

CMB Anisotropy and Polarization



Kin-Wang Ng (吳建宏)

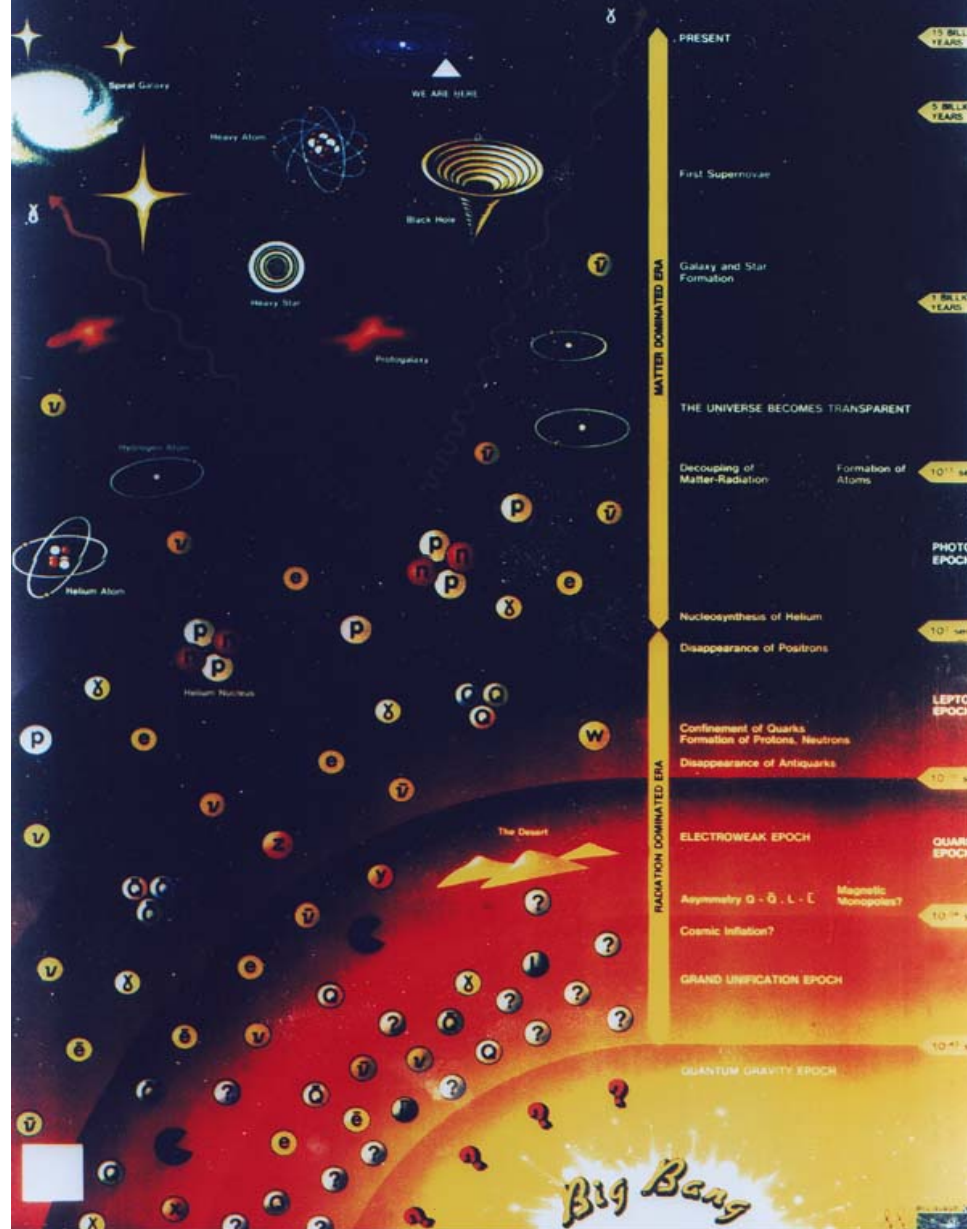
**Institute of Physics &
Institute of Astronomy and Astrophysics
Academia Sinica, Taiwan**

**APCTP School/Workshop on Gravitational-Wave Cosmology
Nov 30 - Dec 4, 2019
Taipei**

Outline

- The 3K cosmic microwave background
 - the relic photons of Hot Big Bang
- CMB anisotropy and polarization
- How to probe the early Universe and measure the cosmological parameters
- Current status and future directions

History of the Universe



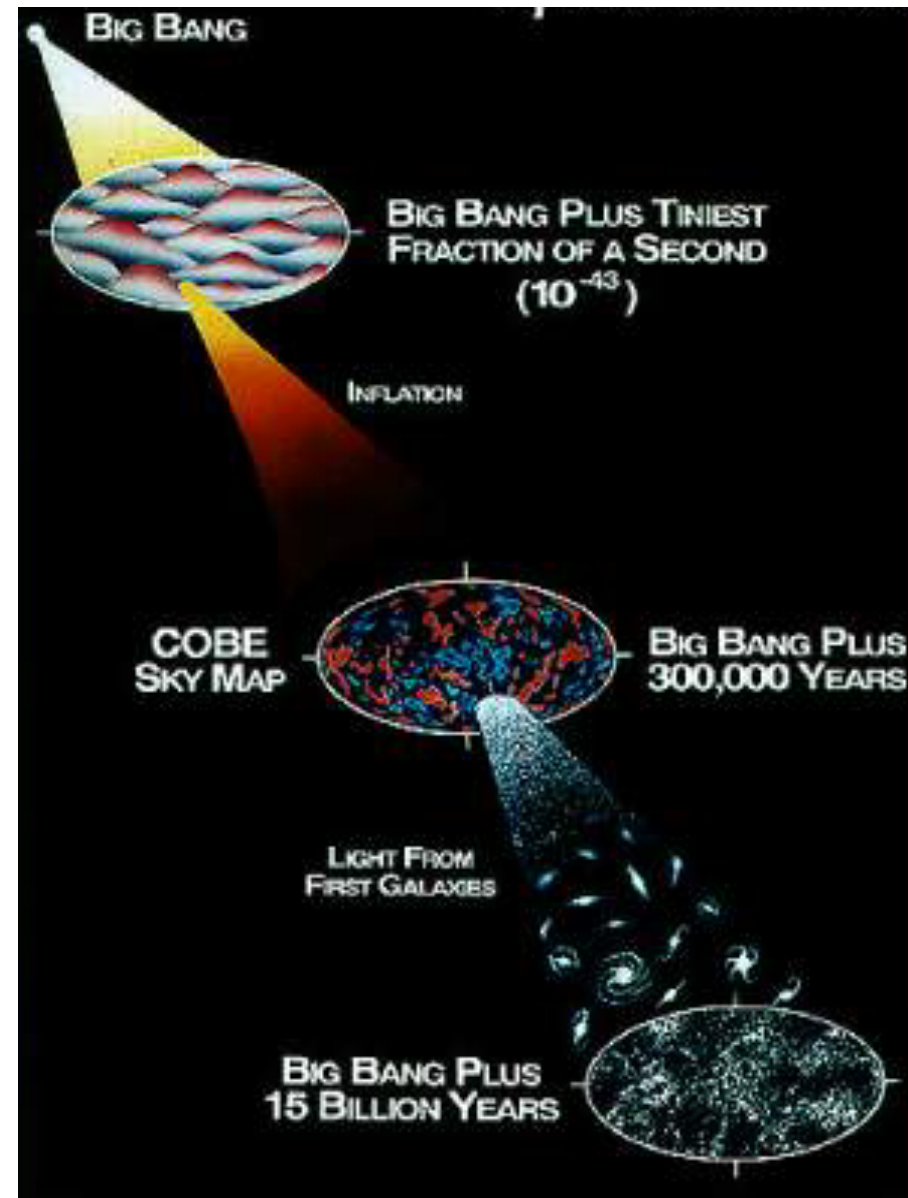
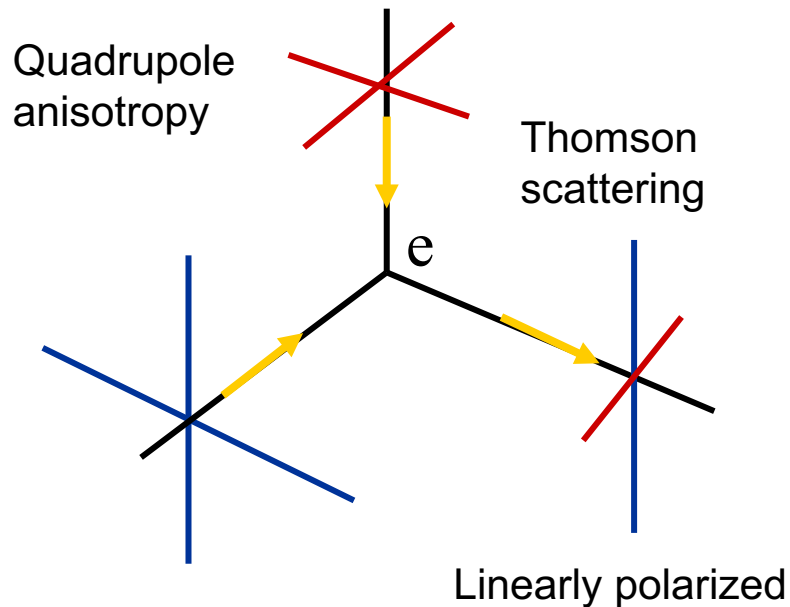
Fermilab Image 01-582D

Cosmic Microwave Background

- Relic photons of hot big bang
- First observed in 1965
- Black body radiation of temperature about 3K
- Coming from last scatterings with electrons at redshift of about 1100 or 400,000 yrs after the big bang (age of the Universe is about 14 Gyrs)
- Slightly anisotropic ($10\mu\text{K}$) and linearly polarized (μK)

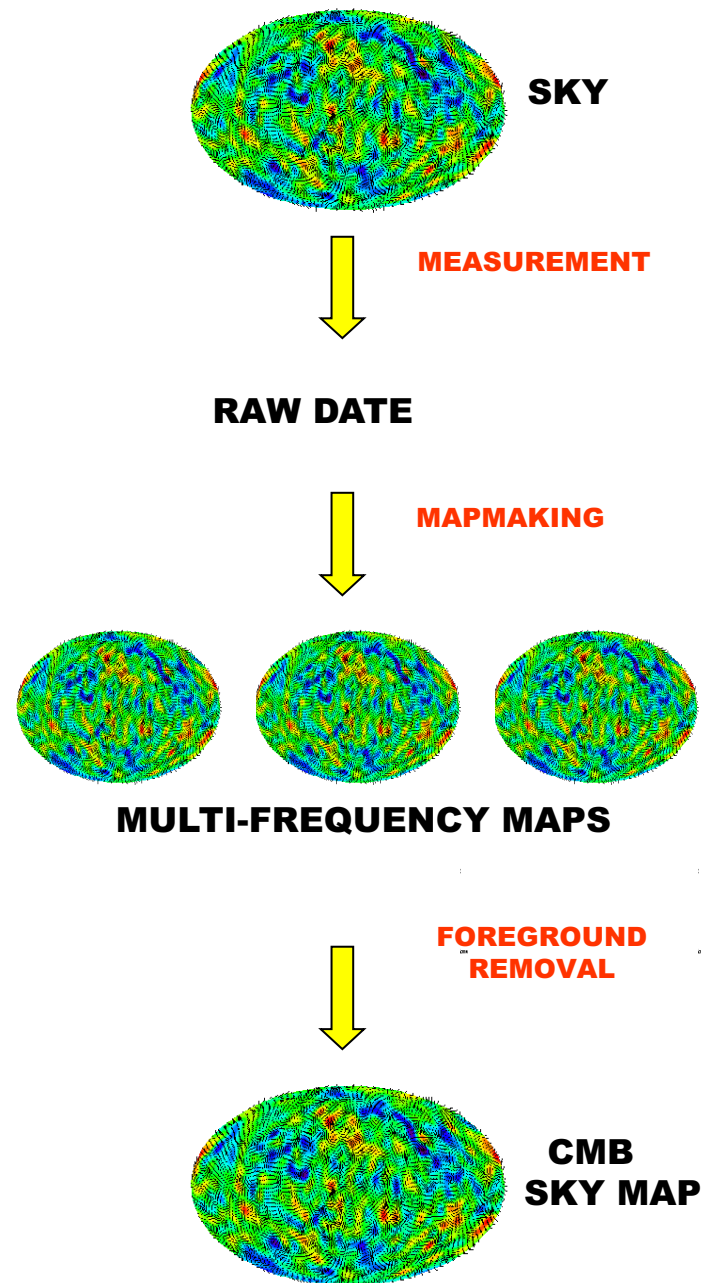
CMB Anisotropy and Polarization

- On large angular scales, matter inhomogeneities or primordial gravitational waves generate gravitational redshifts
- On small angular scales, acoustic oscillations in plasma on last scattering surface generate Doppler shifts
- Thomson scatterings with electrons generate polarization



CMB Measurements

- Point the telescope to the sky
- Measure CMB Stokes parameters:
 $T = T_{\text{CMB}} - T_{\text{mean}}$,
 $Q = T_{\text{EW}} - T_{\text{NS}}$, $U = T_{\text{SE-NW}} - T_{\text{SW-NE}}$
- Scan the sky and make a sky map
- Sky map contains CMB signal, system noise, and foreground contamination including polarized galactic and extra-galactic emissions
- Remove foreground contamination by multi-frequency subtraction scheme
- Obtain the CMB sky map



CMB Anisotropy and Polarization Angular Power Spectra

Decompose the CMB sky into a sum of spherical harmonics:

$$T(\theta, \varphi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \varphi)$$

$$(Q - iU)(\theta, \varphi) = \sum_{lm} a_{2,lm} Y_{2,lm}(\theta, \varphi)$$

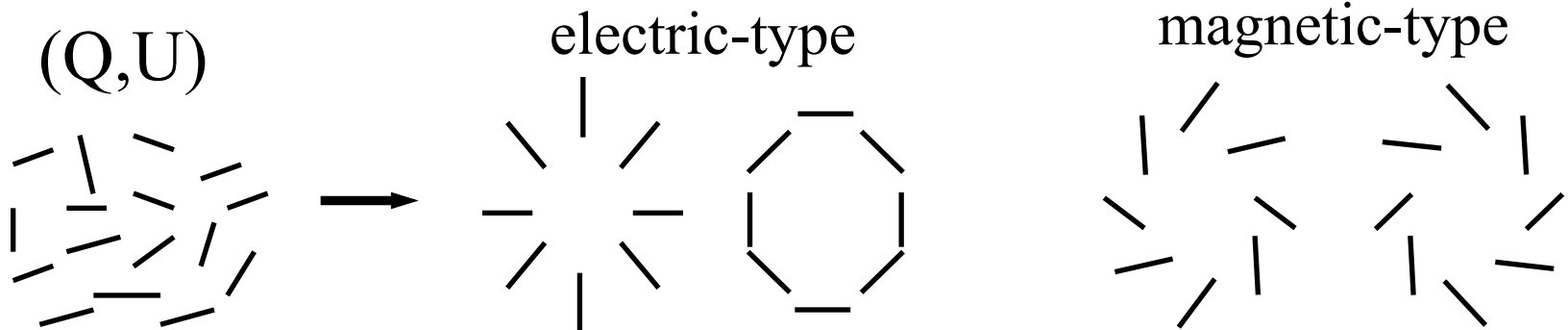
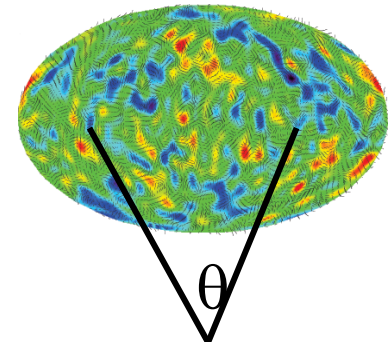
$$(Q + iU)(\theta, \varphi) = \sum_{lm} a_{-2,lm} Y_{-2,lm}(\theta, \varphi)$$

$$C_l^T = \sum_m (a_{lm}^* a_{lm}) \quad \text{anisotropy power spectrum} \quad l = 180 \text{ degrees} / \theta$$

$$C_l^E = \sum_m (a_{2,lm}^* a_{2,lm} + a_{2,lm}^* a_{-2,lm}) \quad \text{E-polarization power spectrum}$$

$$C_l^B = \sum_m (a_{2,lm}^* a_{2,lm} - a_{2,lm}^* a_{-2,lm}) \quad \text{B-polarization power spectrum}$$

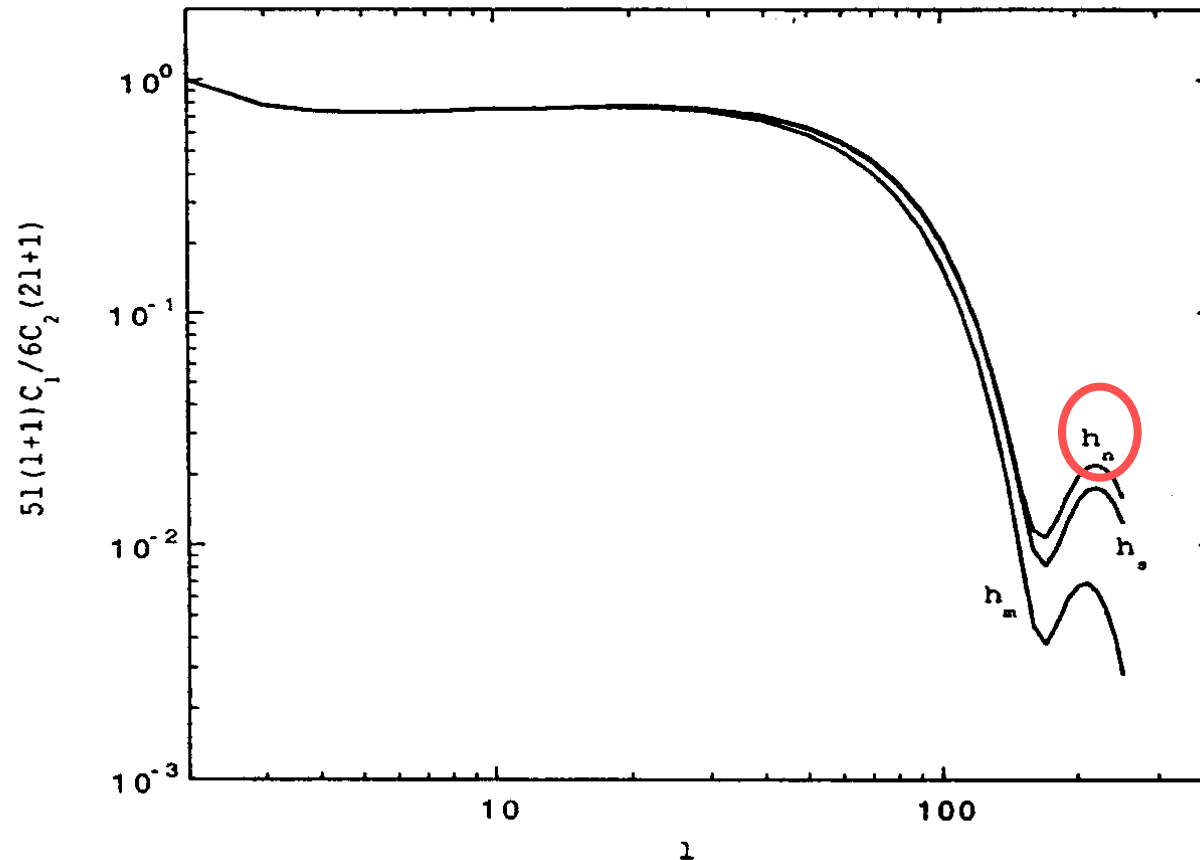
$$C_l^{TE} = - \sum_m (a_{lm}^* a_{2,lm}) \quad \text{TE correlation power spectrum}$$



PGWs and CMB temperature anisotropy

Sachs–Wolfe effect:
$$\frac{\delta T}{T}(\mathbf{e}) = -\frac{1}{2} \int_e^r d\Lambda e^i e^j \frac{\partial}{\partial \eta'} h_{ij}(\eta', \mathbf{x})$$

$$C_l = \frac{32\pi v}{3} (2l+1)(l+2)(l+1)l(l-1) \int_0^\infty \frac{dk'}{k'} \left[\int_{\eta'_{\text{dec}}}^{\eta'_0} d\eta' \frac{dh(\eta')}{d\eta'} \frac{j_l[k'(\eta'_0 - \eta')]}{k'^2(\eta'_0 - \eta')^2} \right]^2$$



$$\left(\frac{\partial}{\partial \eta} + \mathbf{e} \cdot \frac{\partial}{\partial \mathbf{x}} \right) \mathbf{n} = -\frac{1}{2} \frac{\partial \mathbf{n}}{\partial \ln \nu} \frac{\partial h_{ij}}{\partial \eta} e^i e^j - \sigma_T N_e a \times \left[\mathbf{n} - \frac{1}{4\pi} \int_{-1}^1 \int_0^{2\pi} P(\mu, \phi, \mu', \phi') \mathbf{n} d\mu' d\phi' \right], \quad (1)$$

where σ_T is the Thomson scattering cross section, N_e is the number of free electrons per unit volume, $(\mu = \cos \theta, \phi)$ are the polar angles of the propagation direction $\mathbf{e}$ of the photon with a comoving frequency ν , and P is the phase matrix for Thomson scattering.

