

Tests of gravity with gravitational waves

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Dec. 2-4, 2019 @ Academia Sinica, Taipei, Taiwan
APCTP School: Gravitational-Wave Cosmology

1. Test of general relativity
2. Cosmological observations and modification of gravity
3. GW observation and testing gravity
4. GW generation
 - 4.1. Inspiral-Merger-Ringdown (IMR) consistency test
 - 4.2. Post-Newtonian (PN) consistency test
5. GW propagation
 - 5.1. GW speed and modified dispersion relation
 - 5.2. Generalized framework of GW propagation
6. GW polarization
7. Parity violating gravity

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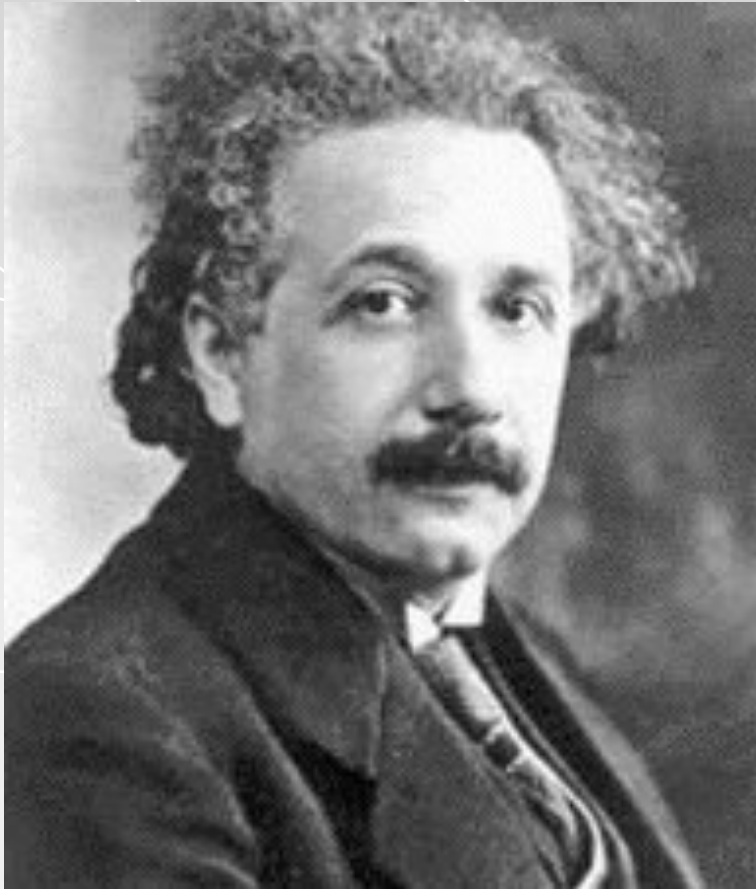
5.2. Generalized framework of GW propagation

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General Relativity

In 1915, Einstein proposed.

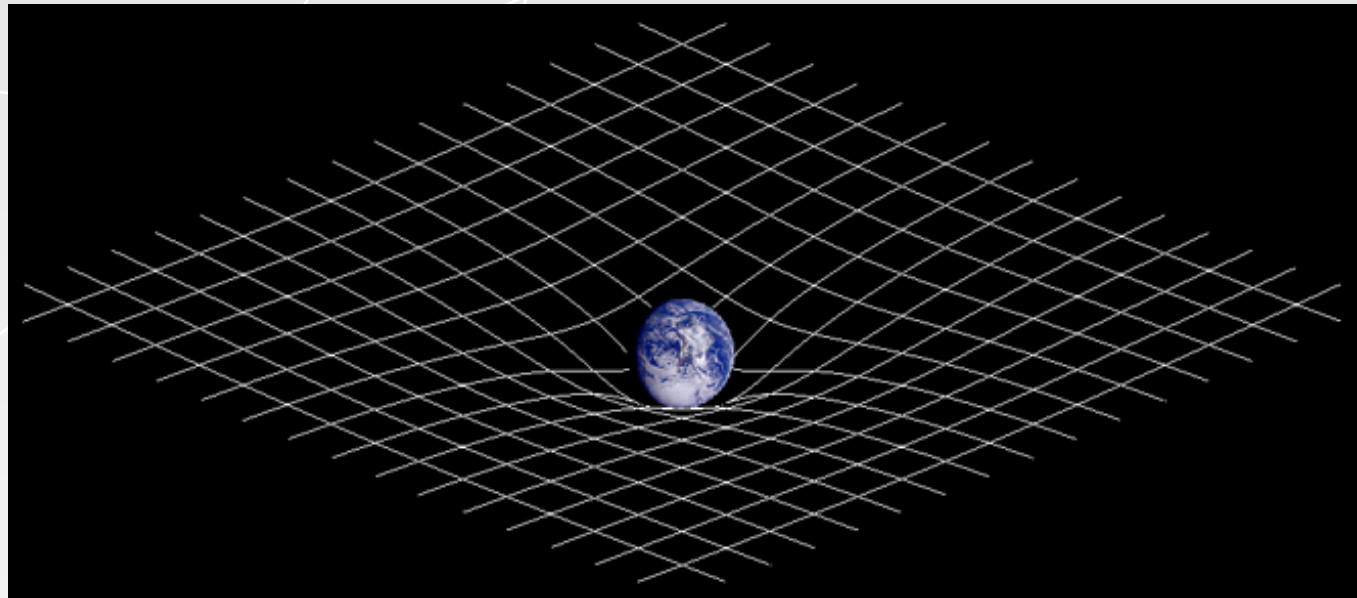


$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

gravity
(spacetime)

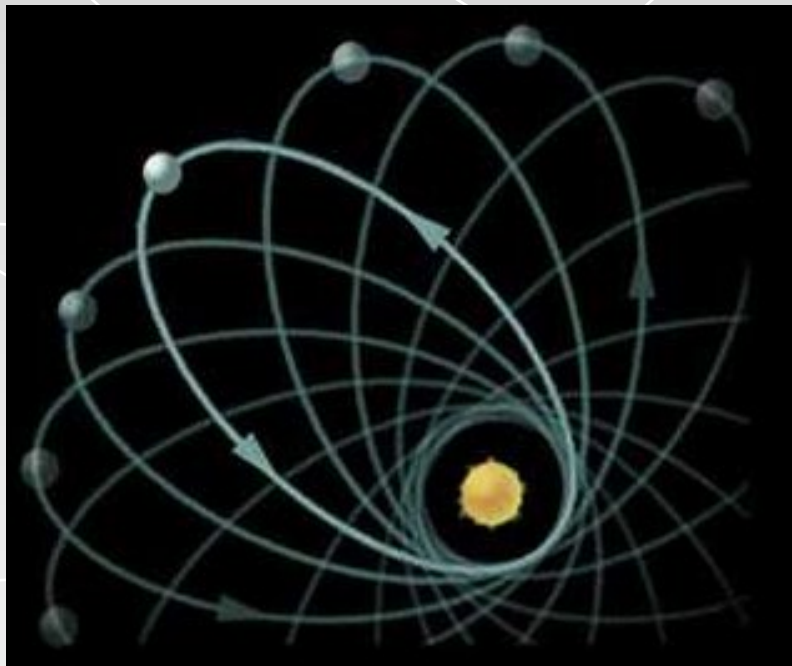
matter

Spacetime is not solid object
but dynamical one.



Test of GR in the solar system

perihelion shift of Mercury



prediction in Newtonian

$5557''.0/\text{century}$

observed value

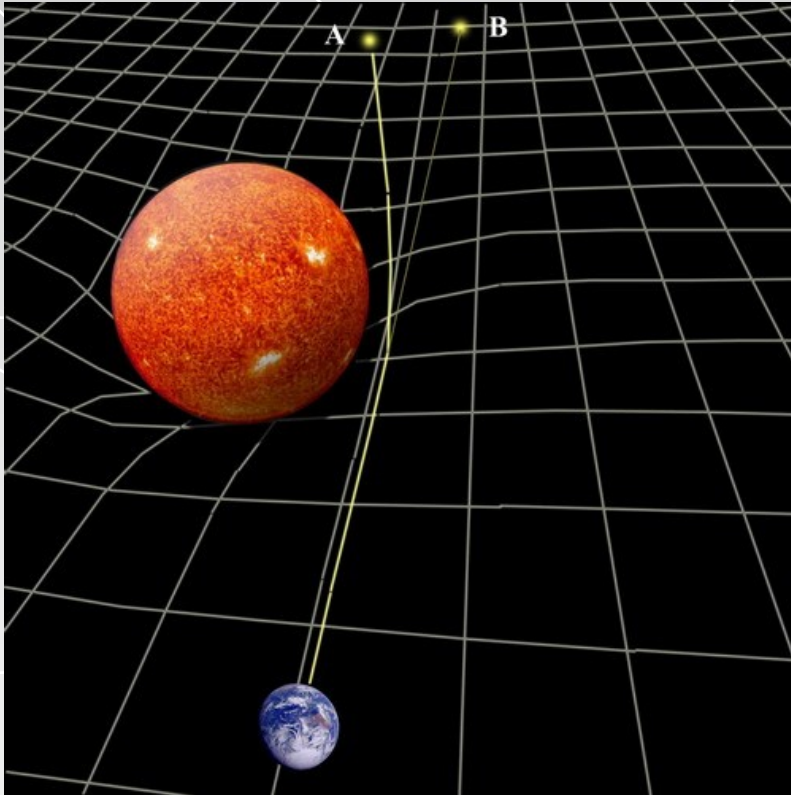
$5599''.7/\text{century}$

GR correctly predicted the
observed value.

(correction to Newtonian is
 $42''.7/\text{century}$)

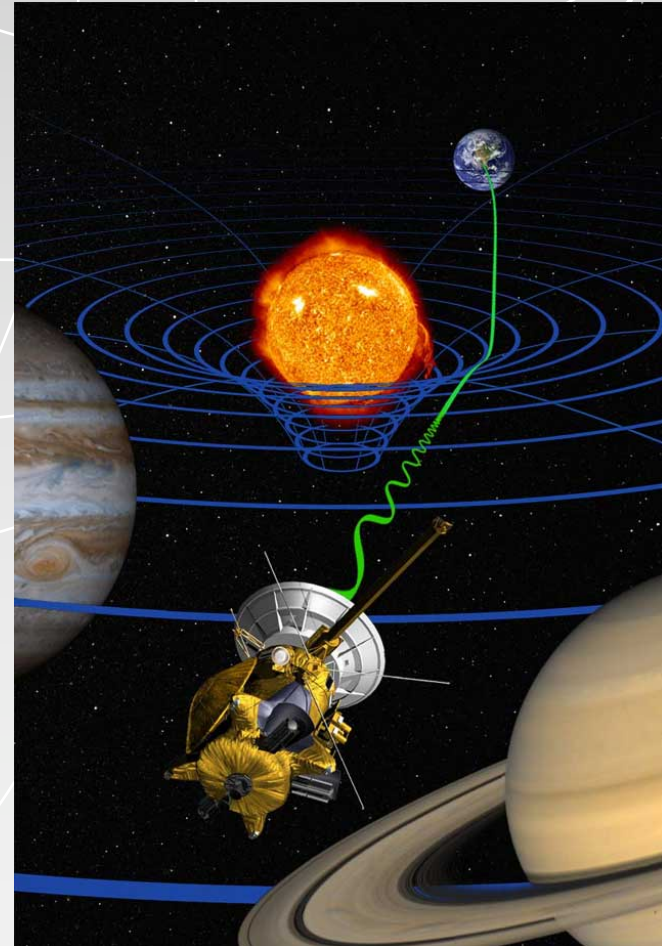
Test of GR in the solar system

deflection of light



In 1919, firstly observed
by Eddington et al.

