Gravitational waves as a probe of the history of the universe

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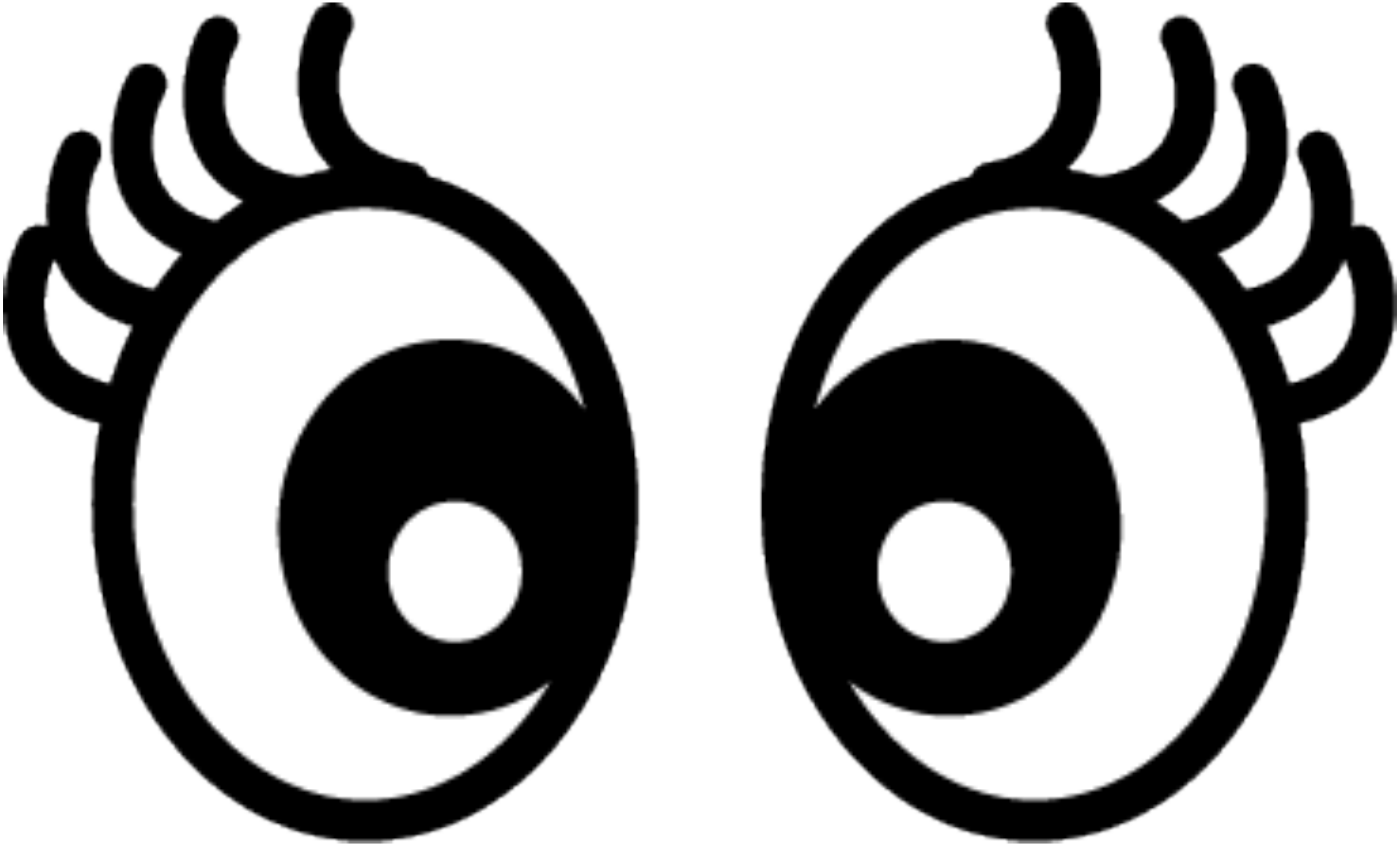
Gravitational wave Physics and Astronomy
New Eyes to Observe the Universe



Gravitational wave Physics and Astronomy Science Goals

- Cosmology
  - Can we directly see before the CMB era?

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INVITATION

TO

Gravitational Wave Cosmology

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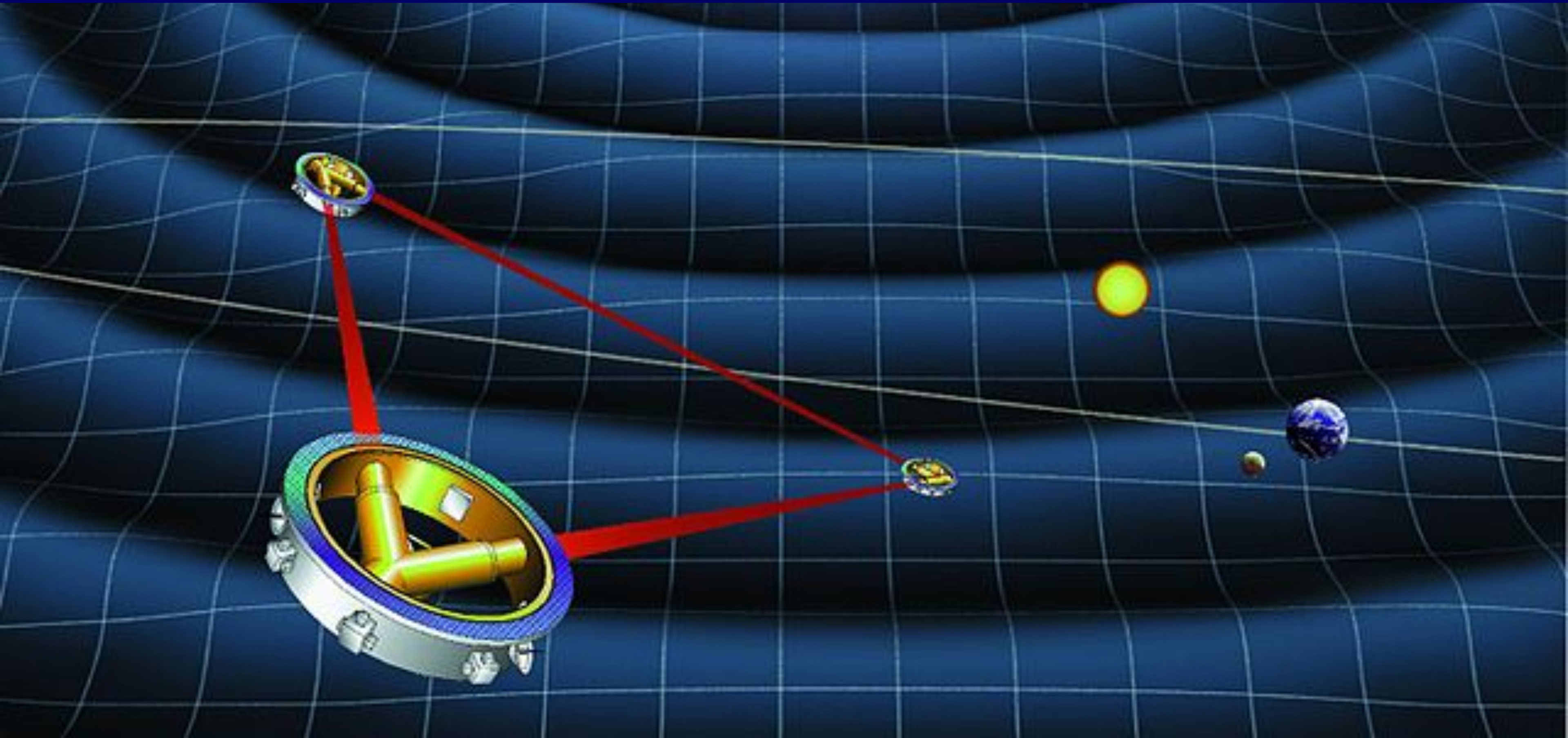
Direct detection of cosmological Gravitational waves can be achieved by space-based laser interferometers Such as LISA, DECIGO, TianQin, Taiji…



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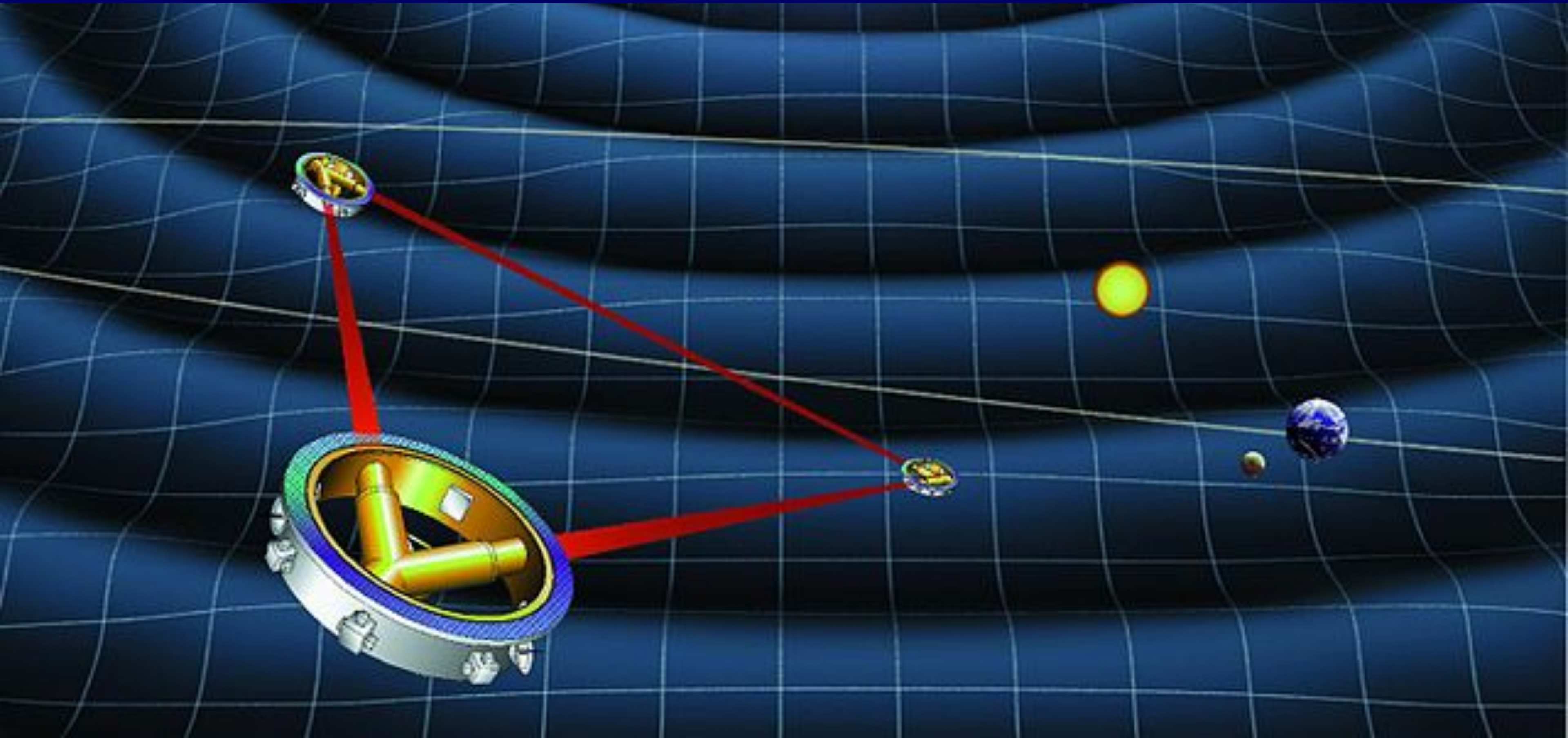
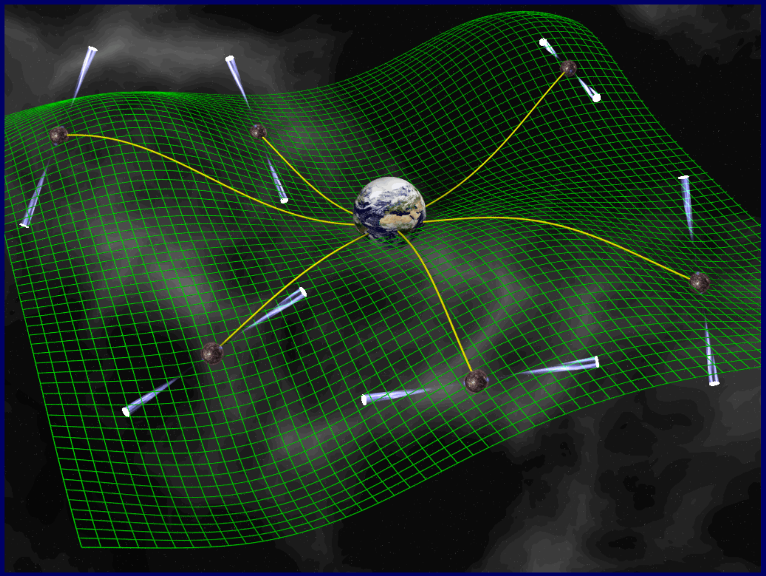


Other means of detection of Cosmological gravitational waves

Pulsar timing



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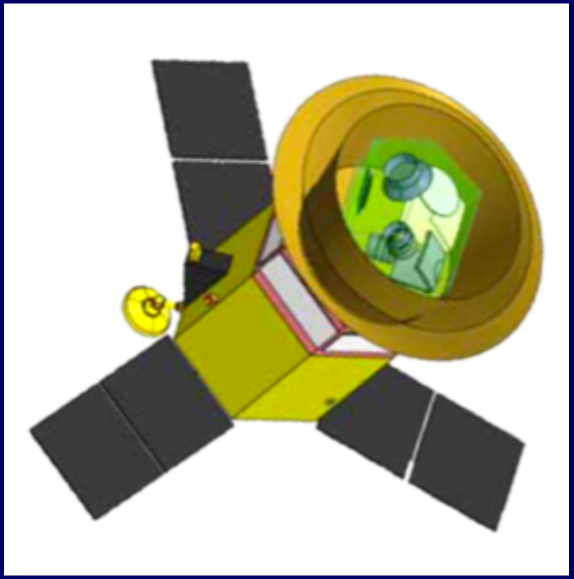
B-mode polarization of Cosmic Microwave Background (CMB)

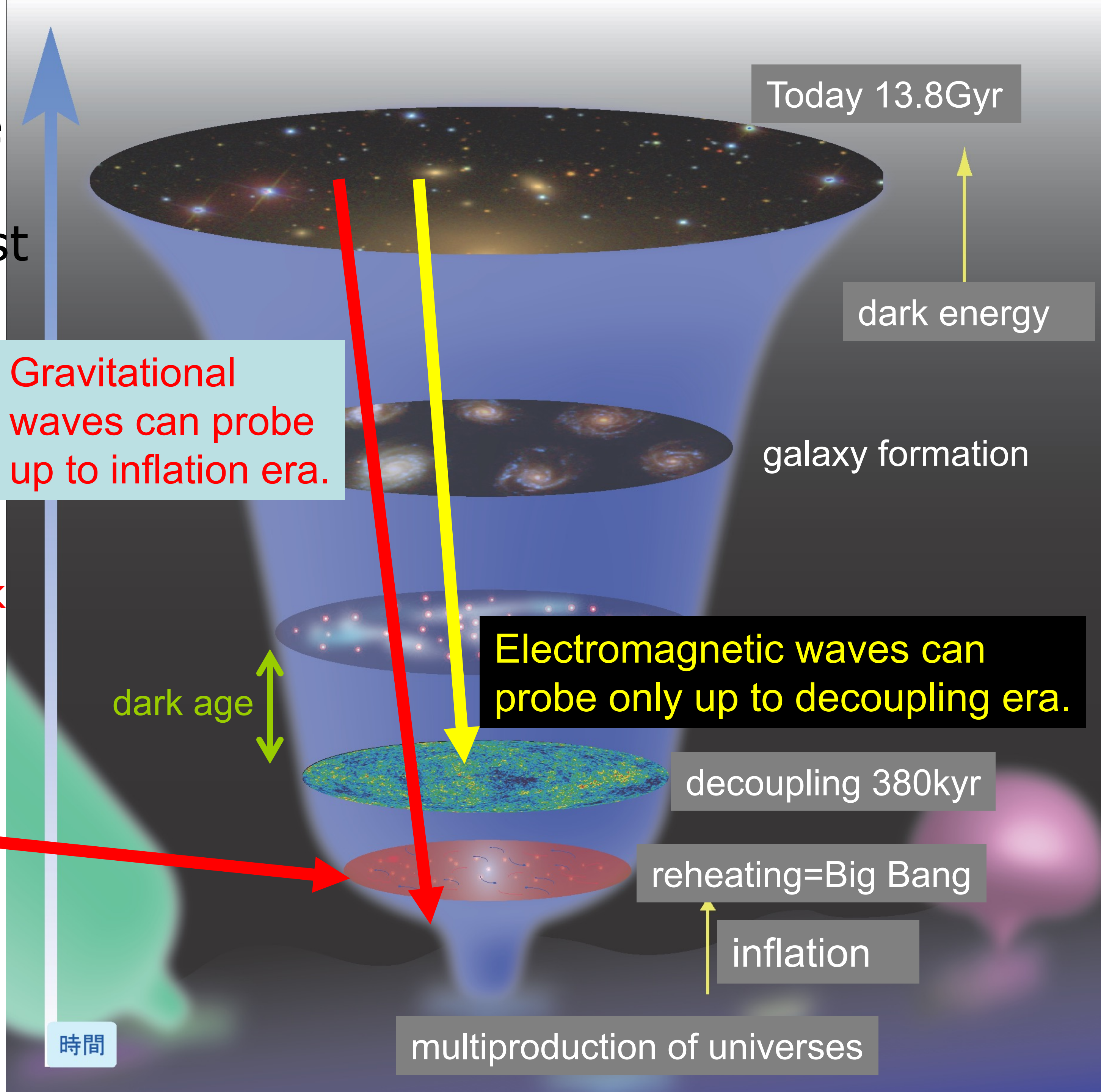


LiteBIRD: approved by JAXA in May 2019!

Gravitational waves provide new eyes to see the earliest Universe.

We can probe another tiny dark age between inflation and Big Bang Nucleosynthesis

Shedding new “light” on this epoch



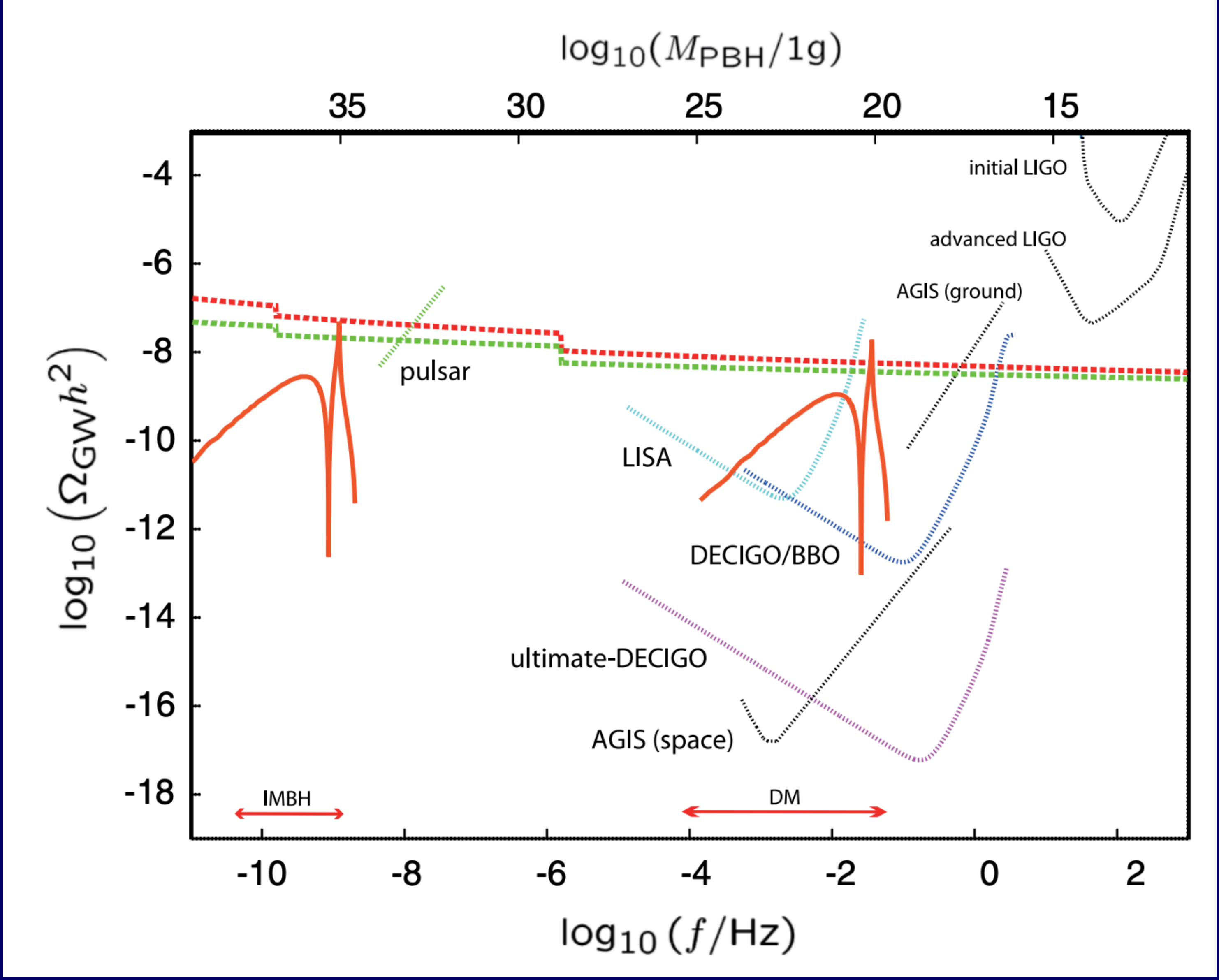
Sources of gravitational waves from the early universe

① Quantum tensor perturbations from inflation

② GWs from second-order scalar perturbations

PBH DM scenario can be tested by LISA & DECIGO/BBO.

Saito & JY PRL107(2011)069901



Sources of gravitational waves from the early universe

① Quantum tensor perturbations from inflation

② GWs from second-order scalar perturbations

③ GWs from bubble collisions after phase transition

④ GWs from self-ordering scalar fields

⑤ GWs from topological defects esp. cosmic strings

⑥ Gravitational particle production of gravitons

⑦ 。。。

Tensor Perturbations (Quantum Gravitational Waves)

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

$$h_{ij} = h_+ \varepsilon_{ij}^+ + h_\times \varepsilon_{ij}^\times$$

transverse
-traceless

They are equivalent with two massless scalar fields.

$$h_{ij}(t, \boldsymbol{x}) = \sum_{\lambda=+,\times} \int \frac{d^3k}{(2\pi)^{3/2}} \epsilon_{ij}^\lambda(\boldsymbol{k}) h_{\boldsymbol{k}}^\lambda(t) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

satisfies massless Klein-Gordon eqn

Quantization in De Sitter background yields nearly scale-invariant long-wave perturbations during inflation.

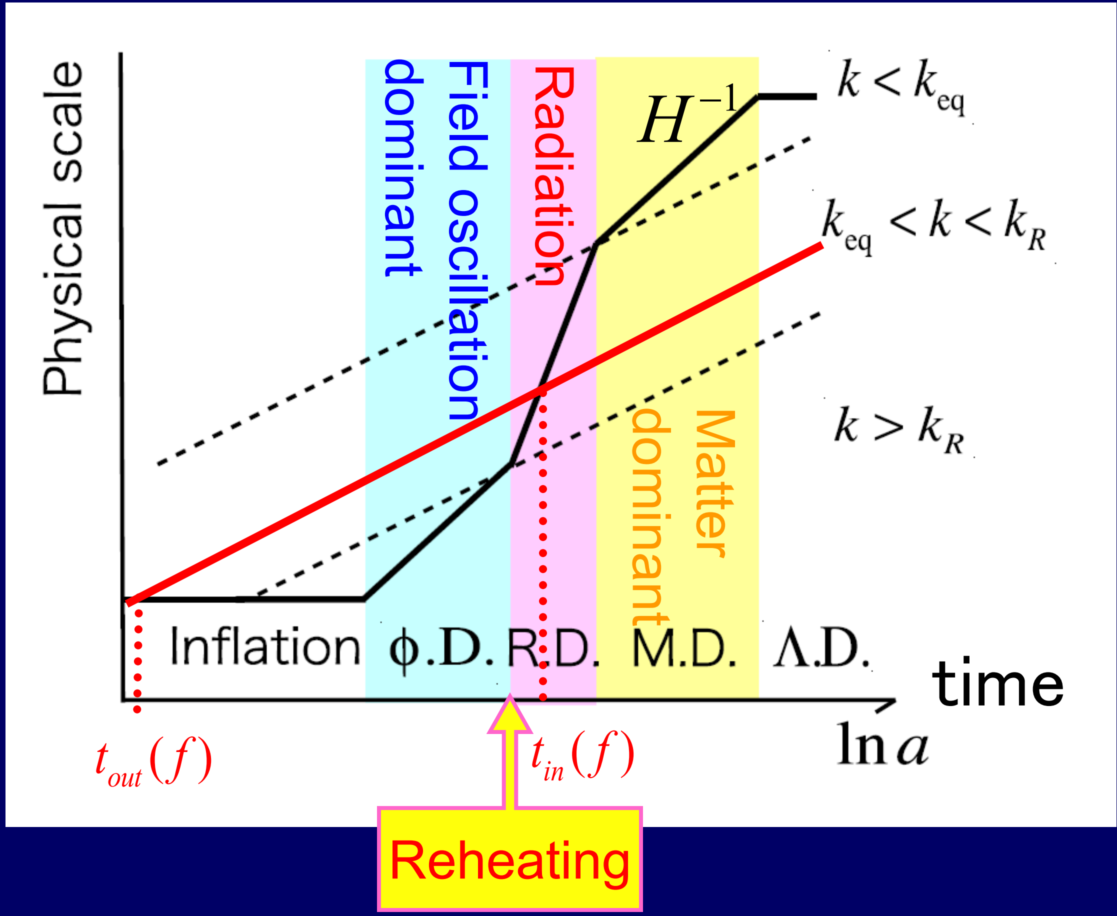
$$\Delta_h^2(k) = \langle h_{ij} h^{ij}(k) \rangle = 64\pi G \left(\frac{H(t_k)}{2\pi} \right)^2$$

Grishchuk (1974)
Starobinsky (1979)

Evolution of gravitational waves in the standard inflationary Universe

- Amplitude of GW is constant when its wavelength is longer than the Hubble radius between $t_{out}(f)$ and $t_{in}(f)$.

- After entering the Hubble radius, the amplitude decreases as $\propto a^{-1}(t)$ and the energy density as $\propto a^{-4}(t)$.



When $a(t) \propto t^p \quad (p<1)$, the tensor perturbation evolves as

$$h(f,a) \propto a(t)^{\frac{1-3p}{2p}} J_{\frac{3p-1}{2(1-p)}}\left(\frac{p}{1-p}\frac{k}{a(t)H(t)}\right), \quad k = 2\pi f a(t_0)$$

Density parameter in GW per logarithmic frequency interval

$$\Omega_{GW}(f,t) = \frac{1}{\rho_{cr}(t)} \frac{d\rho_{GW}(f,t)}{d\ln f}$$

When the mode reentered the Hubble horizon at $t \equiv t_{in}(f)$, the angular frequency is equal to $\omega = H\left(t_{in}(f)\right)$, so we find

$$\frac{d\rho_{GW}(f,t_{in}(f))}{d\ln f} = \frac{\omega^2}{32\pi G} h_{\inf}^2(f) = \frac{H^2\left(t_{in}(f)\right)}{32\pi G} h_{\inf}^2(f) = \frac{1}{24}\rho_{cr}(t_{in}(f))\Delta_h^2(f)$$

$$\boxed{\Omega_{GW}(f,t_{in}(f)) = \frac{1}{24}\Delta_h^2(f)}$$

at horizon reentry

After entering the Hubble horizon at $t_{in}(f)$,

$$\Omega_{GW}(f,t) = \frac{\rho_{GW,\ln f}(f,t)}{\rho_{tot}(t)} \quad \begin{matrix} \propto a^{-4}(t) \\ \propto a^{-3(1+w)}(t) \end{matrix}$$

$$w \equiv \frac{p}{\rho_{tot}}$$: equation of state parameter

$$\Omega_{GW}(f,t) \approx \frac{1}{24}\Delta_h^2(f)\left(\frac{a(t_{in}(f))}{a(t)}\right)^{1-3w}$$

Radiation dominated era: $w = \frac{1}{3}$ $\quad \Omega_{GW}(f,t) = \text{const.}$

Matter dominated era: $w = 0$ $\quad \Omega_{GW}(f,t)\,] \quad \propto a^{-1}(t)$

$t_{in}(f)$ as a function of frequency

$$2\pi f = aH \propto t^{\frac{2}{3(1+w)}} \frac{2}{3(1+w)t} \propto t^{-\frac{3w+1}{3w+3}} \quad \text{at} \quad t = t_{in}(f)$$

$$\therefore t_{in}(f) \propto f^{-\frac{3w+3}{3w+1}}$$

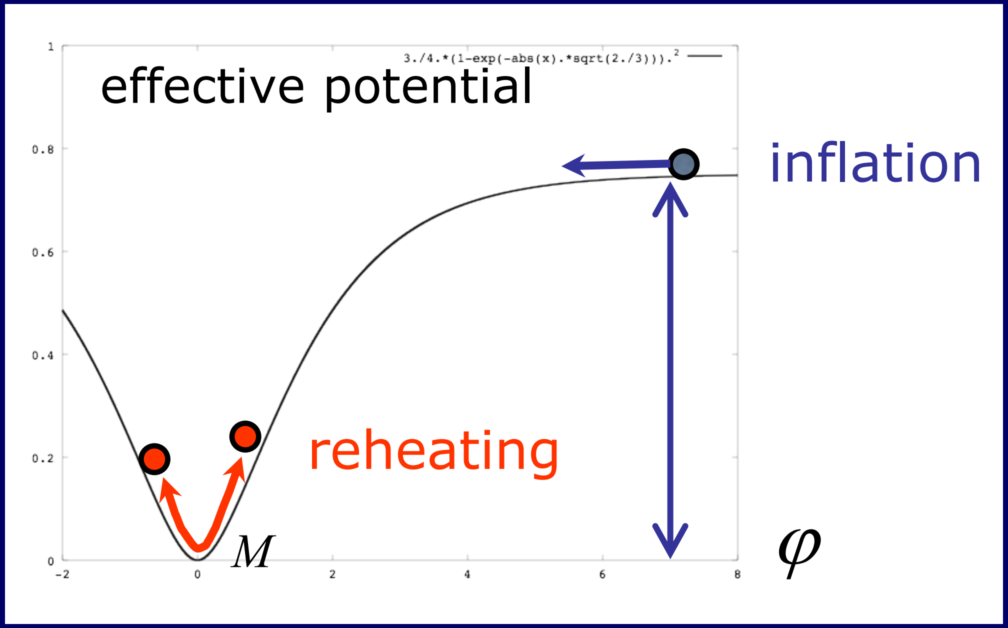
$$\Omega_{GW}(f,t) \approx \frac{1}{24}\Delta_h^2(f)\left(\frac{a(t_{in}(f))}{a(t)}\right)^{1-3w} \propto \Delta_h^2(f) f^{\frac{6w-2}{3w+1}}$$

We may determine the equation of state in the early Universe.

We may determine thermal history of the early Universe.

N. Seto & JY (03), Boyle & Steinhardt (08), Nakayama, Saito, Suwa, JY (08), Kuroyanagi et al (11)…

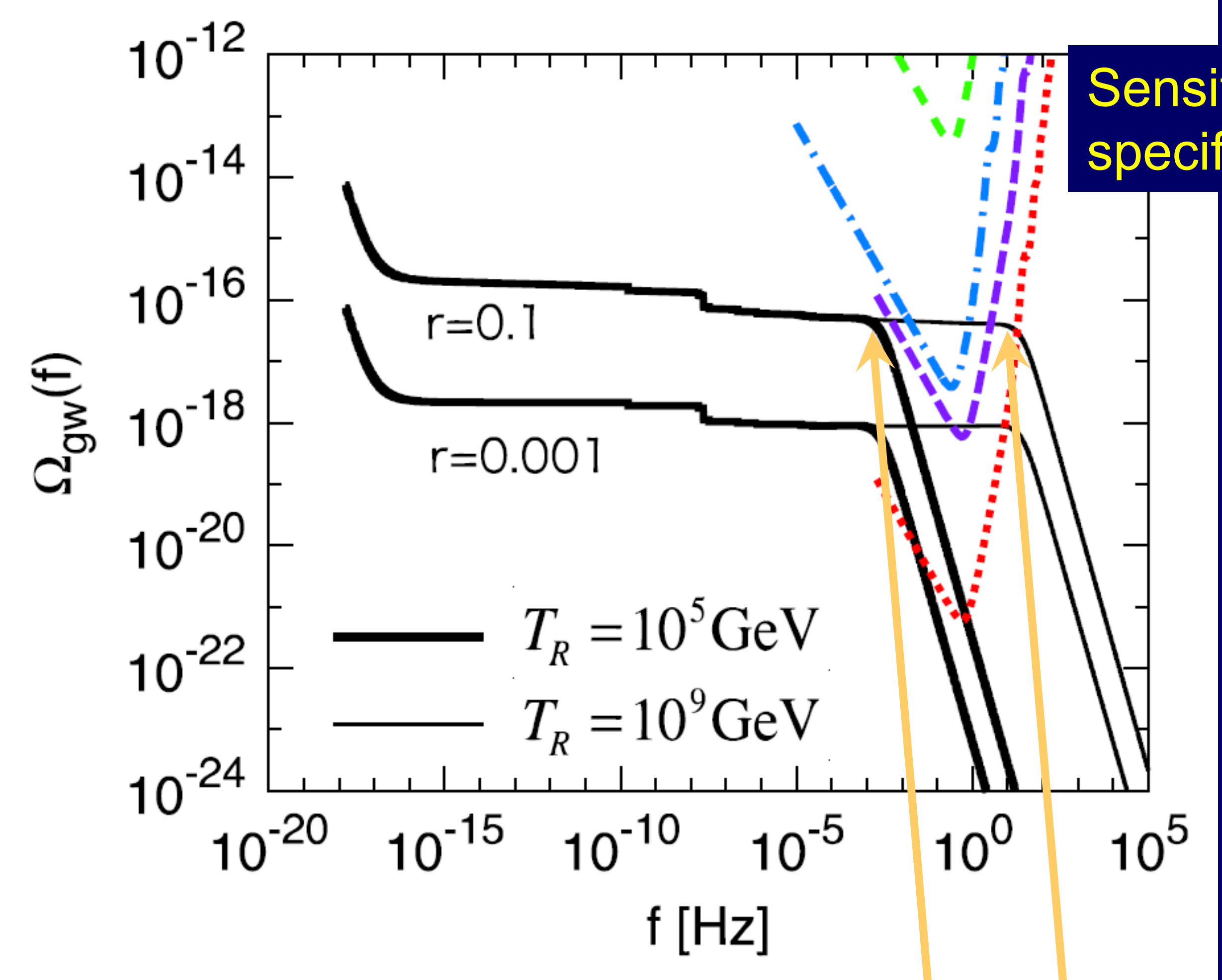
Standard potential-driven inflation models



Inflation is followed by a coherent oscillation of the inflaton which behaves like non-relativistic matter with EOS parameter $w=0$ if the mass term dominates the potential.

$$\Omega_{GW}(f) \propto \Delta_h^2(f) f^{\frac{6w-2}{3w+1}} \propto f^{-2}$$ in high frequency region

Thermal History is inprinted on the spectrum of GWs.