The solution $\mathbf{n}$ for the equation of transfer assumes the form $\mathbf{n} = \mathbf{n}_0 + (n_0 \delta \mathbf{n} / 2)$, where $\mathbf{n}_0$ and $\delta \mathbf{n}$ are the unperturbed solution and perturbation, respectively. We expand $\delta \mathbf{n} = \int d\mathbf{k} \mathbf{n}' e^{i\mathbf{k} \cdot \mathbf{x}}$, where $\mathbf{n}' = \alpha \mathbf{a} + \beta \mathbf{b}$. For the scalar-mode solution, the Stokes components n_u and n_v both decouple from n_l and n_r , and it suffices to consider only the first two components of $\mathbf{n}$ with $\mathbf{a} = (1, 1)$ and $\mathbf{b} = (1, -1)$. Substituting the solution $\mathbf{n}$ and the Fourier expansion for h_{ij} into equation (1), and expanding α and β in terms of Legendre polynomials,

$$\begin{aligned} \alpha(\mu) &= \sum_l (2l+1) \alpha_l P_l(\mu), \\ \beta(\mu) &= \sum_l (2l+1) \beta_l P_l(\mu), \end{aligned} \quad (3)$$

scalar modes/
density
perturbations

$$\begin{aligned}
\dot{\alpha} + ik\mu\alpha &= \frac{1}{3}[T(k)]^{1/2}k^2\eta(1 + 2P_2) - q \\
&\times [\alpha - \alpha_0 - \frac{1}{2}P_2(\alpha_2 - \beta_0 + \beta_2)] , \\
\dot{\beta} + ik\mu\beta &= -q[\beta - \frac{1}{2}(1 - P_2)(\beta_0 - \alpha_2 - \beta_2)] ,
\end{aligned}$$

$$h_{ij} = \int dk \delta_k e^{ik \cdot x} k_i k_j, \quad \delta_k \propto k^2 \eta^2,$$

$T(k)$ CDM transfer function

$$q = \sigma_T N_e a.$$



$$\dot{\alpha}_0 = -ik\alpha_1 + \frac{1}{3} [T(k)]^{1/2}k^2\eta ,$$

$$\dot{\alpha}_1 = -\frac{i}{3} k(\alpha_0 + 2\alpha_2) - q\alpha_1 ,$$

$$\begin{aligned}
\dot{\alpha}_2 &= -\frac{i}{5} k(2\alpha_1 + 3\alpha_3) + \frac{2}{15} [T(k)]^{1/2}k^2\eta \\
&\quad - \frac{q}{10} (9\alpha_2 + \beta_0 - \beta_2) ,
\end{aligned}$$

$$\dot{\beta}_0 = -ik\beta_1 - \frac{q}{2} (\beta_0 + \beta_2 + \alpha_2) ,$$

$$\dot{\beta}_1 = -\frac{i}{3} k(\beta_0 + 2\beta_2) - q\beta_1 ,$$

$$\dot{\beta}_2 = -\frac{i}{5} k(2\beta_1 + 3\beta_3) - \frac{q}{10} (9\beta_2 + \beta_0 - \alpha_2) ,$$

$$\left. \begin{aligned}
\dot{\alpha}_l &= -q\alpha_l - \frac{ik}{2l+1} [l\alpha_{l-1} + (l+1)\alpha_{l+1}] \\
\dot{\beta}_l &= -q\beta_l - \frac{ik}{2l+1} [l\beta_{l-1} + (l+1)\beta_{l+1}]
\end{aligned} \right\} l \geq 3$$

tensor
modes/
GWs

In the case of the tensor mode, only n_v decouples, and we thus choose the basis (Polnarev 1985) $\mathbf{a} = [\frac{1}{2}(1 - \mu^2) \cos 2\phi]$ (1, 1, 0) and $\mathbf{b} = \frac{1}{2}[(1 + \mu^2) \cos 2\phi, -(1 + \mu^2) \cos 2\phi, 4\mu \sin 2\phi]$ for the + mode solution. The $\times$ mode solution is given by the same expressions with $\cos 2\phi$ and $\sin 2\phi$ interchanged, and an additional minus sign in the last component of the basis vector $\mathbf{b}$. Defining $\xi = \alpha + \beta$, we obtain a system of coupled differential equations in a similar way:

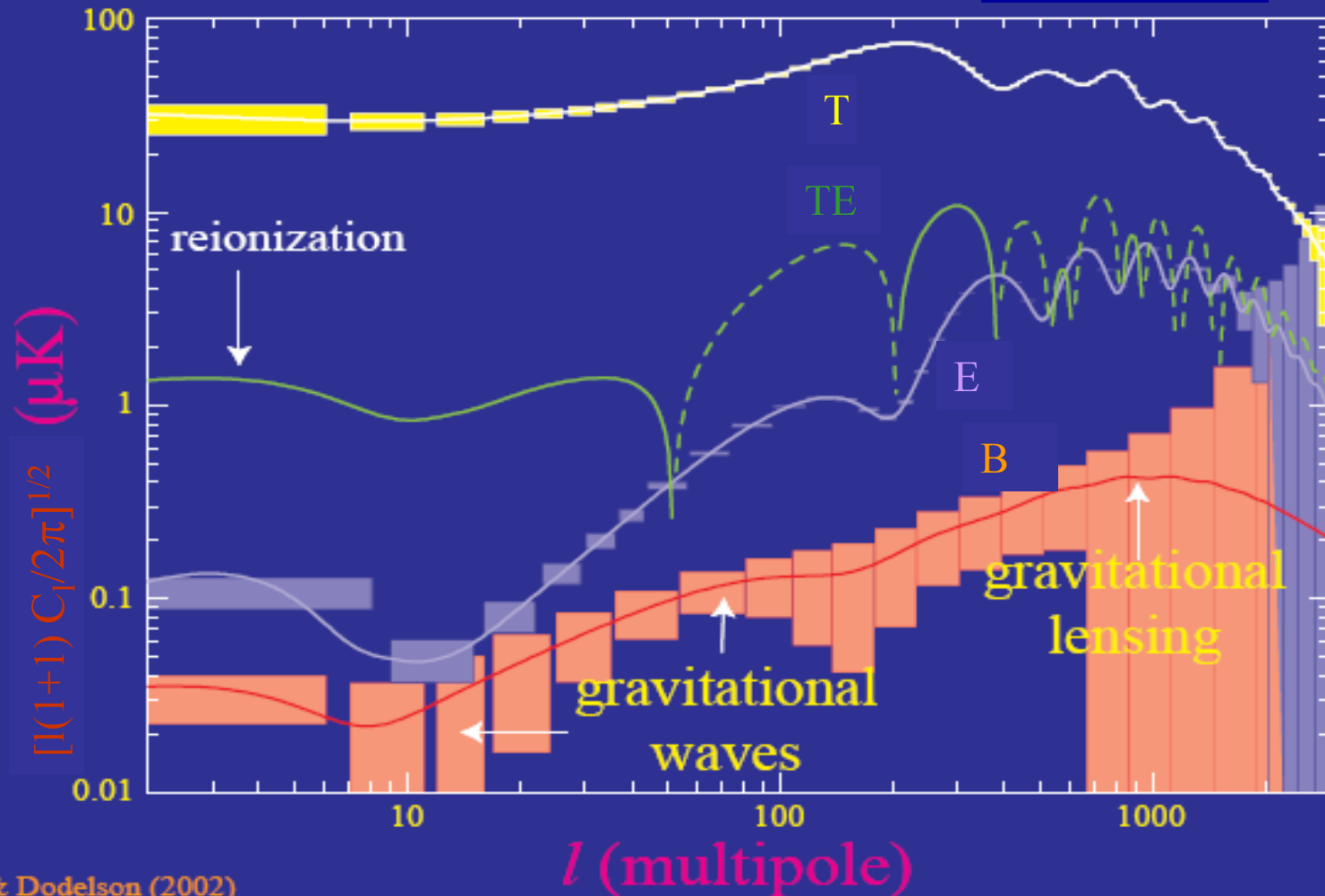
$$\begin{aligned} \dot{\xi}_0 &= -q\xi_0 - ik\xi_1 + \dot{h}_k, \\ \dot{\beta}_0 &= -\frac{3}{10} q\beta_0 - ik\beta_1 + q \\ &\quad \times (\frac{5}{7}\beta_2 + \frac{3}{35}\beta_4 - \frac{1}{10}\xi_0 + \frac{1}{7}\xi_2 - \frac{3}{70}\xi_4), \\ \dot{\xi}_l &= -q\xi_l - \frac{ik}{2l+1} [l\xi_{l-1} + (l+1)\xi_{l+1}] \\ \dot{\beta}_l &= -q\beta_l - \frac{ik}{2l+1} [l\beta_{l-1} + (l+1)\beta_{l+1}] \end{aligned} \quad \left. \vphantom{\begin{aligned} \dot{\xi}_l \\ \dot{\beta}_l \end{aligned}} \right\} l \geq 1, \quad (7)$$

$$\begin{aligned} h_{ij}^\lambda &= \int dk h_k e^{ik \cdot x} \epsilon_{ij}^\lambda \\ \ddot{h}_k + (2\dot{a}\dot{h}_k/a) + k^2 h_k &= 0, \end{aligned}$$

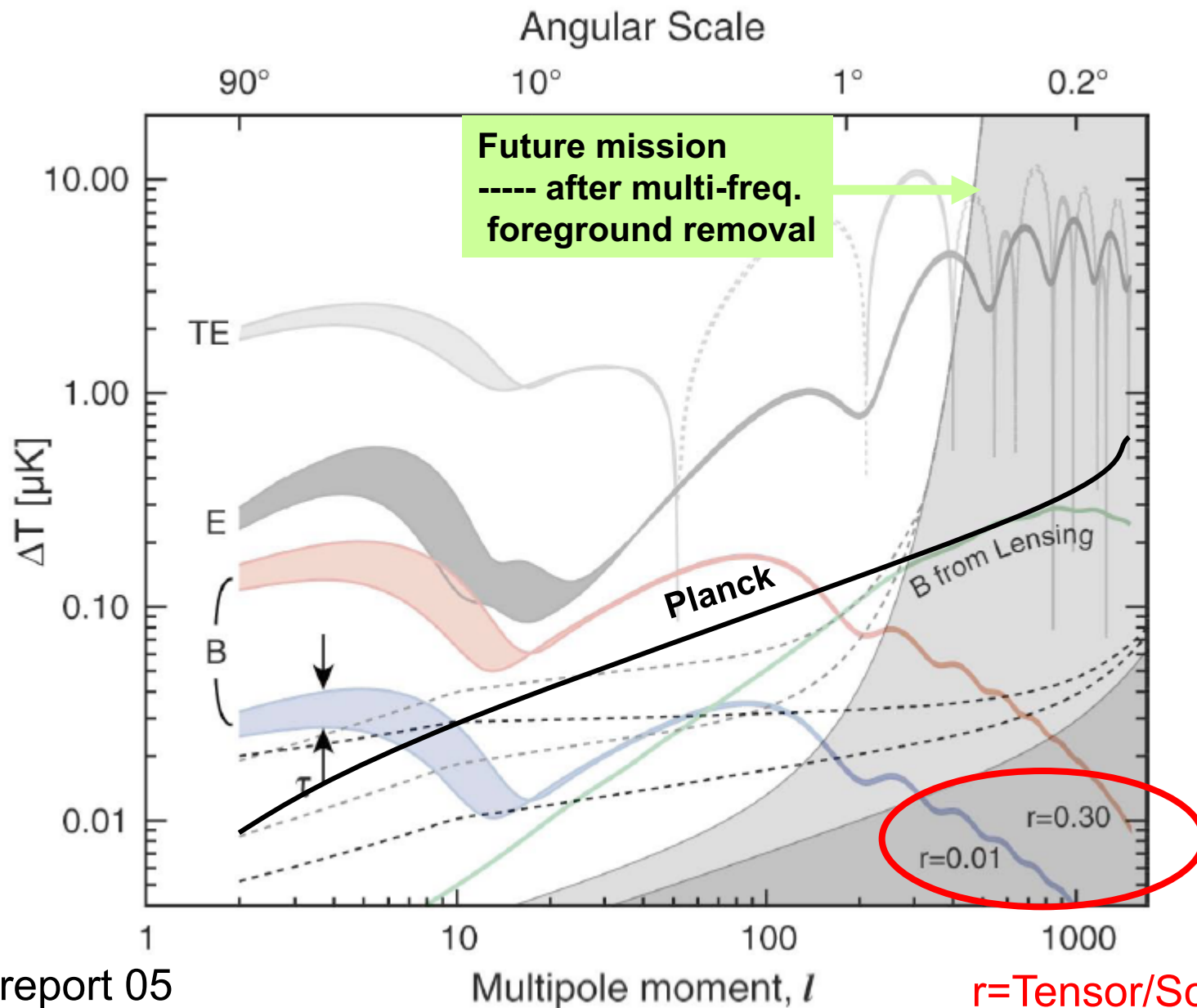
+ multi-component fluid perturbation evolution $\rightarrow$ CMB numerical codes :
COSMICS Ma+Bertschinger, CMBFAST Seljak+Zaldarriaga, CAMB,...

