Tests of gravity with gravitational waves

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APCTP School: Gravitational-Wave Cosmology

1. Test of general relativity
2. Cosmological observations and modification of gravity
3. GW observation and testing gravity
4. GW generation
    4.1. Inspiral-Merger-Ringdown (IMR) consistency test
    4.2. Post-Newtonian (PN) consistency test
5. GW propagation
    5.1. GW speed and modified dispersion relation
    5.2. Generalized framework of GW propagation
6. GW polarization
7. Parity violating gravity

Generalized framework of GW propagation

Amplitude damping in MG

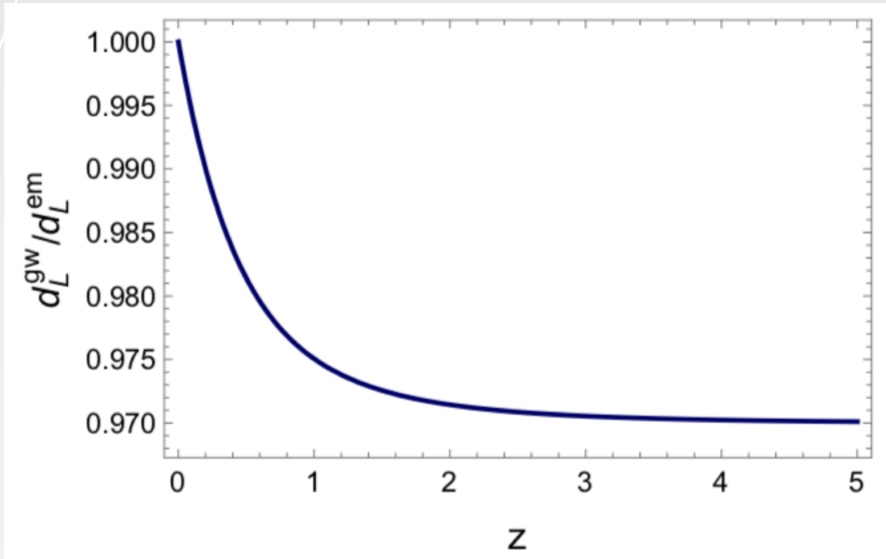
F(R) gravity   Hwang &Noh 1996

$$h''_{ij} + \left(2\mathcal{H} + \frac{\dot{F}}{F}\right) h'_{ij} + c^2 k^2 h_{ij} = 0 \qquad F \equiv \frac{df(R)}{dR}$$

nonlocal RR gravity   Belgacem et al. 2018

$$\mathcal{L} \supset m^2 R \Box^{-2} R$$

$$h''_{ij} + \{2 - \delta(\tau)\}\, \mathcal{H} h'_{ij} + c^2 k^2 h_{ij} = 0$$



Parametrization for the amplitude damping

$$h''_{ij} + (2+\nu)\mathcal{H}h'_{ij} + k^2 h_{ij} = 0 \,,$$

prime: derivative w.r.t. conformal time

$$\mathcal{H} \equiv a'/a$$

$$\nu = \mathcal{H}^{-1}\frac{d\ln M_*^2}{dt}$$

: effective Planck mass run rate (running G)

- change of gravity strength
- effective friction of spacetime (escape to extra dim, anomalous diffusion)

Needs of a generalized framework for GW propagation

- It is difficult to treat all the theory of modified gravity. Model-independent test is necessary.
- It should be independent of GW sources and background spacetimes (NS, BH, supernova, stochastic background etc.)
- It needs to be able to be combined with other observations. (cosmology, binary pulsar, Solar system)
- Parametrization should be directly related to physics behind them to interpret the results easily and transparently.

Generalized propagation of GW

GW propagation eq. in the effective field theory at the linear level

Saltas et al., PRL (2014)

$$h''_{ij} + (2+\nu)\mathcal{H}h'_{ij} + (c_T^2 k^2 + a^2\mu^2)h_{ij} = a^2\Gamma\gamma_{ij}$$

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C_T : GW propagation speed

- violation of Lorentz sym.
- violation of equivalence principle
- modified dispersion relation

Generalized propagation of GW

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Saltas et al., PRL (2014)

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μ : graviton mass

- massive gravity
- compactified extra dim.

Generalized propagation of GW

GW propagation eq. in the effective field theory at the linear level

Saltas et al., PRL (2014)

$$h''_{ij} + (2+\nu)\mathcal{H}h'_{ij} + (c_T^2 k^2 + a^2\mu^2)h_{ij} = a^2\Gamma\gamma_{ij}$$

Γ : source for GW

- energy injection from extra dim.
- nonminimal coupling to other fields

Classification of gravity theories

gravity theory	ν	$c_T^2 - 1$	μ	Γ
general relativity	0	0	0	0
Horndeski theory	α_M	α_T	0	0
f(R) gravity	$F'/\mathcal{H}F$	0	0	0
Einstein-aether theory	0	$c_\sigma/(1+c_\sigma)$	0	0
bimetric massive gravity theory	0	0	$m^2 f_1$	$m^2 f_1$
quantum gravity phenom.	0	$(n_{\rm QG}-1)\mathbb{A}E^{n_{\rm QG}-2}$	when $n_{\rm QG}=0$	0

$$E^2 = p^2\left[1+\xi\left(\frac{E}{E_{\rm QG}}\right)^{n_{\rm QG}-2}\right]$$

doubly special relativity
extra dimensional theories
Horava-Lifshitz theory
gravitational SME
⋮

Analytical solution

Nishizawa, PRD 2018

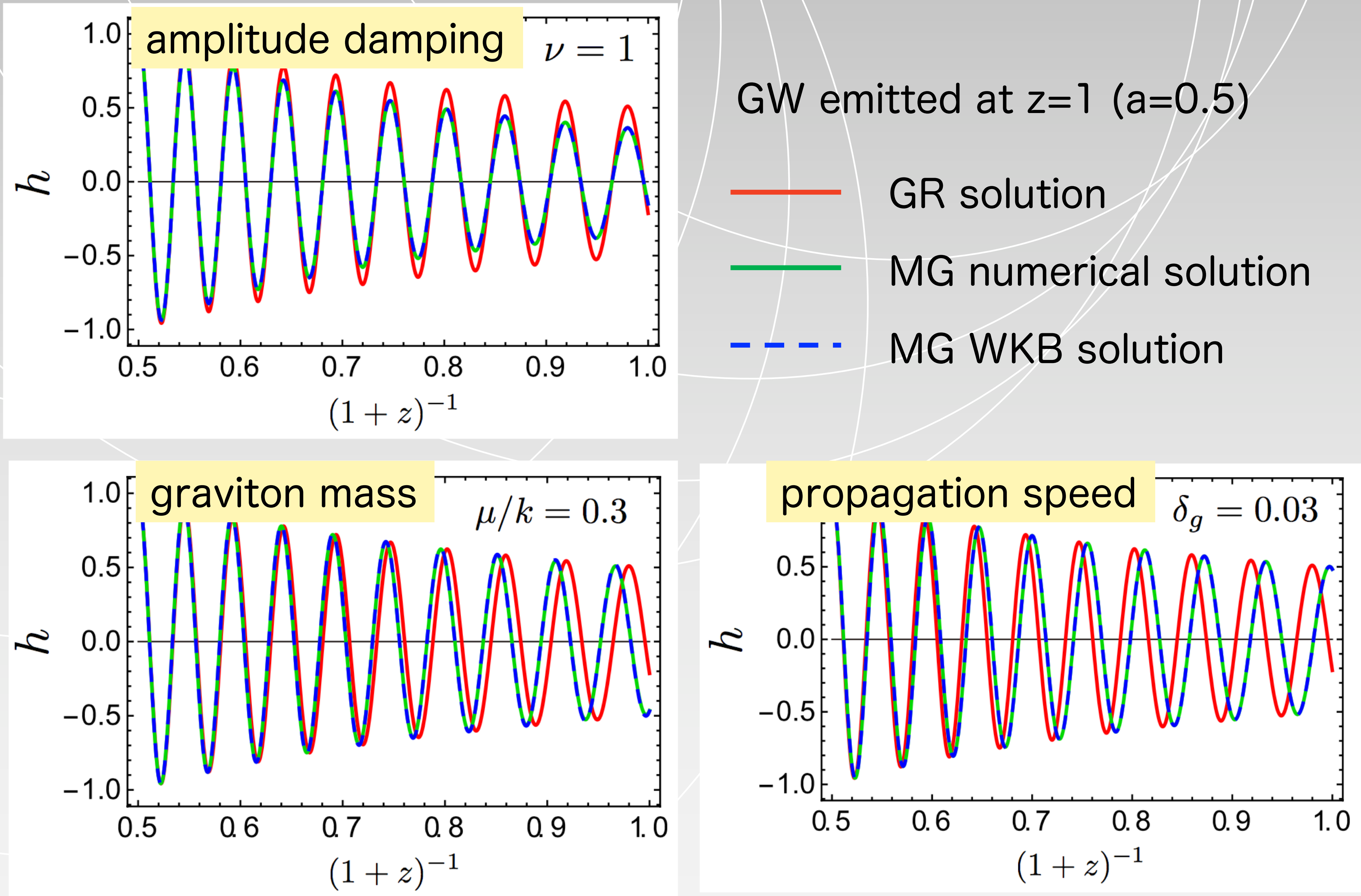
For $\Gamma = 0$, the eq. can be solved analytically, if the amplitude is a slowly varying function with cosmo timescale.

$$h = \mathcal{C}_{\rm MG} h_{\rm GR} \qquad \mathcal{C}_{\rm MG} = e^{-\mathcal{D}} e^{-ik\Delta T}$$

damping factor $\mathcal{D} = \frac{1}{2}\int_0^z \frac{\nu}{1+z'} dz'$ $\qquad c_T \equiv 1 - \delta_g$

extra time delay $\Delta T = \int_0^z \frac{1}{\mathcal{H}} \left(\frac{\delta_g}{1+z'} - \frac{\mu^2}{2k^2(1+z')^3} \right) dz'$

Even when $\Gamma \neq 0$, an analytical solution is also obtained.


amplitude damping
$\nu = 1$
GW emitted at z=1 (a=0.5)
GR solution
MG numerical solution
MG WKB solution
graviton mass
$\mu/k = 0.3$
propagation speed
$\delta_g = 0.03$
h
$(1+z)^{-1}$
1.0
0.5
0.0
−0.5
−1.0
0.5
0.6
0.7
0.8
0.9
1.0

Relation to ppE framework

$$h(f) = \left(1 + \sum_i \alpha_i u^i\right) e^{i \sum_j \beta_j u^j} h_{\mathrm{GR}}(f)$$

$$u \equiv (\pi \mathcal{M} f)^{1/3}$$

$$\alpha_0 = -\frac{1}{2} \int_0^z \frac{\nu}{1+z'} dz'$$

Newtonian in amplitude

$$\beta_3 = -\frac{2}{\mathcal{M}} \int_0^z \frac{\delta_g}{(1+z')\mathcal{H}} dz'$$

4PN in phase

$$\beta_{-3} = \frac{\mathcal{M}}{2} \int_0^z \frac{\mu^2}{(1+z')^3 \mathcal{H}} dz'$$

1PN in phase

How to measure the modifications

phase modification

$$t_c + \delta_g \frac{d_L(z)}{1+z}$$

δ_g is degenerated with t_c

➡ info on t_c (EM counterpart) is necessary.