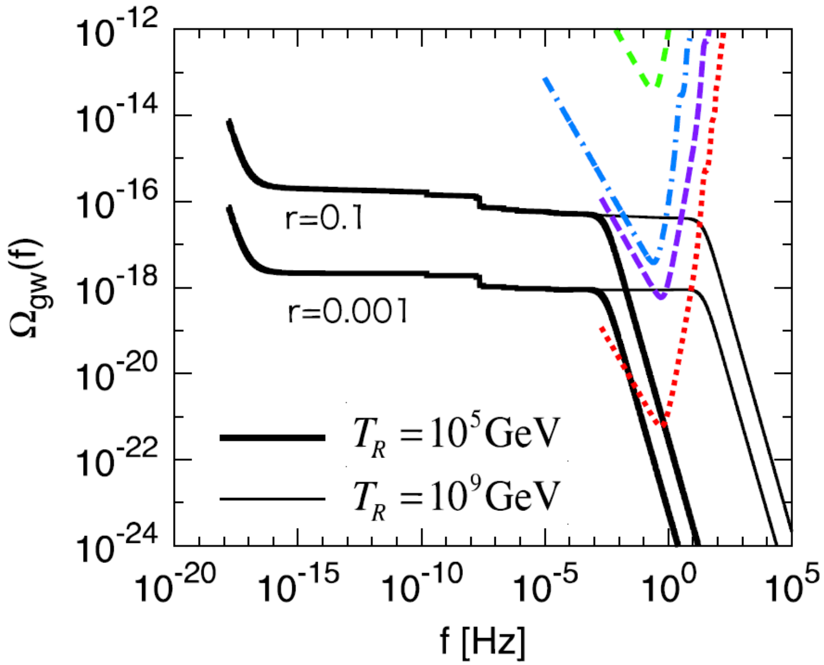
Sensitivity curves of various specifications of DECIGO

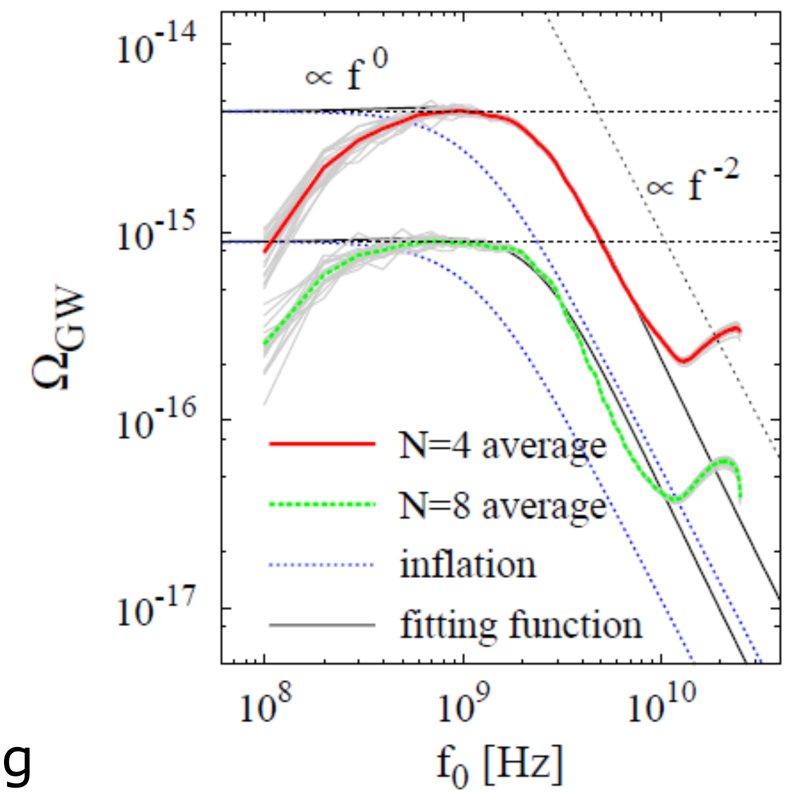
(Nakayama, Saito, Suwa & JY 08)

$$f_{\rm R} = \frac{k_{\rm R}}{2\pi a_0} \simeq 0.26{\rm Hz} \left(\frac{g_{*s}(T_{\rm R})}{106.75}\right)^{1/6} \left(\frac{T_{\rm R}}{10^7{\rm GeV}}\right)$$

characteristic frequency at reheating time

Comparison between inflationary tensor perturbations and GWs from self-ordering N-component scalar fields





Transfer functions due to reheating

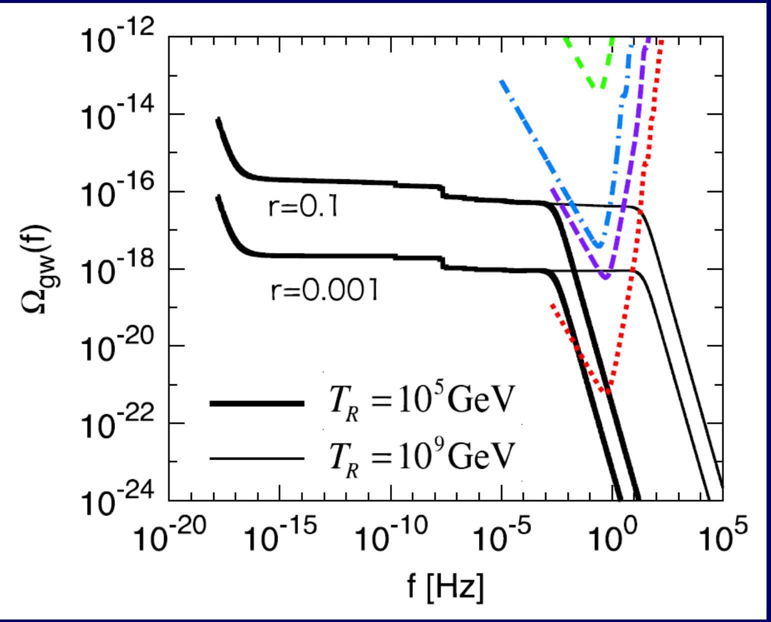
$$T^2_{\rm inflation}(x_R) = (1 - \boxed{0.22} x_R^{1.5} + 0.65 x_R^2)^{-1} \qquad T^2_{O(N)}(x_{R'}) = (1 - \boxed{0.6} x_{R'}^{1.5} + 0.65 x_{R'}^2)^{-1}$$

$$x_R = \frac{f}{f_R} \qquad f_R = 0.26 \left(\frac{g_*(T_R)}{106.75}\right)^{\frac{1}{6}} \left(\frac{T_R}{10^7\,\mathrm{GeV}}\right) \mathrm{Hz}$$

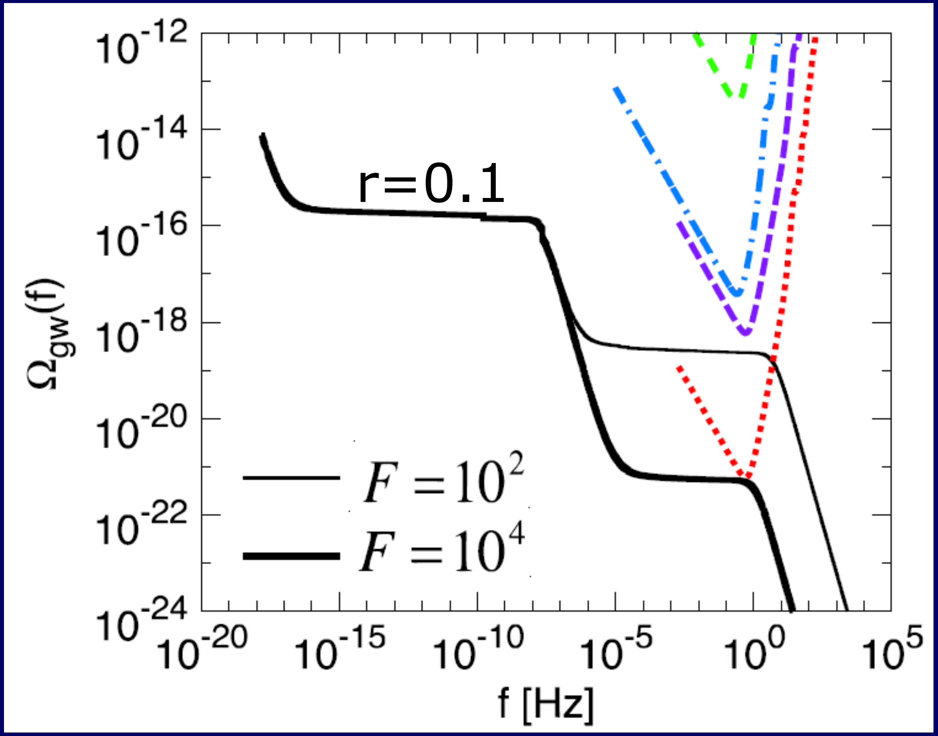
can be distinguished

(Kuroyanagi, Hiramatsu & JY 16)

If we could measure the GWs at two different frequencies, we could probe entropy production between two regimes, too.



No entropy production after reheating



Entropy production w/ dilution factor

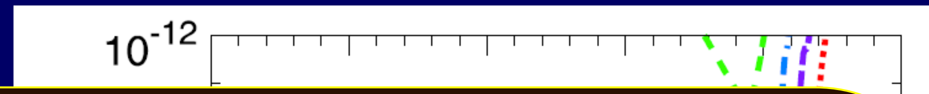
$$F \equiv \frac{s(T_d)a^3(T_d)}{s(T_R)a^3(T_R)}$$

T_d: temperature after entropy production

High frequency part is modified as

$$\Omega_{GW}(f,t) \rightarrow F^{-4/3}\Omega_{GW}(f,t) \text{ and } f_R \rightarrow F^{-1/3} f_R$$

If we could measure the GWs at two different frequencies, we could probe entropy production between two regimes, too.



We do not consider entropy production after reheating hereafter.

No
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$s(T_R)a^3(T_R)$

e after entropy production

High frequency part is modified as

$$\Omega_{GW}(f,t) \rightarrow F^{-4/3}\Omega_{GW}(f,t) \text{ and } f_R \rightarrow F^{-1/3} f_R$$

Broadband Approach to Inflationary Cosmology

Generalized G-inflation

The most general single-field inflation with second order field equations

$$S=\sum_{i=2}^{5}\int \mathcal{L}_i\sqrt{-g}\,d^4x$$

Generalized Galileon = Horndeski Theory

(Kobayashi, Yamaguchi & JY 2011)

4 arbitrary functions of ϕ and $X\equiv-\frac{1}{2}(\partial\phi)^2$

$$\mathcal{L}_2=\boxed{K(\phi,X)}$$

$$\mathcal{L}_3=-\boxed{G_3(\phi,X)}\Box\phi$$

$$\mathcal{L}_4=\boxed{G_4(\phi,X)}R+G_{4X}\left[(\Box\phi)^2-\left(\nabla_\mu\nabla_\nu\phi\right)^2\right]$$

$$\mathcal{L}_5=\boxed{G_5(\phi,X)}G_{\mu\nu}\nabla^\mu\nabla^\nu\phi-\frac{1}{6}G_{5X}\left[(\Box\phi)^3-3(\Box\phi)\left(\nabla_\mu\nabla_\nu\phi\right)^2+2\left(\nabla_\mu\nabla_\nu\phi\right)^3\right]$$

Generalized G-inflation

The most general single-field inflation
with second order field equations

$$S=\sum_{i=2}^{5}\int \mathcal{L}_i\sqrt{-g}\,d^4x$$

Generalized Galileon = Horndeski Theory

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = \boxed{G_4(\phi, X)}R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$G_4 \supset M_{Pl}^2/2$ gives the Einstein action

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

Generalized G-inflation is a framework to study the most general single-field inflation model with second-order field equations.

G-inflation model

$$K(\phi, X) - G(\phi, X)\Box\phi$$

Inflation driven by kinetic energy

Ex. G-inflation (k-inflation is its special case w/ G=0)

(Kobayashi, Yamaguchi & JY 10) (Armendariz-Picon, Damour & Mukhanov 99)

$$\mathcal{L}_\phi = K(\phi, X) - G(\phi, X)\Box\phi, \quad X \equiv -\frac{1}{2}(\partial\phi)^2$$ canonical kinetic function

Energy momentum tensor

$$T_{\mu\nu} = \left(K_X - G_X\right)\nabla_\mu\phi\nabla_\nu\phi + g_{\mu\nu}\left(K + \nabla_\lambda G\nabla^\lambda\phi\right) - 2\nabla_{(\nu}G\nabla_{\nu)}\phi$$

Flat RW background $ds^2 = -dt^2 + a^2(t)d\boldsymbol{x}^2$

$$T^\mu_\nu = \mathrm{diag}\left(-\rho, p, p, p\right)$$ with

$$\rho = 2K_X X - K + 3G_X H\dot{\phi}^3 - 2G_\phi X$$

$$p = K - 2\left(G_\phi + G_X\ddot{\phi}\right)X$$

leading to

$$3M_{Pl}^2H^2 = \rho, \quad M_{Pl}^2\left(3H^2 + 2\dot{H}\right) = -p$$

Simplest de Sitter k-inflation Solution

- We seek for a solution with $H = \text{const.}$ and $\dot{\phi} = \text{const.}$ in the simplest k-inflation model with

$$K(\phi, X) \equiv K(X), \quad G(\phi, X) \equiv 0.$$

$\boxed{\text{No scalar potential}}$ shift symmetry $\phi \to \phi + c$

- $3M_{Pl}^2 H^2 = \rho = 2K_X X - K, \quad M_{Pl}^2\left(3H^2 + \cancel{2\dot{H}}\right) = -p = -K$

A simple choice $K(X) \equiv -X + \dfrac{X^2}{2M^4} \Rightarrow K_X = 0$ at $X = M^4$

We find de Sitter solution $H^2 = \dfrac{M^4}{6M_{Pl}^2}$ with $p = -\rho = -\dfrac{M^4}{2}$.

(Armendariz-Picon, Damour & Mukhanov 99)

G-de Sitter solution can also be found with $G(\phi, X) \neq 0$.

(Kobayashi, Yamaguchi & JY 10)

Essential difference between k-inflation & G-inflation

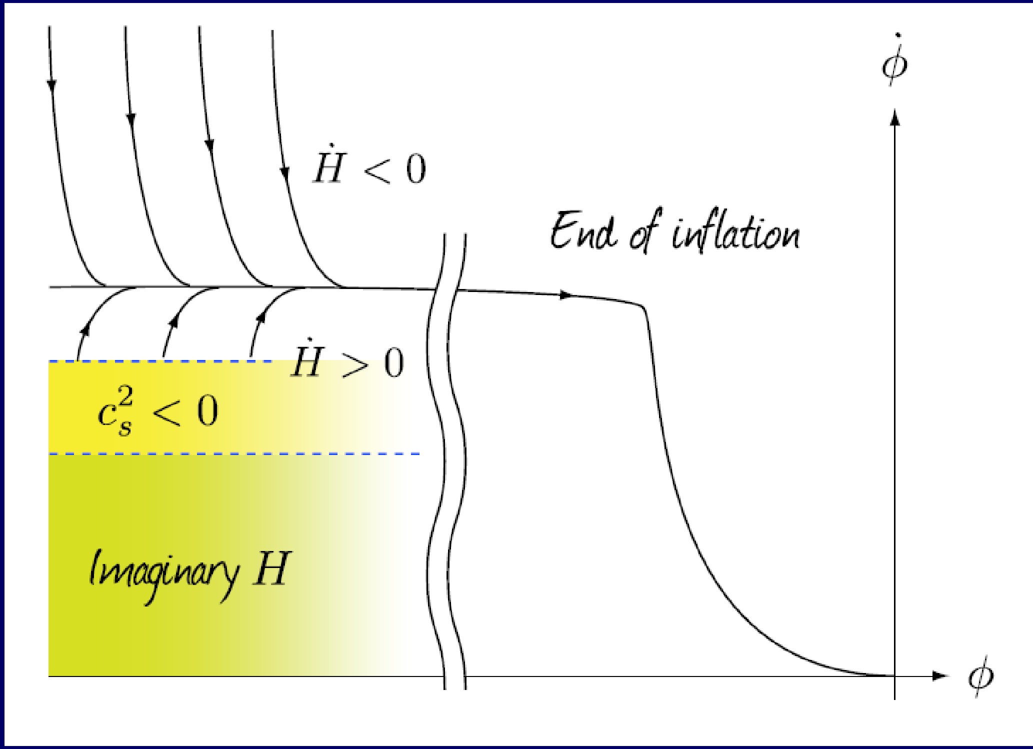
In G-inflation, the null energy condition may be violated, $2M_{Pl}^2\dot{H} = -(\rho + p) > 0.$

It can be violated without instabilities, keeping $c_s^2 > 0.$

The tensor spectral index can be positive,

$$n_T = -2\varepsilon = 2\frac{\dot{H}}{H^2} > 0.$$

Short wave tensor fluctuations may have a larger amplitude at formation.

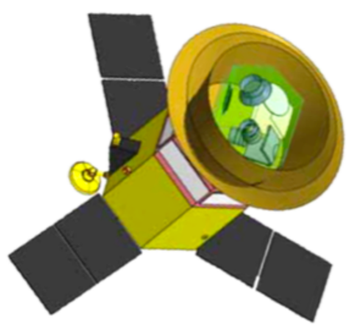


NB. In k-inflation, null energy condition cannot be violated, since it would cause gradient instability.

Many people say positive tensor spectral ind... wou... ...tely falsi... inflation.

This is not true.

We hope LiteBIRD will measure $n_t>0$!!



See also Mishima and Kobayashi 1911.02143 for more cases

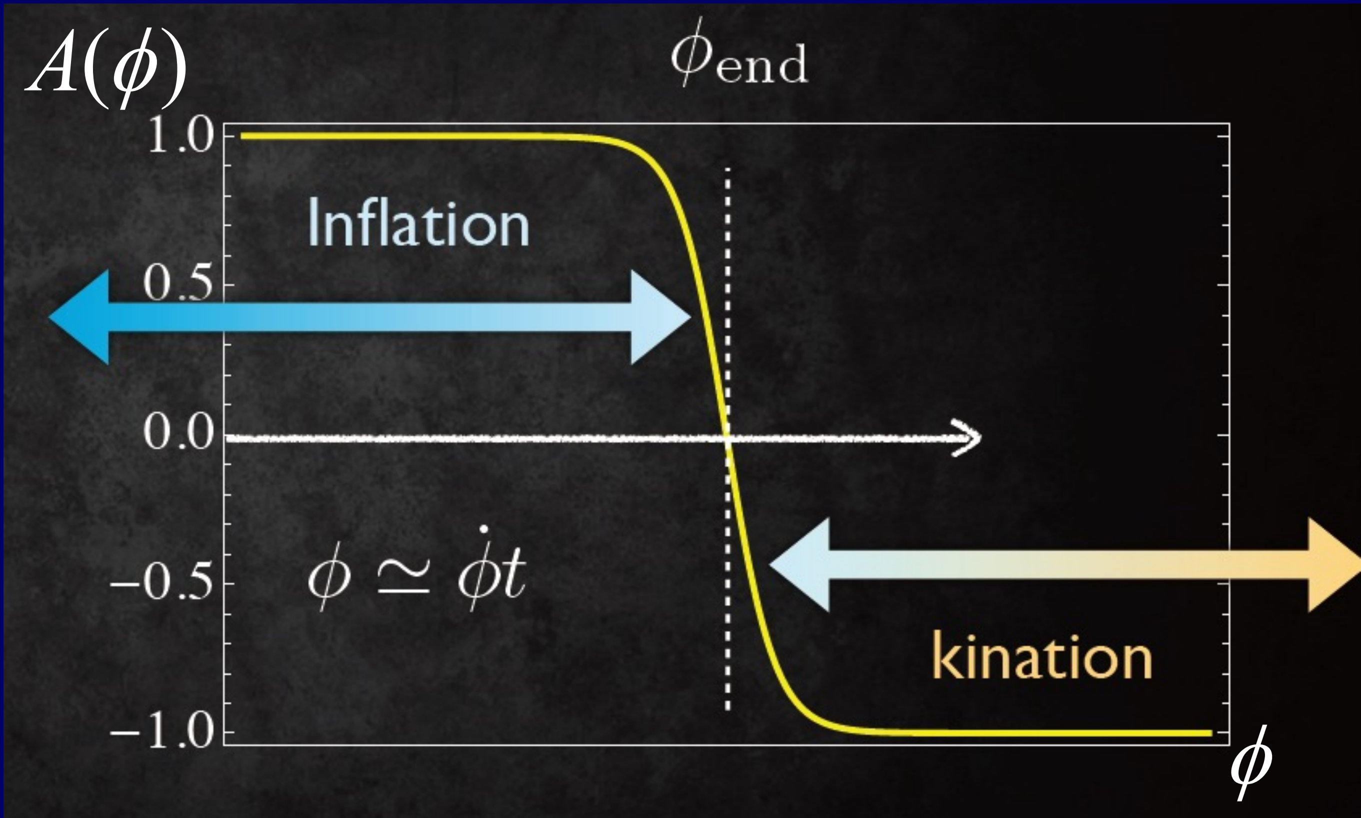
★Inflation can be terminated by flipping the sign of $-X$.

$$K(\phi, X) \equiv -A(\phi) X + \frac{X^2}{2M^3\mu}$$

($K \cong -\rho < 0$ during inflation.)

A simple choice: $A(\phi) \equiv \tanh\left[\lambda\left(\phi_{end} - \phi\right)/M_{Pl}\right]$ with $\lambda = O(1)$.

Numerical solutions indicate ϕ stalls within one e-fold after crossing ϕ_{end} and all higher derivative terms become negligible.



The Universe after G-inflation

- After inflation the Universe is dominated by the kinetic energy of ϕ, which now behaves as a free massless field,

$$\boxed{\rho = \frac{\dot{\phi}^2}{2} \propto a^{-6}(t).} \qquad w = 1$$

$$\Omega_{GW}(f) \propto \Delta_h^2(f) f^{\frac{6w-2}{3w+1}} \propto f \text{ Z}$$ in high frequency region

(Chiba, Tashiro & Sasaki 04)

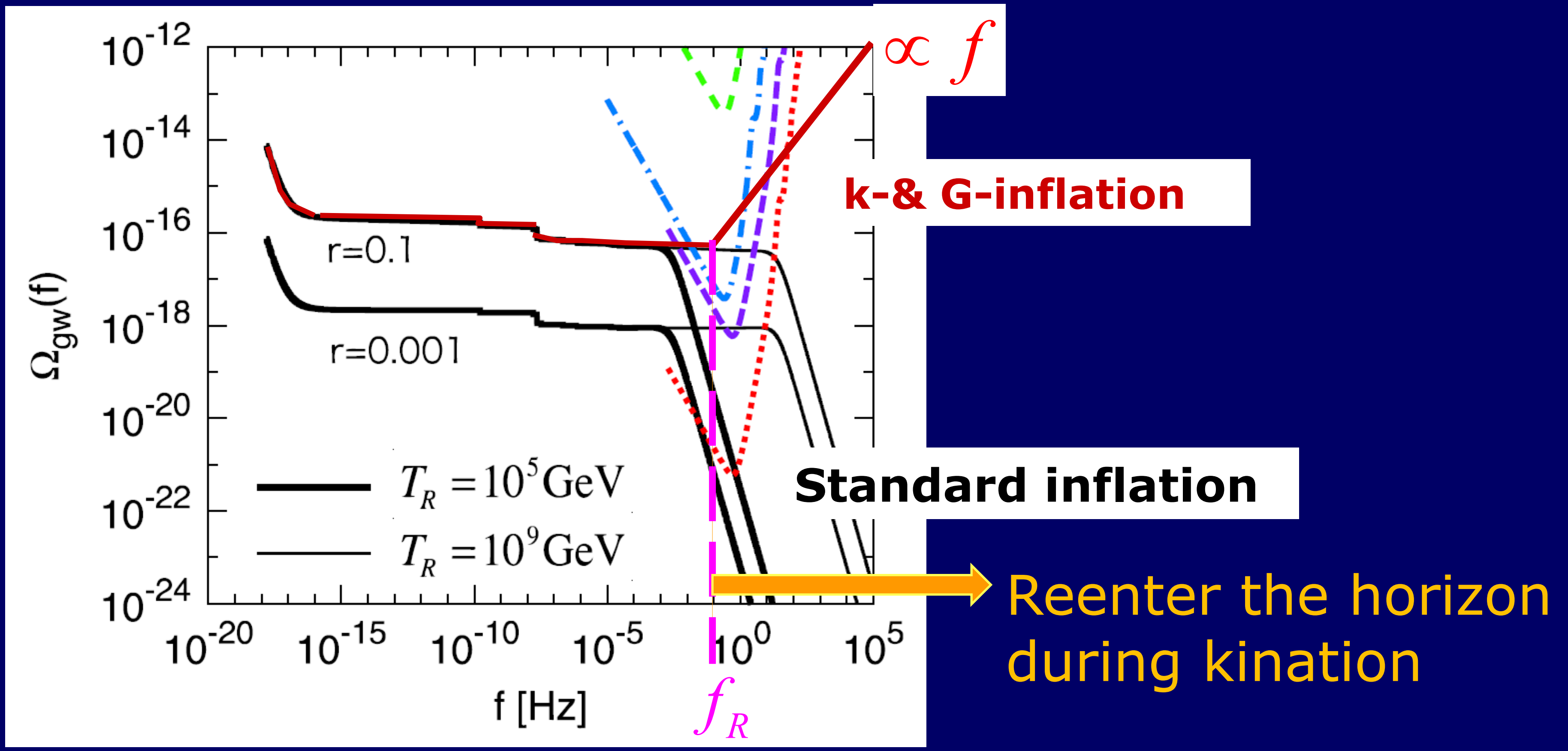
- Shift symmetry of the Lagrangian prevents direct interaction between ϕ and standard particles.

- Reheating proceeds through gravitational particle production due to the change of the cosmic expansion law: $a(t) \propto e^{H_{\inf} t} \rightarrow a(t) \propto t^{\frac{1}{3}}$.

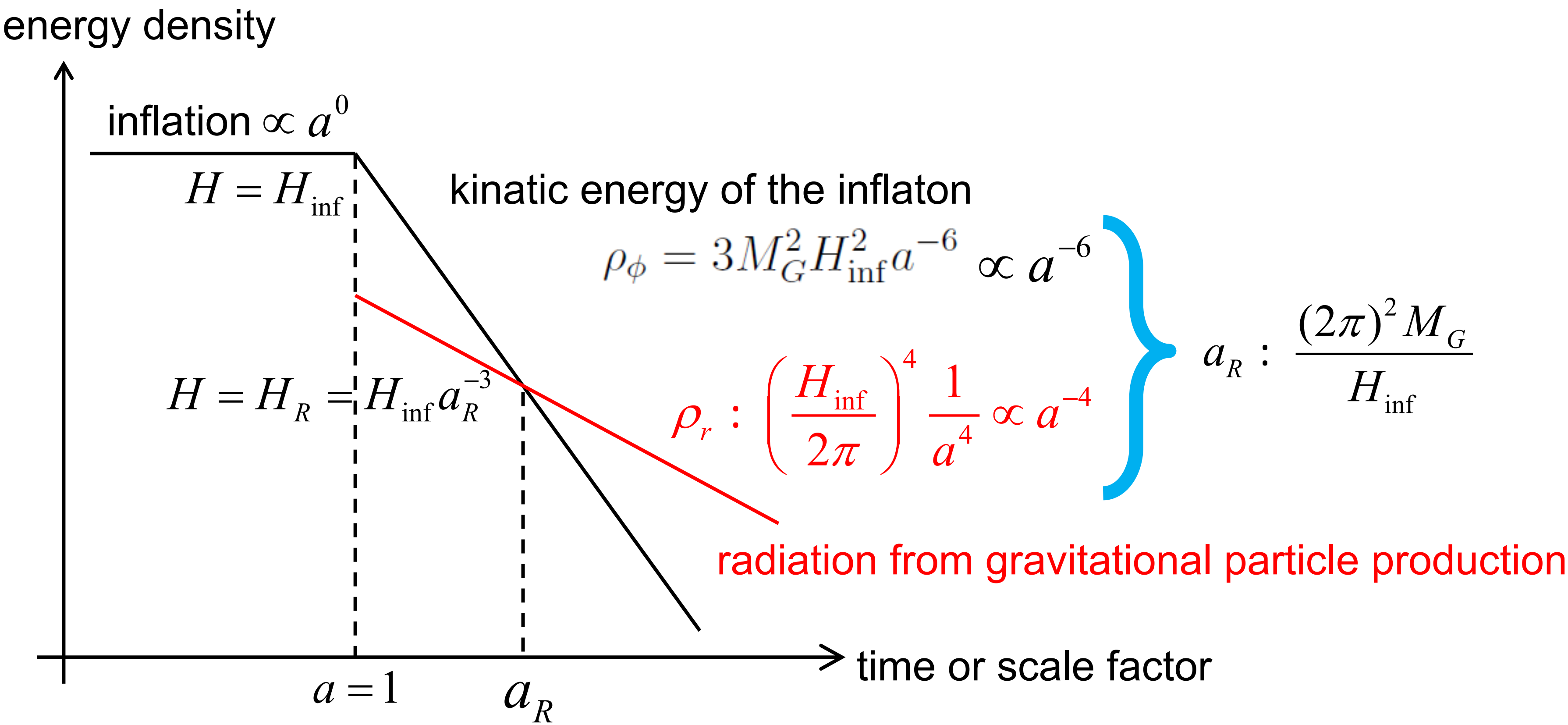
  This process may create radiation energy density of order of (the Hawking temperature)^4, namely, $\rho_r : T_H{}^4 = \left(H_{\inf}/2\pi\right)^4$.
  (We return to this issue more in detail later.)

(Ford 87, Kunimitsu & JY 12)

High frequency tensor perturbation is enhanced.

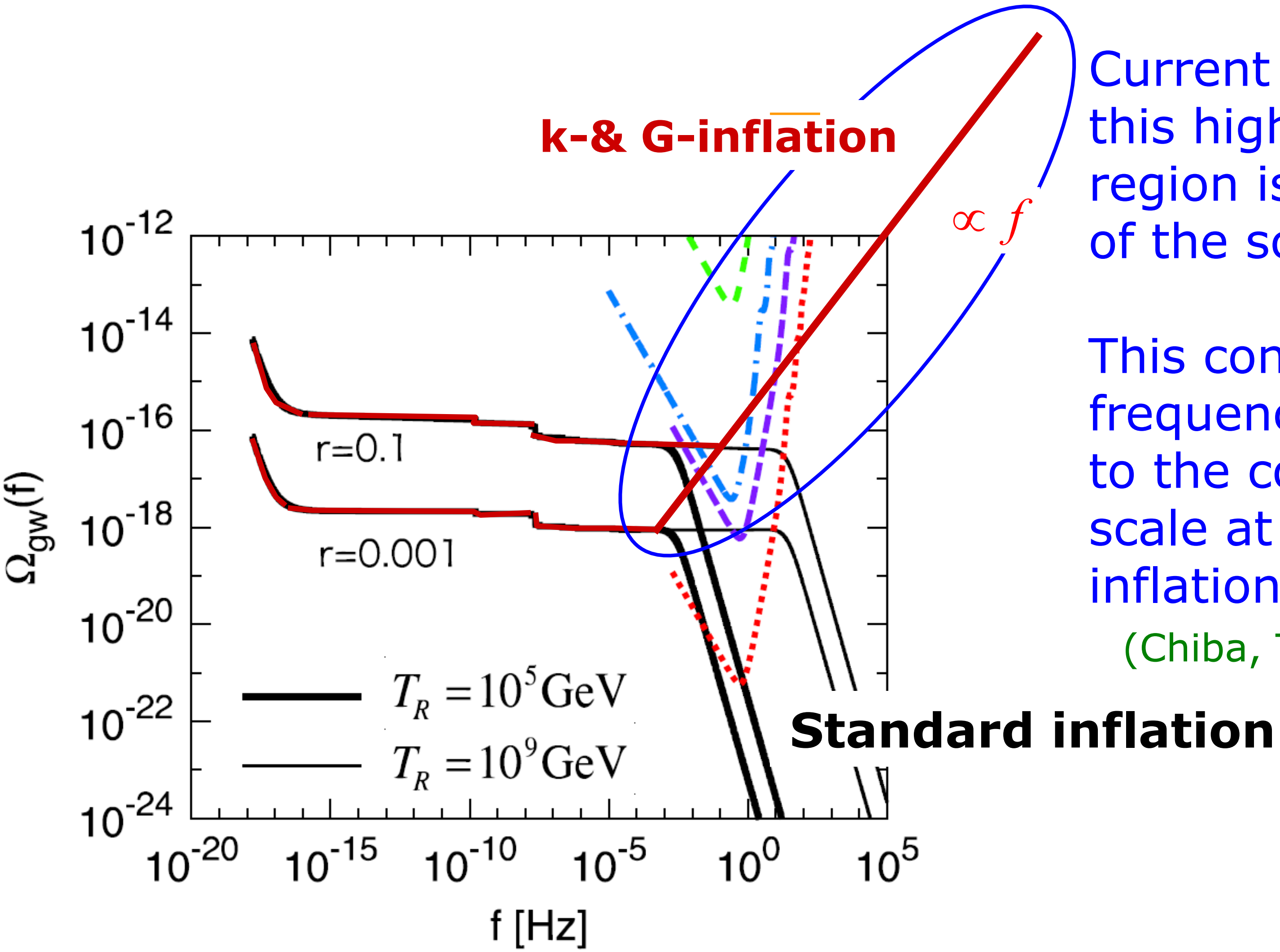


f_R : Frequency that reentered the horizon at reheating



$$f_R = \frac{a_R H_R}{2\pi a_0} \propto \frac{T_0}{T_R} H_R \propto \frac{T_0}{(H_R M_G)^{1/2}} H_R \propto H_R^{1/2} \propto H_{\inf}^2$$

$f > f_R$ $\quad \Omega_{GW}(f) : \Delta_h^2(f) \dfrac{f}{f_R} \propto H_{\inf}^2 \dfrac{f}{H_{\inf}^2}$ is independent of $H_{\inf}$



Current amplitude in this high frequency region is independent of the scale of inflation.

This continues to the frequency corresponding to the comoving horizon scale at the end of inflation.

(Chiba, Tashiro & Sasaki 04)

Hence we study creation of gravitons around the horizon scale at the end of inflation more in detail.

Gravitational Particle Production from Inflation to Kination

A scalar field χ with $\frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\xi R\chi^2$ in $ds^2 = a^2(\eta)(-d\eta^2 + d\mathbf{x}^2)$.

