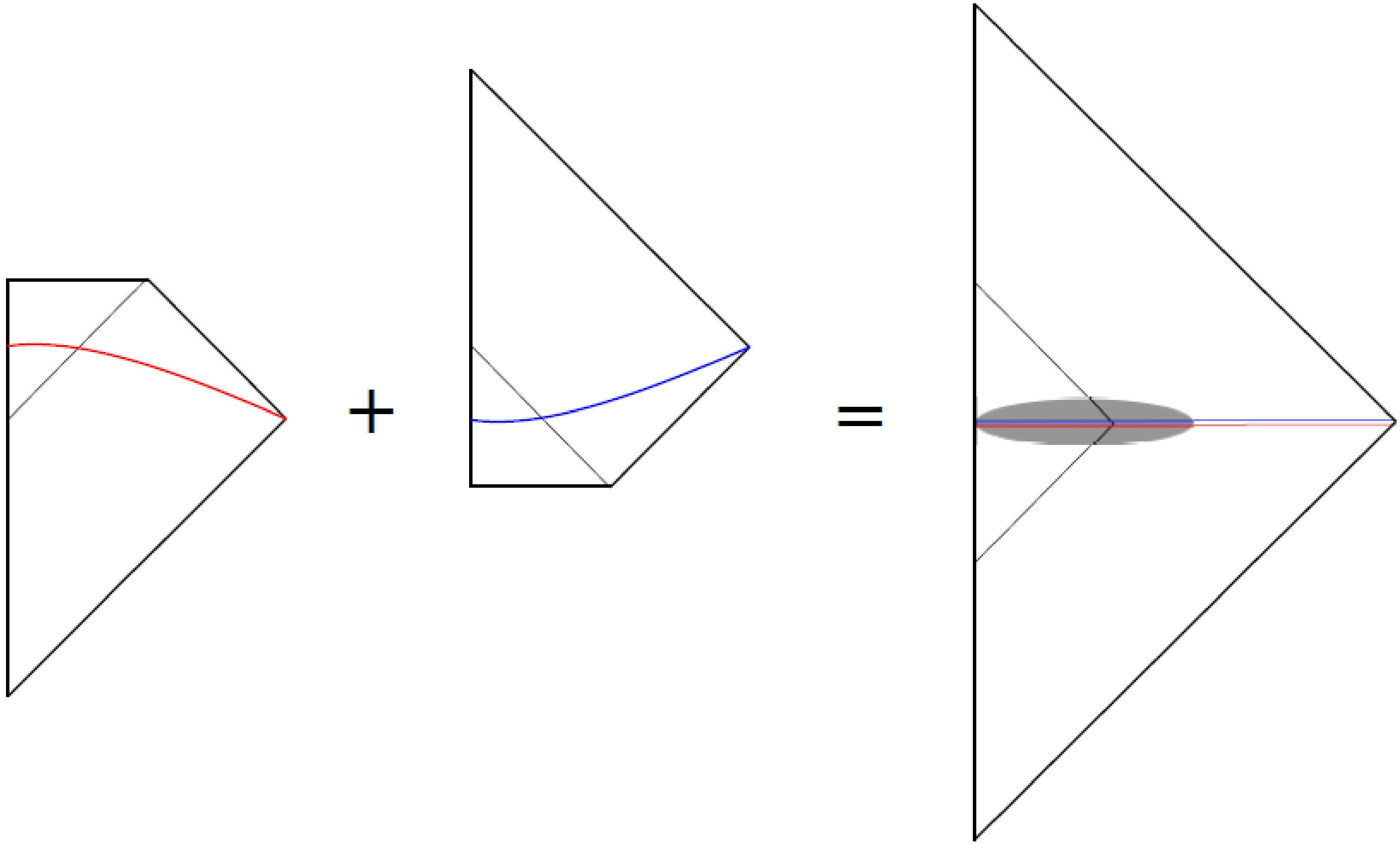


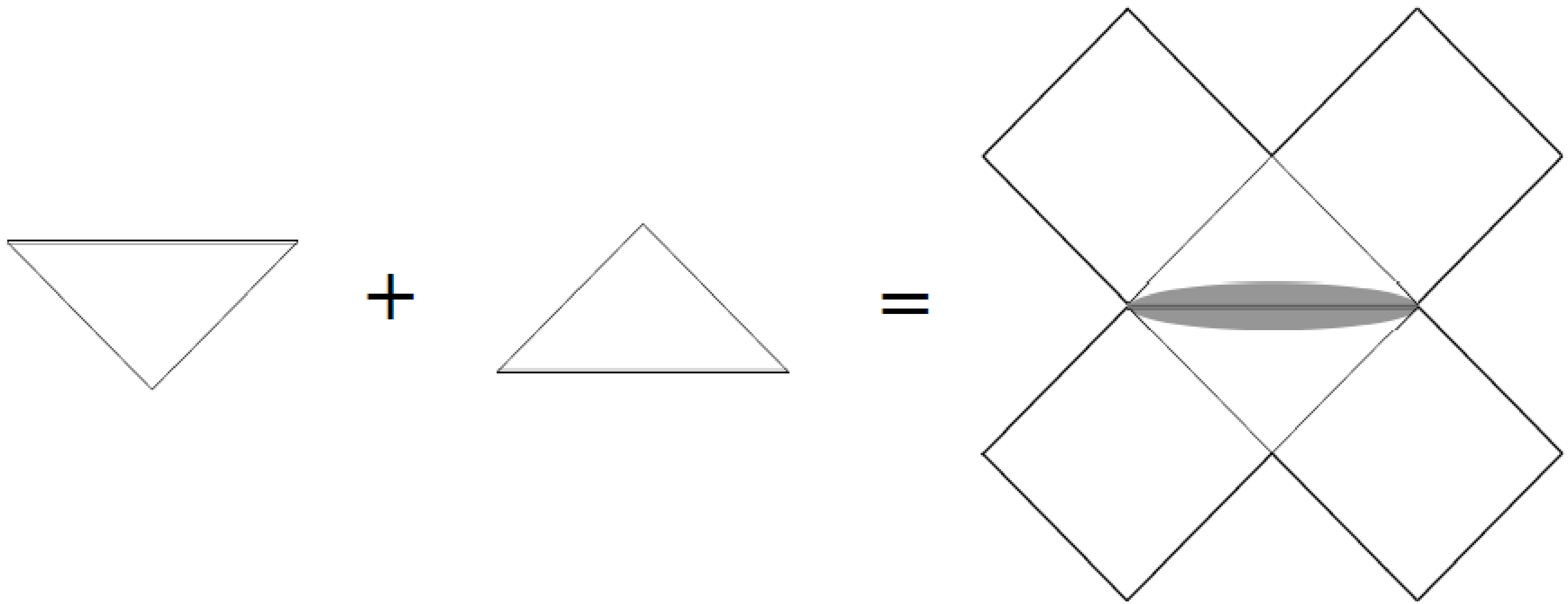

Correct interpretation



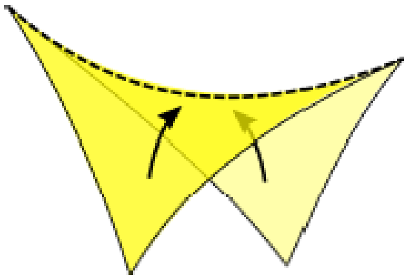
Paradigms of LQG black holes (1):
Ashtekar–Bojowald

