Astrophysical and Cosmological Gravity-Wave Sources

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Outline

- Gravitational Waves
- Detection of GWs by LIGO/VIRGO
- Cosmology using GWS
- Hubble constant, modified gravity, gravitational lensing, phase transitions, cosmic strings, inflation,…..
- Current status and future directions

Metric

[Units: $\hbar = c = k_B = 1$

$1\ \text{GeV} = 5.07 \times 10^{13}\ \text{cm}^{-1} = 1.52 \times 10^{24}\ \text{s}^{-1} = 1.16 \times 10^{13}\ \text{K}$]

$$d\tau^2 = -g_{\mu\nu}\, dx^\mu dx^\nu \qquad \mu, \nu = 0, 1, 2, 3$$

where $g^{\mu\nu} g_{\nu\lambda} = \delta^\mu{}_\lambda$ & $T^{\mu\lambda} g_{\lambda\nu} = T^\mu{}_\nu$ for a tensor $T^{\mu\nu}$

e.g.

A flat metric $\delta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

For light $0 = d\tau^2 = dt^2 - d\vec{x}^2 \Rightarrow \left|\frac{d\vec{x}}{dt}\right| = 1$.

Expanding Universe

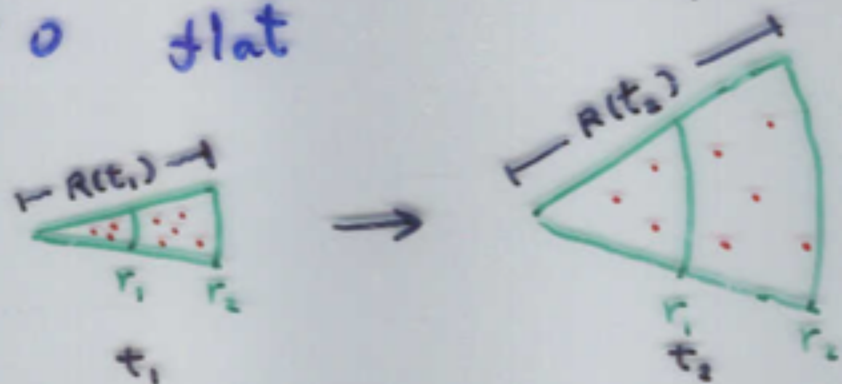
Robertson-Walker metric

$$d\tau^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$$

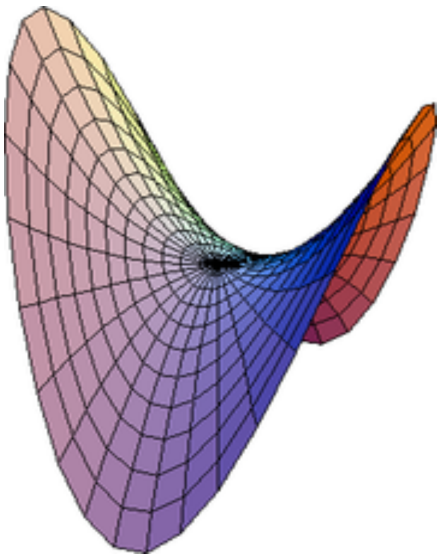
$$k = \begin{cases} +1 & \text{closed} \\ -1 & \text{open} \\ 0 & \text{flat} \end{cases}$$

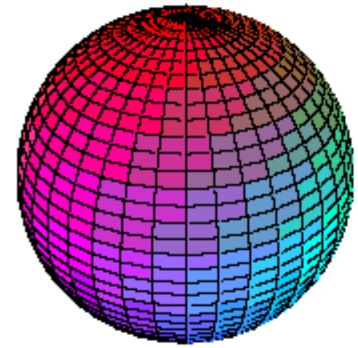
$R(t)$: Scale factor

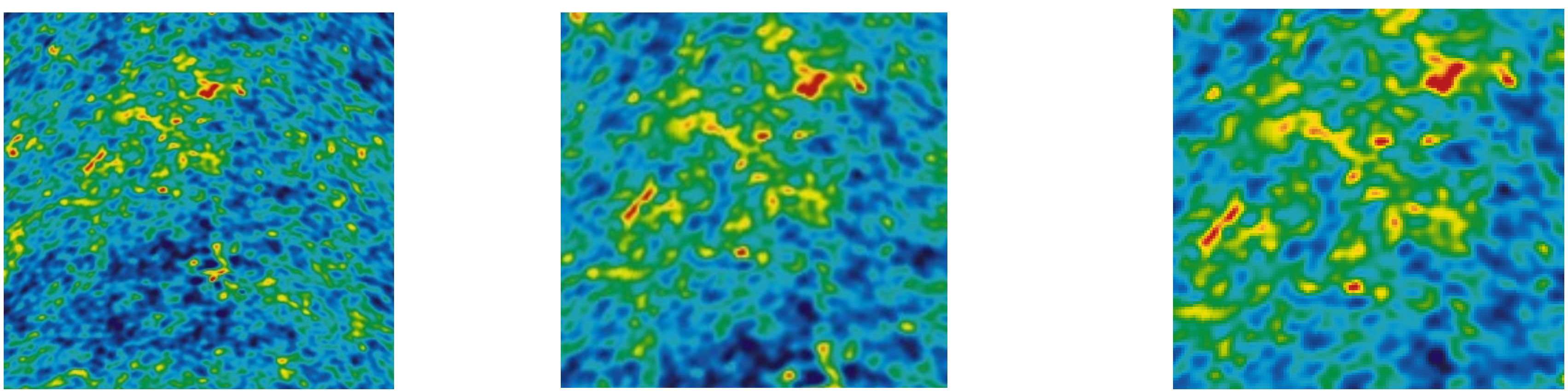
(r, θ, φ) : Comoving coordinates



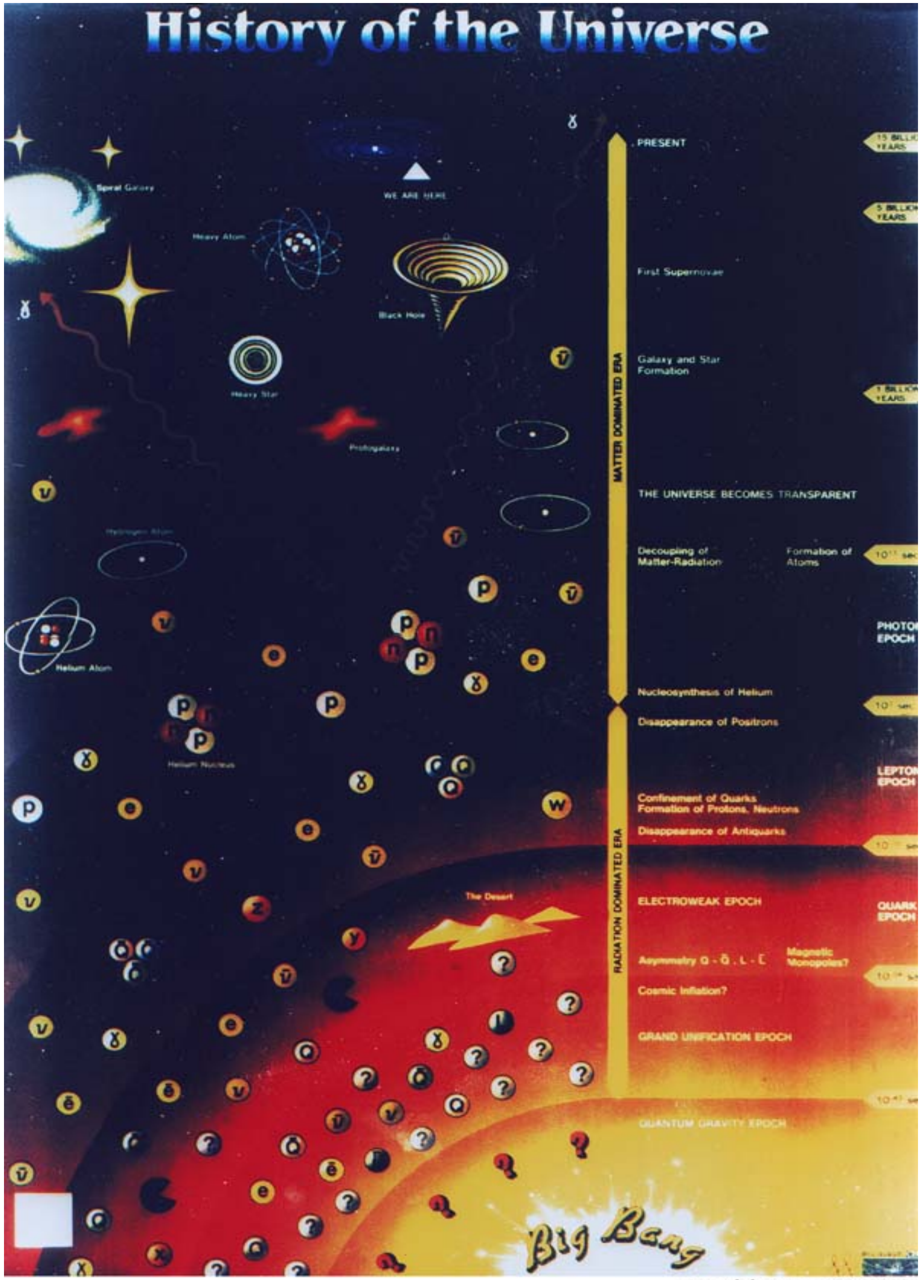
k=-1 k=0 k=1







simulated maps for cosmic microwave background temperature anisotropy



Fermilab Image 01-582D

Cosmic Microwave Background

- Relic photons of hot big bang
- First observed in 1965
- Black body radiation of temperature about 3K
- Coming from last scatterings with electrons at redshift of about 1100 or 400,000 yrs after the big bang (age of the Universe is about 14 Gyrs)
- Slightly anisotropic (10**μ**K) and linearly polarized (**μ**K)

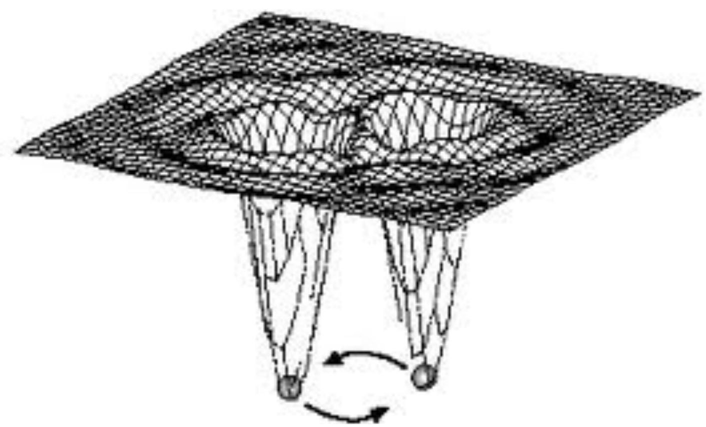
Gravitational Waves in Expanding Universe

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2$$
$$= a^2(\eta)\left(d\eta^2 - d\mathbf{x}^2\right)$$