➡ GW source should be BH-NS or NS-NS binaries

amplitude modification

$$h \propto \underline{(1+z)^{-\nu/2} d_L^{-1}(z)}$$

ν is degenerated with redshift z .

$d_{L,eff}^{-1}$

➡ Need EM counterparts or host galaxy identification

GW170817 was the first opportunity to measure them.

Standard siren

GW from a compact binary can be a cosmological tool to measure distance to a source. Schutz, Nature (1986); Holz & Hughes, ApJ (2005)

GW phase

from $L_{\rm gw} = -\frac{dE_{\rm orbit}}{dt}$

$$\dot{f}(t) \propto \{(1+z)M_c\}^{5/3} f^{11/3}$$

GW amplitude

$$h(t) \propto \frac{\{(1+z)M_c\}^{5/3} f^{2/3}}{D_L}$$

From observational data,

$h, f, \dot{f} \cdots$

⬇

$$M_z \equiv (1+z)M_c$$

⬇

luminosity distance

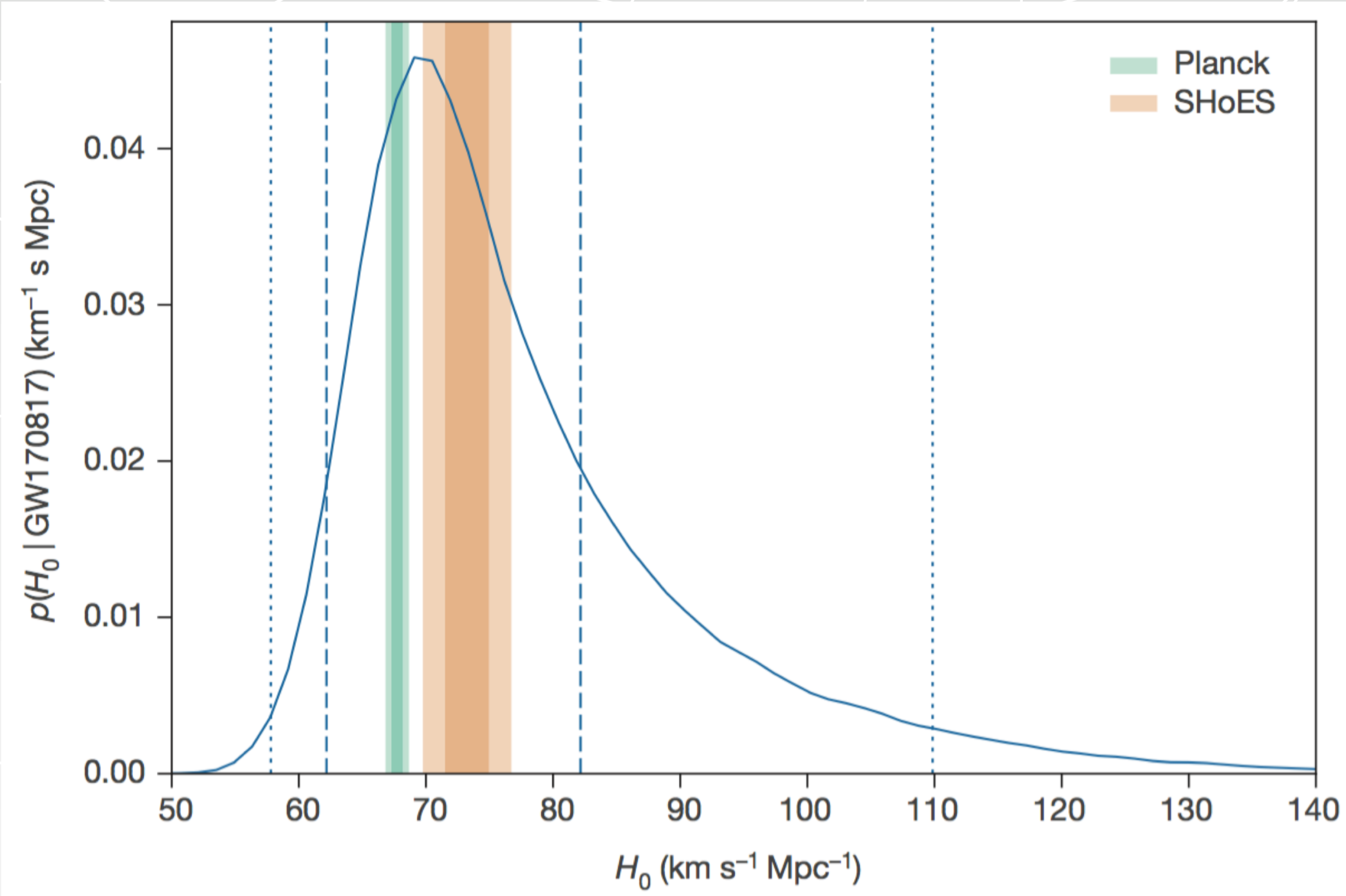
D_L

Hubble constant from GW170817

LSC + optical telescopes, Nature 551, 85

at low redshift ,

from GW observation → $$d_\mathrm{L} \approx \frac{z}{H_0}$$ ← from EM observation of the host galaxy



Constraining the time evolution

GW170817 was the first opportunity to measure them.

- Expansion up to linear order in time

$$\nu = \nu_0 - \nu_1 H_0 t_{\rm LB}$$

$$\delta_g = \delta_{g0} - \delta_{g1} H_0 t_{\rm LB}$$

Arai & Nishizawa, PRD (2018)

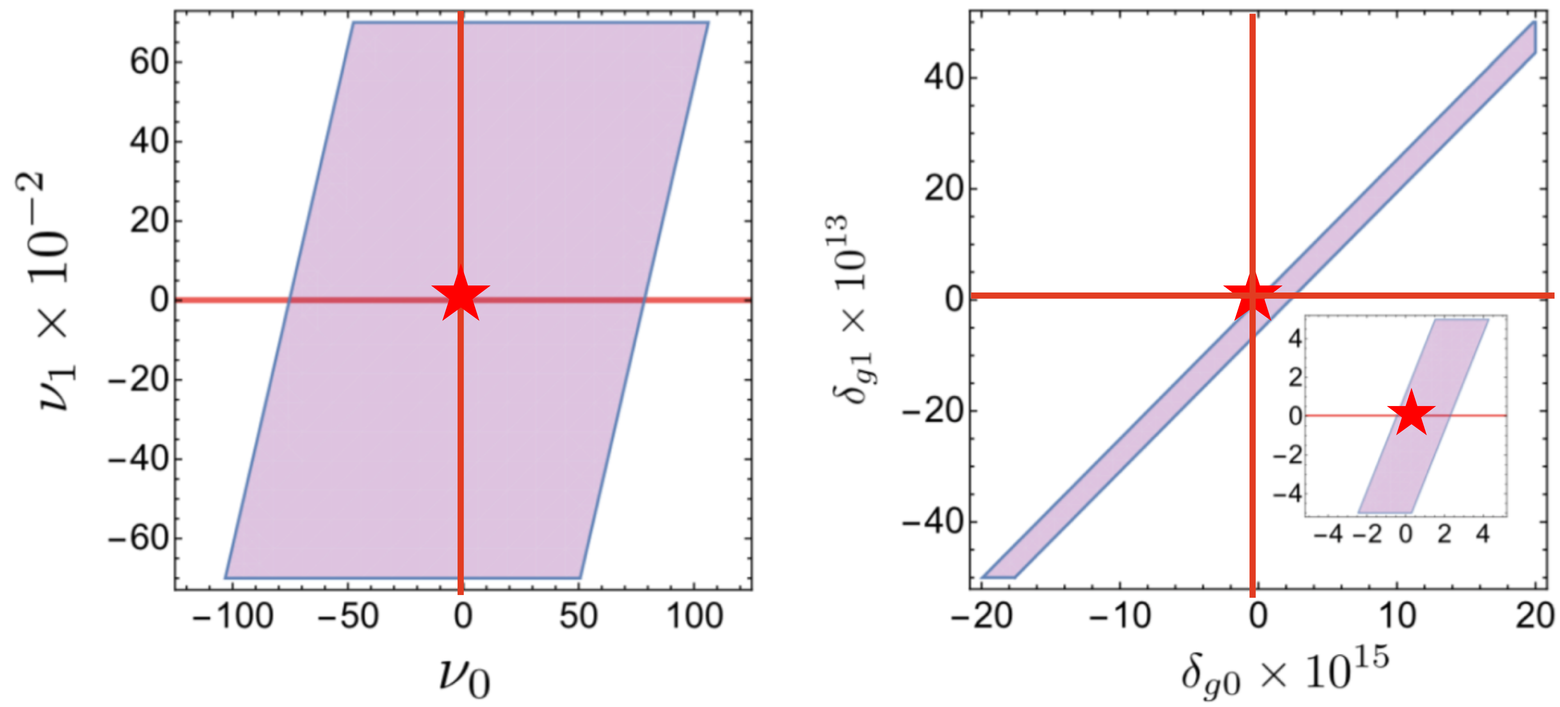
$t_{\rm LB}(t)$: lookback time in the standard ΛCDM universe

- Observables are expressed in terms of new parameters

$$\mathcal{D} \approx \frac{1}{2}\left\{\nu_0 \ln(1+z) - \frac{\nu_1}{2}(H_0 t_{\rm LB})^2\right\}$$

$$\Delta T \approx \frac{1}{H_0}\left\{\delta_{g0} H_0 t_{\rm LB} - \frac{\delta_{g1}}{2}(H_0 t_{\rm LB})^2\right\}$$

Constraining the time evolution



$-75.3 \leq \nu_0 \leq 78.4$

$-4.7 \times 10^{-16} \leq \delta_{g0} \leq 2.2 \times 10^{-15}$

Summary of current constraints

generalized GW propagation — Saltas et al., PRL (2014)

$$h''_{ij} + (2+\nu)\mathcal{H}h'_{ij} + (c_{\mathrm{T}}^2 k^2 + a^2\mu^2)h_{ij} = 0$$

- graviton mass $\mu \leq 7.7 \times 10^{-23}\,\mathrm{eV}$ — LIGO Scientific Collaboration 2017
- From GW170817/GRB170817A, GW speed has been measured so precisely

$$-3 \times 10^{-15} < \frac{c_{\mathrm{T}} - c}{c} < 7 \times 10^{-16}$$

LSC + Fermi + INTEGRAL, ApJL (2017)

- Constraint on amplitude damping rate

$$-75.3 \leq \nu \leq 78.4$$

Arai & Nishizawa, PRD (2018)

Future sensitivity estimation

We estimate an parameter error in the future measuremenet of the amplitude damping with the Fisher information matrix.