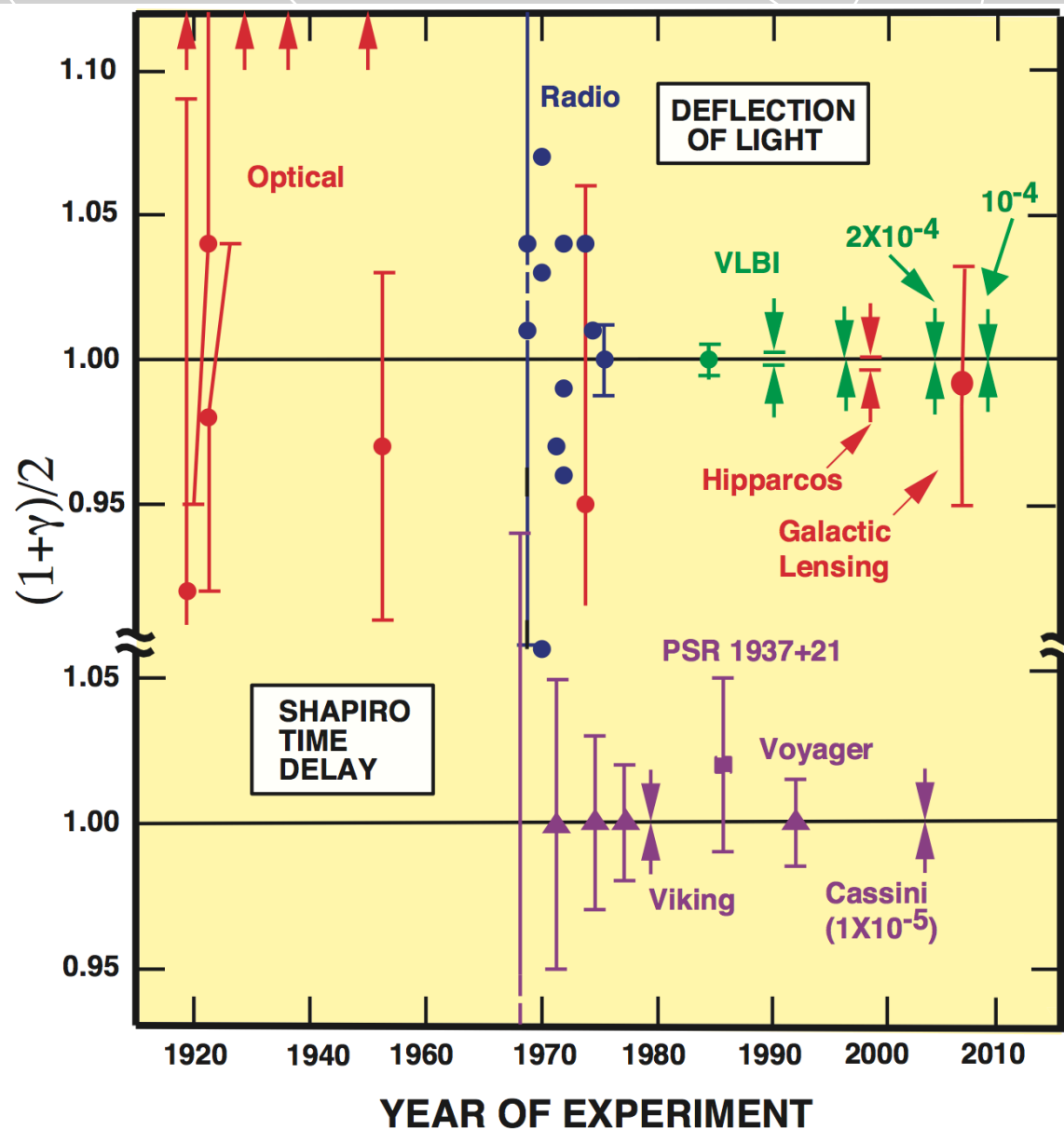
Shapiro time delay



In 1968, firstly observed
by Shapiro et al.

Observational constraint

Will, LRR (2014)



deflection angle
(at the rim of Sun)

$$\delta\theta \approx \frac{1}{2}(1 + \gamma) \times 1.''75 \dots$$

$$\gamma - 1 \leq 1 \times 10^{-4}$$

time delay
(Mercury - Earth)

$$\Delta t \approx \frac{1}{2}(1 + \gamma) \times 240 \mu\text{sec} \dots$$

$$\gamma - 1 \leq 1 \times 10^{-5}$$

In GR $\gamma = 1$

Indirect evidence of GW

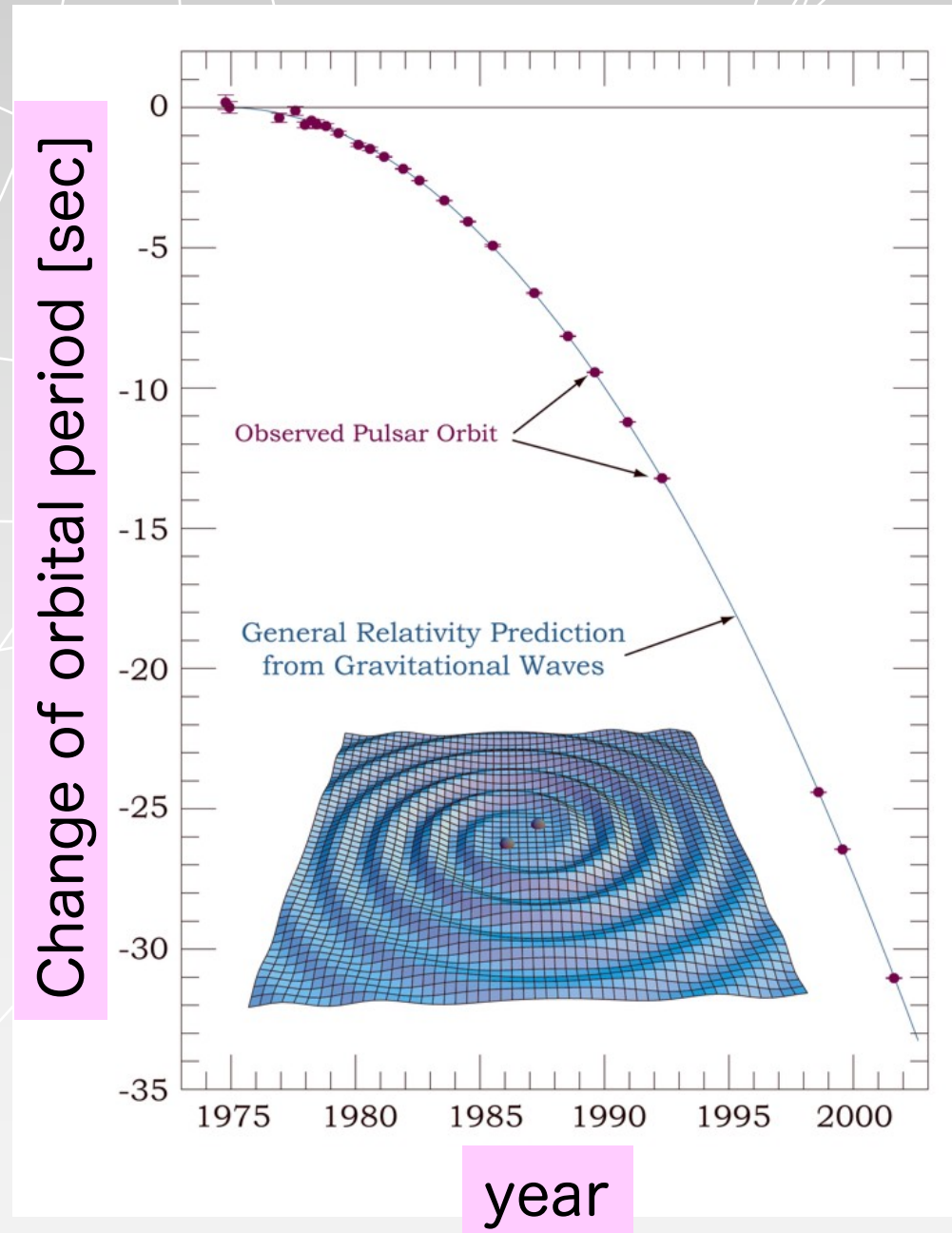
- Indirect evidence from binary pulsar (PSR B1913+16)

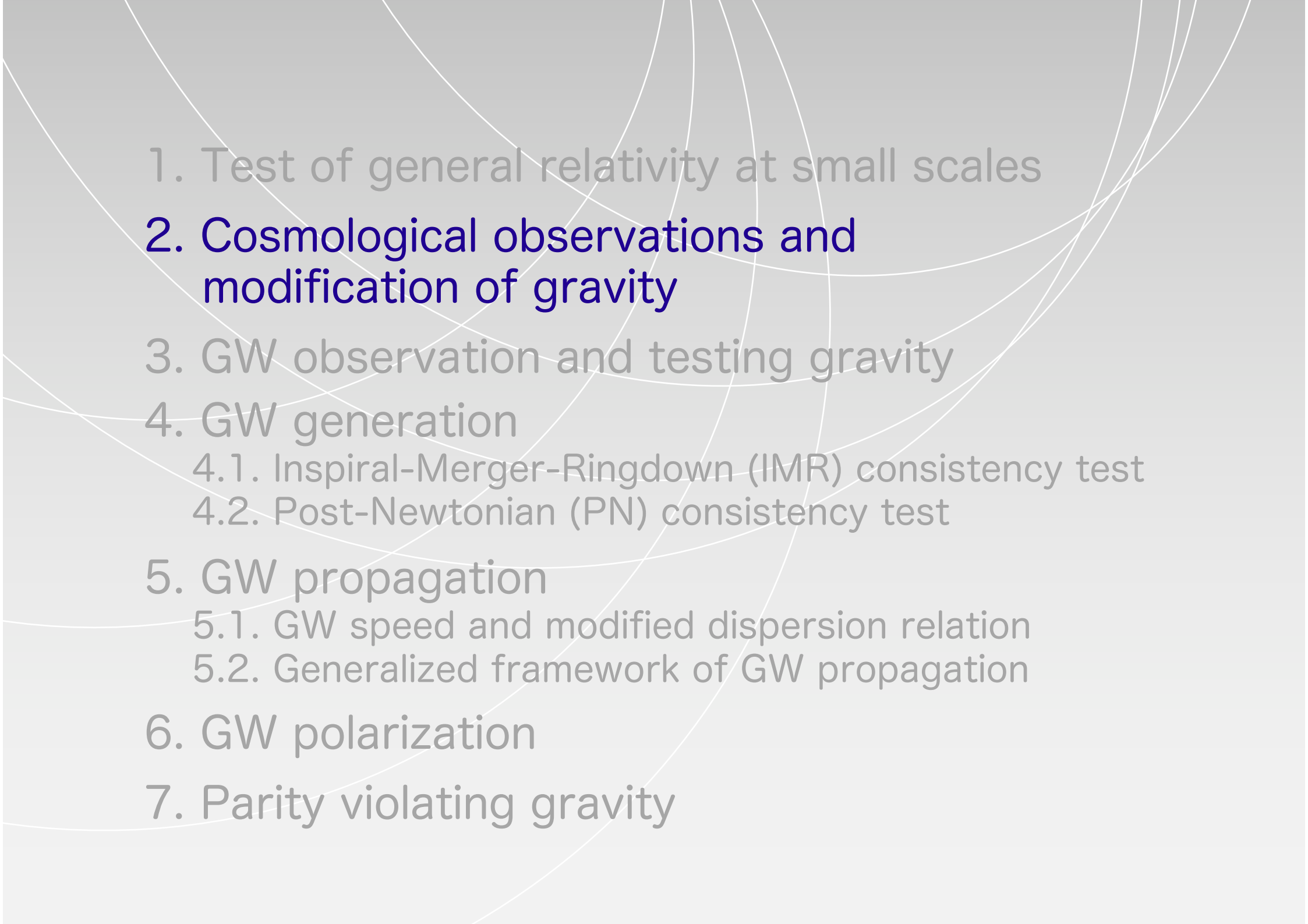
Novel prize in 1993:
Hulse & Taylor

Will, Living Review of Relativity (2014)

$$\frac{\Delta \dot{P}}{\dot{P}} = 0.003 \pm 0.002$$

Good agreement with GR
at the precision of ~0.1%.





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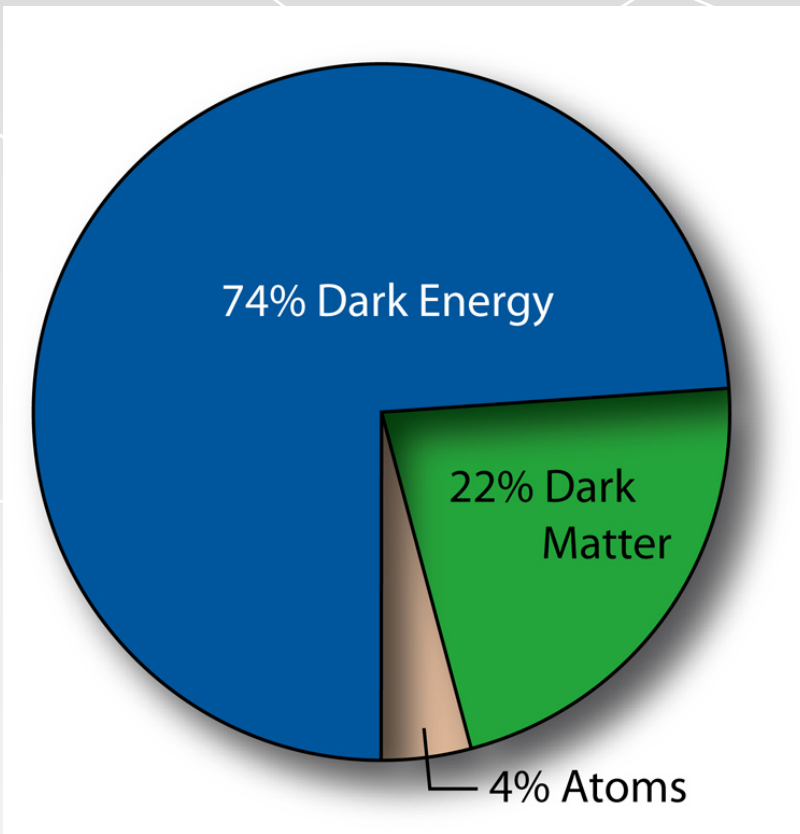
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Why modify gravity?

The current Universe is accelerating, but the physical mechanism is still unknown.