conformal time

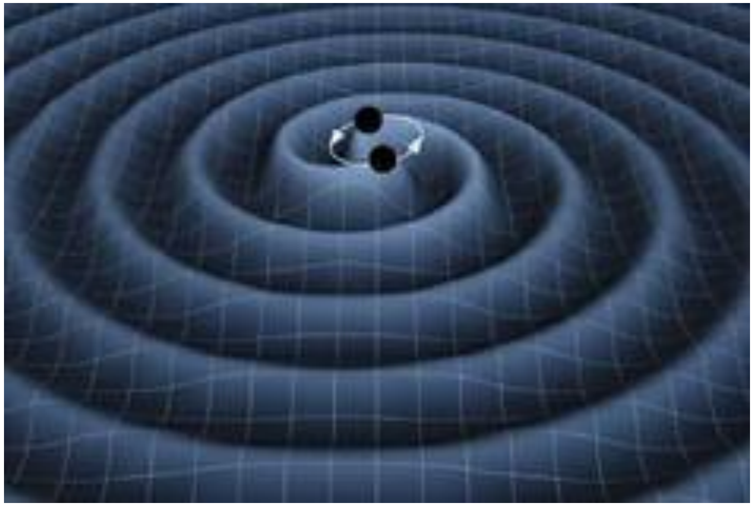
$$d\eta = dt/a(t)$$

$$g_{\mu\nu} = a^2(\eta)(\eta_{\mu\nu} + h_{\mu\nu}), \quad h_{\mu\nu} \ll 1$$



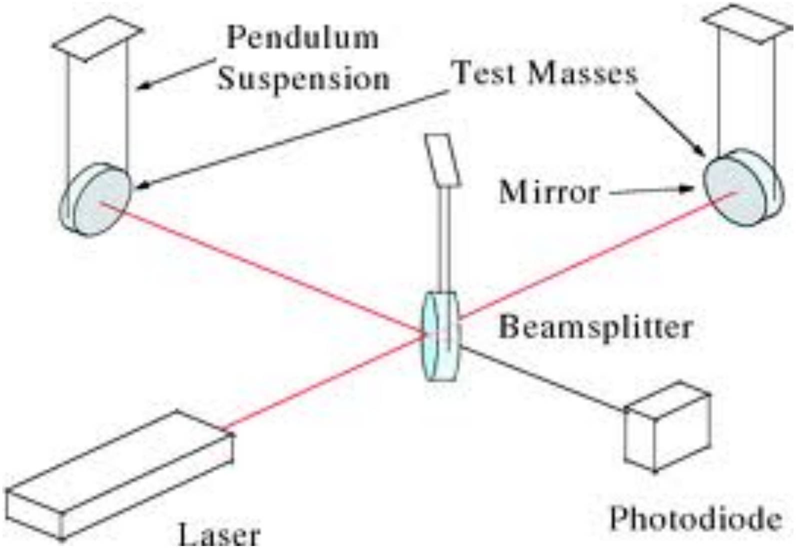
binary black holes

gravitational waves are ripples of space-time

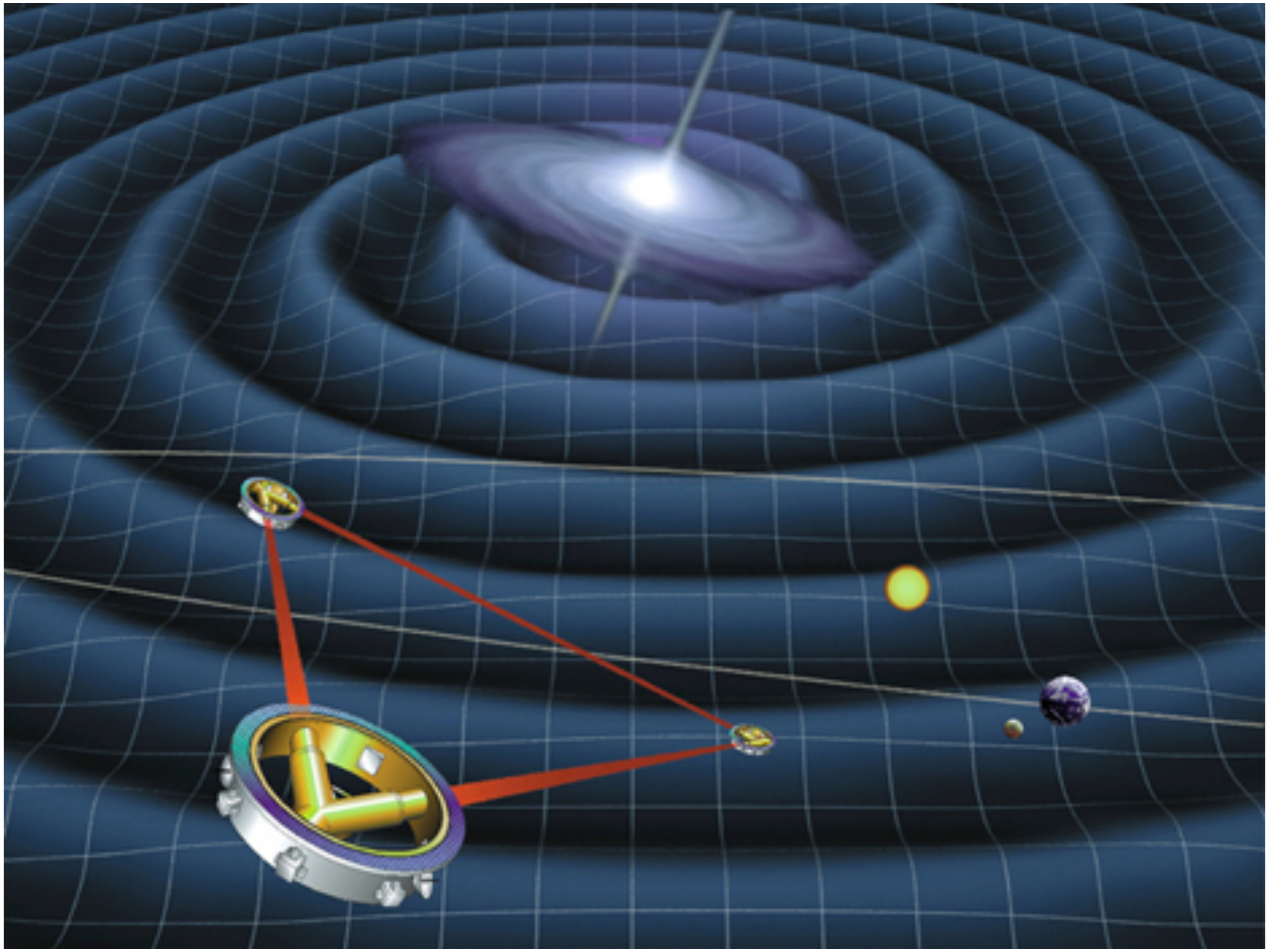


GWs Observation

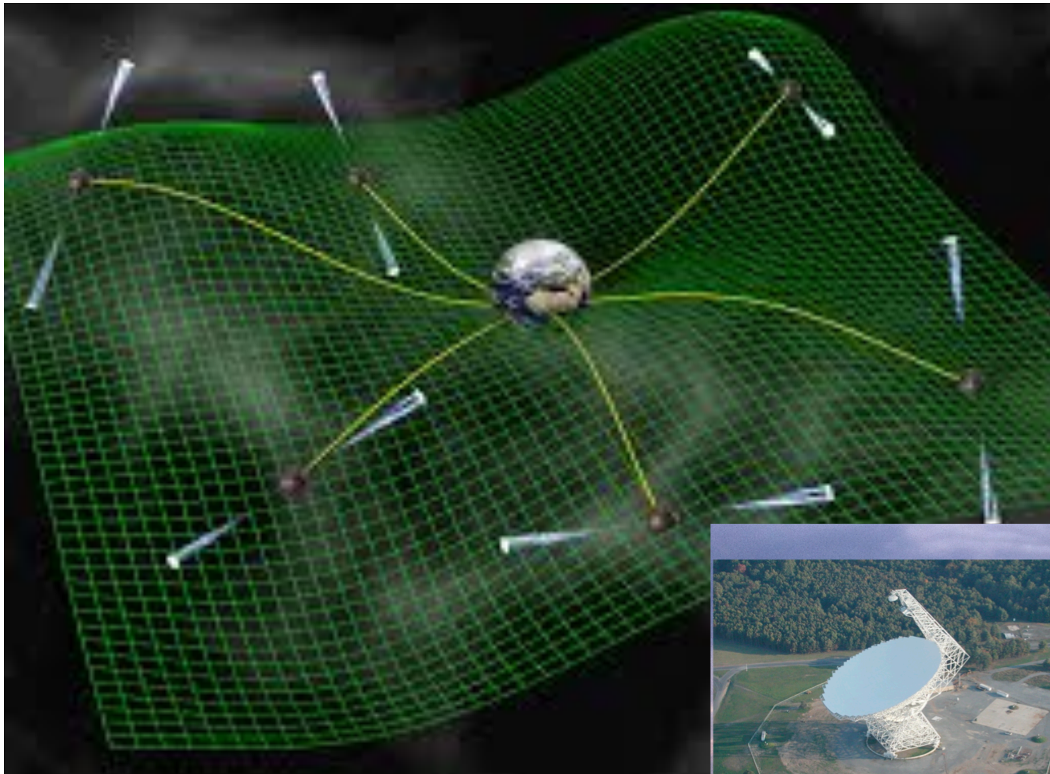
LIGO interferometry experiment



Future GWs space mission LISA



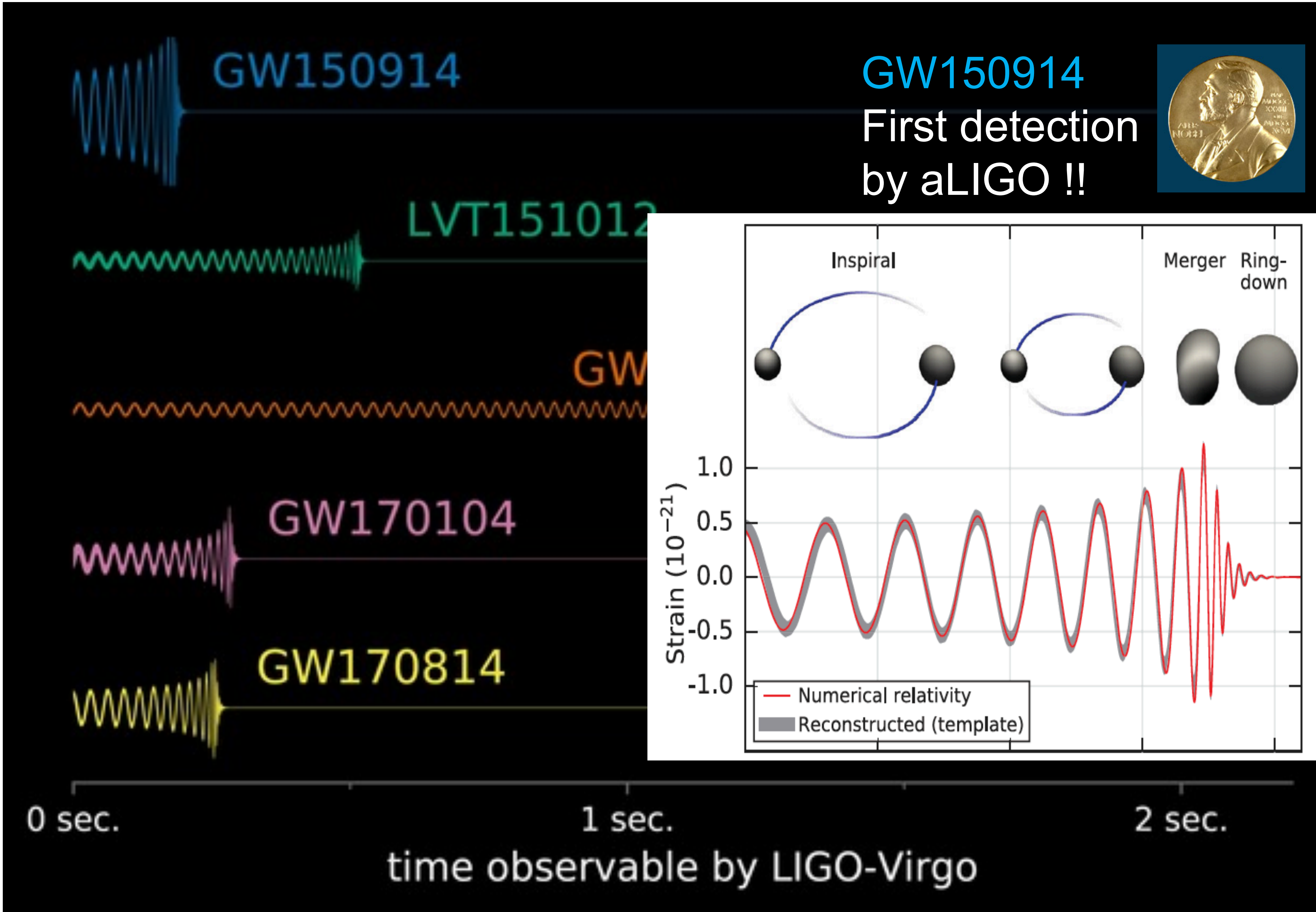
Current Pulsar Timing Arrays

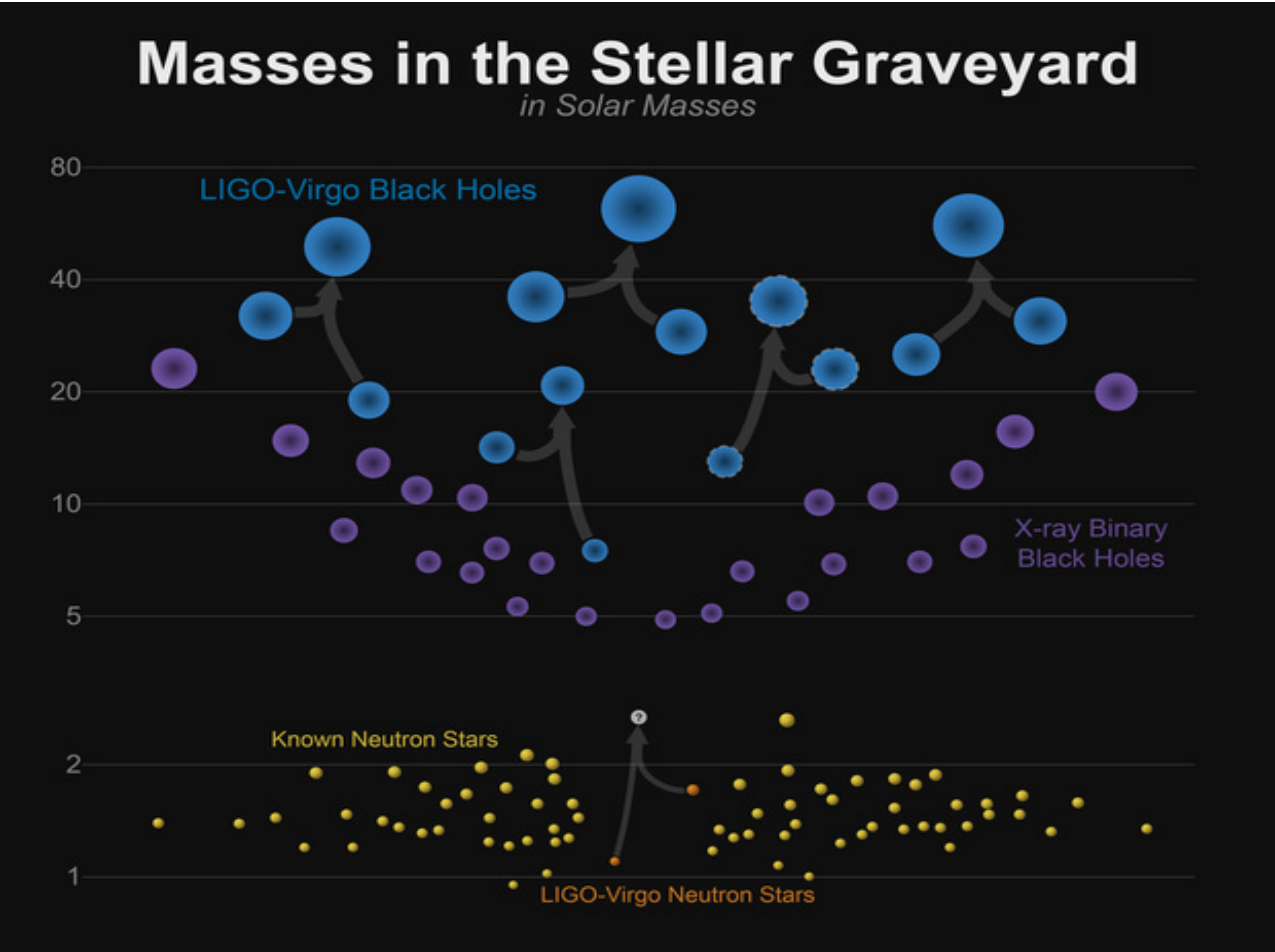


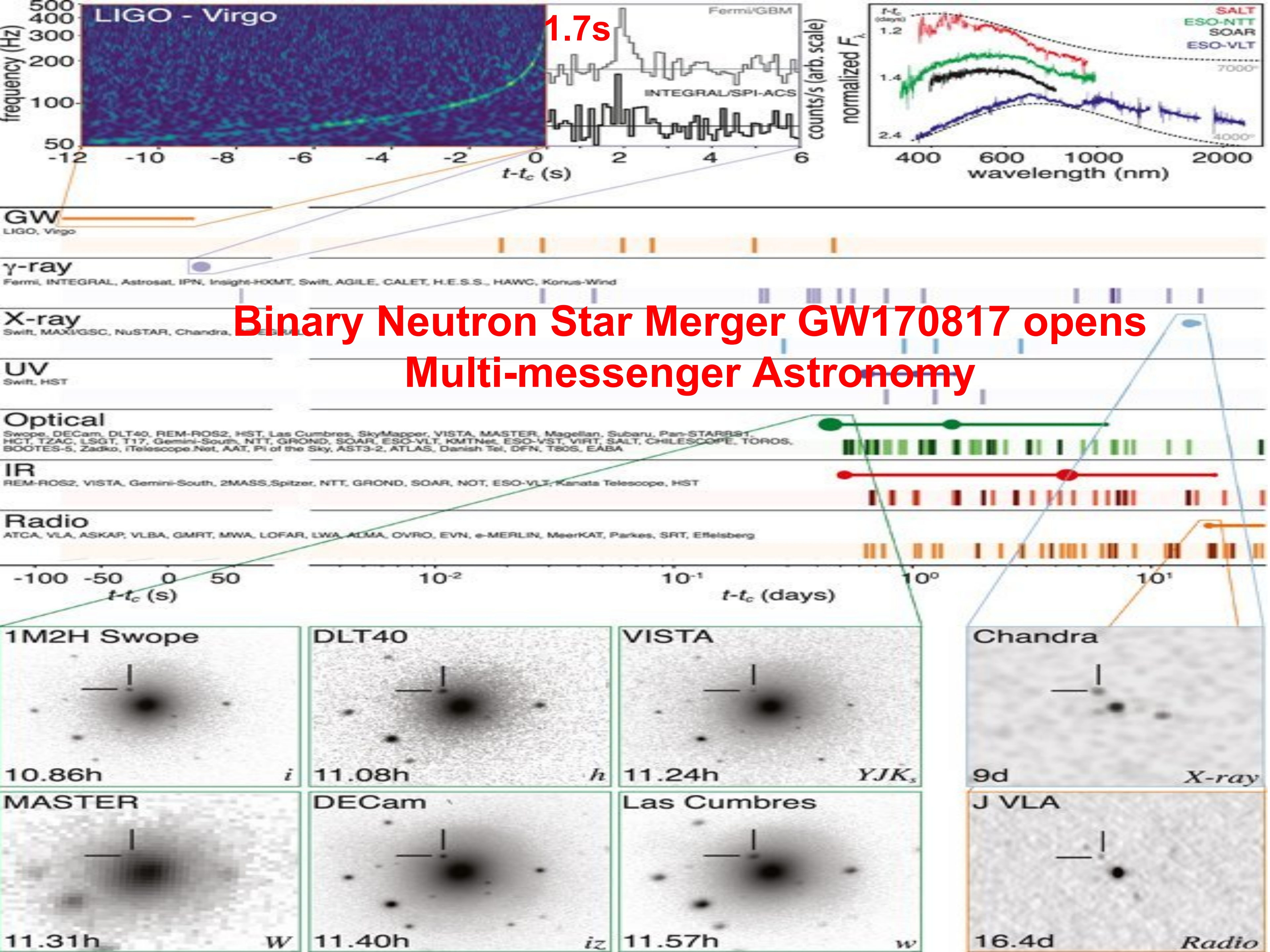
Nano-Hz GWs cause small correlated changes to the times of arrival of radio pulses from millisecond pulsars

International Pulsar Timing Arrays

Detection of Gravitational Waves in Binary Black Hole Merger




Masses in the Stellar Graveyard
in Solar Masses
80
40
20
10
5
2
1
LIGO-Virgo Black Holes
X-ray Binary
Black Holes
Known Neutron Stars
LIGO-Virgo Neutron Stars


Binary Neutron Star Merger GW170817 opens Multi-messenger Astronomy
LIGO - Virgo
1.7s
Fermi/GBM
INTEGRAL/SPI-ACS
frequency (Hz)
counts/s (arb. scale)
t-tc (s)
normalized Fλ
wavelength (nm)
SALT
ESO-NTT
SOAR
ESO-VLT
GW
LIGO, Virgo
γ-ray
Fermi, INTEGRAL, Astrosat, IPN, Insight-HXMT, Swift, AGILE, CALET, H.E.S.S., HAWC, Konus-Wind
X-ray
UV
Swift, HST
Optical
IR
Radio
ATCA, VLA, ASKAP, VLBA, GMRT, MWA, LOFAR, LWA, ALMA, OVRO, EVN, e-MERLIN, MeerKAT, Parkes, SRT, Effelsberg
t-tc (days)
1M2H Swope
10.86h
DLT40
11.08h
VISTA
11.24h
Chandra
9d
X-ray
MASTER
11.31h
DECam
11.40h
Las Cumbres
11.57h
J VLA
16.4d
Radio

Speed of Gravitational Waves

GW170817 with a distance at D=43.8 Mpc ($z \sim 0.01$)

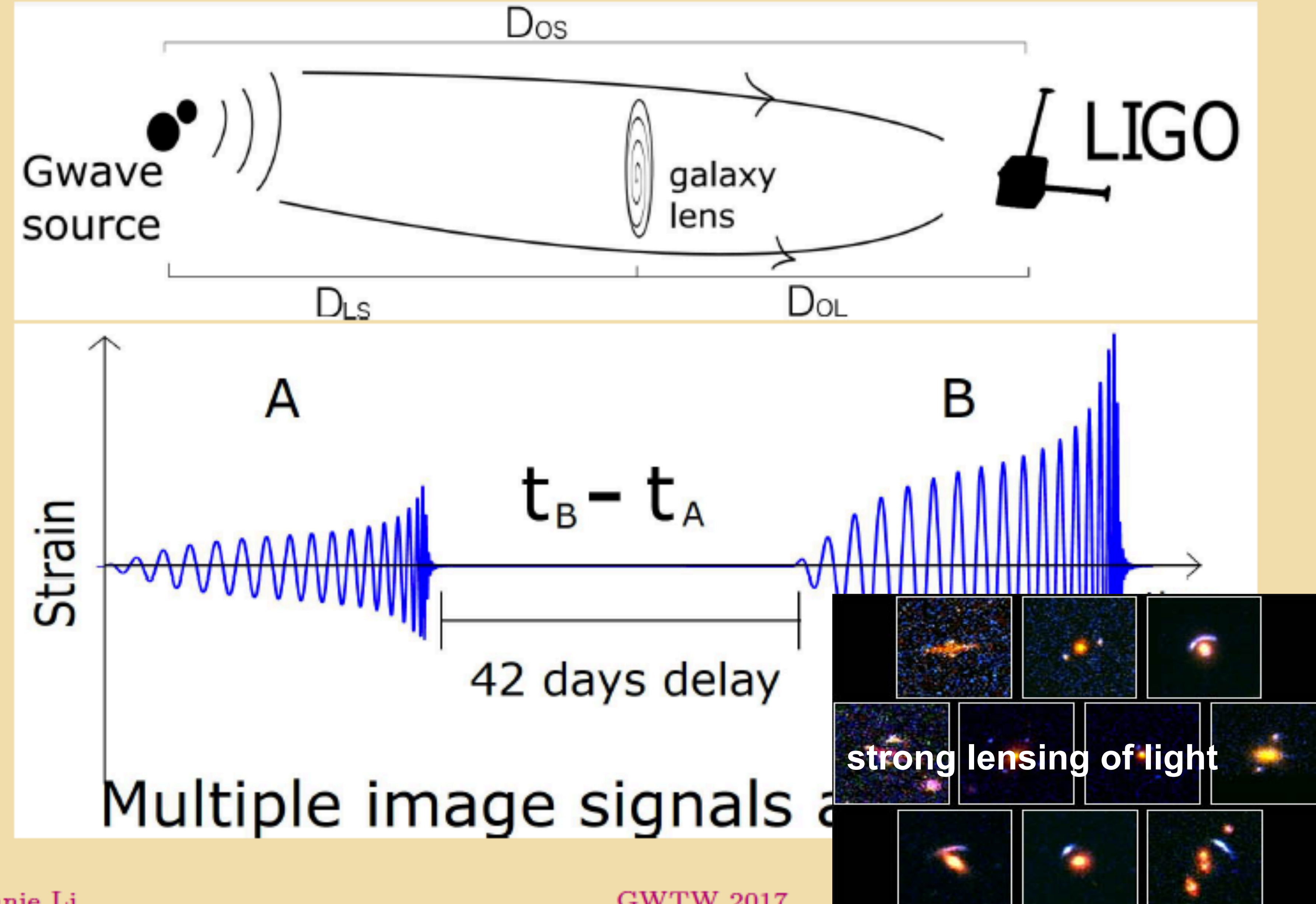
$D/c - D/v_{gw} = 1.7s$ $v_{light}=c=3x10^{10}$ cm/s

$(v_{gw} - c)/c = 1.7s\ (c/D) = 10^{-16}$ at frequency of 100 Hz

This has put a severe constraint on gravity models beyond the Einstein's theory of general relativity such as f(R), MOND,….

Cosmology based on Einstein's gravity works very well.

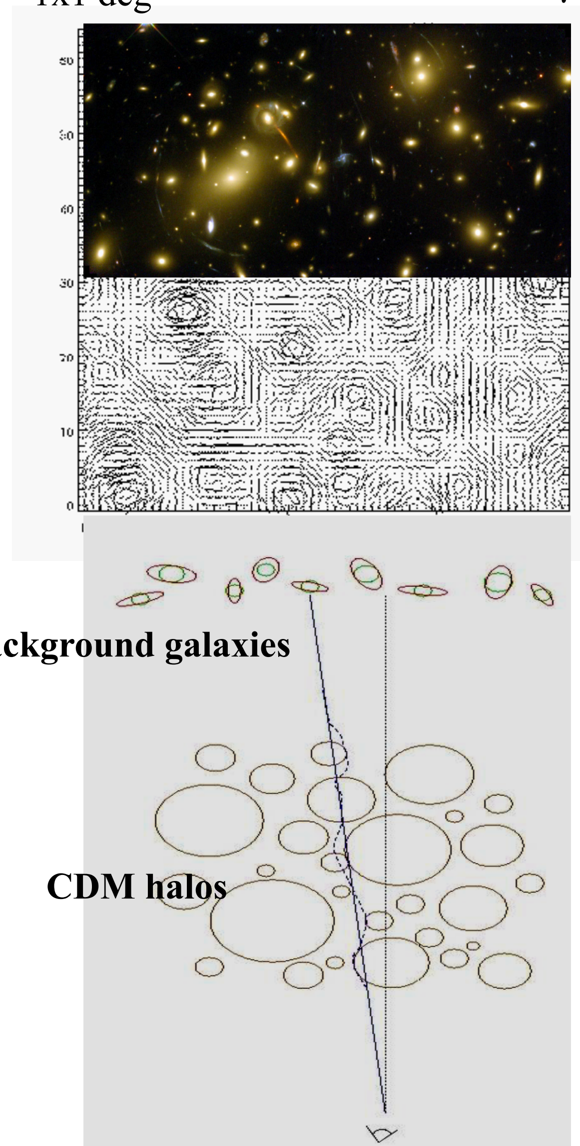
Gravitational Lensing of Gravitational Waves



Weak Lensing by Large-Scale Structure

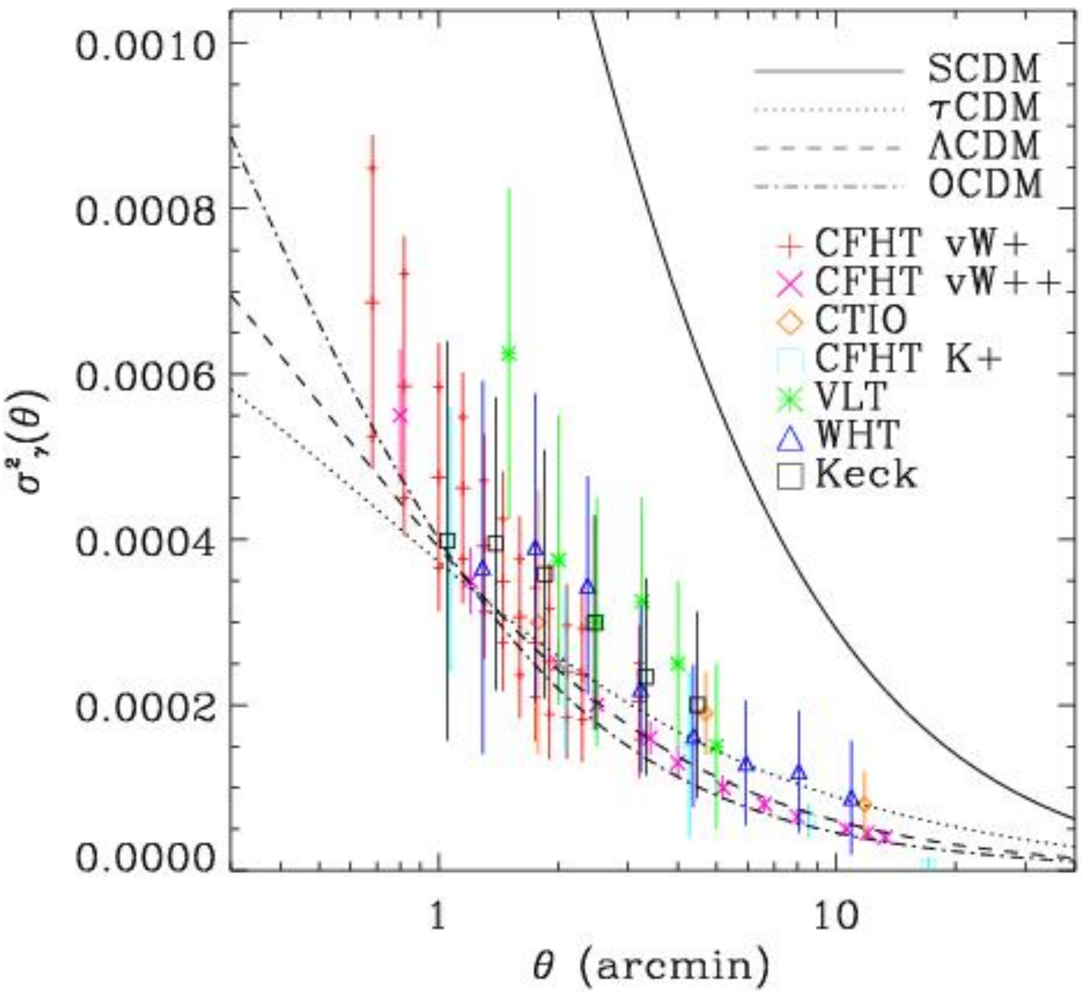
Jain et al. 1997
1x1 deg

Cosmic Shear γ



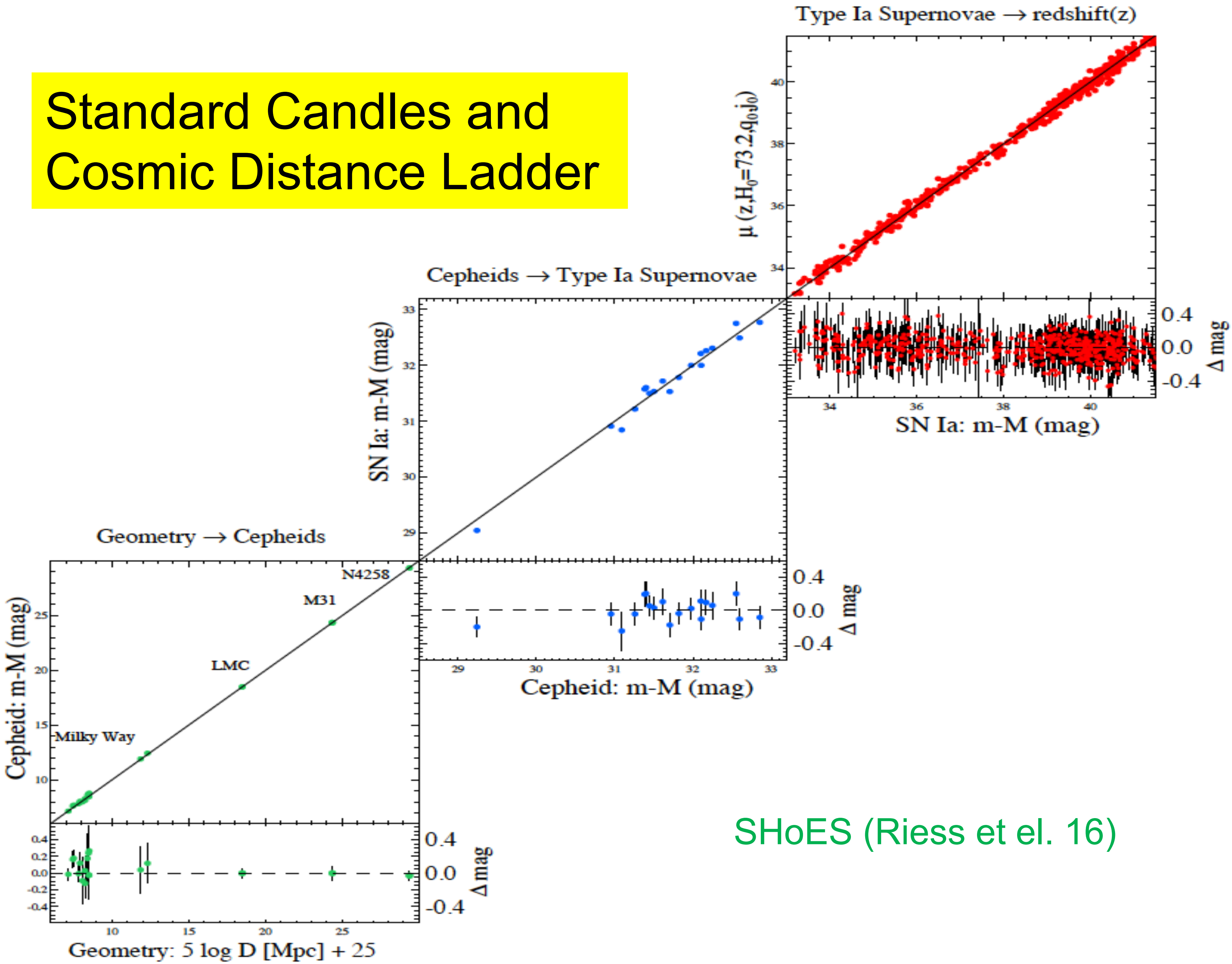
Shear Variance in circular cells with size θ