Fisher information matrix

Parameter measurement errors can be estimated without doing experiments.

likelihood function

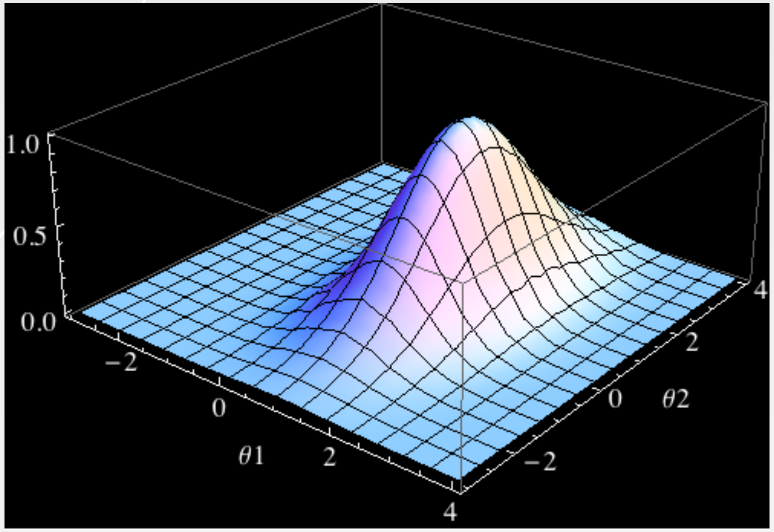
$$p(\vec{\theta}\,|s) = p_0(\vec{\theta}\,)\exp\left(-\frac{1}{2}\Gamma_{ab}\Delta\theta^a\Delta\theta^b\right)$$

Fisher information matrix

$$\Gamma_{ab} = \left(\frac{\partial h}{\partial\theta^a}\middle|\frac{\partial h}{\partial\theta^b}\right) \qquad (A|B) \equiv 4\mathrm{Re}\int_0^\infty \frac{\tilde{A}^*(f)\tilde{B}(f)}{S_h(f)}df$$

parameter estimation error

$$\Delta\theta^a = \sqrt{(\Gamma^{-1})_{aa}}$$



Future sensitivity estimation

We estimate an parameter error in the future measuremenet of the amplitude damping with the Fisher information matrix.

- generate 500 sources with SNR > 8 for each case.
- source direction & inclination angles: uniformly random
- GW waveform:

phenomenological IMR waveform (PhenomD) for BBH

Khan et al. 2016

post-Newtonian inspiral waveform for BH-NS and BNS

- Redshifts are assumed to be determined from identification of a host galaxy or an electromagnetic counterpart.

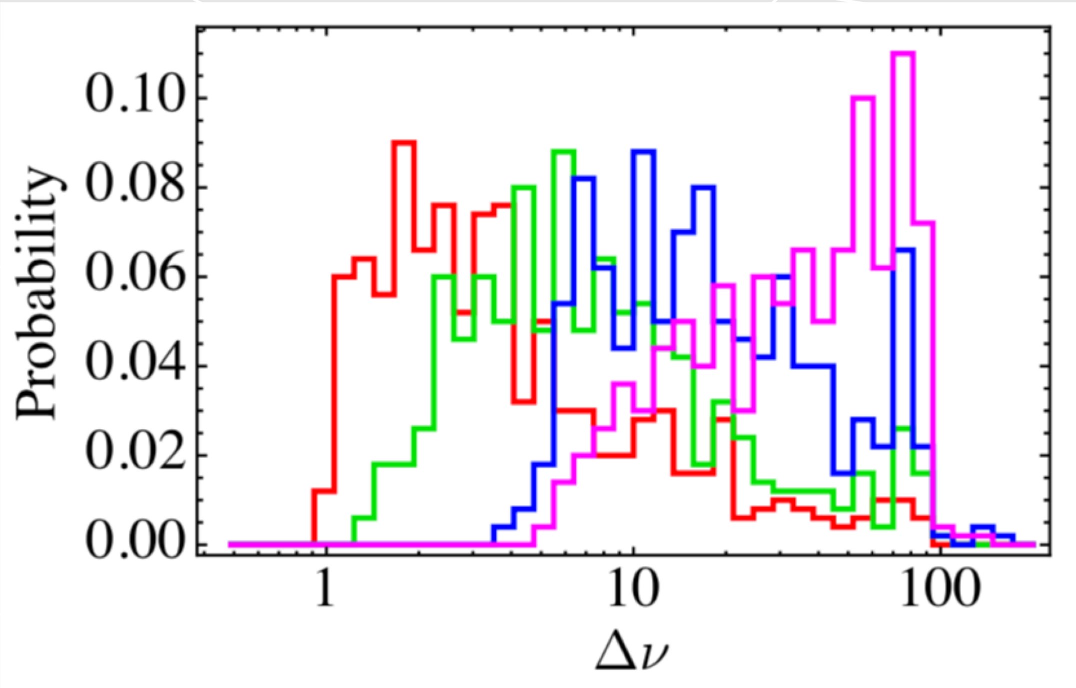
Sensitivity to amplitude damping

We generate Mock GW catalogs and estimate the measurement errors of model parameters with the Fisher information matrix.

current detector network (aLIGO, KAGRA, etc.)

$$\Delta\nu \sim \mathcal{O}(1)$$

Nishizawa, PRD (2018)



$30M_\odot - 30M_\odot$

$10M_\odot - 10M_\odot$

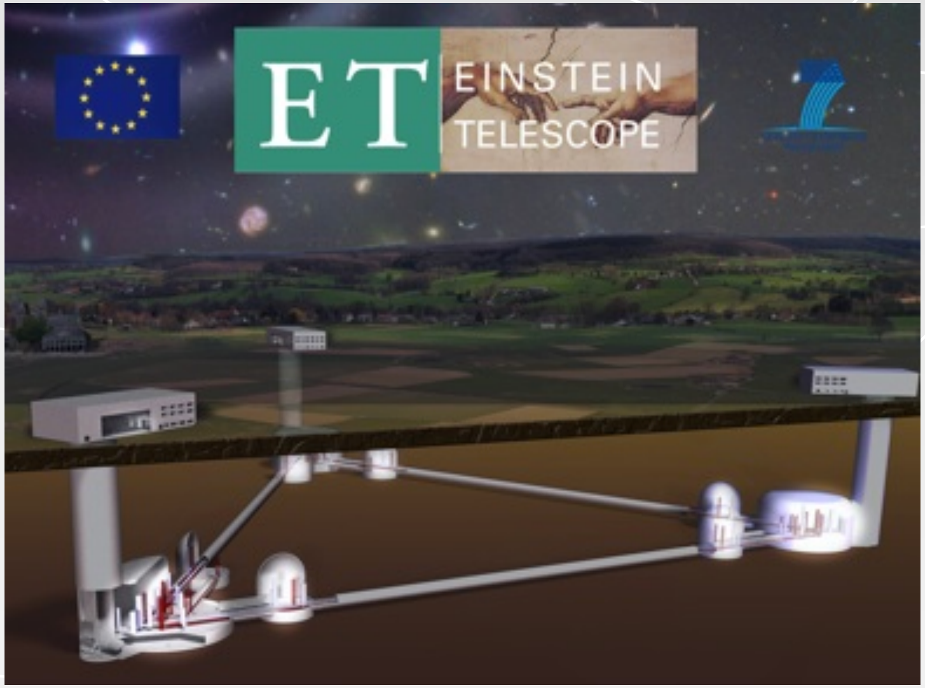
$10M_\odot - 1.4M_\odot$

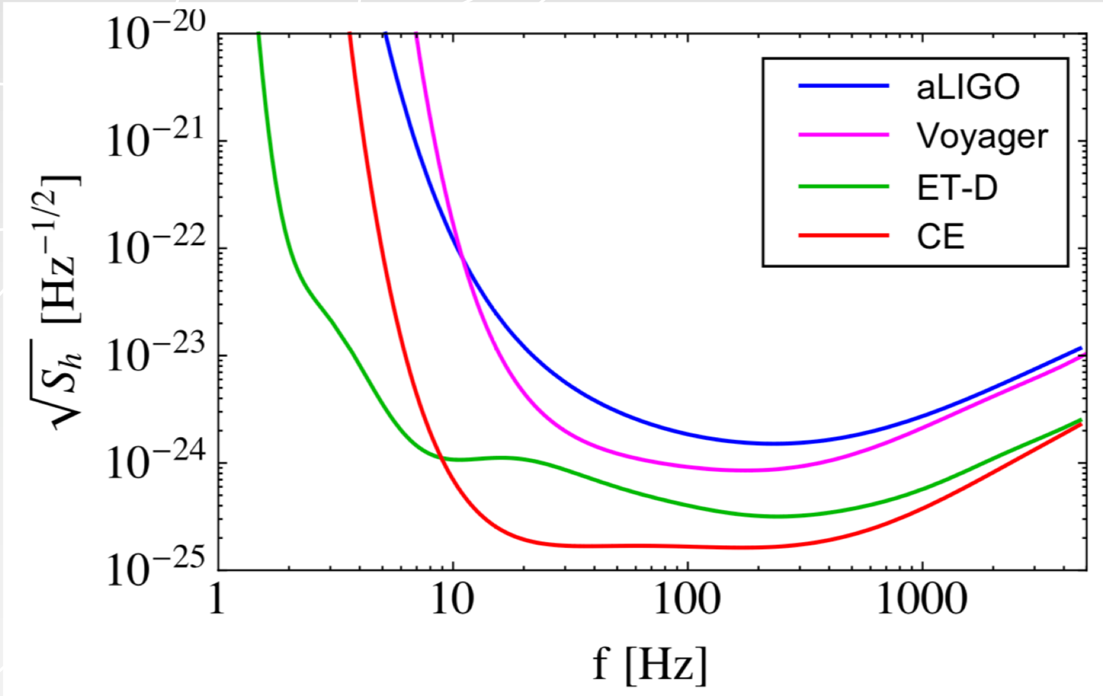
$1.4\,M_\odot - 1.4\,M_\odot$

3rd gen. detectors

Einstein Telescope (ET) is planned to start observation after aLIGO in 2032 or later.