Theoretical Predictions for CMB Power Spectra

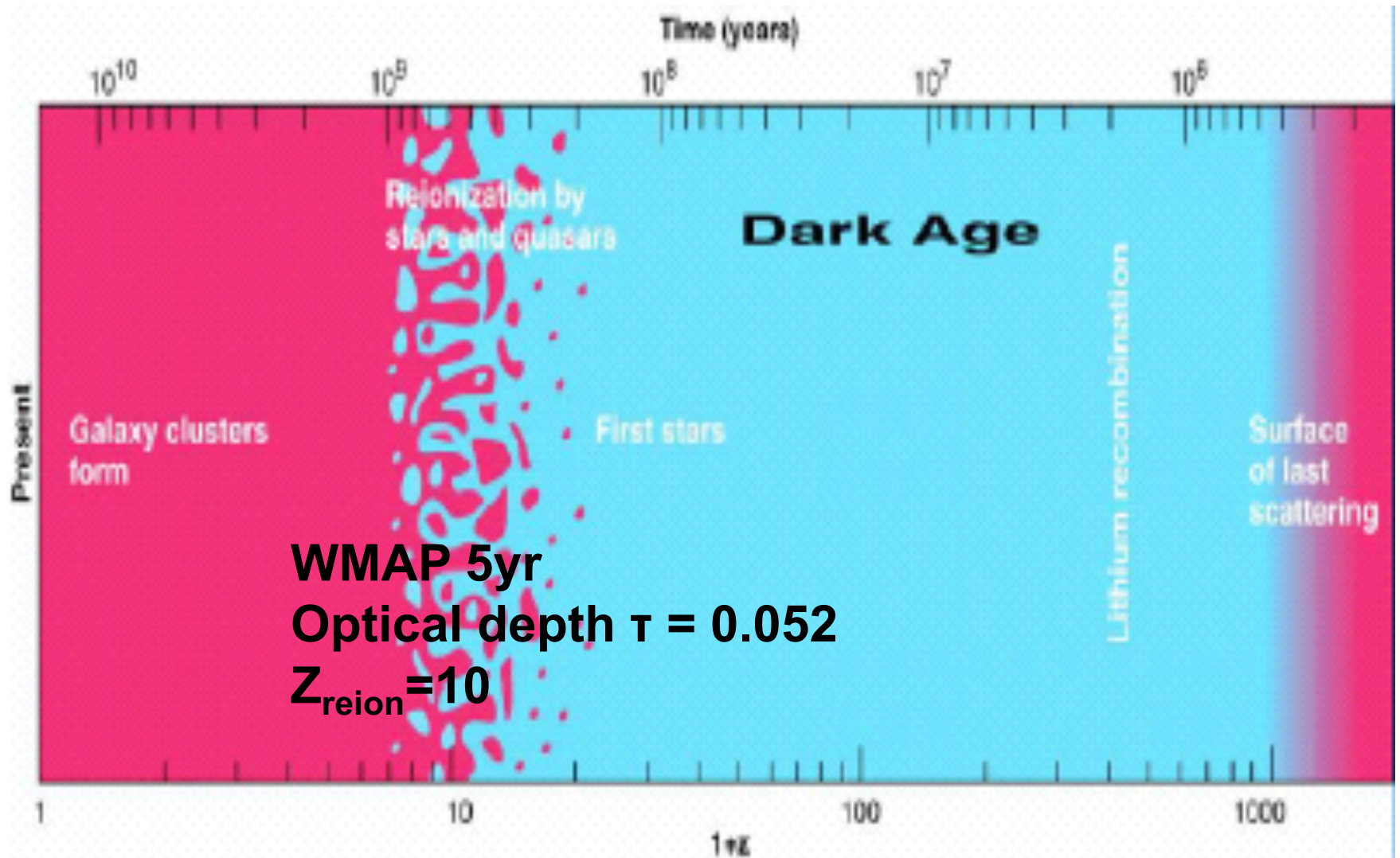
Polarized Issues



Gravity-wave induced B-mode



Large-scale CMB Polarization and Reionization of the Universe

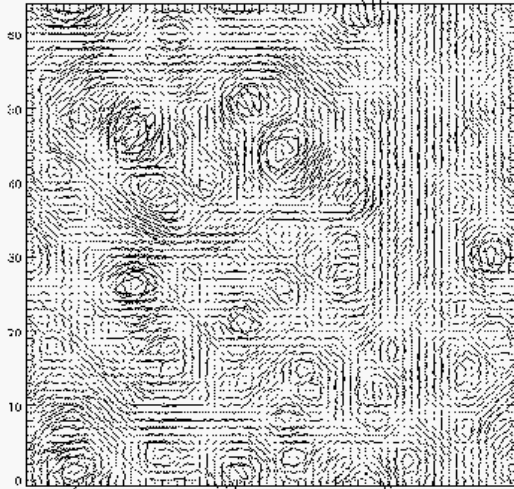


Weak Lensing by Large-Scale Structure

Jain et al. 1997

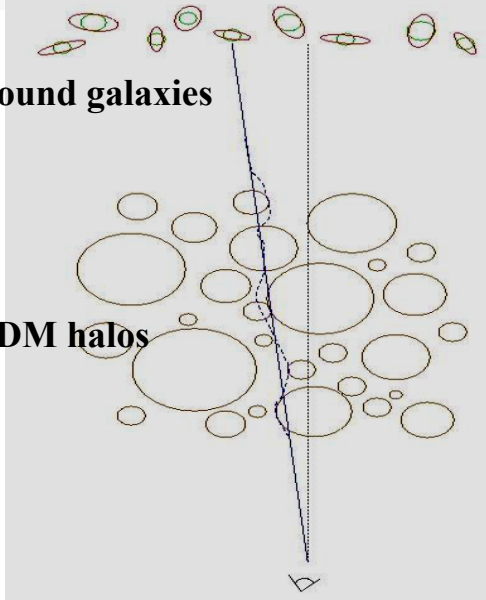
1x1 deg

Cosmic Shear γ



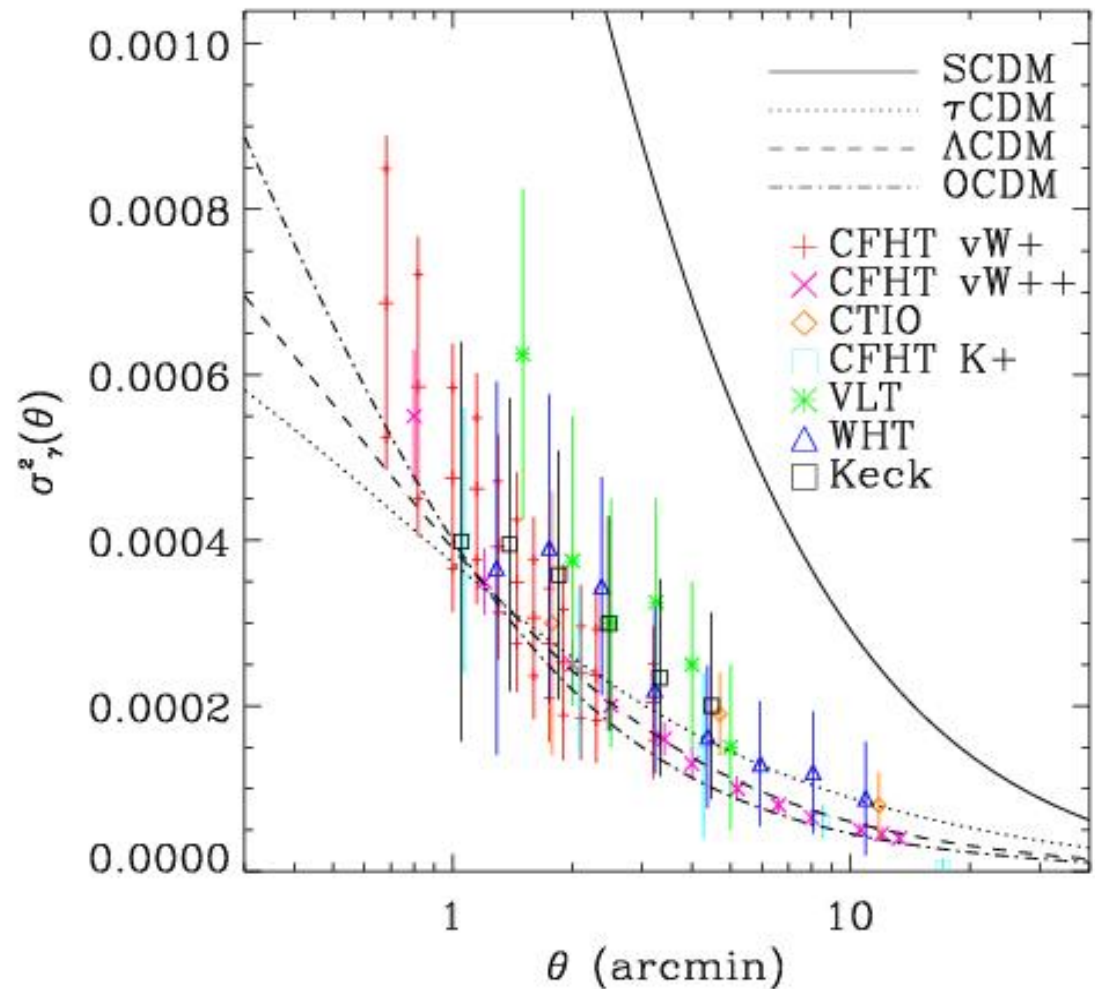
background galaxies

CDM halos



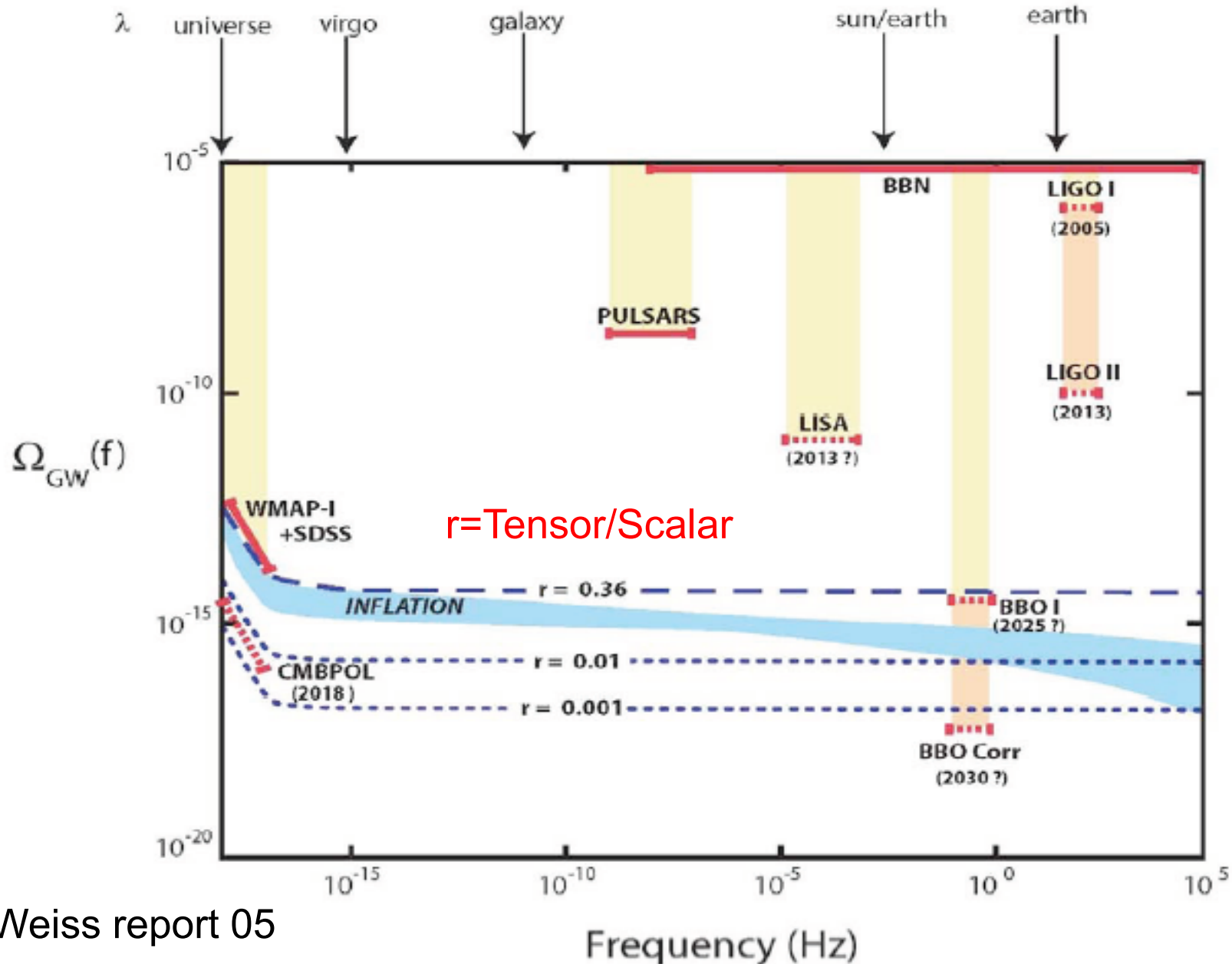
Shear Variance in circular cells with size θ

$$\sigma^2_\gamma(\theta) = \langle \gamma^2 \rangle$$

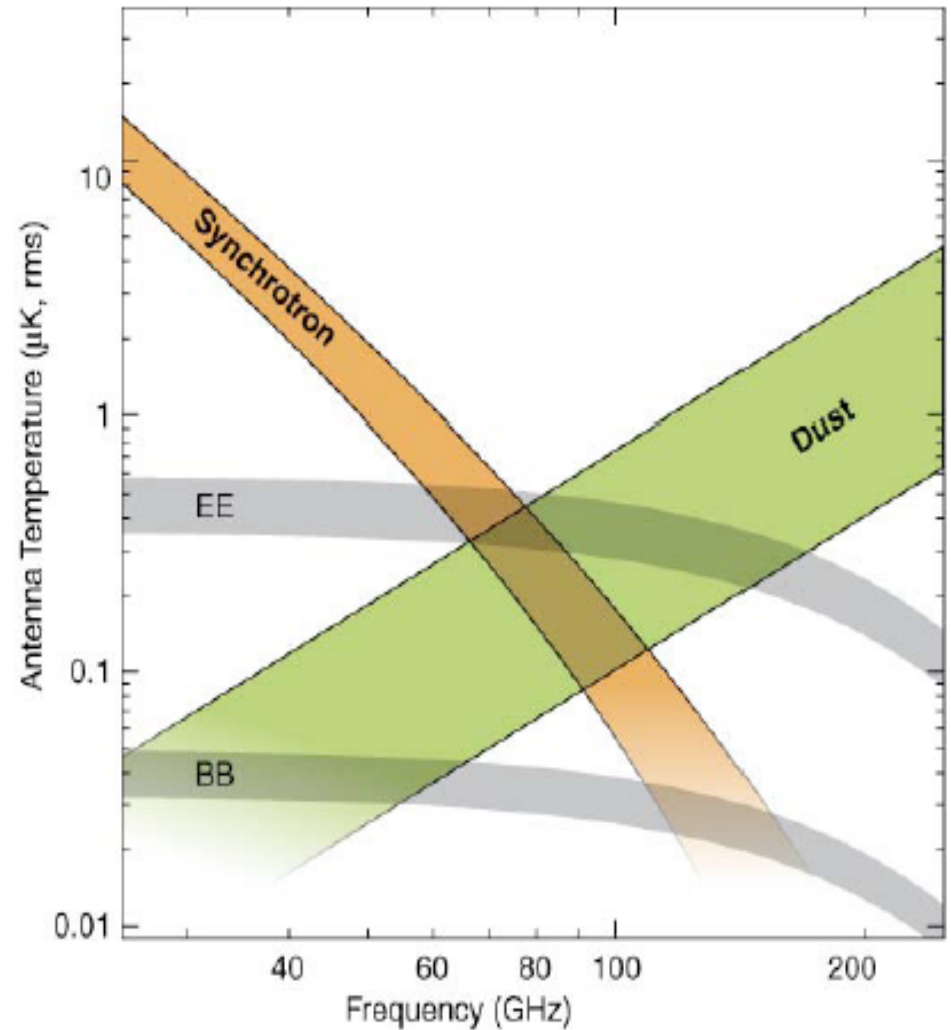
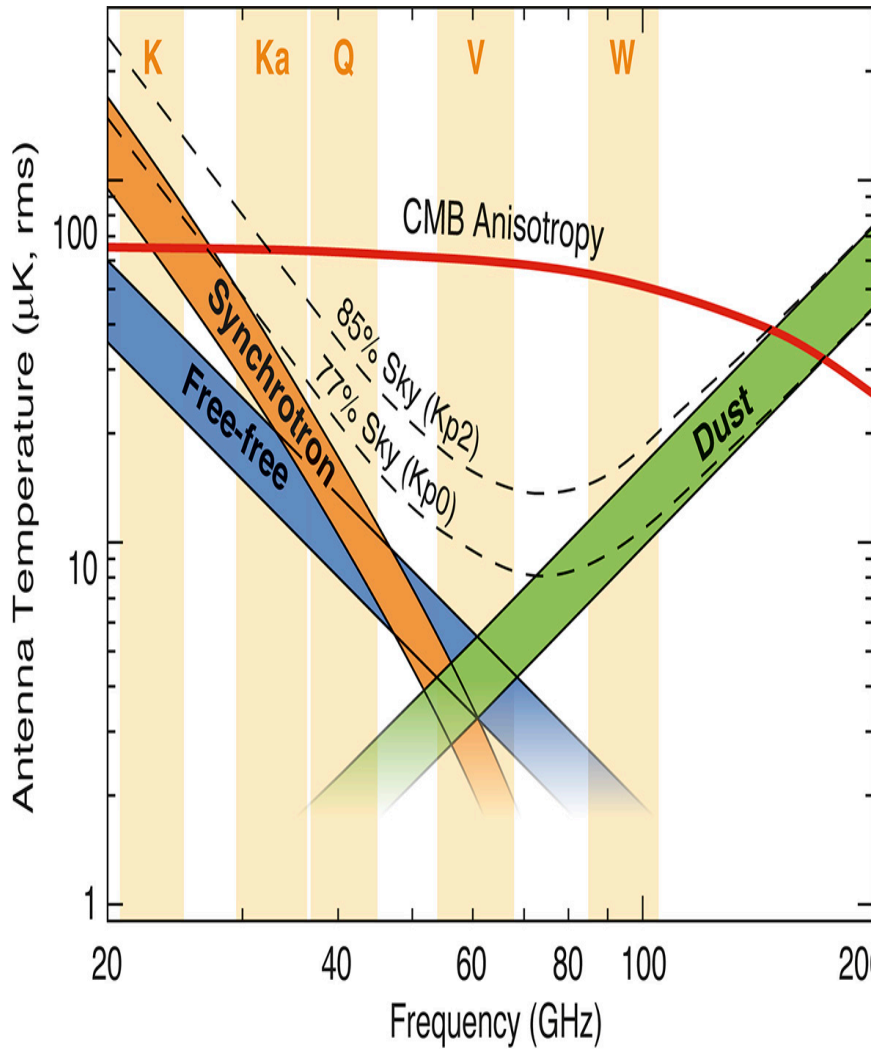


Ellis et al. 02

Primordial gravitational waves



CMB Foreground



1965

CMB Milestones

AT&T Bell

Penzias and
Wilson

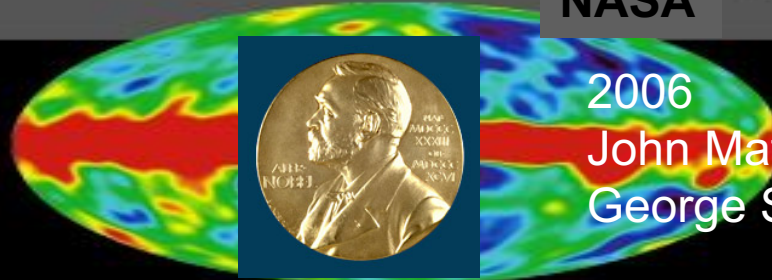


1978
Arno Penzias
Robert Wilson

1992

NASA

COBE

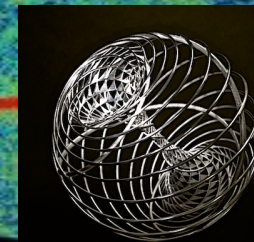
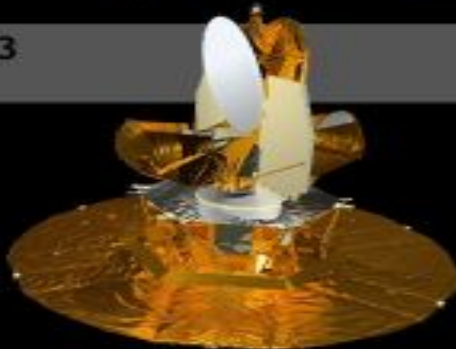


2006
John Mather
George Smoot

2003

NASA

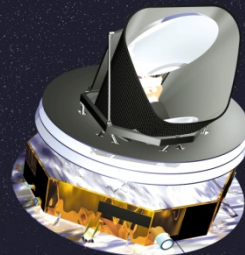
WMAP



2010, 2018
Charles Bennett
Lyman Page
David Spergel

2013

ESA Planck



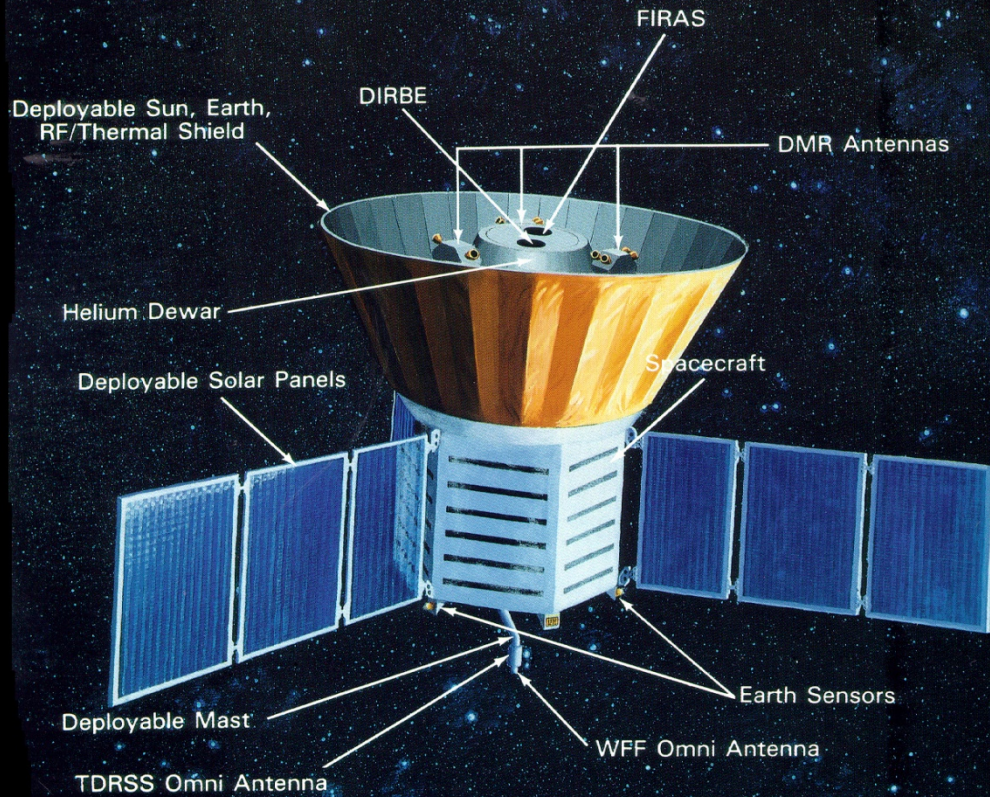
Plus many
ground-based
experiments



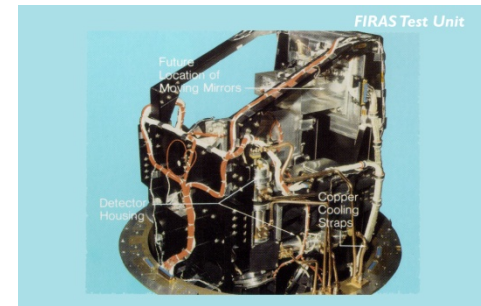
2004, 2019
James Peebles
Theoretical Cosmologist