$$\sigma^2_\gamma(\theta) = \langle\gamma^2\rangle$$



Ellis et al. 02

Standard Candles and Cosmic Distance Ladder



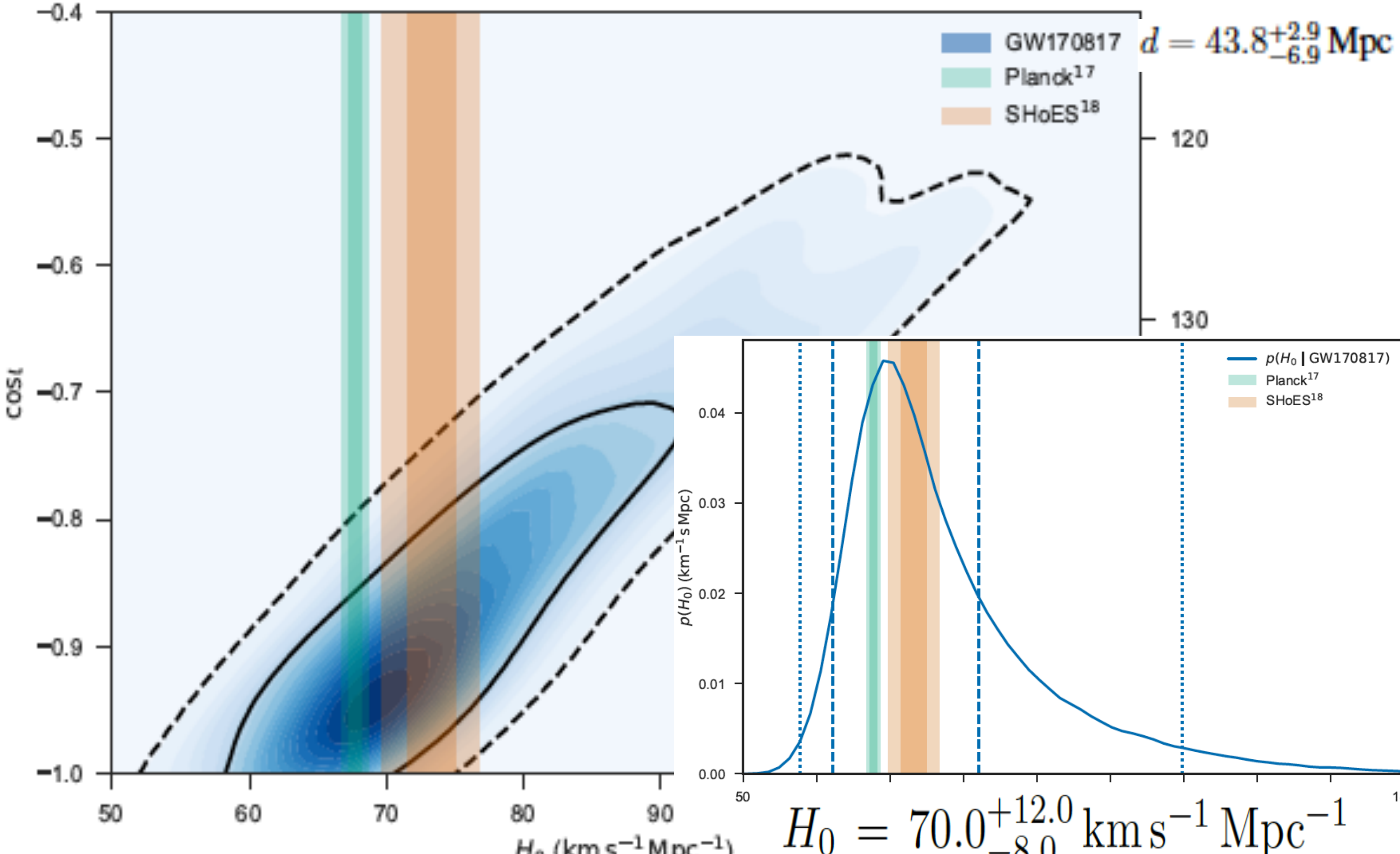
SHoES (Riess et el. 16)

Binary Neutron Star Mergers as Standard Sirens

Hubble law $\quad v_H = H_0 d \quad (d \lesssim 50\,\mathrm{Mpc})$

Multi-messenger astronomy: v_H from EM and *d* from GW/BNS

near 180 deg ($\cos\iota = -1$) indicate that the orbital angular momentum is anti-parallel with the direction from the source to the detector.

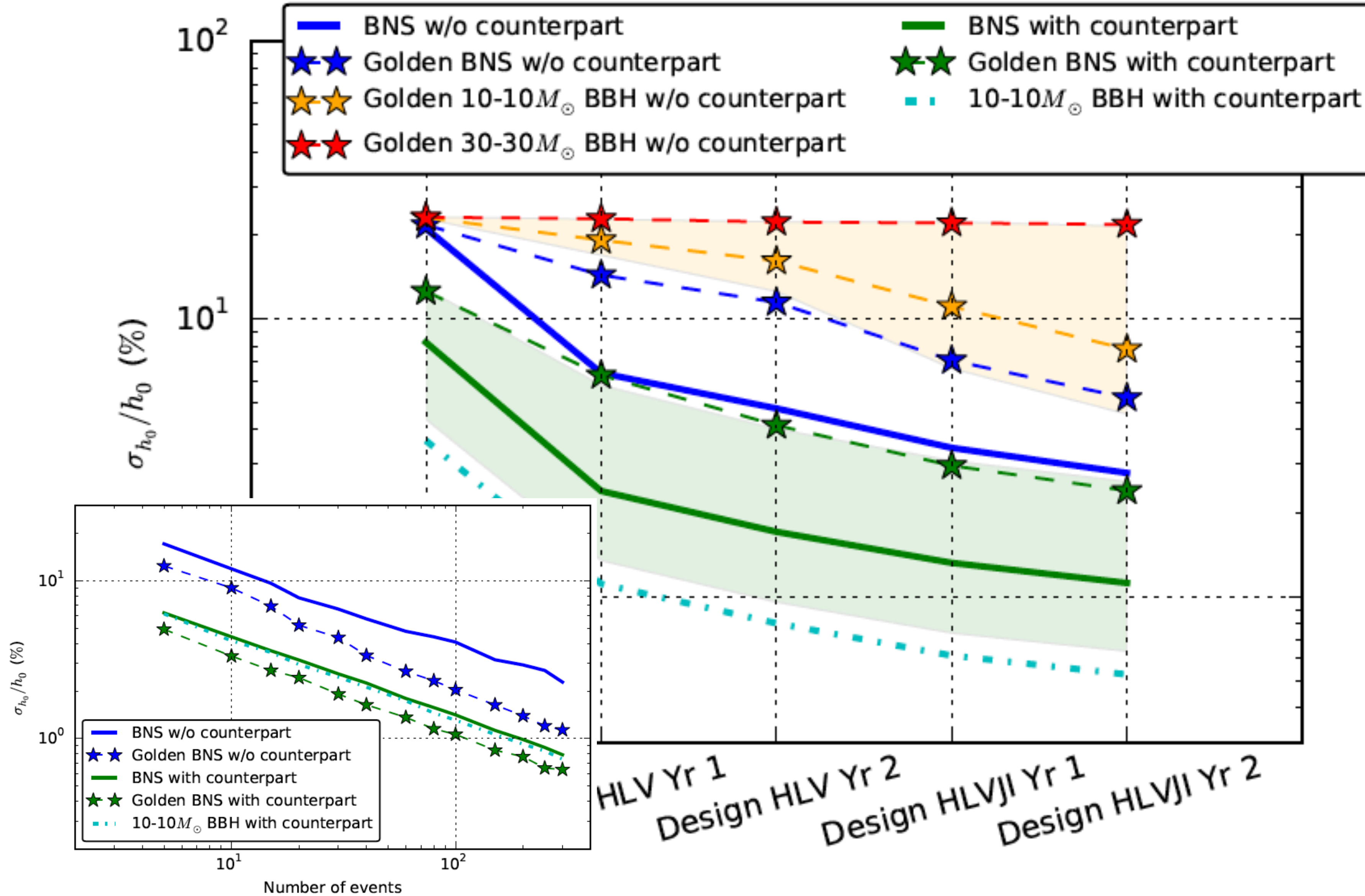


$d = 43.8^{+2.9}_{-6.9}\,\mathrm{Mpc}$

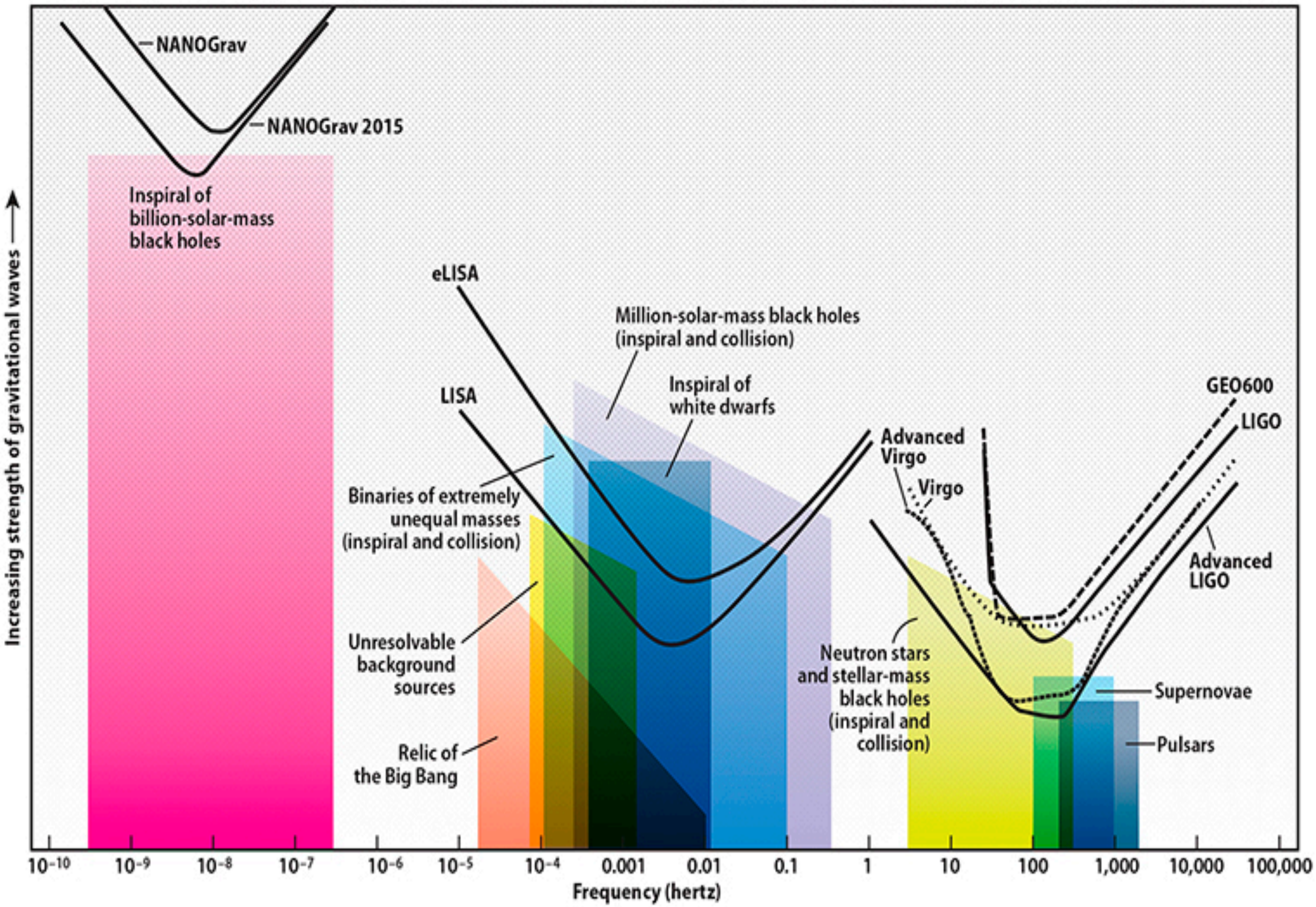
$$H_0 = 70.0^{+12.0}_{-8.0}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

Precision GW standard siren cosmology

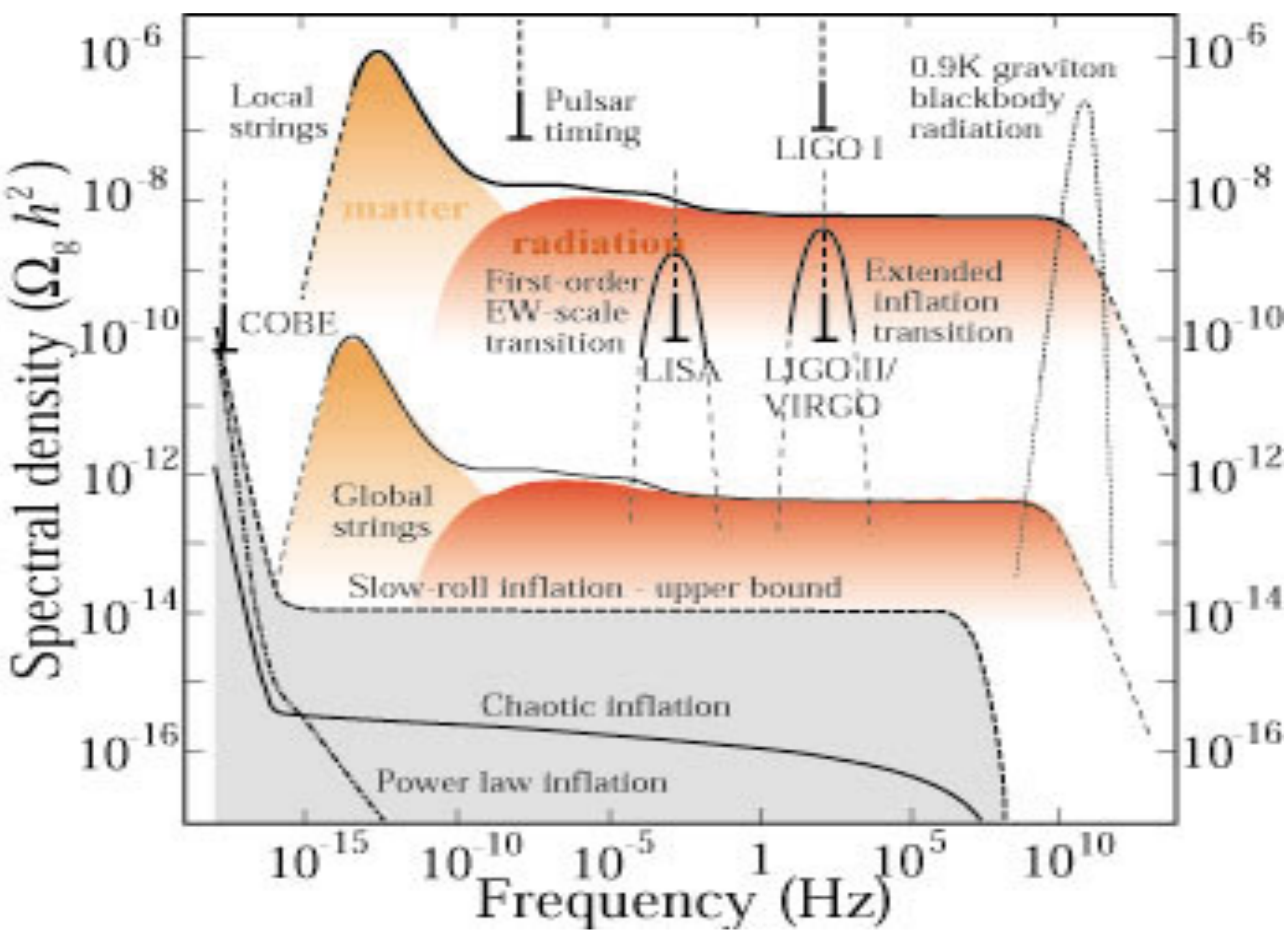
Chen et al. 17



Astrophysical sources for gravitational waves



Cosmological sources for gravitational waves



GW spectral energy density

tion h_{ij} is gauged to be transverse-traceless. The latter can be decomposed into two polarization unit tensors as

$$h_{ij}(\eta,\vec{x}) = \sum_{\lambda=+,\times} \int \frac{d^3\vec{k}}{(2\pi)^{\frac{3}{2}}} h_\lambda(\eta,\vec{k}) \epsilon^\lambda_{ij}(\hat{k}) e^{i\vec{k}\cdot\vec{x}}, \qquad (2)$$

where $h_\lambda(\eta,\vec{k})$ is a Gaussian random field that defines the power spectrum of tensor perturbation,

$$\langle h_\lambda(\eta,\vec{k}) h^*_{\lambda'}(\eta,\vec{k}')\rangle = \delta(\vec{k}-\vec{k}')\frac{2\pi^2}{k^3}\mathcal{P}_h^{\lambda\lambda'}(\eta,k). \qquad (3)$$

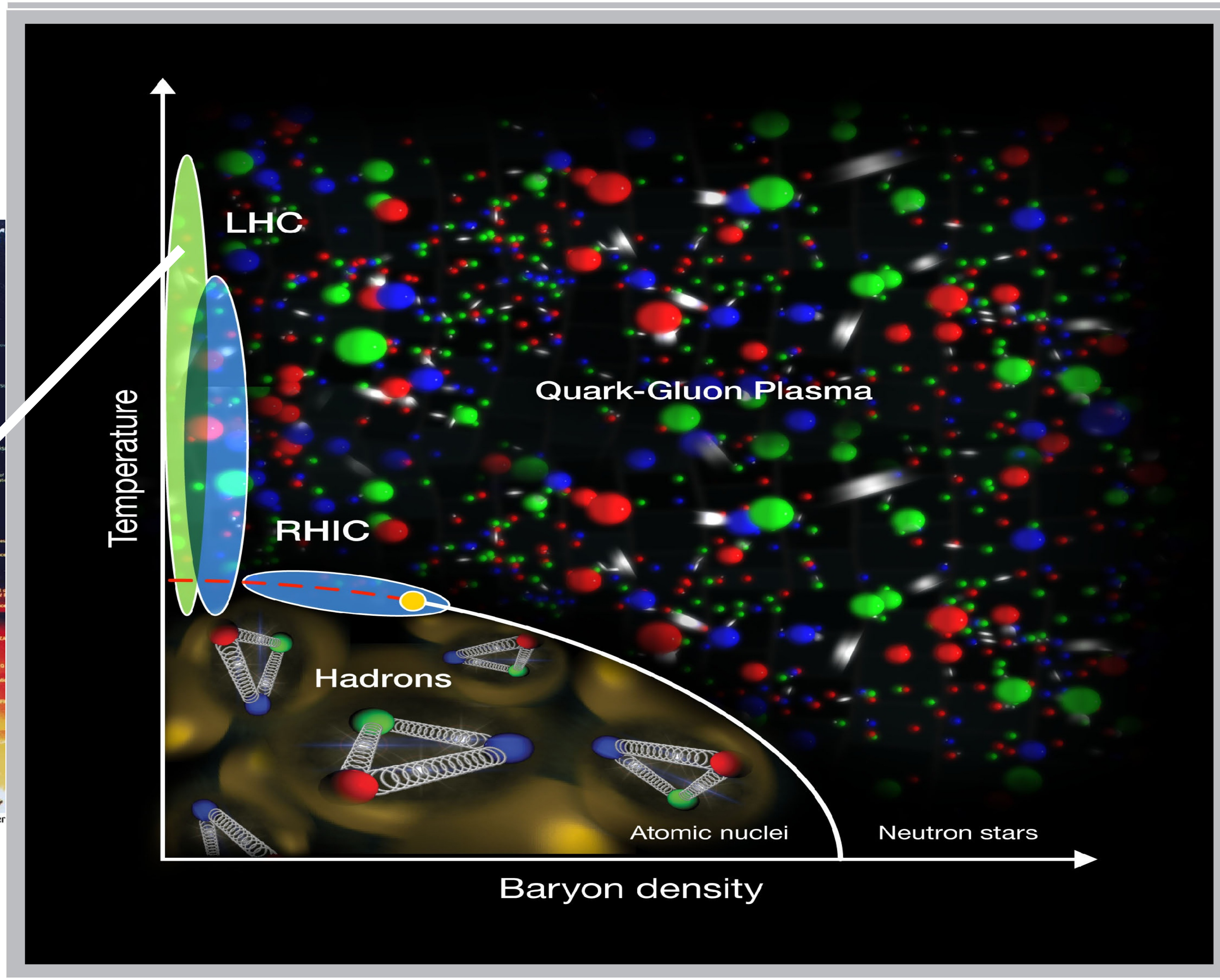
In the following, we will assume that $\mathcal{P}_h^{\lambda\lambda'}(\eta,k) = \delta_{\lambda\lambda'}\mathcal{P}_h(\eta,k)$. Then, the spectral energy density of the GWs relative to the critical density is given by

$$\Omega_{\rm GW}(\eta,k,\hat{k}) \equiv \frac{k}{\rho_c}\frac{d\rho_{\rm GW}}{dk d^2\hat{k}} = \frac{1}{96\pi}\left(\frac{k}{aH}\right)^2 \bar{\mathcal{P}}_h(\eta,k), \quad (4)$$

where $\rho_c = 3M_p^2H^2$ and the overbar denotes taking a time-average. For k-modes that re-enter the horizon

First-order phase transitions

e.g. quark-hadron transition ?