ET has x10 better sensitivity than aLIGO and will detect a million of BBH out to z~20.

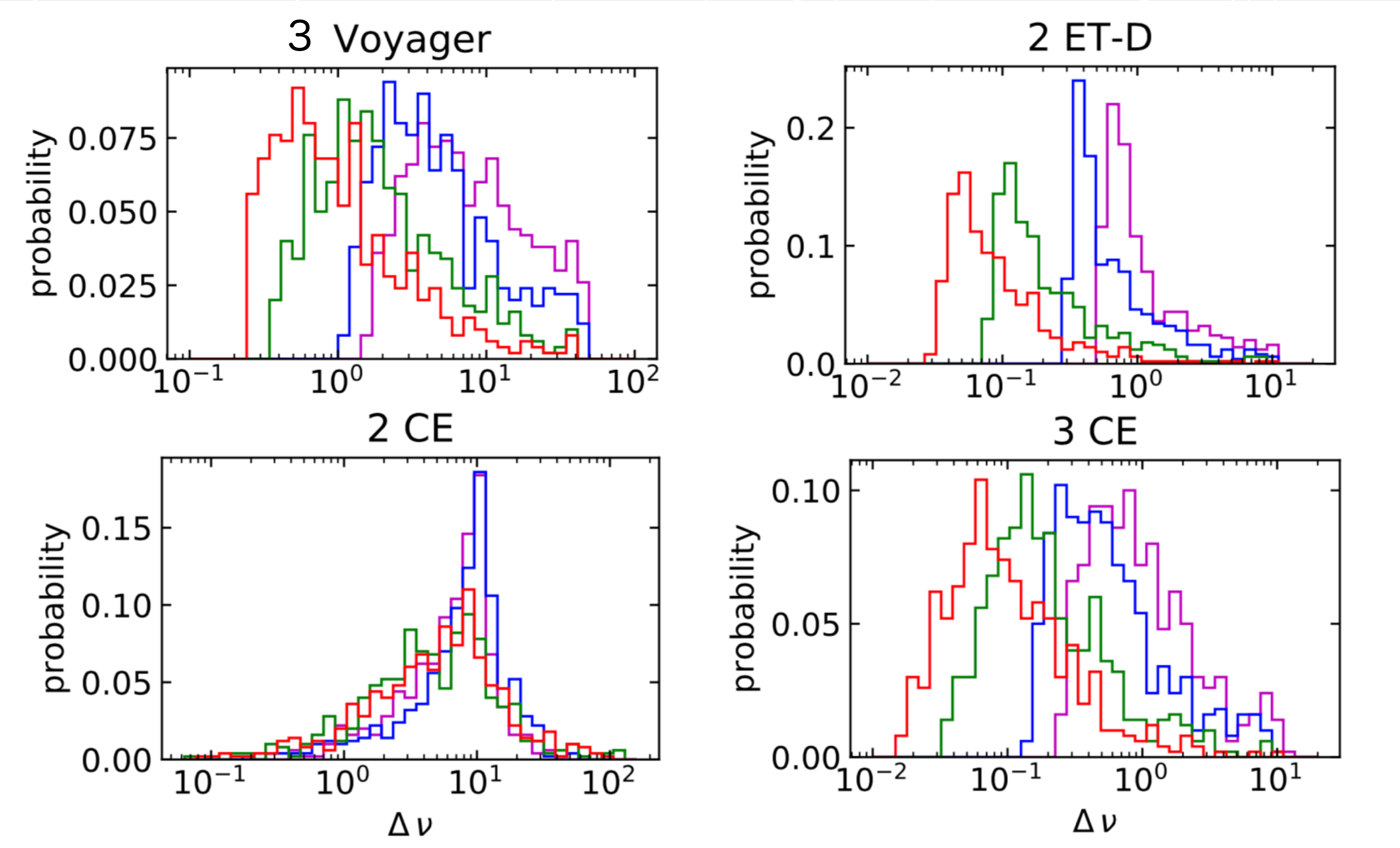




Sensitivity to amplitude damping

$$\Delta\nu \sim \mathcal{O}(0.01)$$

Nishizawa & Arai, PRD (2019)
Belgacem et al., arXiv:1907.01487

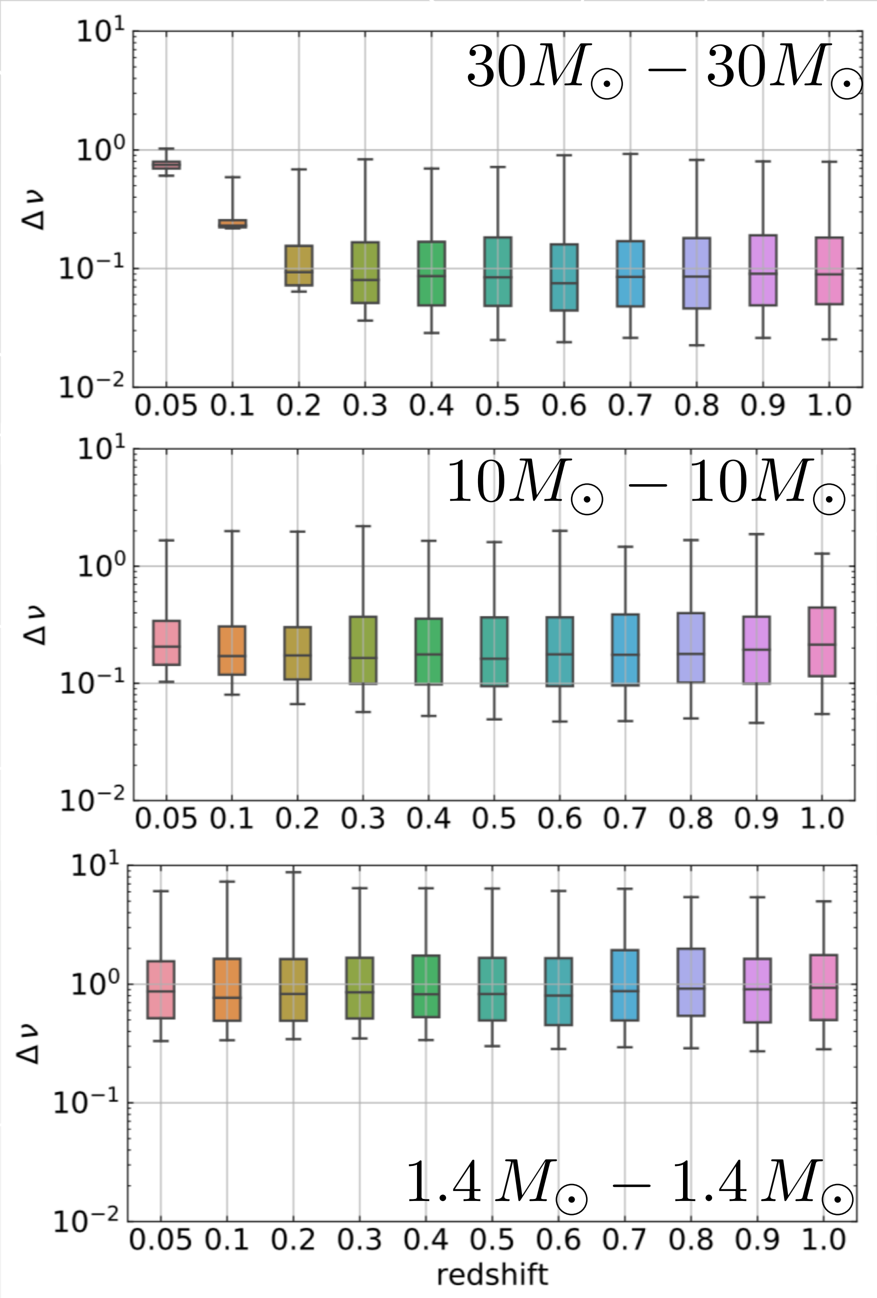


Redshift dependence

$\Delta\nu \sim 0.03$

$\Delta\nu \sim 0.07$

$\Delta\nu \sim 0.3$



detector network
3 CE @ H, L, V

thick bars (25-75%)
thin bars (5-95%)

ν erros are independent of source redshifts

$$\Delta\nu \sim \frac{2}{\log(1+z)\times \mathrm{SNR}}$$

smaller errors for heavier binaries (deu to larger SNR)

References

Generalized framework for testing gravity with GW propagation

I. Formulation

Nishizawa, PRD 97, 104037 (2018)

II. Constraints on Horndeski theory

Arai & Nishizawa, PRD 97, 104038 (2018)

III. Future prospect

Nishizawa & Arai, PRD 99, 104038 (2019)