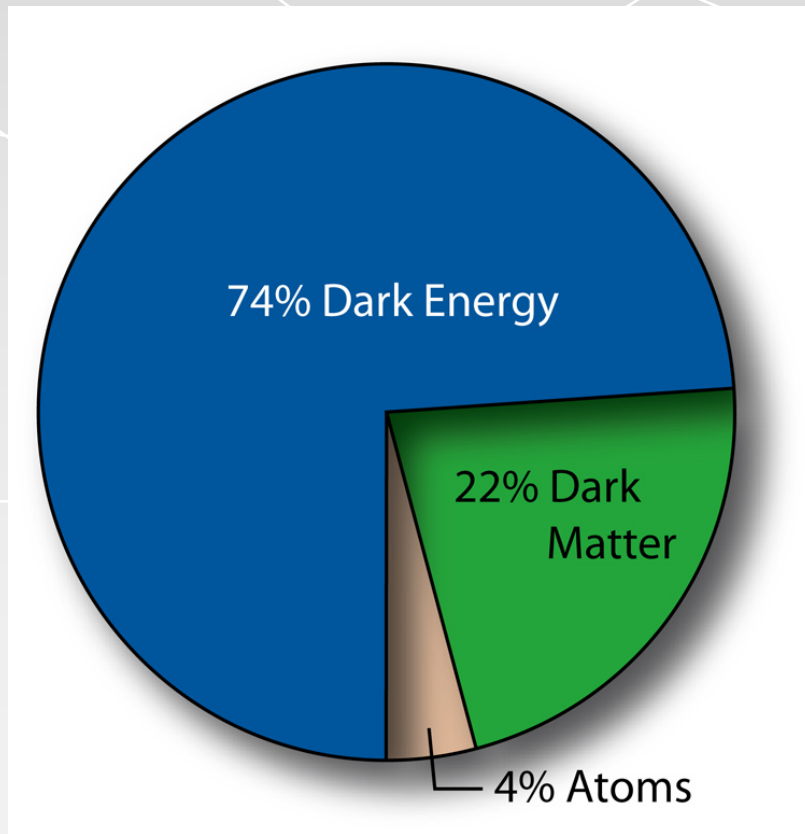


$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- cosmological constant
- Introducing a new energy component in the Universe (e.g. scalar field)
- Modifying general relativity at cosmological scales

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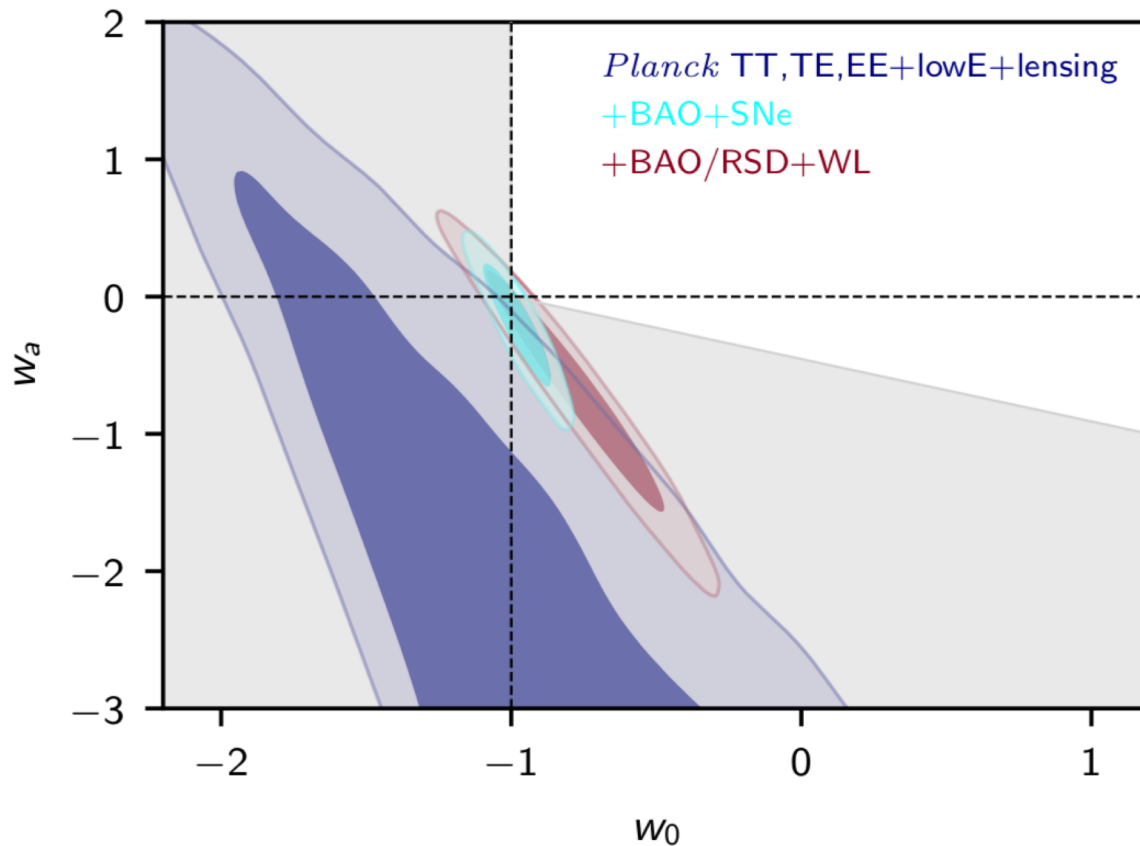


$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- cosmological constant
- Introducing a new energy component in the Universe (e.g. scalar field)
- **Modifying general relativity at cosmological scales**

Observation of cosmic expansion

Planck Collaboration 2018



dark-energy equation of state

$$p = w\rho$$

$$w(z) = w_0 + w_a \frac{z}{1+z}$$

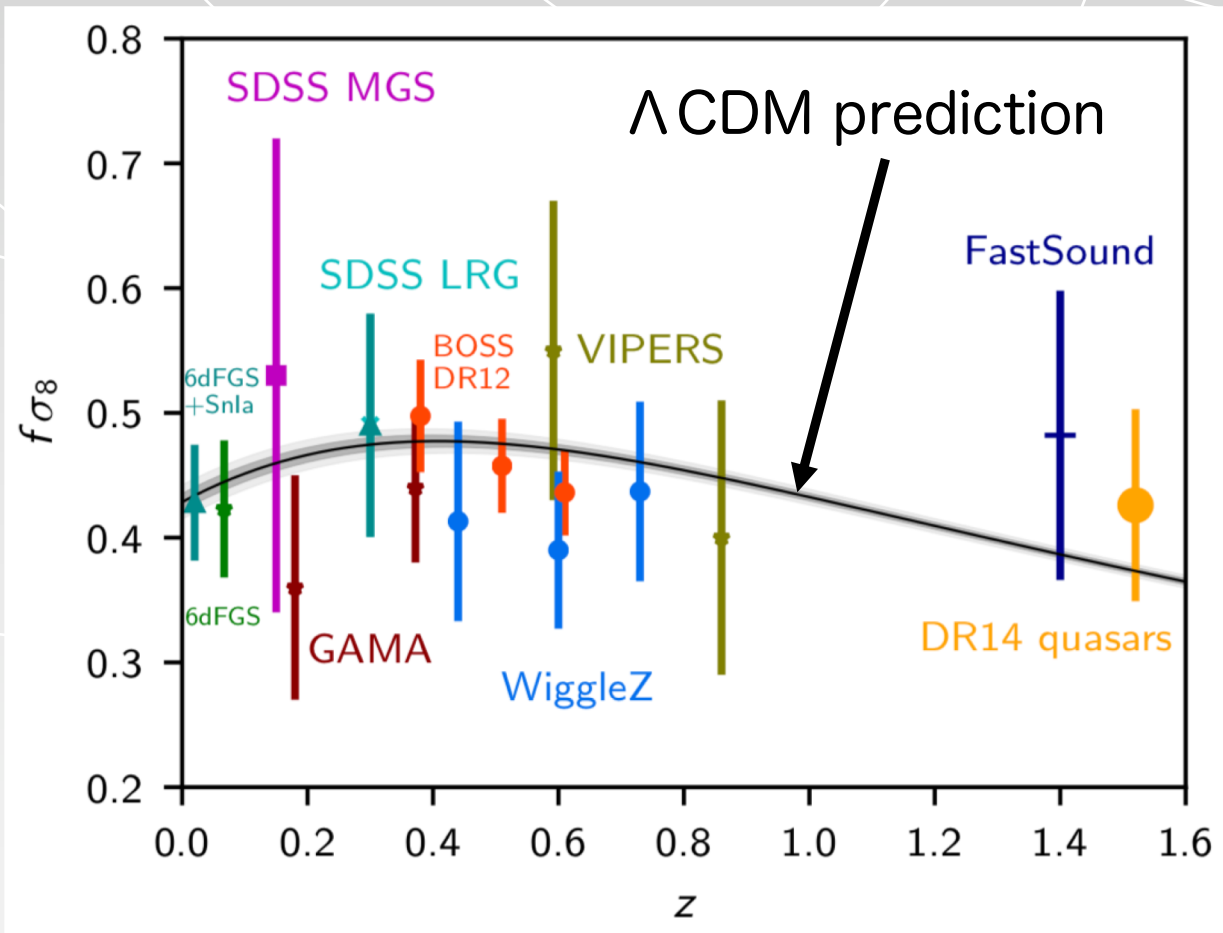
$$w = -1$$

for cosmological constant

The cosmic expansion needs to be close (~10%) to the Λ CDM model.

Observation of large scale structure

Planck Collaboration 2018



Current observational data from galaxy surveys limit the strength of matter clustering in the Universe.

The growth rate needs to be close (~20-30%) to the Λ CDM model.

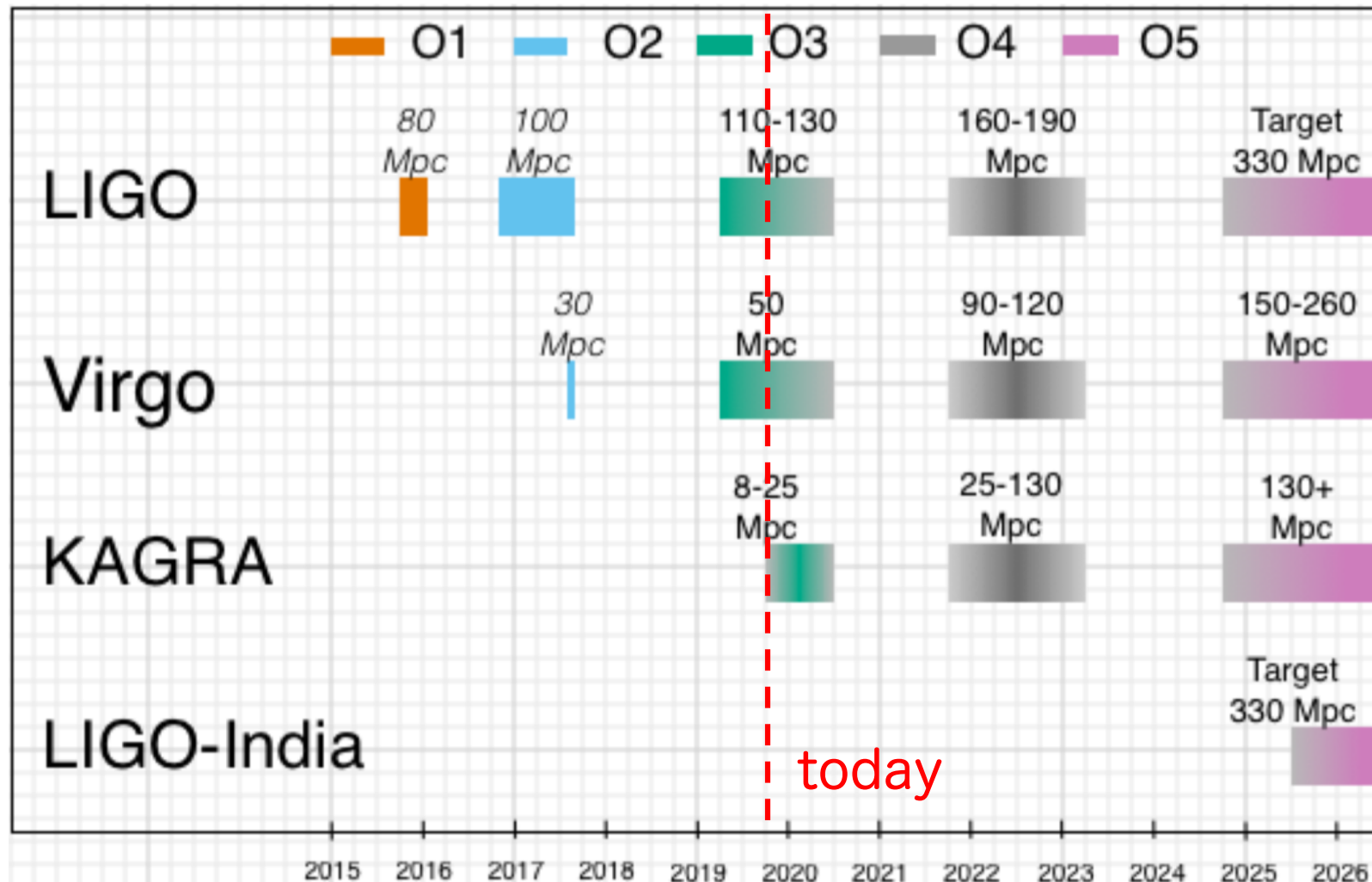
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GW detector network