Non-topological soliton model for quark-hadron transition

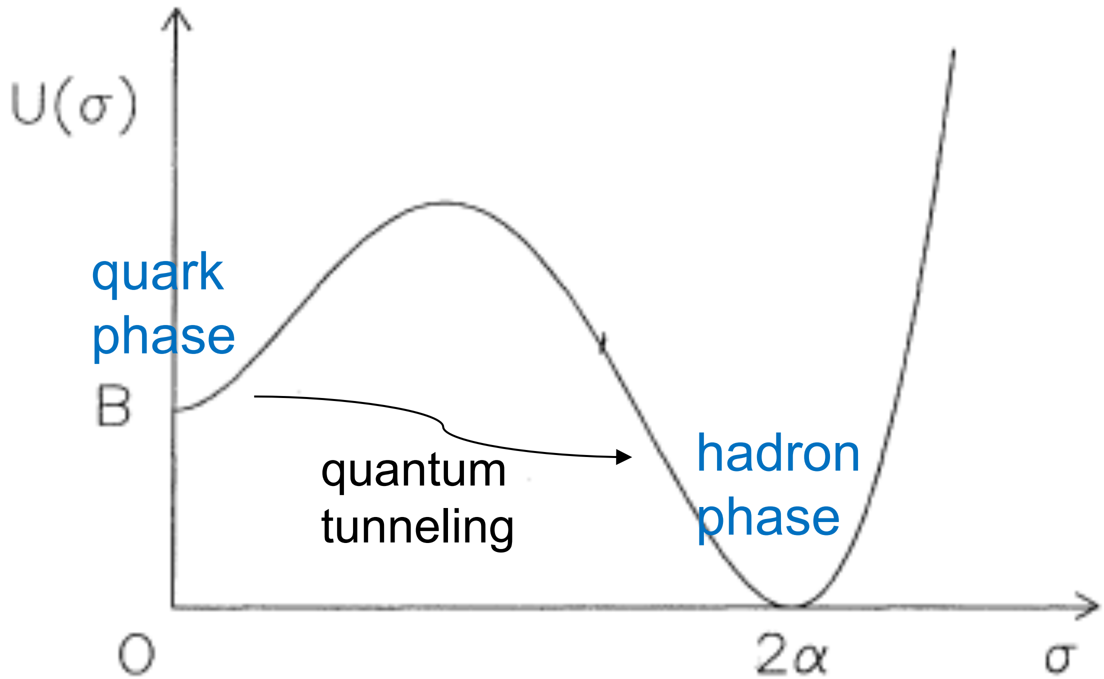
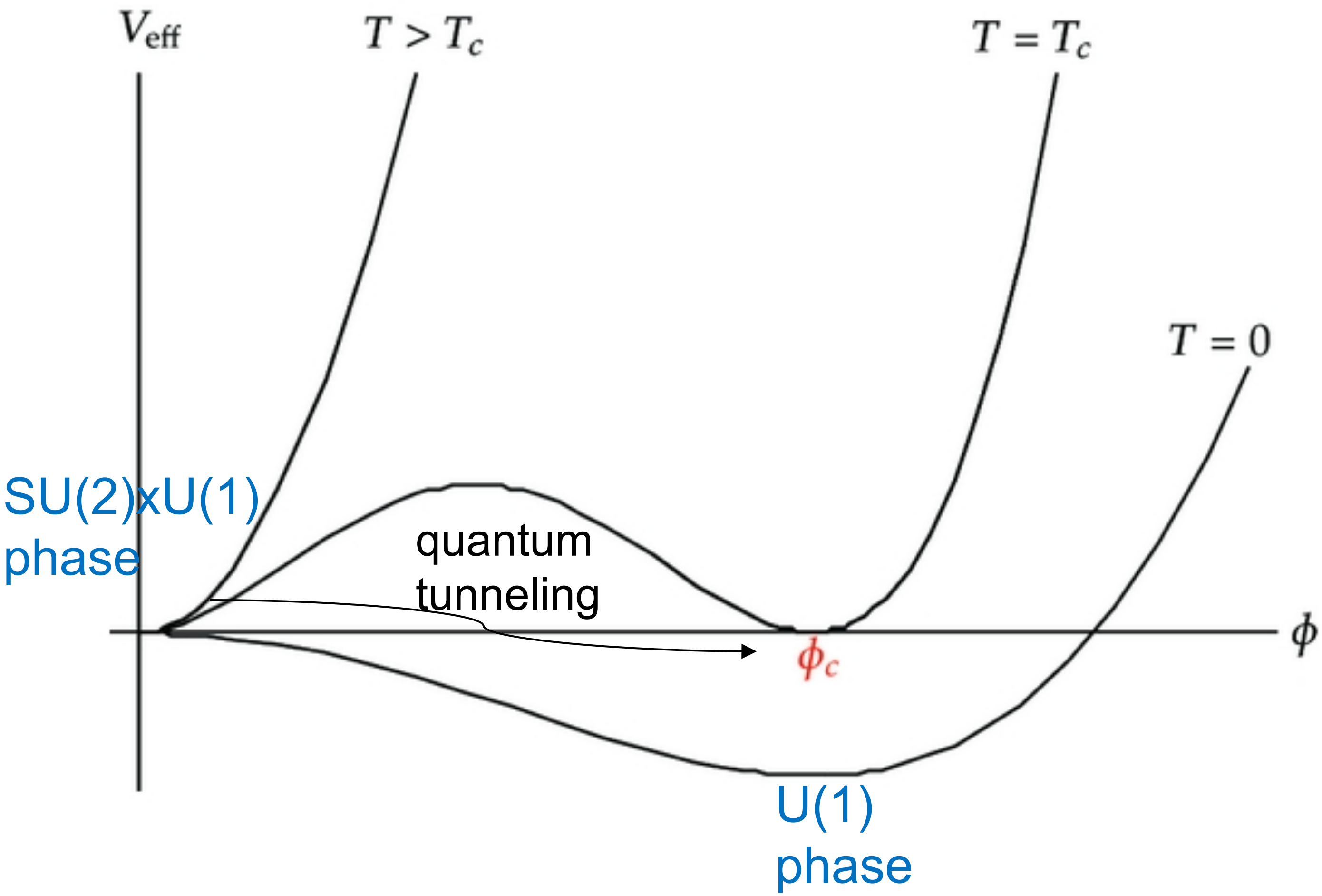
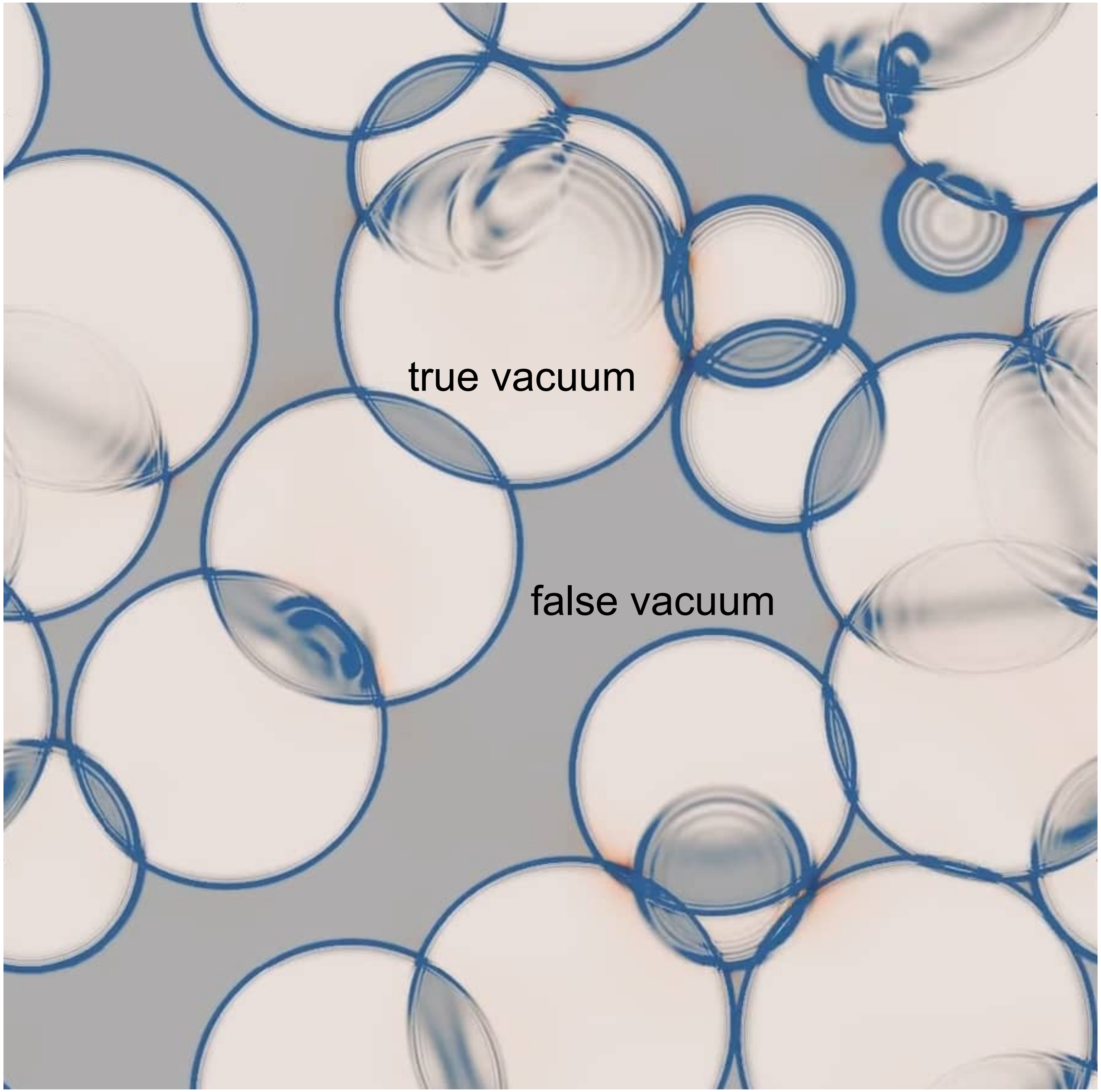


FIG. 1. Typical potential $U(\sigma)$ in the nontopological soliton model.

e.g. Electroweak symmetry breaking ?



Bubble nucleation



Contributions to the GW spectrum

- Collisions of bubble walls and shocks in the plasma
- Sound waves in the plasma after the bubbles have collided
- Magneto-hydrodynamic (MHD) turbulence in the plasma forming after the bubbles have collided

$$h^2\Omega_{\rm GW} \simeq h^2\Omega_\phi + h^2\Omega_{\rm sw} + h^2\Omega_{\rm turb}$$

Relevant physical quantities for GW production

bubble nucleation rate $\Gamma(t) = A(t)e^{-S(t)}$

aT=constant

inverse time duration $\beta \equiv - \left.\frac{dS}{dt}\right|_{t=t_*} \simeq \frac{\dot{\Gamma}}{\Gamma}$ $\Rightarrow$ $\frac{\beta}{H_*} = T_* \left.\frac{dS}{dT}\right|_{T_*}$

ratio of the vacuum energy density released $\alpha = \frac{\rho_{\rm vac}}{\rho^*_{\rm rad}}$ $\rho^*_{\rm rad} = g_* \pi^2 T_*^4/30,$

g_* degrees of freedom

fraction of vacuum energy that gets converted into bulk motion $\kappa_v = \frac{\rho_v}{\rho_{\rm vac}}$

fraction of vacuum energy into gradient energy of the soliton field $\kappa_\phi = \frac{\rho_\phi}{\rho_{\rm vac}}$

bubble wall velocity in the rest frame of the plasma v_w

Collisions of bubble walls and shocks

- envelope (thin wall)-approximation numerical simulations

$$h^2\Omega_{\rm env}(f) = 1.67\times10^{-5}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{0.11\,v_w^3}{0.42+v_w^2}\right)S_{\rm env}(f)$$

fraction κ of the latent heat deposited in a thin shell close to the PT front.

$$S_{\rm env}(f) = \frac{3.8\,(f/f_{\rm env})^{2.8}}{1+2.8\,(f/f_{\rm env})^{3.8}}$$

$$h^2\Omega_\phi(f) = h^2\Omega_{\rm env}(f)\big|_{\kappa=\kappa_\phi}$$

peak frequency (at t_*) $$\frac{f_*}{\beta} = \left(\frac{0.62}{1.8-0.1v_w+v_w^2}\right)$$

peak frequency today $$f_{\rm env} = 16.5\times10^{-3}\,{\rm mHz}\left(\frac{f_*}{\beta}\right)\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

Acoustic production – sound waves numerical simulations

$$h^2\Omega_{\rm sw}(f) = 2.65\times10^{-6}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}} v_w\, S_{\rm sw}(f)$$

amplified by a factor of β/H_* for long-lasting sources

$$S_{\rm sw}(f) = (f/f_{\rm sw})^3\left(\frac{7}{4+3\,(f/f_{\rm sw})^2}\right)^{7/2}$$

peak frequency (at t_*) $\quad f_* = (2/\sqrt{3})(\beta/v_w)$,

peak frequency today $\quad f_{\rm sw} = 1.9\times10^{-2}\,{\rm mHz}\,\frac{1}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$

MHD turbulence - Kolmorgorov-type numerical simulations

$$h^2\Omega_{\rm turb}(f) = 3.35\times10^{-4}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm turb}\,\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{100}{g_*}\right)^{1/3} v_w\, S_{\rm turb}(f)$$

amplified by a factor of β/H$_*$ for long-lasting sources

$$S_{\rm turb}(f) = \frac{(f/f_{\rm turb})^3}{[1+(f/f_{\rm turb})]^{\frac{11}{3}}\,(1+8\pi f/h_*)}$$

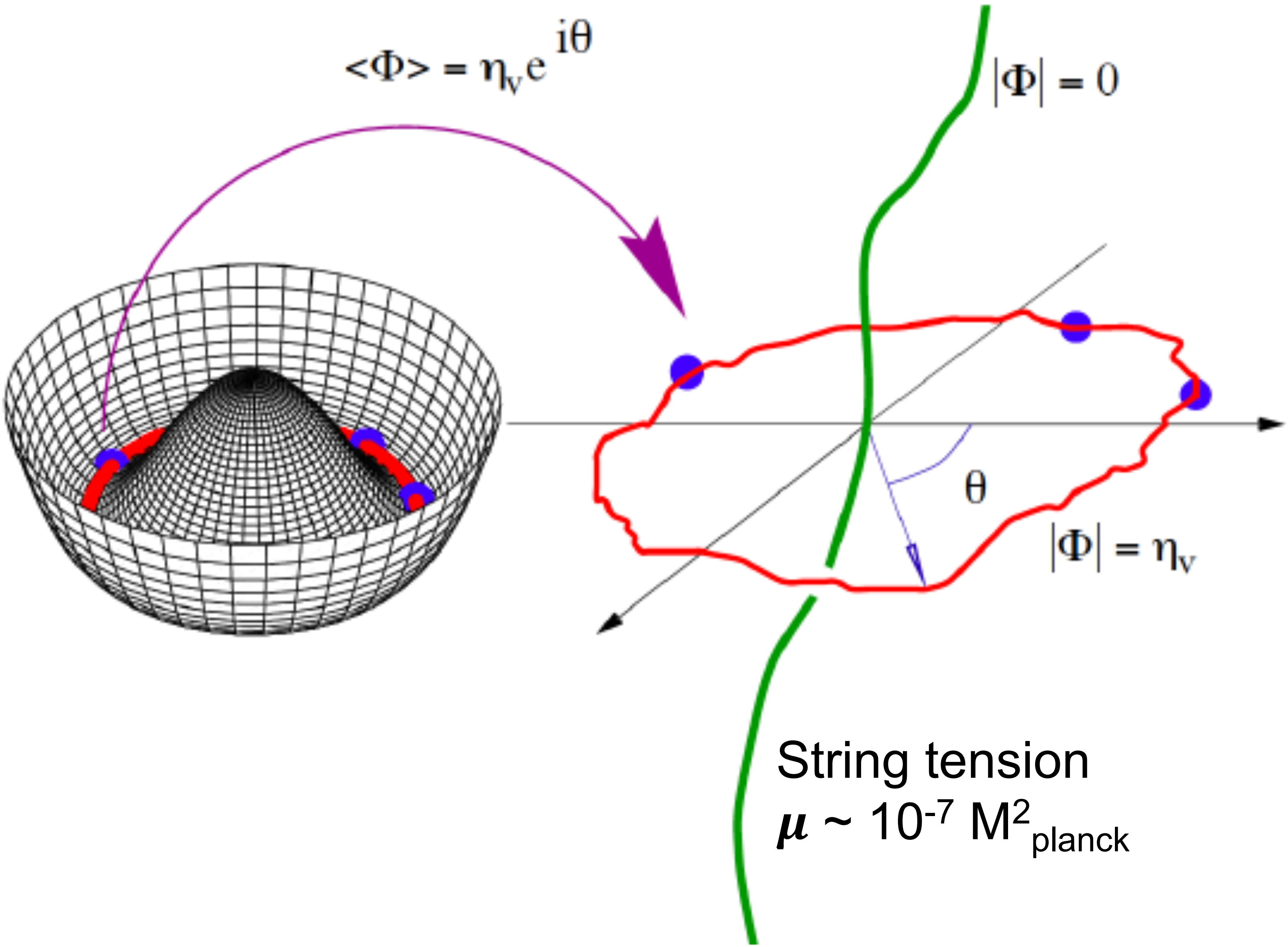
$$h_* = 16.5\times10^{-3}\,{\rm mHz}\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

H$_*$ redshifted to today

peak frequency (at t_*) $\quad f_* \simeq (3.5/2)(\beta/v_w)$

peak frequency today $\quad f_{\rm turb} = 2.7\times10^{-2}\,{\rm mHz}\,\frac{1}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$

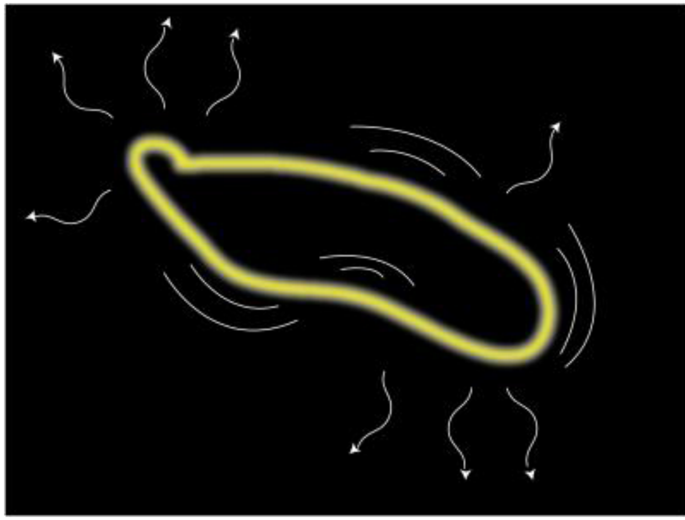
Defects in phase transitions – cosmic strings