The COBE Satellite

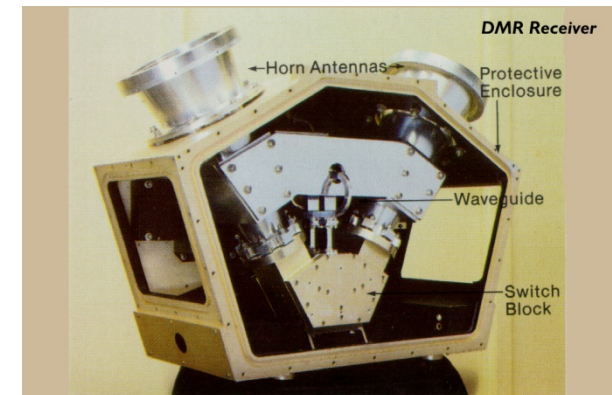
launched on Nov 18, 1989



Diffuse
InfraRed
Background
Experiment



Far InfraRed Absolute
Spectrophotometer

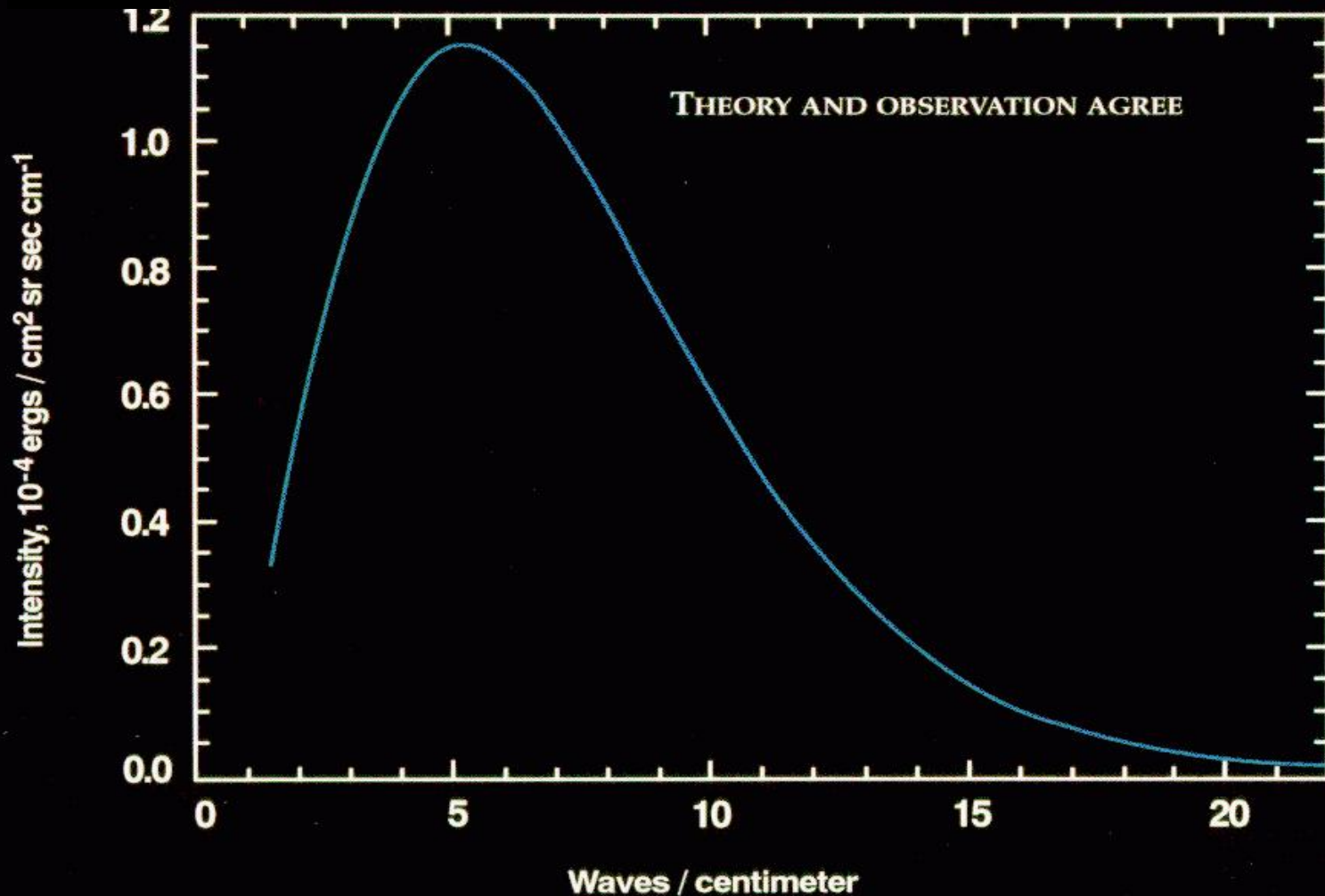


Differential Microwave Radiometer

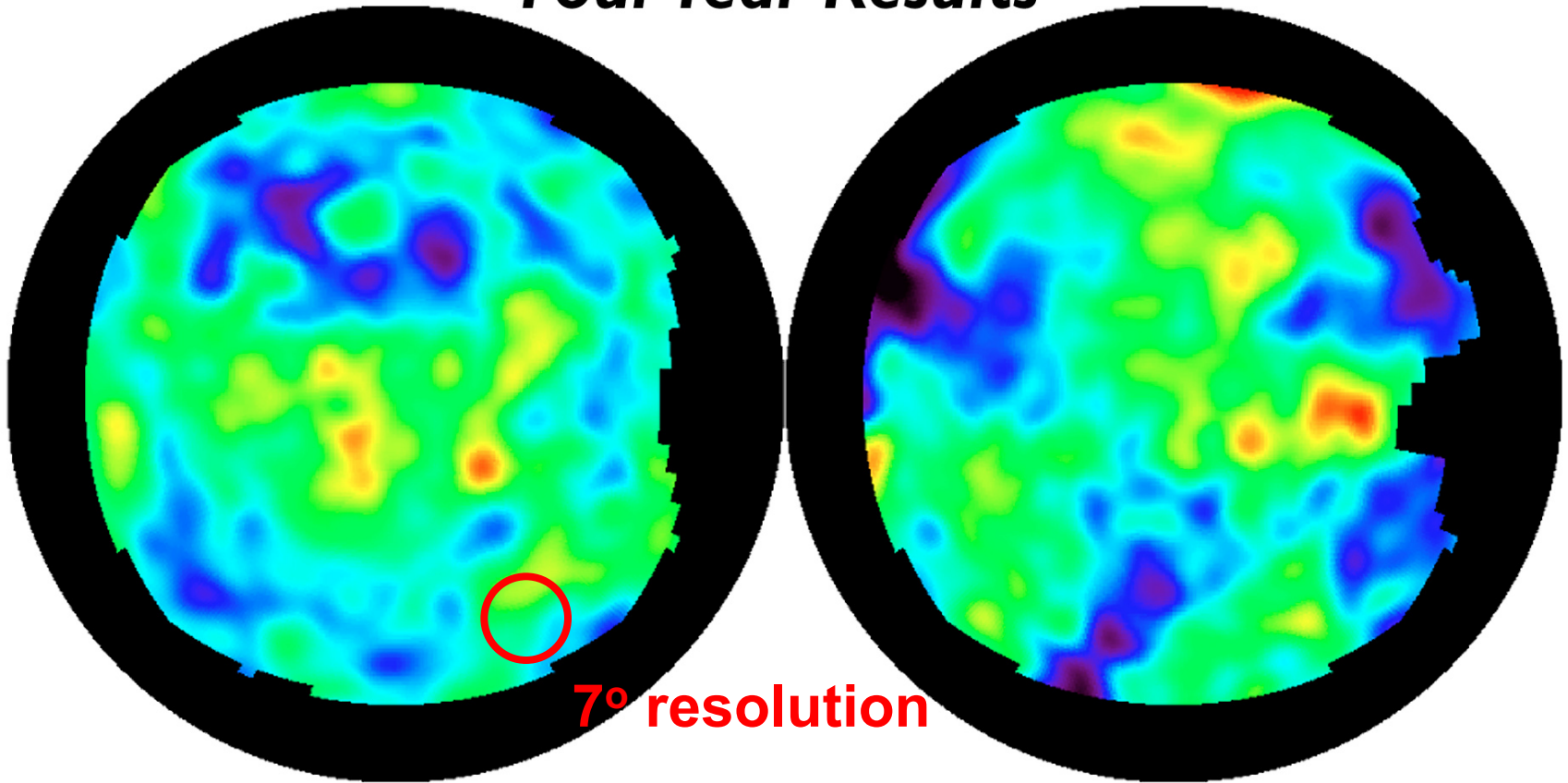
CMB Spectrum $T=2.725 \pm 0.002$

FIRAS
1992

COSMIC MICROWAVE BACKGROUND SPECTRUM FROM COBE



COBE - DMR Map of CMB Anisotropy Four Year Results



7° resolution

North Galactic Hemisphere

South Galactic Hemisphere

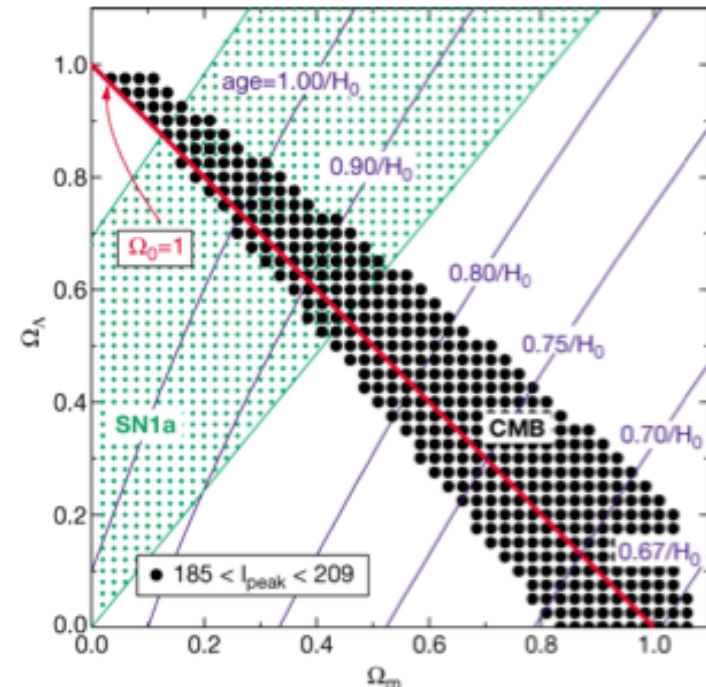
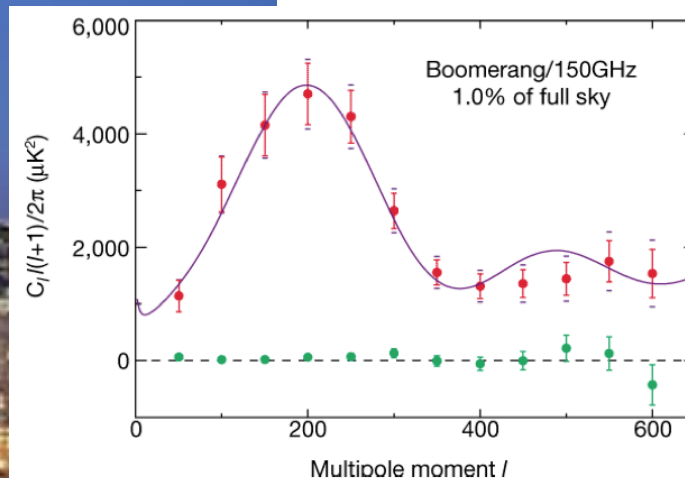
$-100 \mu\text{K}$  $+100 \mu\text{K}$

Location of first Doppler peak determining the curvature of the Universe (2000)

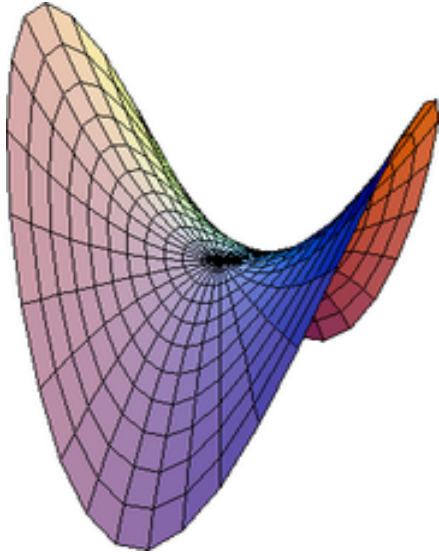


BOOMERANG
(pipeline headquarter@CITA)

MAXIMA
(headquarter@Berkeley
with Protty Wu)



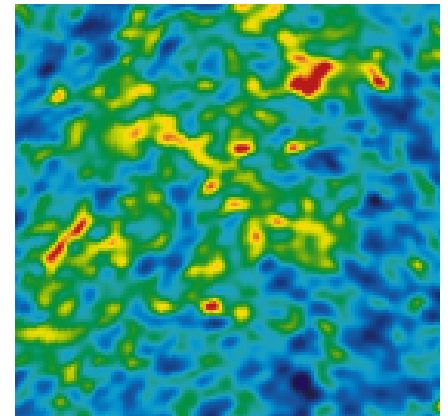
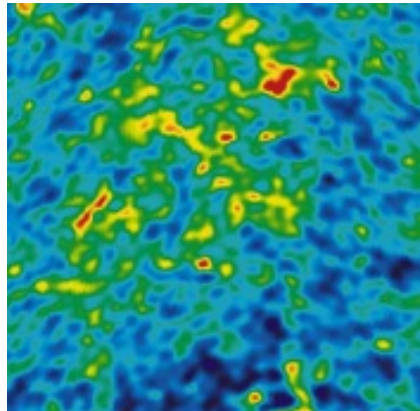
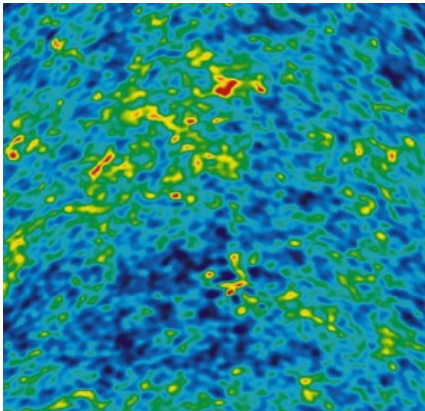
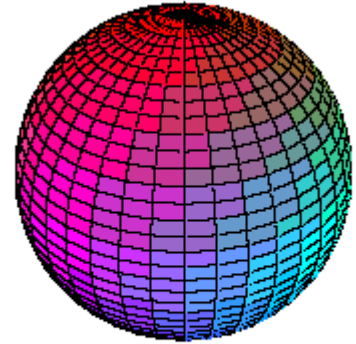
$k=-1$



$k=0$



$k=1$



$$1 = \Omega_{\text{M}} + \Omega_{\text{DE}} - k / (R^2 H^2)$$

$$\Omega_{\text{M}} + \Omega_{\text{DE}} = 1.099^{+0.100}_{-0.085}$$

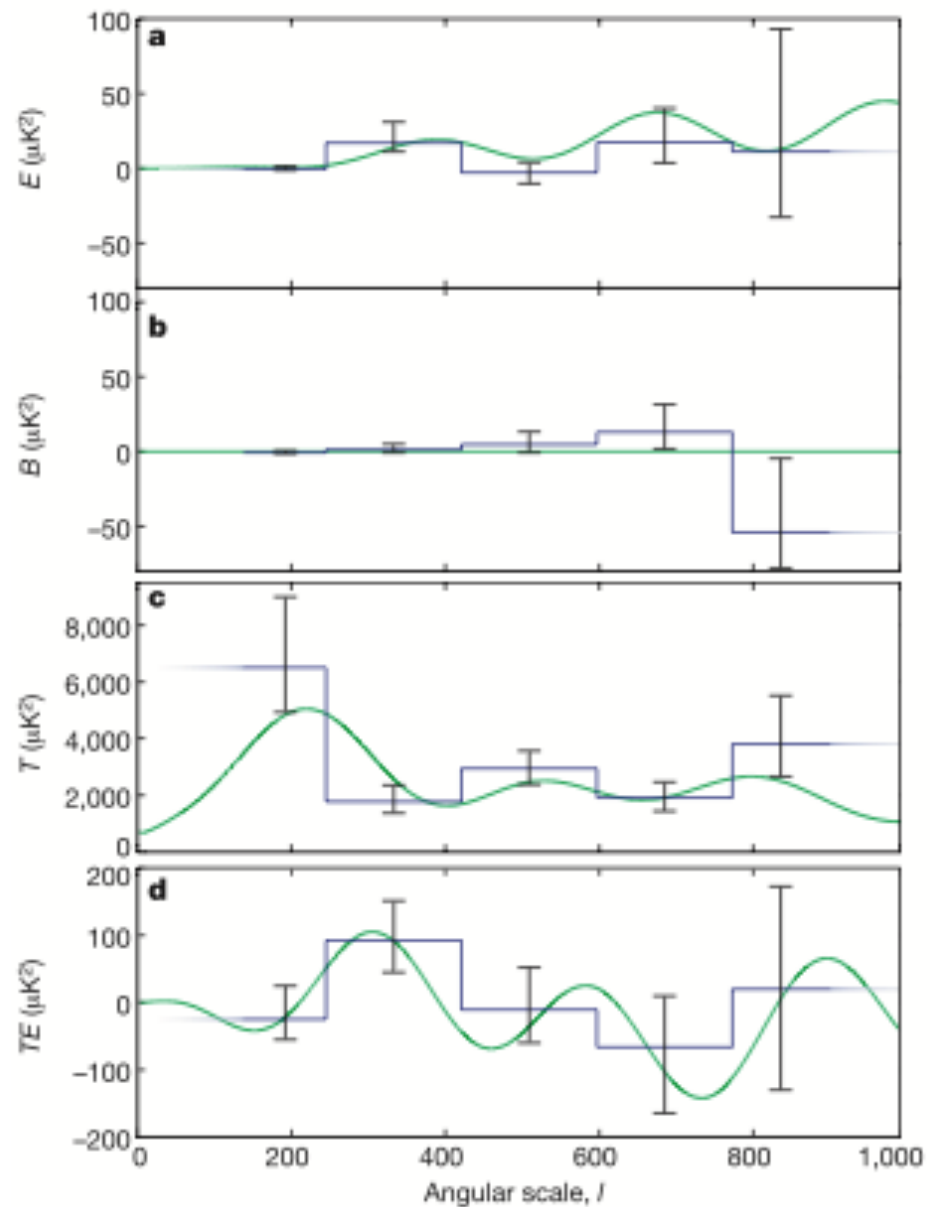
First detection of polarization by DASI (2002)



DASI@South Pole
Interferometry



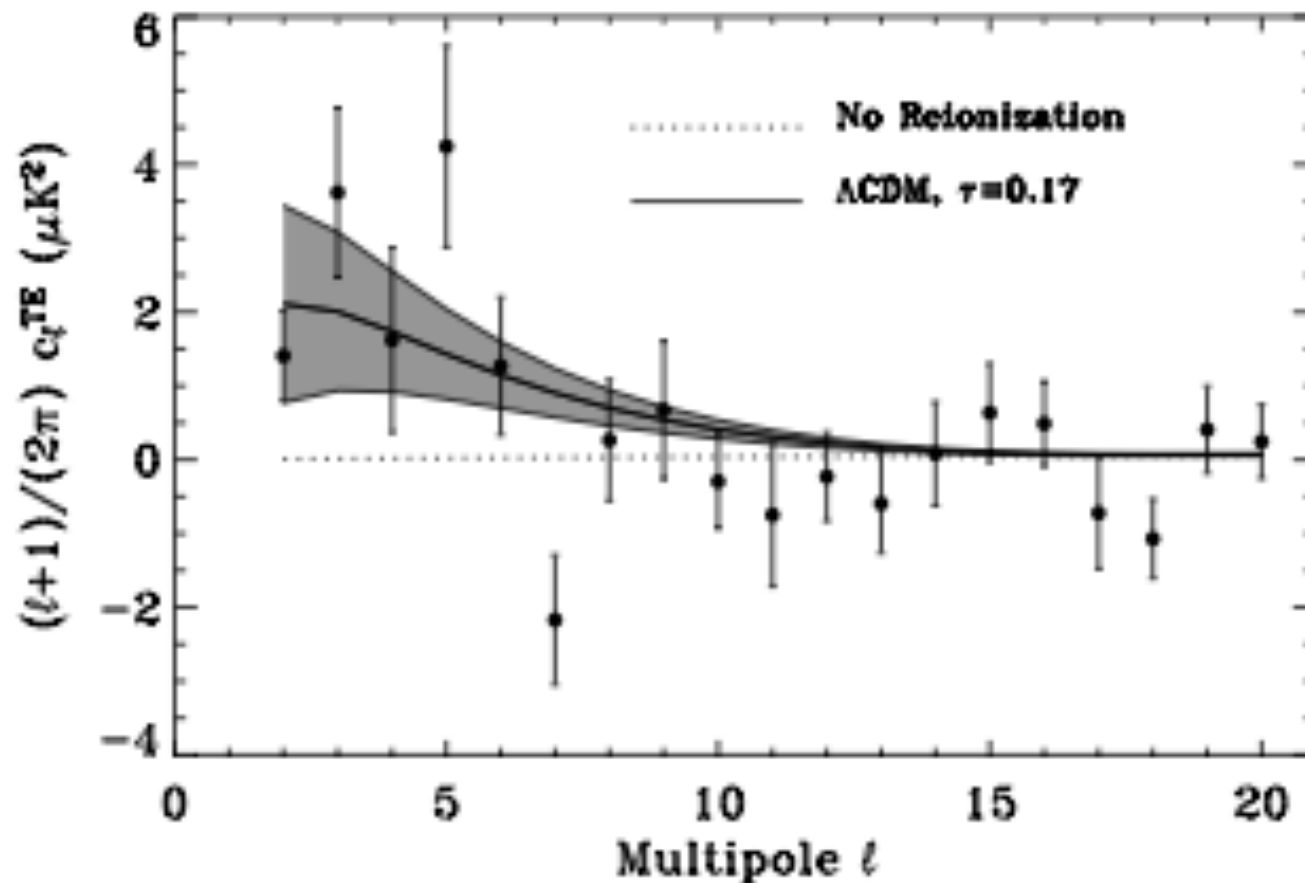
AMiBA



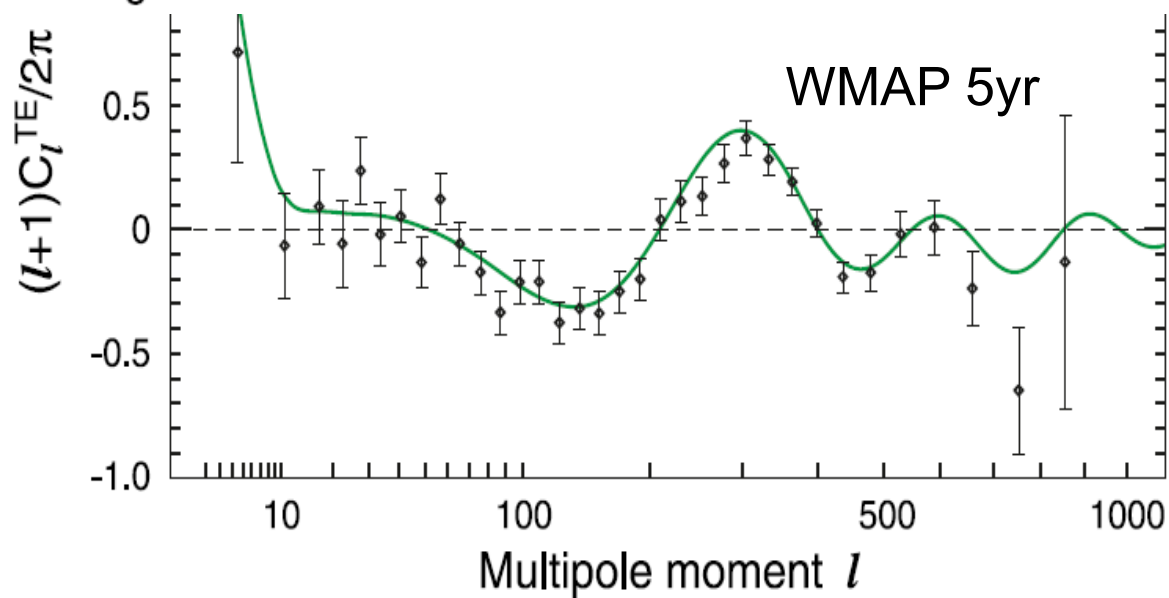
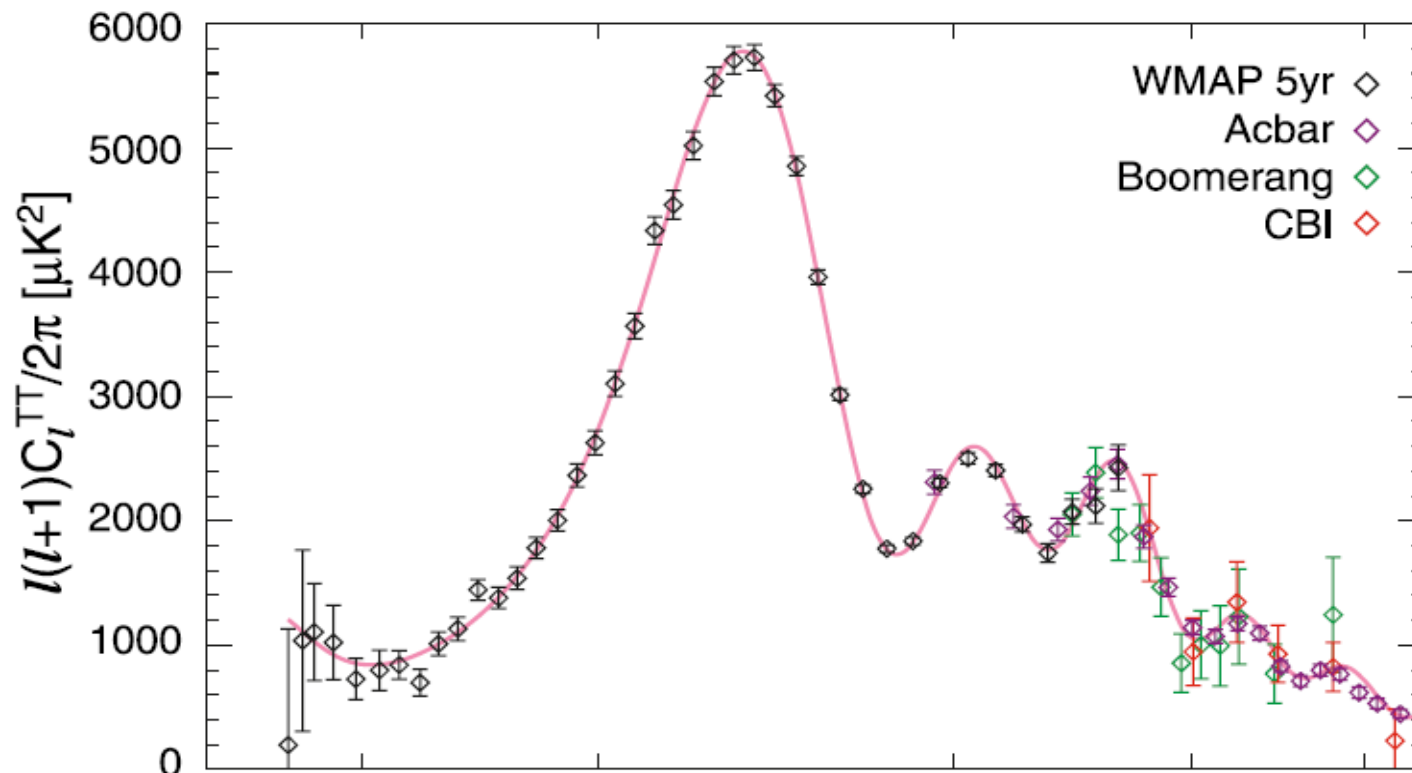
FIRST-YEAR *WILKINSON MICROWAVE ANISOTROPY PROBE (WMAP)*¹ OBSERVATIONS:
TEMPERATURE-POLARIZATION CORRELATION