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GW polarizations

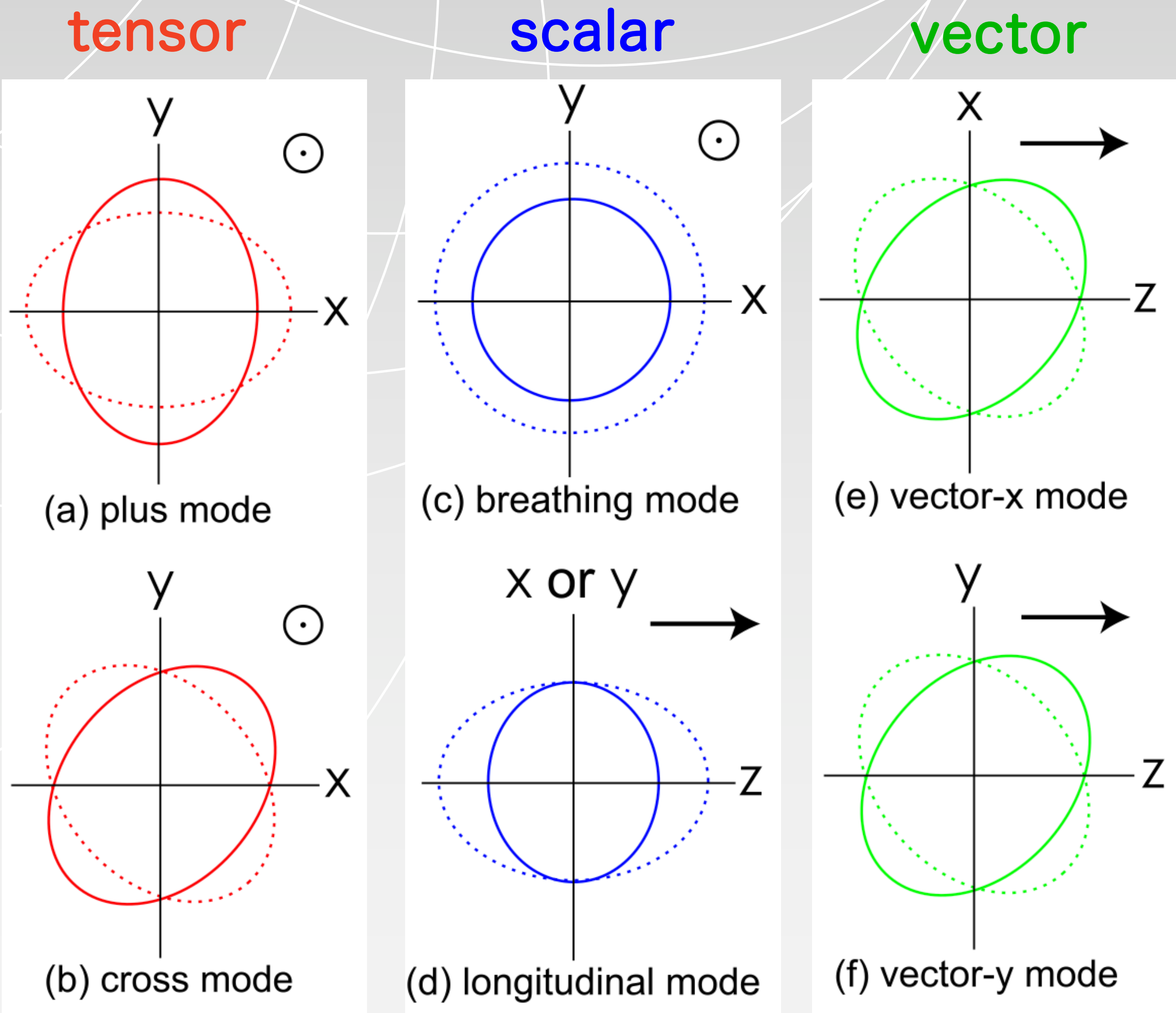
In general metric theory of gravity, 6 pols. are allowed.

Eardley et al., PRL (1973)
Will, textbook (1993)

6 pols. are classified into tensor, vector, and scalar modes, depending on the rotational symmetry.

(More rigorously, Newman-Penrose formalism)

For a GW propagating in the z direction.



(a) plus mode (c) breathing mode (e) vector-x mode
(b) cross mode (d) longitudinal mode (f) vector-y mode

GW polarizations in a specific theory

➢ General relativity

two polarization modes (+ and × modes = tensor pol.)

➢ Scalar-tensor theory, f (R) gravity theory

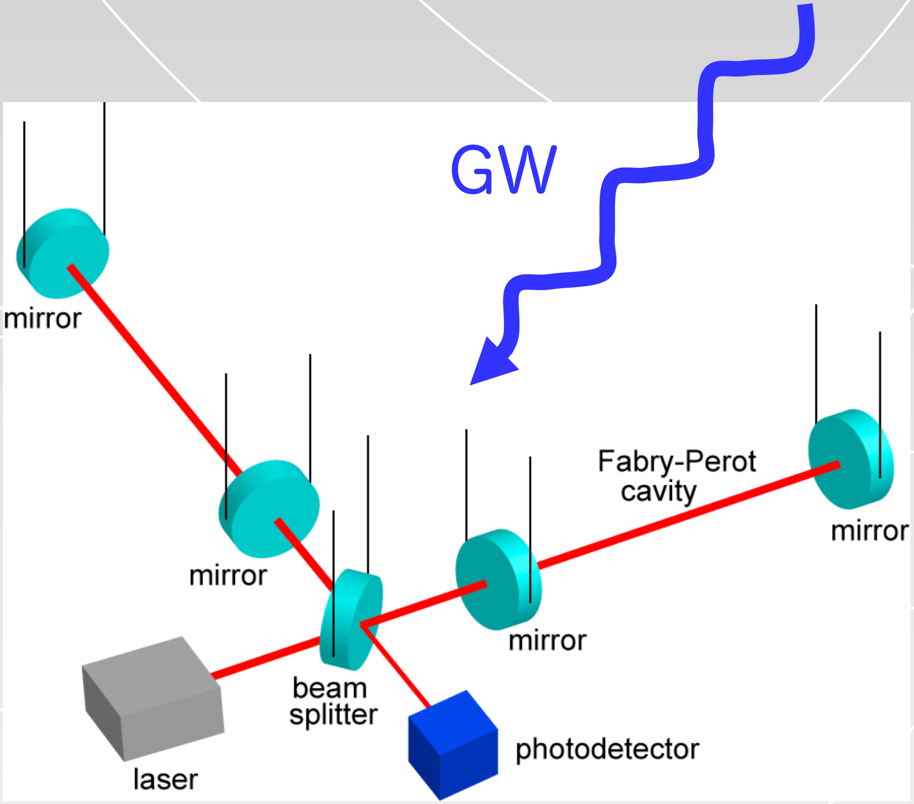
Scalar degree of freedom ⟶ scalar pol.

➢ Massive gravity theory, bimetric theory

Scalar & vector degree of freedom ⟶ scalar & vector pols.

Searching for additional pols. allows us to distinguish gravity theories in terms of d.o.f.

Antenna pattern function



Response of a detector to GW propagating in a direction.

Definition

$$F_A(\hat{\Omega}) = D_{ij}\, e_A^{ij}(\hat{\Omega})$$

detector tensor

$$D_{ij} = \frac{1}{2}\left\{\hat{u}_i\hat{u}_j - \hat{v}_i\hat{v}_j\right\}$$

polarization tensor: e_A^{ij}

Antenna pattern functions for non-GR pols.

Tobar, Suzuki & Kuroda 1999

tensor

$$F_I^+(\hat{\Omega}) = \frac{1}{2}(1+\cos^2\theta_I)\cos 2\phi_I,$$

$$F_I^\times(\hat{\Omega}) = -\cos\theta_I \sin 2\phi_I,$$

scalar

$$F_I^\circ(\hat{\Omega}) = -\frac{1}{2}\sin^2\theta_I \cos 2\phi_I,$$

$$F_I^\ell(\hat{\Omega}) = \frac{1}{\sqrt{2}}\sin^2\theta_I \cos 2\phi_I,$$

vector

$$F_I^x(\hat{\Omega}) = -\frac{1}{2}\sin 2\theta_I \cos 2\phi_I,$$

$$F_I^y(\hat{\Omega}) = \sin\theta_I \sin 2\phi_I.$$

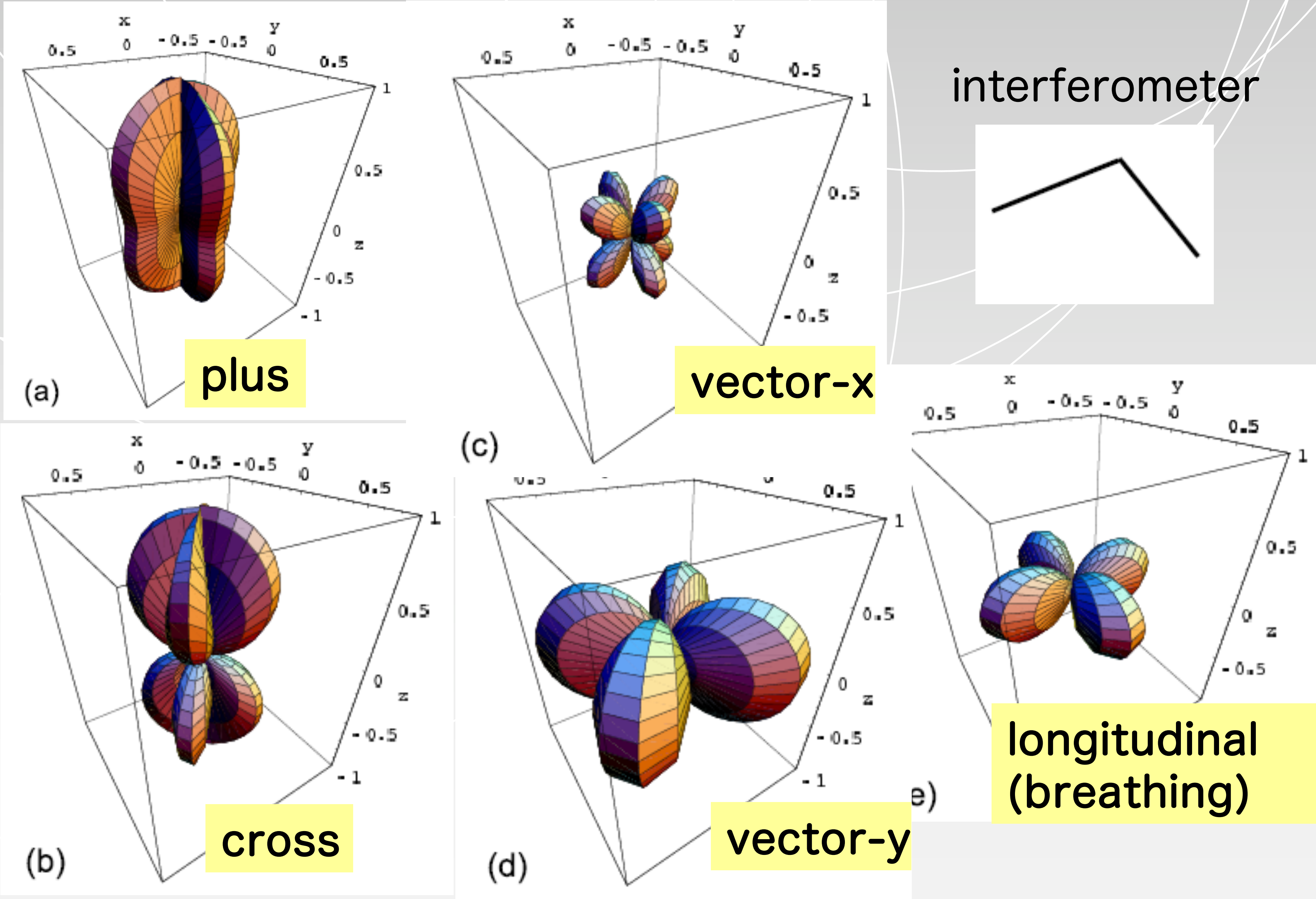
Two scalar modes are degenerated.

⇩

GW interferometers cannot distinguish these two modes.

(Pulsar timing array can do.)

Nishizawa et al., PRD (2009)



Observational constraint from GW170817

LVC, PRL123, 011102 (2019)

Sky position is fixed to NGC4993.