Its mode function satisfies $\frac{d^2\chi_k}{d\eta^2} + \left[k^2 - V(\eta)\right]\chi_k = 0$

$$V(\eta) = -a^2(\eta)\left[m_\chi^2 + \left(\xi - \tfrac{1}{6}\right)R(\eta)\right] \xrightarrow{\xi = 0,\, m_\chi = 0} V(\eta) = \tfrac{1}{6}a^2(\eta)R(\eta)$$

$$\chi_k(\eta) = \chi_k^{(in)}(\eta) + \frac{1}{\omega}\int_{-\infty}^{\eta} V(\eta')\sin\omega(\eta-\eta')\chi_k(\eta')d\eta'$$

$$\chi_k^{(in)}(\eta) = \frac{e^{-i\omega\eta}}{\sqrt{2\omega}} \quad (\eta \to -\infty) \qquad \chi_k(\eta) = \frac{1}{\sqrt{2\omega}}\left(\alpha_k e^{-i\omega\eta} + \beta_k e^{i\omega\eta}\right) \quad (\eta \to \infty)$$

Bogoliubov coefficient $\beta_\omega = \frac{i}{2\omega}\int_{-\infty}^{\infty} e^{-2i\omega\eta}V(\eta)d\eta$

"Radiation" energy density $\rho_r = \frac{1}{2\pi^2 a^4}\int_0^\infty |\beta_\omega|^2\omega^3 d\omega.$

Gravitational Particle Production from Inflation to Kination

$$\rho_r = -\frac{1}{32\pi^2 a^4}\int_{-\infty}^{\eta_0} d\eta_1 \int_{-\infty}^{\eta_0} d\eta_2 \ln(|\eta_1 - \eta_2|\mu) V'(\eta_1) V'(\eta_2)$$

Energy density of created massless minimally coupled field

$$\rho_r = \frac{H_{\rm inf}^4}{128\pi^2 a^4} I, \qquad I = -\int_{-\infty}^{x} dx_1 \int_{-\infty}^{x} dx_2 \ln(|x_1 - x_2|) \tilde{V}'(x_1) \tilde{V}'(x_2)$$

where $x \equiv H_{\rm inf}\eta$ $\qquad \tilde{V}(x) = \dfrac{f'' f - \frac{1}{2}(f')^2}{f^2} \qquad f(x) \equiv f(H_{\rm inf}\eta) \equiv a^2(\eta)$

We model the transition from inflation to kination as

$$f(x) = \begin{cases} 1/x^2 & \text{De Sitter} \quad (x < -1) \\ a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 & (-1 < x < -1 + x_0) \\ b_0(x + b_1) & \text{Kination} \quad (-1 + x_0 < x) \end{cases}$$

continuous transition up to $f^{(3)}$

We find $I \simeq 50 x_0^{-0.262} \qquad x_0 \cong H_{\rm inf}\Delta t \approx 1$

$$\boxed{\rho_r \cong \frac{9 H_{\rm inf}^4}{64\pi^2 a^4}}$$

(Nakama & JY 19)

Since each polarization mode of the graviton satisfies the same field equation as a minimally coupled massless scalar field.

$$\rho_{graviton} = \rho_{GW} \cong \frac{9H_{\mathrm{inf}}^4}{32\pi^2 a^4} \qquad a = 1 \text{ at the end of inflation}$$

It behaves as dark radiation similar to additional sterile massless neutrinos only to speed up cosmic expansion.

The energy density of GW is quantified in terms of an extra effective species of neutrinos, $N_{\mathrm{eff,GW}}$, and constrained by Big Bang Nucleosynthesis (BBN) and Cosmic Microwave Background (CMB) anisotropy.

The effective number appears in the expression of the total radiation energy density as follows.

$$\rho_{\rm tot} = \frac{\pi^2}{30}\left(2 + \frac{7}{8}\cdot 2\cdot\left(\frac{4}{11}\right)^{4\epsilon/3}[2(1-\epsilon) + N_\nu + N_{\rm eff,GW}]\right)T^4$$

$\varepsilon = 0$: at BBN (before neutrino decoupling)

$\varepsilon = 1$: at photon decoupling $N_\nu = 3 + 0.046\epsilon,$

$$\left.\frac{\rho_{\rm GW}}{\rho_{\rm rad}}\right|_{t>t_{\rm th}}\left(= \left[\frac{g_*}{g_*(t_{\rm th})}\right]^{1/3}\left.\left(\frac{\rho_{\rm GW}}{\rho_{\rm rad}}\right)\right|_{t=t_{\rm th}}\right) = g_*^{-1}\cdot\frac{7}{4}\left(\frac{4}{11}\right)^{4\epsilon/3} N_{\rm eff,GW}$$

At BBN $N_{\rm eff,GW} = 2.86\dfrac{\rho_{\rm GW}}{\rho_{\rm rad}}$ $g_* = 10.75$

At photon decoupling $N_{\rm eff,GW} = 2.36\dfrac{\rho_{\rm GW}}{\rho_{\rm rad}}$ $g_* = 3.38$

Big Bang Nucleosynthesis constraint $N_{\rm eff,GW} = 2.86 \dfrac{\rho_{GW}}{\rho_{\rm rad}} < 1.65$

CMB power spectrum (Planck) constraint $N_{\rm eff,GW} = 2.36 \dfrac{\rho_{GW}}{\rho_{\rm rad}} < 0.72$

In order to achieve reheating through minimally coupled massless scalar field χ, for which we find $\rho_\chi = \rho_{\rm GW}/2$, we need 7 or more species of them.

Since massless particles cannot decay, thermalization must proceed by scattering.
But χ field acquires a large vev during inflation due to quantum fluctuations, which means that particles coupled to χ are heavy if they are coupled to χ with sufficient strength.
Otherwise, they cannot thermalize and remain as a dark radiation.

Two working scenarios of gravitational reheating that can evade GW constraints with bonuses:

I. The universe is reheated by decay of long-lived massive scalar particles “A” and dark matter is explained by another heavy scalar particle “X”, both of which are created gravitationally.
This serves as an ideal realization mechanism of the PGDM (Purely Gravitational Dark Matter) scenario.

Garny, Sandora & Sloth (2016) Tang & Wu (2016) Ema, Nakayama & Tang (2018)

Hashiba & JY PRD99(2019)043008

II. Three species of massive right-handed neutrinos are created gravitationally, reheat the universe, generate lepton asymmetry and dark matter, evading the GW constraint.

Hashiba & JY Phys Lett B 798(2019)135024

We consider reheating through production of massive (conformally coupled) scalar fields $\chi = A, X$.

A scalar field χ with $\frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\xi R\chi^2$ in $ds^2 = a^2(\eta)(-d\eta^2 + d\boldsymbol{x}^2)$.

Its mode function satisfies $\frac{d^2\chi_k}{d\eta^2} + \left[k^2 - V(\eta)\right]\chi_k = 0$

$$V(\eta) = -a^2(\eta)\left[m_\chi^2 + \left(\xi - \tfrac{1}{6}\right)R(\eta)\right] \xrightarrow{\xi = \frac{1}{6},\, m_\chi = m_{A,X}} V(\eta) = -a^2(\eta)m_{A,X}^2$$

Inflation

$$\frac{d^2\chi_k}{d\eta^2} + \left(k^2 + \frac{m^2}{H_{\rm inf}^2\eta^2}\right)\chi_k = 0.$$

Bunch-Davies vacuum

$$\chi_k^{\rm BD}(\eta) = \frac{\sqrt{-\pi\eta}}{2}e^{-i\frac{2\nu+1}{4}\pi}H_\nu^{(1)}(-k\eta).$$

Kination

$$\frac{d^2\chi_k}{d\eta^2} + \left[k^2 + m^2(2H_{\rm inf}\eta + 3)\right]\chi_k = 0$$

Kination vacuum

$$\chi_k^{\rm K}(\eta) = \sqrt{\frac{\pi}{6}}\,(2m^2H_{\rm inf})^{-1/6}\exp\left[\left(\frac{k^3}{3m^2H_{\rm inf}} + \frac{3k}{2H_{\rm inf}} - \frac{5}{12}\pi\right)i\right]\sqrt{x}H_{1/3}^{(2)}\left(\frac{2}{3}x^{3/2}\right)$$

$$x(k,\eta) = \frac{k^2 + 3m^2 + 2m^2H_{\rm inf}\eta}{(2m^2H_{\rm inf})^{2/3}}$$

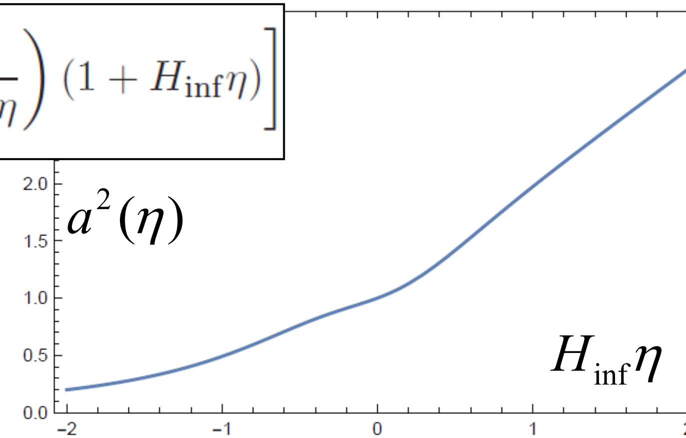
$$\chi_k(\eta) = \begin{cases} \chi_k^{\rm BD}(\eta) & \text{Inflation} \\ \alpha_k\chi_k^{\rm K}(\eta) + \beta_k\chi_k^{\rm K*}(\eta) & \text{Kination} \end{cases}$$

Bogoliubov coefficients

The mode function is numerically solved throughout a smooth transition from inflation to kination.

$$a^2(\eta) = \frac{1}{2}\left[\left(1 - \tanh\frac{\eta}{\Delta\eta}\right)\frac{1}{1+H_{\rm inf}^2\eta^2} + \left(1 + \tanh\frac{\eta}{\Delta\eta}\right)(1+H_{\rm inf}\eta)\right]$$

$\Delta\eta = 0.5 H_{\rm inf}^{-1}$ $\Rightarrow$



to calculate β_k and obtain

$$\rho_\chi = \int_0^\infty \frac{4\pi k^2 dk}{(2\pi)^3}\sqrt{m^2+k^2}|\beta_k|^2$$

We have numerically calculated many different cases

A. vary $H_{\rm inf}$, $(\Delta\eta, m)$ while fixing $H_{\rm inf}\Delta\eta$ and $m/H_{\rm inf}$.

B. vary $H_{\rm inf}$ while fixing $\Delta\eta$ and m.

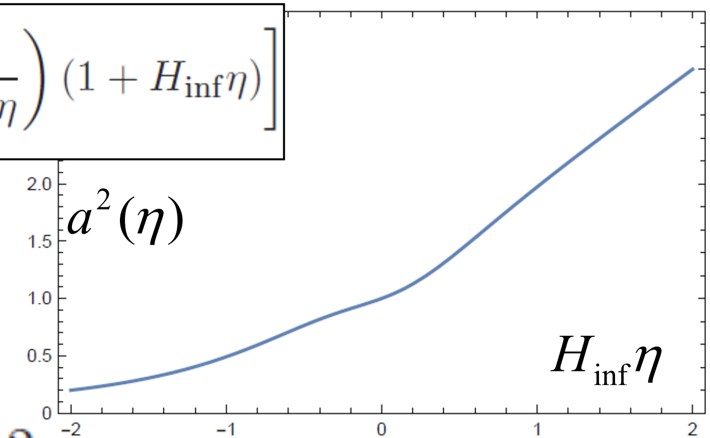
C. vary m while fixing $\Delta\eta$ and $H_{\rm inf}$.

D. vary $\Delta\eta$ while fixing m and $H_{\rm inf}$.

The mode function is numerically solved throughout a smooth transition from inflation to kination.

$$a^2(\eta) = \frac{1}{2}\left[\left(1 - \tanh\frac{\eta}{\Delta\eta}\right)\frac{1}{1 + H_{\rm inf}^2\eta^2} + \left(1 + \tanh\frac{\eta}{\Delta\eta}\right)(1 + H_{\rm inf}\eta)\right]$$

$\Delta\eta = 0.5 H_{\rm inf}^{-1}$ ⇨



to obtain

$$\rho_\chi = \int_0^\infty \frac{4\pi k^2 dk}{(2\pi)^3}\sqrt{m^2 + k^2}|\beta_k|^2$$

$$\chi = A, X$$

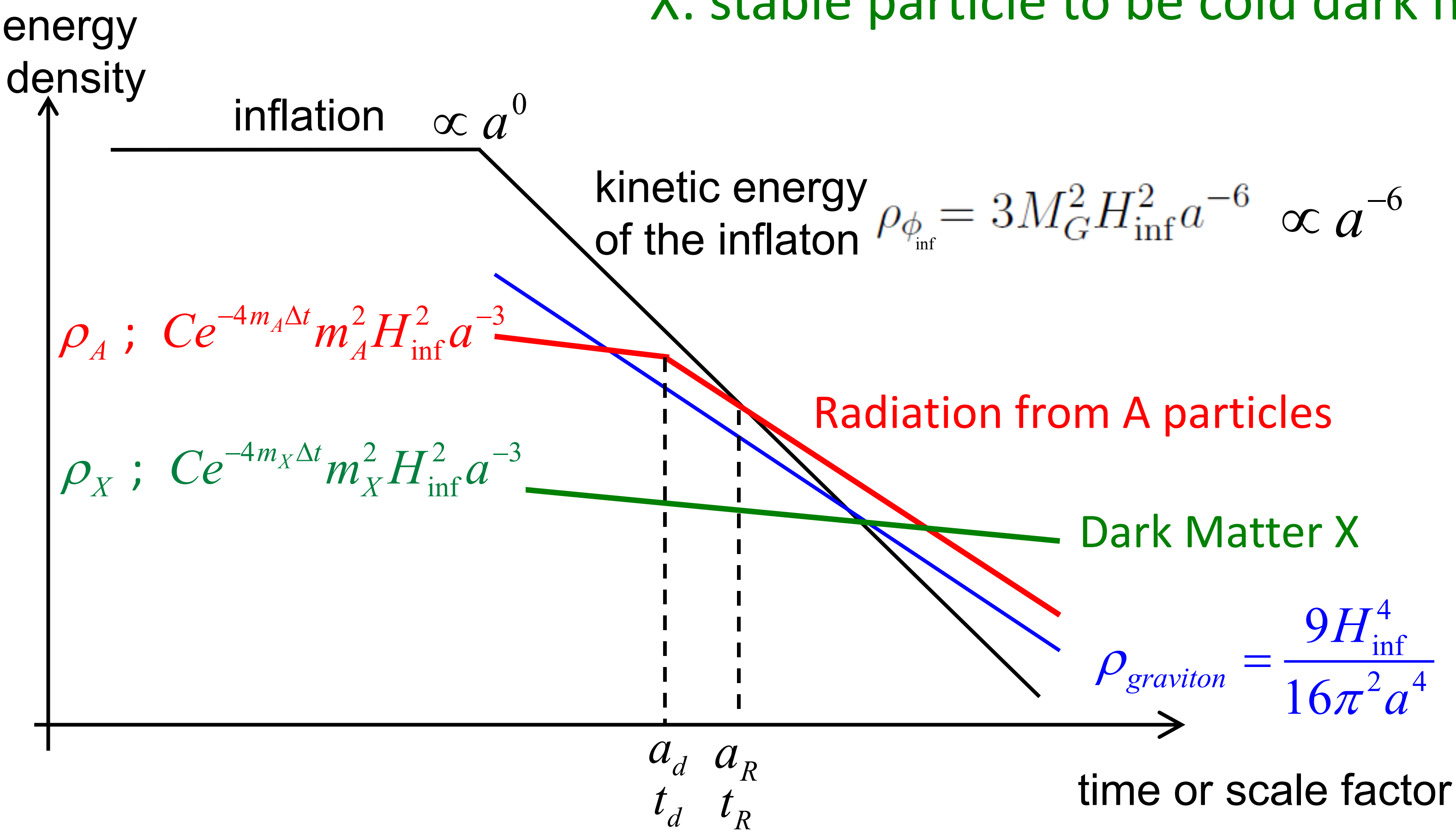
We have numerically calculated many different cases and found that the results are well approximated as

$$\rho_\chi = C e^{-4 m_\chi \Delta t} m_\chi^2 H_{\rm inf}^2 a^{-3}(t)$$

$$C \simeq 2\times 10^{-4}$$

$$\Delta t \simeq H_{\rm inf}^{-1}$$

★ We consider gravitational particle production of two conformally coupled massive scalar. A: decay into radiation to reheat the universe
X: stable particle to be cold dark matter today



A particles decay at $a = a_d$ to radiation with the decay rate $\Gamma_A \equiv \alpha m_A$.

The Universe became radiation dominant at $a = a_R$: reheating time.

★ The dominant part of the entropy is produced at t_d, when we find

$$s|_d = \frac{2\pi^2}{45} g_{*d} T_d^3 \quad \text{with} \quad T_d = 5 \times 10^{-2} \alpha^{1/4} e^{-m_A \Delta t} m_A^{3/4} H_{\rm inf}^{1/4},$$

$$g_{*d} = 106.75$$

$$\rho_X|_d = C\alpha e^{-4m_X \Delta t} m_A m_X^2 H_{\rm inf}.$$

$$\Rightarrow \quad \frac{\rho_X}{s} = 4 \times 10^{-2} \alpha^{1/4} e^{(3m_A - 4m_X)\Delta t} \frac{m_X^2 H_{\rm inf}^{1/4}}{m_A^{5/4}} \approx 4 \times 10^{-10}\ \mathrm{GeV}$$

should hold to explain CDM

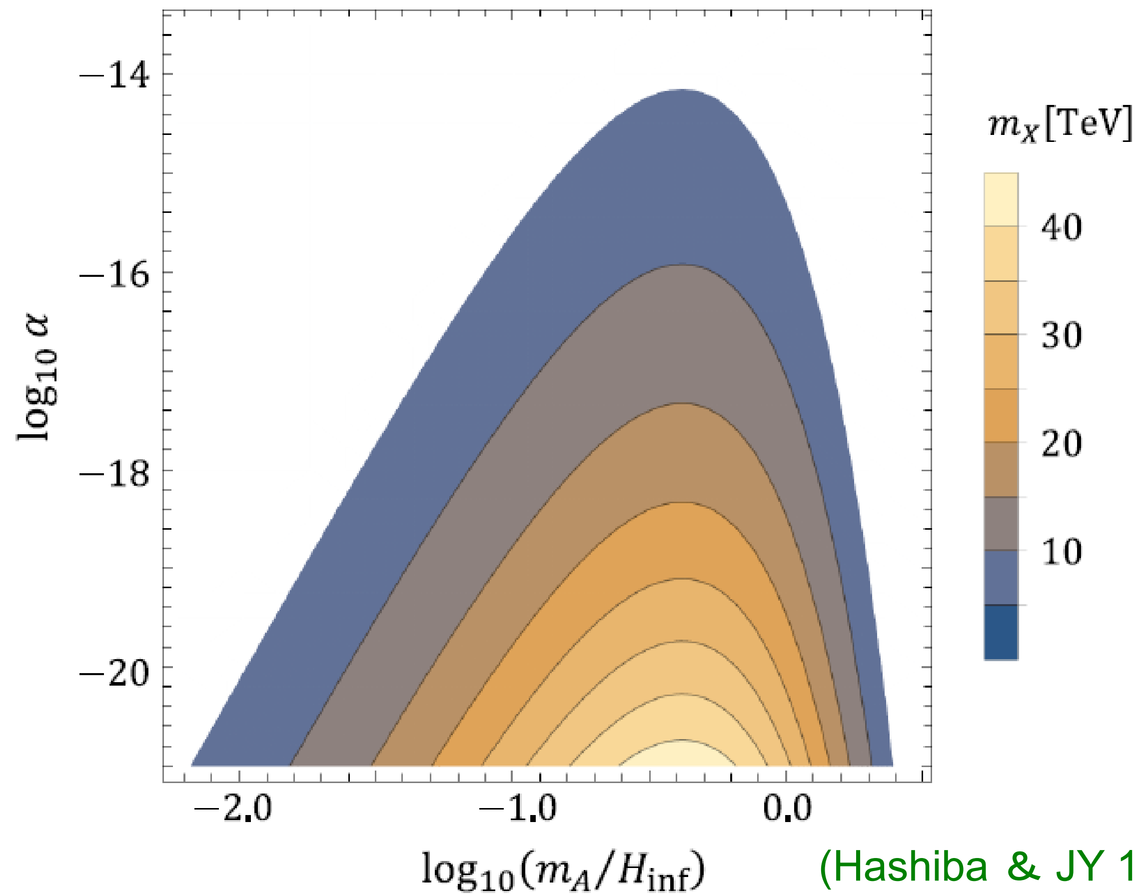
★ Gravitons (quantum GWs) must be sufficiently diluted

$$N_{\rm eff,GW} = 2.36 \frac{\rho_{GW}}{\rho_{\rm rad}} = 2.36 \left.\frac{\rho_{GW}}{\rho_A}\right|_d < 0.72$$

$$\Rightarrow \quad \alpha^{-1/3} e^{-4m_A \Delta t} \left(\frac{m_A}{H_{\rm inf}}\right)^{5/3} > 2.3 \times 10^3.$$

A severe constraint on α in $\Gamma_A = \alpha m_A$

Allowed region to explain CDM and reheating



(Hashiba & JY 19)

$\Gamma_A = \alpha m_A$ with $\alpha \ll 10^{-14}$ can be easily realized if A is coupled to the standard model with a

Planck-suppressed interaction $e^{-\lambda\frac{A}{M_{Pl}}}\mathscr{L}_{std}$ with $\lambda = O(1)$.

This model is a nightmare scenario for particle DM experiments as our DM interacts only gravitationally.

There is an astrophysical implication, though;
Comoving free streaming scale at equality time t_{eq} is very small

$$\lambda_{fs} = \int_{t_*}^{t_{eq}} \frac{v(t)}{a(t)} \sim \frac{ct_{eq}}{a_{eq} T_*} \ln\left(\frac{T_*}{T_{eq}}\right) \qquad T_* \geq T_R$$

and the minimum mass of DM halo is also very small.

$M_{\min} \sim 10^{-15} M_{\odot}$ for $T_* = 1\mathrm{TeV}$ and even smaller for higher temperature.

This may be tested by pulsar timing measurements.

Kashiyama & Oguri (2018)

Key Question:

What are A and X, after all?

II Neutrinos produce everything!

Hashiba & JY Phys Lett B 798(2019)135024

- We consider 3 hierarchical right-handed neutrinos

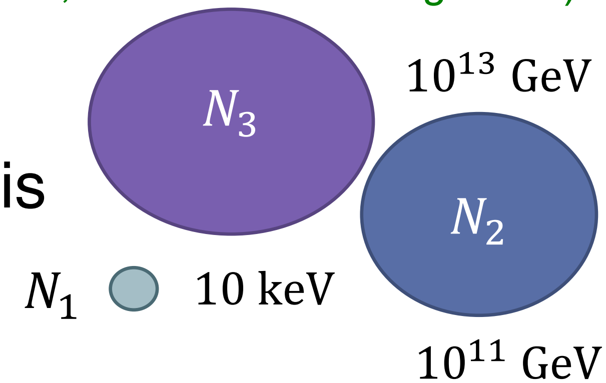
(Kusenko, Takahashi & Yanagida 10)

$N_3 : M_3 \sim 10^{13}$ GeV $\longrightarrow$ Reheating

$N_2 : M_2 \sim 10^{11}$ GeV

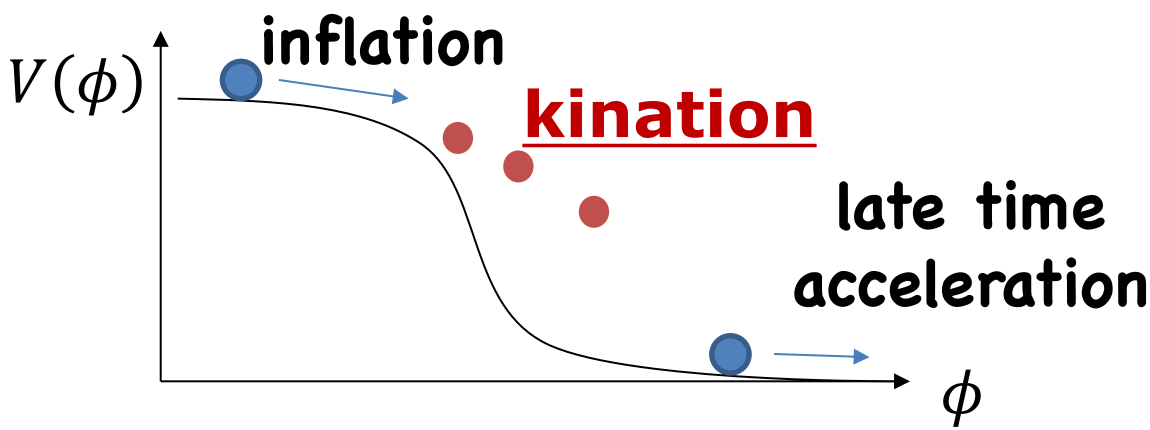
$\left.\begin{matrix} N_3 \\ N_2 \end{matrix}\right\} \longrightarrow$ Baryogenesis

$N_1 : M_1 \sim 10$ keV $\longrightarrow$ Dark matter



- $$\mathcal{L}_N = M_i \bar{N}_i^c N_i + h_{i\alpha} N_i L_\alpha H^\dagger$$

- Quintessential inflation which is followed by kination



(Peebles & Vilenkin 99)

Gravitational particle creation of fermions

- Produced fermion energy density

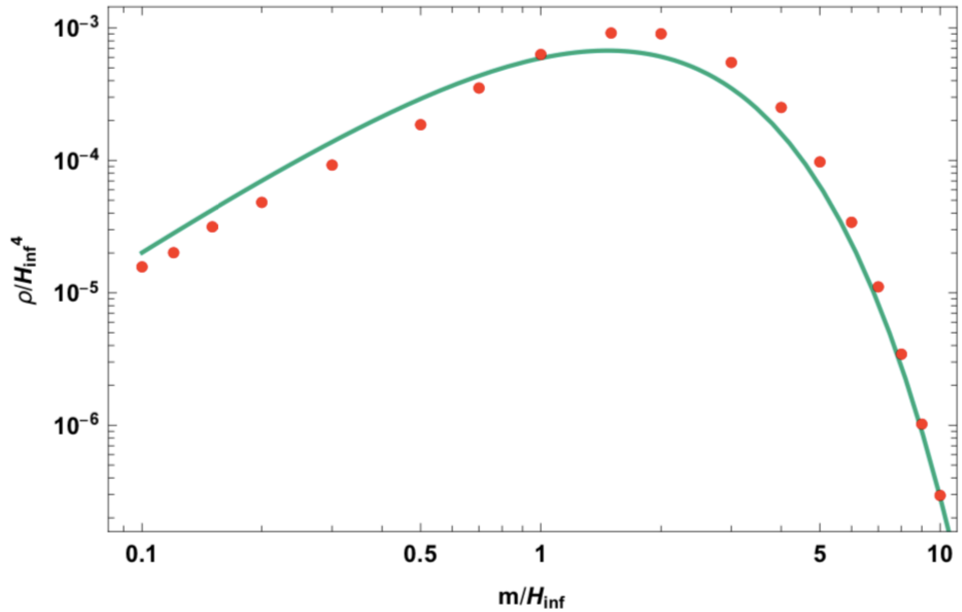
$$\rho \cong C' e^{-4m\Delta t} m^2 H_{\rm inf}^2 a^{-3}$$

m : Fermion mass

Δt : Transition time scale

$H_{\rm inf}$: Hubble parameter during inflation

$$C' \cong 2\times 10^{-3} \approx 10\mathrm{C}_{\rm scalar}$$



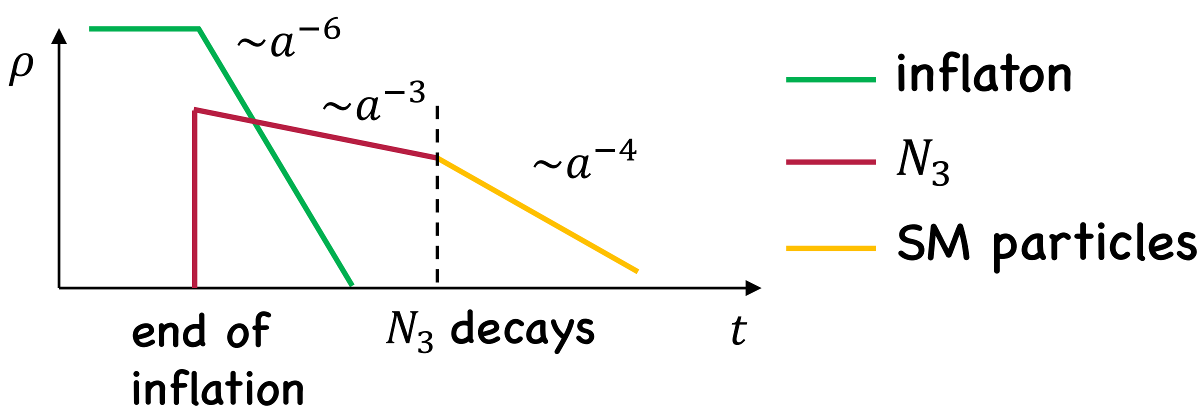
N_3 for reheating

- Decay of N_3

  N_3 decays into SM particles with decay rate Γ_3

$$\Gamma_3 = \frac{1}{4\pi} \sum_{\alpha} \left| \tilde{h}_{i\alpha} \right|^2 M_3$$

  Since N_3 is much heavier than any SM particles, resultant SM particles are relativistic



- Reheating temperature

$$T_{RH} \cong 6\times 10^{7} \left(\frac{\sum_\alpha \left|\tilde{h}_{3\alpha}\right|^2}{10^{-12}}\right)^{-\frac{1}{4}} e^{-3M_3\Delta t} \left(\frac{M_3}{10^{13}\mathrm{GeV}}\right)^{\frac{5}{4}} \left(\frac{H_{\mathrm{inf}}}{10^{13}\mathrm{GeV}}\right)^{\frac{3}{4}} \mathrm{GeV}$$

- Gravitons (quantum GWs) must be sufficiently diluted

$$N_{\mathrm{eff,GW}} = 2.36\frac{\rho_{GW}}{\rho_{\mathrm{rad}}} = 2.36\left.\frac{\rho_{GW}}{\rho_A}\right|_d < 0.72$$

$$\Rightarrow \left(\sum_\alpha \left|\tilde{h}_{3\alpha}\right|^2\right)^{-\frac{1}{3}} e^{-4M_3\Delta t}\left(\frac{M_3}{H_{\mathrm{inf}}}\right)^{\frac{5}{3}} > 1.0\times 10^{2}$$

$$\Rightarrow \sum_\alpha \left|\tilde{h}_{3\alpha}\right|^2 < 8.5\times 10^{-11}$$

~ Yukawa coupling of electron

Baryogenesis through leptogenesis

(Fukugita & Yanagida 86)

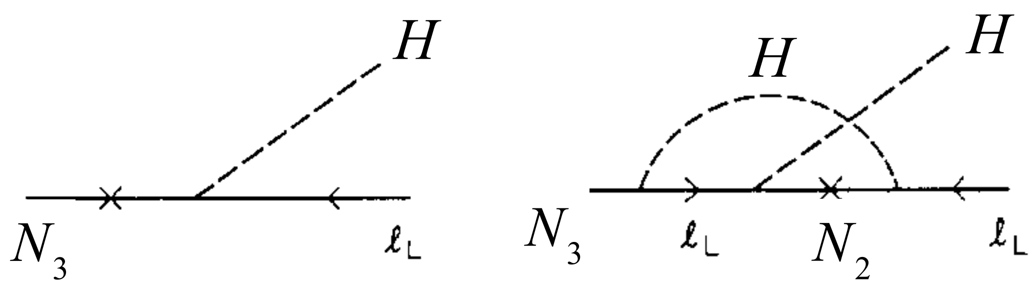
$$\frac{n_B}{s} = \frac{28}{79}\frac{n_L}{s}$$

$$\approx 1\times 10^{-3}\,\frac{\mathrm{Im}\left[\left\{(\tilde{h}\tilde{h}^\dagger)_{32}\right\}^2\right]}{(\tilde{h}\tilde{h}^\dagger)_{33}}\left(e^{-M_3\Delta t}\ln\frac{M_3}{M_2}\right)\left(\sum_\alpha |\tilde{h}_{3\alpha}|^2\right)^{\frac{1}{4}}\frac{M_2}{M_3}\left(\frac{M_3}{H_{\mathrm{inf}}}\right)^{-\frac{1}{4}}$$

$$= 9\times 10^{-11}$$

CP violation

Interference between N_3 & N_2



$$M_2 \gtrsim 10^{11}\ \mathrm{GeV} \quad \text{and} \quad \tilde{h}_{22}\ \text{or}\ \tilde{h}_{23} \gtrsim 10^{-3}\sqrt{M_3/M_2}$$

N_1 For Dark Matter in split seasaw scenario

(Kusenko, Takahashi & Yanagida 10)

$\sim 10\mathrm{keV}$ sterile neutrino can account for whole dark matter!

$$\theta^2 \simeq \sum_\alpha \left|\tilde{h}_{1\alpha}\right|^2 \frac{v^2}{2M_1^2} \sim 10^{-11}$$

$$v = 246\mathrm{GeV}$$

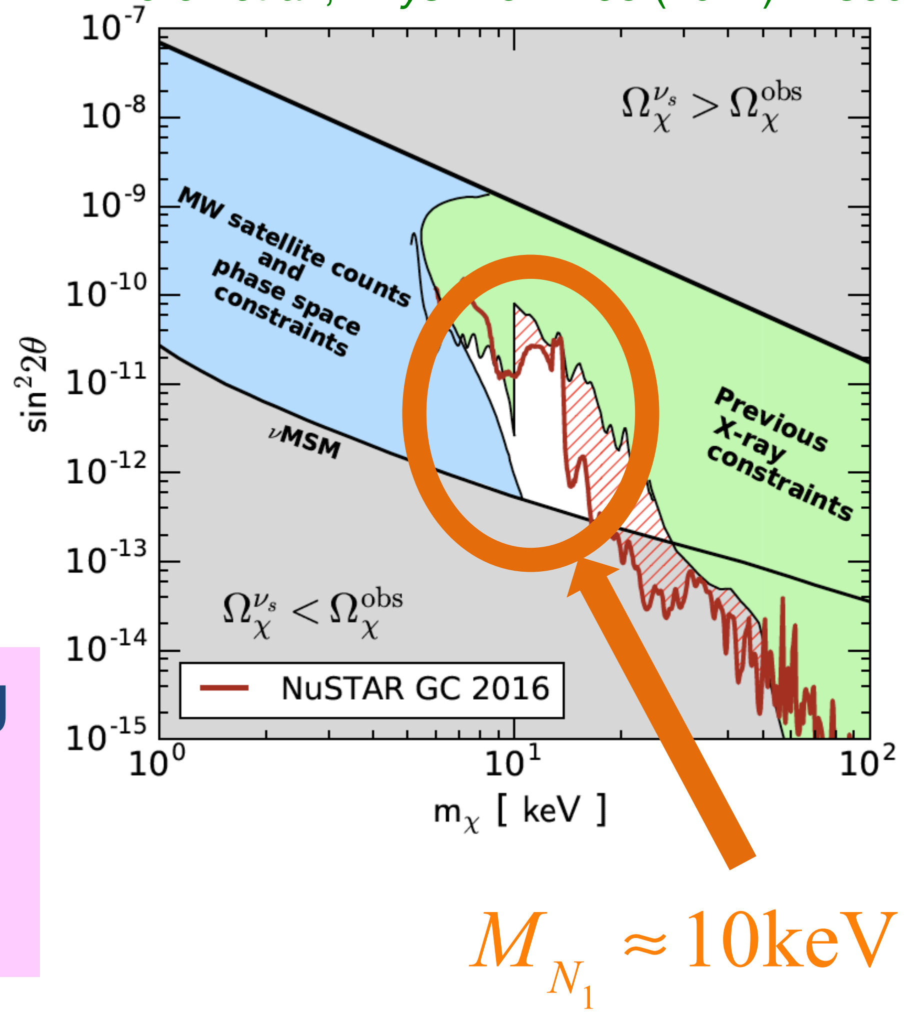
For $M_1 \sim 10\ \mathrm{keV}$,

$$\sum_\alpha \left|\tilde{\boldsymbol{h}}_{1\alpha}\right|^2 < 10^{-26}$$

This small Yukawa coupling can be naturally explained in Randall-Sundrum type Brance world scenario.