A. KOGUT,² D. N. SPERGEL,³ C. BARNES,⁴ C. L. BENNETT,² M. HALPERN,⁵ G. HINSHAW,² N. JAROSIK,⁴
M. LIMON,^{2,4,6} S. S. MEYER,⁷ L. PAGE,³ G. S. TUCKER,^{2,6,8} E. WOLLACK,² AND E. L. WRIGHT⁹

Received 2003 February 11; accepted 2003 May 20

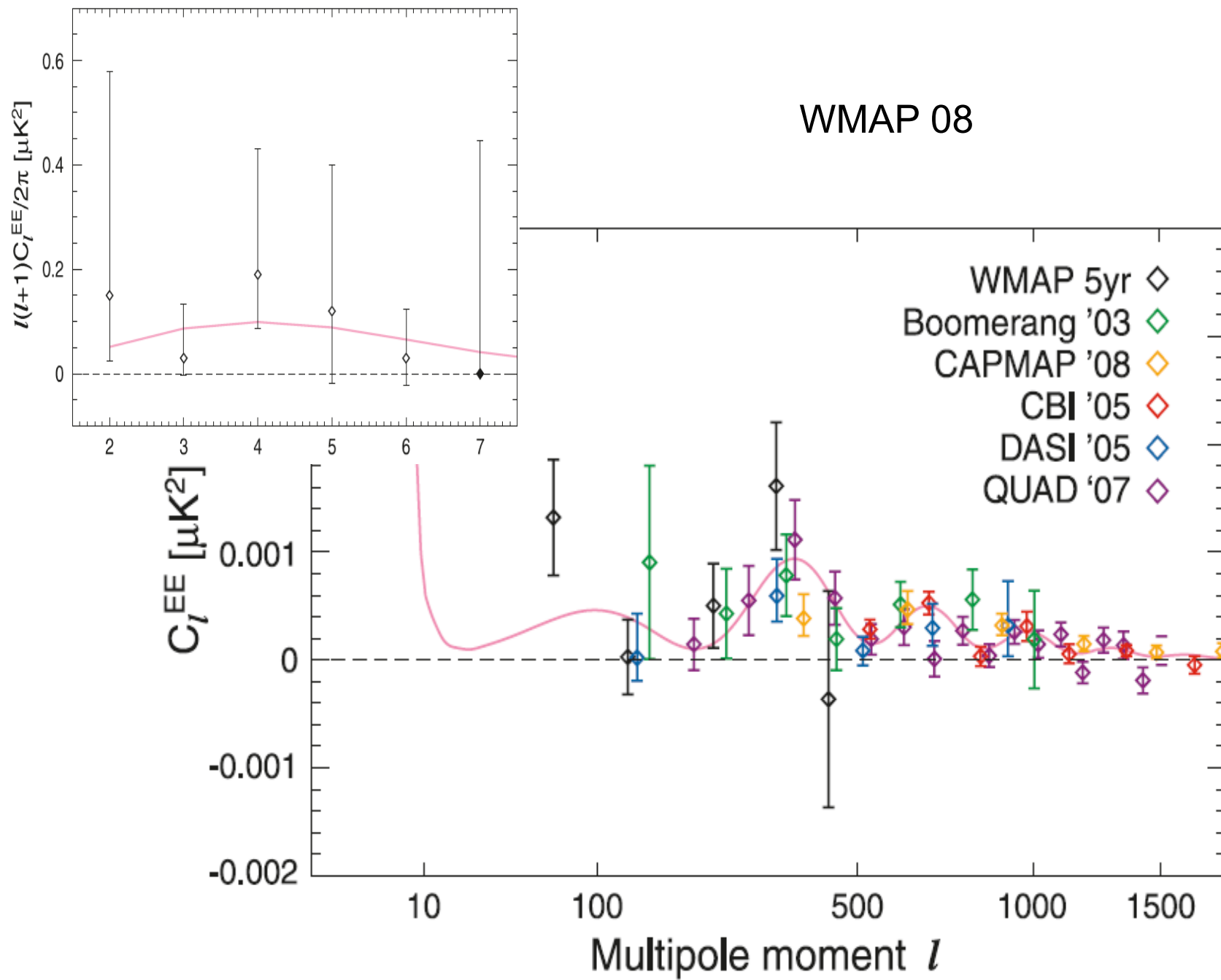


$Z_{\text{reionization}} \sim 20$ (too big to be true?)



WMAP 08

WMAP 08



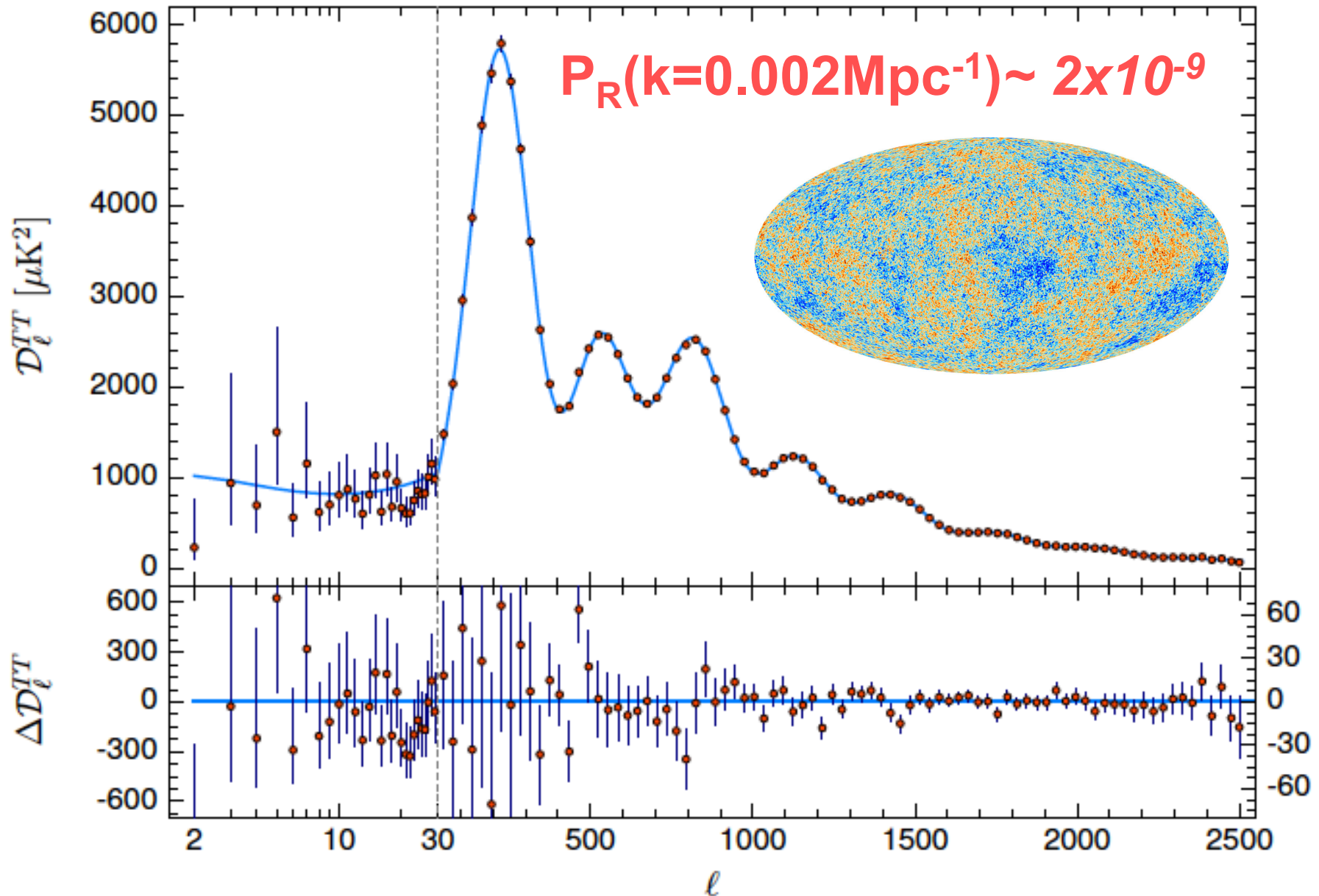
WMAP Cosmological Parameters

Model: Λ cdm+sz+lens

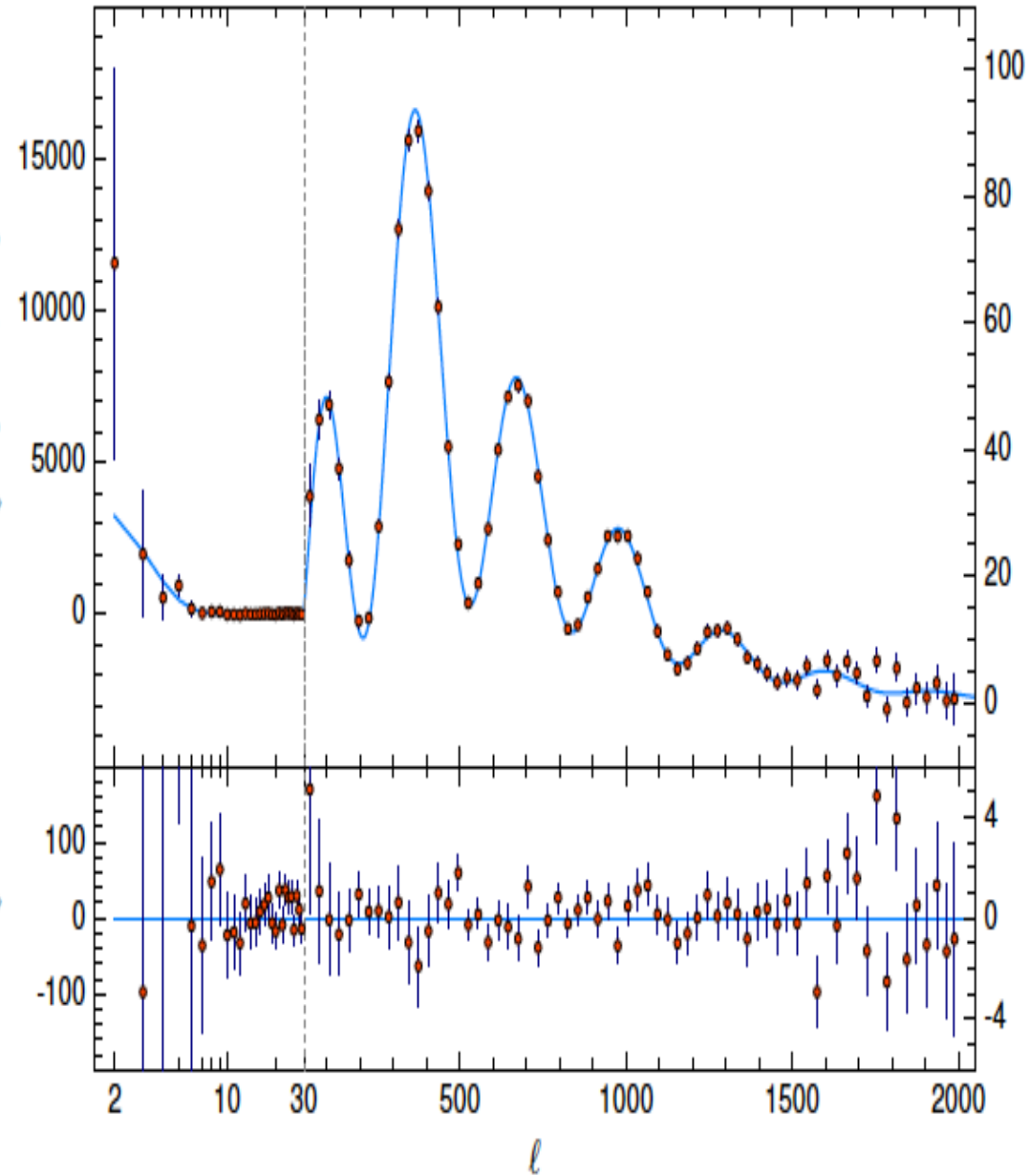
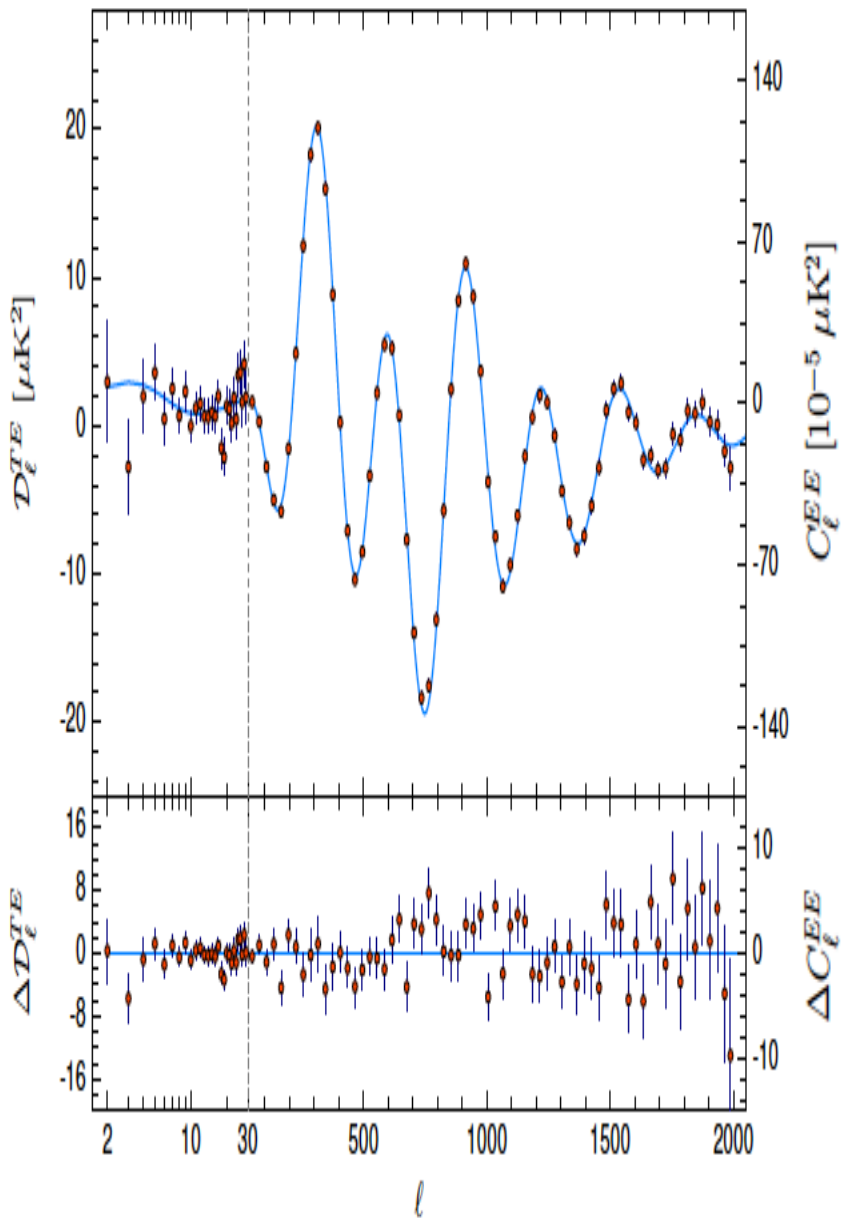
Data: wmap5+cmb

$10^2\Omega_b h^2$	2.250 ± 0.061	$1 - n_s$	0.040 ± 0.014
$1 - n_s$	$0.013 < 1 - n_s < 0.067$ (95% CL)	$\mathcal{A}_{\text{BAO}}(z = 0.35)$	0.451 ± 0.021
C_{220}	5753^{+42}_{-44}	$d_A(z_{\text{eq}})$	14351^{+186}_{-181} Mpc
$d_A(z_*)$	14188^{+188}_{-183} Mpc	$\Delta_{\mathcal{R}}^2$	$(2.41 \pm 0.11) \times 10^{-9}$
h	$0.725^{+0.026}_{-0.027}$	H_0	$72.5^{+2.6}_{-2.7}$ km/s/Mpc
k_{eq}	0.00952 ± 0.00045	ℓ_{eq}	135.0 ± 4.7
ℓ_*	$302.14^{+0.84}_{-0.83}$	n_s	0.960 ± 0.014
Ω_b	$0.0430^{+0.0028}_{-0.0029}$	$\Omega_b h^2$	0.02250 ± 0.00061
Ω_c	$0.207^{+0.025}_{-0.026}$	$\Omega_c h^2$	$0.1079^{+0.0061}_{-0.0060}$
Ω_Λ	$0.750^{+0.029}_{-0.028}$	Ω_m	$0.250^{+0.028}_{-0.029}$
$\Omega_m h^2$	0.1304 ± 0.0061	$r_{\text{hor}}(z_{\text{dec}})$	287.2 ± 3.4 Mpc
$r_s(z_d)$	$154.2^{+2.0}_{-1.9}$ Mpc	$r_s(z_d)/D_v(z = 0.2)$	$0.1969^{+0.0079}_{-0.0078}$
$r_s(z_d)/D_v(z = 0.35)$	0.1178 ± 0.0042	$r_s(z_*)$	$147.5^{+1.8}_{-1.7}$ Mpc
R	1.708 ± 0.020	σ_8	0.783 ± 0.035
A_{SZ}	$0.66^{+0.57}_{-0.66}$	t_0	$13.70^{+0.14}_{-0.13}$ Gyr
τ	$0.086^{+0.017}_{-0.016}$	θ_*	0.010398 ± 0.000029
θ_*	$0.5958^{+0.0016}_{-0.0017} \circ$	t_*	382028^{+5752}_{-5791} yr
z_{dec}	1087.9 ± 1.2	z_d	1019.8 ± 1.5
z_{eq}	3122 ± 147	z_{reion}	10.9 ± 1.4
z_*	1090.62 ± 0.95		

Planck CMB Anisotropy $D_{\ell}^{TT} = 1(l+1) C_{\ell}^T$ 2018



Planck CMB Polarization Power Spectra 2018



Best-fit 6-parameter Λ CDM model 2018

Density perturbation (scalar)

$$\text{Spectral index } \mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_0} \right)^{n_s-1}$$

$$k_0=0.05\text{Mpc}^{-1}$$

Parameter	TT+lowE 68% limits	TE+lowE 68% limits	EE+lowE 68% limits	TT,TE,EE+lowE 68% limits	TT,TE,EE+lowE+lensing 68% limits	TT,TE,EE+lowE+lensing+BAO 68% limits
$\Omega_b h^2$	0.02212 ± 0.00022	0.02249 ± 0.00025	0.0240 ± 0.0012	0.02236 ± 0.00015	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1206 ± 0.0021	0.1177 ± 0.0020	0.1158 ± 0.0046	0.1202 ± 0.0014	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{\text{MC}}$	1.04077 ± 0.00047	1.04139 ± 0.00049	1.03999 ± 0.00089	1.04090 ± 0.00031	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0522 ± 0.0080	0.0496 ± 0.0085	0.0527 ± 0.0090	$0.0544^{+0.0070}_{-0.0081}$	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10} A_s)$	3.040 ± 0.016	$3.018^{+0.020}_{-0.018}$	3.052 ± 0.022	3.045 ± 0.016	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9626 ± 0.0057	0.967 ± 0.011	0.980 ± 0.015	0.9649 ± 0.0044	0.9649 ± 0.0042	0.9665 ± 0.0038
z_{re}	7.50 ± 0.82	$7.11^{+0.91}_{-0.75}$	$7.10^{+0.87}_{-0.73}$	7.68 ± 0.79	7.67 ± 0.73	7.82 ± 0.71