- tensor mode vs scalar mode

$s = F_T h_{GR}$ $\qquad$ $s = F_S h_{GR}$

$$\frac{L(D|F_T)}{L(D|F_S)} > 10^{23.0}$$

Tensor mode is much more likely than scalar mode.

- tensor mode vs vector mode

$s = F_T h_{GR}$ $\qquad$ $s = F_V h_{GR}$

$$\frac{L(D|F_T)}{L(D|F_V)} > 10^{20.8}$$

Tensor mode is much more likely than vector mode.

Problem of replacing the antenna patterns

In general relativity,

$$h_I = \mathcal{G}_{\mathrm{T},I} h_{\mathrm{GR}}$$

$$\mathcal{G}_{\mathrm{T},I} = \frac{5}{2}\{(1+\cos^2\iota)F_{+,I}(\vec{\theta}_s,\vec{\theta}_e)$$

$$+ 2i\cos\iota F_{\times,I}(\vec{\theta}_s,\vec{\theta}_e)\}e^{i\phi_{\mathrm{D},I}(\theta_s,\phi_s,\theta_e,\phi_e)}$$

geometrical factor

This is obtained from the quadrupole formula

$$h_{ab}(t,\mathbf{x}) = \frac{2G}{rc^4}\Lambda_{abcd}\ddot{M}^{cd}(t-r/c)$$

M_{ab} : mass moment of the system

Λ_{abcd} : transverse-traceless projection

Geometrical factors for other pols.

Takeda, Nishizawa et al., PRD (2018)

For scalar (dipole) mode

$$\mathcal{G}_{\mathrm{S}_1,I} = \sqrt{\frac{45}{2}} \sin\iota F_{\mathrm{b},I}(\vec{\theta}_s, \vec{\theta}_e) e^{i\phi_{\mathrm{D},I}(\theta_s,\phi_s,\theta_e,\phi_e)}$$

For vector x & y modes

$$\mathcal{G}_{\mathrm{V}_x,I} = \sqrt{\frac{525}{56}} \sin 2\iota F_{\mathrm{V}_x,I}(\vec{\theta}_s, \vec{\theta}_e) e^{i\phi_{\mathrm{D},I}(\theta_s,\phi_s,\theta_e,\phi_e)}$$

$$\mathcal{G}_{\mathrm{V}_y,I} = \sqrt{\frac{15}{2}} \sin\iota F_{\mathrm{V}_y,I}(\vec{\theta}_s, \vec{\theta}_e) e^{i\phi_{\mathrm{D},I}(\theta_s,\phi_s,\theta_e,\phi_e)}$$

Inclination angle dependence is different for each pol.

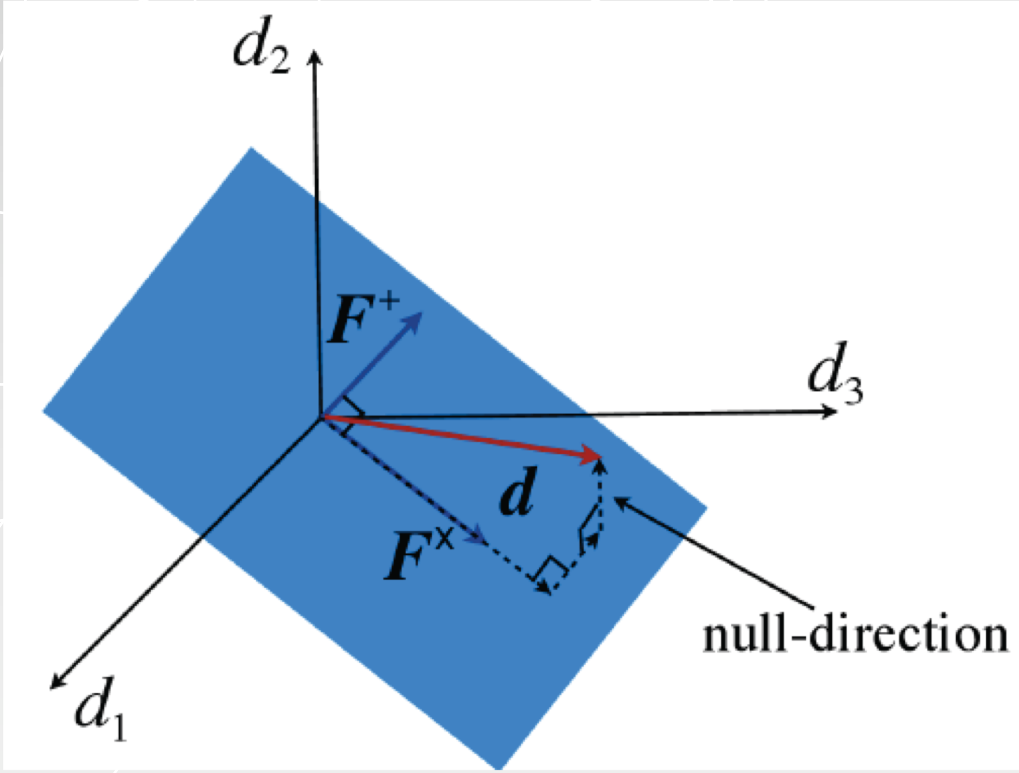
Null stream in GR

Guersel & Tinto, PRD (1989)
Chatterji et al., PRD (2006)

for + and × pols

A null stream is constructed from 3 detector output.

$$x_{\rm null} = \frac{\epsilon^{cab} F_a^+ F_b^\times}{|\delta^{ab} F_a^+ F_b^\times|} x_c$$



Yunes & Siemens, LRR (2013)

Null stream in modified gravity

Chatziioannou et al., PRD (2012)

for arbitrary number of pols.

With M detector outputs, one can eliminate N (M > N) pols.
from the data and construct (M-N) null streams.

1) for HLV detectors, 1 null stream is available and
   can search for 1 additional (scalar) pol.

2) for HLVK detectors, 2 null streams are available and
   can search for 2 additional (vector) pols.

Note that these statements are in principle and do not mean
pol modes are separated in reality.

Mode separation

In the presence of 3 modes

$$h_I = F_{I,+}h_+ + F_{I,\times}h_\times + F_{I,s}h_s$$

From 3 detectors

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} F_{1,+} & F_{1,\times} & F_{1,S} \\ F_{2,+} & F_{2,\times} & F_{2,S} \\ F_{3,+} & F_{3,\times} & F_{3,S} \end{pmatrix} \begin{pmatrix} h_+ \\ h_\times \\ h_S \end{pmatrix}$$

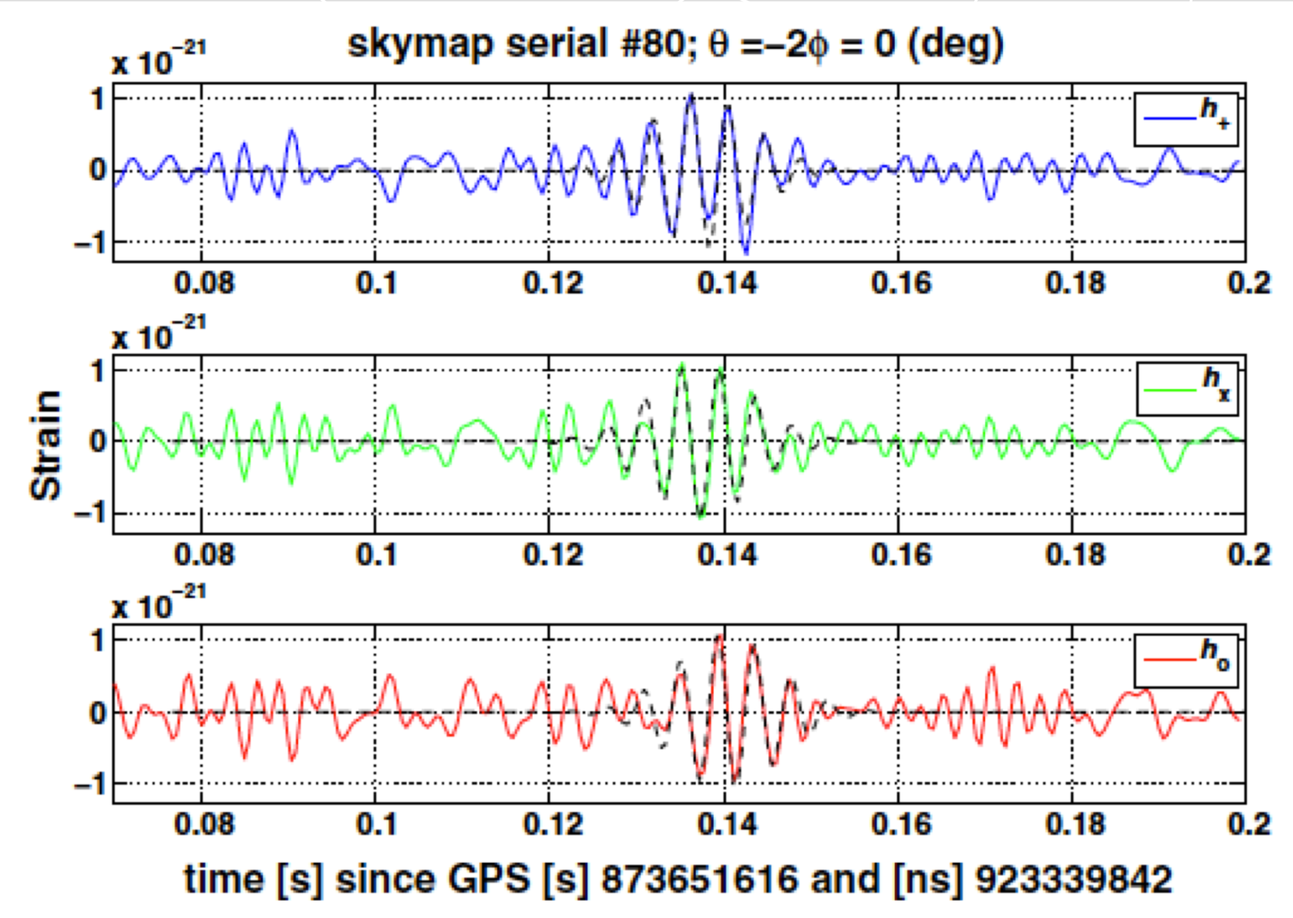
⬇

$$\begin{pmatrix} h_+ \\ h_\times \\ h_S \end{pmatrix} = \Pi \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} \qquad \Pi \equiv \begin{pmatrix} F_{1,+} & F_{1,\times} & F_{1,S} \\ F_{2,+} & F_{2,\times} & F_{2,S} \\ F_{3,+} & F_{3,\times} & F_{3,S} \end{pmatrix}^{-1}$$

Studies on mode separability

For burst GWs   Hayama & Nishizawa, PRD (2013)

3 pol modes can be separated with 3 detectors?



black dached :
injected GW signal

colored :
reconstructed signal

For most of sky locations with ground -based detectors, pol modes can be separated very well.