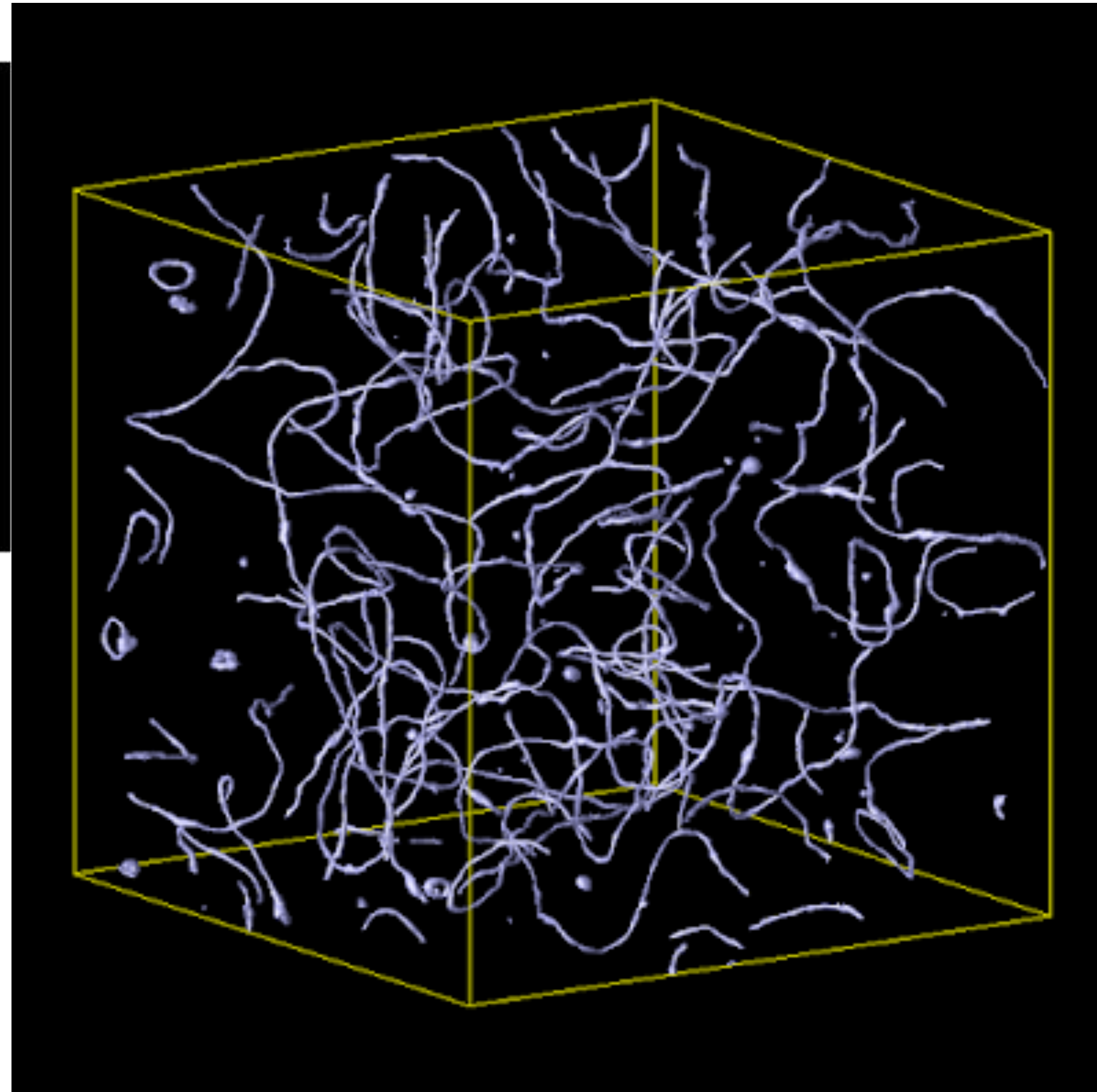


String tension

$\boldsymbol{\mu} \sim 10^{-7}\ M^2_{planck}$

A network of cosmic strings

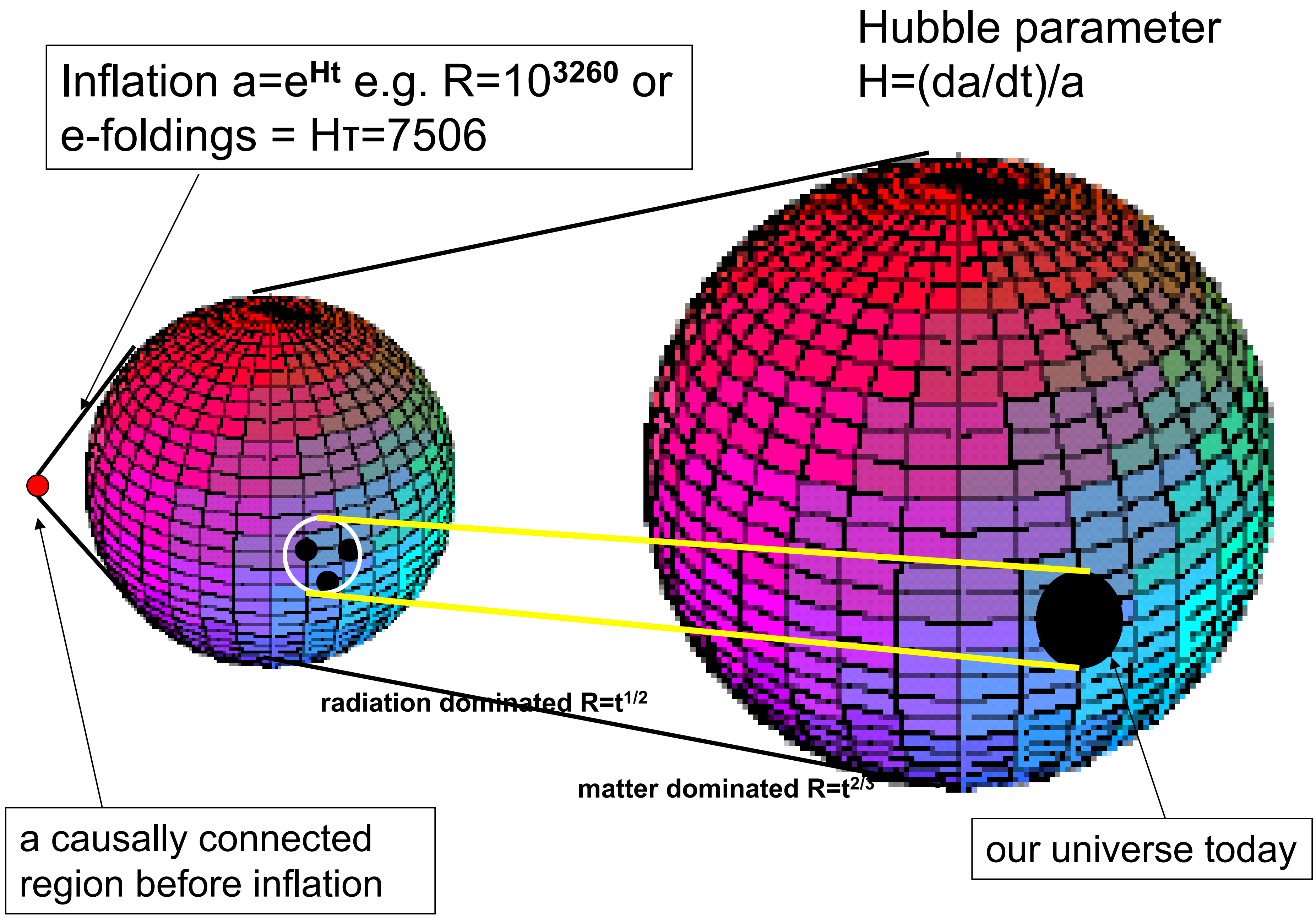




Cosmic-string loops wiggle and oscillate, producing gravitational waves, then slowly shrink as they lose energy until they disappear. (Image: © Matt DePies/UW)

Inflation

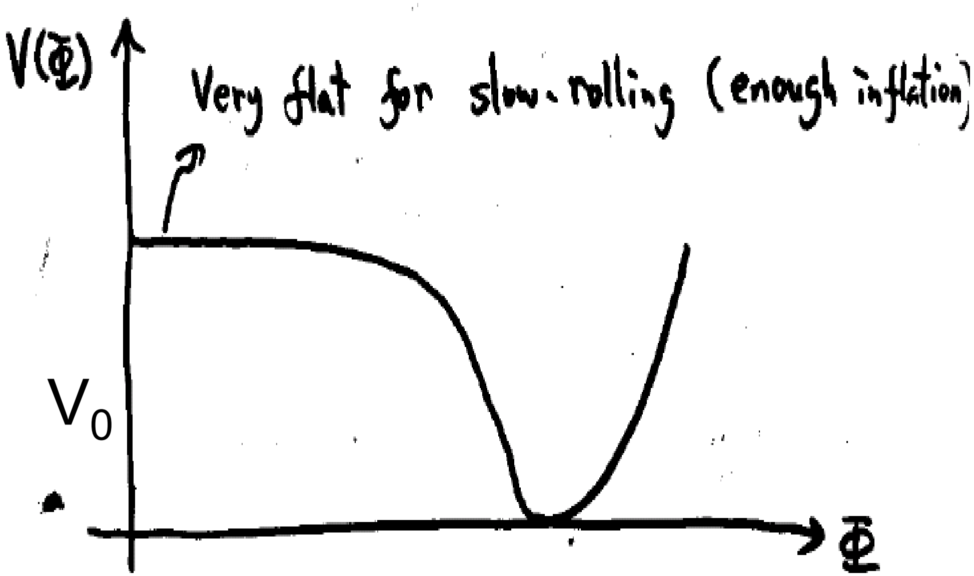
- Homogeneity and flatness problems in Hot Big Bang
- Inflation solves these problems
- What drove inflation?
- Inflation models
- Density (Scalar) perturbation $\rightarrow$ seeds for structure formation
- Tensor perturbation $\rightarrow$ primordial gravitational waves
- Preheating and reheating $\rightarrow$ Hot Big Bang


Hubble parameter
H=(da/dt)/a
Inflation $a=e^{Ht}$ e.g. $R=10^{3260}$ or
e-foldings = Hτ=7506
radiation dominated $R=t^{1/2}$
matter dominated $R=t^{2/3}$
a causally connected
region before inflation
our universe today

Spectrum of the primordial curvature perturbation

Scalar mode

A generic $V(\Phi)$ for inflation



scalar field equation of motion

$$\ddot{\phi} + 3\frac{\dot{R}}{R}\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \qquad \frac{\dot{R}}{R} = H$$

Φ Inflaton

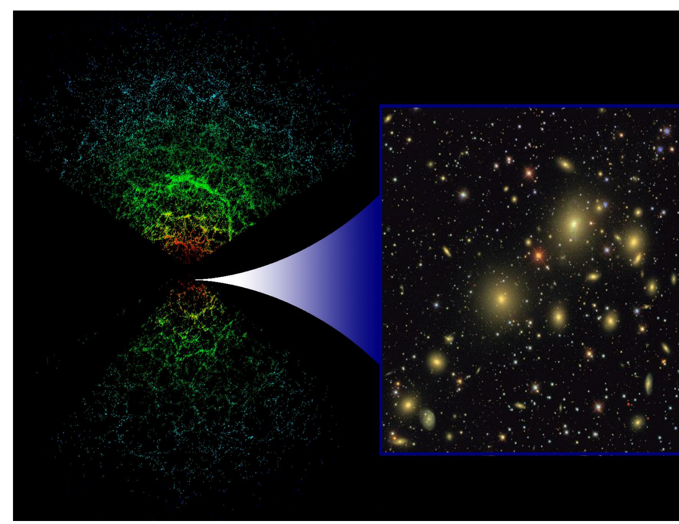
Slow-roll inflation

$$H^2 \simeq \frac{V(\phi)}{3M_{\rm Pl}^2},$$

$$3H\dot{\phi} \simeq -V'(\phi),$$

$$\mathcal{P}_{\mathcal{R}}(k) = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2$$

$$P_R(k) = (\delta\rho/\rho)^2$$



large scale structure of the Universe

Gravitational Wave Equation

Einstein–Hilbert action $$I_G = \frac{1}{16\pi G}\int d^4x\sqrt{g}\,R$$

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2$$
$$= a^2(\eta)\left(d\eta^2 - d\mathbf{x}^2\right)$$

$$g_{\mu\nu} = a^2(\eta)(\eta_{\mu\nu} + h_{\mu\nu})\,, \quad h_{\mu\nu} \ll 1\,, \quad a(\eta) = e^{\sigma(\eta)}\,, \quad \sigma(\eta) \ll 1$$

$$I_G = \frac{1}{16\pi G}\int d^4x\ a^2\left(\frac{1}{4}\partial_\mu h_{\alpha\beta}\partial^\mu h^{\alpha\beta} - \frac{1}{4}\partial_\mu h\partial^\mu h + \frac{1}{2}\partial_\alpha h^{\alpha\beta}\partial_\beta h\right.$$
$$\left. - \frac{1}{2}\partial_\alpha h^{\alpha\mu}\partial^\beta h_{\beta\mu} - 2\partial_\mu\sigma\partial^\mu h + 2\partial_\alpha\sigma\partial_\beta h^{\alpha\beta} - 6\partial_\mu\sigma\partial^\mu\sigma\right),$$

Synchronous
Transverse
Traceless gauge

$$h_{00} = h_{0i} = 0, \quad h_k^k = \partial_i h^{ij} = 0,$$

$$I_{\text{graviton}} = \frac{1}{16\pi G}\int d^4x\ a^2\ \frac{1}{4}\partial_\mu h_{ij}\partial^\mu h^{ij}$$

Tensor mode

$$h_{ij}(x) = h(x;\mathbf{k},\lambda)\,\epsilon_{ij}(\mathbf{k};\lambda)\,,$$

polarization tensor and $\lambda = +, \times$.

$$\epsilon_{ij}(\mathbf{k};\lambda)\epsilon^{ij}(\mathbf{k};\lambda') = 2\delta_{\lambda\lambda'}$$

$$I_{\mathrm{graviton}} = \frac{1}{16\pi G}\int d^4x\, a^2(\eta)\,\frac{1}{2}\left[(\partial_\mu h(x;\mathbf{k},+))^2 + (\partial_\mu h(x;\mathbf{k},\times))^2\right]$$

$$h(x;\mathbf{k},\lambda) = (2\pi)^{-\frac{3}{2}} h_\lambda(\eta;\mathbf{k})\, e^{i\mathbf{k}\cdot\mathbf{x}} + \mathrm{H.c.}$$

equation of motion for the wave amplitude

$$\ddot{h}_\lambda + 2\frac{\dot{a}}{a}\dot{h}_\lambda + k^2 h_\lambda = 0$$

Inflation ⟶ Radiation-dominated ⟶ matter-dominated

choose
Bunch-Davis vacuum
or positive-energy solution

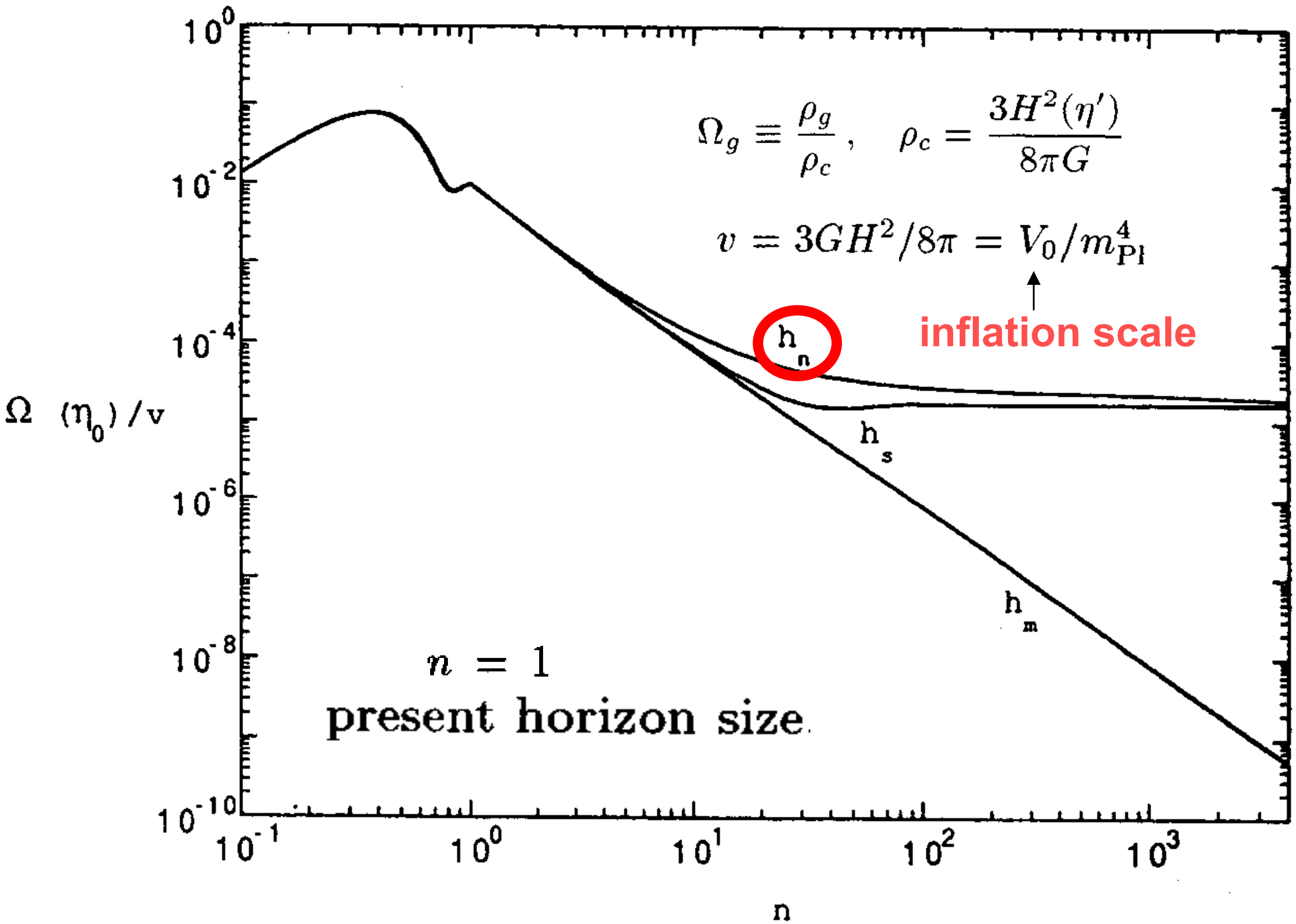
Almost
scale-invariant
power spectrum
$P_h(k)$

$$|h_\lambda(\eta;k)|^2 k^3 \simeq 8\pi G H^2 \left[\frac{3j_1(k\eta)}{k\eta}\right]^2$$

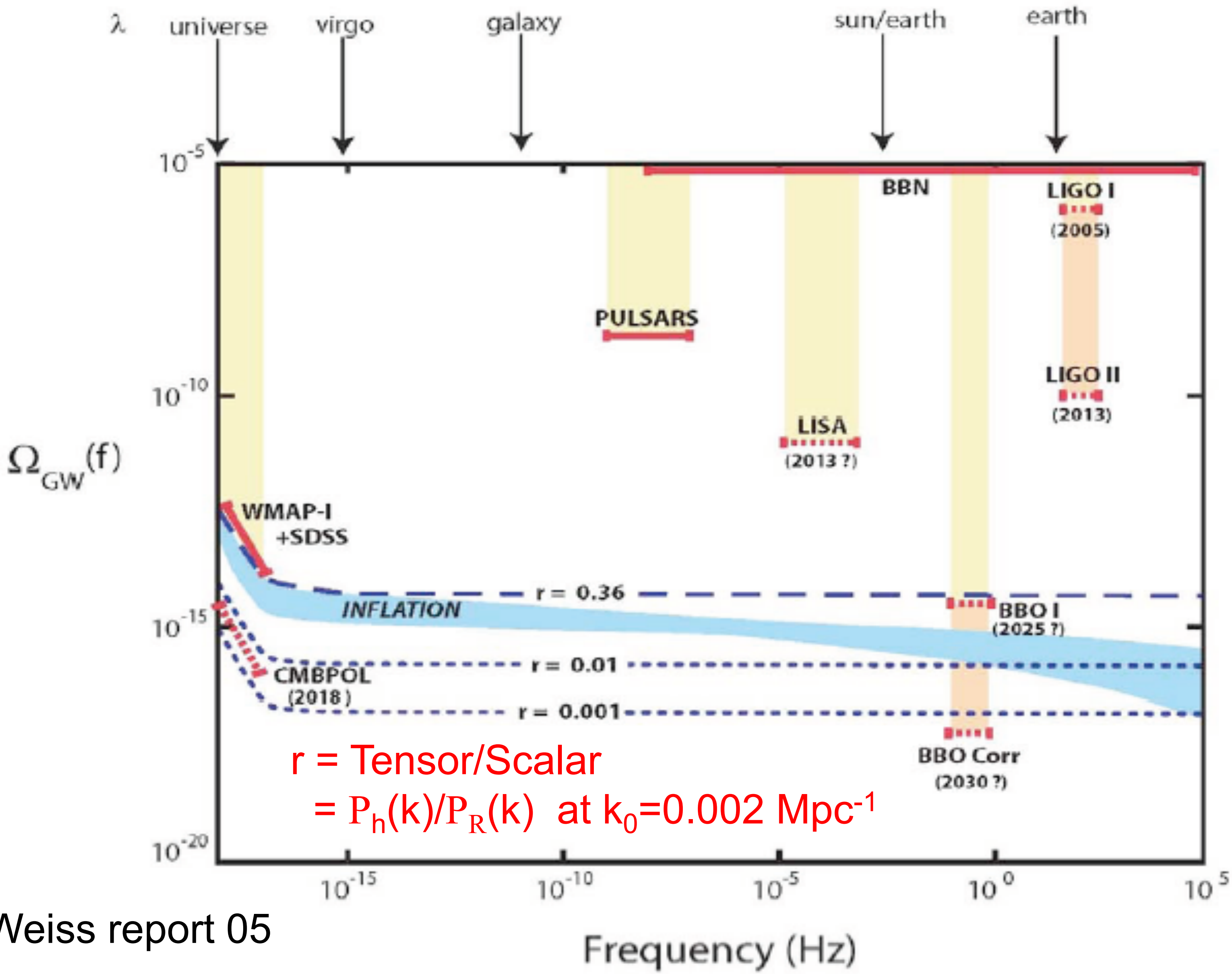
$$\frac{3j_1(k\eta)}{k\eta} \simeq 1 \quad \text{as} \quad k\eta \ll 1\,.$$

spectral energy density of gravitational waves

$$\rho_g \equiv \sum_{\lambda=+,\times} k' \frac{d\rho_\lambda}{dk'} = \sum_{\lambda=+,\times} \frac{1}{32\pi G a^2(\eta')} \left(\frac{k'}{2\pi}\right)^3 \left[k'^2 |h_\lambda|^2 + \left| \frac{dh_\lambda}{d\eta'} \right|^2 \right]$$