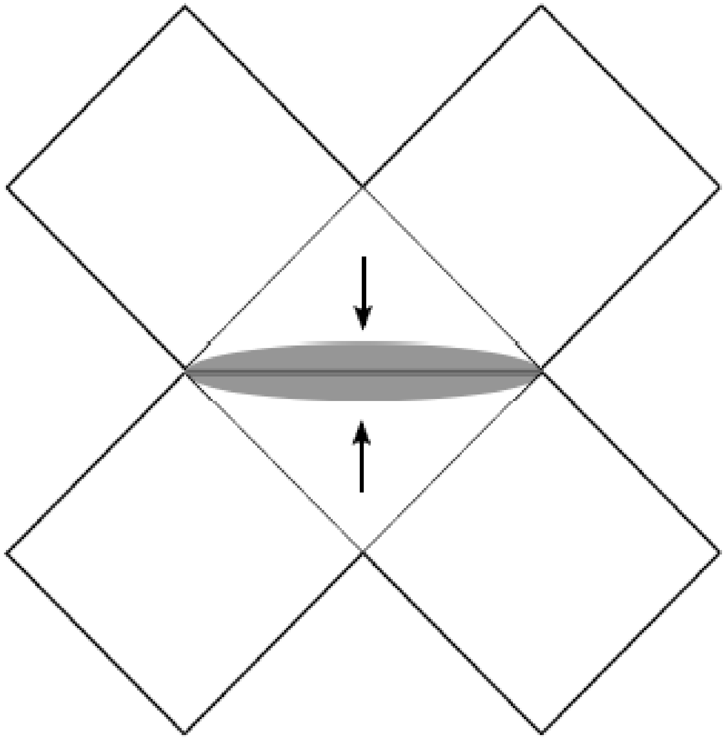


Paradigms of LQG black holes (2):
Haggard–Rovelli



Paradigms of LQG black holes (3):
Ashtekar–Olmedo–Singh





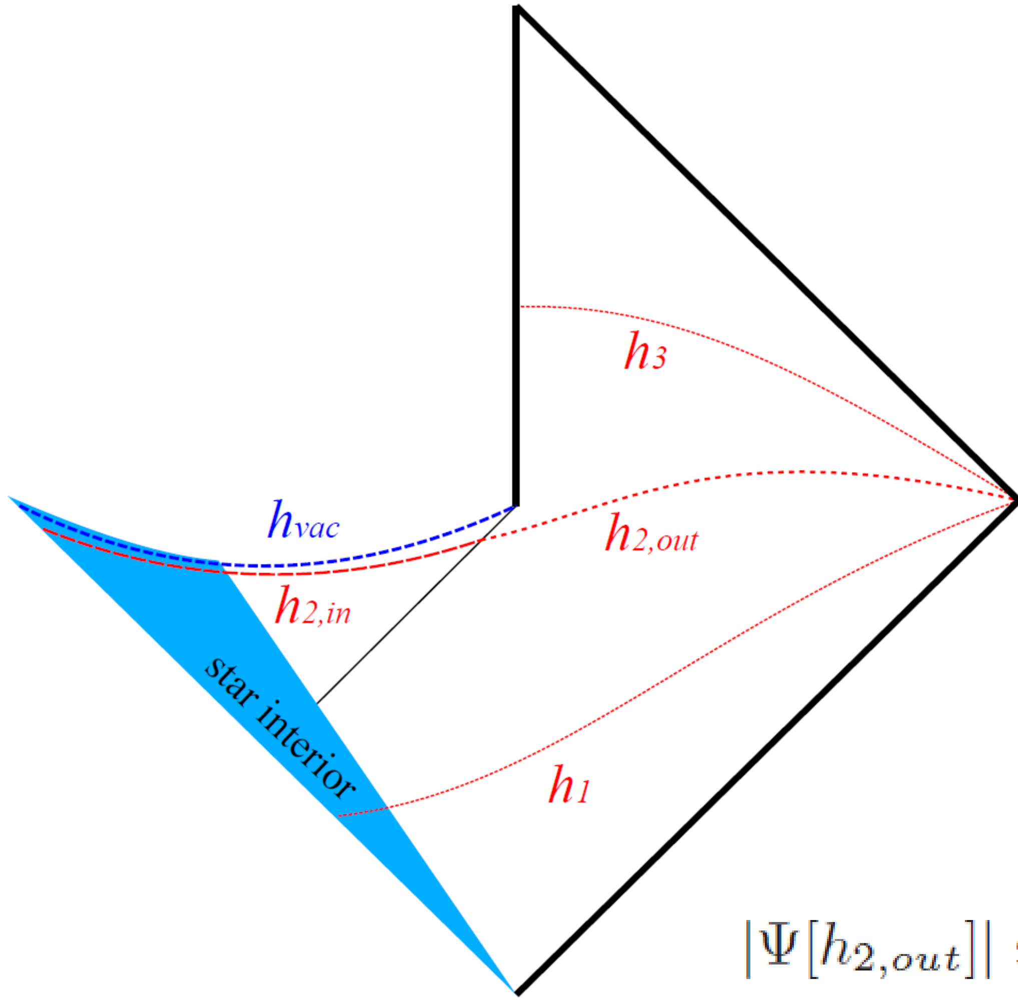
Annihilation-to-Nothing interpretation