Observation schedule

LVK collaboration, LRR (2020?), arXiv:1304.0670v8



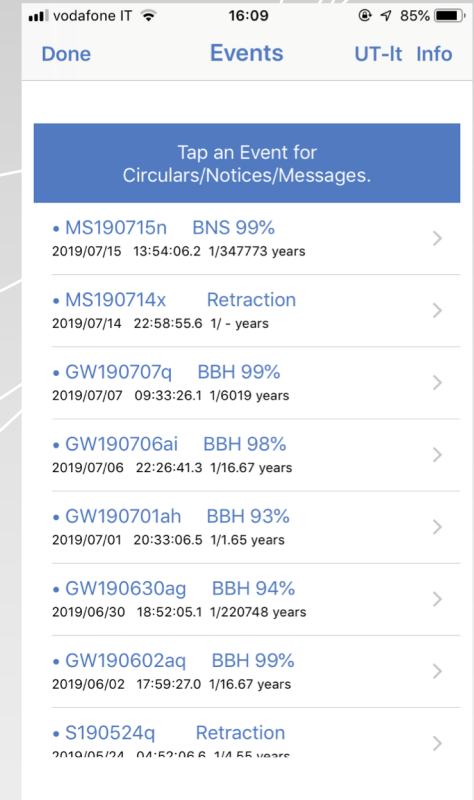
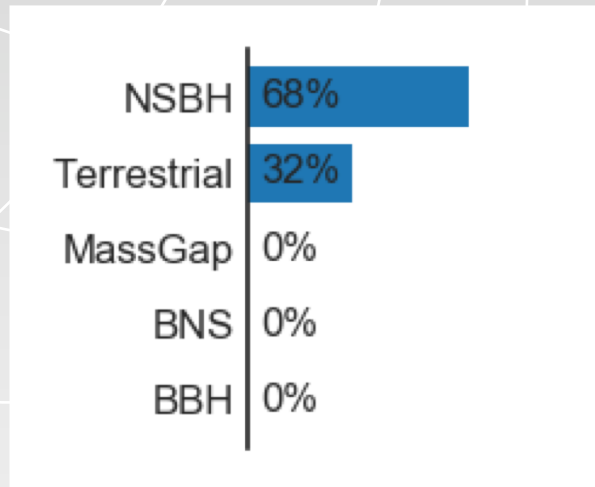
Current status of GW obs

- GW from 31 BBHs, 3-4 BNSs, 3.5 NSBHs have been detected so far (as of Sep. 30)

s190923

GraceDB (webpage)

Gravitational Wave Events (iphone app.)

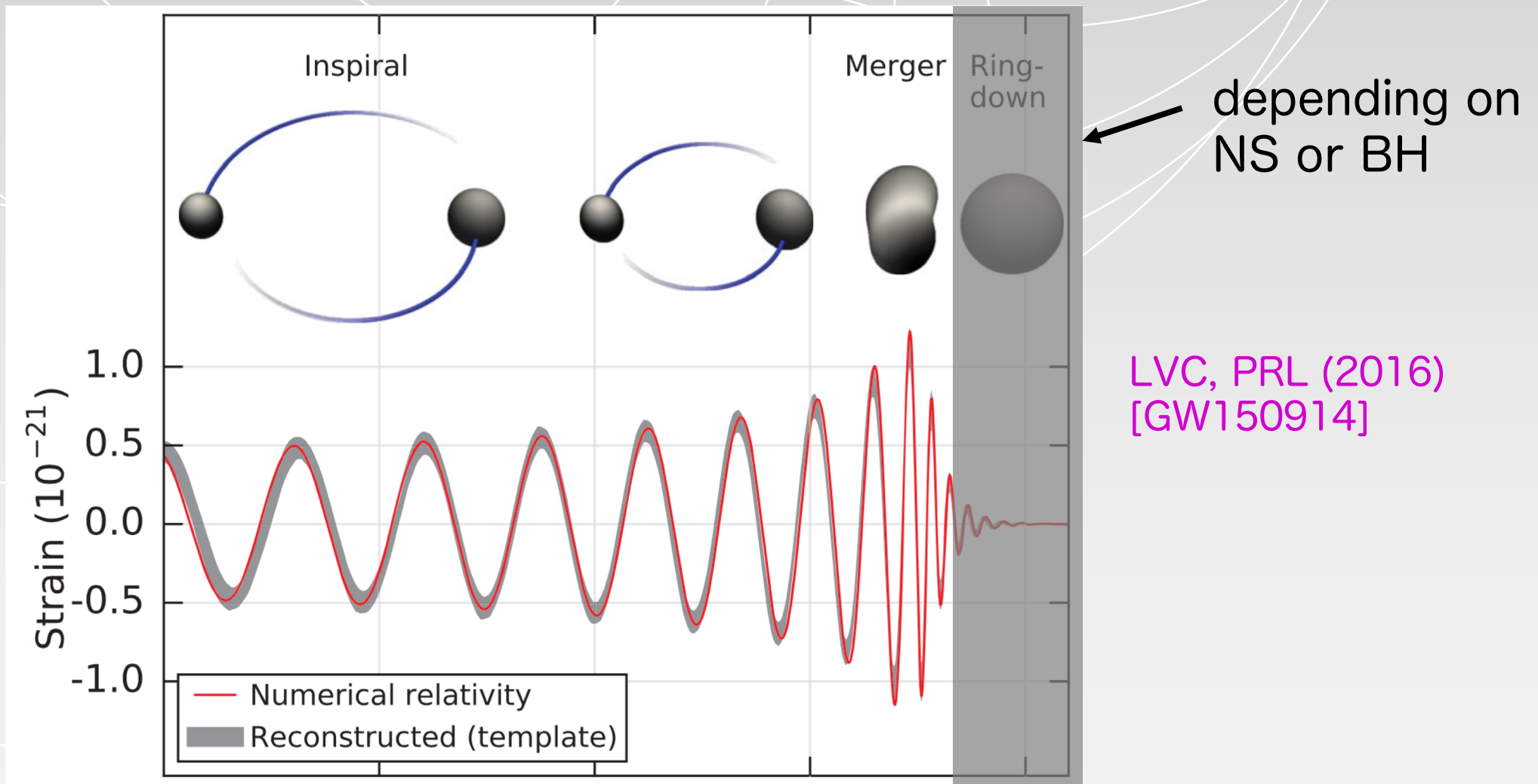


- The number of events expected (aLIGO, aVIRGO, KAGRA)
 - $\sim 100 - 1000$ BBH/yr up to $z \sim 1$
 - $\sim 20 - 400$ BNS/yr up to $z \sim 0.1$

GW will be a daily tool to probe for the Universe!

GW waveform

compact binaries (BBH, BNS, NS-BH)



GW in modified gravity

There are many modified gravity theories suggested to explain the cosmic acceleration expansion. These theories modify gravity at cosmological distance and also affect GW propagation.

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general features of modification on GWs

- different propagation speed
- different amplitude damping (variation of G)
- graviton mass
- additional GW polarizations (scalar & vector pols.)

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GW observation can be utilized for

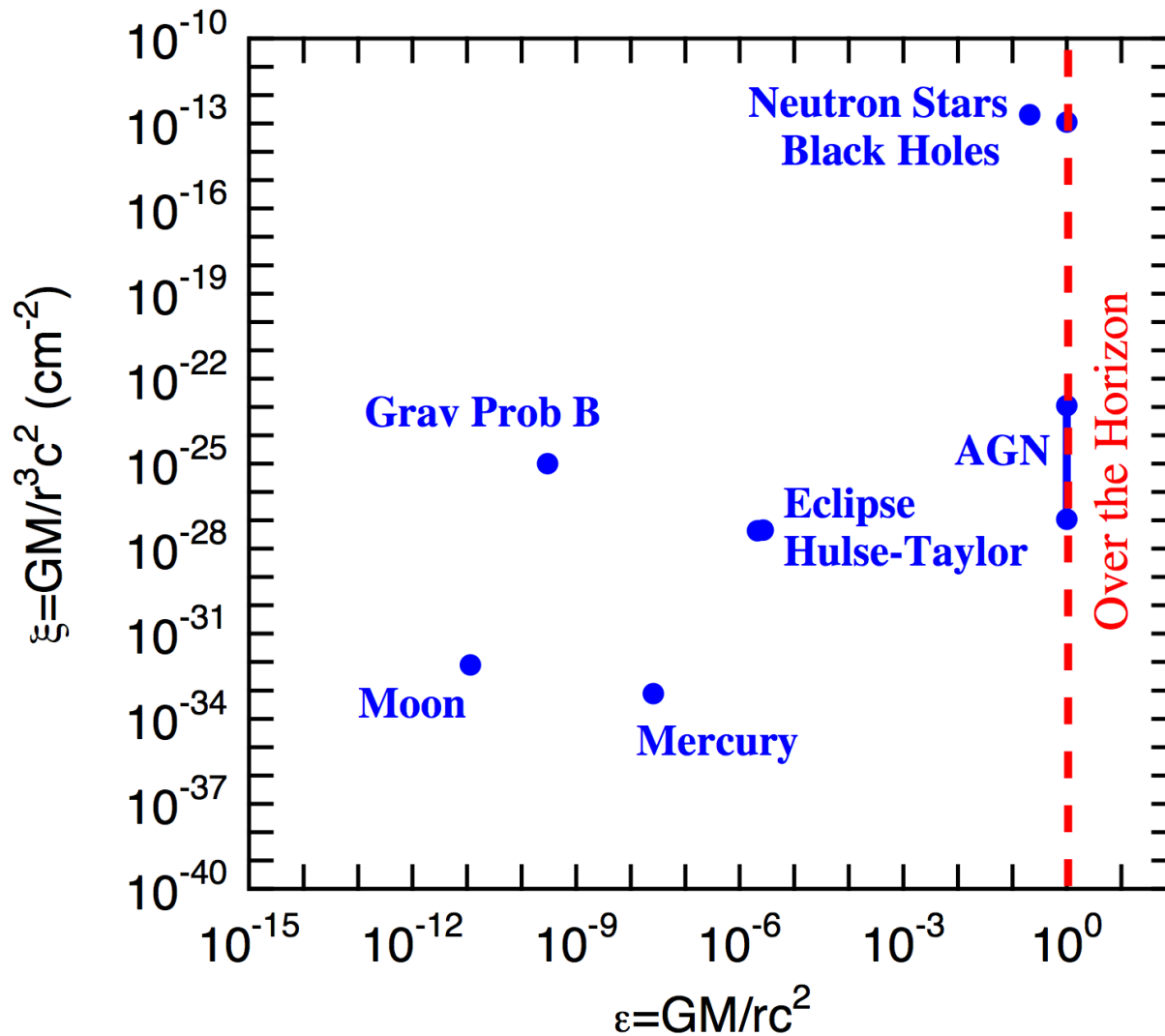
- testing general relativity directly
- resolving the cosmic acceleration problem

Gravity test with GW

Psaltis 2008
(modified)