Λ CDM model + 1-parameter extension

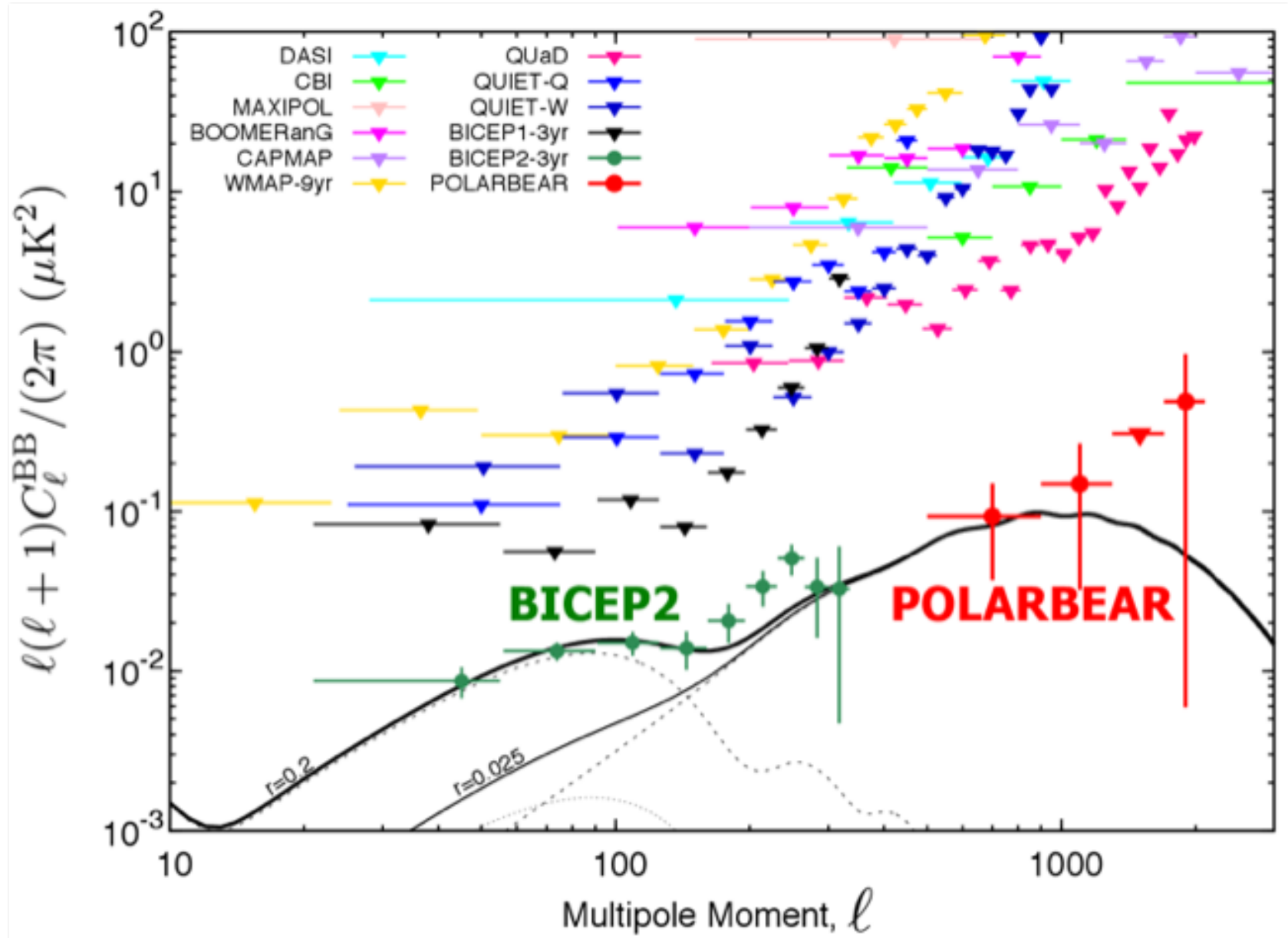
Spectral index $\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_0} \right)^{n_s-1}$

r = Tensor/Scalar

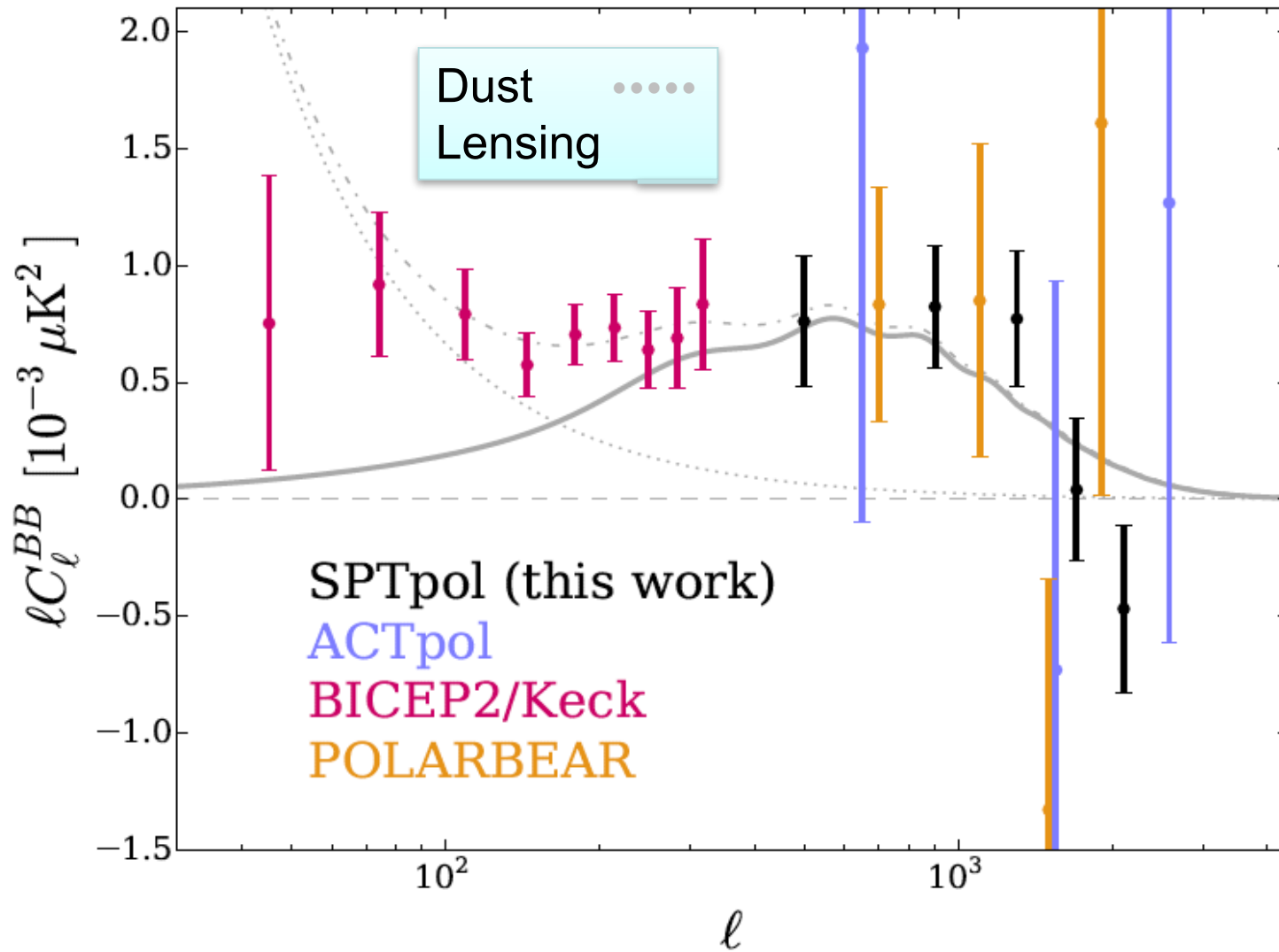
= $P_h(k)/P_R(k)$ at $k=0.002 \text{ Mpc}^{-1}$

Parameter	TT+lowE	TT, TE, EE+lowE	TT, TE, EE+lowE+lensing	TT, TE, EE+lowE+lensing+BAO
Ω_K	$-0.056^{+0.044}_{-0.050}$	$-0.044^{+0.033}_{-0.034}$	$-0.011^{+0.013}_{-0.012}$	$0.0007^{+0.0037}_{-0.0037}$
$\Sigma m_\nu [\text{eV}]$	< 0.537	< 0.257	< 0.241	< 0.120
N_{eff}	$3.00^{+0.57}_{-0.53}$	$2.92^{+0.36}_{-0.37}$	$2.89^{+0.36}_{-0.38}$	$2.99^{+0.34}_{-0.33}$
Y_p	$0.246^{+0.039}_{-0.041}$	$0.240^{+0.024}_{-0.025}$	$0.239^{+0.024}_{-0.025}$	$0.242^{+0.023}_{-0.024}$
$dn_s/d \ln k$	$-0.004^{+0.015}_{-0.015}$	$-0.006^{+0.013}_{-0.013}$	$-0.005^{+0.013}_{-0.013}$	$-0.004^{+0.013}_{-0.013}$
$r_{0.002}$	< 0.102	< 0.107	< 0.101	< 0.106
w_0	$-1.56^{+0.60}_{-0.48}$	$-1.58^{+0.52}_{-0.41}$	$-1.57^{+0.50}_{-0.40}$	$-1.04^{+0.10}_{-0.10}$

POLARBEAR+BICEP2 B-mode Detection



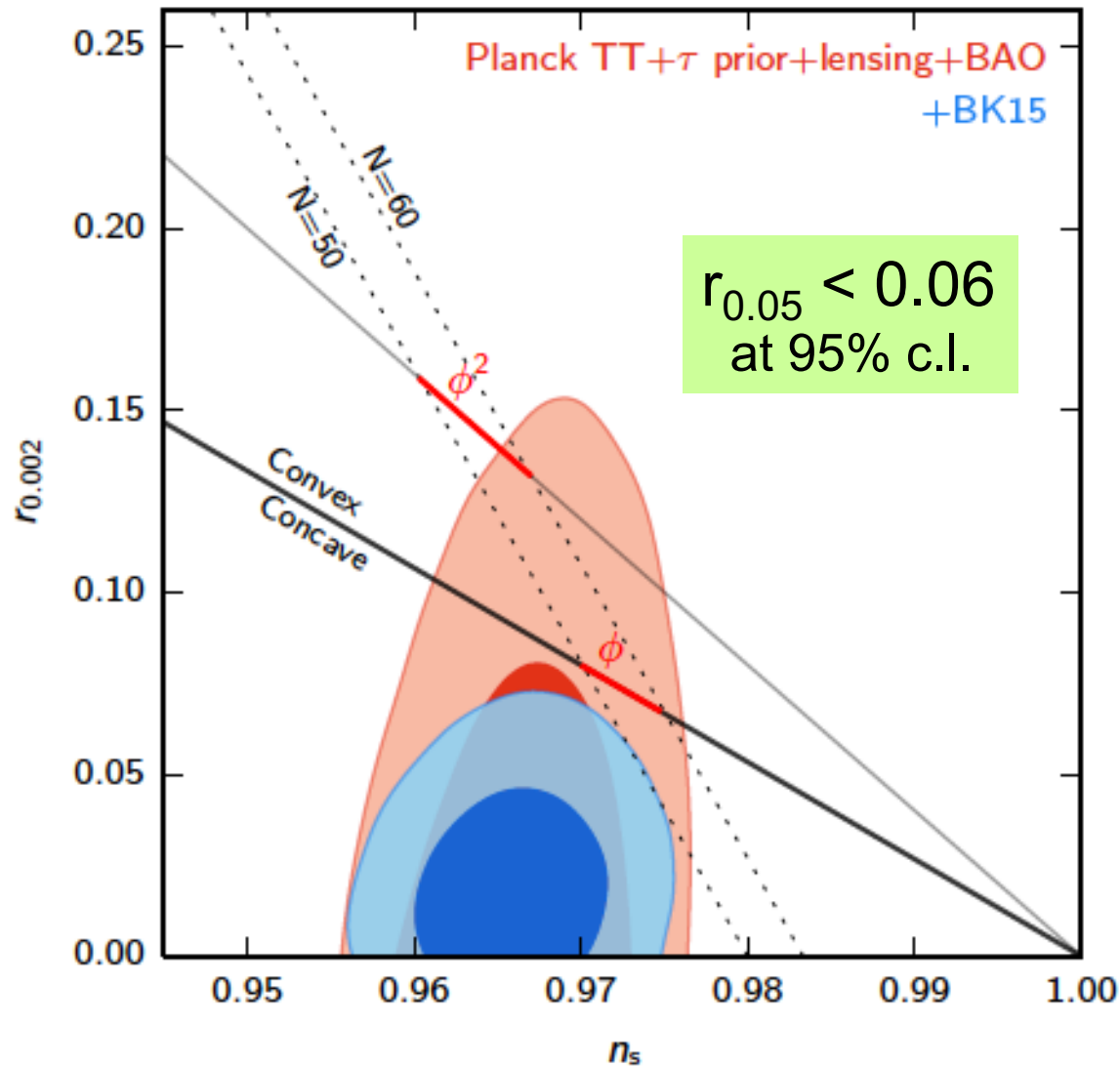
Current B-mode measurements



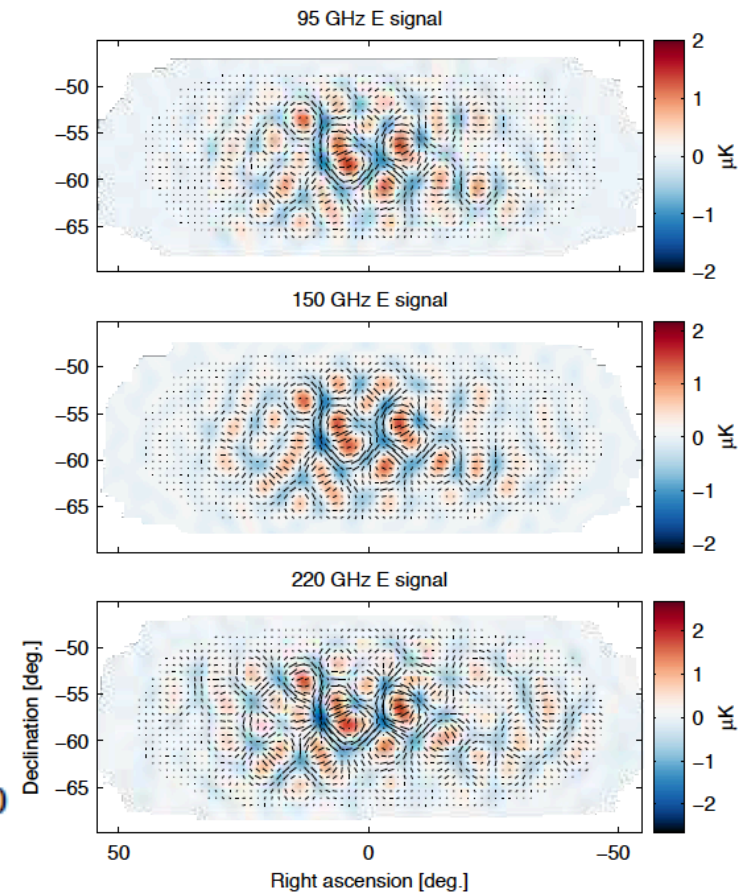
Galactic polarized dust emission
B-mode power spectrum at 150 GHz

$$\frac{l(l+1)}{2\pi} C_{l \text{ dust}}^{BB} = 0.0118 \left(\frac{l}{80} \right)^{-0.42} \mu K^2$$

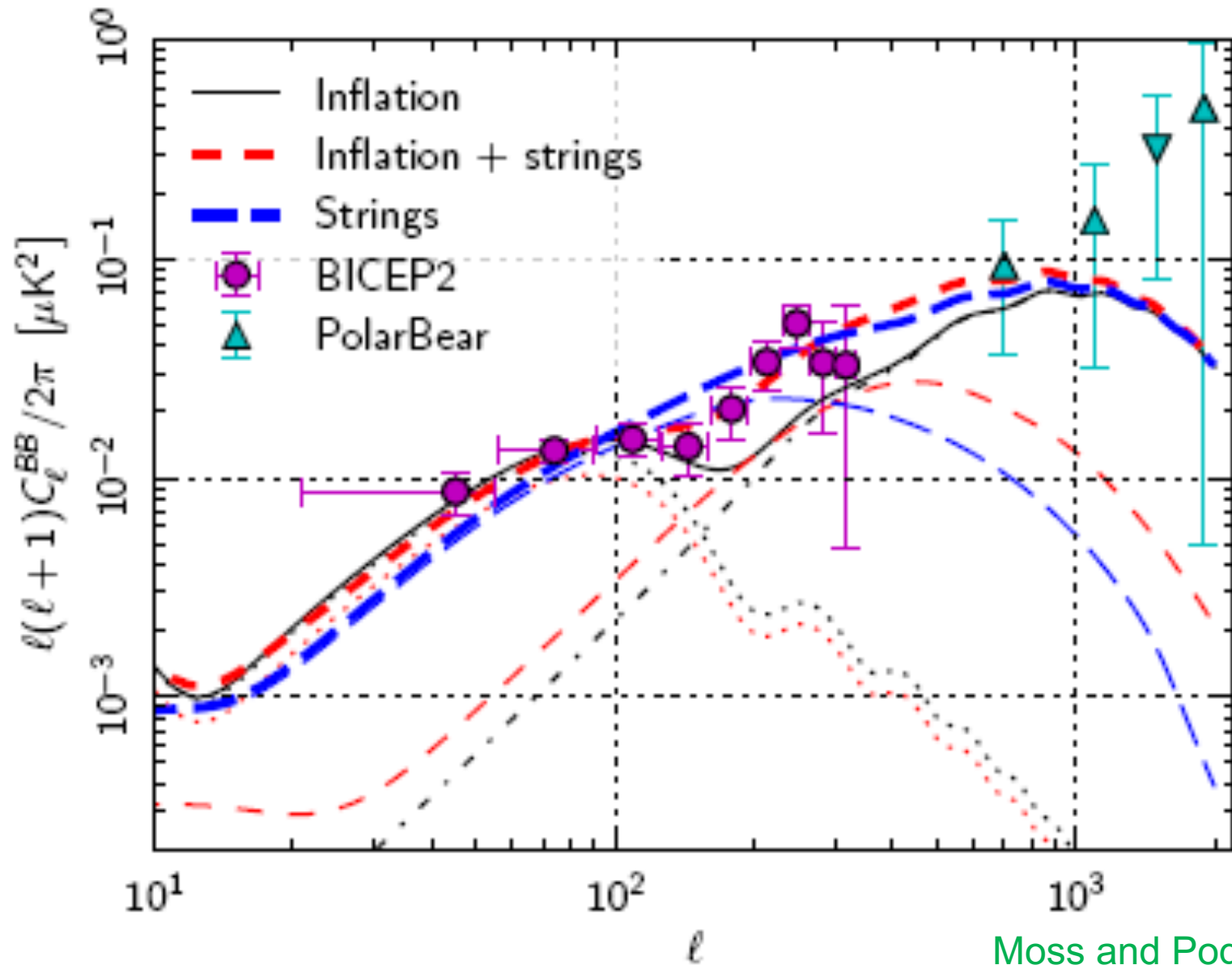
Joint Planck+BICEP2/Keck Array constraint on r by removal of dust contamination (2018)



Keck Array 2015 E-polarization data

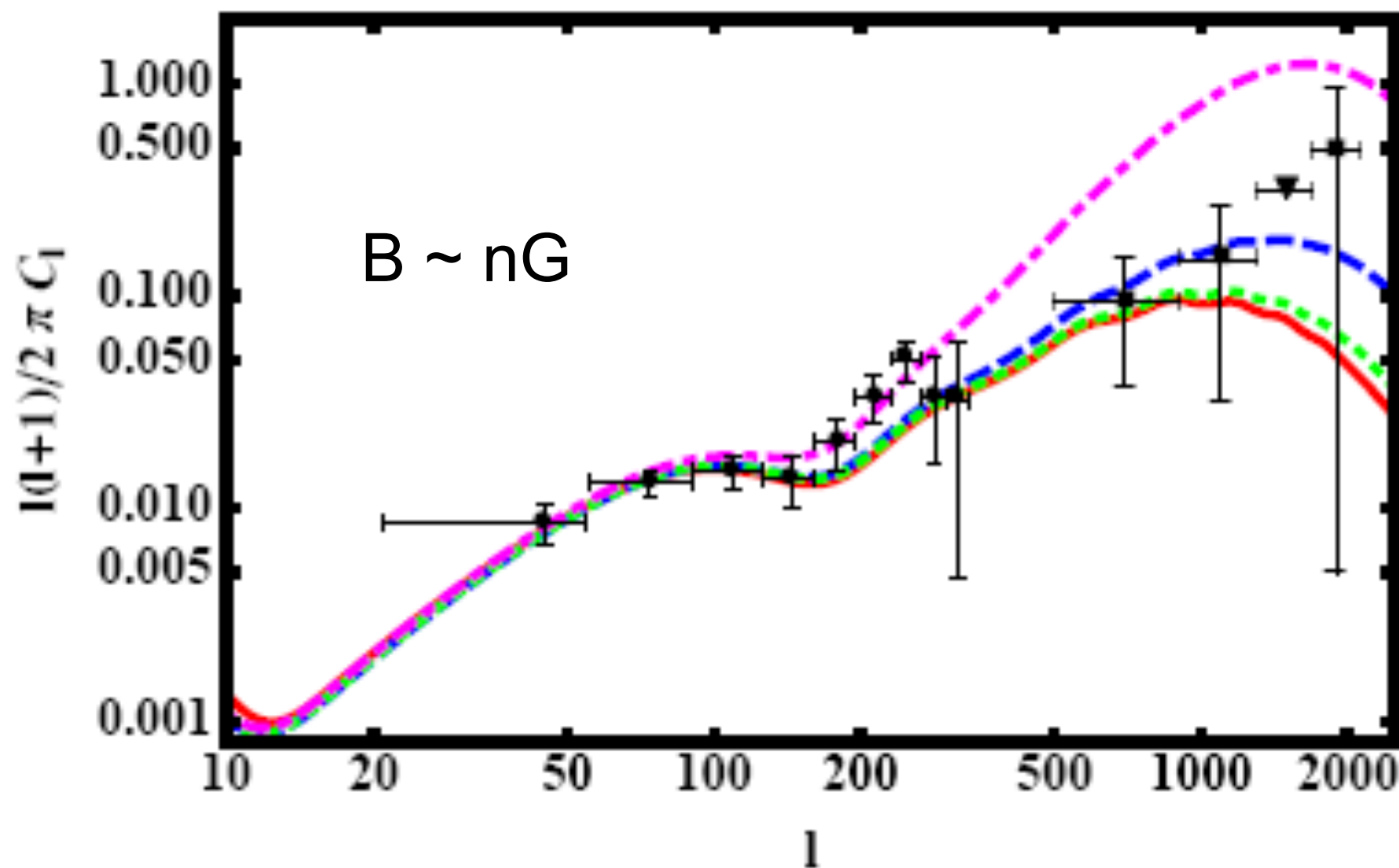


Alternative sources?



Can primordial magnetic fields be the origin of the BICEP2 data?