Studies on mode separability

For inspiral GWs Takeda, Nishizawa et al., PRD (2018)

Why is the case of compact binary more difficult?

- The waveform evolves in time.
- The parameter correlation at each time should be considered when solving the inverse problem.

Model TS1 : GR + scalar (dipole)

$$h_I = \{\mathcal{G}_{\mathrm{T},I} + A_{\mathrm{S}_1}\mathcal{G}_{\mathrm{S}_1,I}\}h_{\mathrm{GR}}$$

The measurement error of A_{S_1} is estimated with the Fisher matrix. If $\Delta A_{\mathrm{S}_1}/A_{\mathrm{S}_1} < 1$, the modes are separable.

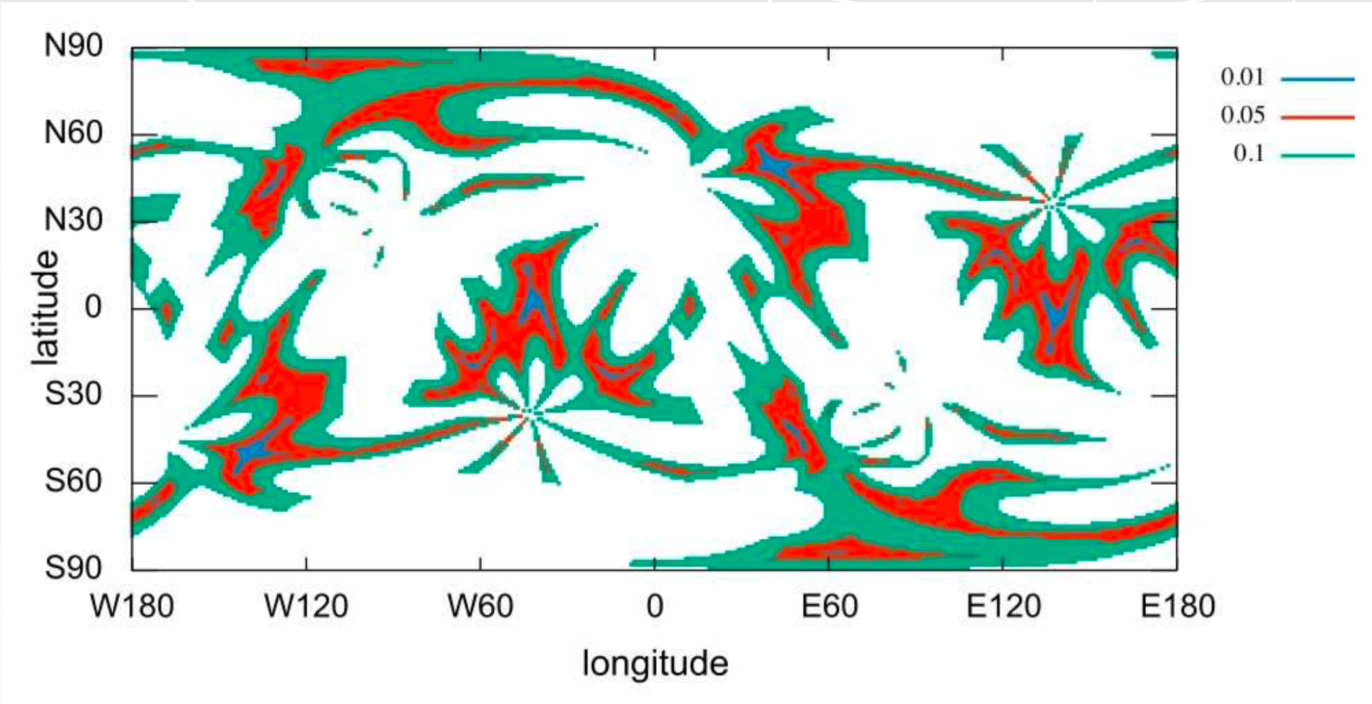
The number of detectors need to be at least equal to the number of pols searched for.

Case with EM counterpart

Hagihara et al., PRD (2018, 2019)

With info about a source direction from an EM counterpart, more pols (e.g. 5 modes with 4 detectors) can be probed by constructing null streams.

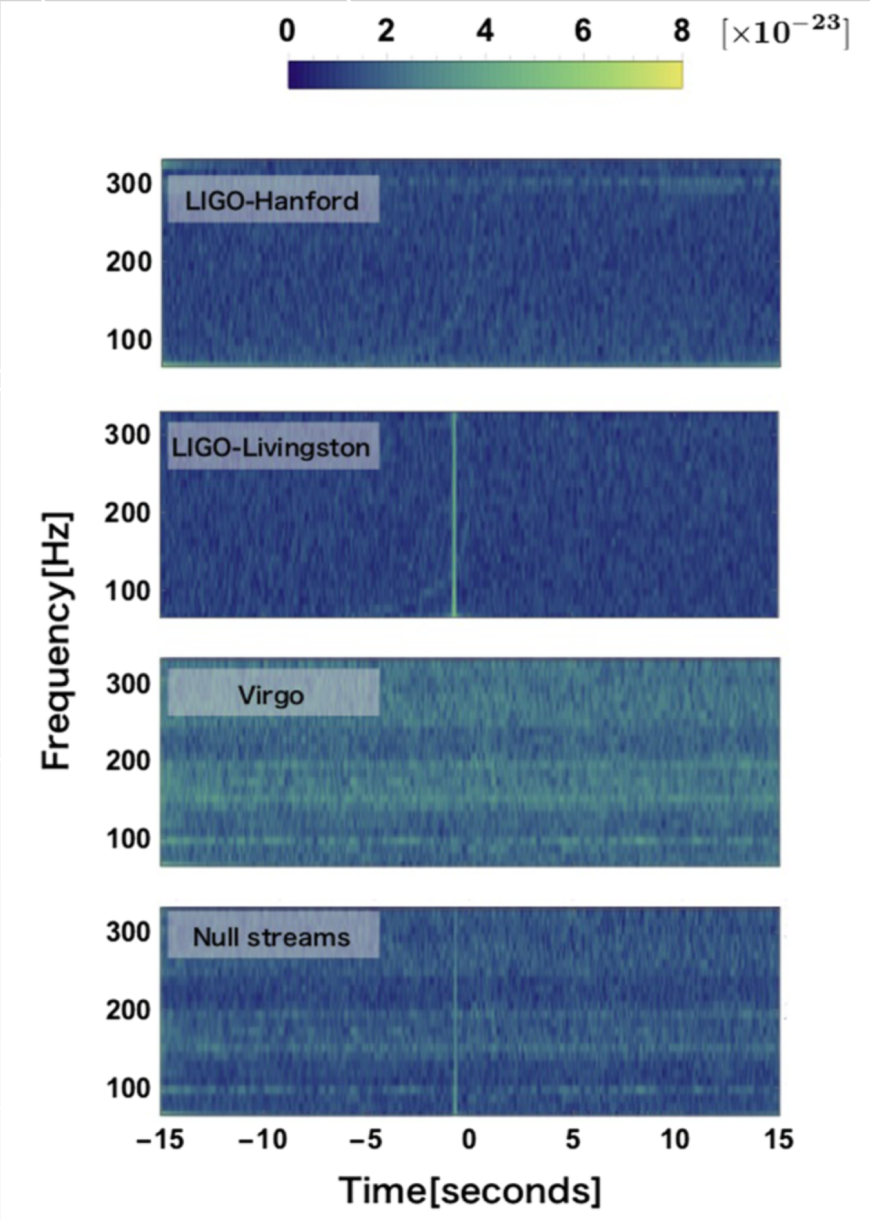
The coefficient for the scalar mode in a null stream



1 of 10 events has 90% suppression of a scalar mode coefficient.

Only vector mode is in the null stream.

Constraint on vector modes from GW170817



HLV signals → 1 null stream

Coefficient for the scalar mode

$$P_a F_a^{\mathrm{S}} = -0.0738$$

(1-of-5-events suppression)

Coefficient for the vector modes

$$\sqrt{(P_a F_a^{\mathrm{V}})^2 + (P_a F_a^{\mathrm{W}})^2} = 0.4976$$

Upper limit on vector GW amp.

$$|h^{\mathrm{V}} + h^{\mathrm{W}}| < 6 \times 10^{-23}$$

Studies on mode separability

For stochatic GWs

Nishizawa et al. 2009

Energy density of stochastic GWB $\Omega_{\rm gw}(f) \equiv \frac{1}{\rho_{\rm c}} \frac{d\rho_{\rm gw}}{d\ln f}$

$$\Omega^{\rm T}_{\rm gw} \equiv \Omega^{+}_{\rm gw} + \Omega^{\times}_{\rm gw} \quad (\Omega^{+}_{\rm gw} = \Omega^{\times}_{\rm gw})$$

$$\Omega^{\rm V}_{\rm gw} \equiv \Omega^{x}_{\rm gw} + \Omega^{y}_{\rm gw} \quad (\Omega^{x}_{\rm gw} = \Omega^{y}_{\rm gw})$$

assumptions

$$\Omega^{\rm S}_{\rm gw} \equiv \Omega^{b}_{\rm gw} + \Omega^{\ell}_{\rm gw} = \Omega^{b}_{\rm gw}(1+\kappa) \qquad \kappa \equiv \Omega^{\ell}_{\rm gw}/\Omega^{b}_{\rm gw}$$

Correlation signal between detectors

$$\mu_{\rm IJ}(f) \propto \gamma^{T}_{\rm IJ}(f)\Omega^{T}_{\rm gw}(f) + \gamma^{V}_{\rm IJ}(f)\Omega^{V}_{\rm gw}(f) + \xi\gamma^{S}_{\rm IJ}(f)\Omega^{S}_{\rm gw}(f)$$