small scale

spacetime curvature



large scale

gravitational potential

weak

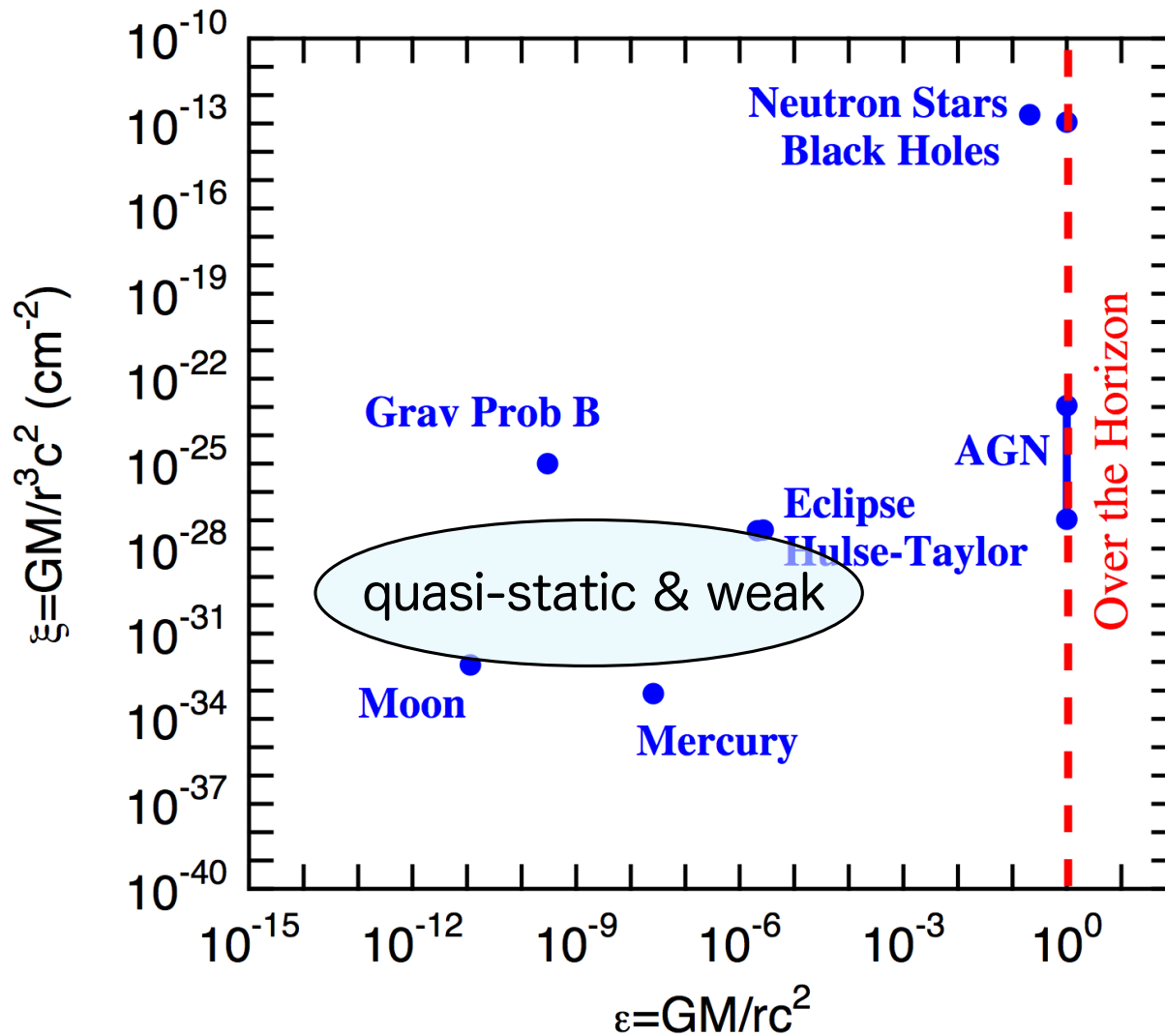
strong

Gravity test with GW

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large scale

gravitational potential

weak

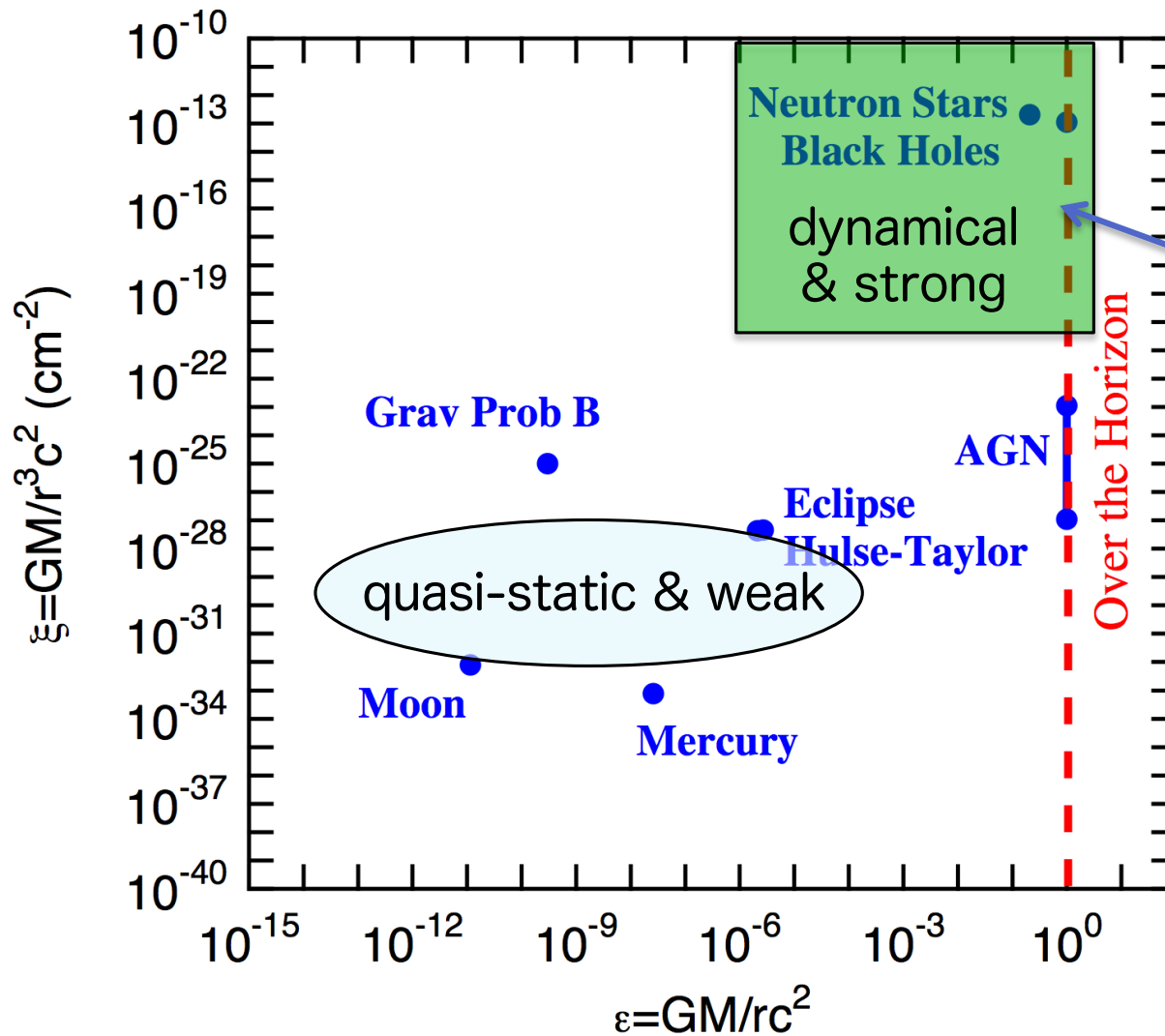
strong

Gravity test with GW

small scale

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Psaltis 2008
(modified)

GW generation

gravitational potential

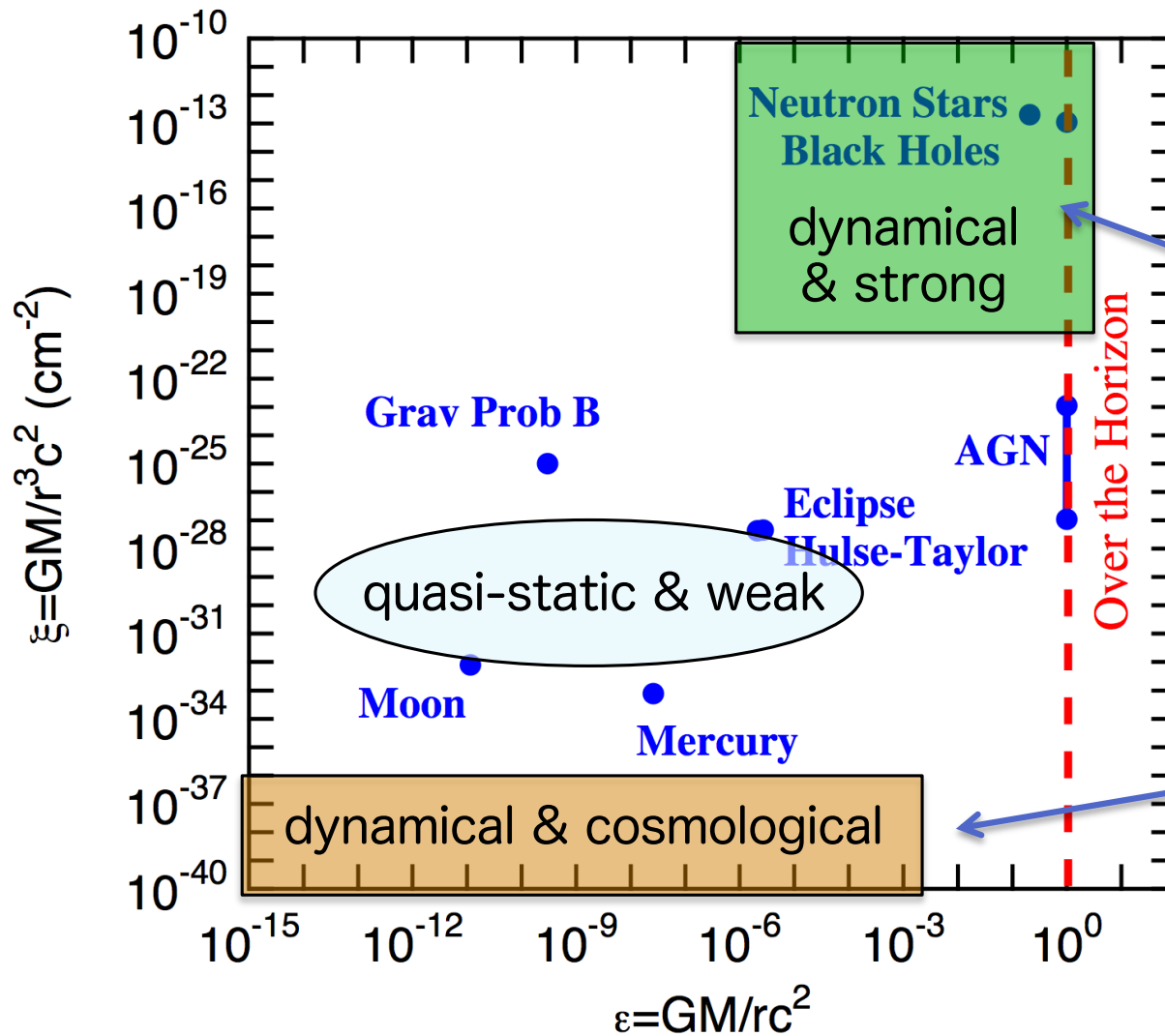
weak

strong

Gravity test with GW

small scale

spacetime curvature



Psaltis 2008
(modified)

GW generation

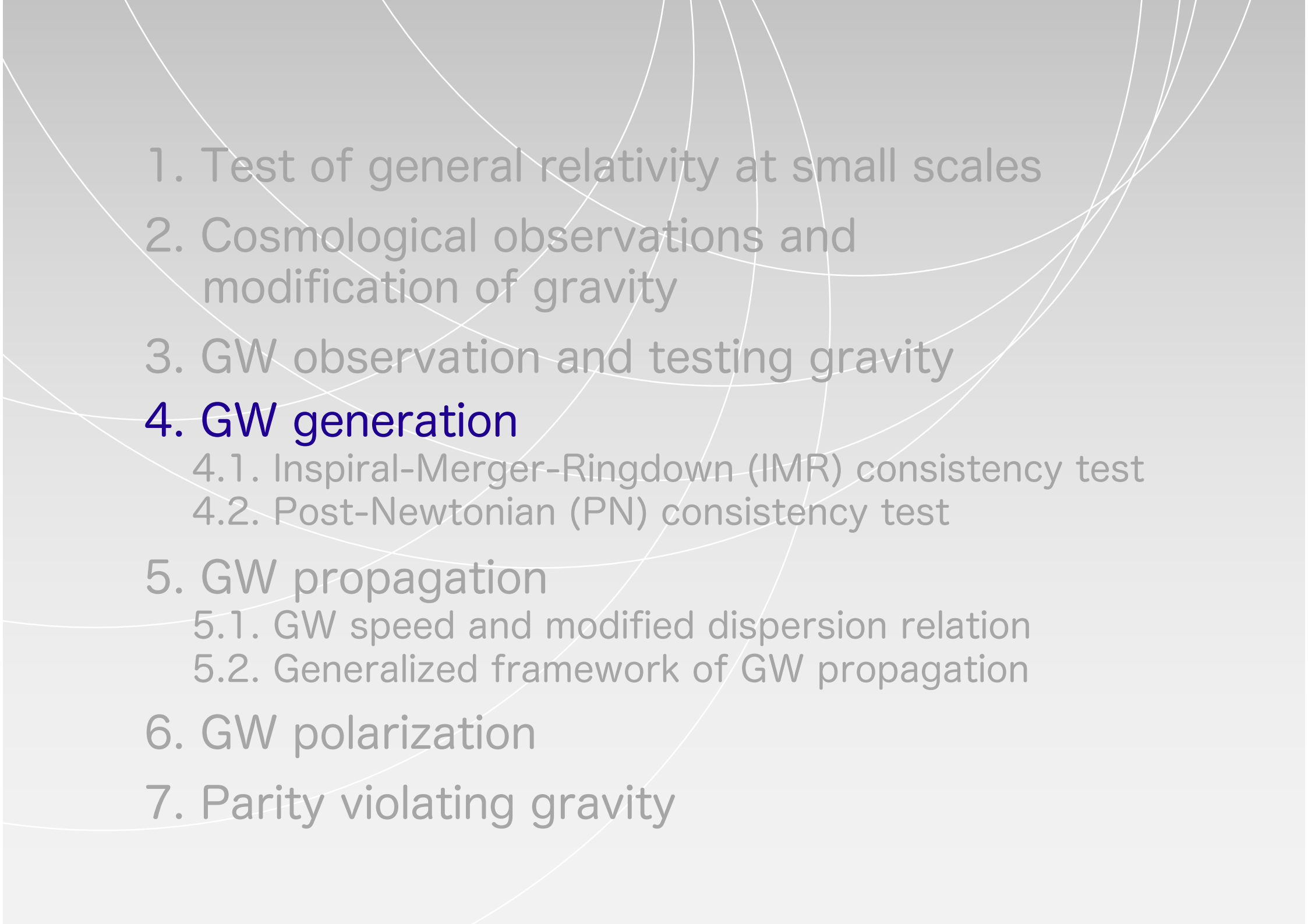
GW propagation

large scale

gravitational potential

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strong

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Inspiral-Merger-Ringdown (IMR) consistency test

Idea

Given masses and spins during inspiral phase, one can theoretically estimate the mass and spin after the coalescence.

$$M_f^I(m_1^I, m_2^I, \vec{\chi}_1^I, \vec{\chi}_2^I) , \quad a_f^I(m_1^I, m_2^I, \vec{\chi}_1^I, \vec{\chi}_2^I)$$

Fitting formula from numerical relativistic simulation.