Allowed region for sterile neutrino dark matter

K. Perez *et al.*, *Phys. Rev.* **D95** (2017) 123002.



Since the mass of N_1 is too light, its gravitational production is too inefficient to account for dark matter.

This problem can be solved by introducing a coupling to the scalar curvature as

$$\frac{R}{\mu}\bar{\psi}\psi \qquad \mu \text{ : constant with unit mass dimension}$$

Then the abundance is given by $n \cong 1.1\times10^{-1}\, H_{\rm inf}^5/\mu^2$ right after inflation with $\Delta t \approx H_{\rm inf}^{-1}$.

Taking $\mu \approx 10^{15}\mathrm{GeV}$, we can realize sufficient production for dark matter.

Conclusion

Cosmological gravitational wave is a useful probe of the early universe even if it is not detected yet.

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- RS brane-world scenario

We identify the zero mode of a 5D bulk field $\overline{\Psi}_i$ with the 4D right-handed neutrino N_i

$$S = \int d^4x dy \{M(i\overline{\Psi}_i \gamma^A \partial_A \Psi_i + m_i \overline{\Psi}_i \Psi_i)$$
$$+\delta(y)\left(\frac{\kappa_i}{2} v_{\mathrm{B-L}} \overline{\Psi}_i^c \Psi_i + \lambda_{i\alpha} \overline{\Psi}_i L_\alpha H^\dagger + \mathrm{h.c.}\right)\}$$

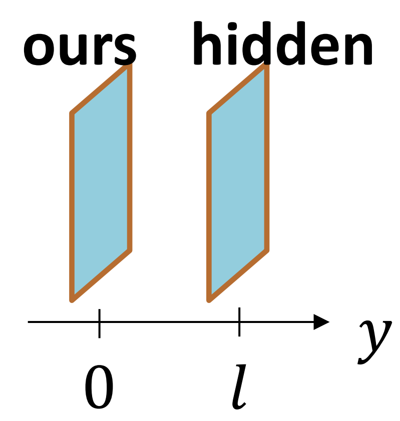
M : 5D fundamental scale $\sim 5\times 10^{17}$ GeV

m_i : bulk mass

l : size of extra dimension $\sim (10^{16}\ \mathrm{GeV})^{-1}$

κ_i : numerical constant of order unity

$v_{\mathrm{B-L}}$: VEV of $\mathrm{B-L}$ gauge boson $\sim 10^{16}$ GeV



L. Randall and R. Sundrum, *Phys. Rev. Lett.* 83 (1999) 3370.

- RS brane-world scenario

RS brane-world scenario can explain

	4D		5D
Large mass hierarchy	M_i	$=$	$\kappa_i v_{\mathrm{B-L}} \dfrac{2m_i}{M(e^{2m_i l}-1)}$
Extremely small coupling	$\tilde{h}_{i\alpha}$	$=$	$\dfrac{\lambda_{i\alpha}}{\sqrt{M}} \sqrt{\dfrac{2m_i}{e^{2m_i l}-1}}$

4D parameters

$M_3 \sim 10^{13}$ GeV $\quad \tilde{h}_{3\alpha} < 3\times 10^{-6}$

$M_2 \sim 10^{11}$ GeV $\quad \tilde{h}_{22,\,23} \sim 10^{-2}$

$M_1 \sim 10$ keV $\quad \tilde{h}_{1\alpha} < 10^{-13}$

5D parameters

$m_3 \sim 2.3 l^{-1} \quad \lambda_{3\alpha} < 3\times 10^{-4}$

$m_2 \sim 3.6 l^{-1} \quad \lambda_{22,\,23} \sim 1$

$m_1 \sim 24 l^{-1} \quad \lambda_{1\alpha} < 10^{-2}$

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Tensor Perturbations

The quadratic action $\alpha = \beta = \mathcal{R} = 0$

$$S_T^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\vec{\nabla} h_{ij})^2\right]$$

$$\mathcal{F}_T := 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right],$$
$$\mathcal{G}_T := 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right] = \frac{1}{2}\sum_{i=2}^{5}\frac{\partial \mathcal{P}_i}{\partial \dot{H}}.$$

The "sound" velocity $c_T^2 \equiv \mathcal{F}_T / \mathcal{G}_T$ deviates from unity if $G_{4X} \neq 0$, $G_{5X} \neq 0$ or $G_{5\phi} \neq 0$.

No ghosts, No gradient instability if $\mathcal{G}_T > 0$, $c_T^2 = \dfrac{\mathcal{F}_T}{\mathcal{G}_T} > 0$.

Introducing new variables $dy_T = \frac{c_T}{a}dt,\ \ z_T = \frac{a}{2}\left(\mathcal{F}_T\mathcal{G}_T\right)^{1/4},\ \ v_{ij} = z_T h_{ij},$

the action reads
$$S_T^{(2)} = \frac{1}{2}\int \mathrm{d}y_T \mathrm{d}^3x \left[(v'_{ij})^2 - (\vec{\nabla} v_{ij})^2 + \frac{z''_T}{z_T}v_{ij}^2\right]$$

The normalized mode function in k space reads

$$v_{ij} = \frac{\sqrt{\pi}}{2}\sqrt{-y_T}H^{(1)}_{\nu_T}(-ky_T)\,\mathrm{e}_{ij}, \qquad \nu_T := \frac{3-\epsilon+g_T}{2-2\epsilon-f_T+g_T}$$

where we have defined the slow-variation parameters:

$$\varepsilon \equiv -\frac{\dot{H}}{H^2},\quad f_T \equiv \frac{\dot{\mathcal{F}}_T}{H\mathcal{F}_T},\quad g_T \equiv \frac{\dot{\mathcal{G}}_T}{H\mathcal{G}_T},\quad s_T \equiv \frac{\dot{c}_T}{Hc_T}.$$

(assumed to be constant, if not small.)

The tensor power spectrum and spectral index read

$$\mathcal{P}_T(k) = \frac{k^3}{2\pi^2}\left|\frac{v_{ij}}{z_T}\right|^2 = 2^{\nu_T}\left|\frac{\Gamma(3/2)}{\Gamma(\nu_T)}\right|\frac{(1-\varepsilon-s_T)}{4\pi^2}\frac{H^2}{\mathcal{F}_T c_T}\Bigg|_{ky_T=-1}$$

$$n_T = 3-2\nu_T = -\frac{4\varepsilon+3f_T-g_T}{2(1-\varepsilon-s_T)}$$

Blue spectrum if $4\varepsilon+3f_T-g_T<0.$

Curvature Perturbations

We adopt the unitary gauge in which ϕ is homogeneous, $\delta\phi = 0$.

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\partial_i\beta\, dt\, dx^i + a^2(1+2\mathcal{R})d\mathbf{x}^2$$

Curvature Perturbation

As usual,

① Expand the action to the second order.

② Eliminate α and β using constraint equations.

③ Obtain a quadratic action for $\mathcal{R}$.

$$\mathcal{G}_T\dot{\mathcal{R}} = \Theta\alpha,$$

$$\frac{\nabla^2}{a^2}\left(\mathcal{G}_T\mathcal{R} + a^2\Theta\beta\right) = \Sigma\alpha + 3\Theta\dot{\mathcal{R}}$$

$$S_S^{(2)} = \frac{1}{2}\int dt\, d^3x\, a^3\left[\mathcal{G}_S\dot{\mathcal{R}}^2 - \frac{\mathcal{F}_S}{a^2}(\nabla\mathcal{R})^2\right],$$

where

$$\mathcal{F}_S := \frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a}{\Theta}\mathcal{G}_T^2\right) - \mathcal{F}_T,$$

$$\mathcal{G}_S := \frac{\Sigma}{\Theta^2}\mathcal{G}_T^2 + 3\mathcal{G}_T.$$

with

$$\begin{aligned}
\Sigma := {}& XK_X + 2X^2K_{XX} + 12H\dot{\phi}XG_{3X} \\
&+6H\dot{\phi}X^2G_{3XX} - 2XG_{3\phi} - 2X^2G_{3\phi X} - 6H^2G_4 \\
&+6\Big[H^2\left(7XG_{4X} + 16X^2G_{4XX} + 4X^3G_{4XXX}\right) \\
&-H\dot{\phi}\left(G_{4\phi} + 5XG_{4\phi X} + 2X^2G_{4\phi XX}\right)\Big] \\
&+30H^3\dot{\phi}XG_{5X} + 26H^3\dot{\phi}X^2G_{5XX} \\
&+4H^3\dot{\phi}X^3G_{5XXX} - 6H^2X\big(6G_{5\phi} \\
&+9XG_{5\phi X} + 2X^2G_{5\phi XX}\big), \\
\Theta := {}& -\dot{\phi}XG_{3X} + 2HG_4 - 8HXG_{4X} \\
&-8HX^2G_{4XX} + \dot{\phi}G_{4\phi} + 2X\dot{\phi}G_{4\phi X} \\
&-H^2\dot{\phi}\left(5XG_{5X} + 2X^2G_{5XX}\right) \\
&+2HX\left(3G_{5\phi} + 2XG_{5\phi X}\right).
\end{aligned}$$

$$S_S^{(2)} = \frac{1}{2}\int dt d^3 x a^3 \left[\mathcal{G}_S \dot{\mathcal{R}}^2 - \frac{\mathcal{F}_S}{a^2}(\nabla \mathcal{R})^2 \right],$$

No ghosts, No gradient instability if $\mathcal{G}_S > 0, \quad c_S^2 = \dfrac{\mathcal{F}_S}{\mathcal{G}_S} > 0.$

In k inflation where $G_3 = G_5 = 0,\ G_4 = M_{Pl}^2/2$ hold,
we find $\mathcal{F}_S = M_{Pl}^2 \varepsilon = -M_{Pl}^2 \dot{H}/H^2$, which means that
$\dot{H} > 0$ is prohibited by the stability condition.
But in G-inflation $\dot{H} > 0$ is possible.

Introducing new variables $dy_S = \dfrac{c_S}{a} dt,\ \ z_S = \dfrac{a}{2}(\mathcal{F}_S \mathcal{G}_S)^{1/4},\ \ u = z_S \mathcal{R},$
the action reads

$$S_S^{(2)} = \frac{1}{2}\int \mathrm{d}y_S \mathrm{d}^3 x \left[(u')^2 - (\vec{\nabla} u)^2 + \frac{z_S''}{z_S} u^2 \right]$$

The normalized mode function in k space reads

$$u_k = \frac{\sqrt{\pi}}{2}\sqrt{-y_S} H_{\nu_S}^{(1)}(-k y_S), \quad \nu_S = \frac{3 - \varepsilon + g_S}{2 - 2\varepsilon - f_S + g_S}$$

with $f_S \equiv \dfrac{\dot{\mathcal{F}}_S}{H \mathcal{F}_S}, \quad g_S \equiv \dfrac{\dot{\mathcal{G}}_S}{H \mathcal{G}_S}.$

The scalar power spectrum and spectral index read

$$\mathcal{P}_S(k) = \frac{k^3}{2\pi^2}\left|\frac{u_k}{z_S}\right|^2 = 2^{2\nu_S - 3}\left|\frac{\Gamma(\nu_S)}{\Gamma(3/2)}\right|\frac{(1-\varepsilon-s_S)}{4\pi^2}\frac{H^2}{\mathcal{F}_S c_S}\Bigg|_{ky_S=-1}$$

$$n_S - 1 = 3 - 2\nu_S = -\frac{4\varepsilon + 3f_S - g_S}{2(1-\varepsilon-s_S)}$$

scale invariant if $4\varepsilon + 3f_S - g_S = 0$

The tensor-to-scalar ratio for small variation parameters

$$r = 16\left(\frac{\mathcal{F}_S}{\mathcal{F}_T}\right)^{3/2}\left(\frac{\mathcal{G}_S}{\mathcal{G}_T}\right)^{-1/2} = 16\frac{\mathcal{F}_S}{\mathcal{F}_T}\frac{c_S}{c_T}.$$

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The case of cosmic strings produced after inflation

(Cui, Lewicki, Morrissey, Wells 18)

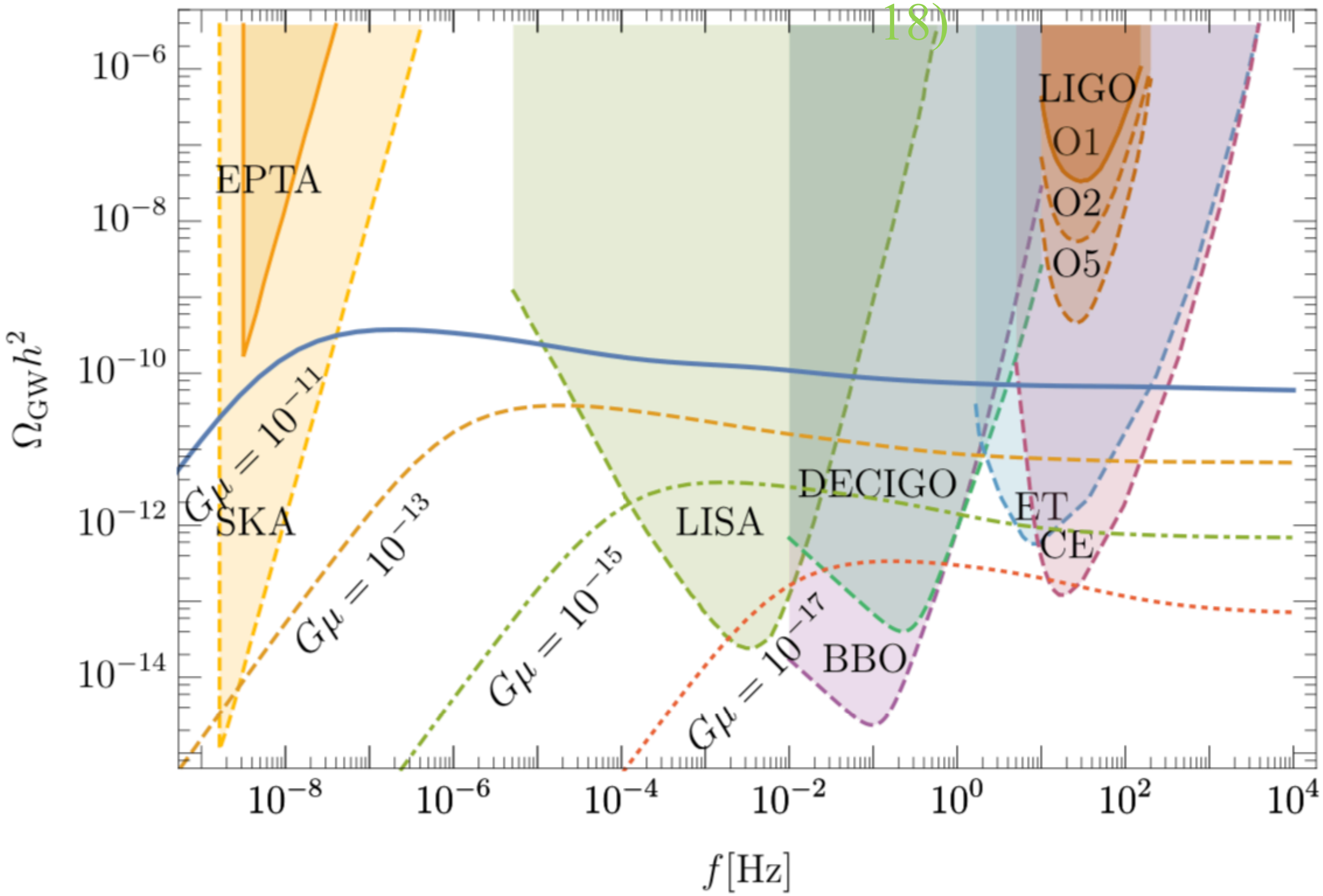
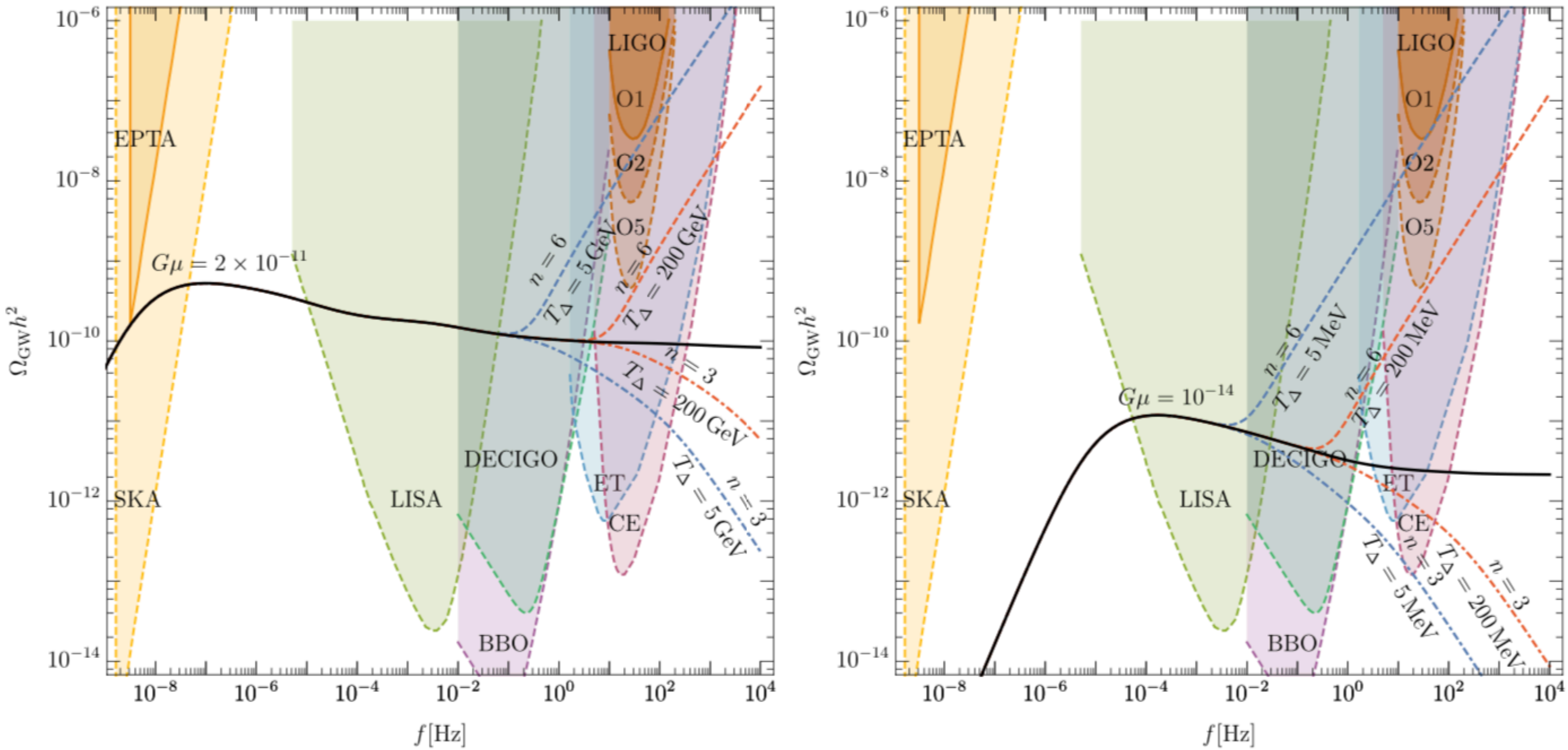


Figure 2. Gravitational wave spectrum from a cosmic string network with $\alpha = 0.1$ and $G\mu = 10^{-11}, 10^{-13}, 10^{-15}, 10^{-17}$. Also shown are the current sensitivities of LIGO and EPTA (solid bounded regions), and the projected future sensitivities of LISA, DECIGO/BBO, ET/CE, and SKA (dash bounded regions).

(Cui, Lewicki, Morrissey, Wells 18)



上州人

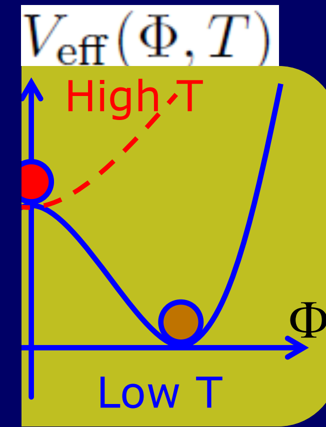
上州無智亦無才
剛毅木訥易被欺
唯以正直接萬人
至誠依神期勝利

鑑三

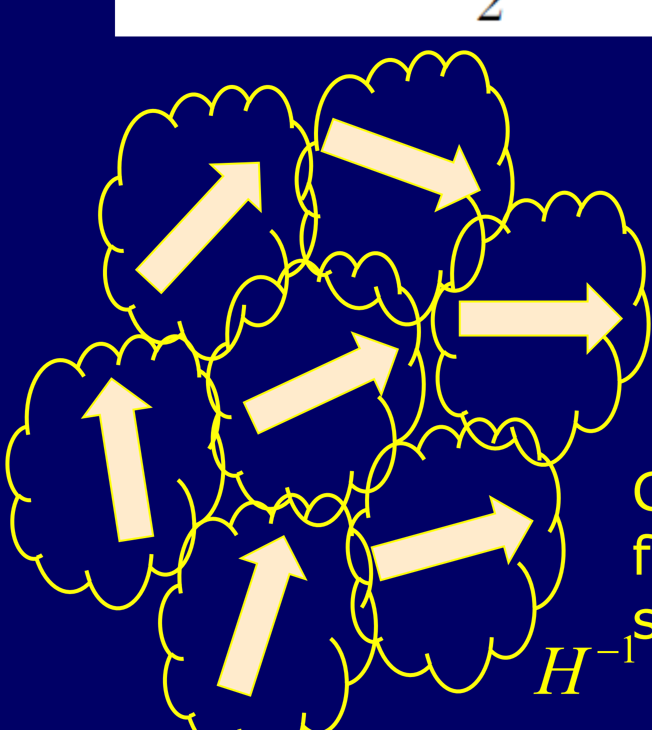
Another source of scale-invariant stochastic GWs: The self-ordering scalar field

Consider a global phase transition of an N-component scalar field

$$\Phi = (\phi_1, \phi_2, ..., \phi_a, ..., \phi_N) \qquad \Phi^2 = \sum_a \phi_a^2$$

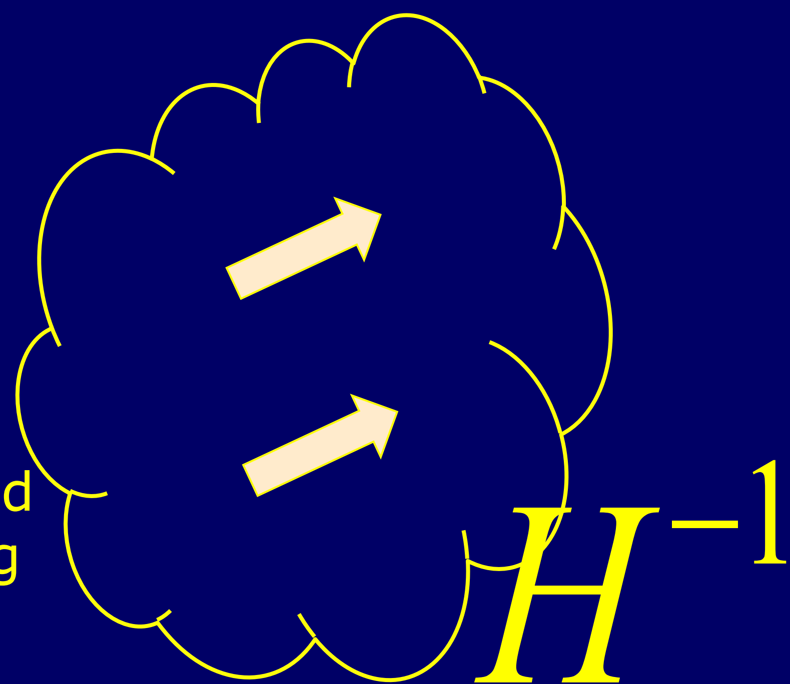


$$V_{\rm eff}(\Phi, T) = \frac{\lambda}{2}(\Phi^2 - v^2)^2 + \frac{\lambda}{3}T^2\Phi^2$$



Scalar field is homogenized as the universe expands

GWs are produced from self ordering scalar field



$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

$$\phi_a''(\tau, \boldsymbol{x}) + 2\mathcal{H}\phi_a'(\tau, \boldsymbol{x}) - \nabla^2\phi_a(\tau, \boldsymbol{x}) = -a^2\frac{\partial V_{\rm eff}}{\partial \phi_a}$$

$$h_{ij}''(\tau, \boldsymbol{x}) + 2\mathcal{H}h_{ij}'(\tau, \boldsymbol{x}) - \nabla^2 h_{ij}(\tau, \boldsymbol{x}) = \frac{2}{M_{\rm pl}^2}\Pi_{ij}^{\rm TT}(\tau, \boldsymbol{x})$$

$$\Pi_{ij}^{\rm TT}(\tau, \boldsymbol{k}) = \Lambda_{ij,k\ell}(\hat{k})\Pi_{k\ell}(\tau, \boldsymbol{k})$$

$$\Pi_{ij}(\tau, \boldsymbol{x}) = \sum_a \left[\partial_i\phi_a(\tau, \boldsymbol{x})\partial_j\phi_a(\tau, \boldsymbol{x}) - \frac{1}{3}\delta_{ij}\partial_k\phi_a(\tau, \boldsymbol{x})\partial^k\phi_a(\tau, \boldsymbol{x})\right]$$

GWs are generated when each Fourier mode enters the Hubble horizon, in contrast with inflationary tensor fluctuations generated when they go out of the horizon.

Krauss(1992), Krauss et al(2010), Dent et al(2014), Durrer et al(2014)

We analyze time evolution of rescaled variables $\psi_a = a\phi_a$ and $h_{ij} = a\Lambda_{ij,kl}\chi_{kl}$ on a 3D lattice.

$$\psi_a''(\tau,\boldsymbol{x}) - \frac{a''}{a}\psi_a(\tau,\boldsymbol{x}) - \nabla^2\psi_a(\tau,\boldsymbol{x}) = -2\lambda a^2\left(\Phi^2 - v^2 + \frac{T^2}{3}\right)\psi_a(\tau,\boldsymbol{x})$$

$$\chi_{ij}''(\tau,\boldsymbol{x}) - \frac{a''}{a}\chi_{ij}(\tau,\boldsymbol{x}) - \nabla^2\chi_{ij}(\tau,\boldsymbol{x}) = \frac{2}{M_{\rm pl}^2 a}\sum_a \left[\partial_i\psi_a(\tau,\boldsymbol{x})\partial_j\psi_a(\tau,\boldsymbol{x})\right]$$

to calculate the energy density of GWs

$$\rho_{\rm GW}(\tau) = \frac{M_{\rm pl}^2}{4a^2}\langle h_{ij}'(\tau,\boldsymbol{x})h_{ij}'(\tau,\boldsymbol{x})\rangle_V = \frac{M_{\rm pl}^2}{4a^4}\frac{1}{V}\int\frac{d^3k}{(2\pi)^3}\left[\Lambda_{ij,kl}(\hat{k})\chi_{kl}'(\tau,\boldsymbol{k})\right]^2$$ and

its density parameter per logarithmic frequency interval

$$\Omega_{\rm GW}(k) \equiv \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\log k} = \frac{k^3}{96\pi^3\mathcal{H}^2a^2V}\int d\Omega\,\chi_{ij}'(\tau,\boldsymbol{k})\chi_{ij}'^*(\tau,\boldsymbol{k})$$

again in the reheating regime after inflation

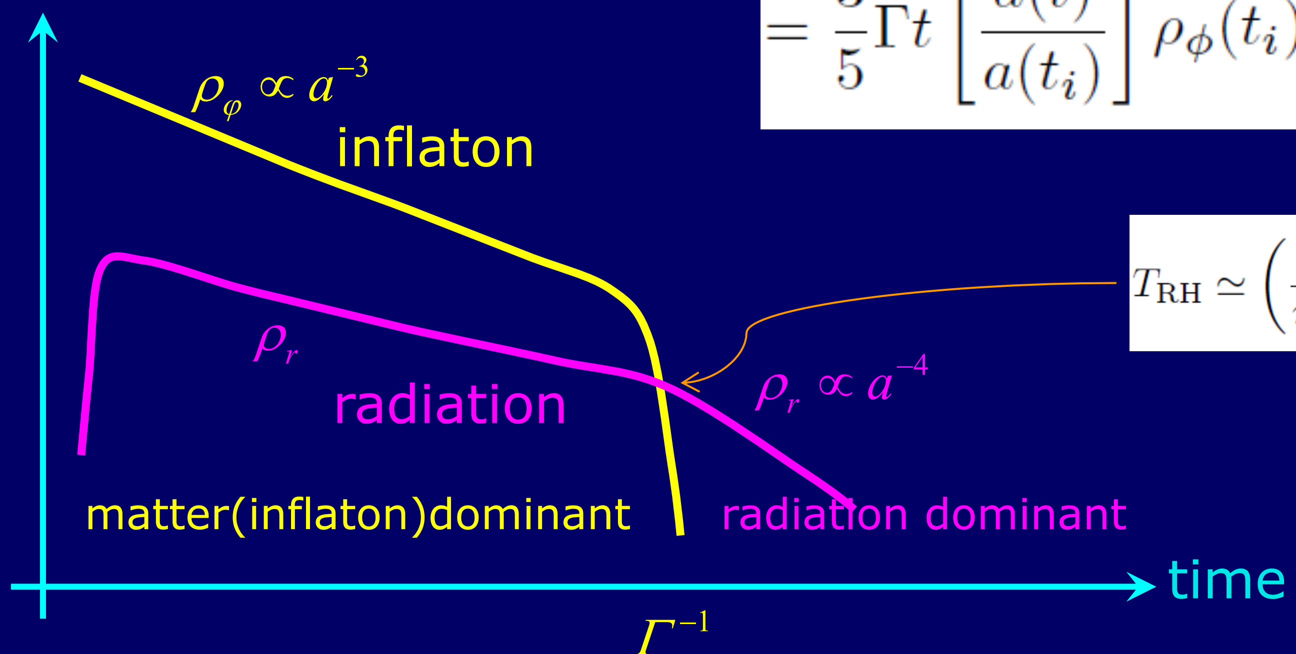
$$\rho'_\varphi + 3\mathcal{H}\rho_\varphi = -a\Gamma\rho_\varphi, \qquad \boxed{\mathcal{H}^2 = \frac{a^2}{3M_{pl}^2}(\rho_\varphi + \rho_r)} \qquad \rho'_r + 4\mathcal{H}\rho_r = a\Gamma\rho_\varphi$$

$$\text{inflaton} \xrightarrow[\text{decay rate } \Gamma]{} \text{radiation}$$

$$\rho_\phi(t) = \rho_\phi(t_i)\left[\frac{a(t)}{a(t_i)}\right]^{-3} e^{-\Gamma(t-t_i)}$$

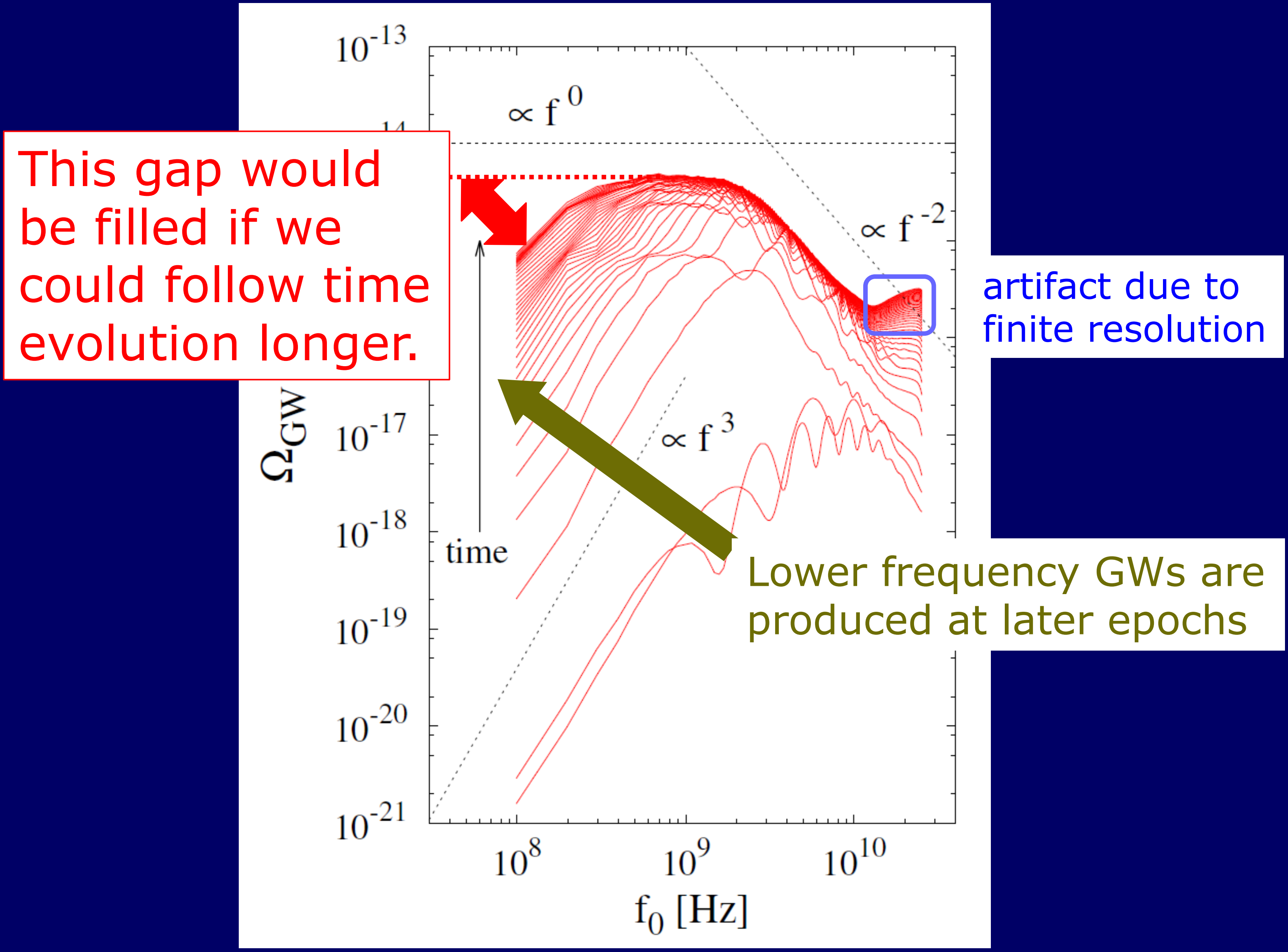
$$\rho_r(t) = \Gamma\int_{t_i}^{t}\left[\frac{a(t)}{a(t')}\right]^{-3}\rho_\phi(t')dt'$$

$$= \frac{3}{5}\Gamma t\left[\frac{a(t)}{a(t_i)}\right]^{-3}\rho_\phi(t_i) \cong \frac{6}{5}\Gamma H M_{pl}^2$$