Camille Bonvin¹, Ruth Durrer² and Roy Maartens^{3,4}



Search for Cosmic Parity Violation

- T. D. Lee and C. N. Yang, C. S. Wu
Parity symmetry is broken in sub-atomic world - weak interaction is left-handed
- Is there any parity violation in the cosmos on the sky?
CMB polarization, polarized radio galaxies,...
- BICEP to AliCPT@China

CMB power spectra

- $\langle TT \rangle$, $\langle EE \rangle$, $\langle BB \rangle$, and $\langle TE \rangle$ correlations exist in standard Λ CDM model
- Since B is odd under parity symmetry, we expect that $\langle TB \rangle = \langle EB \rangle = 0$
- Any trace of $\langle TB \rangle \neq \langle EB \rangle \neq 0$ may indicate parity violation

DE/DM Coupling to Electromagnetism

$$\mathcal{L}_N = -\frac{1}{4}\sqrt{-g}B_{F\tilde{F}}(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu}, \quad \text{where} \quad \phi \equiv \frac{\Phi}{M}, \quad M = M_{Pl}/\sqrt{8\pi}$$

This leads to photon dispersion relation

Carroll,
Field,
Jackiw 90

$$n_{\pm} = \varepsilon \mp \frac{1}{2} \frac{\partial B_{F\tilde{F}}}{\partial \phi} \left(\frac{\partial \phi}{\partial \eta} + \vec{\nabla} \phi \cdot \hat{n} \right) \quad (\varepsilon, \vec{n}) \text{ is the photon four-momentum}$$

± left/right handed η conformal time

vacuum birefringence

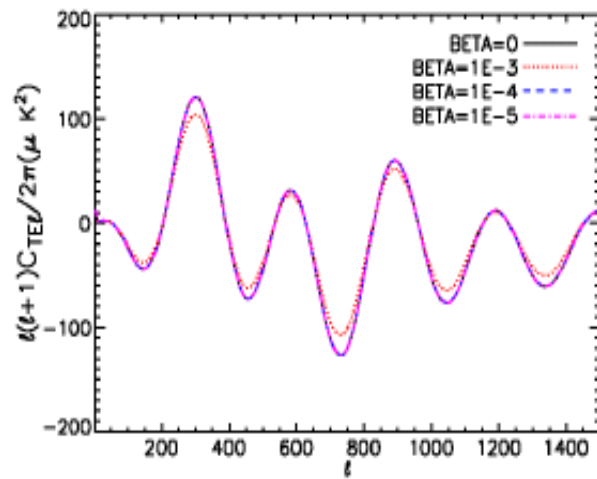
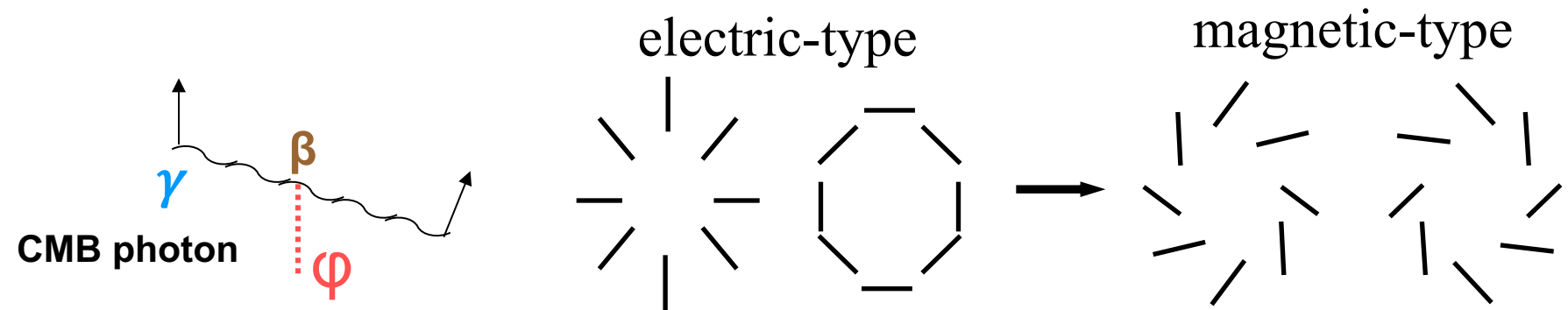
then, a rotational speed of polarization plane

$$\omega = \frac{1}{2}(n_+ - n_-) = -\frac{1}{2} \frac{\partial B_{F\tilde{F}}}{\partial \phi} \left(\frac{\partial \phi}{\partial \eta} + \vec{\nabla} \phi \cdot \hat{n} \right)$$

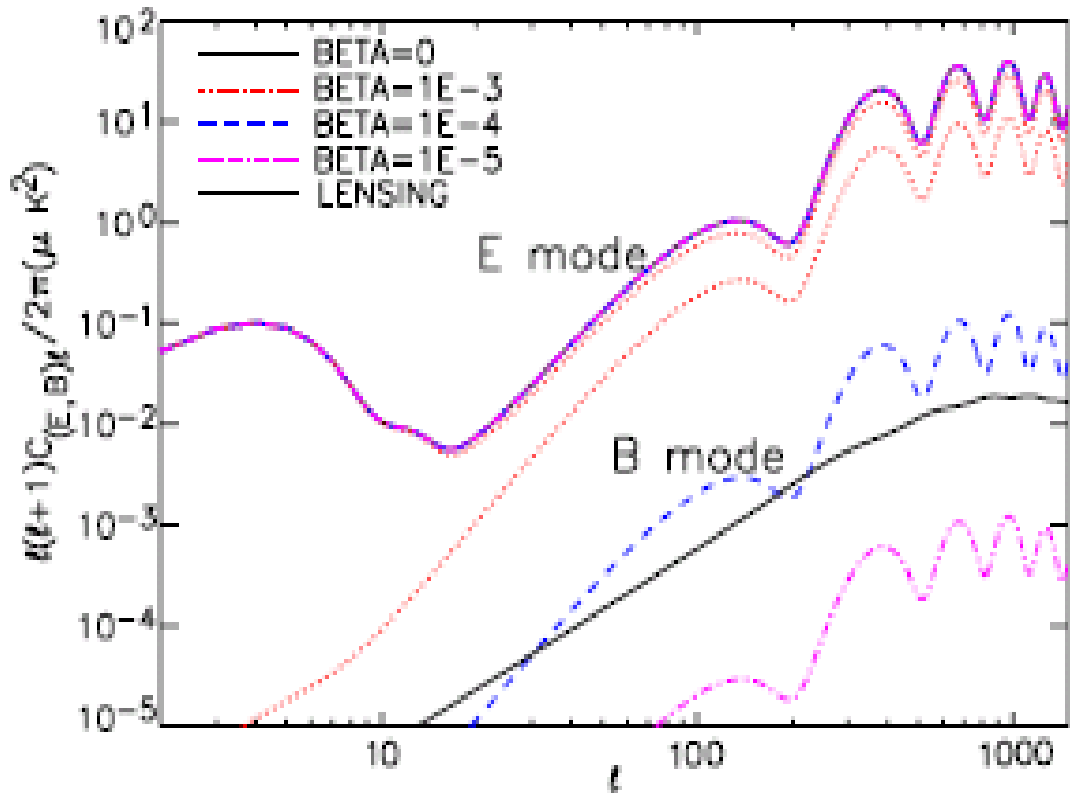
If $B=\beta\phi$, cooling of horizontal branch stars would imply $\beta < 10^7$

DE mean field induced vacuum birefringence – cosmic rotation of CMB polarization

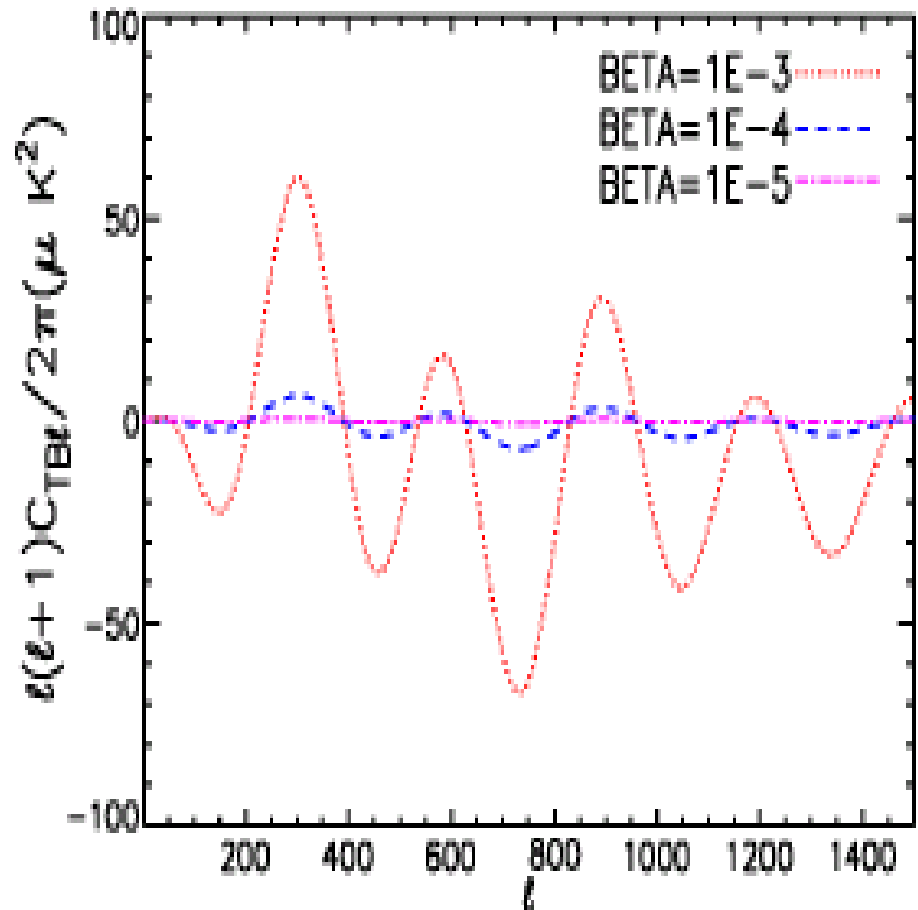
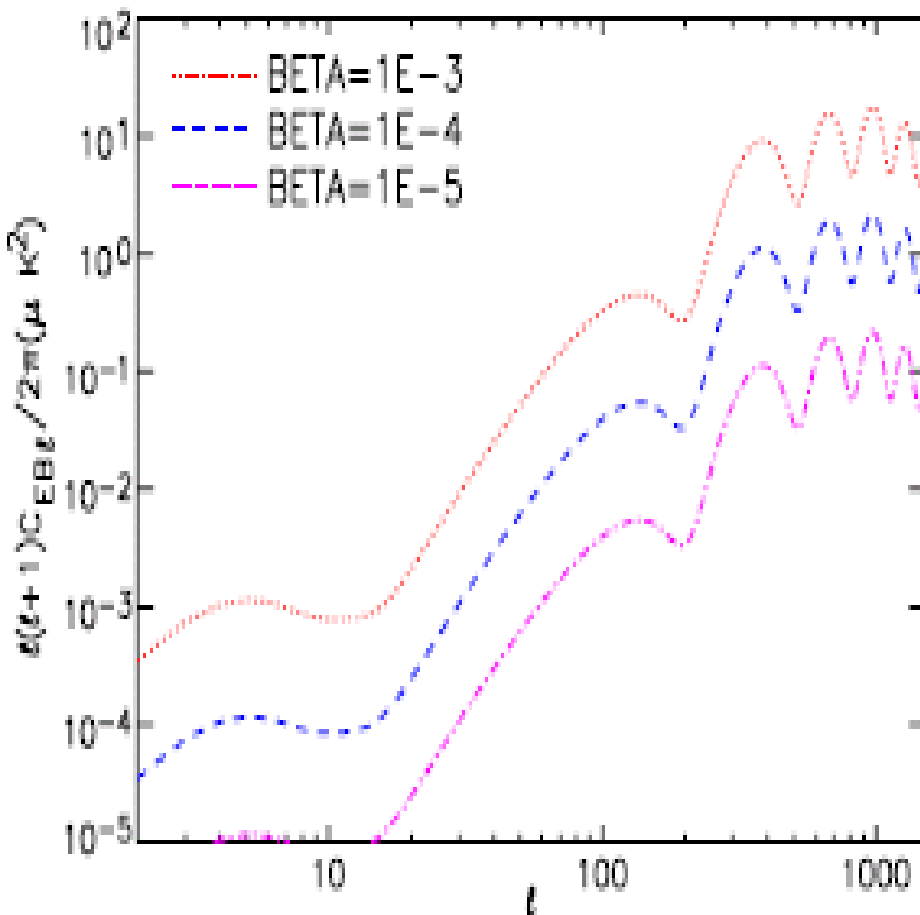
Zhang et al 06
Liu, Lee, Ng 06



TE spectrum

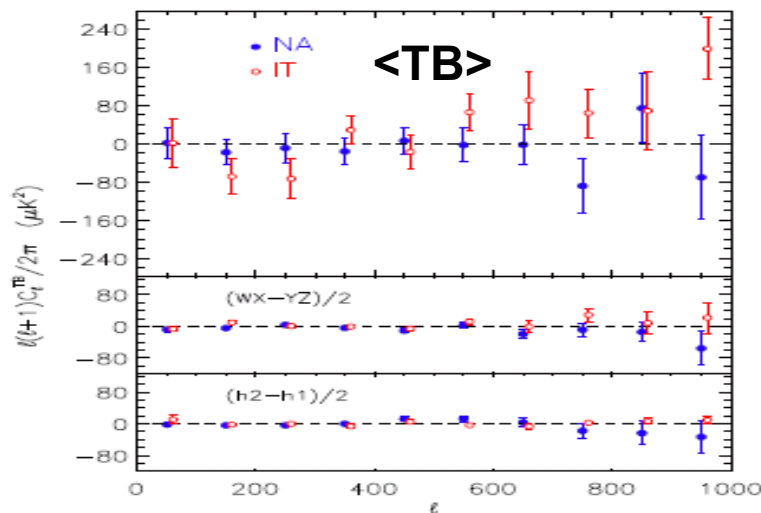
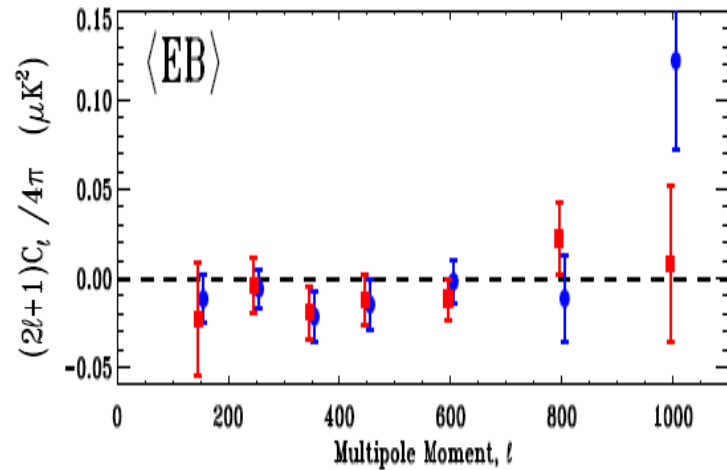


Parity violating EB,TB cross power spectra – cosmic parity violation



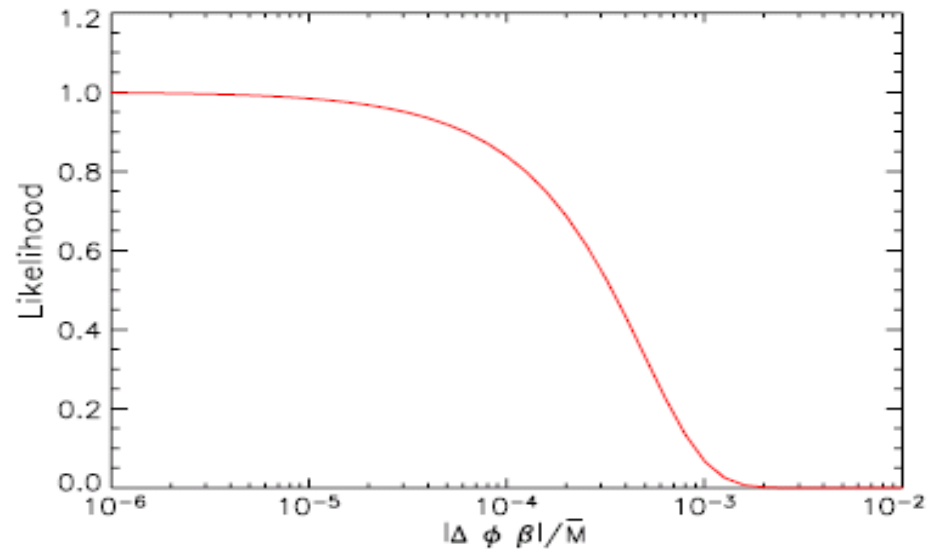
Constraining β by CMB polarization data

2003 Flight of BOOMERANG



Likelihood analysis assuming reasonable quintessence models

$$\bar{V}(\phi) = V_0 \exp(\lambda \phi^2 / 2 \bar{M}^2) \quad \bar{V}(\phi) = V_0 \cosh(\lambda \phi / \bar{M})$$



$|\beta_{FF} \Delta \phi| / \bar{M} < 8.32 \times 10^{-4}$ at 95% c.l.
where $\Delta \phi$ is the total change of ϕ until today.

$\bar{M}$ reduced Planck mass

More stringent limits
from...WMAP...to Planck

Including Dark Energy Perturbation

Dark energy
perturbation

$$\phi(\eta, \vec{x}) = \bar{\phi}(\eta) + \delta\phi(\eta, \vec{x}) \quad \delta\phi(\eta, \vec{x}) = \frac{1}{\sqrt{(2\pi)^3}} \int \delta\phi(\vec{k}', \eta) e^{i\vec{k}' \cdot \vec{x}} d^3 k'$$

time and space
dependent rotation

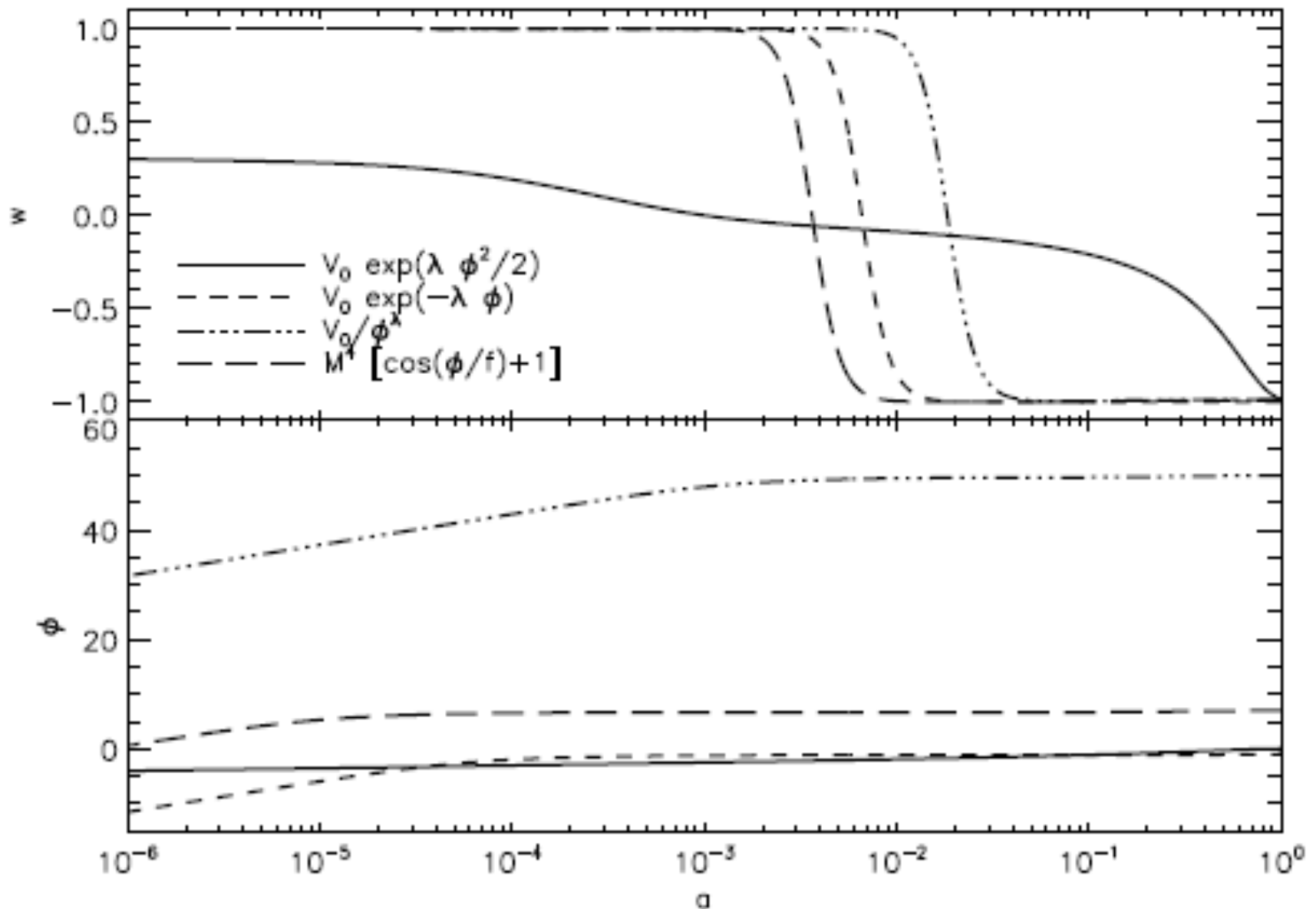
$$\omega = -\frac{1}{2} \frac{\partial B_{F\tilde{F}}}{\partial \phi} \left(\frac{\partial \phi}{\partial \eta} + \vec{\nabla} \phi \cdot \hat{n} \right)$$

$$\dot{\Delta}_{Q\pm iU}(\vec{k}, \eta) + ik\mu \Delta_{Q\pm iU}(\vec{k}, \eta) = n_e \sigma_T a(\eta) \left[-\Delta_{Q\pm iU}(\vec{k}, \eta) \times \right. \\ \left. \sum_m \sqrt{\frac{6\pi}{5}} {}_{\pm 2}Y_2^m(\hat{n}) S_P^{(m)}(\vec{k}, \eta) \right] \mp i2 \frac{1}{\sqrt{(2\pi)^3}} \int d\vec{k}' \tilde{\omega}(\vec{k} - \vec{k}', \eta) \Delta_{Q\pm iU}(\vec{k}', \eta)$$

$$\tilde{\omega}(\vec{k}, \eta) = -\frac{1}{2} \frac{\partial B_{F\tilde{F}}}{\partial \bar{\phi}} \left[\dot{\delta\phi}_{\vec{k}}(\eta) + i\vec{k} \cdot \hat{n} \delta\phi_{\vec{k}}(\eta) \right]$$