Primordial gravitational waves



r = Tensor/Scalar

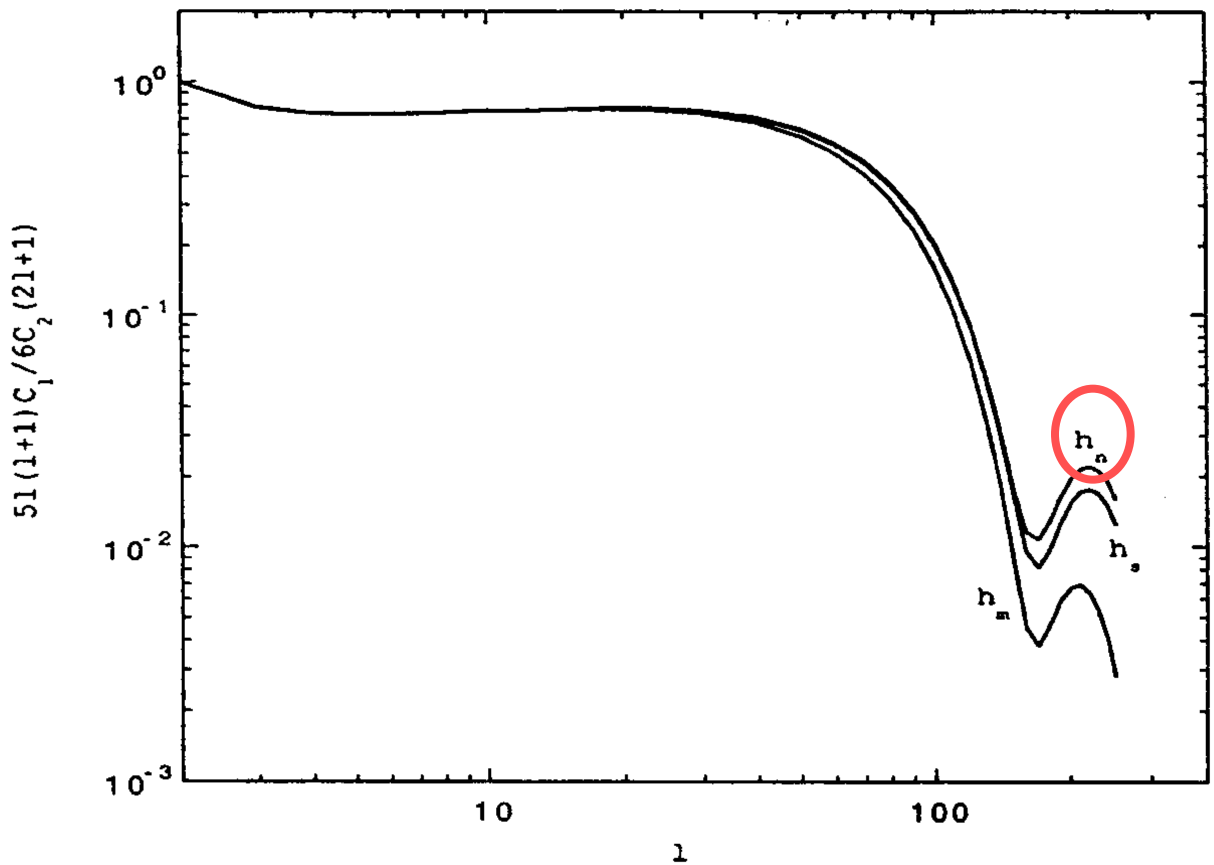
= $P_h(k)/P_R(k)$ at k_0=0.002 Mpc^{-1}

Weiss report 05

PGWs and CMB temperature anisotropy

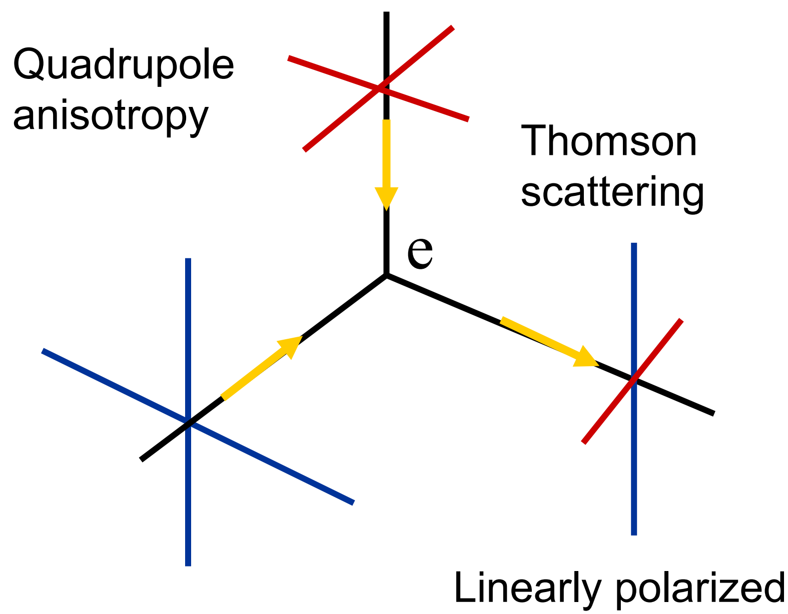
Sachs–Wolfe effect: $$\frac{\delta T}{T}(\mathbf{e}) = -\frac{1}{2}\int_e^r d\Lambda e^i e^j \frac{\partial}{\partial \eta'} h_{ij}(\eta', \mathbf{x})$$

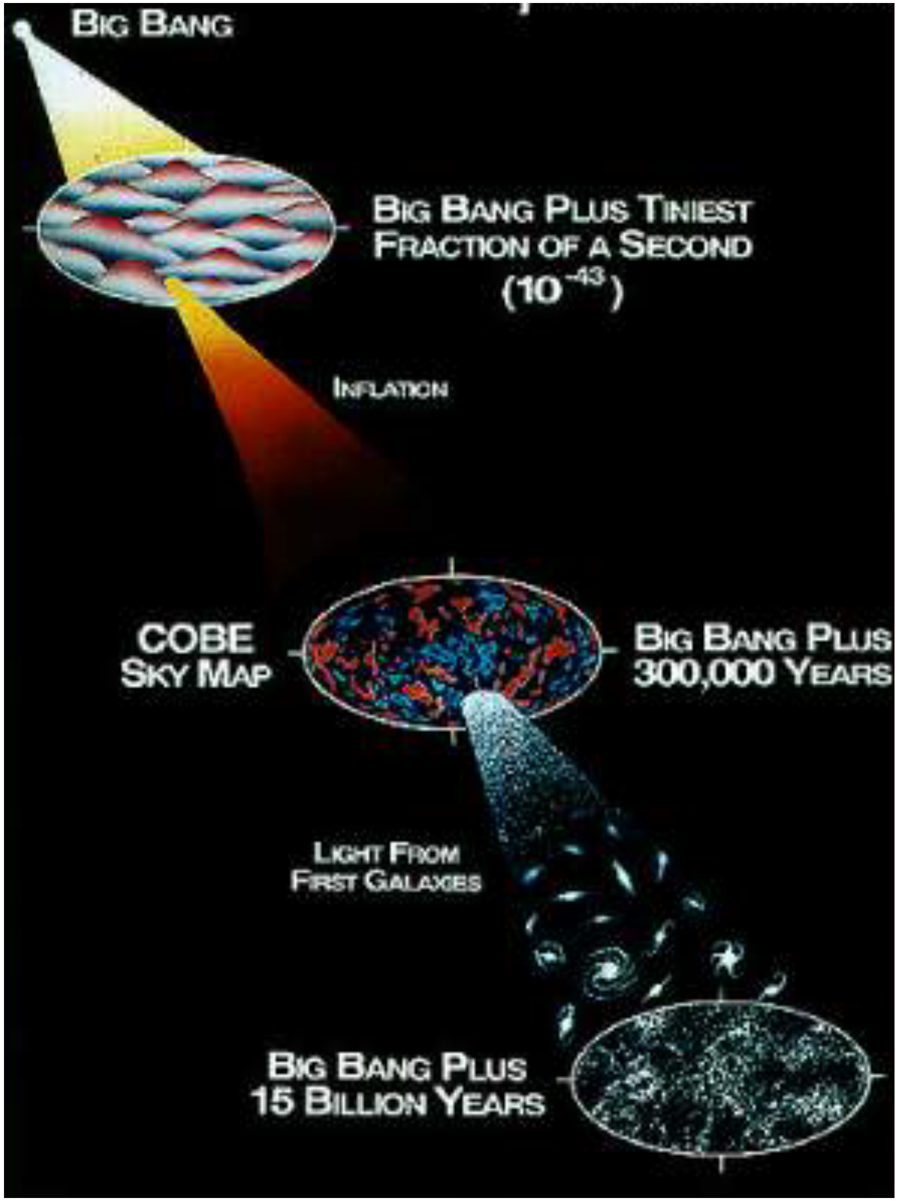
$$C_l = \frac{32\pi v}{3}(2l+1)(l+2)(l+1)l(l-1)\int_0^\infty \frac{dk'}{k'}\left[\int_{\eta'_{\rm dec}}^{\eta'_0} d\eta' \frac{dh(\eta')}{d\eta'}\frac{j_l[k'(\eta'_0-\eta')]}{k'^2(\eta'_0-\eta')^2}\right]^2$$



CMB Anisotropy and Polarization

- On large angular scales, matter imhomogeneities or primordial gravitational waves generate gravitational redshifts
- On small angular scales, acoustic oscillations in plasma on last scattering surface generate Doppler shifts
- Thomson scatterings with electrons generate polarization





Collisional Boltzmann Equation

Ng&Ng 96

$$\left(\frac{\partial}{\partial\eta}+\boldsymbol{e}\cdot\frac{\partial}{\partial\boldsymbol{x}}\right)\boldsymbol{n}=-\frac{1}{2}\frac{\partial\boldsymbol{n}}{\partial\ln\nu}\frac{\partial h_{ij}}{\partial\eta}e^ie^j-\sigma_T N_e a$$
$$\times\left[\boldsymbol{n}-\frac{1}{4\pi}\int_{-1}^{1}\int_0^{2\pi}P(\mu,\phi,\mu',\phi')\boldsymbol{n}\,d\mu'\,d\phi'\right], \quad (1)$$

where σ_T is the Thomson scattering cross section, N_e is the number of free electrons per unit volume, ($\mu = \cos\theta$, ϕ) are the polar angles of the propagation direction $\boldsymbol{e}$ of the photon with a comoving frequency ν, and P is the phase matrix for Thomson scattering.

scalar modes/density perturbations

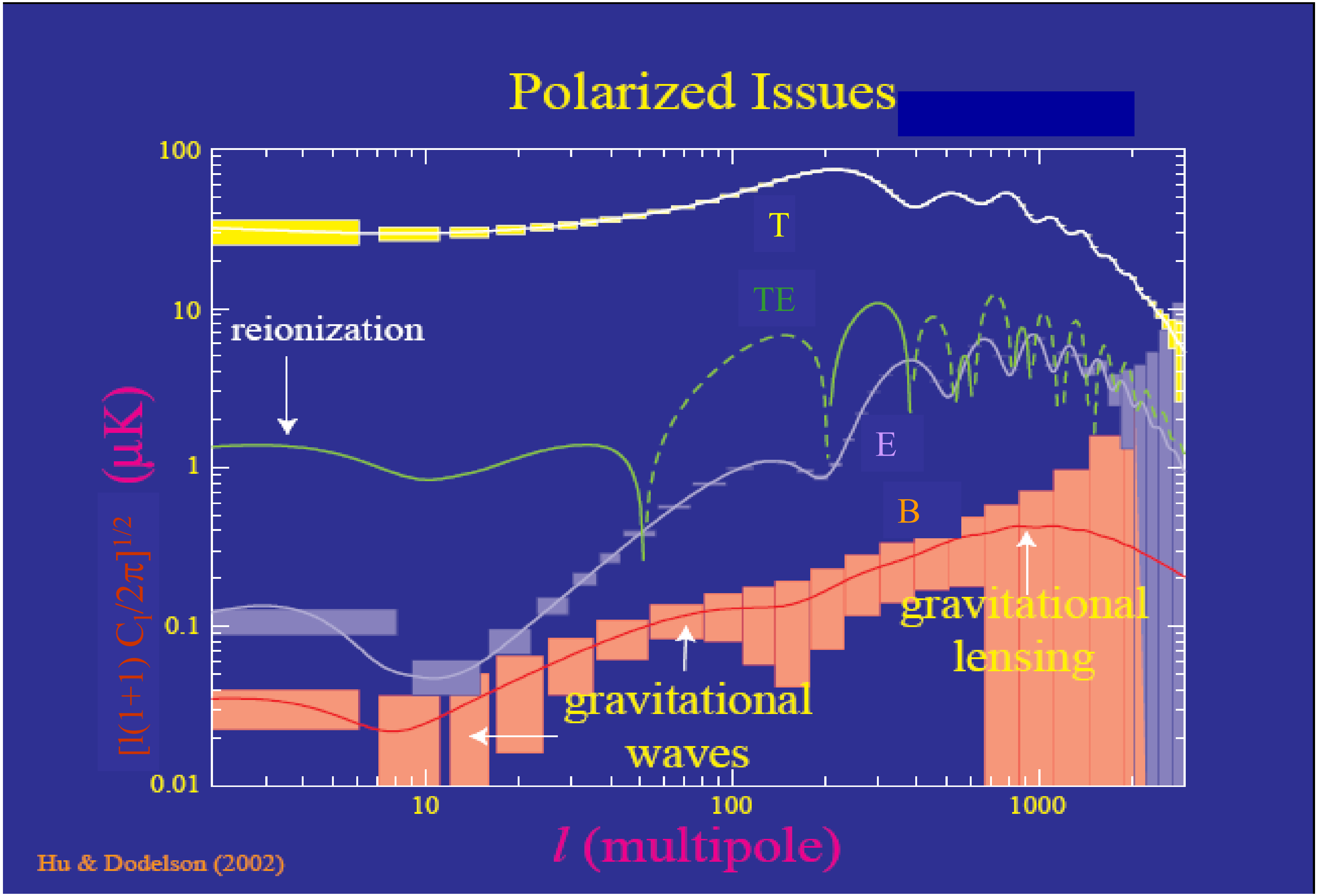
The solution $\boldsymbol{n}$ for the equation of transfer assumes the form $\boldsymbol{n} = \boldsymbol{n}_0 + (n_0\,\delta\boldsymbol{n}/2)$, where $\boldsymbol{n}_0$ and $\delta\boldsymbol{n}$ are the unperturbed solution and perturbation, respectively. We expand $\delta\boldsymbol{n} = \int d\boldsymbol{k}\,\boldsymbol{n}'e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$, where $\boldsymbol{n}' = \alpha\boldsymbol{a} + \beta\boldsymbol{b}$. For the scalar-mode solution, the Stokes components n_u and n_v both decouple from n_l and n_r, and it suffices to consider only the first two components of $\boldsymbol{n}$ with $\boldsymbol{a} = (1, 1)$ and $\boldsymbol{b} = (1, -1)$. Substituting the solution $\boldsymbol{n}$ and the Fourier expansion for h_{ij} into equation (1), and expanding α and β in terms of Legendre polynomials,

$$\alpha(\mu)=\sum_l(2l+1)\alpha_l P_l(\mu)\,,$$
$$\beta(\mu)=\sum_l(2l+1)\beta_l P_l(\mu)\,, \quad (3)$$

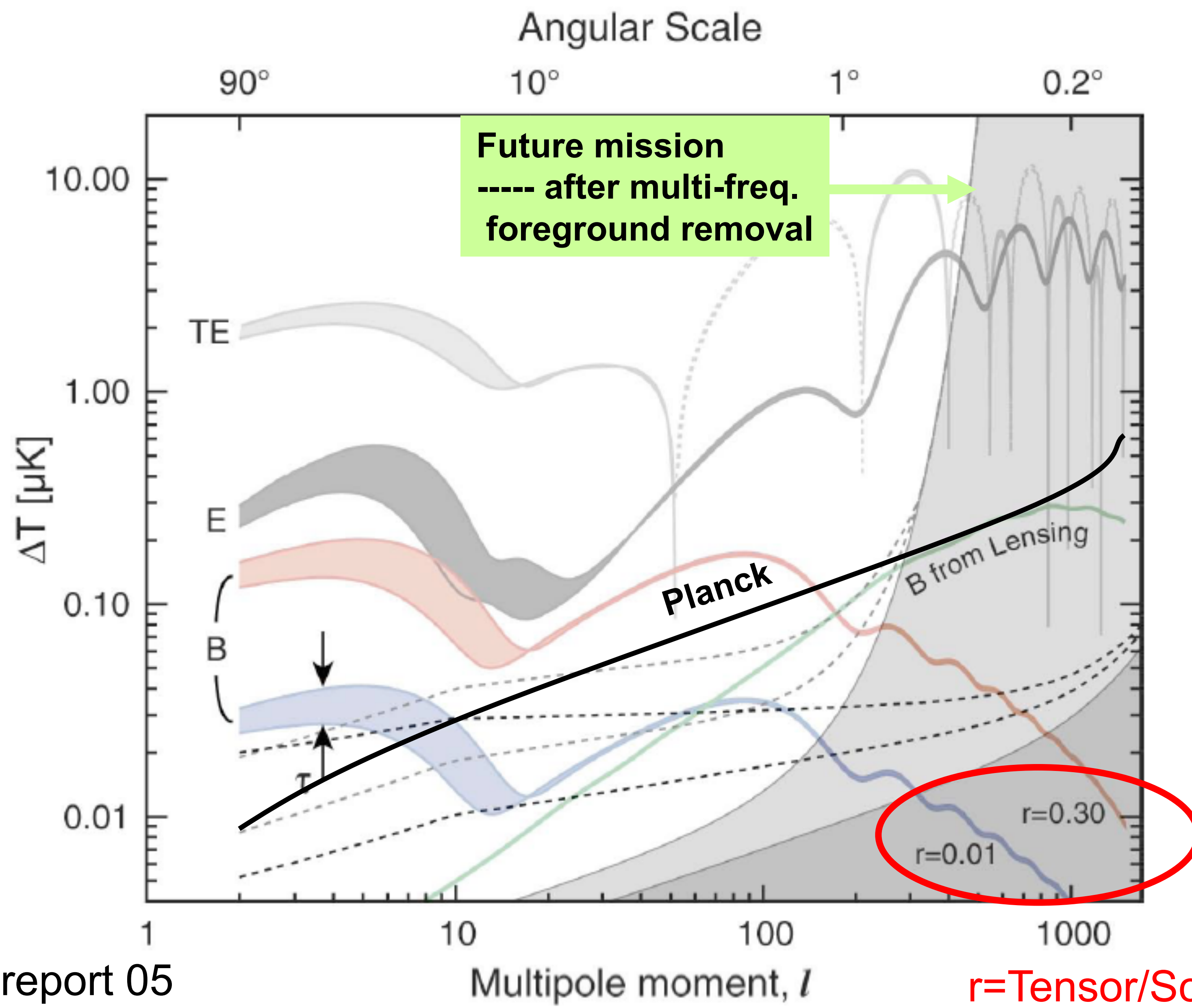
tensor modes/GWs

In the case of the tensor mode, only n_v decouples, and we thus choose the basis (Polnarev 1985) $\boldsymbol{a} = [\frac{1}{2}(1-\mu^2)\cos 2\phi]$ (1, 1, 0) and $\boldsymbol{b} = \frac{1}{2}[(1+\mu^2)\cos 2\phi, -(1+\mu^2)\cos 2\phi, 4\mu \sin 2\phi]$ for the + mode solution. The × mode solution is given by the same expressions with cos 2ϕ and sin 2ϕ interchanged, and an additional minus sign in the last component of the basis vector b. Defining $\xi = \alpha + \beta$, we obtain a system of coupled

Theoretical Predictions for CMB Power Spectra



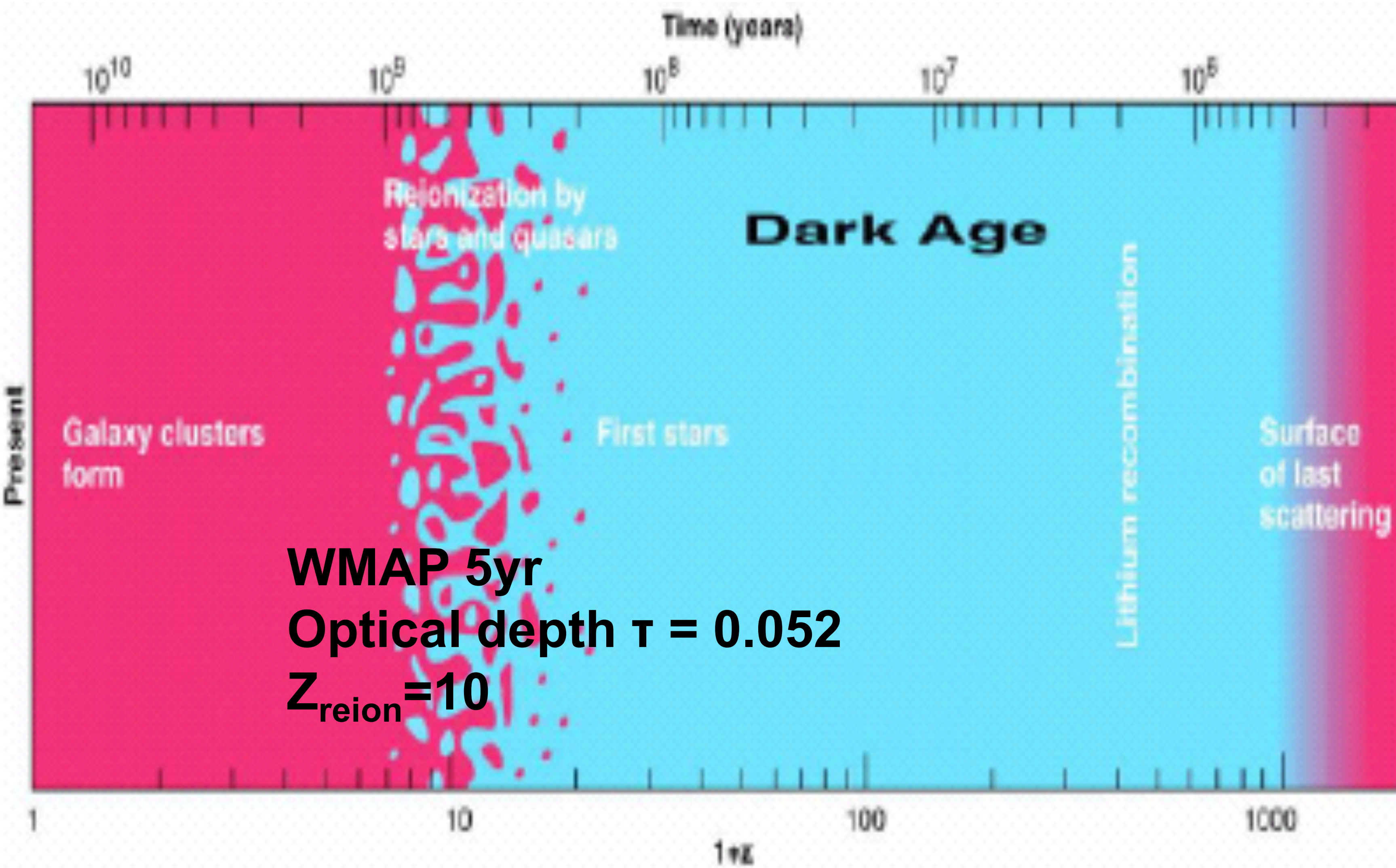
Gravity-wave induced B-mode



Weiss report 05

r=Tensor/Scalar

Large-scale CMB Polarization and Reionization of the Universe

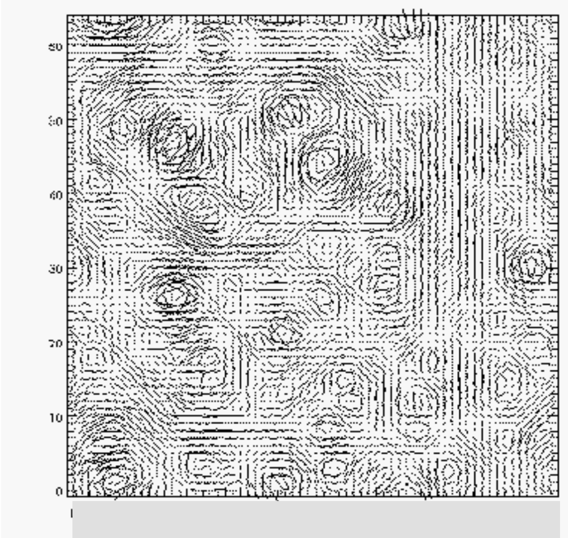


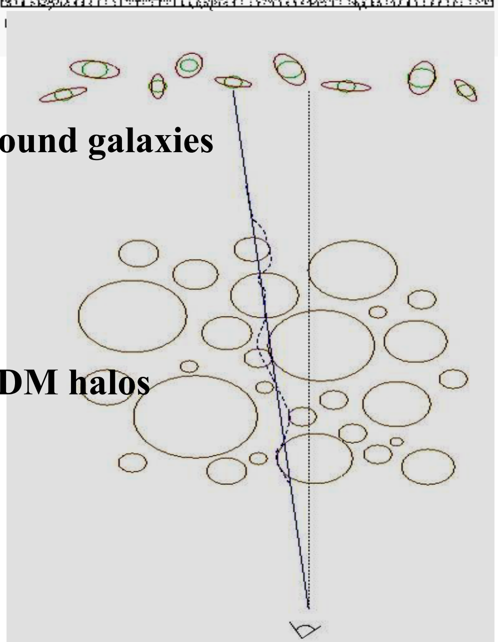
WMAP 5yr
Optical depth τ = 0.052
Z_{reion}=10