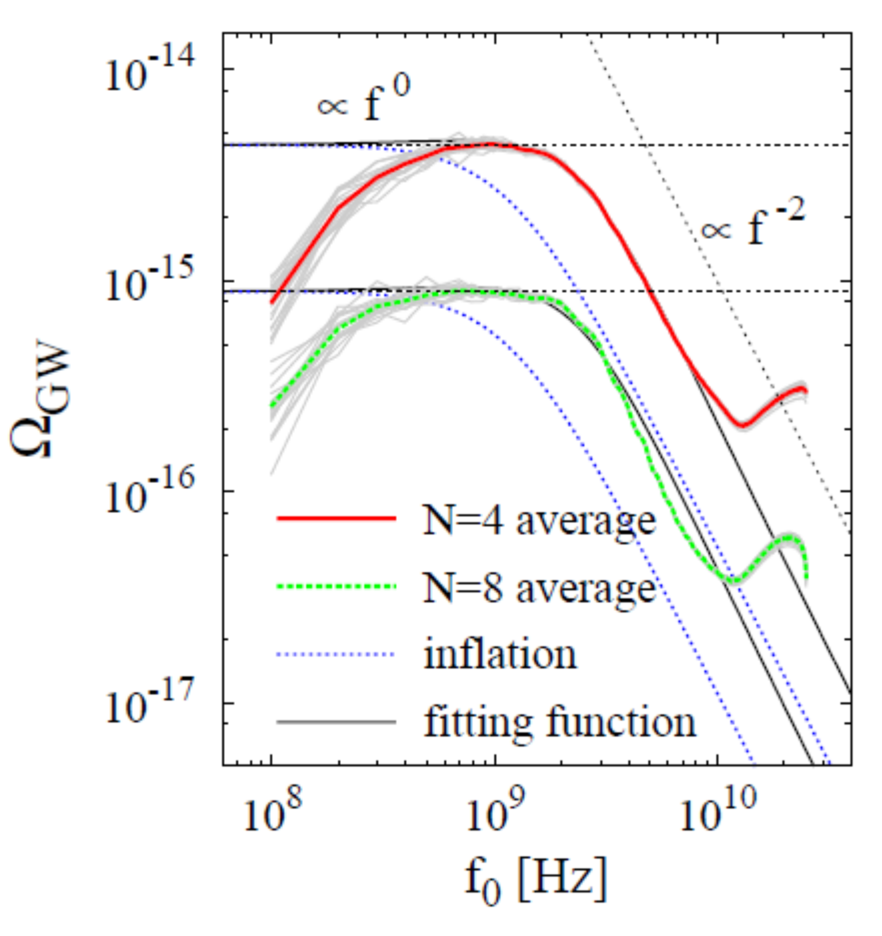


$$T_{\rm RH} \simeq \left(\frac{10}{\pi^2 g_*}\right)^{\frac{1}{4}} (M_{\rm pl}\Gamma)^{\frac{1}{2}}$$

Typical result



Comparison with inflationary tensor perturbations



$$T^2_{\text{inflation}}(x_R) = \left(1 - \boxed{0.22}\,x_R^{1.5} + 0.65x_R^2\right)^{-1}$$

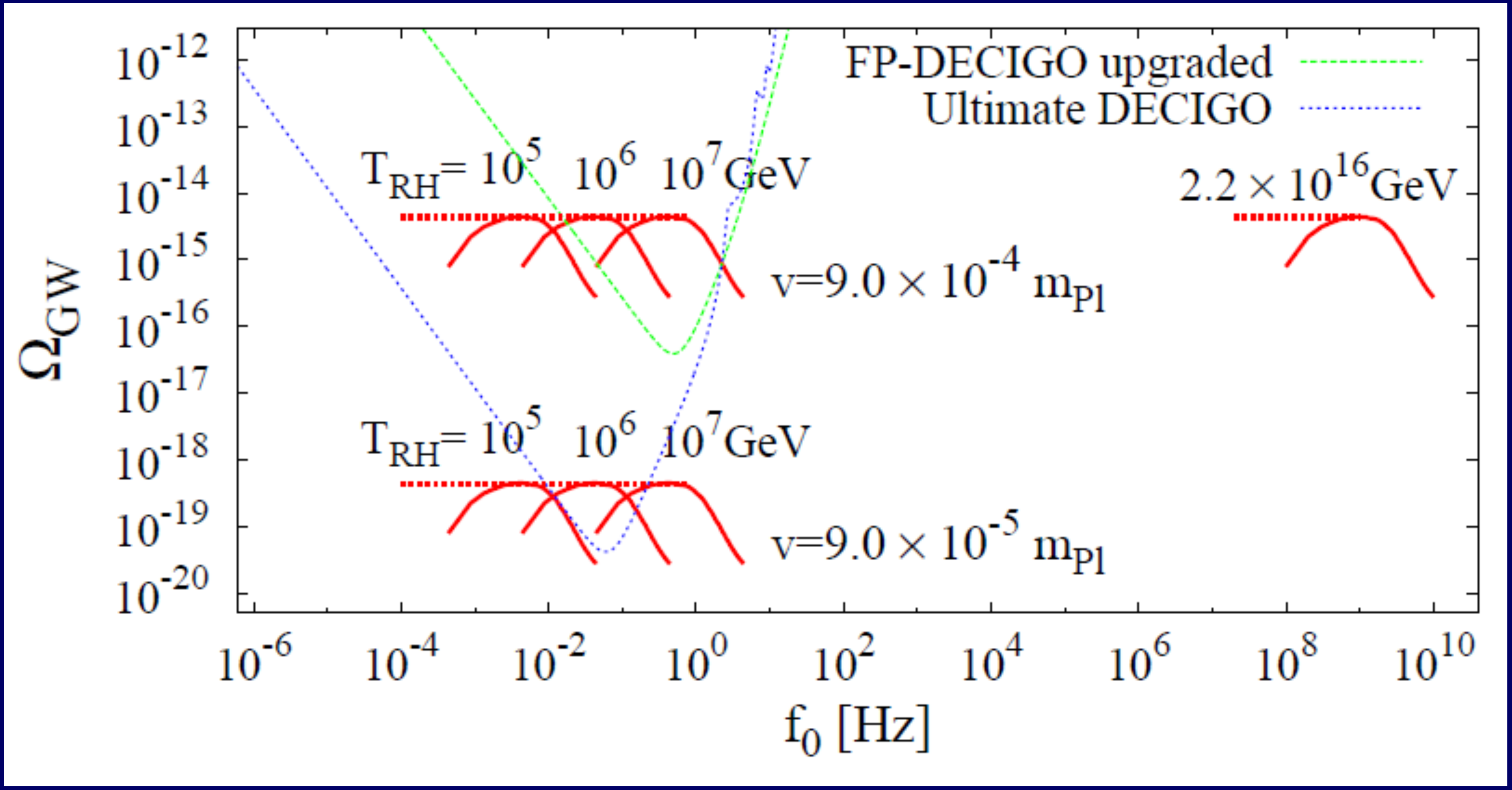
B

$$T^2_{O(N)}(x_{R'}) = \left(1 - \boxed{0.6}\,x_{R'}^{1.5} + 0.65x_{R'}^2\right)^{-1}$$

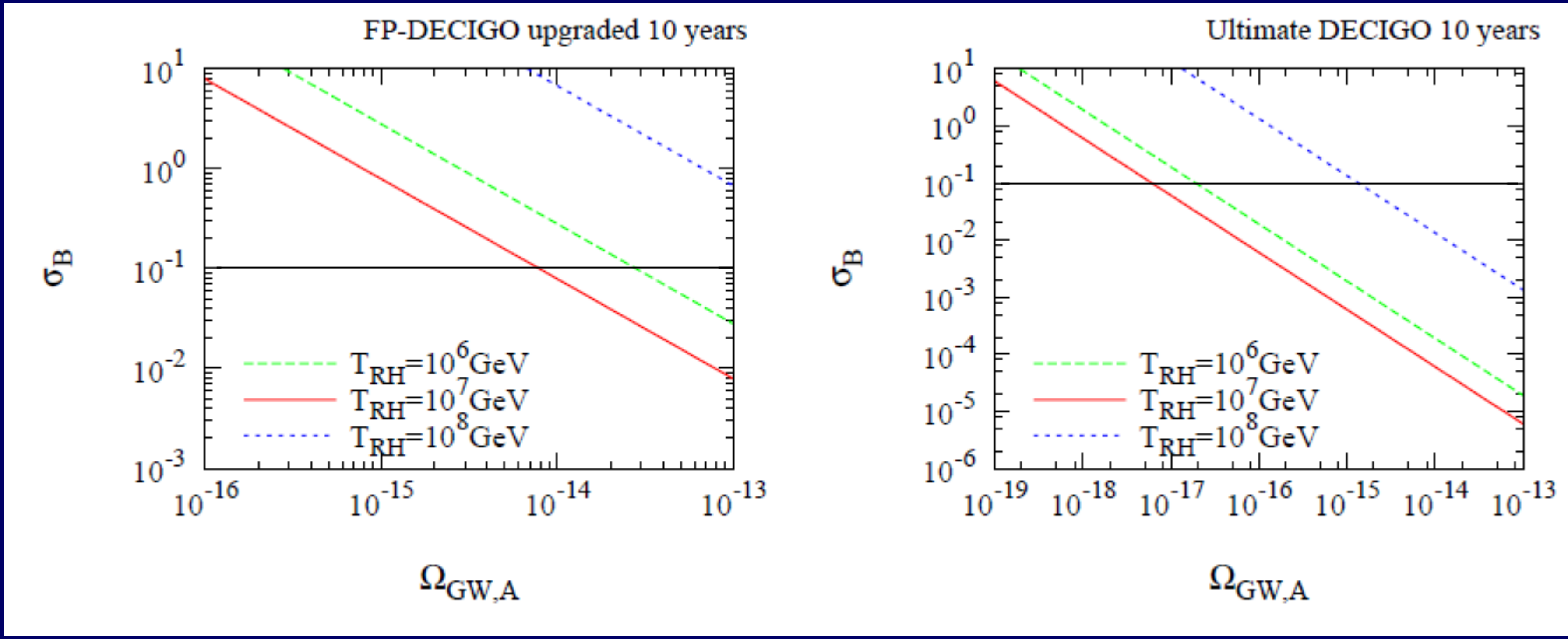
$$x_R = \frac{f}{f_R}$$

$$f_R = 0.26\left(\frac{g_*(T_R)}{106.75}\right)^{\frac{1}{6}}\left(\frac{T_R}{10^7\,\text{GeV}}\right)\text{Hz}$$

We extrapolate numerical results to more realistic values of reheating temperature.



Inflationary tensor perturbations (B=0.22) and GWs from self-ordering scalar fields (B=0.6) may be distinguished from the reheating signature if σ_B<0.1.



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唯以正直接萬人
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鑑三

★ Gravitational particle production takes place due to this rapid change of the expansion law.

Vacuum state in de Sitter space is different from that in the power-law expanding Universe.

$$\langle 0_{dS} | a^{\dagger}_{\mathrm{powerlaw}\boldsymbol{k}} a_{\mathrm{powerlaw}\boldsymbol{k}} | 0_{dS} \rangle \neq 0$$

★ The energy density of created particles:

Massless minimally coupled scalar particle $\rho \simeq \dfrac{9H_{\inf}^4}{32\pi^2 a^4}$ (Ford 1987, Kunimitsu & JY 2012)

Gravitons (2 polarization modes) $\rho_{graviton} \simeq \dfrac{9H_{\inf}^4}{16\pi^2 a^4}$

behave as dark radiation; must be diluted

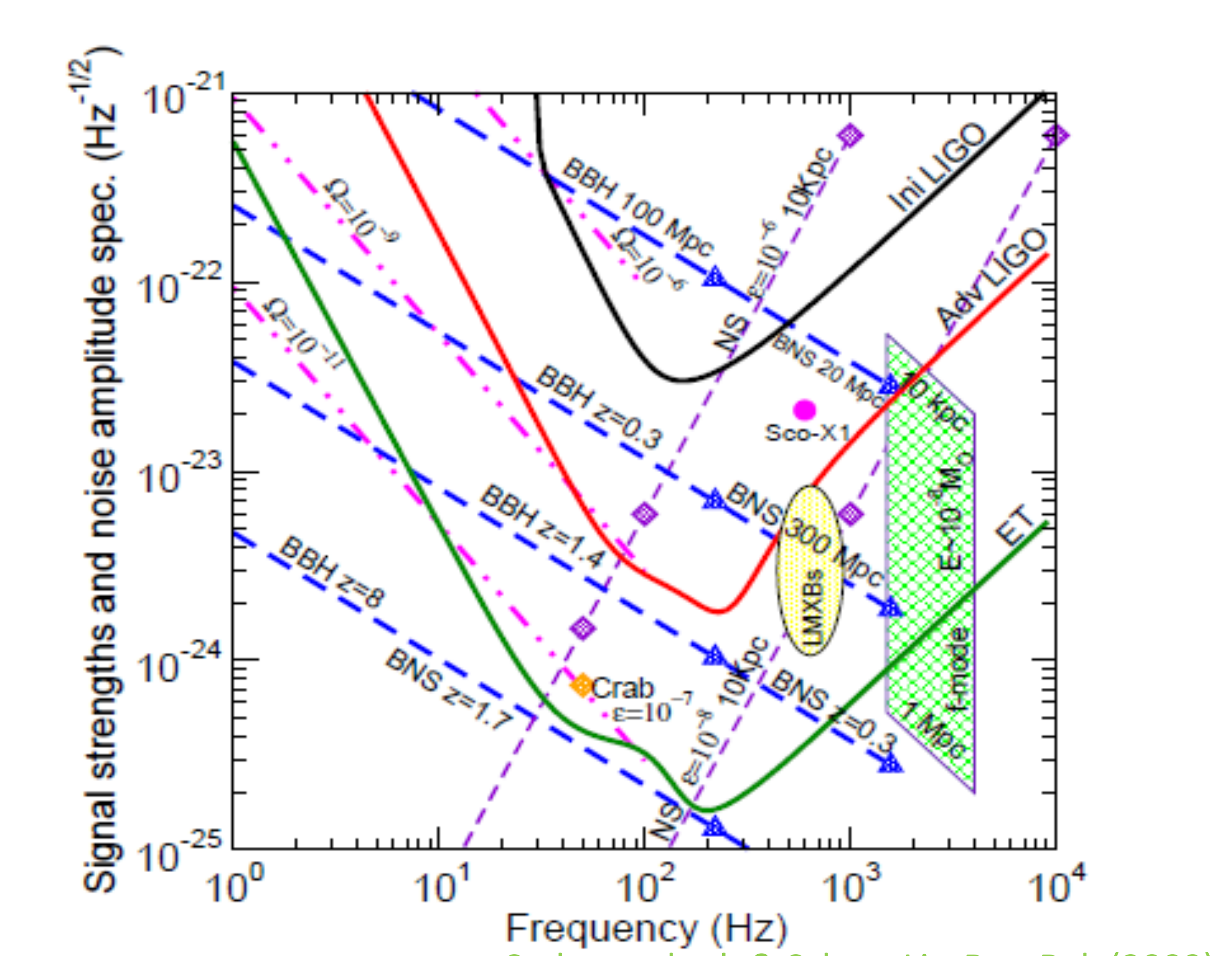
Massive conformally coupled scalar $\rho ; \ Ce^{-4m\Delta t} m^2 H_{\inf}^2 a^{-3}$ (Hashiba & JY 2018)

$$C \simeq 2\times 10^{-4}$$
$$\Delta t \approx H_{\inf}^{-1}$$

taking $a=1$ at the end of inflation.

$$\left\langle h_{ij}h^{ij}\right\rangle = \int_{-\infty}^{\infty} df\, S_h(f) \quad S_h^{\frac{1}{2}}(f) \equiv h(f)$$: signal strength in Fourier space
unit: $1/\sqrt{\mathrm{Hz}}$

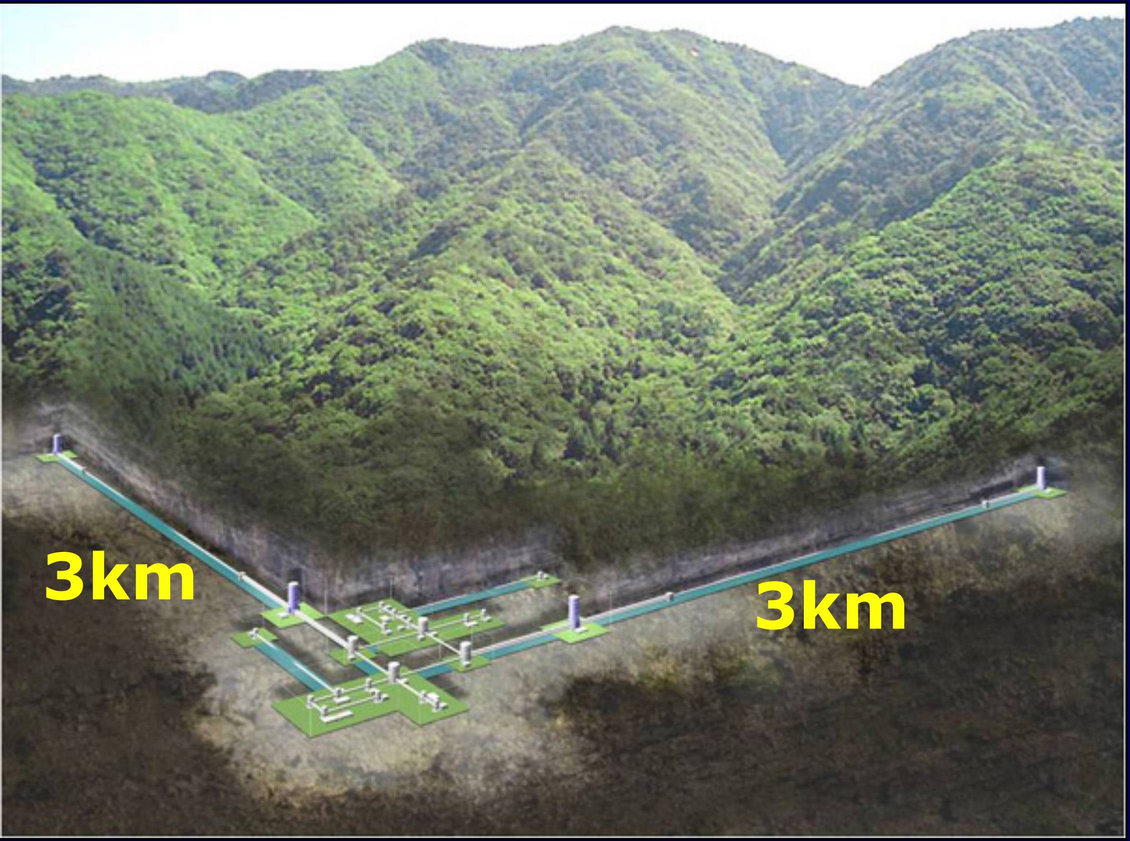
$$\left\langle n^2(t)\right\rangle = \int^{\infty} df\, S_n(f) \quad S_n^{\frac{1}{2}}(f)$$: noise amplitude, unit $1/\sqrt{\mathrm{Hz}}$



Sathyaprakash & Schutz Liv. Rev. Rel. (2009)

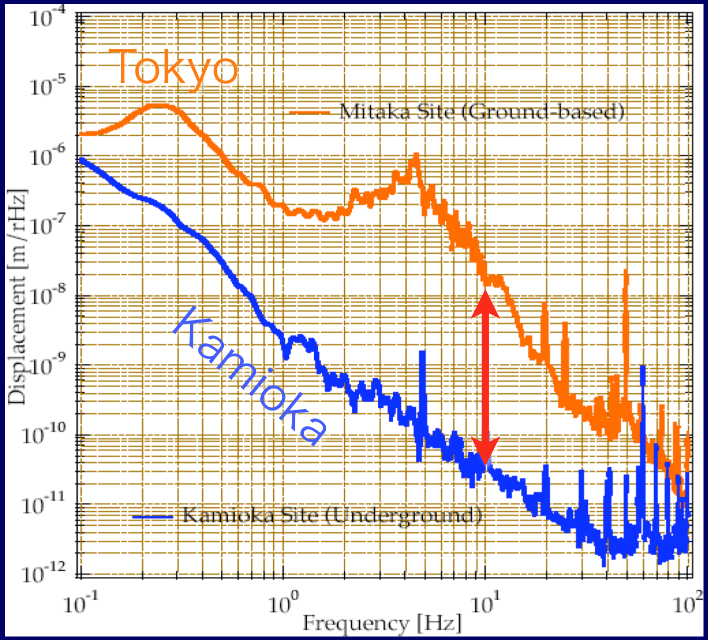
KAGRA

World's first
2.5 generation GW detector



Cryogenic to reduce thermal noises (T=20K @mirrors)
Underground to reduce seismic noises

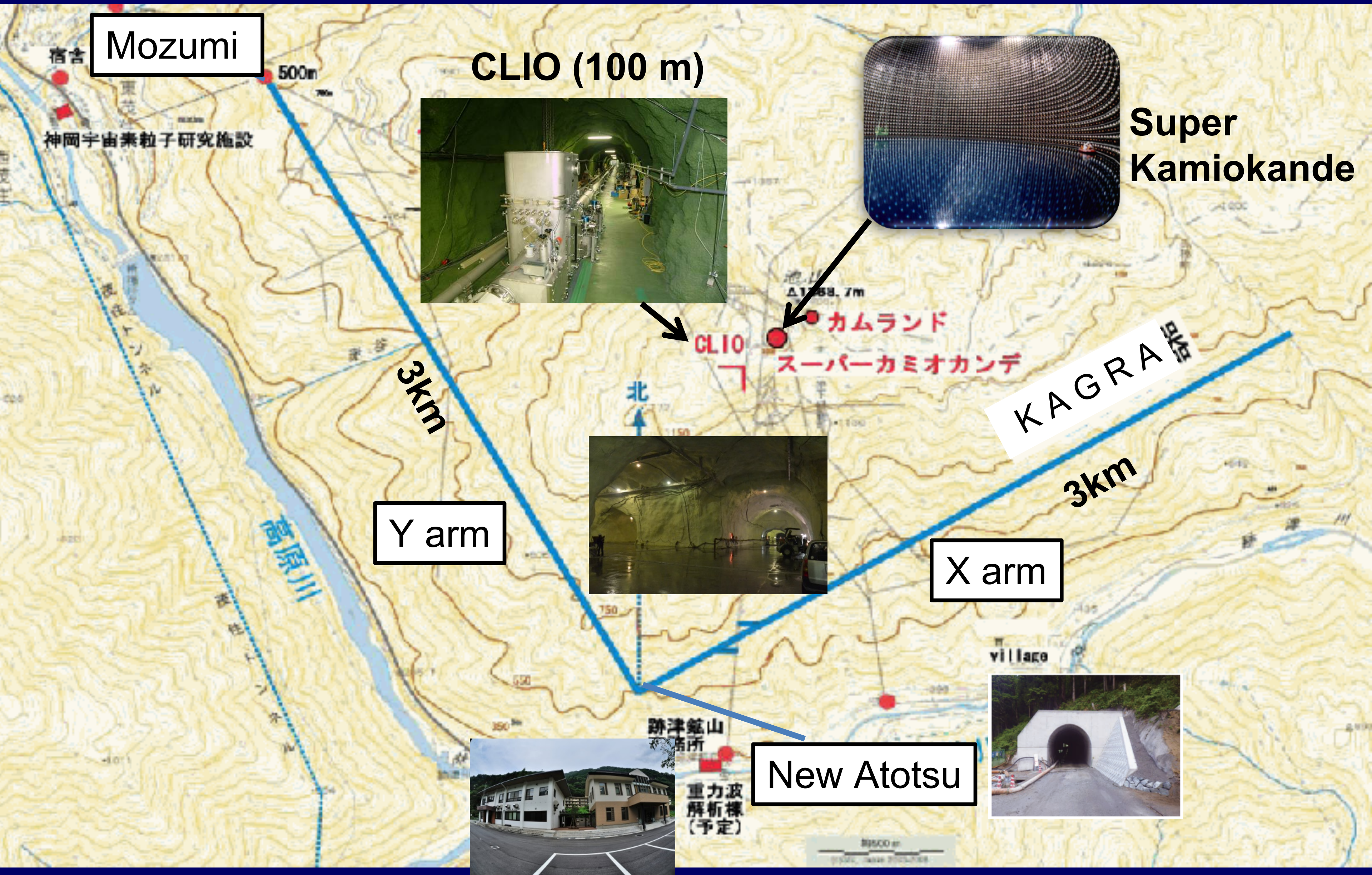
Fabry-Perot Michelson type
Laser interferometer

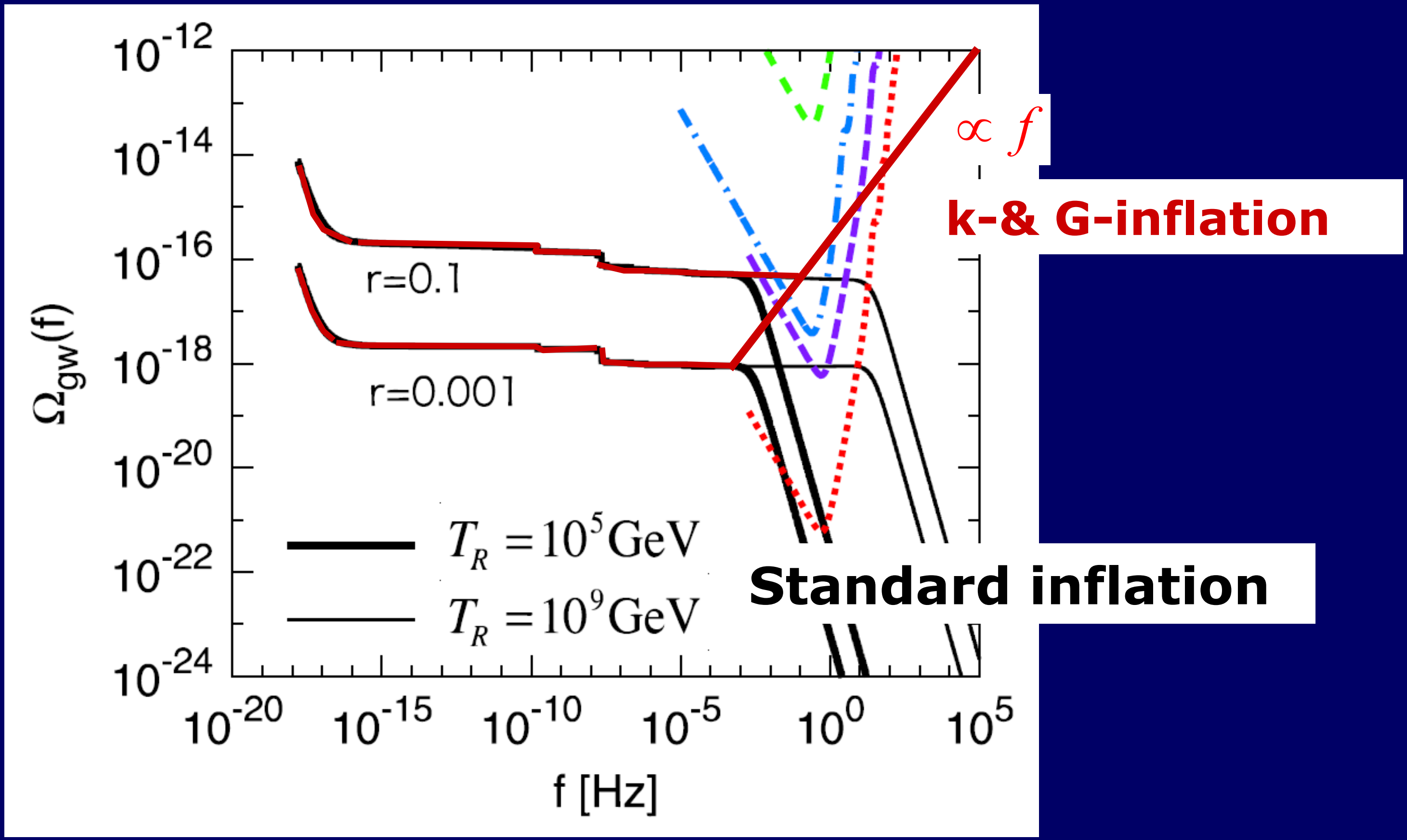


Amplitude of seismic motion

Map of KAGRA site (Kamioka mine, Gifu Pref.)

Map of KAGRA site (Kamioka mine, Gifu Pref.)





10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
10^{-24}
$\Omega_{gw}(f)$
r=0.1
r=0.001
$T_R = 10^5$GeV
$T_R = 10^9$GeV
10^{-20}
10^{-15}
10^{-10}
10^{-5}
10^{0}
10^{5}
f [Hz]
$\propto f$
k-& G-inflation
Standard inflation

Conceptual design of DECIGO

DECihertz Interferometer Gravitational-wave Observatory

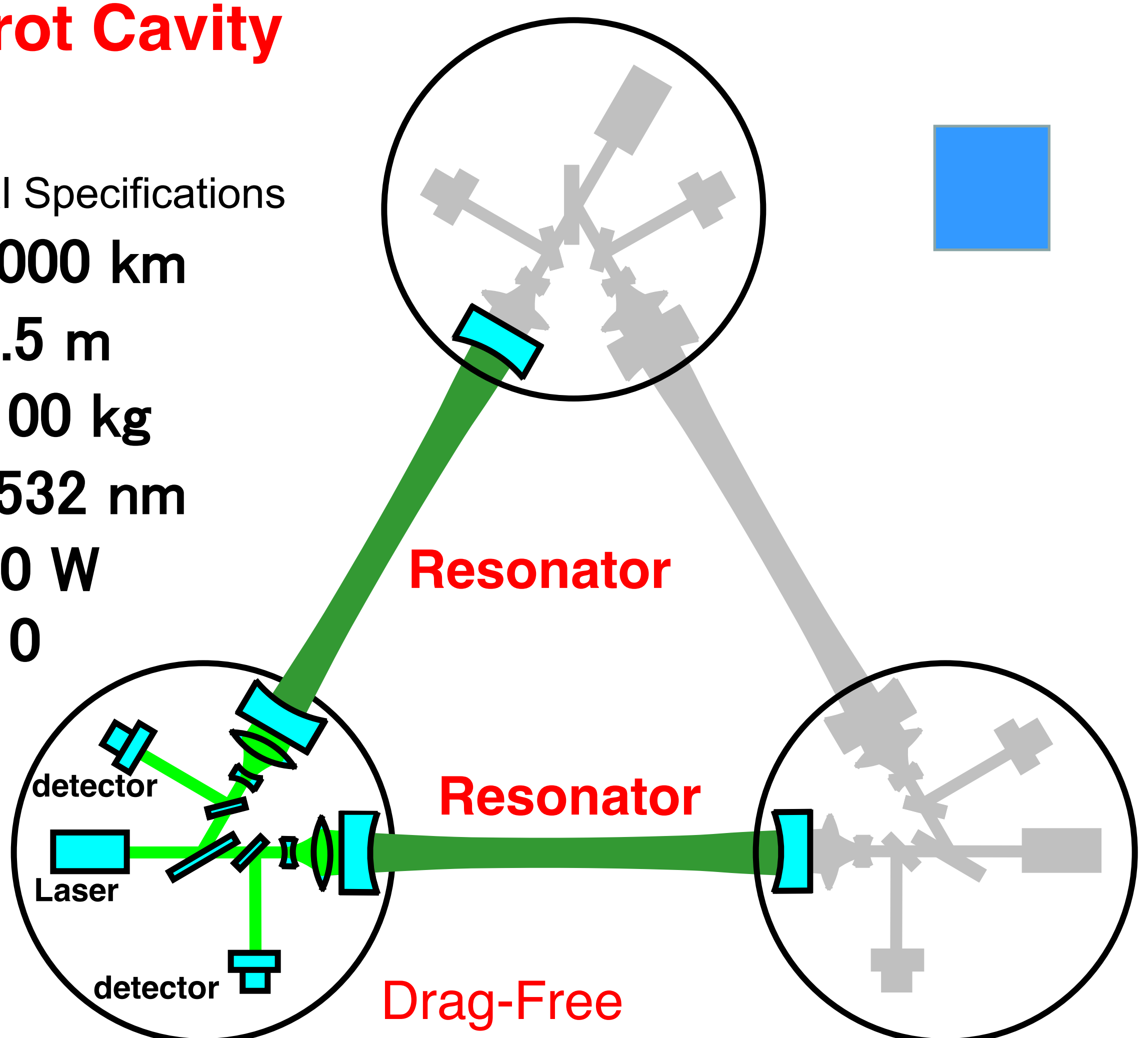
N. Seto, S. Kawamura, & T. Nakamura, PRL 87(2001)221103

Now include Fabry-Perot Cavity =Light Resonator

	Original Specifications
Arm length:	**L=1000 km**
Mirror Diameter:	**R=0.5 m**
Mirror Mass:	**M=100 kg**
Laser Wavelength:	**λ=532 nm**
Laser Power:	**P=10 W**
Finesse:	**$\mathcal{F}$=10**

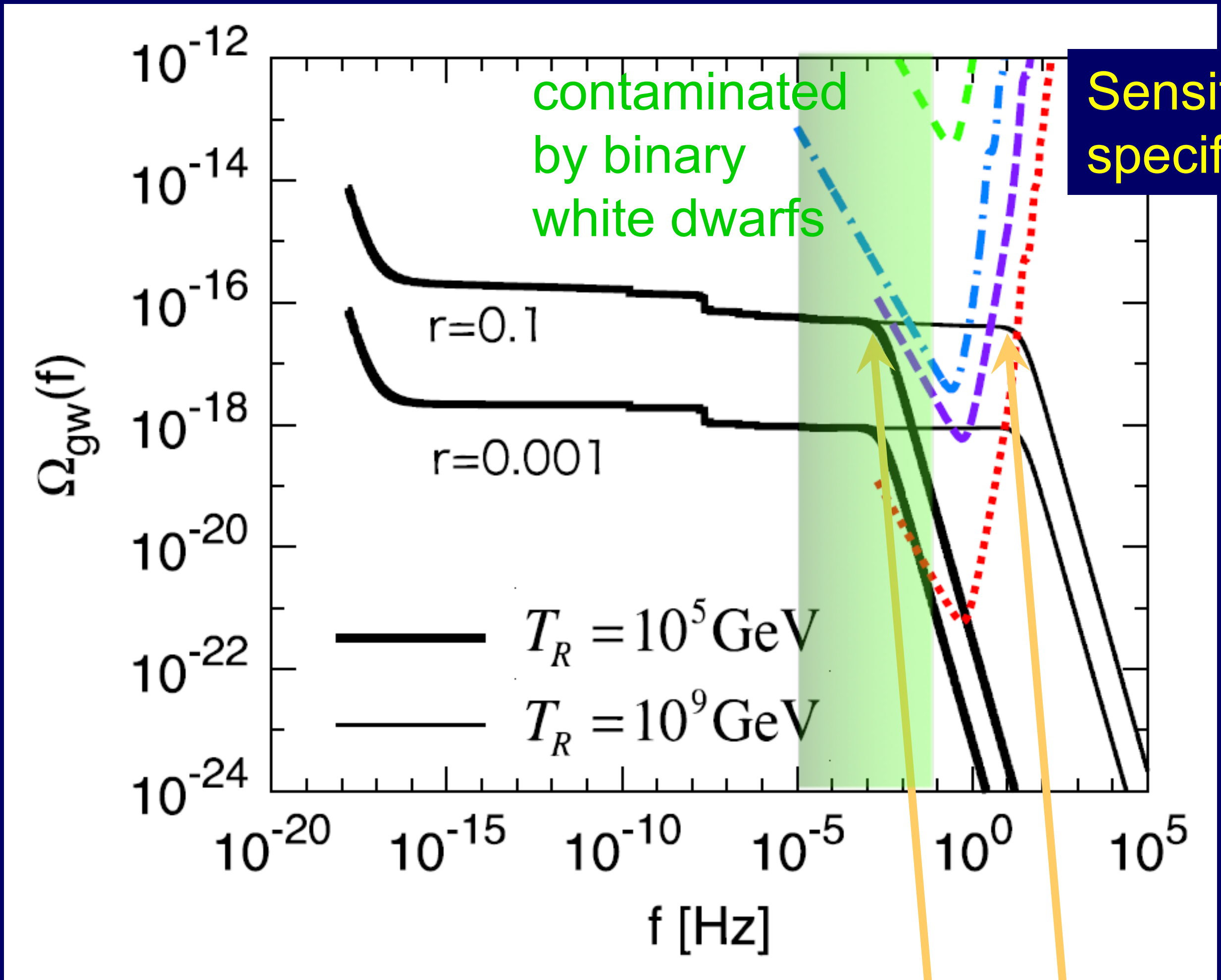
Average time photons spend inside the resonator cavity

$$\tau_s \approx \frac{L}{c}\frac{\mathcal{F}}{\pi}$$



© S. Kawamura

Thermal History is inprinted on the spectrum of GWs.