for gravitational collapsing cases.

This provides new interpretation for LQG black holes.

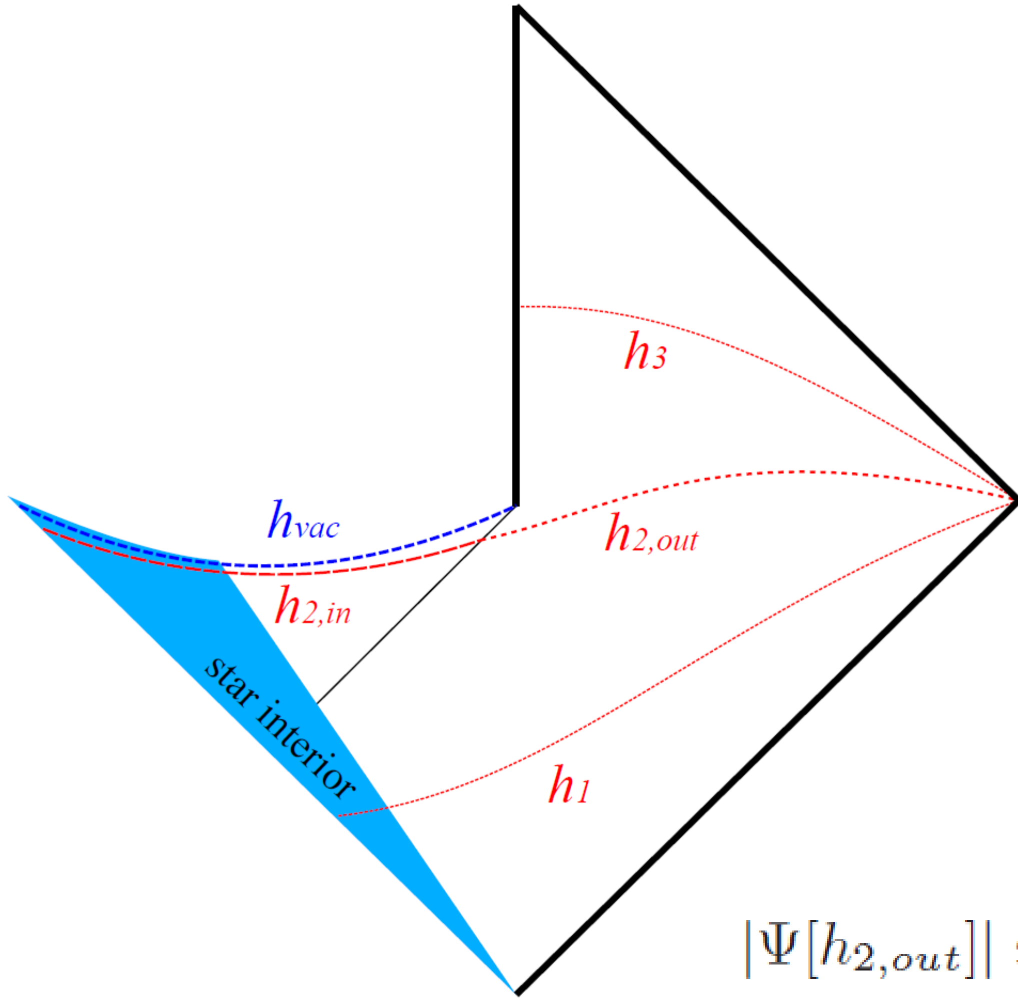
Time−reversal symmetry was the guiding principle.
Indeed, this is related to the DeWitt boundary condition.