Hofmann et al., ApJL (2016).

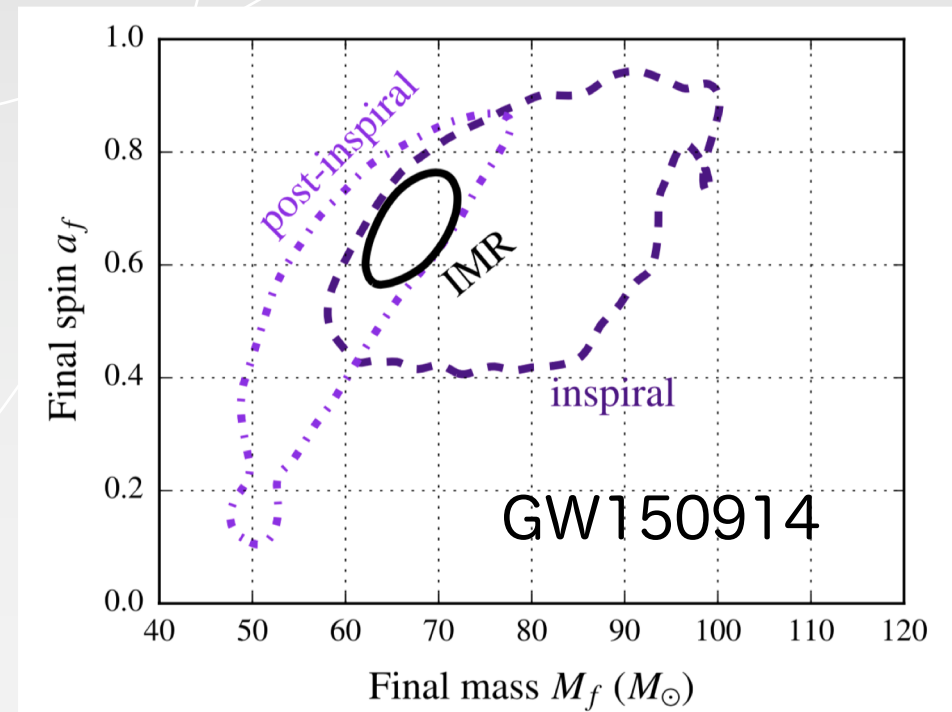
Healy and Lousto, PRD (2017).

Jimenez-Forteza et al., PRD (2017).

LVC, PRL 116, 221101 (2016)

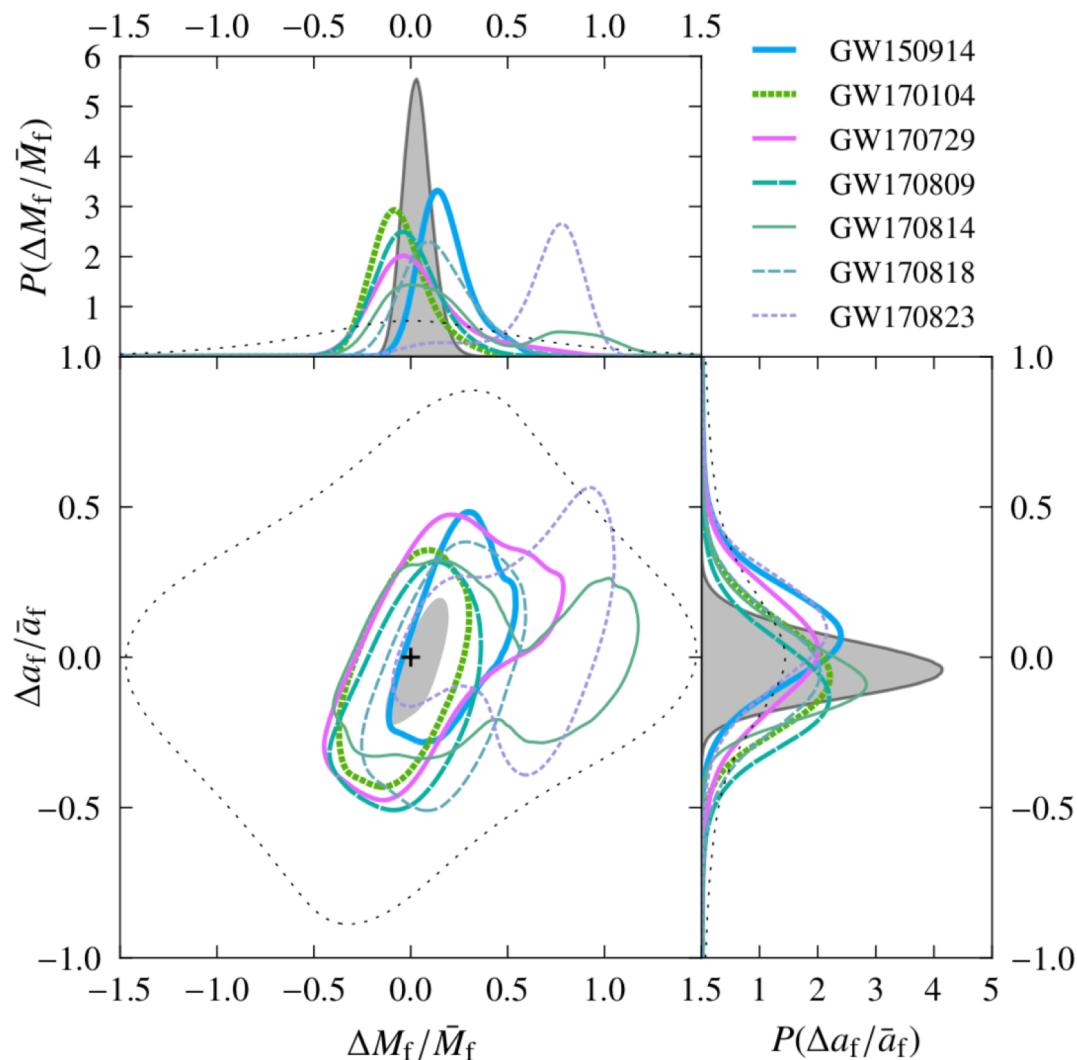
On the other hand, the final mass and spin are directly measured from the merger and ringdown phases.

$$M_f^{MR} , \quad a_f^{MR}$$



Recent results from LVC

LVC, arxiv:1903.04467.



$$\frac{\Delta M_f}{M_f} = 2 \frac{M_f^I - M_f^{\text{MR}}}{M_f^I + M_f^{\text{MR}}}$$

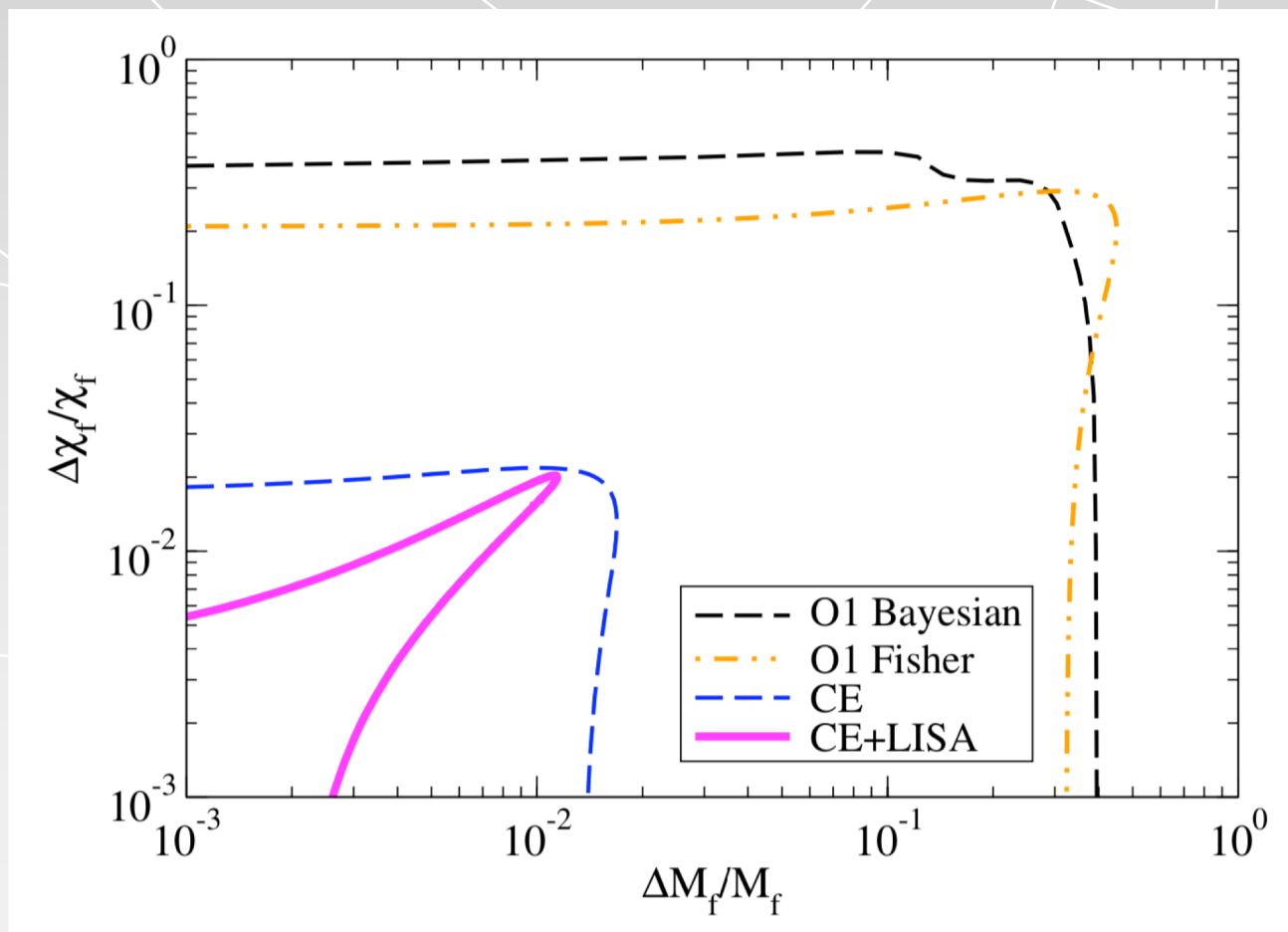
$$\frac{\Delta a_f}{a_f} = 2 \frac{a_f^I - a_f^{\text{MR}}}{a_f^I + a_f^{\text{MR}}}$$

Combined posterior gives the result consistent with GR at ~20% precision.

The second peaks?

Future prospect

Carson & Yagi, arxiv:1905.13155

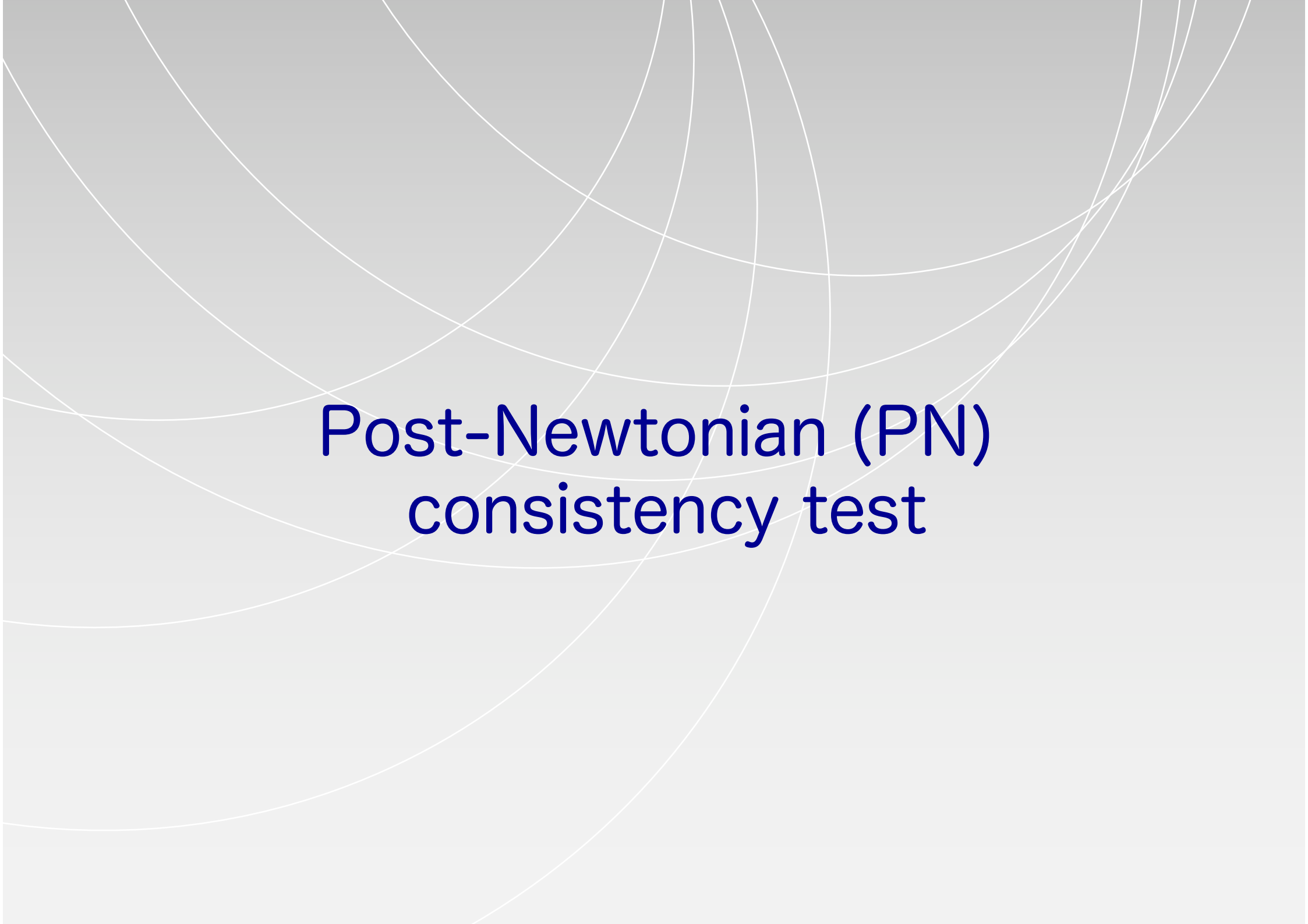


2G detectors
(LIGO, VIRGO, KAGRA)