Sensitivity curves of various specifications

In order to probe higher reheating temperature we need sufficient sensitivity at higher frequency.

$$f_{\rm R} = \frac{k_{\rm R}}{2\pi a_0} \simeq 0.26{\rm Hz}\left(\frac{g_{*s}(T_{\rm R})}{106.75}\right)^{1/6}\left(\frac{T_{\rm R}}{10^7{\rm GeV}}\right)$$

At the present time, the energy density of GW is given by

$$\Omega_{GW}(f,t_0)=\frac{(2\pi f)^2}{12H_0^2}\Delta_h^2(f)T_h^2(f)=\frac{4\pi^2}{3H_0^2}f^3 S_h^2(f)$$

Amplitude per logarithmic frequency interval $d\ln f$

Transfer function depending on thermal history

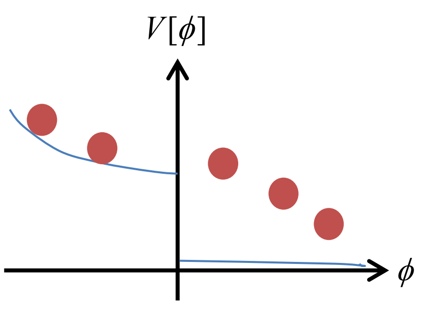
Strain power spectrum with a measure df

This f^3 dependence makes it very difficult to detect higher frequency stochastic GWs.

$$\left\langle h_{ij}h^{ij}\right\rangle=\int_{-\infty}^{\infty} d\ln f\,\Delta_h^2(f)T_h^2(f)=2\int_{-\infty}^{\infty} df\,S_h^2(f)=4\int_{-\infty}^{\infty} d\ln f\, fS_h^2(f)$$

Thermal History after k-(G-) or Quintessential Inflation

- In these models, inflation ends abruptly without being followed by its coherent field oscillation.
- Inflation is followed by a "kination" regime when the universe is dominated by kinetic energy of the inflaton.
- Reheating proceeds only through gravitational particle creation.



Quintessential inflation
(Peebles & Vilenkin 99)

A simple k-inflation example (Armendariz-Picon, Damour & Mukhanov 99)

$$\mathcal{L} = K_1(\phi_{\rm inf})X + K_2(\phi_{\rm inf})X^2, \quad X \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi_{\rm inf}\partial_\nu\phi_{\rm inf}$$

$\phi_{\rm inf}$: inflaton

★ If K_1 and K_2 are constants with opposite sign, the kinetic function has an attractor solution $X = -\frac{K_1}{2K_2} > 0$.

★ Then the energy density and pressure are given by

$$\rho = 2X\frac{\partial\mathcal{L}}{\partial X} - \mathcal{L} = -P = \text{constant},$$

➡ inflation with $H_{\rm inf}^2 = \dfrac{K_1^2}{12M_G^2K_2}$

★ k-inflation ends when K_1 and K_2 both becomes positive. Then the kinetic energy starts to redshift quickly and only the first term of the Lagrangian becomes relevant.

$K_1K_2 < 0$ | k-inflation | $a(t) \propto e^{Ht}$

⬇

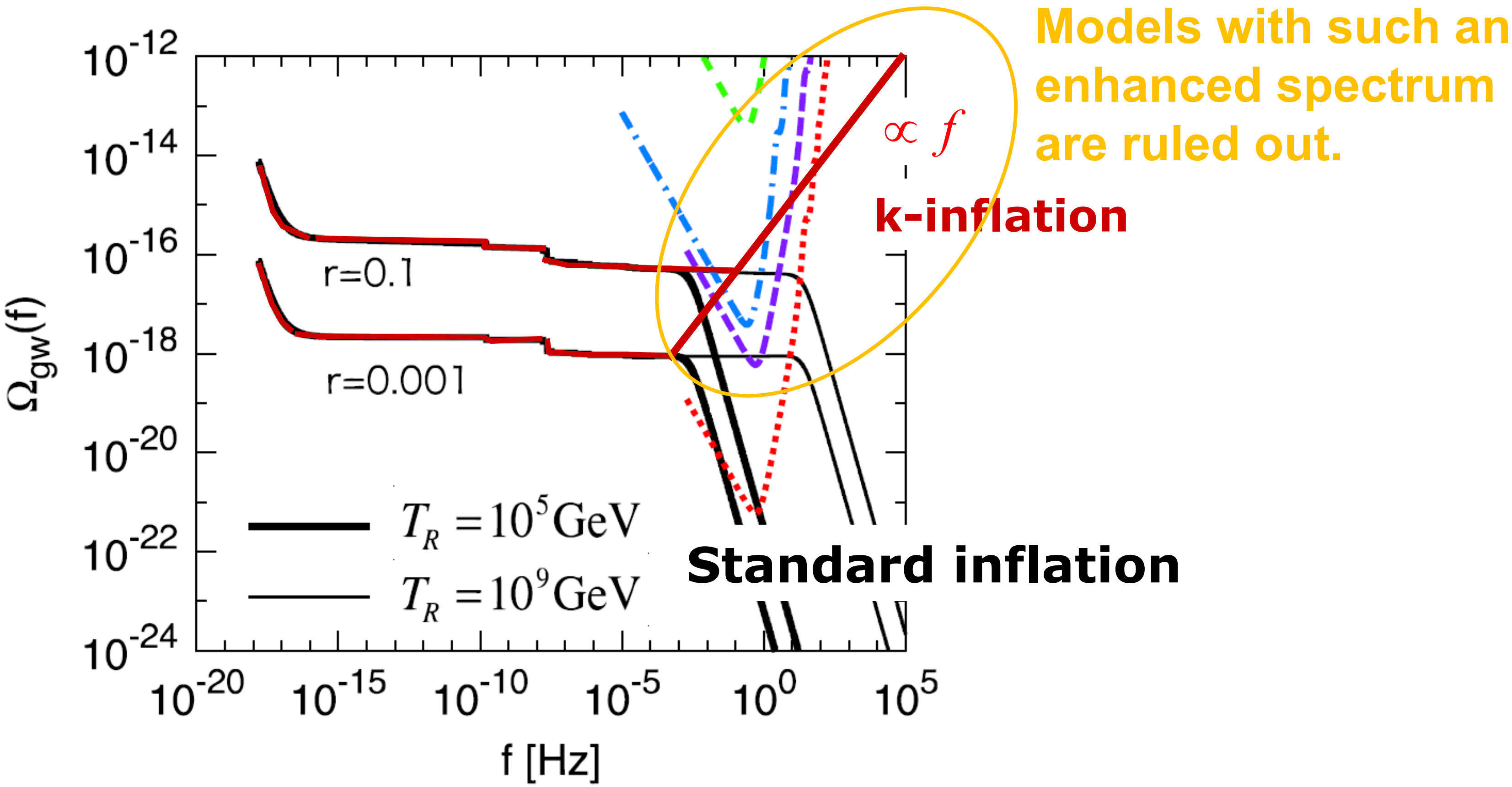
$K_1K_2 > 0$ | Free scalar field | $a(t) \propto t^{1/3}$ | kination

$\Delta t \approx H_{\rm inf}^{-1}$

If this change occurs abruptly,
numerical calculation shows that this transition occurs within Hubble time.

Implication of the Higgs Condensation

- Inflation models followed by kination regime with gravitatio
  reheating is incompatible with the Higgs condensation.



Stochastic gravitational wave background from inflation

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$$\left\langle 0_{dS} \left| a^{\dagger}_{powerlaw\boldsymbol{k}} a_{powerlaw\boldsymbol{k}} \right| 0_{dS} \right\rangle \neq 0$$

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Massless minimally coupled scalar particle $\rho \simeq \dfrac{9H_{\rm inf}^4}{32\pi^2 a^4}$ (Ford 1987, Kunimitsu & JY 2012)

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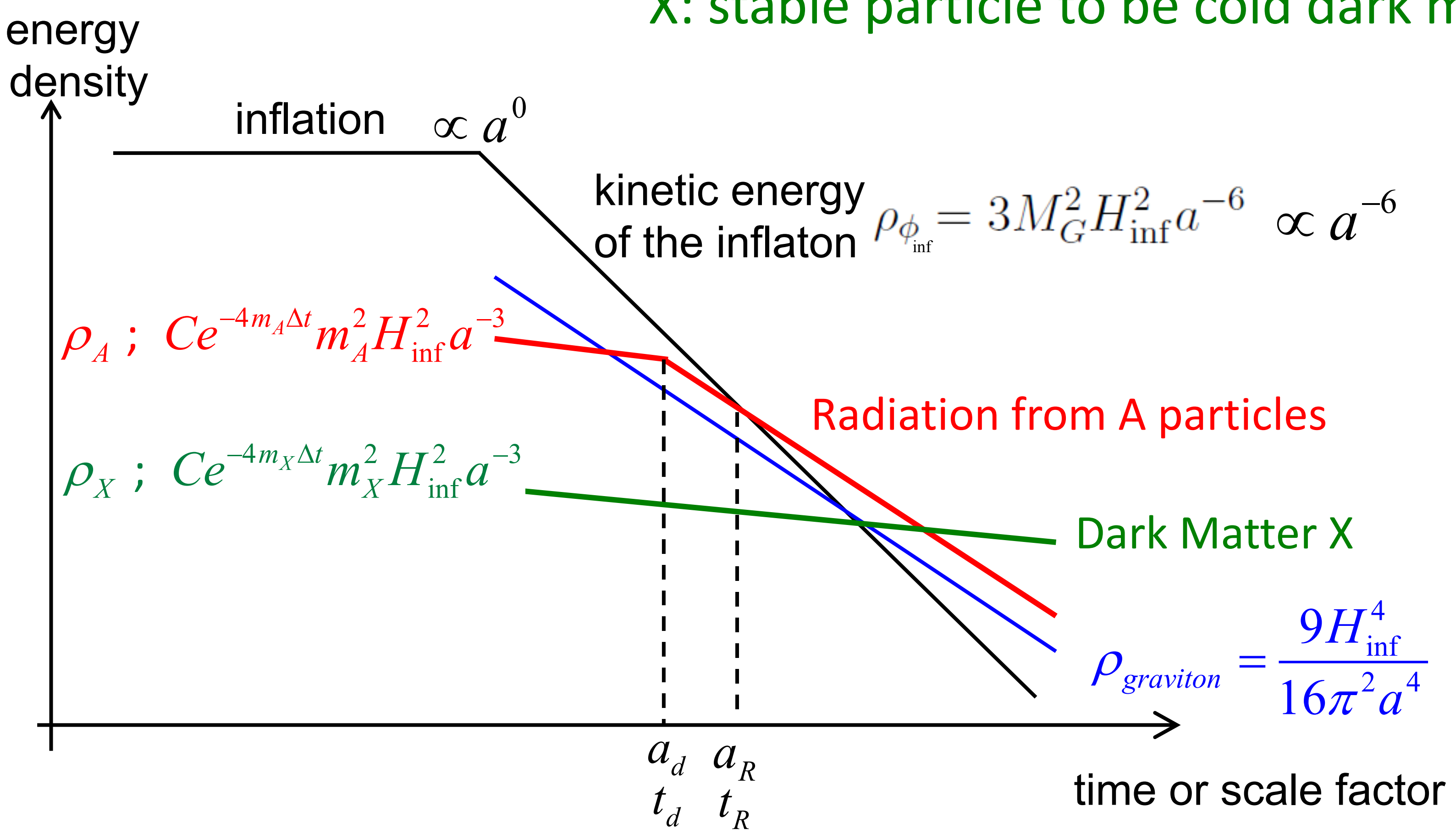
$$C \simeq 2\times 10^{-4}$$
$$\Delta t \approx H_{\rm inf}^{-1}$$

taking $a=1$ at the end of inflation.

★ We consider gravitational particle production of two conformally coupled massive scalar. A: decay into radiation to reheat the universe

X: stable particle to be cold dark matter today



A particles decay at $a = a_d$ to radiation with the decay rate $\Gamma_A \equiv \alpha m_A$

The Universe became radiation dominant at $a = a_R$: reheating time.

★ The dominant part of the entropy is produced at t_d , when we find

$$s|_d = \frac{2\pi^2}{45} g_{*d} T_d^3 \quad \text{with} \quad T_d = 5 \times 10^{-2} \alpha^{1/4} e^{-m_A \Delta t} m_A^{3/4} H_{\rm inf}^{1/4},$$

$$g_{*d} = 106.75$$

$$\rho_X|_d = C \alpha e^{-4 m_X \Delta t} m_A m_X^2 H_{\rm inf}.$$

$$\Rightarrow \quad \frac{\rho_X}{s} = 4 \times 10^{-2} \alpha^{1/4} e^{(3 m_A - 4 m_X)\Delta t} \frac{m_X^2 H_{\rm inf}^{1/4}}{m_A^{5/4}} \approx 4 \times 10^{-10}\ \mathrm{GeV}$$

should hold to explain CDM

★ Gravitons must be sufficiently diluted

$$N_{GW,\rm eff} = \frac{4}{7}\left(\frac{4}{11}\right)^{-4/3} g_{*\rm DC} \left(\frac{g_{*\rm DC}}{g_{*d}}\right)^{1/3} \left(\frac{\rho_{GW}}{\rho_A}\right)\bigg|_d = 2.4 \left(\frac{\rho_{GW}}{\rho_A}\right)\bigg|_d < 0.72$$

$$g_{*\rm DC} = 3.38$$

$$\Rightarrow \quad \alpha^{-1/3} e^{-4 m_A \Delta t} \left(\frac{m_A}{H_{\rm inf}}\right)^{5/3} > 2.3 \times 10^3.$$

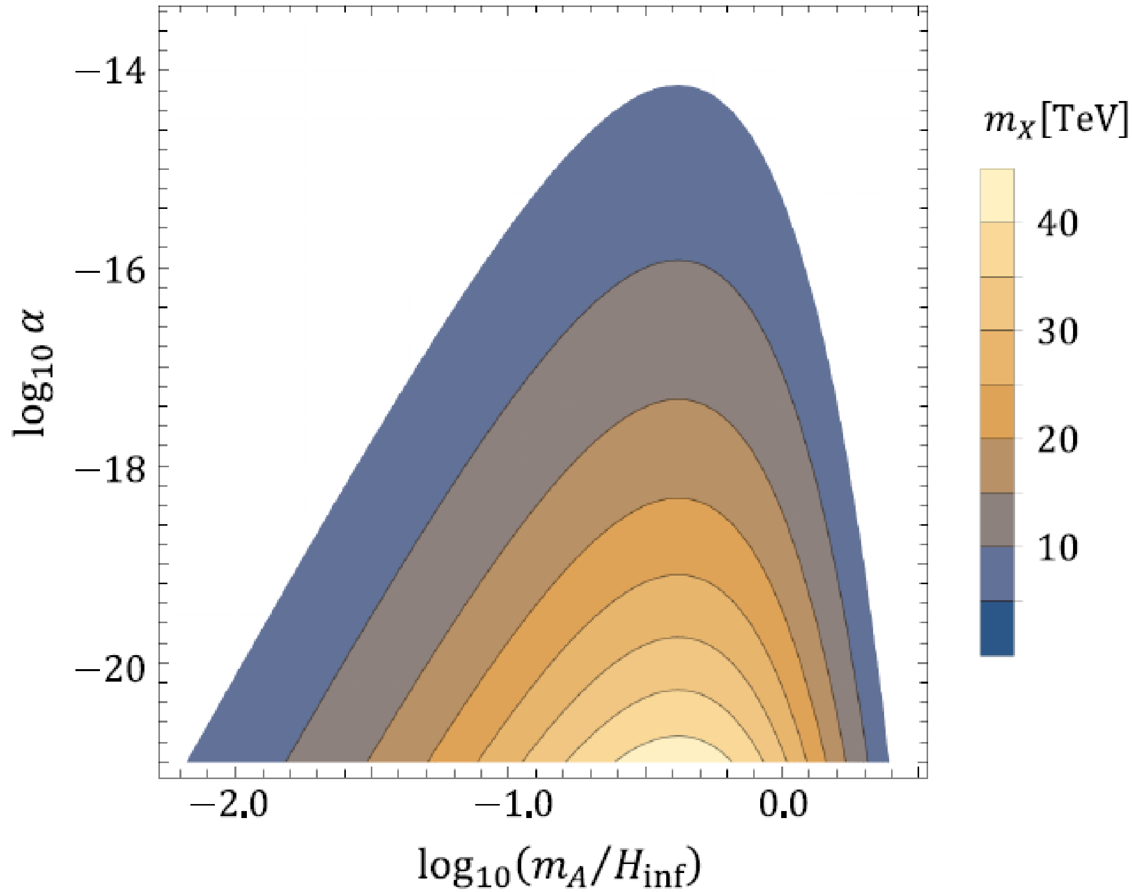


FIG. 1. Parameter values realizing the appropriate abundance of CDM while concealing the effect of gravitationally produced gravitons with $H_{\rm inf} = 10^{13}$ GeV and $\Delta t = 1.0 H_{\rm inf}^{-1}$. The colored region is consistent with the CMB observation [1]. The maximum allowed value of α is 7.0×10^{-15} with $m_A = 0.42 H_{\rm inf}$. The minimum value of m_X is 5.8 TeV on the edge of the allowed region.

Summary and Discussion

We have considered two scenarios of cold dark matter formation, micro black hole remnants and purely gravitational dark matter.

Both work under some conditions.

Both are nightmare scenarios for particle physics experiments as they interact only gravitationally.

There is an astrophysical implication, though;
Comoving free streaming scale at equality time t_{eq} is very small

$$\lambda_{fs} = \int_{t_*}^{t_{eq}} \frac{v(t)}{a(t)} \sim \frac{c t_{eq}}{a_{eq} T_*} \ln\left(\frac{T_*}{T_{eq}}\right) \qquad T_* \geq T_R$$

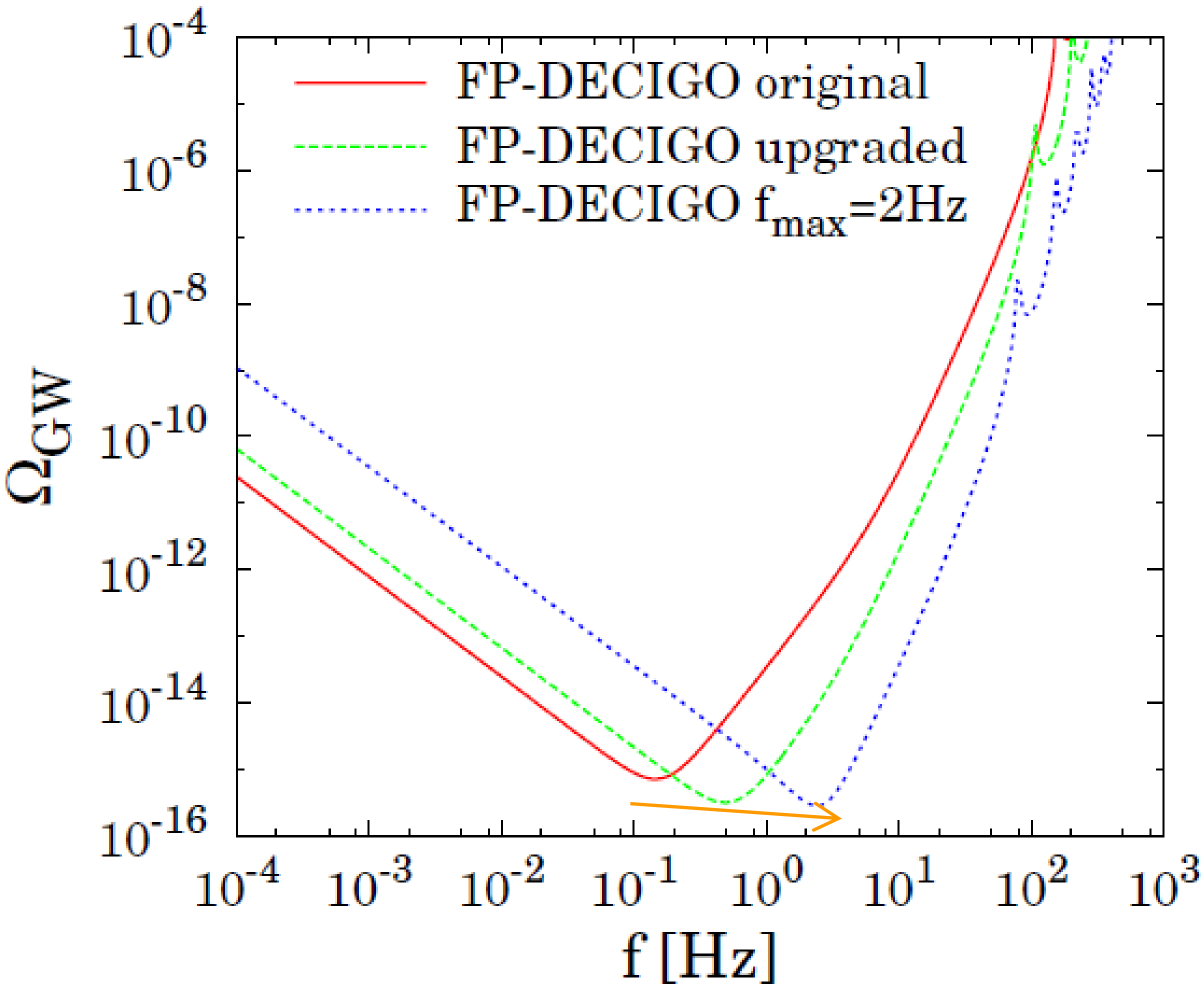
and the minimum mass of DM halo is also very small.

$M_{\min} \sim 10^{-15} M_{\odot}$ for $T_* = 1\mathrm{TeV}$ and even smaller for higher temperature.

This may be tested by pulsar timing measurements.

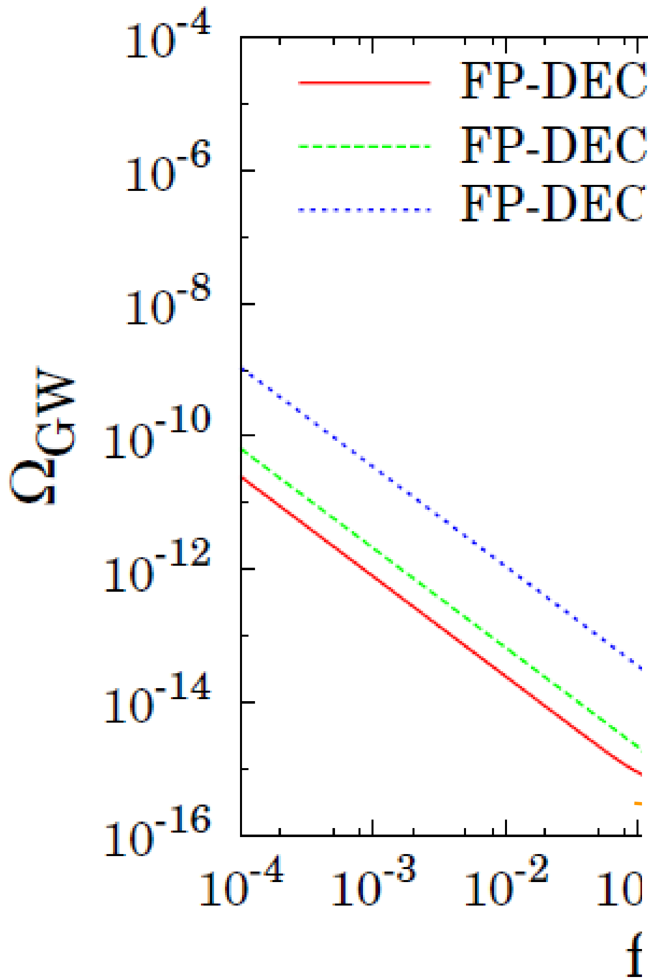
Kashiyama & Oguri (2018)

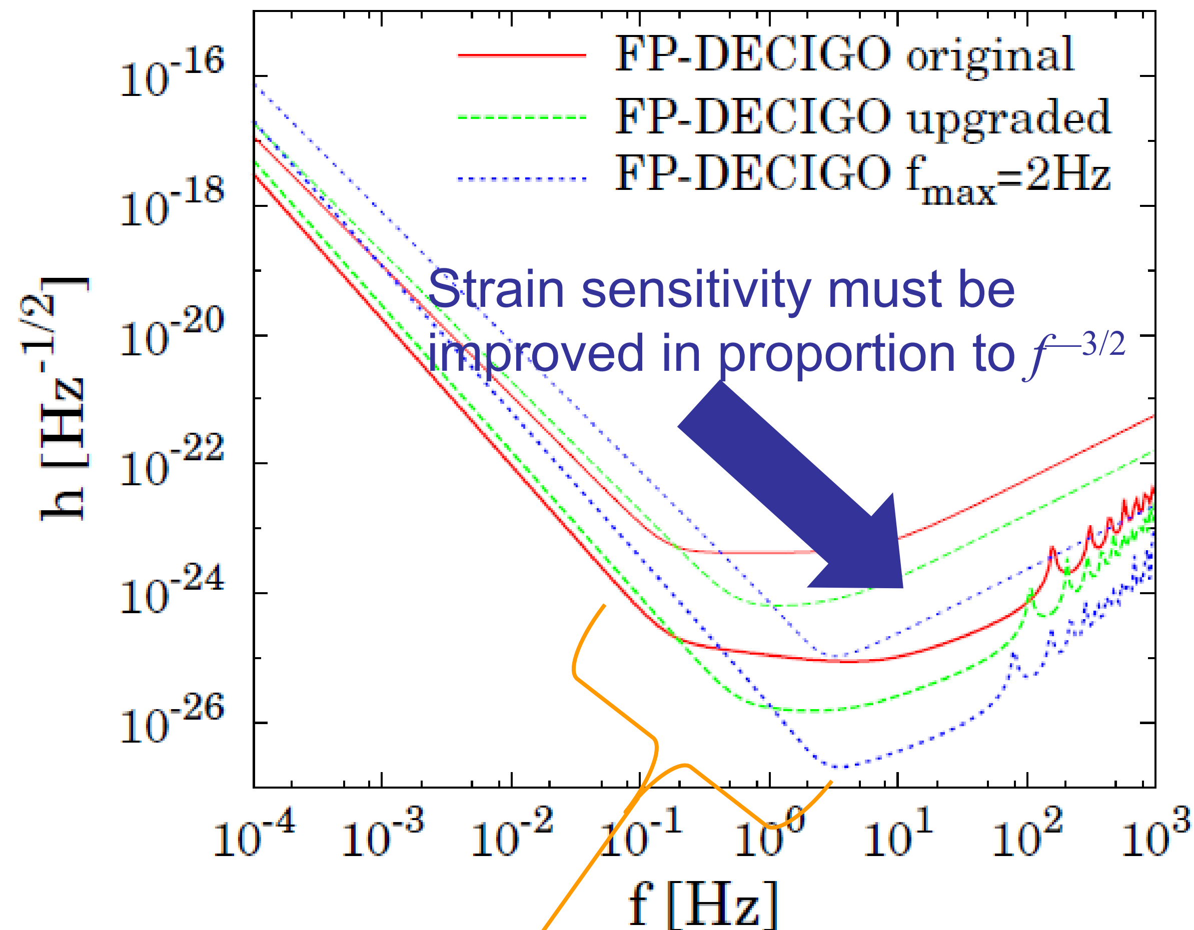
In order to probe higher frequency with the same sensitivity to Ω_{GW},



In order to probe higher frequency with the same sensitivity to Ω_{GW},

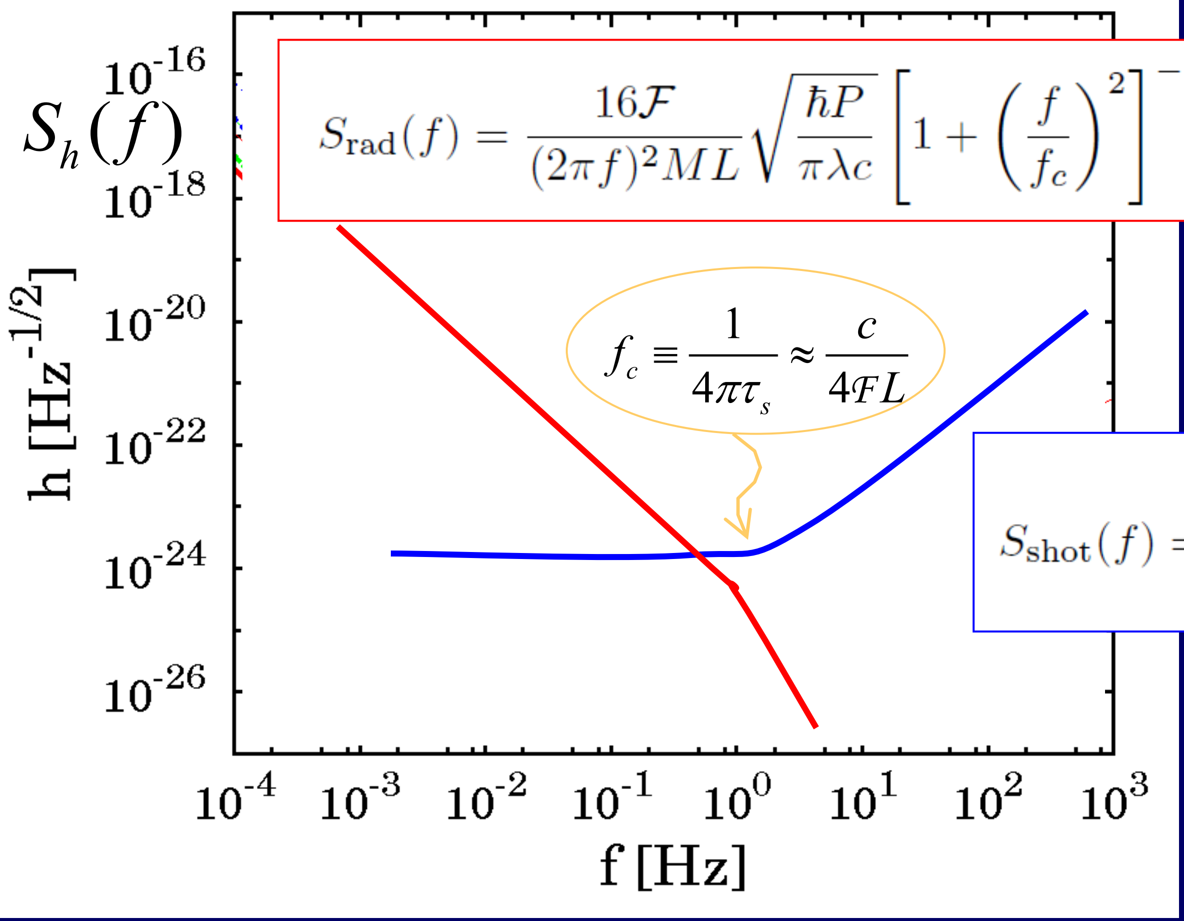
$$h \equiv S_h(f) \propto \sqrt{\frac{\Omega_{GW}(f,t_0)}{f^3}} \propto f^{-3/2}$$





Lower thicker curves indicate sensitivity achieved by 3yr correlation analysis

On the basis of BICEP2 result, we reconsider sensitivity curves of DECIGO for direct detection of inflationary GW & determination of the reheating temperature.



$$S_{\rm rad}(f) = \frac{16\mathcal{F}}{(2\pi f)^2 ML}\sqrt{\frac{\hbar P}{\pi\lambda c}}\left[1+\left(\frac{f}{f_c}\right)^2\right]^{-1/2}$$

$$f_c \equiv \frac{1}{4\pi\tau_s} \approx \frac{c}{4\mathcal{F}L}$$

$$S_{\rm shot}(f) = \frac{\sqrt{\hbar\pi c\lambda}}{4\mathcal{F}L\sqrt{\tilde{P}}}\left[1+\left(\frac{f}{f_c}\right)^2\right]^{1/2}$$

Radiation Pressure Noise

Fluctuations in radiation pressure induces unwanted motion of the mirror

Shot Noise

Poisson noise due to quantum nature of laser

In order to achieve sufficient sensitivity at higher frequency, it is important to suppress shot noise

$$S_{\rm shot}(f) = \frac{\sqrt{\hbar \pi c \lambda}}{4\mathcal{F} L \sqrt{\tilde{P}}} \left[1 + \left(\frac{f}{f_c} \right)^2 \right]^{1/2}$$

by λ] P Z $\mathcal{F}$ Z L Z .

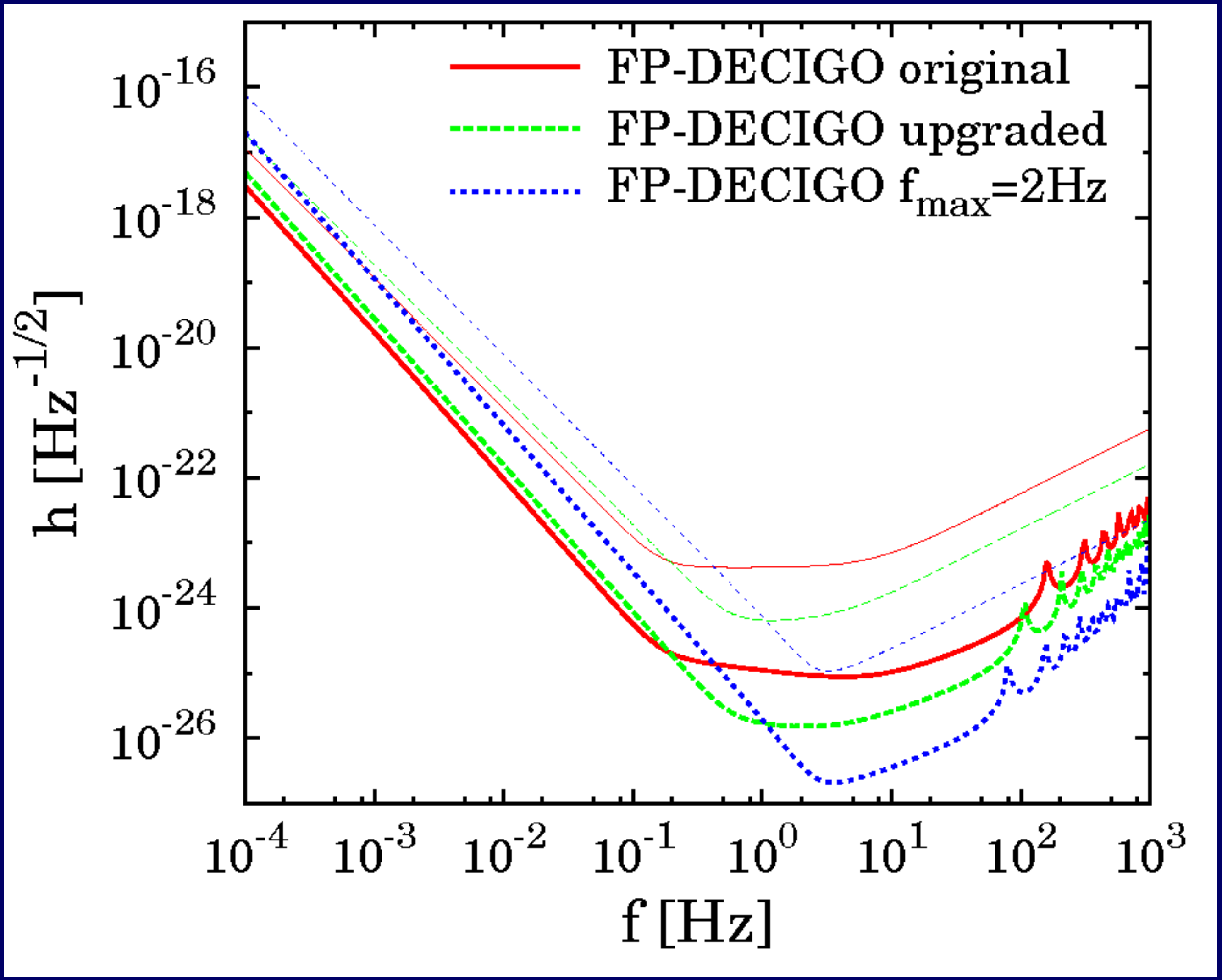
$$f_c \equiv \frac{1}{4\pi\tau_s} \approx \frac{c}{4\mathcal{F}L}$$

But $\mathcal{F}$ Z L Z would also lowers f_c] and the frequency range of our interest would fall above f_c where we find

$$S_{shot}(f) \cong \sqrt{\frac{h\pi\lambda}{cP\%}} f^2$$

Hence we can control the shot noise only by λ] P Z .