What is the correct physical meaning of the DeWitt boundary condition?




$$|\Psi[h_{2,out}]| \simeq |\Psi[h_3]|$$

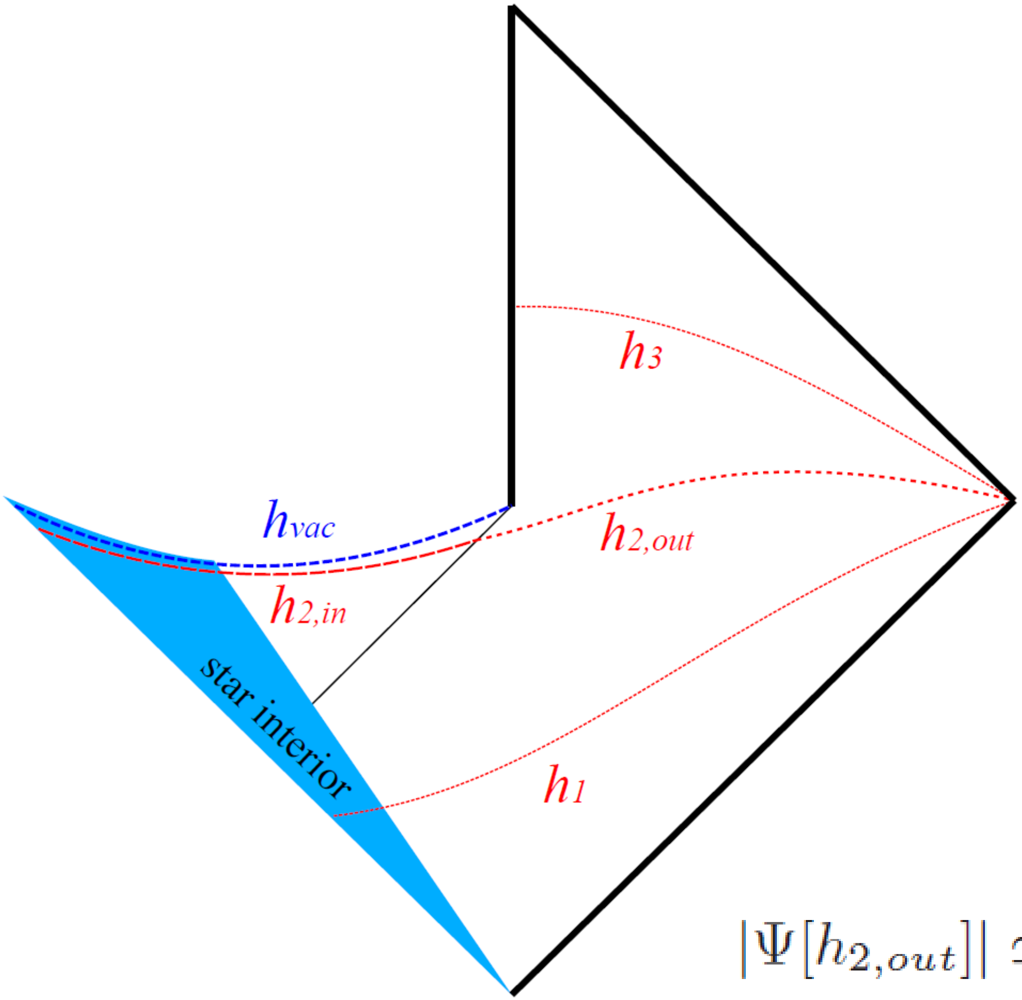
Now let us see both of inside and outside.
If we only focus outside, then it is semi−classical and the probability of each slice will not vary.




$$|\Psi[h_{2,out}]| \simeq |\Psi[h_3]|$$

$$\Psi[h_{vac} \cup h_3] \to 0$$

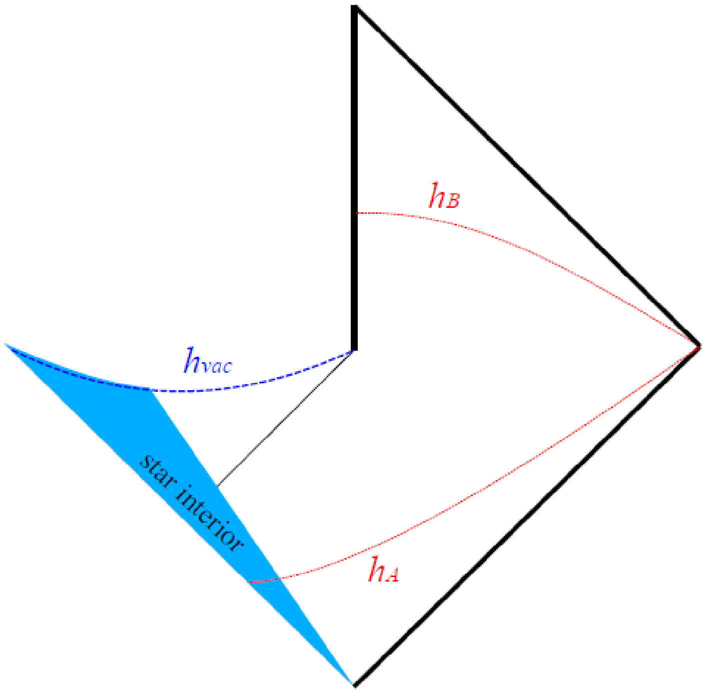
However, if we evaluate the probability of outside and inside together, it will approach to zero.