- Perturbation induced polarization power spectra in general quintessence models are small
- Interestingly, in nearly Λ CDM models (no time evolution of the mean field), birefringence generates $\langle BB \rangle$ while $\langle TB \rangle = \langle EB \rangle = 0$

We Tried Many Scalar Dark Energy Models



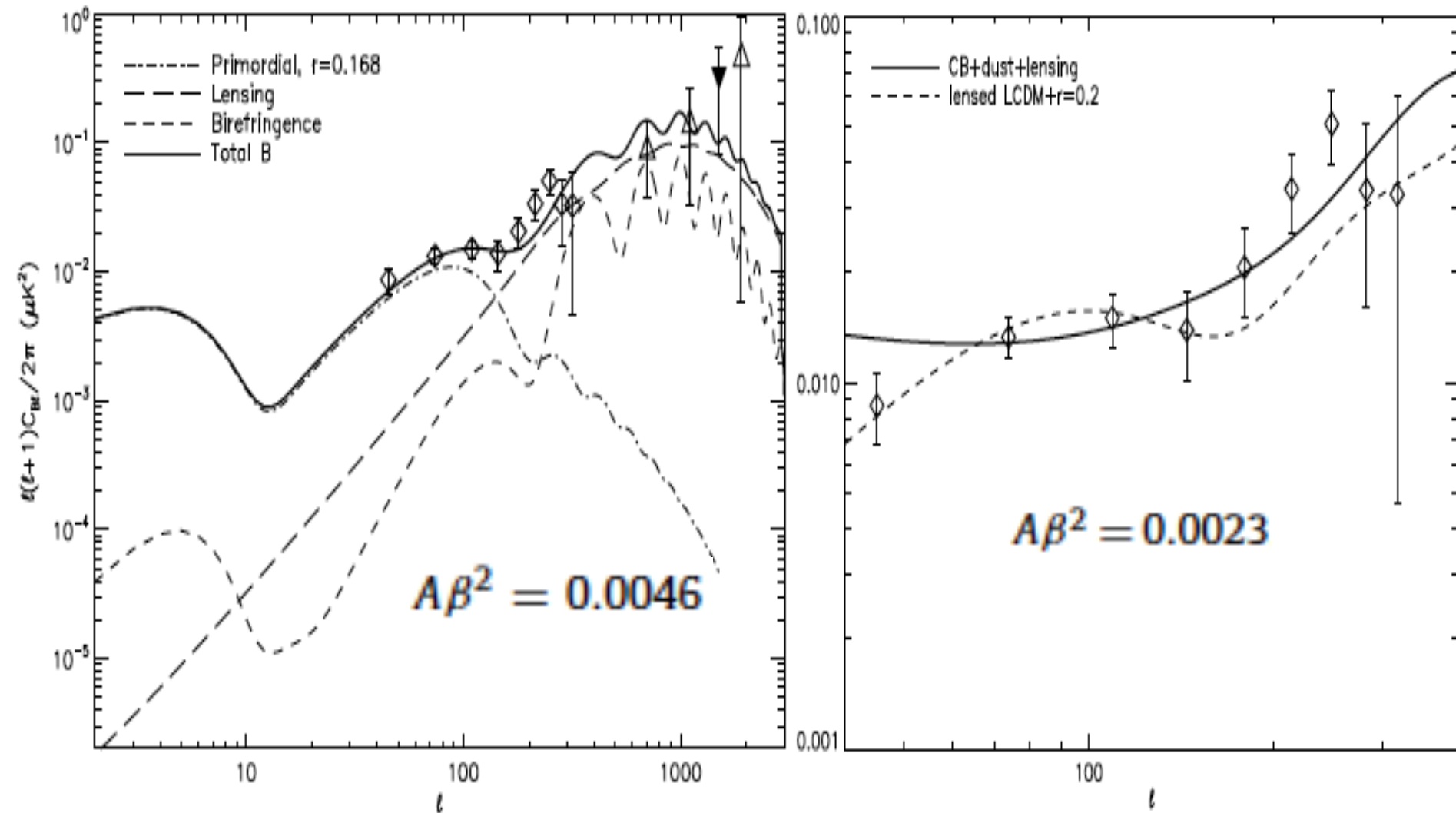
Cosmic Birefringence (CB) Fluctuations

Nearly massless
pseudo scalar

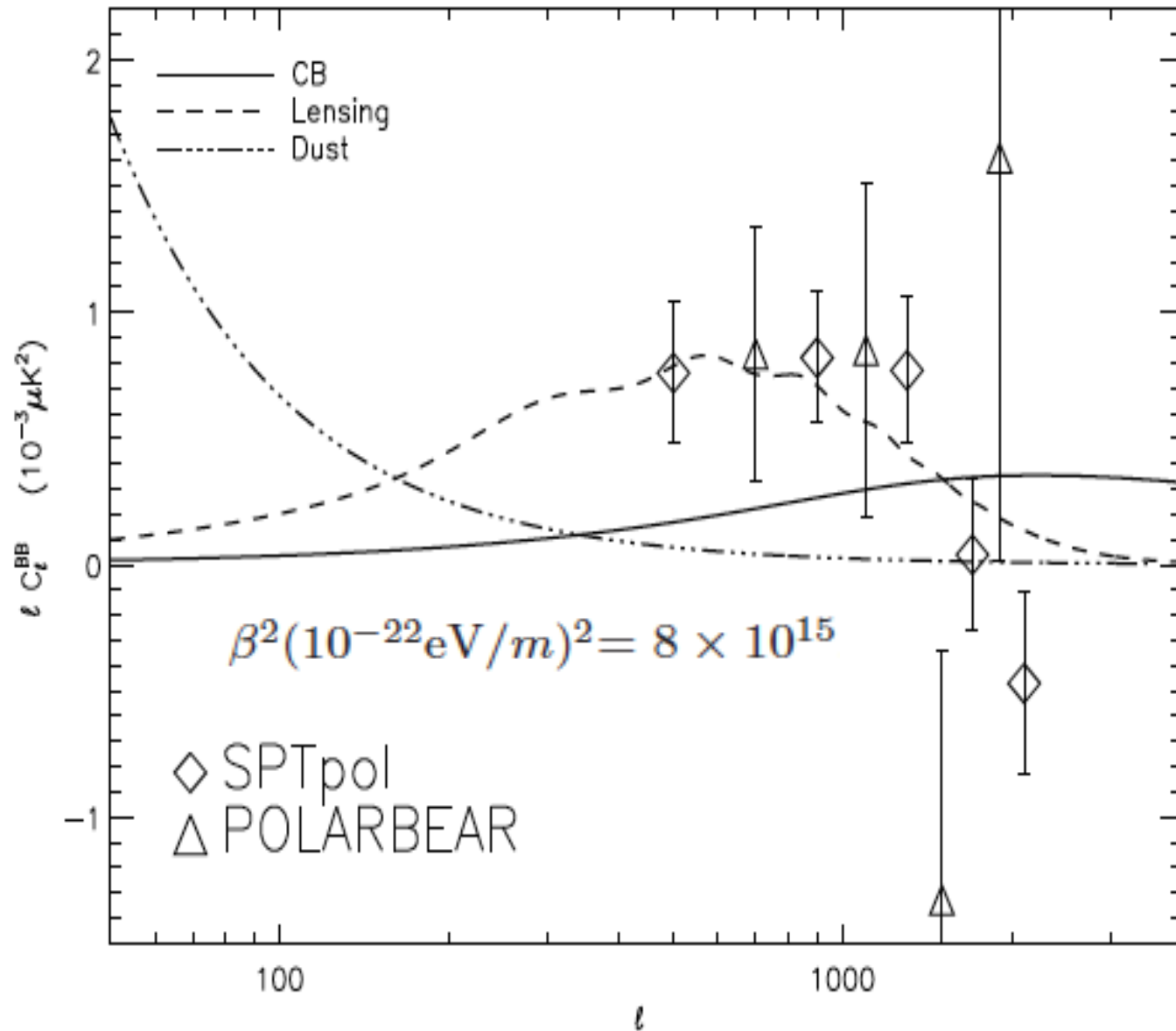
$$\langle \delta\phi_{\vec{k},i} \delta\phi_{\vec{k}',i} \rangle = (2\pi^2/k^3) P_{\delta\phi}(k) \delta(\vec{k} - \vec{k}')$$

$$P_{\delta\phi}(k) = A k^{n-1}$$

Pospelov et al. 08
Lee, Liu, Ng14



Axion ($m \sim 10^{-22} \text{eV}$) CDM curvature perturbation



$$\delta\rho_\phi/\rho_\phi = \delta\rho/\rho$$

$$\rho_\phi = m_\phi^2 \phi^2$$

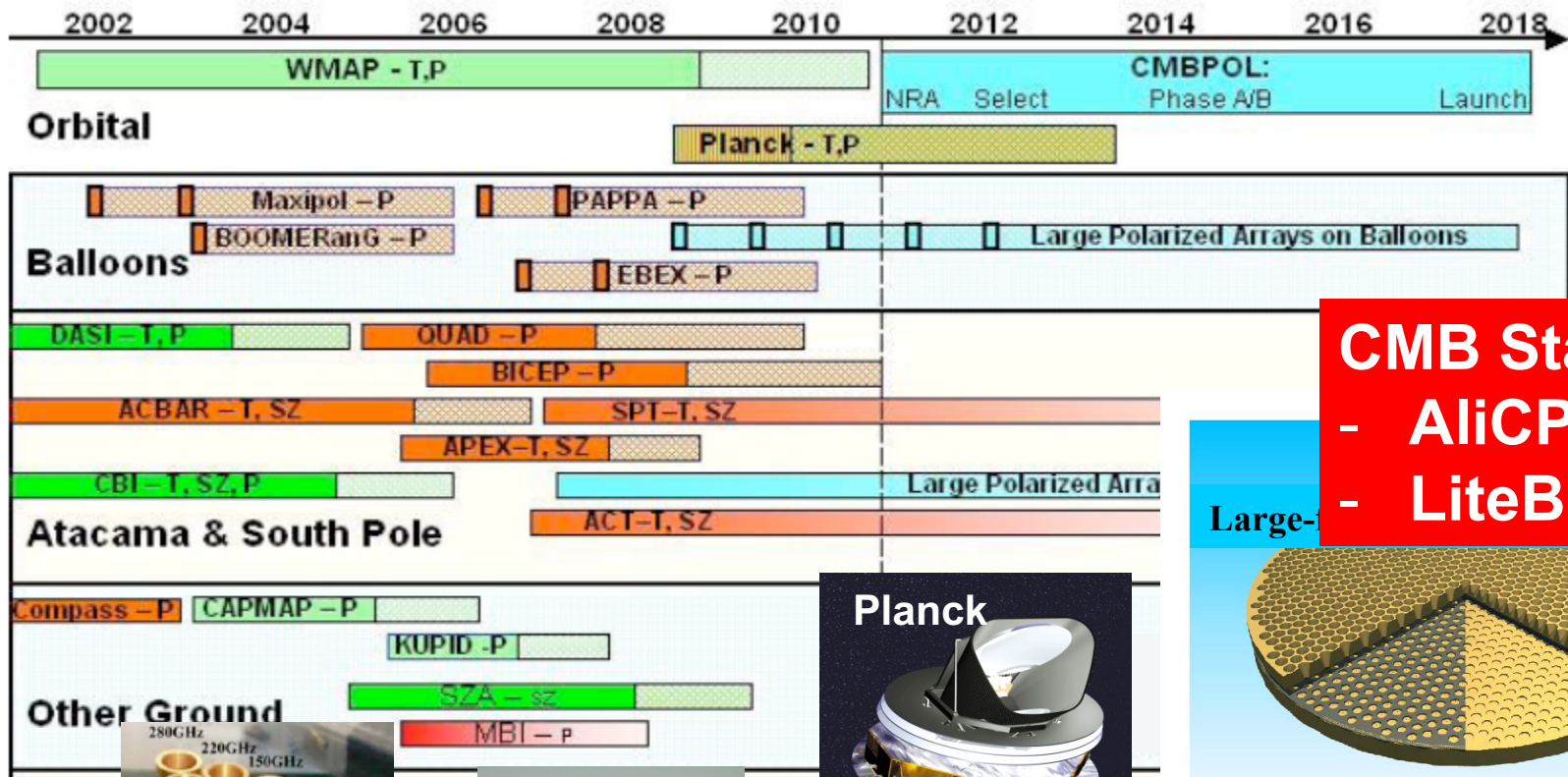
$$\frac{\delta\rho_\phi}{\rho_\phi} = 2 \frac{\delta\phi}{\phi}$$

$$\frac{\delta\phi}{M} = \frac{\delta\phi}{\phi} \frac{\phi}{M}$$

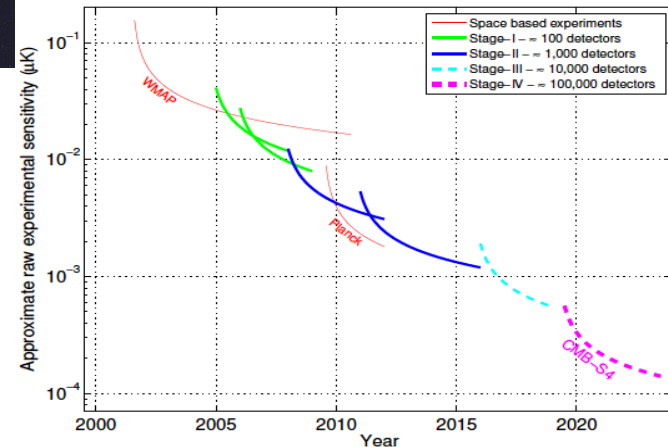
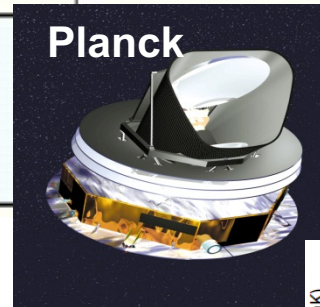
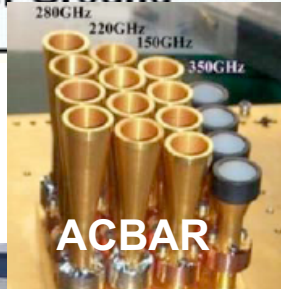
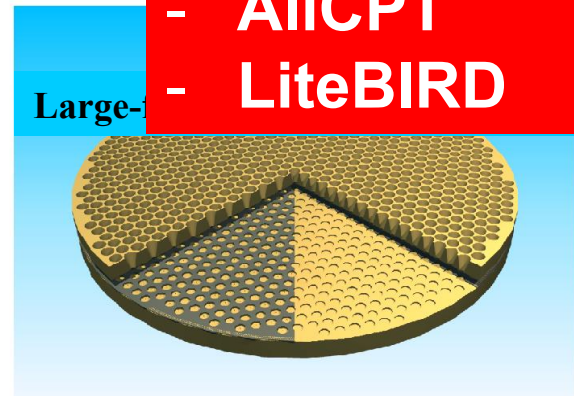
$$\phi = \phi_m \left(\frac{a_m}{a} \right)^{\frac{3}{2}}$$

$$H(a_m) = m$$

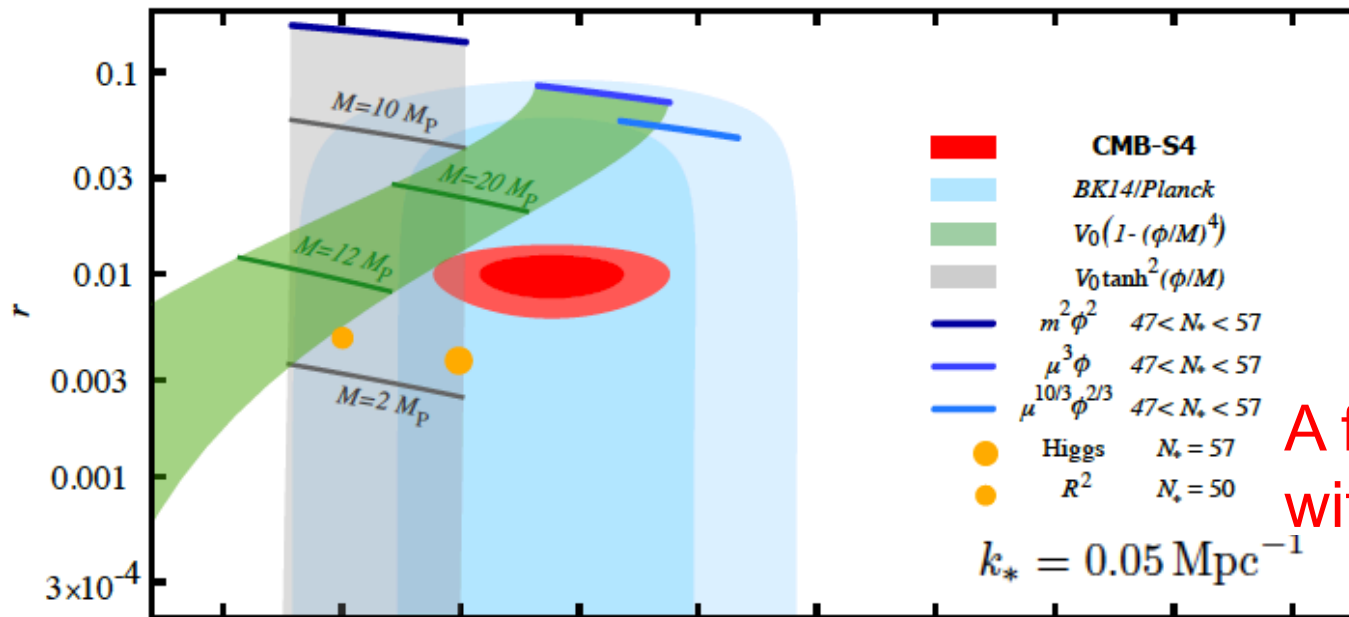
Recent, On-going, and Future CMB Space Missions and Experiments



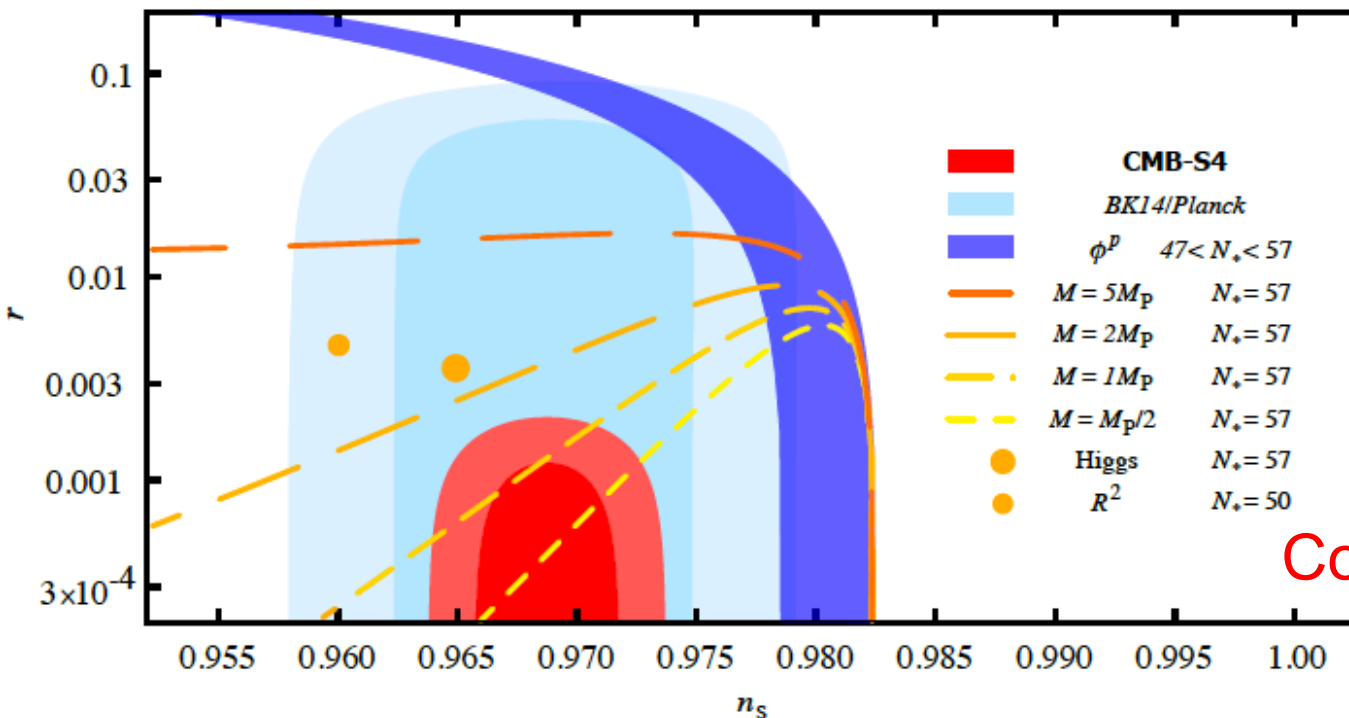
CMB Stage-4
 - AliCPT
 - LiteBIRD



CMB-S4 forecasts on inflation models



A fiducial model with $r=0.01$



Constraints on r

Summary

- Cosmic microwave background is a powerful tool for unveiling the **initial conditions** of the early Universe
- Anisotropy power spectrum has been well measured
- Combined with other observations such as supernova, large-scale structure formation, gravitational lensing, **cosmological parameters** have been measured at **~1% accuracy**
- Polarization signal is a sensitive probe of the formation of **first stars** and **reionization** history
- B-mode polarization is a clean signal of **primordial gravitational waves and inflation**
- Future and next-generation observations will lead us to an era of **high-precision cosmology**