On the basis of BICEP2 result, we reconsider sensitivity curves of DECIGO for direct detection of inflationary GW & determination of the reheating temperature.



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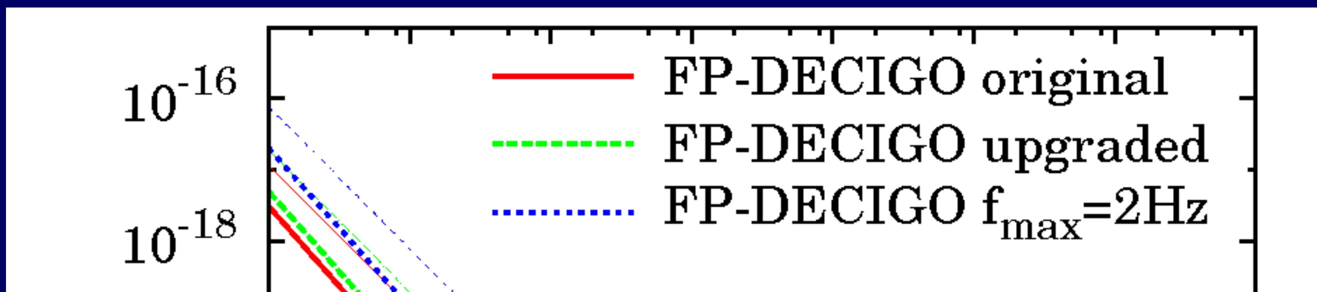


Specifications

	Original	Upgraded	$f_{\max} = 2\text{Hz}$
Arm length:	L=1000 km	1500km	1500km
Mirror Diameter:	R=0.5 m	0.75m	
Mirror Mass:	M=100 kg		
Laser Wavelength:	λ=532 nm		157nm
Laser Power:	P=10 W	30W	300W
Finesse:	$\mathcal{F}$=10		

f [Hz]

We consider quadratic chaotic inflation $V[\phi] = \frac{1}{2}m^2\phi^2$ $r \approx 0.14$

(Linde 83)

and natural inflation

(Freese, Frieman, Olinto 90)

$$V[\phi] = \Lambda^4\left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

with $f = 7M_{Pl}$ yielding $r \approx 0.07$

as fiducial models.

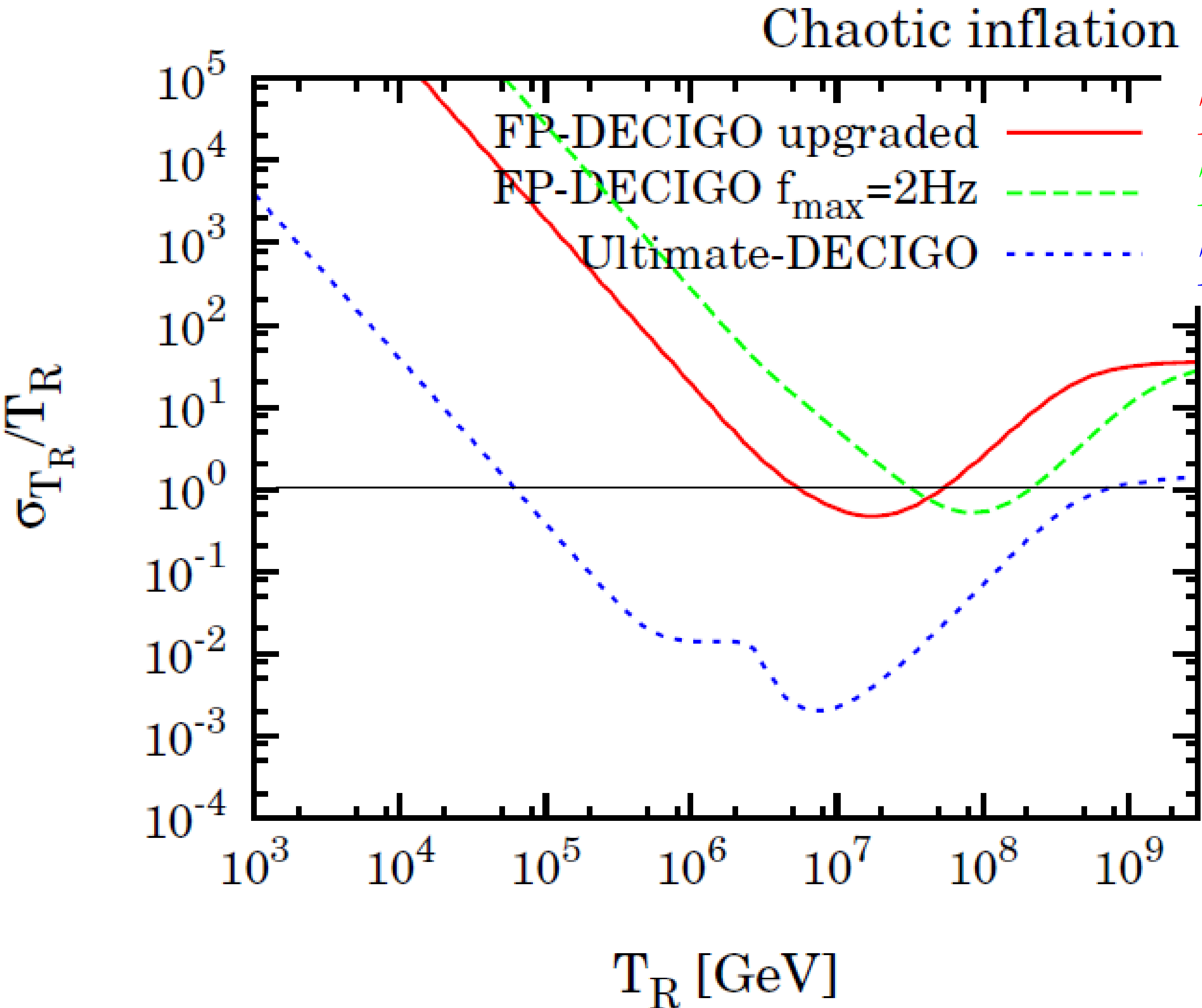
The original DECIGO does not have sufficient sensitivity to detect the stochastic GW background predicted by these models.

We determine maximum possible reheat temperature DECIGO can measure by Fisher matrix analysis for upgraded, $f_{\max} = 2\mathrm{Hz}$ and ultimate versions.

noises are assumed to be quantum limited.

Marginalized 1σ uncertainty in T_R as a fraction of T_R for quadratic chaotic inflation

T_R can be determined within 1σ if



$T_R < 5.2\times10^{7}\,\mathrm{GeV}$

$T_R < 2.1\times10^{8}\,\mathrm{GeV}$

$T_R < 7.0\times10^{8}\,\mathrm{GeV}$

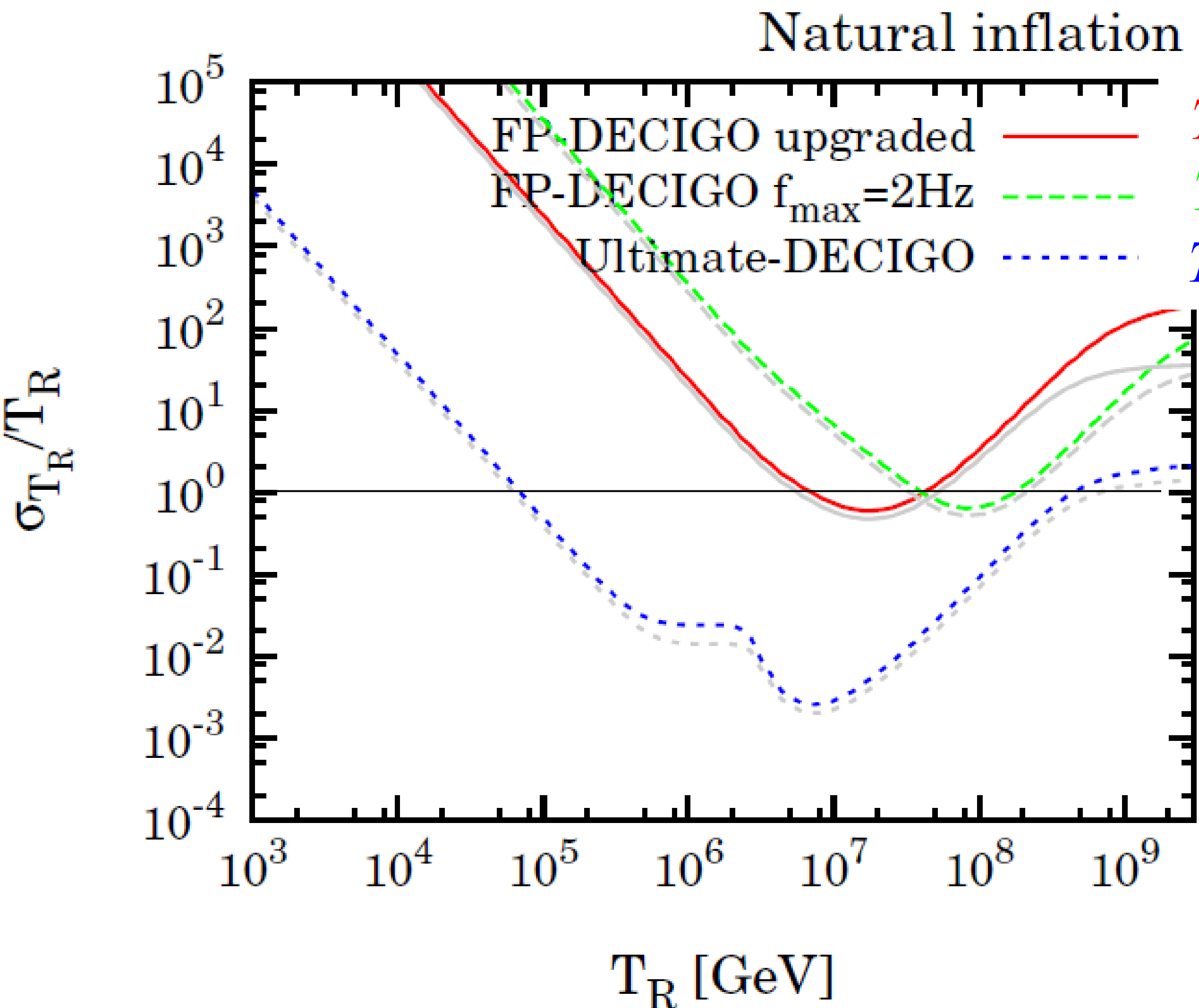
Marginalized 1σ uncertainty in T_R as a fraction of T_R for natural inflation with $f = 7M_{Pl}$

T_R can be determined within 1σ if

$T_R < 4.5\times10^7$ GeV

$T_R < 1.8\times10^8$ GeV

$T_R < 4.6\times10^8$ GeV



DECIGO can measure the reheat temperature T_R if it lies in the range $5\times10^6\,\mathrm{GeV} < T_R < 2\times10^8\,\mathrm{GeV}$

The ultimate DECIGO can measure the reheat temperature T_R if it lies in the range $6\times10^4\,\mathrm{GeV} < T_R < 7\times10^8\,\mathrm{GeV}$

DECIGO can measure the reheat temperature T_R if it lies in the range $5\times10^{6}\,\mathrm{GeV} < T_R < 2\times10^{8}\,\mathrm{GeV}$

The ultimate DECIGO can measure the reheat temperature T_R if it lies in the range $6\times10^{4}\,\mathrm{GeV} < T_R < 7\times10^{8}\,\mathrm{GeV}$

One may naïvely think that high-scale inflation predicts high reheat temperature, and the upper bound we obtained is too low.

However, in order to realize high-scale inflation with a large r and a large field excursion $\phi_i - \phi_e$? M_{Pl} (Lyth-Turner Bound) we often introduce symmetries in model building

Chaotic inflation: Shift symmetry (Kawasaki, Yamaguchi, Yanagida 00)

Natural inflation: Nambu-Goldstone (Freese, Frieman, Olinto 90)

which also constrain coupling of the inflaton and delay reheating.

★ An example of Chaotic inflation in Supergravity

$$K = \frac{1}{2}(\phi + \phi^\dagger)^2 + |X|^2 + |H_u|^2 + |H_d|^2,$$
$$W = mX\phi + yXH_uH_d,$$

$$\Rightarrow \quad V[\phi] = \frac{1}{2} m^2 \left(\mathrm{Im}\,\phi\right)^2$$

$\mathrm{Im}\,\phi$ has a shift symmetry and act as the inflaton.
The Universe is reheated through Higgs bosons & Higgsinos.

$$T_R \simeq 4\times 10^8 \left(\frac{y}{10^{-6}}\right) \mathrm{GeV}$$

(Nakayama, Takahashi, Yanagida 13)

$y < 10^{-6}$ is required for the stability of the inflaton's trajectory.

★ The natural inflation model

$$V[\phi] = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

$$\Gamma_\phi \approx g^2 \frac{M^3}{f^2} \approx g^2 \frac{\Lambda^6}{f^5} \quad \Rightarrow \quad T_R \approx 5\times 10^7 \left(\frac{g}{0.1}\right) \mathrm{GeV} \quad \text{for } f = 7M_{Pl}$$

$$M \equiv \frac{\Lambda^2}{f}$$

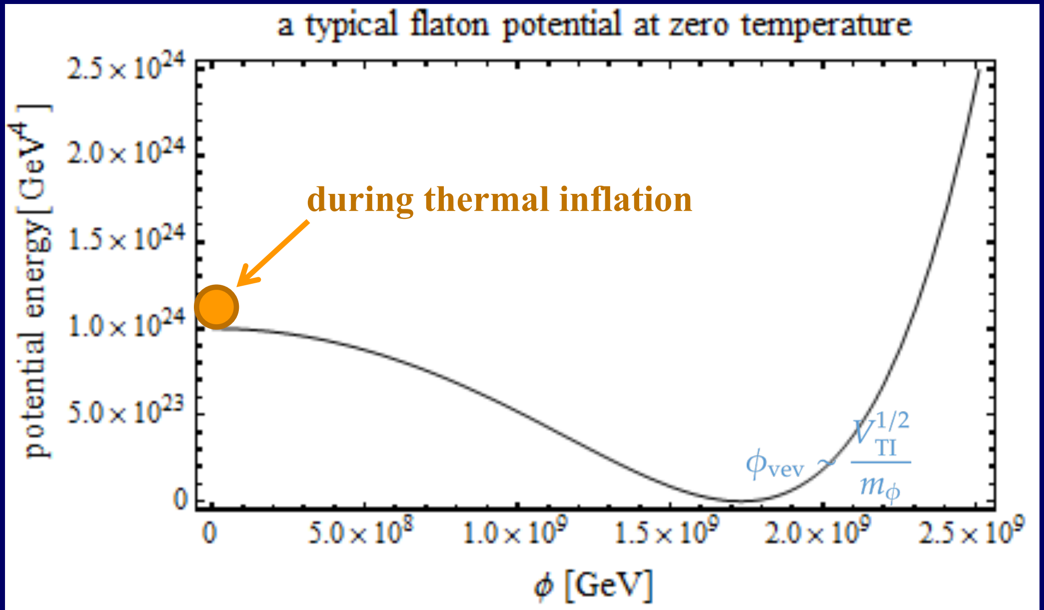
(Freese, Frieman, Olinto 90)

All these high frequency GWs would be diluted away if there existed entropy production after their horizon reentry, such as thermal inflation…

Thermal Inflation

Yamamoto (1986), Lyth & stewart (1995)

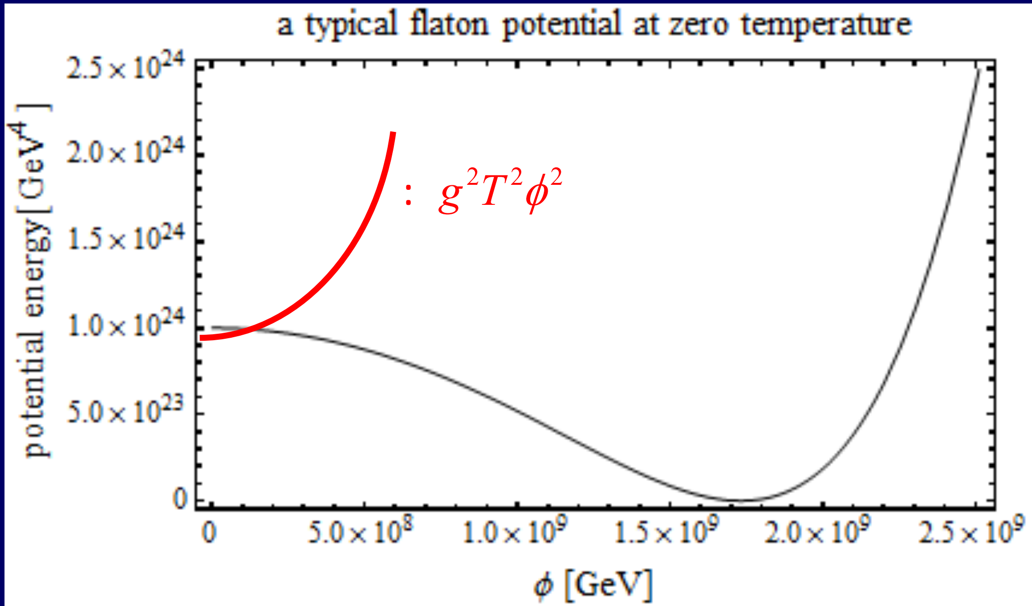
A short period of inflation driven by a scalar potential whose symmetry is restored by finite-temperature effects with a large vacuum energy density there.



flaton: a scalar field w/ a very flat potential near the origin

$$V_0[\phi] = V_{\rm TI} - \frac{1}{2}m_\phi^2\phi^2 + c\,\phi^6$$

$V_{\rm TI}^{\frac{1}{4}} \sim 10^{6-7}\mathrm{GeV}$ $\quad m_\phi \sim 10^3\mathrm{GeV}$



$$V_0[\phi] = V_{\rm TI} - \frac{1}{2}m_\phi^2\phi^2 + c\,\phi^6$$

$V_{\rm TI}^{\frac{1}{4}} \sim 10^{6-7}\mathrm{GeV}$ $\quad m_\phi \sim 10^3\mathrm{GeV}$

For $gT > m_0 : 10^3\,\mathrm{GeV}$ the symmetry is restored.

$\rho_{\rm rad} : T^4$ can be smaller than $V_{\rm TI}$

THERMAL INFLATION

Inflation supported by thermal correction may occur for a short period until high tempereture correction becomes ineffective.

😃Dilutes gravitinos and moduli

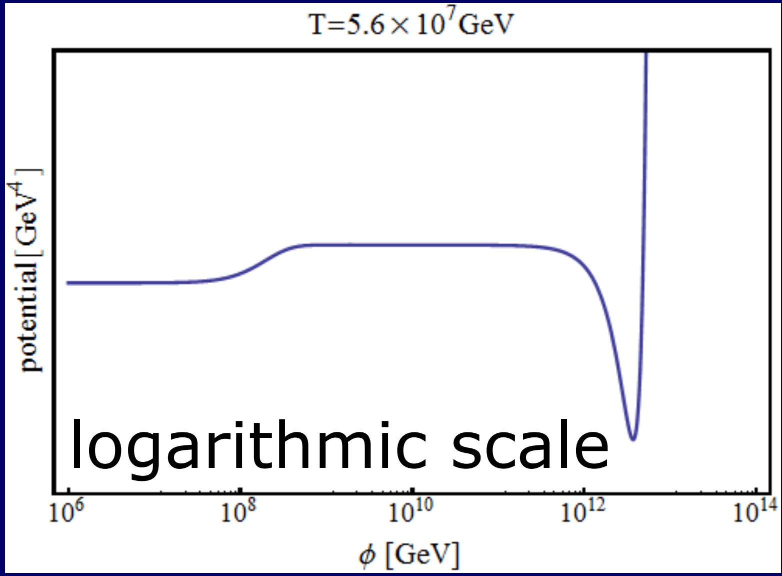
😢Dilutes subhorizon gravitational waves

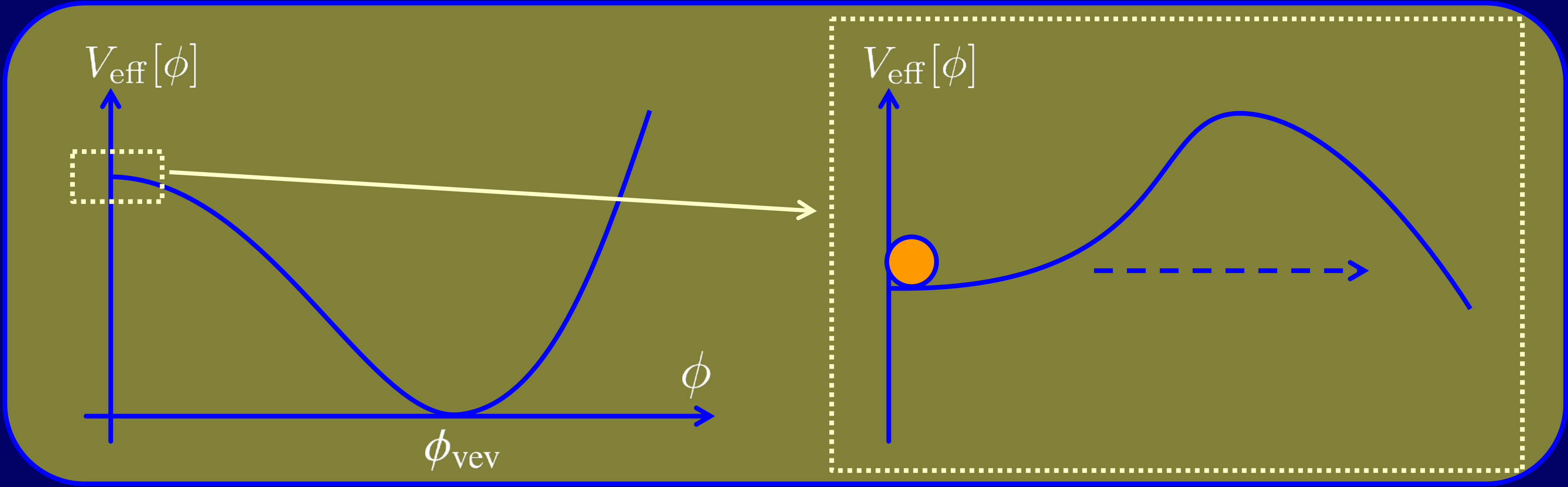
One-loop effective potential predicts a first-order phase transition

$$V_T^{1-\mathrm{loop}}[\phi] = T^4 \sum_p g_p J_p\left(\frac{m_p^2(\phi,T)}{T^2}\right)$$

$$J_\pm(y^2) = \pm\frac{1}{2\pi^2}\int_0^\infty dx\, x^2 \ln\left(1 \mp e^{-\sqrt{x^2+y^2}}\right)$$

$$m_p^2(\phi,T) \approx \begin{cases} m^2 + \frac{1}{2}\lambda^2\phi^2 + (\frac{1}{4}\lambda^2 + \frac{2}{3}g^2)T^2 & \text{boson} \\ \frac{1}{2}\lambda^2\phi^2 + \frac{1}{6}g^2T^2 & \text{fermion} \end{cases}$$





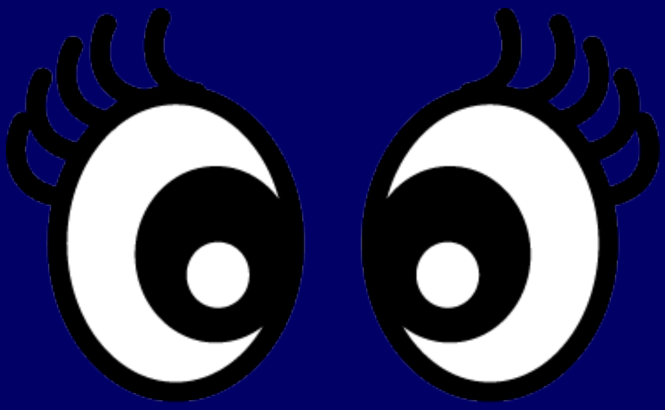
The potential barrier becomes smaller as T decreases.

If thermal inflation was terminated by bubble nucleation and percolation, colliding bubbles would produce GWs.

Conclusion

Gravitational waves provide us with new eyes to probe

* Deep inside compact celestial objects
* Very early Universe



上州無智自高無大
剛毅木訥易被欺
唯以一直接萬人
至誠依神期勝利

鑑

After entering the Hubble horizon,

$$\Omega_{GW}(f,t) = \frac{\rho_{GW,\ln f}(f,t)}{\rho_{tot}(t)} \quad \begin{matrix} \propto a^{-4}(t) \\ \propto a^{-3(1+w)}(t) \end{matrix}$$

$$w \equiv \frac{p}{\rho_{tot}}$$: equation of state in the early Universe

$$\Omega_{GW}(f,t) \approx \frac{1}{24}\Delta_h^2(f)\left(\frac{a(t_{in}(f))}{a(t)}\right)^{1-3w}_{t_{in}(f)}$$

Radiation dominated era: constant

Field oscillation dominated era: decreases $\propto a^{-1}(t)$

High frequency modes which entered the Hubble radius in the field oscillation regime acquires a suppression $\propto f^{-2}$.

We may determine the equation of state in the early Universe.

We may determine thermal history of the early Universe.

N. Seto & JY (03), Boyle & Steinhardt (08), Nakayama, Saito, Suwa, JY (08), Kuroyanagi et al (11)...

最も単純な問題:　ガウス分布のノイズの下で特定の信号 $h(t)$を検出したい

$x(t_i) \equiv x_i$ と離散化して考えると、likelihood ratioは、

$$\Lambda(x) \equiv \frac{p(x|1)}{p(x|0)} = \exp\left[-\frac{1}{2}(x_i - h_i)(K^{-1})_{ij}(x_j - h_j) + \frac{1}{2}x_i(K^{-1})_{ij}x_j\right] = \exp\left[q_i x_i - \frac{1}{2}h_i q_i\right]$$

となる。ただし、 $K_{ij} = \langle n_i n_j \rangle$ はノイズ相関　　$q_i \equiv (K^{-1})_{ij} h_j$　　$h_i = K_{ij} q_j$

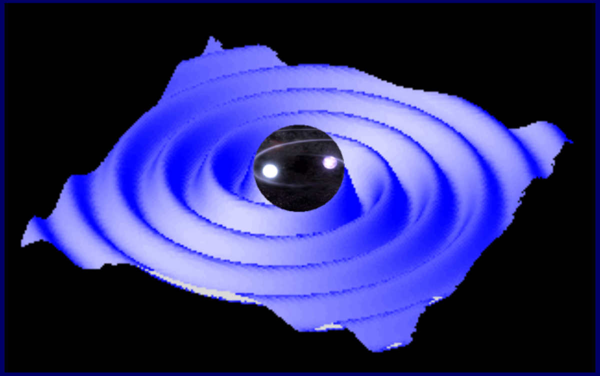
$\Downarrow$　　　　　　$\Downarrow$

連続化する　　$\langle n(t)n(t') \rangle \equiv K(t,t')$　　　$h(t) = \int_0^T K(t,t')q(t')dt$

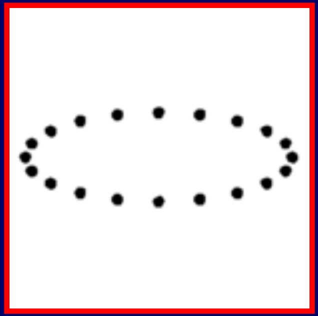
$$\ln \Lambda(x) = \int_0^T q(t)x(t)dt - \frac{1}{2}\int_0^T q(t)h(t)dt$$ を最大化するには、

線型相関 $G \equiv \int_0^T q(t)x(t)dt$ を最大化すればよい

Gravitational Waves



- Transverse waves with
  2 polarizations:

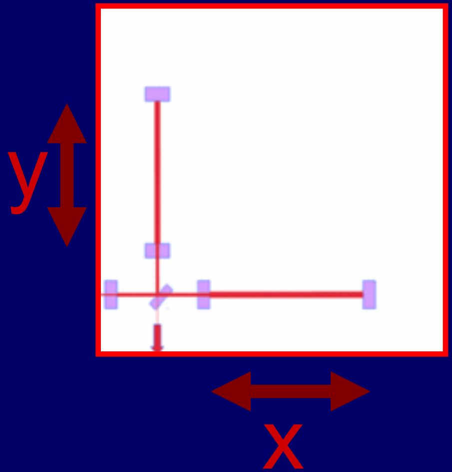


+ polarization

x polarization

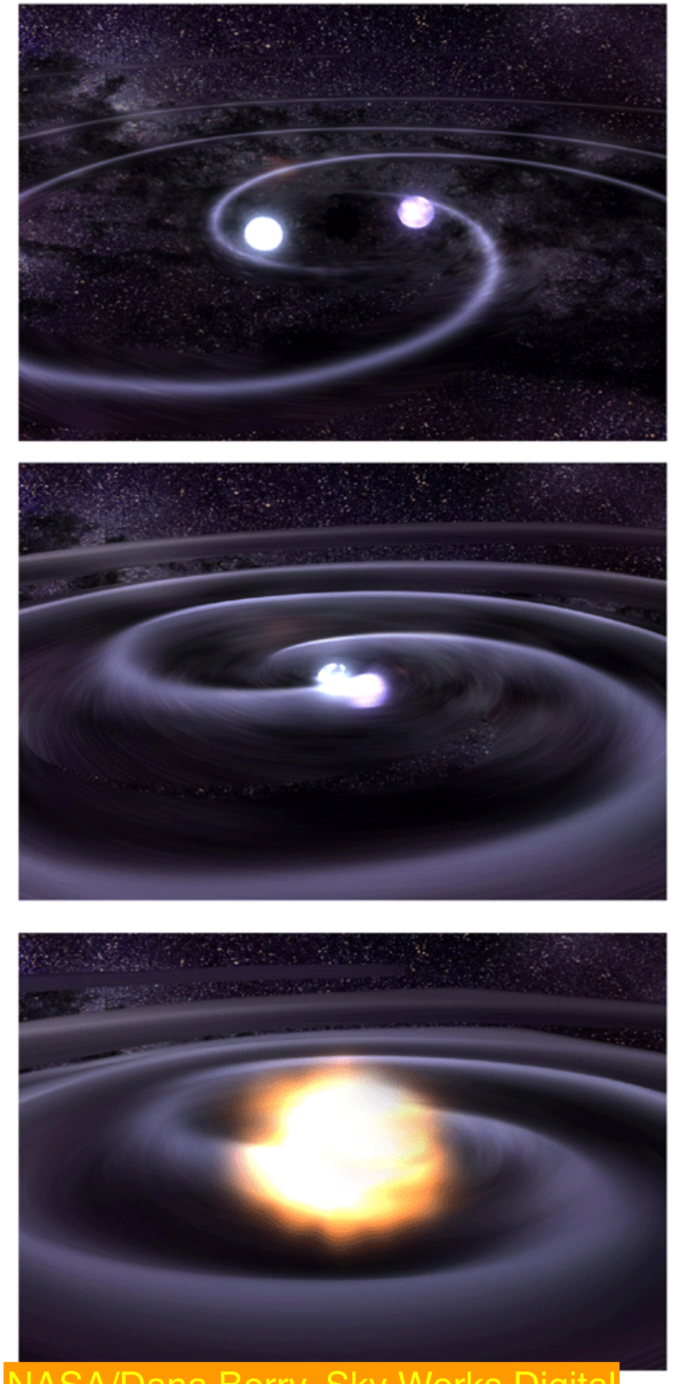
Image credit: Google

- Interferometer measures:
  dL=x-y



- Note: No test particle moves
  - only their separation changes!