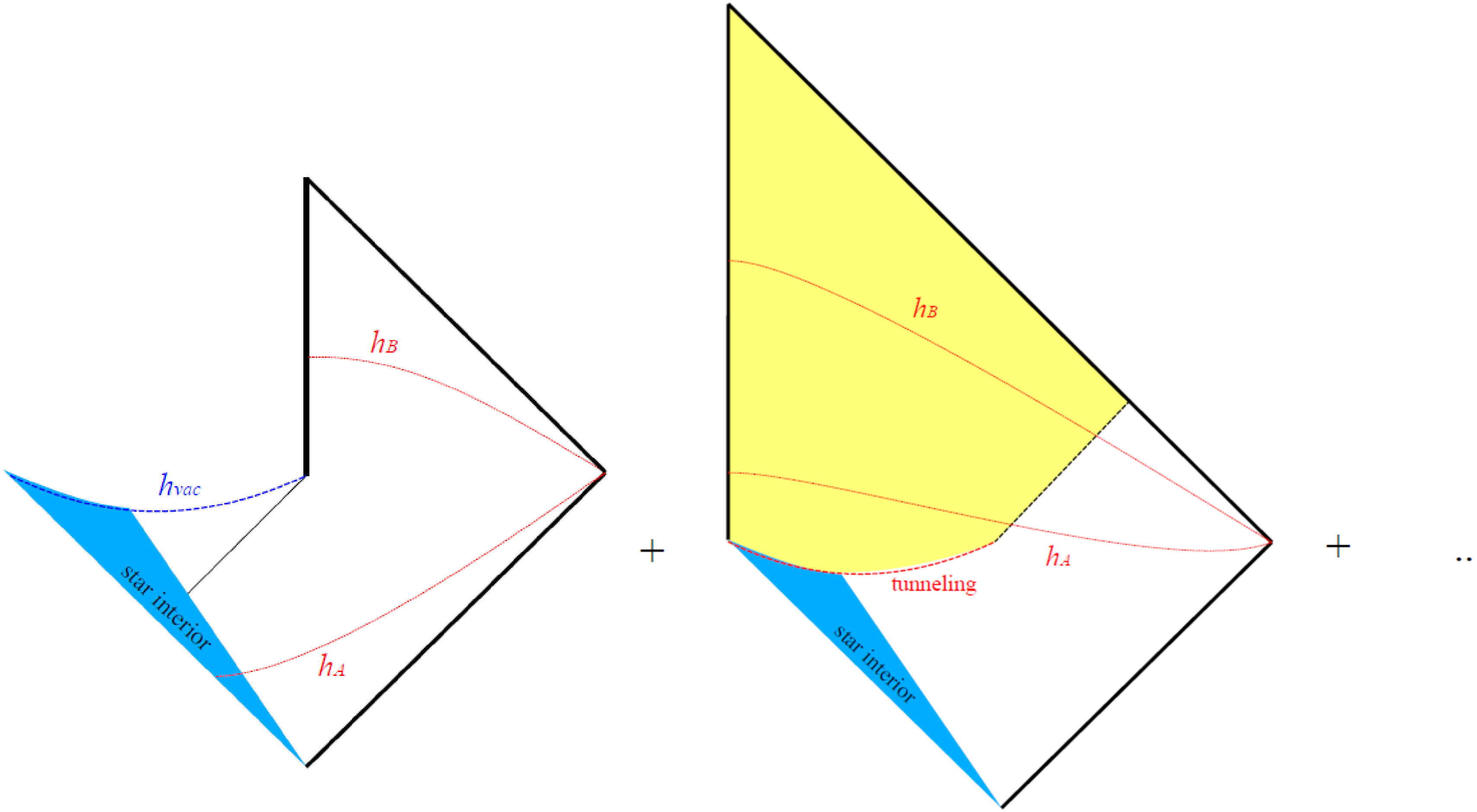

$$|\Psi[h_{2,out}]| \simeq |\Psi[h_3]|$$

$$\Psi[h_{vac} \cup h_3] \to 0$$

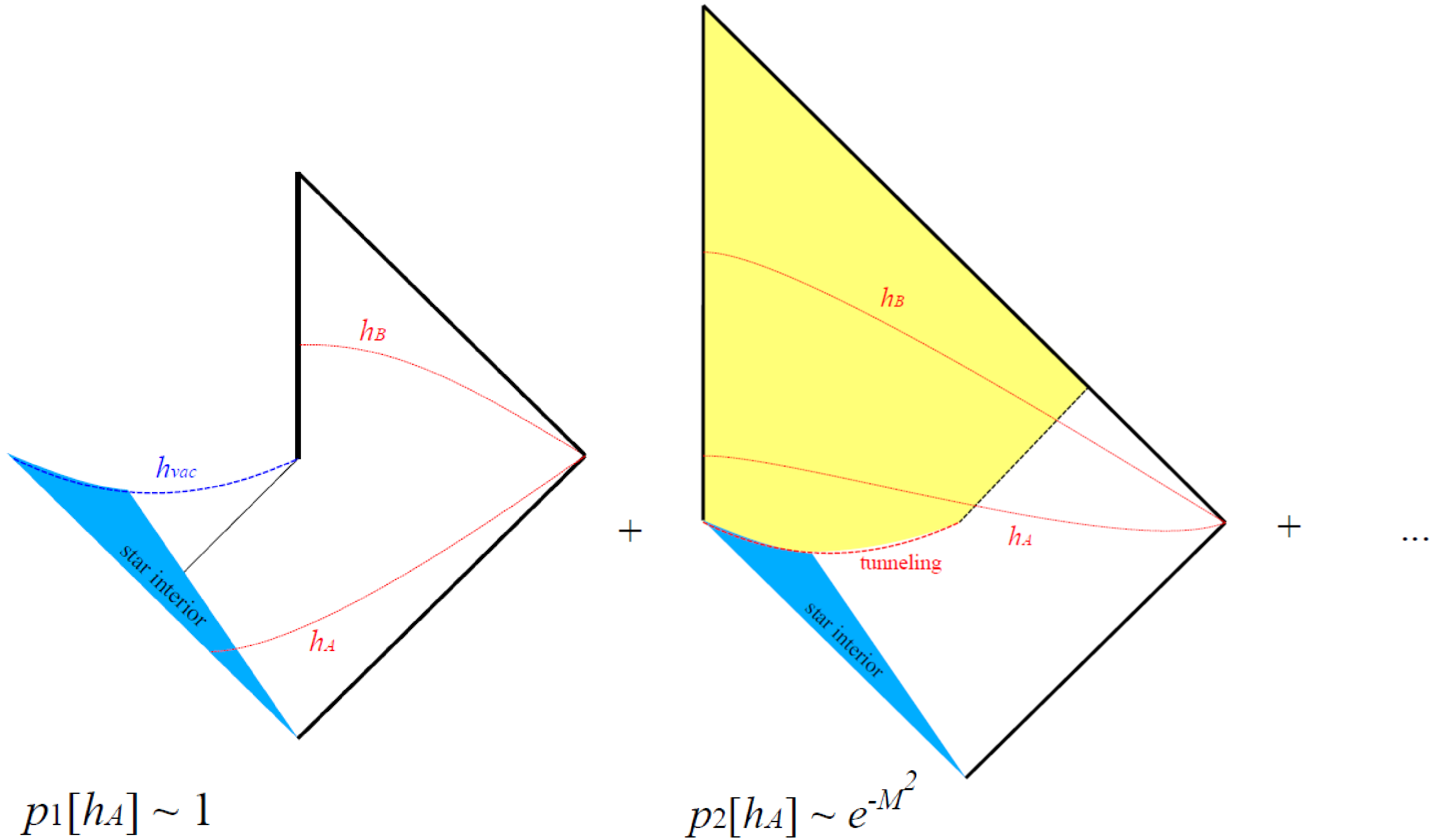
This process is definitely non−unitary and we will lost information.