Weak Lensing by Large-Scale Structure

Jain et al. 1997
1x1 deg

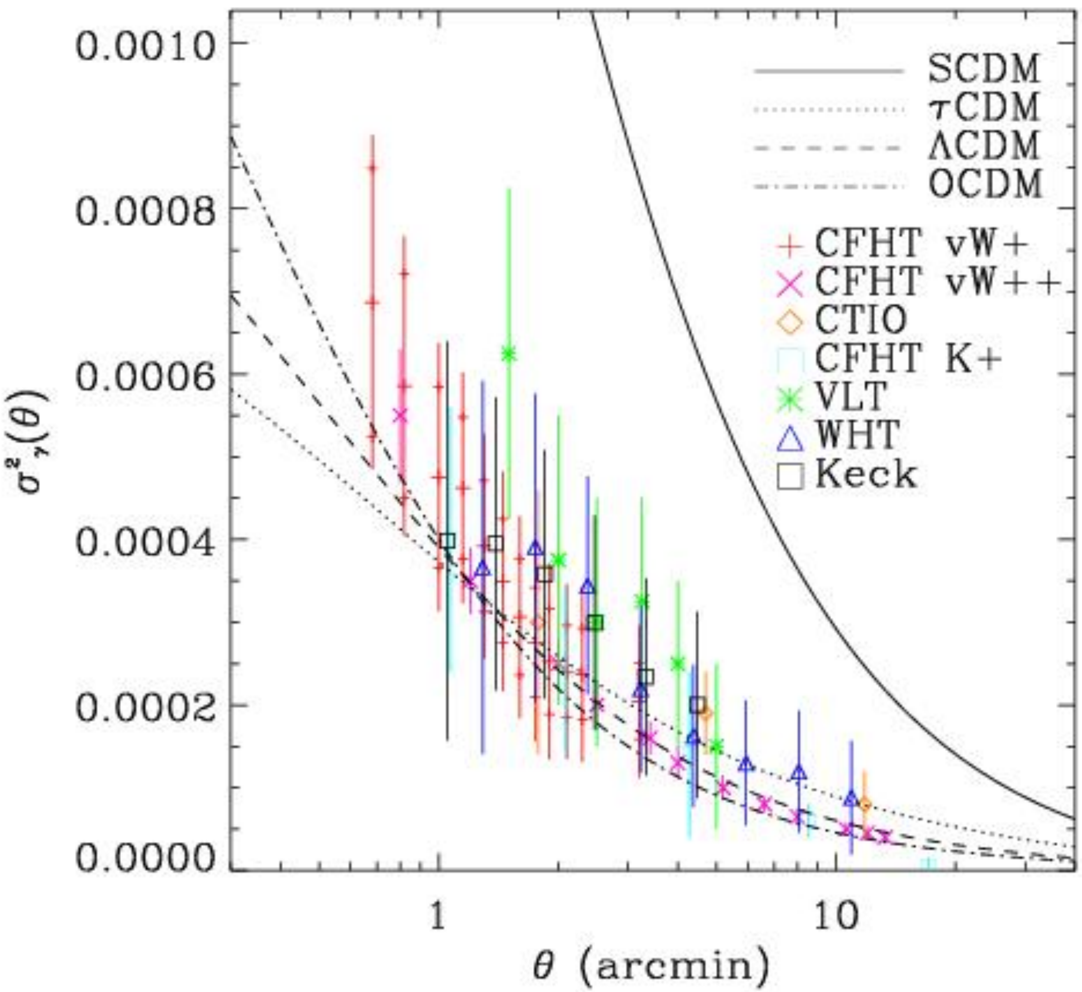
Cosmic Shear γ





Shear Variance in circular cells with size θ

$$\sigma^2_\gamma(\theta) = \langle\gamma^2\rangle$$



Ellis et al. 02

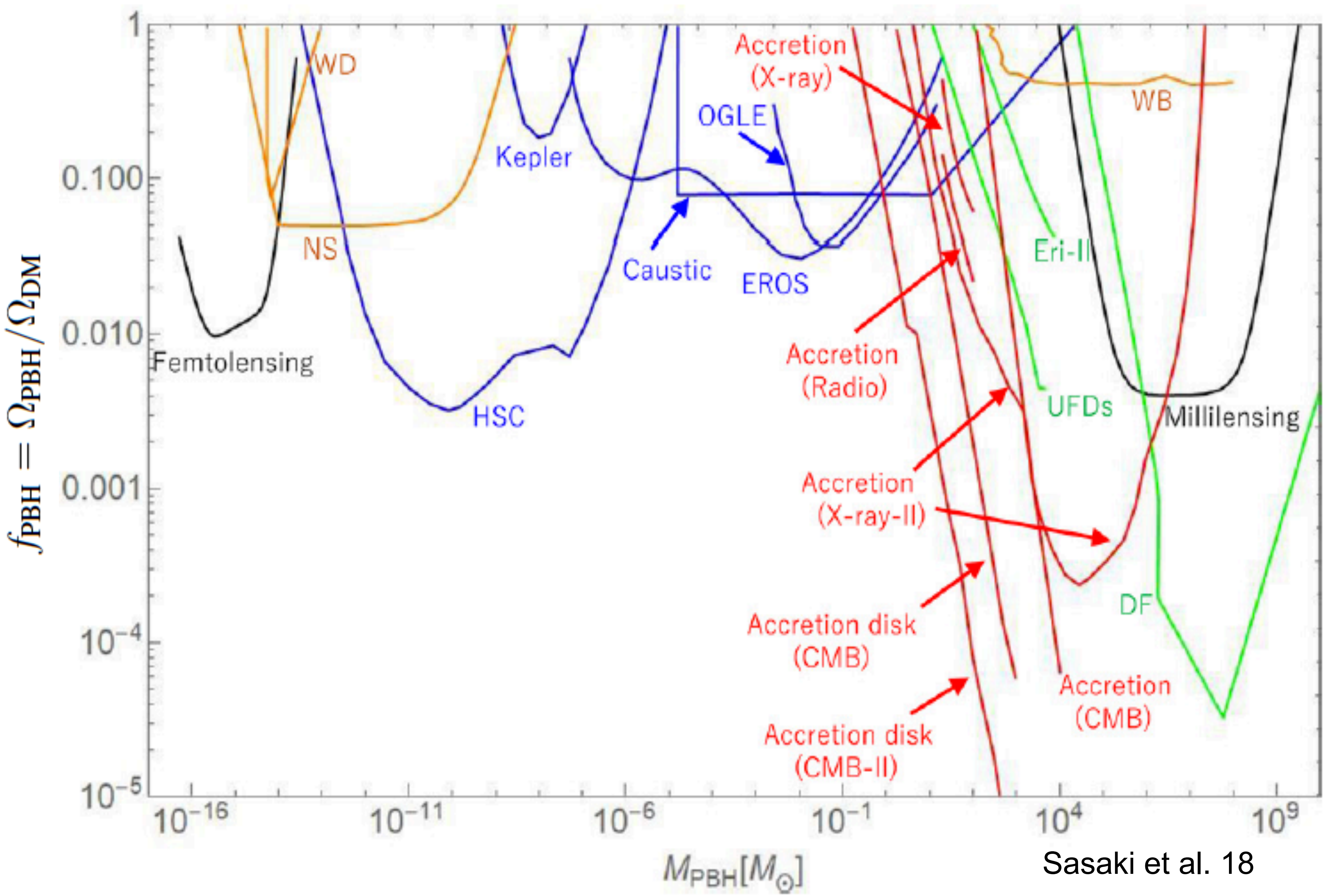
Primordial Black Holes

- Formed at high-density contrasts (*$\delta\rho/\rho \sim 0.5$*) over a wide range of scales or masses in the radiation-dominated Universe
- There have been stringent astrophysical and cosmological constraints on M_{PBH}
- $10M_\odot$ PBHs could be the binary BHs observed by aLIGO gravity-wave detectors

  Bird et al. 16., Clesse et al. 16, Sasaki et al. 16
- PBHs behave like cold dark matter

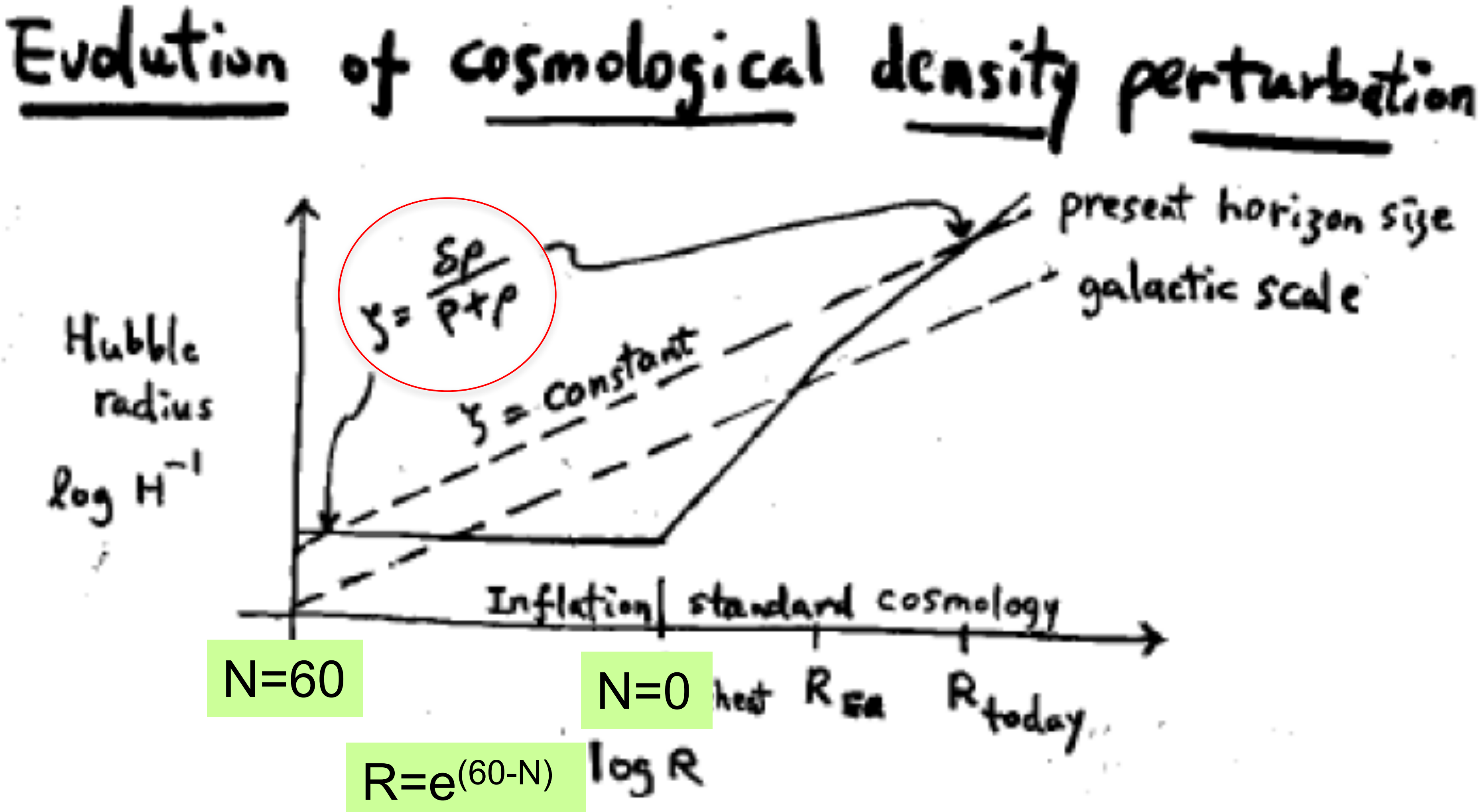
  García-Bellido, Linde, Wands 96
- They, although being of baryonic origin, do not participate in big-bang nucleosynthesis

Astrophysical and Cosmological Constraints on PBHs



Sasaki et al. 18

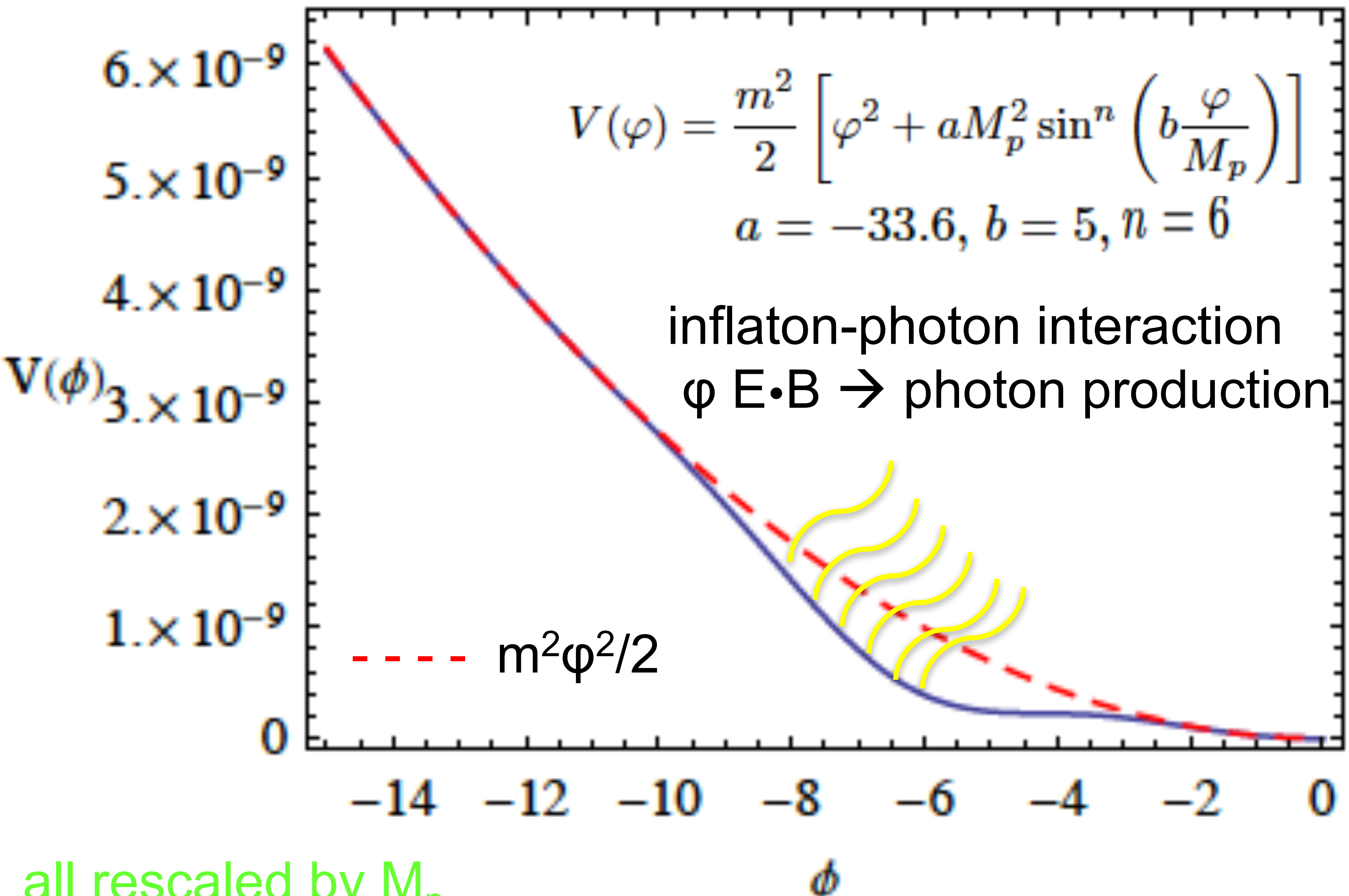
Production of PBHs in inflation



$$M_{\rm BH} = \frac{4\pi M_p^2}{H} e^{2N} = 2.74 \times 10^{-38} e^{2N} \left(\frac{M_p}{H}\right) M_\odot$$

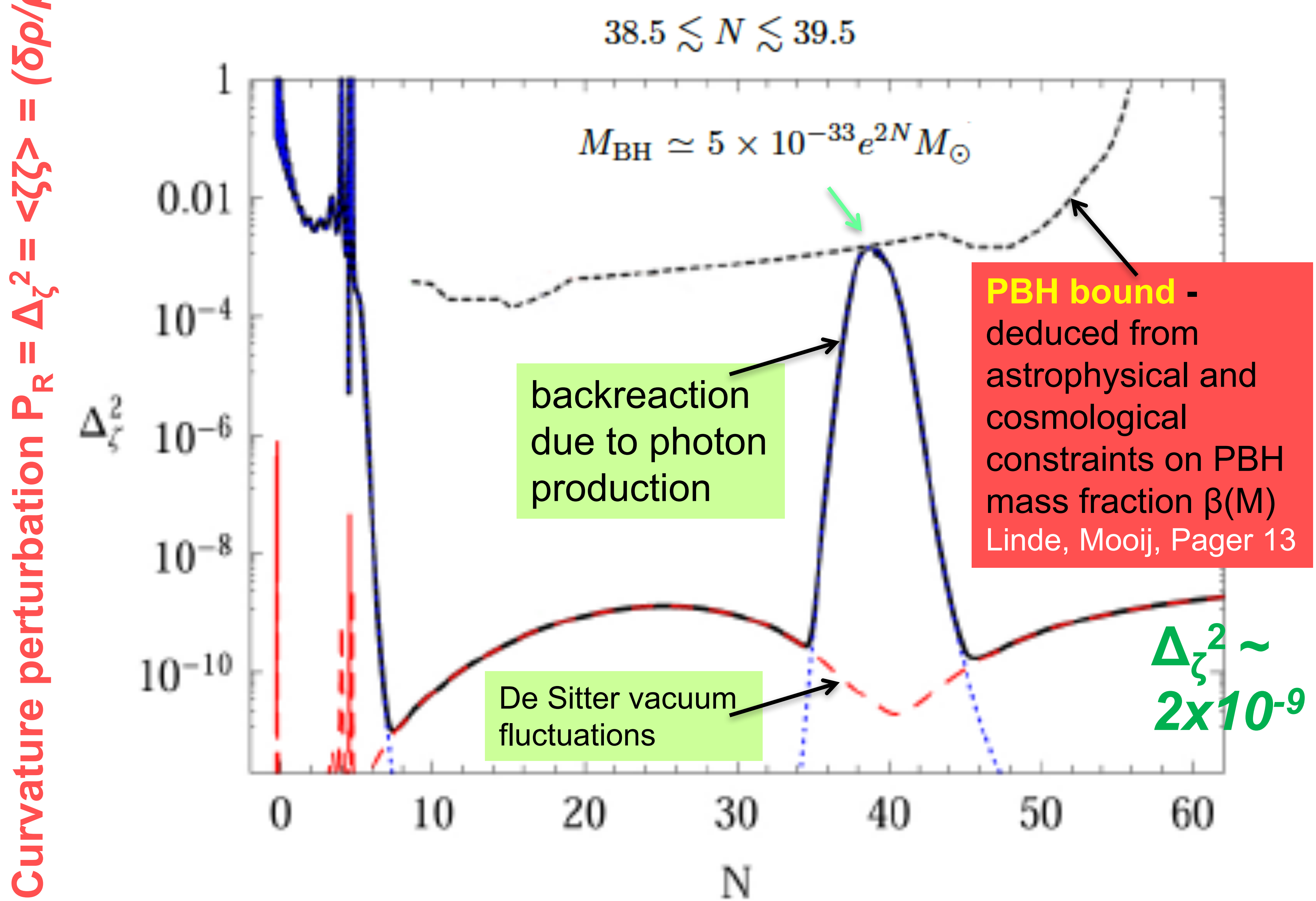
N is e-foldings before the end of inflation

e.g. Trapped axion inflation with a steep potential Cheng, Lee, Ng 16



all rescaled by M_p

Production of 10-100$M_\odot$ PBHs



PBH seeds or Large Curvature Perturbation Associated Gravitational Waves

$$\left[\frac{\partial^2}{\partial\eta^2} + \frac{2}{a}\frac{da}{d\eta}\frac{\partial}{\partial\eta} - \vec{\nabla}^2\right] h_{ij} = 0$$