➡ ~10%

3G detectors
(CE, ET)

➡ ~1%



Post-Newtonian (PN) consistency test

Post-Newtonian expansion

In a binary system, kinetic energy $\sim$ potential energy.

$$\Rightarrow \left(\frac{v}{c}\right)^2 \sim \left| \frac{2GM}{c^2 r} \right| = \frac{R_S}{r}$$

$\Rightarrow v/c$ is a small parameter in the early inspiral phase of a binary.

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \epsilon g_{\mu\nu}^{(1)} + \epsilon^2 g_{\mu\nu}^{(2)} + \epsilon^3 g_{\mu\nu}^{(3)} + \dots$$

$$\epsilon = v/c$$

$$T_{\mu\nu} = T_{\mu\nu}^{(0)} + \epsilon T_{\mu\nu}^{(1)} + \epsilon^2 T_{\mu\nu}^{(2)} + \epsilon^3 T_{\mu\nu}^{(3)} + \dots$$

Einstein eq. at each order can be solved one by one.

Post-Newtonian (PN) waveform

TaylorF2: f-domain waveform up to 3.0PN in amplitude & 3.5PN in phase e.g. Khan et al., PRD (2016)

$$h_{\text{GR}}(f) \propto A_{\text{ins}} e^{i\phi_{\text{ins}}}$$

$$u \equiv (\pi M f)^{1/3} \longleftrightarrow 0.5 \text{ PN}$$

phase

$$\phi_{\text{ins}} = 2\pi f t_c - \phi_c - \pi/4 + \frac{3}{128} (\pi \mathcal{M} f)^{-5/3} \sum_{k=0}^7 \varphi_k u^k$$

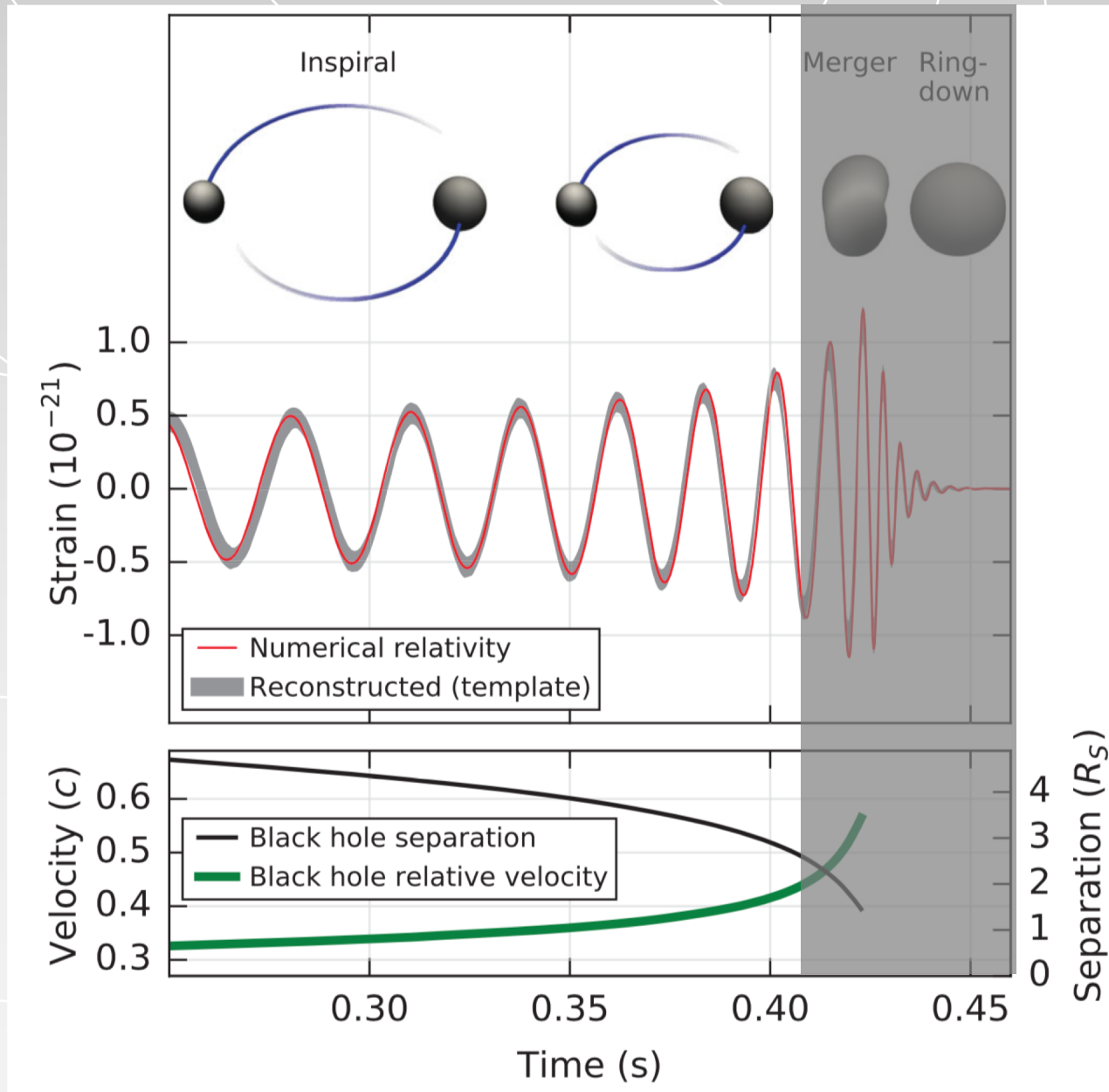
amplitude

$$A_{\text{ins}} = A_0 \sum_{k=0}^6 \mathcal{A}_k u^k, \quad A_0 = \frac{1}{\sqrt{6} \pi^{2/3} d_L} \mathcal{M}^{5/6} f^{-7/6}$$

$$\varphi_0 = 1, \quad \varphi_1 = 0, \quad \varphi_2 = \frac{3715}{756} + \frac{55}{9} \eta, \quad \dots$$

$$\mathcal{A}_0 = 1, \quad \mathcal{A}_1 = 0, \quad \mathcal{A}_2 = -\frac{323}{224} + \frac{451}{168} \eta, \quad \dots$$

Intuitive understanding



LVC, PRL (2016)
[GW150914]

low freq

➡ larger separation

➡ low velocity

➡ lower PN order
is more important

Parametrized-post Einsteinian (ppE) formalism

Yunes and Pretorius, PRD (2009)

$$h(f) = (1 + \alpha u^a) e^{i\beta u^b} h_{\text{GR}}(f) \quad \begin{matrix} a \\ b \end{matrix} \longleftrightarrow \begin{matrix} 0.5a \text{ PN} \\ 0.5(b+5) \text{ PN} \end{matrix}$$

Tahura & Yagi, PRD (2018)

PPE phase parameters

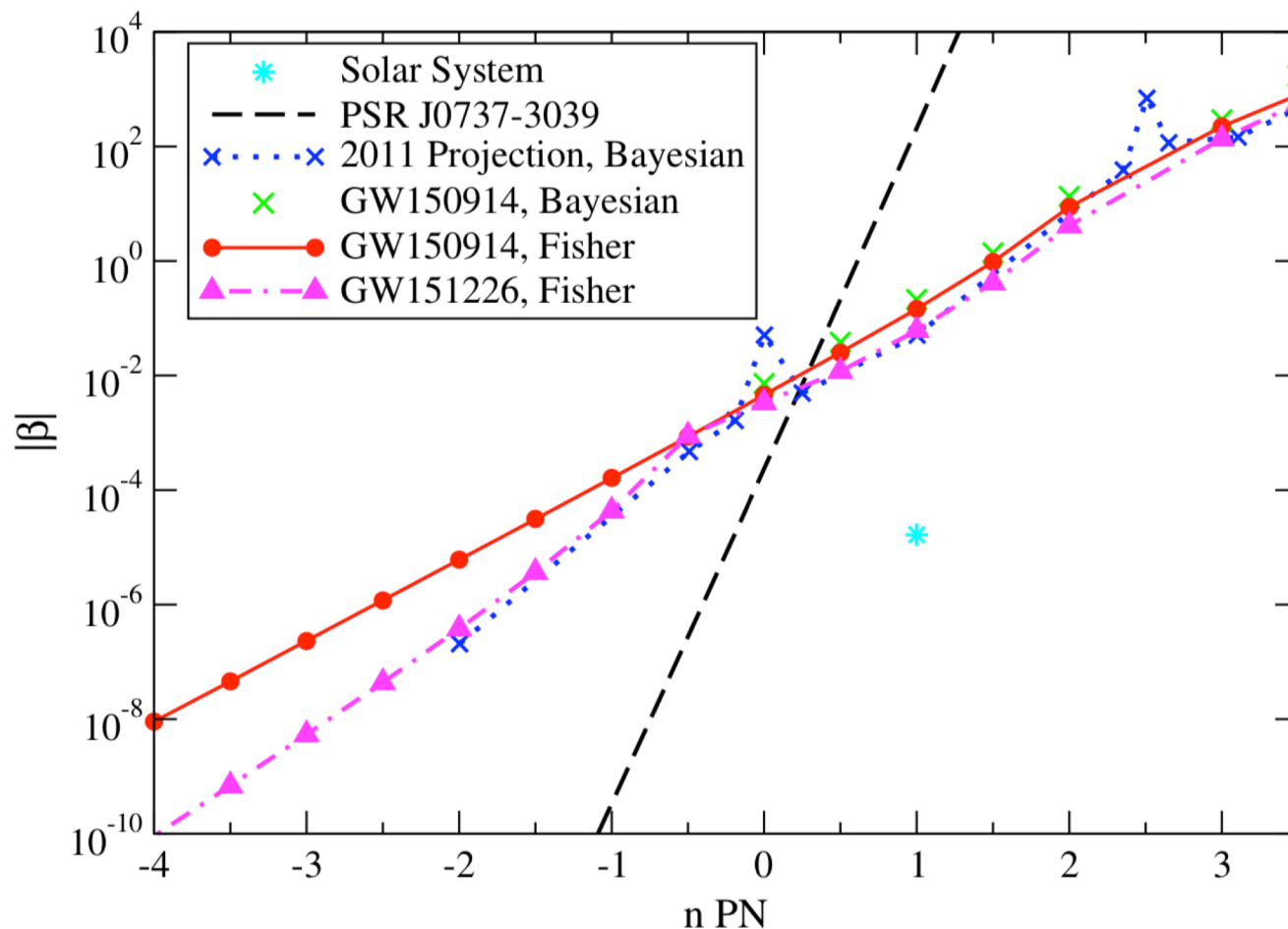
Theories	Magnitude (β)	Exp. (b)
Scalar tensor [95,96]	$-\frac{5}{7168} \eta^{2/5} (\alpha_1 - \alpha_2)^2$	-7
EdGB [97]	$-\frac{5}{7168} \zeta_{\text{EdGB}} \frac{(m_1^2 \tilde{s}_2^{\text{EdGB}} - m_2^2 \tilde{s}_1^{\text{EdGB}})^2}{m^4 \eta^{18/5}}$	-7
DCS [82,98]	$\frac{1549225}{11812864} \eta^{-14/5} \zeta_{\text{dCS}} [-2\delta_m \chi_a \chi_s + (1 - \frac{16068\eta}{61969}) \chi_a^2 + (1 - \frac{231808\eta}{61969}) \chi_s^2]$	-1
Einstein-Æther [99]	$-\frac{5}{3584} \eta^{2/5} \frac{(s_1^{\text{EA}} - s_2^{\text{EA}})^2}{[(1-s_1^{\text{EA}})(1-s_2^{\text{EA}})]^{4/3}} \left[\frac{(c_{14}-2)w_0^3 - w_1^3}{c_{14} w_0^3 w_1^3} \right]$	-7
Khronometric [99]	$-\frac{5}{3584} \eta^{2/5} \frac{(s_1^{\text{kh}} - s_2^{\text{kh}})^2}{[(1-s_1^{\text{kh}})(1-s_2^{\text{kh}})]^{4/3}} \sqrt{\bar{\alpha}_{\text{kh}}} \left[\frac{(\bar{\beta}_{\text{kh}}-1)(2+\bar{\beta}_{\text{kh}}+3\bar{\lambda}_{\text{kh}})}{(\bar{\alpha}_{\text{kh}}-2)(\bar{\beta}_{\text{kh}}+\bar{\lambda}_{\text{kh}})} \right]^{3/2}$	-7
Noncommutative [100]	$-\frac{75}{256} \eta^{-4/5} (2\eta - 1) \Lambda^2$	-1
Varying G [92]	$-\frac{25}{851968} \eta_0^{3/5} \dot{\mathbf{G}}_{\text{C},0} [\mathbf{11m}_0 + \mathbf{3(s}_{1,0} + \mathbf{s}_{2,0} - \delta_{\dot{\mathbf{G}}})\mathbf{m}_0 - \mathbf{41(m}_{1,0}\mathbf{s}_{1,0} + \mathbf{m}_{2,0}\mathbf{s}_{2,0})]$	-13