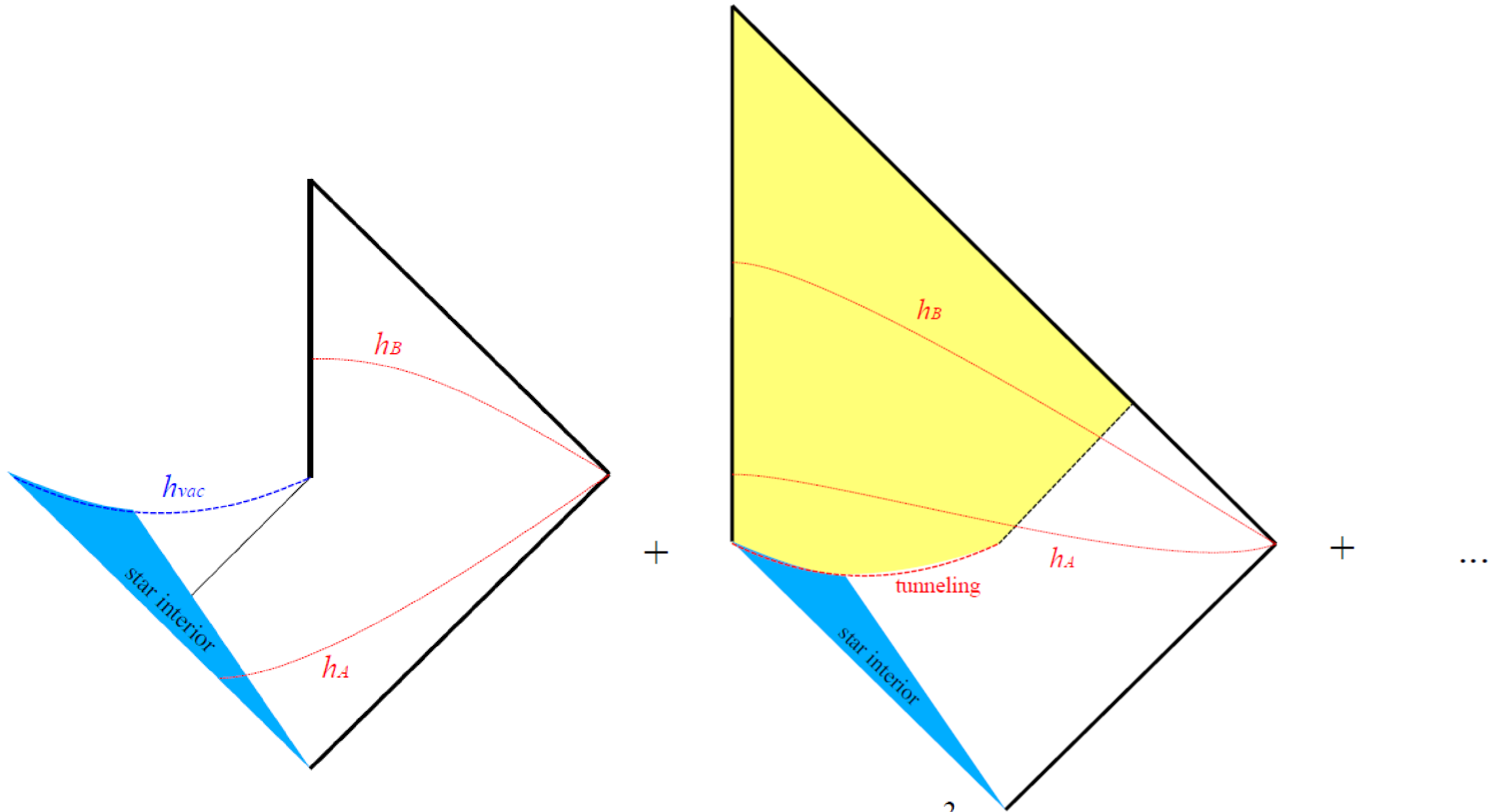

Let us see the entire wave function.




In the path integral, there exists a tunneling channel such that there is no formation of a black hole, even though the probability is very low (Chen, Sasaki and DY, 1806.03766).




From the beginning, the first history is dominant in terms of probability.