Free gravitational wave equation

De Sitter vacuum fluctuations during inflation lead to almost scale-invariant primordial gravitational waves

***$P_h = 8\pi G H^2$* and *$\Delta_\zeta^2 = \langle\zeta\zeta\rangle = (\delta\rho/\rho)^2 \sim 2\times10^{-9}$* on CMB scales**

$$\left[\frac{\partial^2}{\partial\eta^2} + \frac{2}{a}\frac{da}{d\eta}\frac{\partial}{\partial\eta} - \vec{\nabla}^2\right] h_{ij} = 16\pi G T_{ij}$$

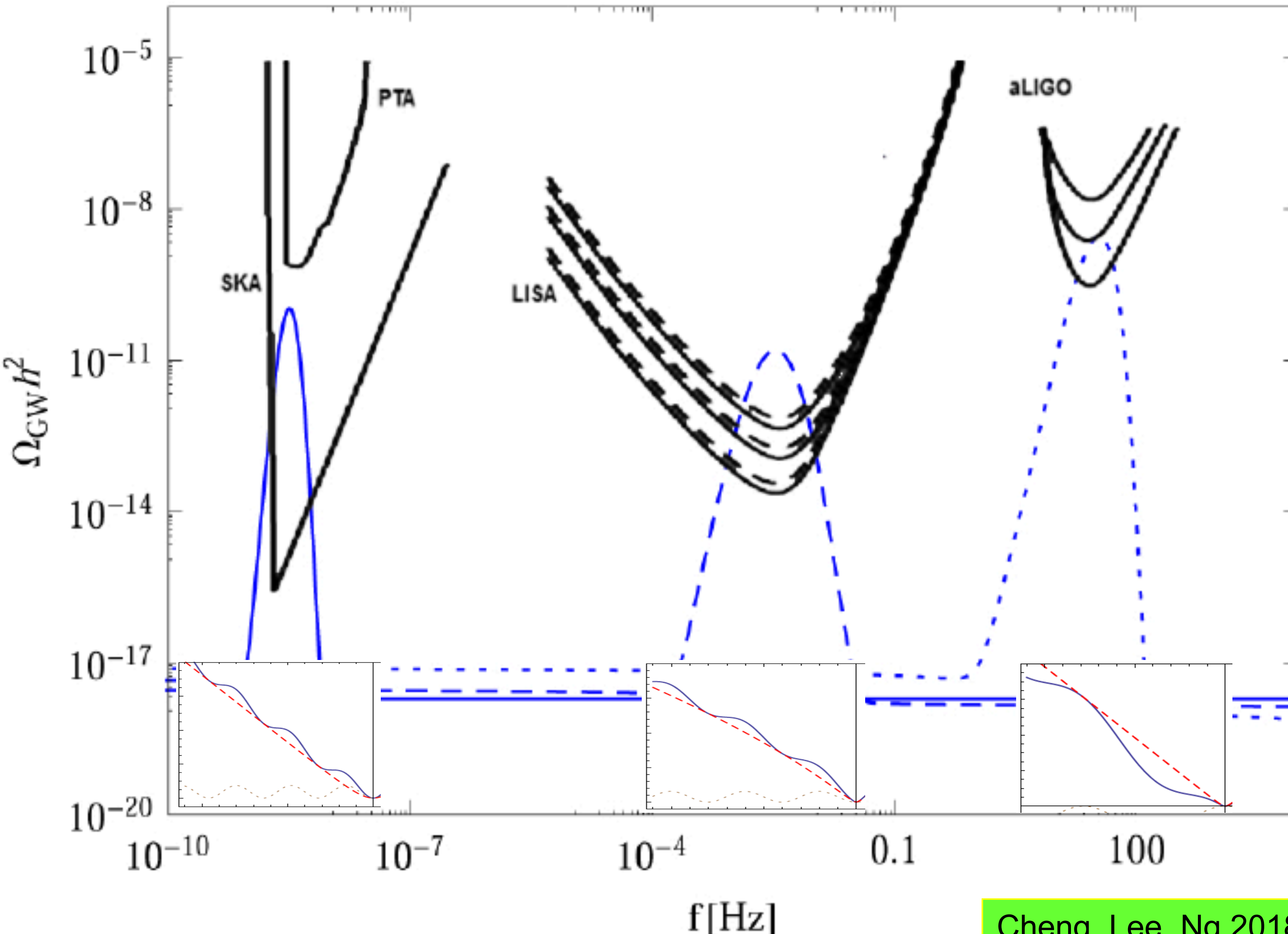
Stress due to transverse traceless part of 2nd order curvature perturbation $T_{ij}(\zeta^2)$

$\Delta_\zeta^2 = \langle\zeta\zeta\rangle = (\delta\rho/\rho)^2 \sim 10^{-3}$

When large curvature perturbation re-enter the horizon during the radiation-dominated era and collapse to form PBHs, they induce gravitational waves at short wavelengths

Ananda, Clarkson,Wands 2007, Baumann, Steinhardt,Takahashi,Ichiki 2007

GWs associated with PBHs in modulated axion inflation



Cheng, Lee, Ng 2018

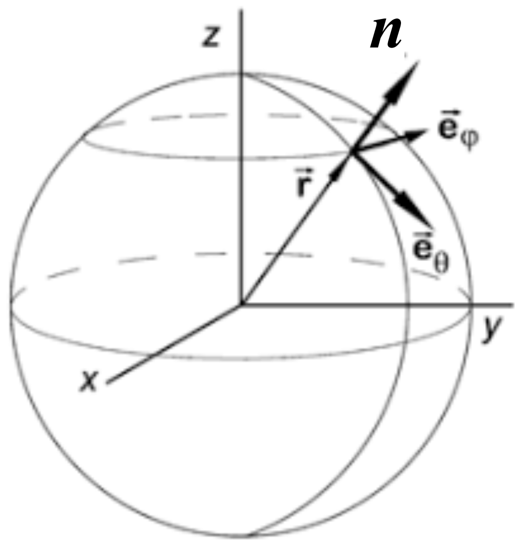
Gravitational Wave Background(GWB) Anisotropies

$$h_{ij}(t,\boldsymbol{x}) = \sum_{P=+,\times} \int_{-\infty}^{\infty} df \int_{S^2} d\boldsymbol{n} h_P(f,\boldsymbol{n}) e^{2\pi i f(-t+\boldsymbol{n}\cdot\boldsymbol{x})} e_{ij}^P(\boldsymbol{n}). \quad (1)$$

Here, the bases for transverse-traceless tensor $\boldsymbol{e}^P$ ($P = +, \times$) are given as

$$\boldsymbol{e}^+ = \hat{\boldsymbol{e}}_\theta \otimes \hat{\boldsymbol{e}}_\theta - \hat{\boldsymbol{e}}_\phi \otimes \hat{\boldsymbol{e}}_\phi, \qquad \boldsymbol{e}^\times = \hat{\boldsymbol{e}}_\theta \otimes \hat{\boldsymbol{e}}_\phi + \hat{\boldsymbol{e}}_\phi \otimes \hat{\boldsymbol{e}}_\theta,$$

$$\begin{pmatrix} \langle h_+(f,\boldsymbol{n}) h_+^*(f',\boldsymbol{n}')\rangle & \langle h_+(f,\boldsymbol{n}) h_\times^*(f',\boldsymbol{n}')\rangle \\ \langle h_\times(f,\boldsymbol{n}) h_+^*(f',\boldsymbol{n}')\rangle & \langle h_\times(f,\boldsymbol{n}) h_\times^*(f',\boldsymbol{n}')\rangle \end{pmatrix} = \frac{1}{2}\delta_D^2(\boldsymbol{n}-\boldsymbol{n}')\delta_D(f-f') \times \begin{pmatrix} I(f,\boldsymbol{n}) + Q(f,\boldsymbol{n}) & U(f,\boldsymbol{n}) - iV(f,\boldsymbol{n}) \\ U(f,\boldsymbol{n}) + iV(f,\boldsymbol{n}) & I(f,\boldsymbol{n}) - Q(f,\boldsymbol{n}) \end{pmatrix},$$



Q= <++> - <xx>
U is the same as in
a frame rotated by **π**/8

$$\boldsymbol{e}^R = \frac{(\boldsymbol{e}^+ + i\boldsymbol{e}^\times)}{\sqrt{2}}, \qquad \boldsymbol{e}^L = \frac{(\boldsymbol{e}^+ - i\boldsymbol{e}^\times)}{\sqrt{2}}$$

$$h_R = \frac{(h_+ - ih_\times)}{\sqrt{2}}, \qquad h_L = \frac{(h_+ + ih_\times)}{\sqrt{2}}$$

$$\begin{pmatrix} \langle h_R(f,\boldsymbol{n}) h_R(f',\boldsymbol{n}')^*\rangle & \langle h_L(f,\boldsymbol{n}) h_R(f',\boldsymbol{n}')^*\rangle \\ \langle h_R(f,\boldsymbol{n}) h_L(f',\boldsymbol{n}')^*\rangle & \langle h_L(f,\boldsymbol{n}) h_L(f',\boldsymbol{n}')^*\rangle \end{pmatrix} = \frac{1}{2}\delta_D(\boldsymbol{n}-\boldsymbol{n}')^2\delta_D(f-f') \times \begin{pmatrix} I(f,\boldsymbol{n}) + V(f,\boldsymbol{n}) & Q(f,\boldsymbol{n}) - iU(f,\boldsymbol{n}) \\ Q(f,\boldsymbol{n}) + iU(f,\boldsymbol{n}) & I(f,\boldsymbol{n}) - V(f,\boldsymbol{n}) \end{pmatrix}.$$

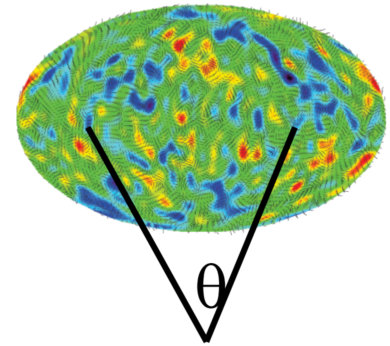
GWB Anisotropy and Polarization Angular Power Spectra

Decompose the GWB sky into a sum of spherical harmonics:

$T(\theta,\varphi) = \Sigma_{lm}\ a_{lm}\, Y_{lm}\,(\theta,\varphi)$, $V(\theta,\varphi) = \Sigma_{lm}\ b_{lm}\, Y_{lm}\,(\theta,\varphi)$

$(Q - iU)\,(\theta,\varphi) = \Sigma_{lm}\ a_{4,lm}\ {}_{4}Y_{lm}\,(\theta,\varphi)$

$(Q + iU)\,(\theta,\varphi) = \Sigma_{lm}\ a_{-4,lm}\ {}_{-4}Y_{lm}\,(\theta,\varphi)$



l = 180 degrees/ θ

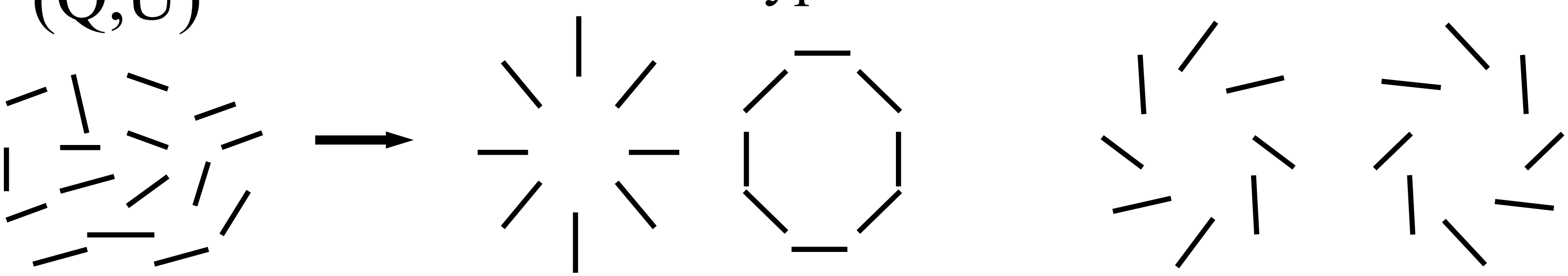
$C^{T}_{l} = \Sigma_{m}\,(a^{*}_{lm}\, a_{lm})$ anisotropy power spectrum

$C^{V}_{l} = \Sigma_{m}\,(b^{*}_{lm}\, b_{lm})$ circular polarization power spectrum

$C^{E}_{l} = \Sigma_{m}\,(a^{*}_{4,lm}\, a_{4,lm} + a^{*}_{4,lm}\, a_{-4,lm})$ E-polarization power spectrum

$C^{B}_{l} = \Sigma_{m}\,(a^{*}_{4,lm}\, a_{4,lm} - a^{*}_{4,lm}\, a_{-4,lm})$ B-polarization power spectrum

(Q,U) electric-type magnetic-type



Collisionless Boltzman Equation for Gravitons

$$ds^2 = a^2(\eta)\left[-e^{2\Phi}d\eta^2 + (e^{-2\Psi}\delta_{ij} + h_{ij})dx^i dx^j\right]$$

$$\frac{\partial f}{\partial \eta} + \frac{\partial f}{\partial x^i}\frac{dx^i}{d\eta} + \frac{\partial f}{\partial q}\frac{dq}{d\eta} + \frac{\partial f}{\partial n^i}\frac{dn^i}{d\eta} = 0$$

Graviton phase space distribution function

$$f = f(\eta, x^i, q, \hat{n}^i)$$

$$\frac{\partial f}{\partial \eta} + n^i \frac{\partial f}{\partial x^i} + \left[\frac{\partial \Psi}{\partial \eta} - n^i \frac{\partial \Phi}{\partial x^i} + \frac{1}{2} n^i n^j \frac{\partial h_{ij}}{\partial \eta}\right] q \frac{\partial f}{\partial q} = 0$$

$$\delta f \equiv -q \frac{\partial \bar{f}}{\partial q} \Gamma(\eta, \vec{x}, q, \hat{n})$$

Newtonian potential k-mode

$$\Psi = \Phi \equiv T_\Phi(\eta, k)\,\hat{\zeta}(\vec{k})$$

Initial scalar power spectrum

Transfer function

GW background k-mode

$$h_{ij} \equiv \sum_{\lambda=\pm 2} e_{ij,\lambda}(\hat{k}) h(\eta, k) \hat{\xi}_\lambda(k^i)$$

Initial tensor power spectrum

Transfer function

$$\Gamma(\hat{n}) = \sum_\ell \sum_{m=-\ell}^{\ell} \Gamma_{\ell m} Y_{\ell m}(\hat{n}) \qquad e^{i\vec{k}\cdot\vec{x}} = 4\pi \sum_{lm} i^l j_l(kx) Y_{lm}^*(\hat{k}) Y_{lm}(\hat{x})$$

Scalar contribution

$$\frac{\Gamma_{\ell m,S}}{4\pi\,(-i)^\ell} = \int \frac{d^3k}{(2\pi)^3}\,\zeta(\vec{k}) Y_{\ell m}^*(\hat{k})\,\mathcal{T}_\ell^{(0)}(k,\,\eta_0,\,\eta_{\rm in})$$

$\vec{x}_0 = 0$

η_0 today

$\eta_{\rm in}$ initial

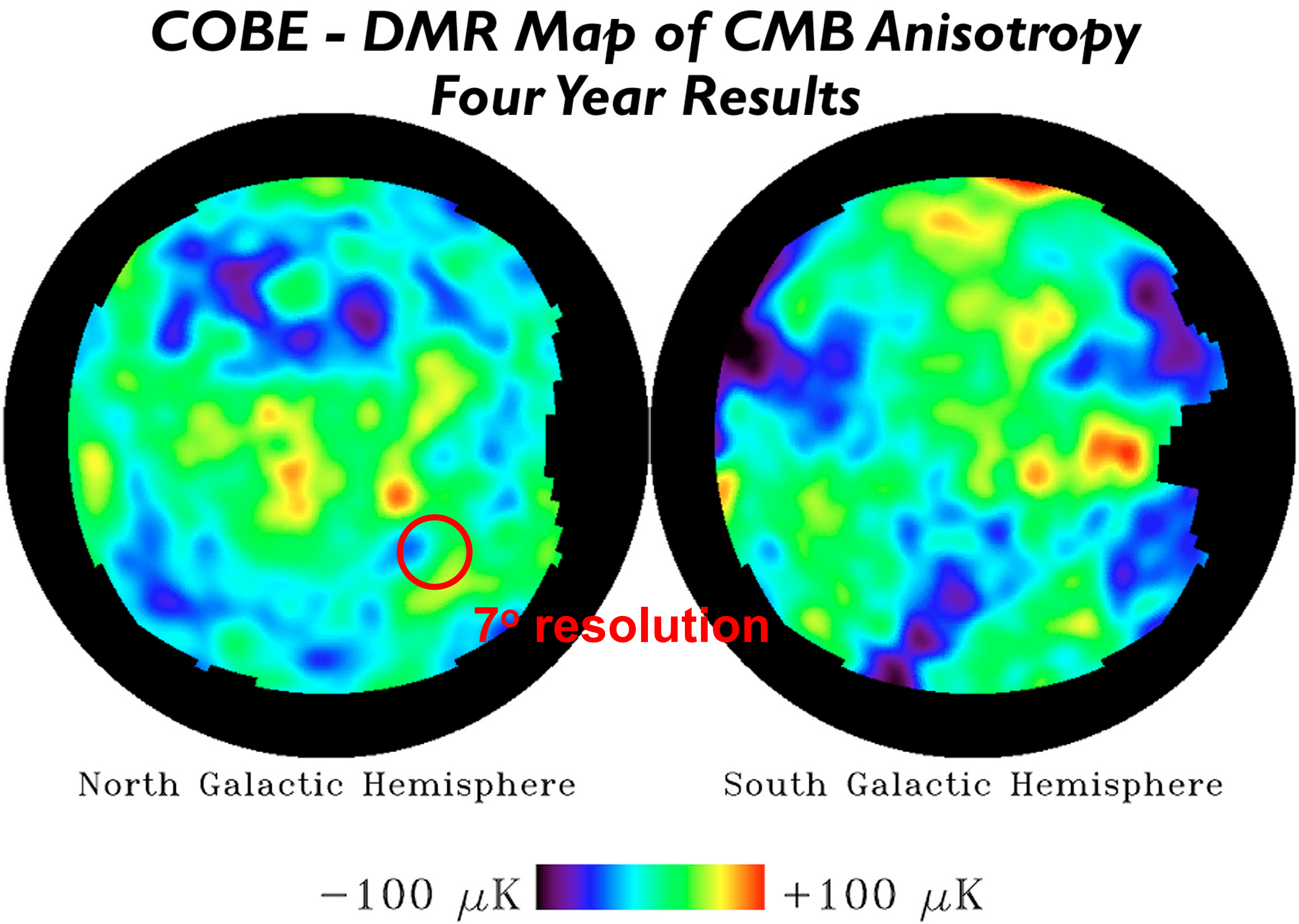
where the scalar transfer function $\mathcal{T}_\ell^{(0)}$ is the sum of a term analogous to the SW effect for CMB photons, $T_\Phi(\eta_{\rm in}, k)\, j_\ell[k(\eta_0 - \eta_{\rm in})]$, plus the analog of the ISW term, $\int_{\eta_{\rm in}}^{\eta_0} d\eta'\, [T'_\Psi(\eta, k) + T'_\Phi(\eta, k)]\, j_\ell[k(\eta - \eta_{\rm in})]$. Finally,

Tensor contribution

$$\mathcal{T}_\ell^{(\pm 2)} = \frac{1}{4}\sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \int_{\eta_{\rm in}}^{\eta_0} d\eta\, h'(\eta,\,k)\, \frac{j_\ell\left[k\left(\eta_0 - \eta\right)\right]}{k^2\left(\eta_0 - \eta\right)^2}$$

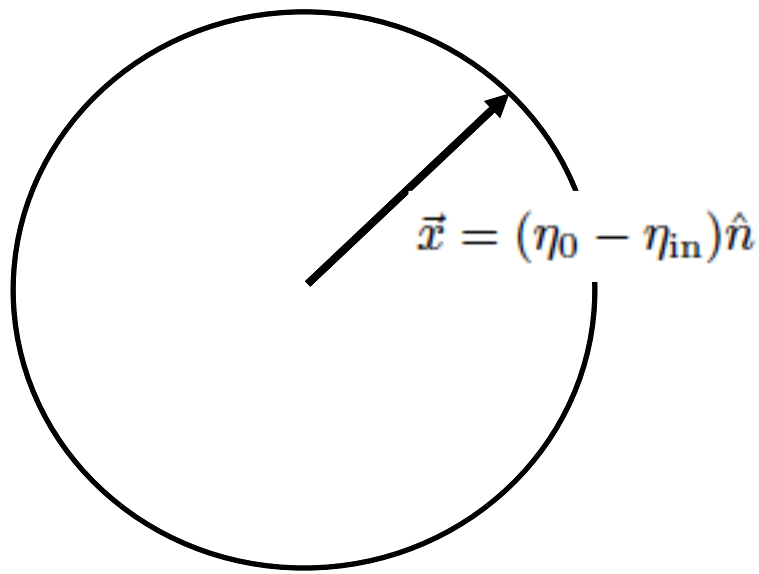
Sachs-Wolfe or Integrated Sachs-Wolfe effects – gravitational redshift of gravitons

GWB Anisotropy Map due to SW and ISW Effects



$$\Gamma(\eta, \vec{x}, q, \hat{n}) = \int \frac{d^3k}{(2\pi)^3} \Gamma(\eta, \vec{k}, q, \hat{n}) e^{i\vec{k}\cdot\vec{x}}$$

k-mode



Initial value $\quad \dfrac{\Gamma_{\ell m,I}(q)}{4\pi(-i)^\ell} = \int \dfrac{d^3k}{(2\pi)^3}\, \Gamma\left(\eta_{\rm in},\, \vec{k},\, q\right) \times Y^*_{\ell m}(\hat{k})\, j_\ell\left[k\left(\eta_0 - \eta_{\rm in}\right)\right]$

Summary

- Measure H_0 to solve the Hubble-constant 4-sigma tension
- CMB B-mode polarization observe low-frequency PGWs, providing a window for seeing inflation scale (grand unified theories?)
- Laser Interferometry and Pulsar Timing Arrays probe high-frequency PGWs - to test scale invariance, primordial black holes,…