$\gamma^{T,V,S}_{\rm IJ}$: overlap reduction function

From 3 detectors (3 correlation signals)

$$\begin{pmatrix} \tilde{\mu}_{12} \\ \tilde{\mu}_{23} \\ \tilde{\mu}_{31} \end{pmatrix} = \begin{pmatrix} \gamma_{12}^{\mathrm{T}} & \gamma_{12}^{\mathrm{V}} & \gamma_{12}^{\mathrm{S}} \\ \gamma_{23}^{\mathrm{T}} & \gamma_{23}^{\mathrm{V}} & \gamma_{23}^{\mathrm{S}} \\ \gamma_{31}^{\mathrm{T}} & \gamma_{31}^{\mathrm{V}} & \gamma_{31}^{\mathrm{S}} \end{pmatrix} \begin{pmatrix} \Omega_{\mathrm{gw}}^{\mathrm{T}} \\ \Omega_{\mathrm{gw}}^{\mathrm{V}} \\ \xi\Omega_{\mathrm{gw}}^{\mathrm{S}} \end{pmatrix}$$

$$\begin{pmatrix} \Omega_{\mathrm{gw}}^{\mathrm{T}} \\ \Omega_{\mathrm{gw}}^{\mathrm{V}} \\ \xi\Omega_{\mathrm{gw}}^{\mathrm{S}} \end{pmatrix} = \Pi \begin{pmatrix} \tilde{\mu}_{12} \\ \tilde{\mu}_{23} \\ \tilde{\mu}_{31} \end{pmatrix} \qquad \Pi = \begin{pmatrix} \gamma_{12}^{\mathrm{T}} & \gamma_{12}^{\mathrm{V}} & \gamma_{12}^{\mathrm{S}} \\ \gamma_{23}^{\mathrm{T}} & \gamma_{23}^{\mathrm{V}} & \gamma_{23}^{\mathrm{S}} \\ \gamma_{31}^{\mathrm{T}} & \gamma_{31}^{\mathrm{V}} & \gamma_{31}^{\mathrm{S}} \end{pmatrix}^{-1}$$

It has been shown that 3 ground-based detectors from HLVK can separate 3 pol modes very well and have almost the same sensitivity to each mode after the separation.

Current constraint

From O1 & O2 data
(basically HL and HLV only in Aug. 2017)

Null detection of a correlation signal

⇓

Constraints on Ω_{gw} in each pol mode

LVC, PRD (2019) [O2, isotropic GWB]

Polarization	Uniform prior	Log-uniform prior
Tensor	8.2×10^{-8}	3.2×10^{-8}
Vector	1.2×10^{-7}	2.9×10^{-8}
Scalar	4.2×10^{-7}	6.1×10^{-8}

Summary for GW polarization

- GW polarizations can be used for the model-independent test of modified gravity theories.
- When N pols. signal exist in GW data from a point source, N detectors can reconstruct N pol. modes.
- Sensitivities to extra pols. are almost the same as those to ordinary tensor pols even if the polarization decomposition is done.
- Stochastic GW backgrounds with 3 pol. modes (tensor, vector, scalar) can be separated by, at least, 3 detectors.

1. Test of general relativity at small scales
2. Cosmological observations and modification of gravity
3. GW observation and testing gravity
4. GW generation
   4.1. Inspiral-Merger-Ringdown (IMR) consistency test
   4.2. Post-Newtonian (PN) consistency test
5. GW propagation
   5.1. GW speed and modified dispersion relation
   5.2. Generalized framework of GW propagation
6. GW polarization
7. Parity violating gravity

Chern-Simons gravity

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[R + \mathcal{L}_{\rm PV} + \mathcal{L}_\phi]$$

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$\mathcal{L}_{\rm PV} = \mathcal{L}_{\rm CS} = f(\phi)P$$

⇓

$$h_A'' + (2+\nu_A)\mathcal{H}h_A' + \underline{c^2}k^2 h_A = 0$$

GW propagates with
the speed of light

circular polarizations

$$A = \mathrm{R}, \mathrm{L}$$

$$e_{ij}^{\mathrm{R}} = \frac{1}{\sqrt{2}}(e_{ij}^{+} + ie_{ij}^{\times})$$

$$e_{ij}^{\mathrm{L}} = \frac{1}{\sqrt{2}}(e_{ij}^{+} - ie_{ij}^{\times})$$

"No bound on gravitational parity violation from GW170817"
Alexander & Yunes, PRD (2018)

"can be constrained by 2nd-gen. detectors"
Yang & Yagi, PRD (2018)

Parity-violating gravity

Crisostomi et al., PRD (2018)

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[R + \mathcal{L}_{\rm PV} + \mathcal{L}_\phi] \qquad \mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 - V(\phi)$$

Up to 1st-order derivative of a scalar field

$$\mathcal{L}_{\rm PV1} = \sum_{A=1}^{4} a_A(\phi, \phi_\mu\phi^\mu) L_A$$

$$L_1 = \epsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R_{\mu\nu}{}^{\rho}{}_{\lambda}\phi^\sigma\phi^\lambda$$

$$L_2 = \epsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R_{\mu\lambda}{}^{\rho\sigma}\phi_\nu\phi^\lambda$$

$$P = \epsilon^{\mu\nu\rho\sigma} R_{\rho\sigma\alpha\beta} R^{\alpha\beta}{}_{\mu\nu}$$

$$L_3 = \epsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R^{\sigma}{}_{\nu}\phi^\rho\phi_\mu$$

$$L_4 = \phi^\lambda\phi_\lambda P$$

(in unitary gauge)

Not to have a ghost, $\quad 4a_1 + 2a_2 + a_3 + 8a_4 = 0$

Up to 2nd-order derivative of a scalar field

$$L_{\mathrm{PV2}} = b_1(\phi, \phi_\lambda \phi^\lambda)\epsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma}\phi^\rho \phi_\mu \phi^\sigma_\nu + \cdots$$

Particularly, in Chern-Simons gravity

$$\mathcal{L}_{\mathrm{PV}} = \mathcal{L}_{\mathrm{CS}} = f(\phi) P$$

GW sector

quadratic action for GW

Nishizawa & Kobayashi, PRD (2018)

$$S^{(2)} = \frac{1}{16\pi G}\int dt d^3x\, a^3 \left[\mathcal{L}^{(2)}_{\rm GR} + \mathcal{L}^{(2)}_{\rm PV}\right]$$

$$\mathcal{L}^{(2)}_{\rm GR} = \frac{1}{4}\left[\dot{h}_{ij}^2 - a^{-2}(\partial_k h_{ij})^2\right]$$

$$\mathcal{L}^{(2)}_{\rm PV} = \frac{1}{4}\left[\frac{\alpha(t)}{a\Lambda}\underline{\epsilon^{ijk}\dot{h}_{il}\partial_j\dot{h}_{kl}} + \frac{\beta(t)}{a^3\Lambda}\underline{\epsilon^{ijk}\partial^2 h_{il}\partial_j h_{kl}}\right]$$

always in this form at the leading order in the perturbative expansion of Λ^{-1}.

$\alpha/\Lambda = \beta/\Lambda \sim \ell_{\rm CS}^2\dot{\vartheta}/4$ ⇨ Chern-Simons gravity

GW propagation

propagation equation

$$h''_A + (2+\nu_A)\mathcal{H}h'_A + \underline{c^2_{\mathrm{T,A}}}k^2 h_A = 0 \qquad A = \mathrm{R, L}$$

amplitude damping rate

$$\nu_A = \frac{\lambda_A \tilde{k}(\alpha - \alpha'\mathcal{H}^{-1})}{1-\lambda_A\tilde{k}\alpha} \qquad \tilde{k} = \frac{k}{a\Lambda}$$

propagation speed

$$c^2_{\mathrm{T,A}} = \frac{1-\lambda_A\tilde{k}\beta}{1-\lambda_A\tilde{k}\alpha} \qquad \lambda_{\mathrm{R}} = +1, \lambda_{\mathrm{L}} = -1,$$

- The amplitude of one mode is enhanced than GR, while the other mode is suppressed.
- In contrast to Chern-Simons gravity, propagation speed is different from the speed of light.

Constraint from GW170817

Expanding in $\tilde{k} = \frac{k}{a\Lambda} \ll 1$,

$$\nu_A = \lambda_A \tilde{k}(\alpha - \alpha' \mathcal{H}^{-1}) + \mathcal{O}(\tilde{k}^2)$$

$$c_{\mathrm{T,A}}^2 = 1 + \underline{\lambda_A \tilde{k}(\alpha - \beta)} + \mathcal{O}(\tilde{k}^2)$$

↓ from GW170817 LSC + Fermi + INTEGRAL, ApJL (2017)

$$\tilde{k}|\alpha - \beta| \lesssim 10^{-15}$$

↓ $k/a \sim 100\,\mathrm{Hz}$

$$\boxed{\Lambda^{-1}|\alpha - \beta| \lesssim 10^{-11}\,\mathrm{km}}$$

Constraint from GW170817

$$\Lambda^{-1}|\alpha - \beta| \lesssim 10^{-11}\,\mathrm{km}$$

$$\mathcal{L}^{(2)}_{\mathrm{PV}} = \frac{1}{4}\left[\frac{\alpha(t)}{a\Lambda}\epsilon^{ijk}\dot{h}_{il}\partial_j\dot{h}_{kl} + \frac{\beta(t)}{a^3\Lambda}\epsilon^{ijk}\partial^2 h_{il}\partial_j h_{kl}\right]$$

three possibilities

(i) $\alpha = \beta$ ➡ parity-violating sector is exactly CS gravity.

(ii) $\alpha \approx \beta$ ➡ GW sector of parity-violating theory is extremely close to CS gravity.

(iii) $\alpha - \beta = \mathcal{O}(1)$, parity violation is suppressed at low energy

$$\Lambda^{-1} \lesssim 10^{-11}\,\mathrm{km}$$

Constraint from binary pulsars

$$\mathcal{L}_{\rm GR}^{(2)} = \frac{1}{4}\left[\dot{h}_{ij}^2 - a^{-2}(\partial_k h_{ij})^2\right]$$

$$\mathcal{L}_{\rm PV}^{(2)} = \frac{1}{4}\left[\frac{\alpha(t)}{a\Lambda}\epsilon^{ijk}\dot{h}_{il}\partial_j\dot{h}_{kl} + \frac{\beta(t)}{a^3\Lambda}\epsilon^{ijk}\partial^2 h_{il}\partial_j h_{kl}\right]$$

gravitational coupling for GW

$$G_{\rm GW}^A = \frac{G}{1-\lambda_A\tilde{k}\alpha}$$

from obs of binary pulsars

Jimenez, Piazza, Velten, PRL (2016)

$$0.995 \lesssim G_{\rm GW}/G_{\rm N} \lesssim 1.00$$

$$\Lambda^{-1}|\alpha| \lesssim 10^6\,{\rm km}$$

$$k/a \sim 4\times 10^{-4}\,{\rm Hz}$$

Summary for parity violating gravity

- The generalized framework for GW propagation is applied to parity violating gravity.

- Parity violation in gravity has constrained tightly from GW170817 and its tensor sector has pinned down to (almost) Chern-Simons gravity.