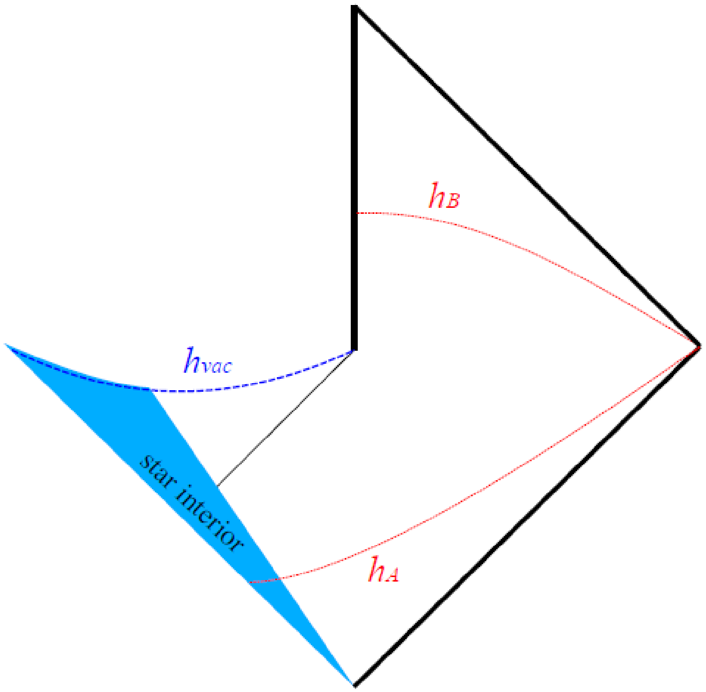



$p_1[h_A] \sim 1$ $\qquad$ $p_2[h_A] \sim e^{-M^2}$

$\rightarrow \quad p_1[h_{vac} \cup h_B] = 0$

However, as the time slice evolves,
the probability of the first history will decay to zero.

$$p_1 + p_2 + \ \dots = 1$$

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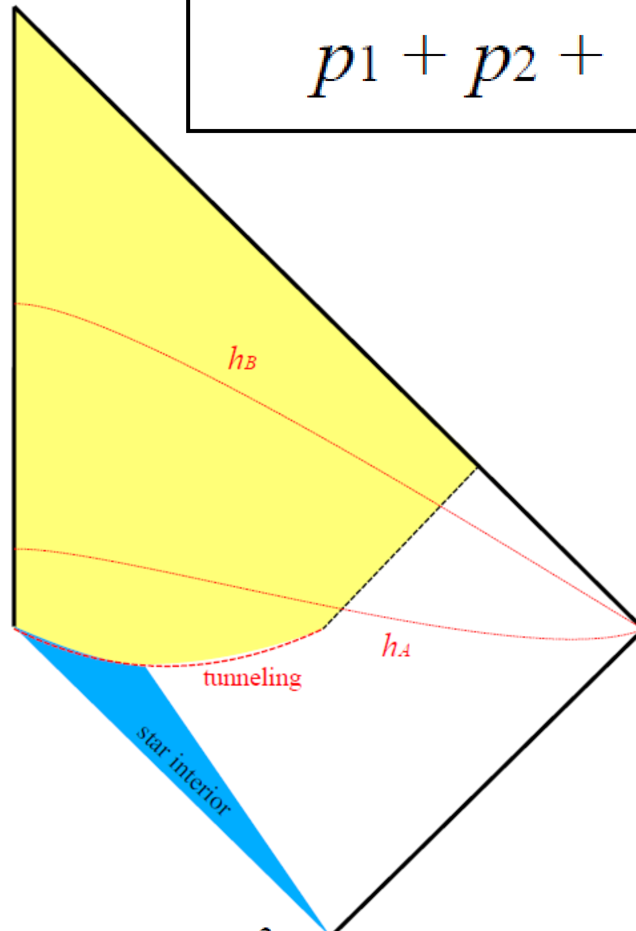