The weakness of Gravity



NASA/Dana Berry, Sky Works Digital

- Gravitational waves produced by orbiting masses:

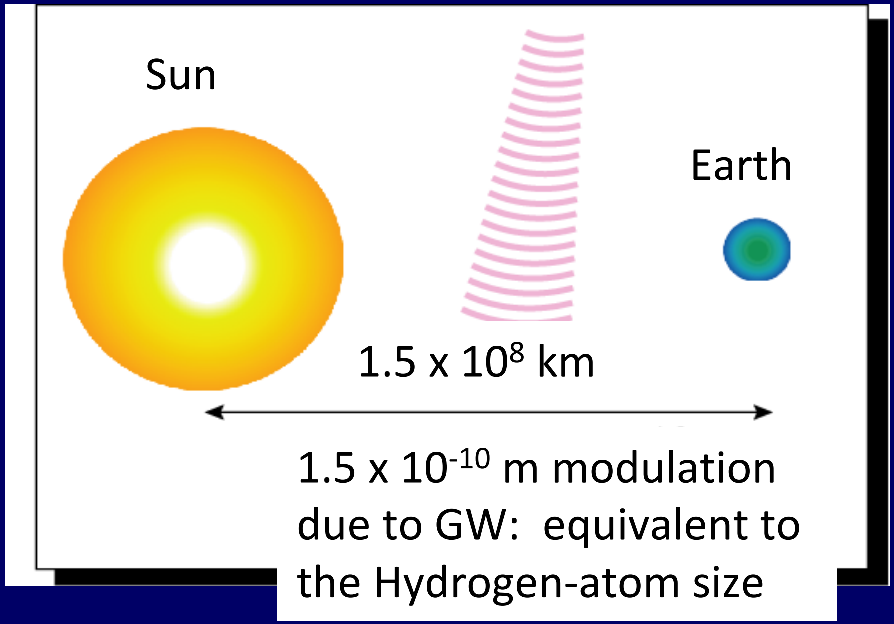
$$\boxed{h_{\mu\upsilon} = \frac{2G}{c^4 d}\ddot{I}_{\mu\upsilon}}$$

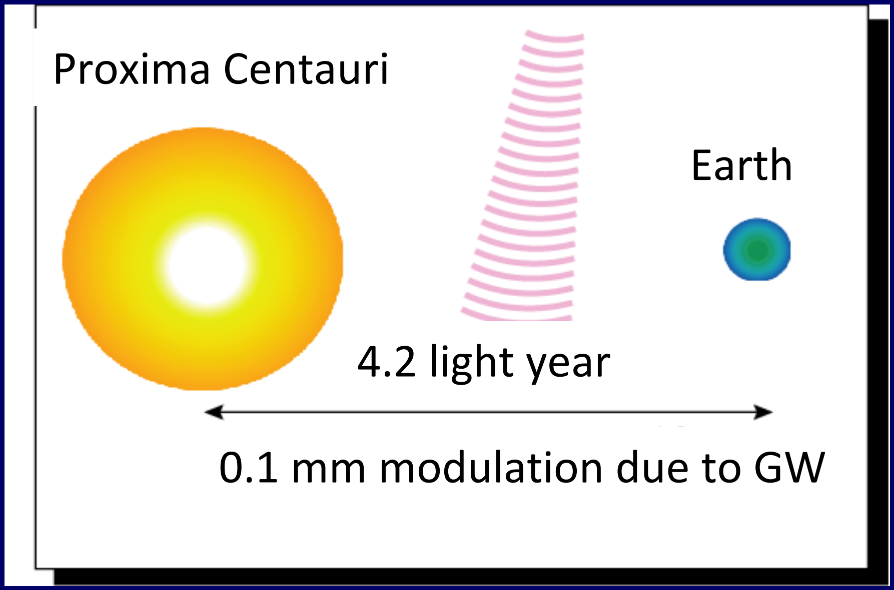
I : Quadratic moment

- For 2 1.4M_{Sun} Neutron stars, at 1 Mpc (3 million light years):

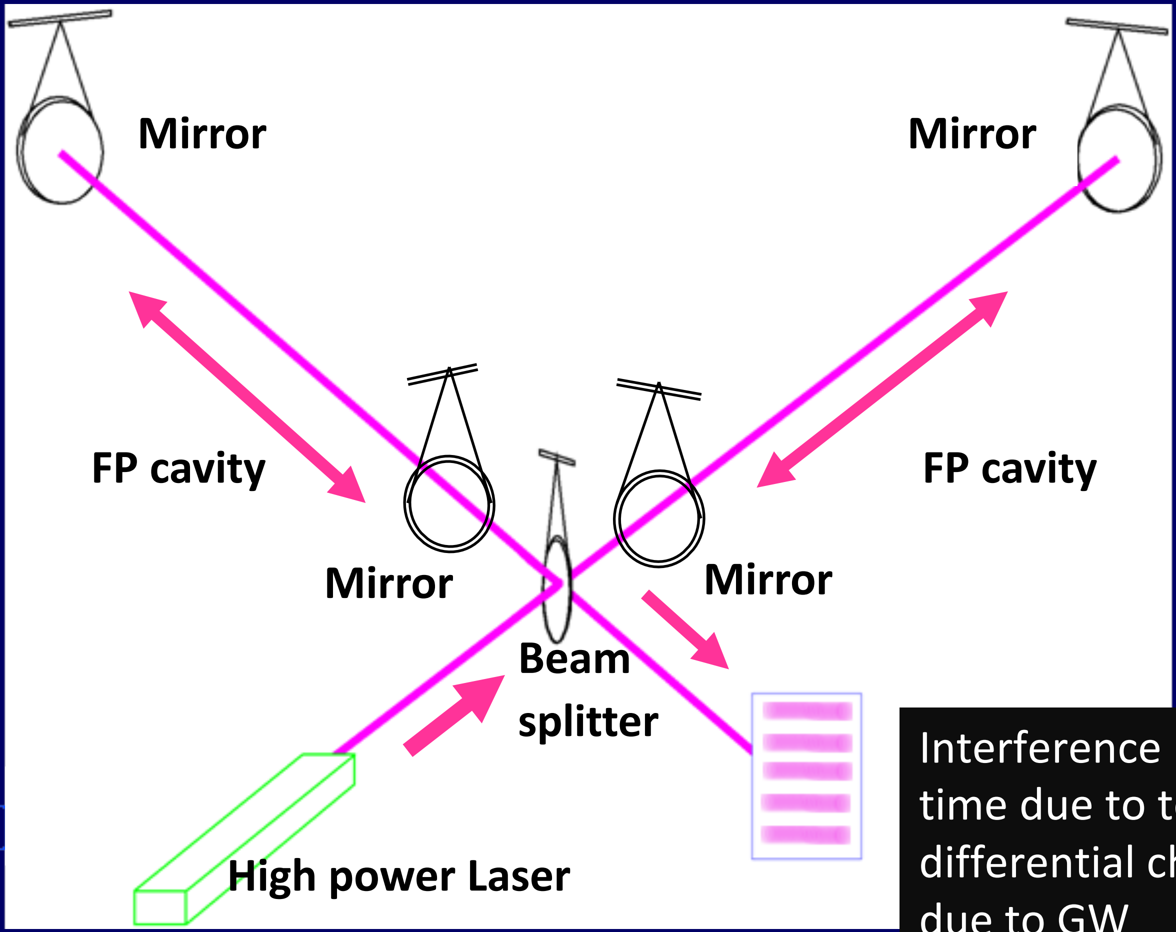
$$\boxed{h = \frac{\Delta L}{L} \approx 3\times10^{-21}}$$

A strain (displacement) of $h = \frac{\Delta L}{L} \approx 3\times 10^{-21}$



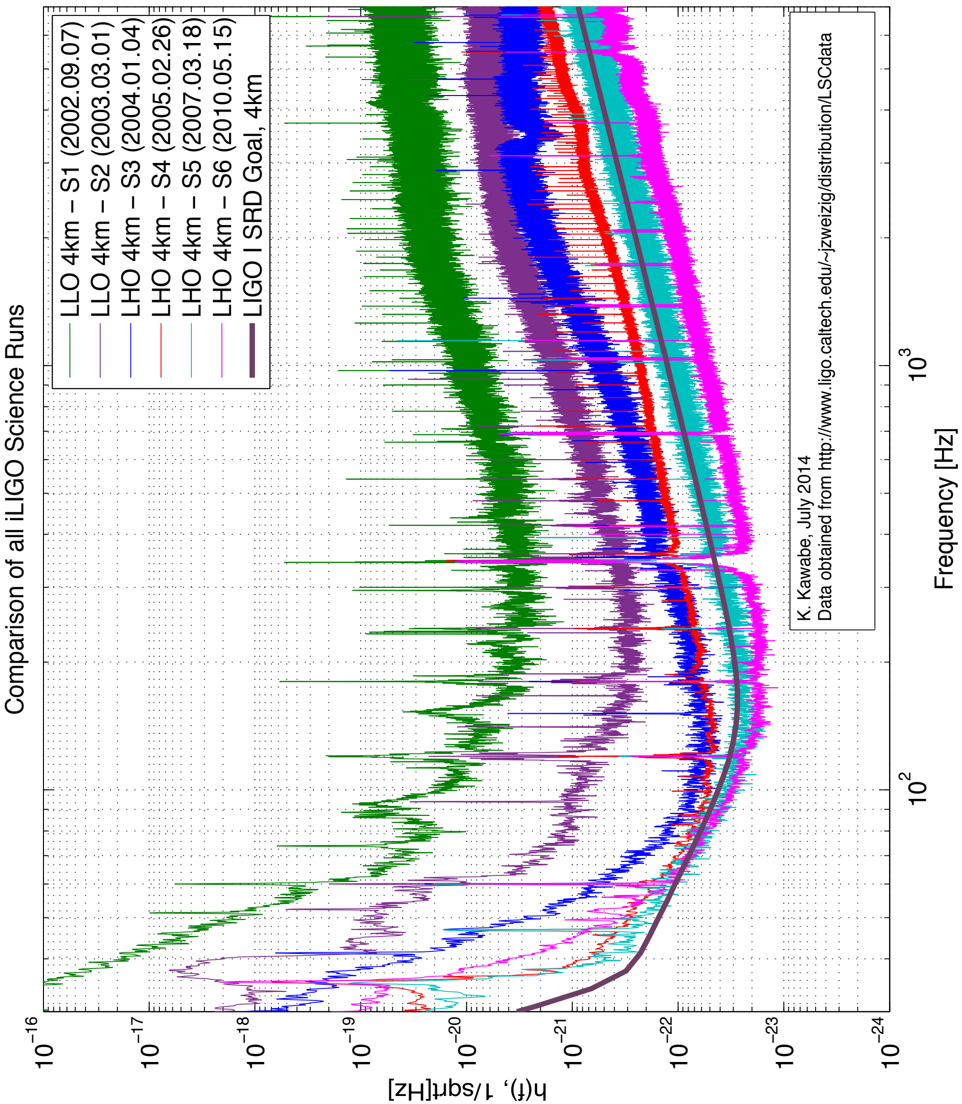


Use a Laser interferometer to detect GWs



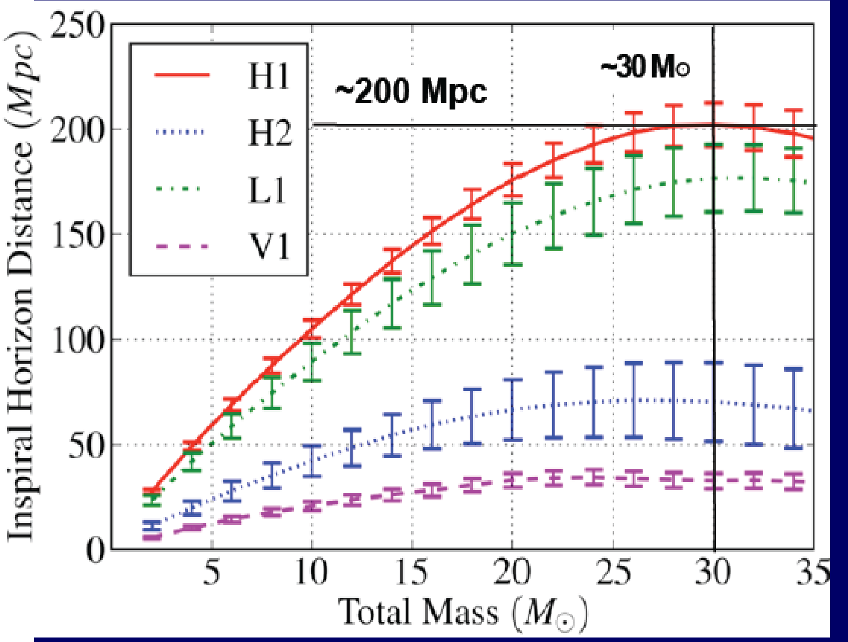
Interference pattern changes in time due to temporal differential change in distances due to GW

Improvement of sensitivity
Initial LIGO Science Run 1-6



courtesy Keita Kawabe

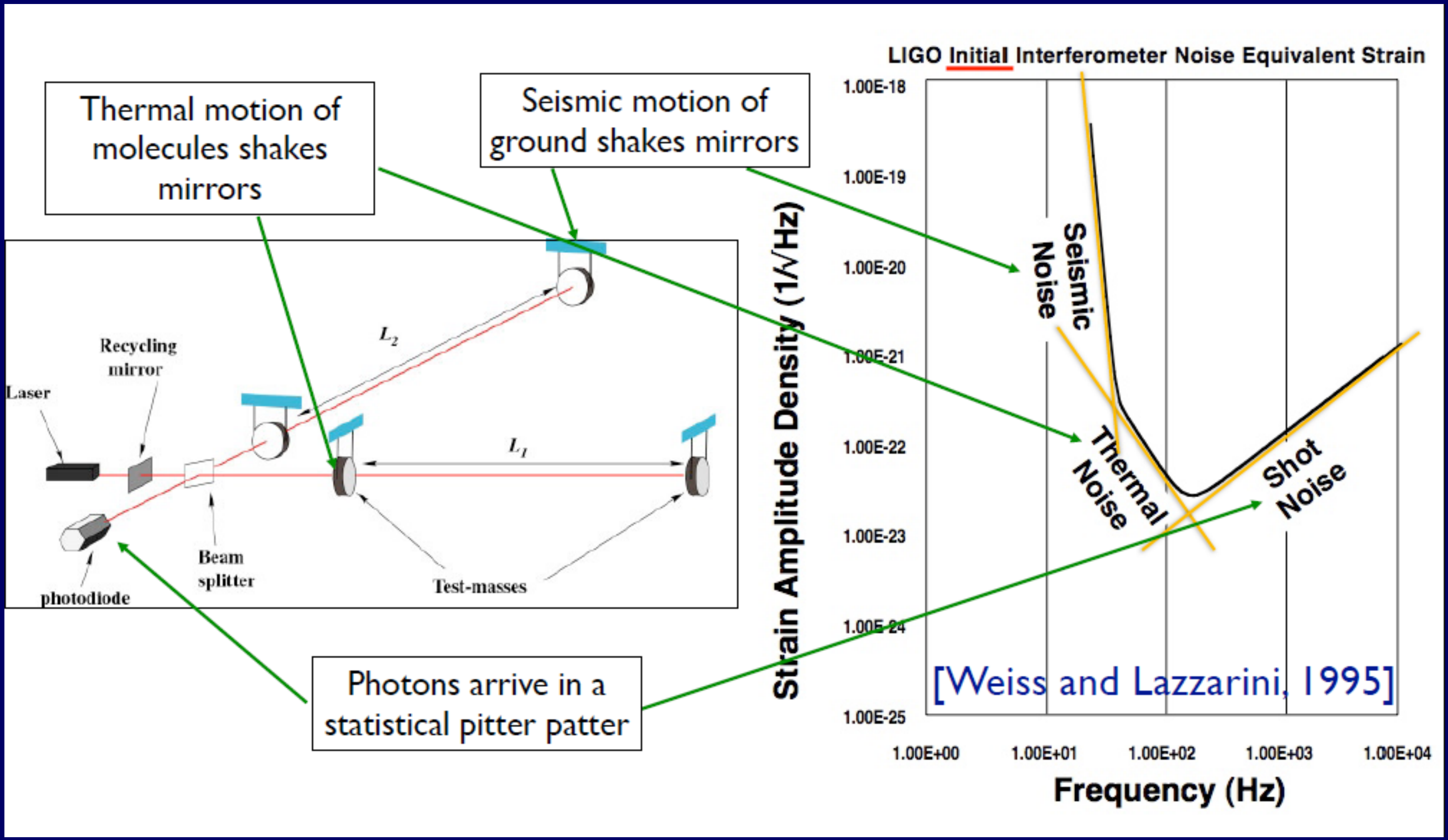
Horizon = Distance at which optimally oriented inspiral yields Signal-to-Noise ratio 8

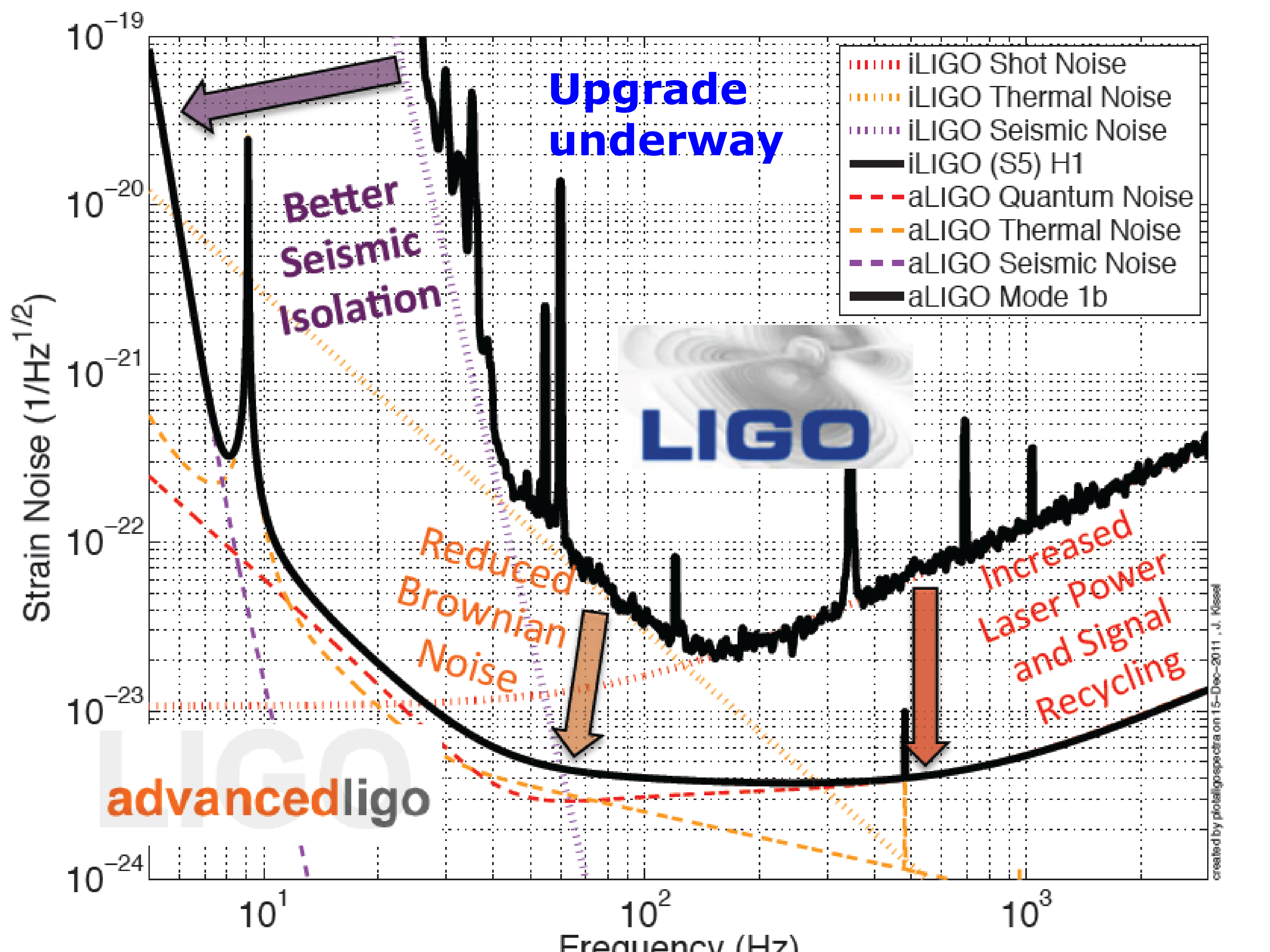


LIGO S5 and Virgo SR1

PRD 82(2010)102001

Origin of the noise curve

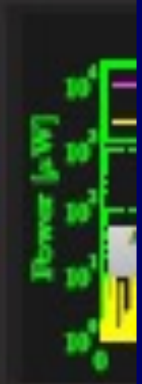




iLIGO Shot Noise
iLIGO Thermal Noise
iLIGO Seismic Noise
iLIGO (S5) H1
aLIGO Quantum Noise
aLIGO Thermal Noise
aLIGO Seismic Noise
aLIGO Mode 1b
Upgrade underway
Better Seismic Isolation
LIGO
Reduced Brownian Noise
Increased Laser Power and Signal Recycling
advancedligo
Strain Noise (1/Hz^1/2)
Frequency (Hz)
10^-19
10^-20
10^-21
10^-22
10^-23
10^-24
10^1
10^2
10^3
created by plotaligospectra on 15-Dec-2011, J. Kissel

Advanced LIGO installation complete!

Major milestone:
First full lock achieved!



2014/5/27

Data Analysis

Mainly led by groups at Osaka U(Tagoshi) & Osaka City U(Kanda) but RESCEU joined the team, too.

If noises are Gaussian distributed, the matched filter which takes a correlation between the detector output and an expected signal gives us a maximum likelihood estimator.

But actual noises are non-Gaussian, so we must deal with them.

Independent Component Analysis (ICA)
独立成分分析

Definition of the Problem

There are N statistically independent sources and N outputs.
Each source obeys non-Gaussian statistics except for at most one Gaussian source.
The purpose is to identify statistically independent sources from mutually correlated outputs making use of the non-Gaussianities.

Cocktail Party Problem

assume that #(sources) = #(microphones)

Independent component analysis is a method to find linear combinations of outputs so that each combination is mutually independent.

In principle we can remove non-Gaussian noises with the help of environmental meters.

For more detail, see a poster.

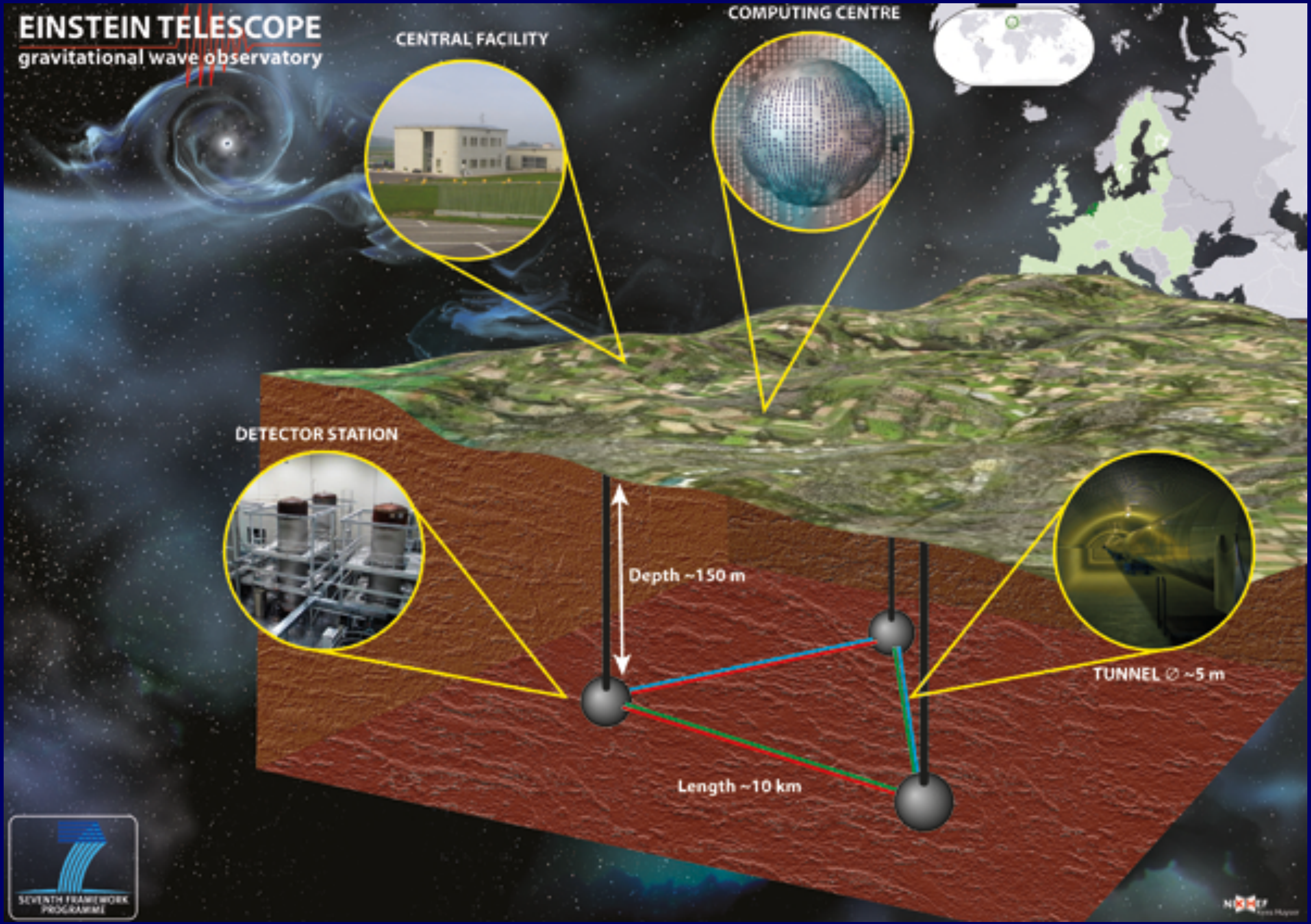
When will we detect GWs?

Some people say 2015, others say 2016, 2018+…

Most likely, GWs from Binary Neutron star coalescence but their predicted rate has a large uncertainties.

IFO	Source[a]	$\dot{N}_{\rm low}$ yr^{-1}	$\dot{N}_{\rm re}$ yr^{-1}	$\dot{N}_{\rm high}$ yr^{-1}	$\dot{N}_{\rm max}$ yr^{-1}
	NS–NS	2×10^{-4}	0.02	0.2	0.6
	NS–BH	7×10^{-5}	0.004	0.1	
Initial LIGO	BH–BH	2×10^{-4}	0.007	0.5	
	IMRI into IMBH			<0.001[b]	0.01[c]
	IMBH-IMBH			10^{-4}[d]	10^{-3}[e]
	NS–NS	0.4	40	400	1000
	NS–BH	0.2	10	300	
Advanced LIGO	BH–BH	0.4	20	1000	
	IMRI into IMBH			10[b]	300[c]
	IMBH-IMBH			0.1[d]	1[e]

Predictions are based on population synthesis theories and small-number statistics of observed binary systems.



EINSTEIN TELESCOPE
gravitational wave observatory
CENTRAL FACILITY
COMPUTING CENTRE
DETECTOR STATION
Depth ~150 m
TUNNEL ∅ ~5 m
Length ~10 km
SEVENTH FRAMEWORK PROGRAMME

BICEP2

A large tensor-to-scalar ratio r from inflation ??

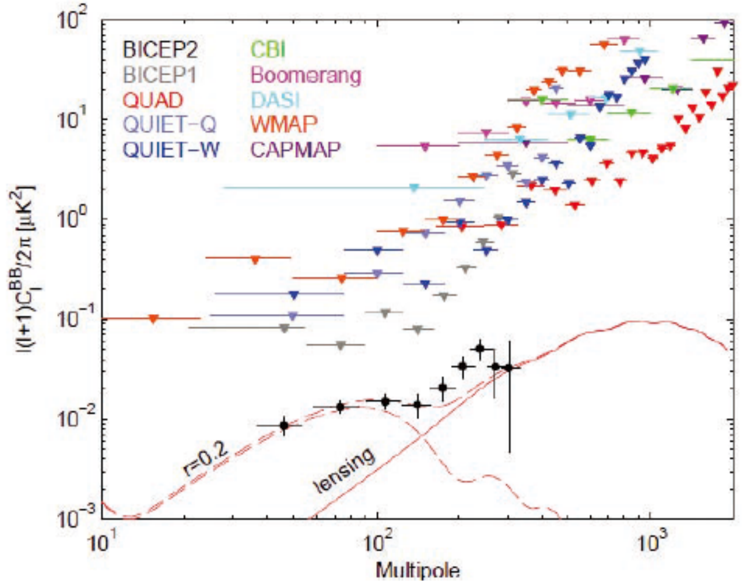


Fig. 3: BICEP2 BB Auto Spectra with 95% Upper Limits from Previous Experiments as a Function of an Angular Multipole l. Also Shown are Theoretical Predictions of Lensed and Primordial (with r=0.2) Components.
Credit: BICEP2 Collaboration

If it is the primordial one,

The physical implications of their result are profound.

0. This is the second indirect proof of the existence of gravitational waves, following the Nobel prize winning analysis of the Hulse-Taylor pulsar.
1. This is most likely a direct proof of inflation.
2. Then the energy scale of inflation is about 2×10^{16}GeV.
3. This energy scale corresponds to the cosmic time 10^{-38}sec, indicating such a tiny time period indeed existed in the beginning of our Universe!
4. This provides another piece of evidence that conventional wisdom of quantum field theory in curved space time is the right approach to calculate observable quantities in the early Universe.
5. This is the first direct evidence of quantum gravity at the perturbative level.

A number of criticisms to point out possibility of dust contamination followed

We were waiting for Planck 2014 result…

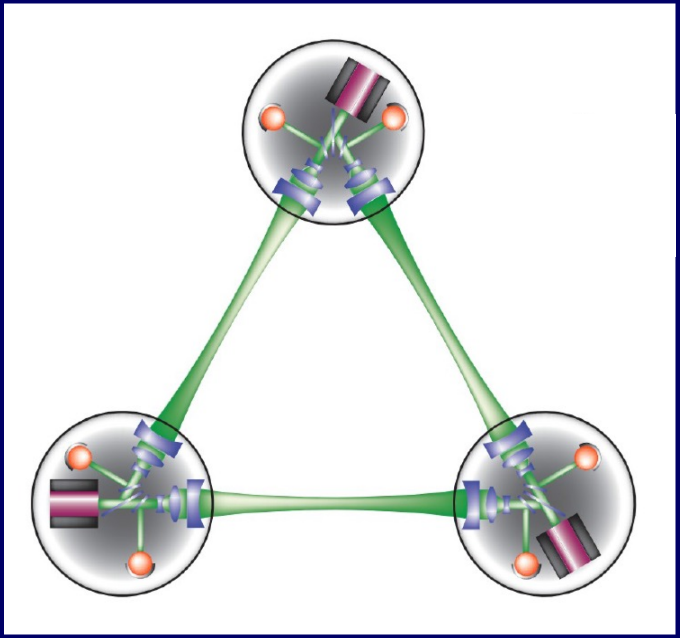
Meanwhile big interest in models with large r arouse in the community

$$r \equiv \frac{\Delta_h^2}{\Delta_{\mathrm{R}}^2}$$

Tensor-to-scalar ratio measures the scale of inflation

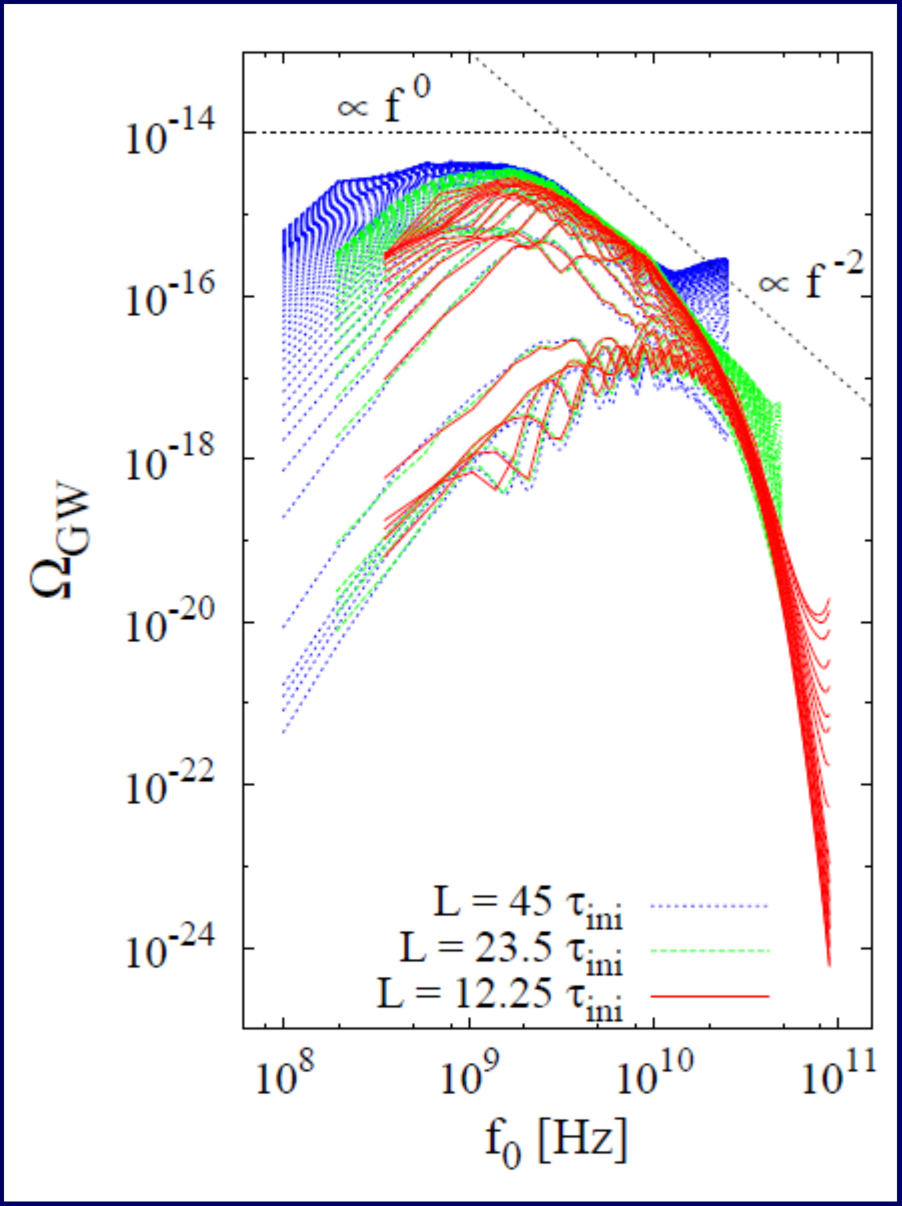
$$V[\phi] = (3.2\times10^{16}\,\mathrm{GeV})^4 r = (7.5\times10^{15}\,\mathrm{GeV})^4\left(\frac{r}{0.003}\right)$$

If r is large $\sim$0.1, we can hope to detect inflationary tensor perturbation directly by future space-based laser interferometer DECIGO at $\sim$1Hz band.

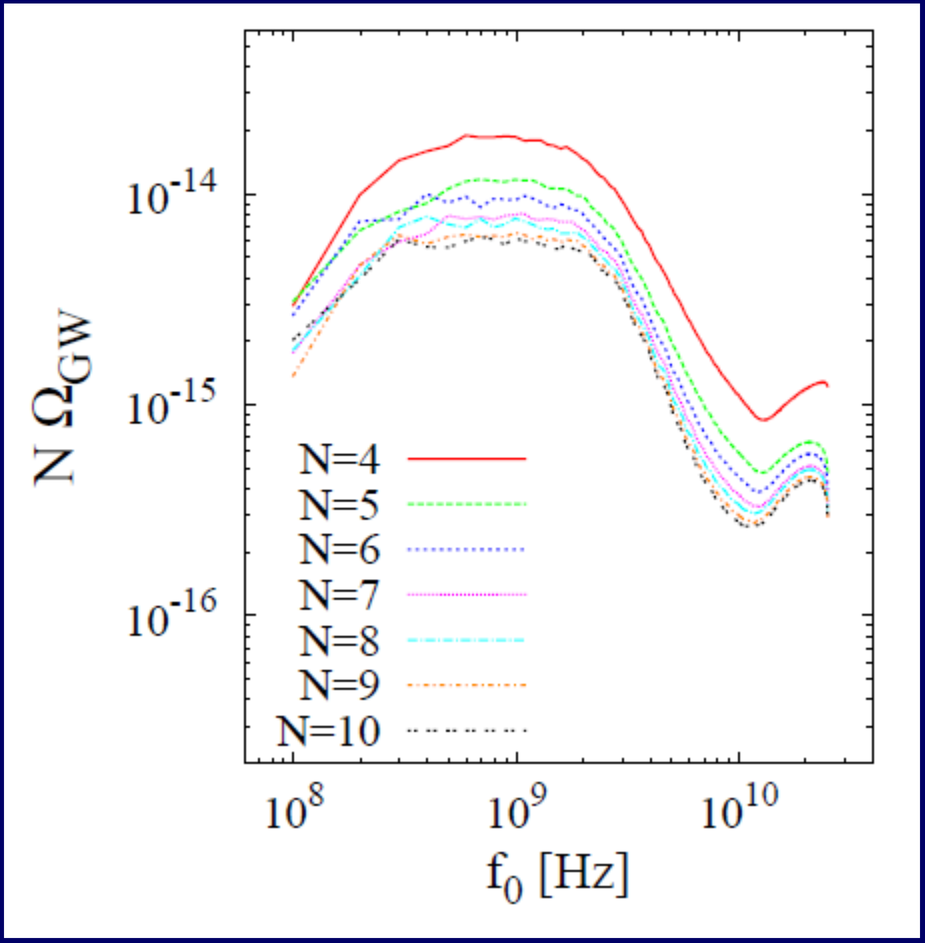


Higher frequency tensor perturbations carry information on the thermal history after inflation.

We discuss prospects of determination of the reheat temperature after inflation by DECIGO.



$\propto f^0$
$\propto f^{-2}$
Ω_{GW}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
10^{-24}
$L = 45\ \tau_{ini}$
$L = 23.5\ \tau_{ini}$
$L = 12.25\ \tau_{ini}$
10^8
10^9
10^{10}
10^{11}
f_0 [Hz]



$N\ \Omega_{GW}$
10^{-14}
10^{-15}
10^{-16}
N=4
N=5
N=6
N=7
N=8
N=9
N=10
10^8
10^9
10^{10}
f_0 [Hz]