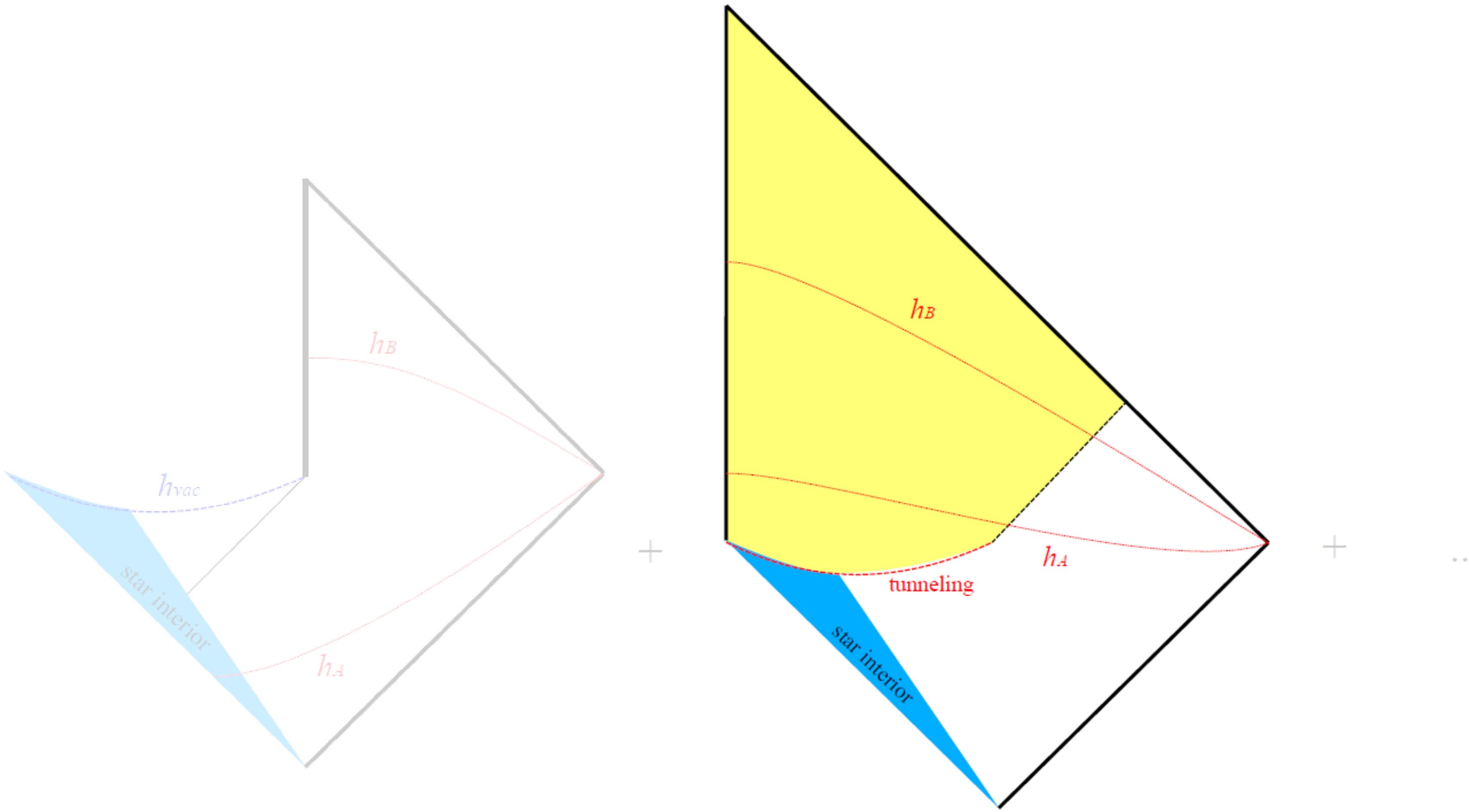

+ ...

$p_1[h_A] \sim 1$

$\rightarrow \quad p_1[h_{vac} \cup h_B] = 0$

$p_2[h_A] \sim e^{-M^2} \quad \rightarrow \quad p_2[h_B] \sim 1$

If the sum of the probabilities should be preserved, then
the probability of the second history should be dominated later.




So, in the late time, the wave function is dominated by the trivial geometry which has no loss of information.

The quantum boundary condition supports the late time dominance!

We need more study to check consistency.

Thank you very much