Tahura & Yagi, PRD (2018)

Theories	PPE amplitude parameters	
	Magnitude (α)	Exponent (a)
Scalar tensor [76,90,101]	$-\frac{5}{192}\eta^{2/5}(\alpha_1 - \alpha_2)^2$	-2
EdGB	$-\frac{5}{192}\zeta_{\text{EdGB}} \frac{(\mathbf{m}_1^2 \bar{s}_2^{\text{EdGB}} - \mathbf{m}_2^2 \bar{s}_1^{\text{EdGB}})^2}{\mathbf{m}^4 \eta^{18/5}}$	-2
DCS	$\frac{185627}{1107456}\eta^{-14/5}\zeta_{\text{dCS}}[-2\delta_{\mathbf{m}}\chi_{\mathbf{a}}\chi_{\mathbf{s}} + (1 - \frac{53408\eta}{14279})\chi_{\mathbf{a}}^2 + (1 - \frac{3708\eta}{14279})\chi_{\mathbf{s}}^2]$	+4
Einstein-Æther [99]	$-\frac{5}{96}\eta^{2/5} \frac{(s_1^{\text{EA}} - s_2^{\text{EA}})^2}{[(1-s_1^{\text{EA}})(1-s_2^{\text{EA}})]^{4/3}} \left[\frac{(c_{14}-2)w_0^3 - w_1^3}{c_{14}w_0^3 w_1^3} \right]$	-2
Khronometric [99]	$-\frac{5}{96}\eta^{2/5} \frac{(s_1^{\text{kh}} - s_2^{\text{kh}})^2}{[(1-s_1^{\text{kh}})(1-s_2^{\text{kh}})]^{4/3}} \sqrt{\bar{\alpha}_{\text{kh}}} \left[\frac{(\bar{\beta}_{\text{kh}}-1)(2+\bar{\beta}_{\text{kh}}+3\bar{\lambda}_{\text{kh}})}{(\bar{\alpha}_{\text{kh}}-2)(\bar{\beta}_{\text{kh}}+\bar{\lambda}_{\text{kh}})} \right]^{3/2}$	-2
Noncommutative	$-\frac{3}{8}\eta^{-4/5}(2\eta - 1)\Lambda^2$	+4
Varying G [92]	$\frac{5}{512}\eta_0^{3/5}\dot{\mathbf{G}}_{\text{C},0}[-7\mathbf{m}_0 + (s_{1,0} + s_{2,0} - \delta_{\dot{\mathbf{G}}})\mathbf{m}_0 + 13(\mathbf{m}_{1,0}s_{1,0} + \mathbf{m}_{2,0}s_{2,0})]$	-8

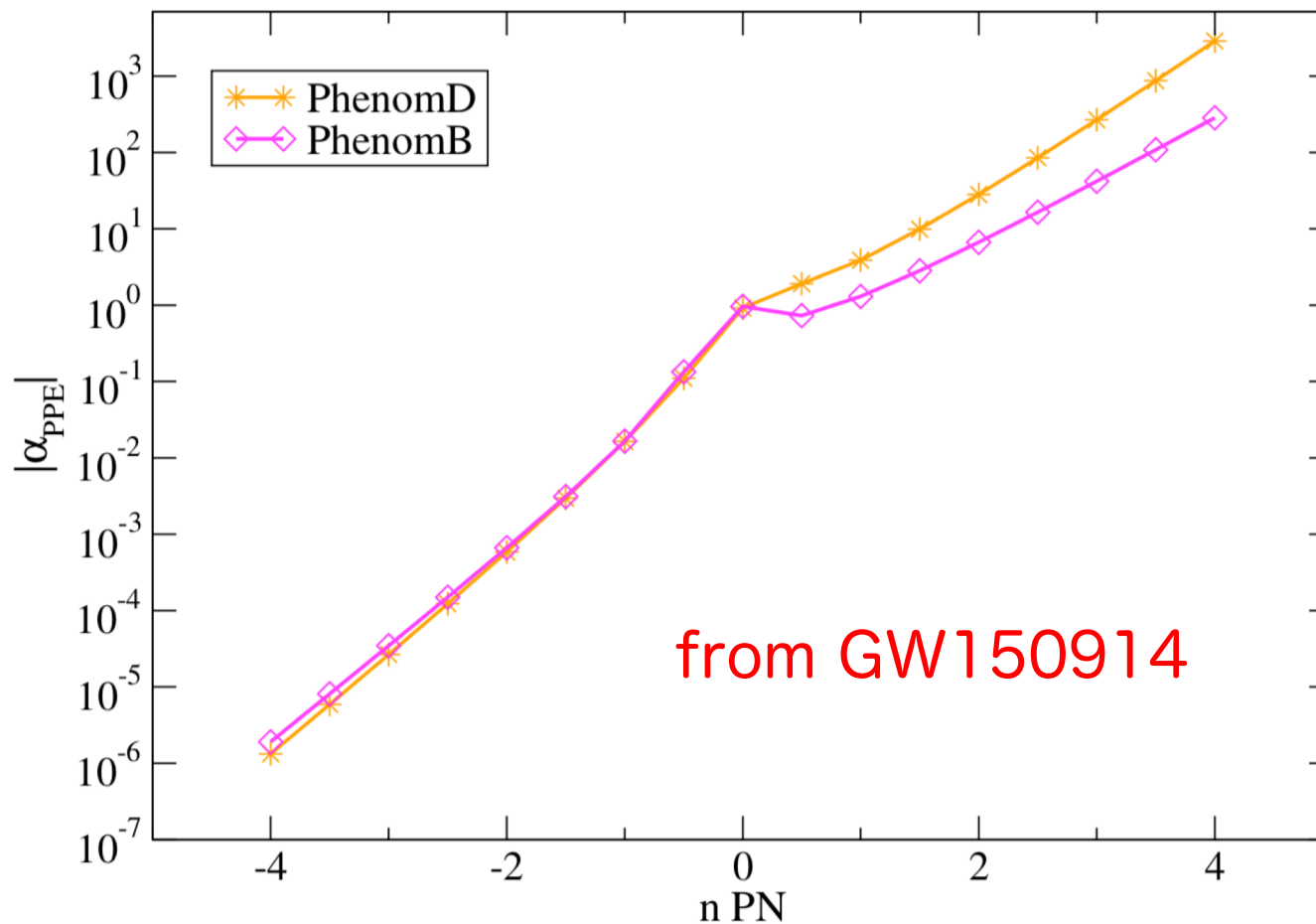
Current constraint on phase modification

Yunes, Yagi, Pretorius, PRD (2016)



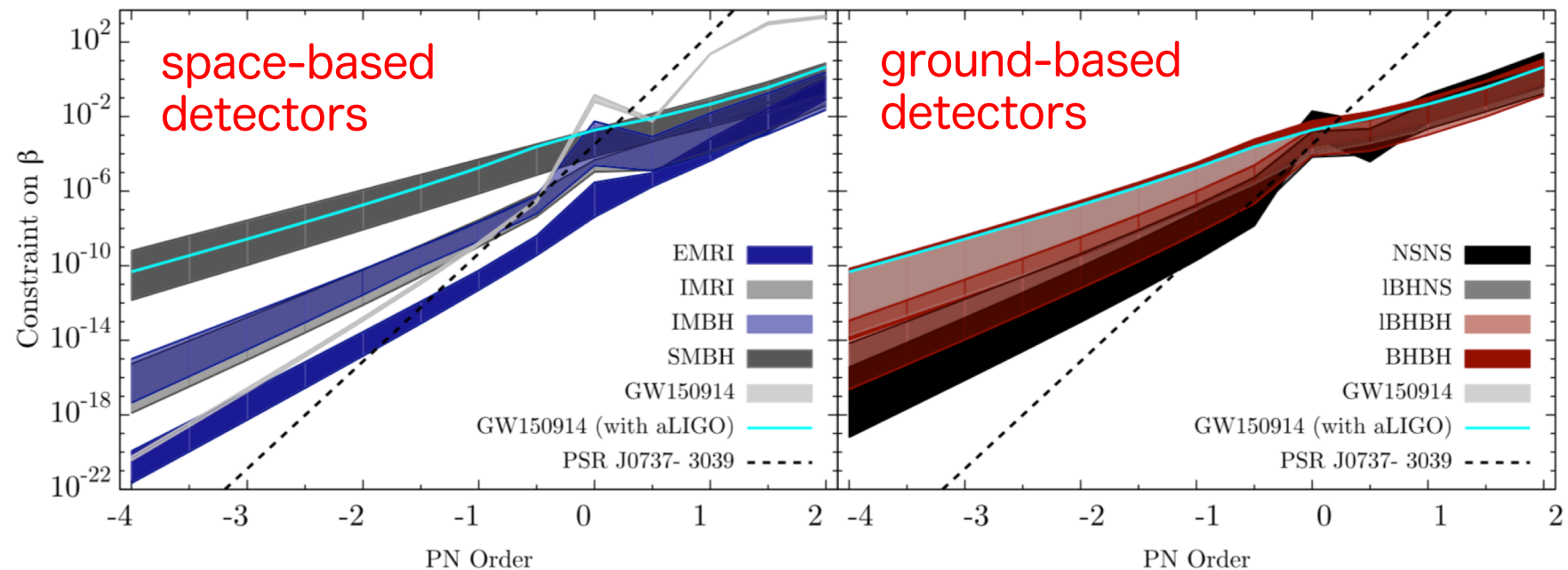
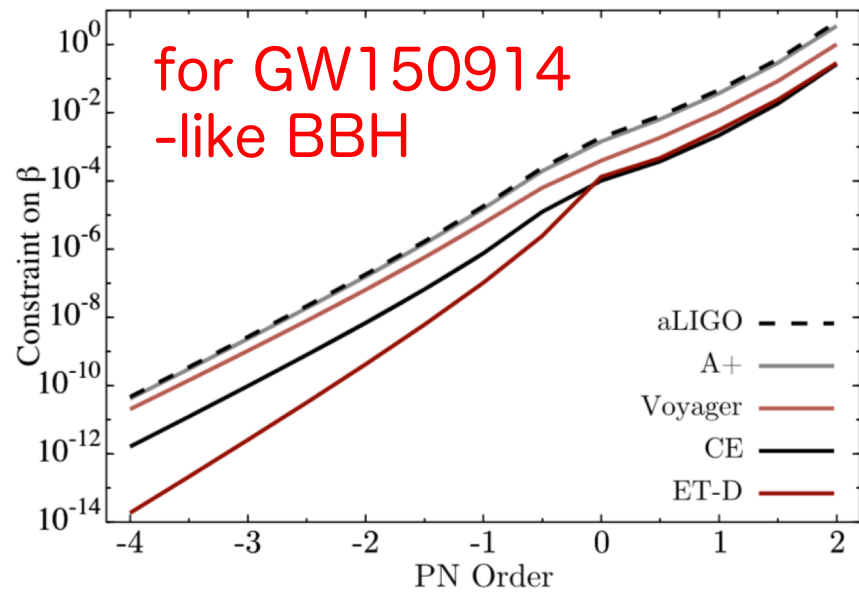
Current constraint on amplitude modification

Tahura et al., arXiv:1907.10059



Future constraint

Chamberlain & Yunes, PRD (2017)



Generalized IMR (gIMR) formalism

phase modification on GW waveform

$$h(f) = e^{i\delta\Phi_{\text{gIMR}}} h_{\text{GR}}(f)$$



coefficients for
each PN order

$$p_i \rightarrow (1 + \delta p_i) p_i$$

- for inspiral PN part

Li, Pozzo, et al., PRD (2012)

$$\delta\Phi_{\text{gIMR}} = \frac{3}{128\eta} \sum_{i=0}^7 \varphi_i (\pi M f)^{(i-5)/3} \delta\varphi_i$$

- for merger-ringdown part

Meidam et al., PRD (2018)

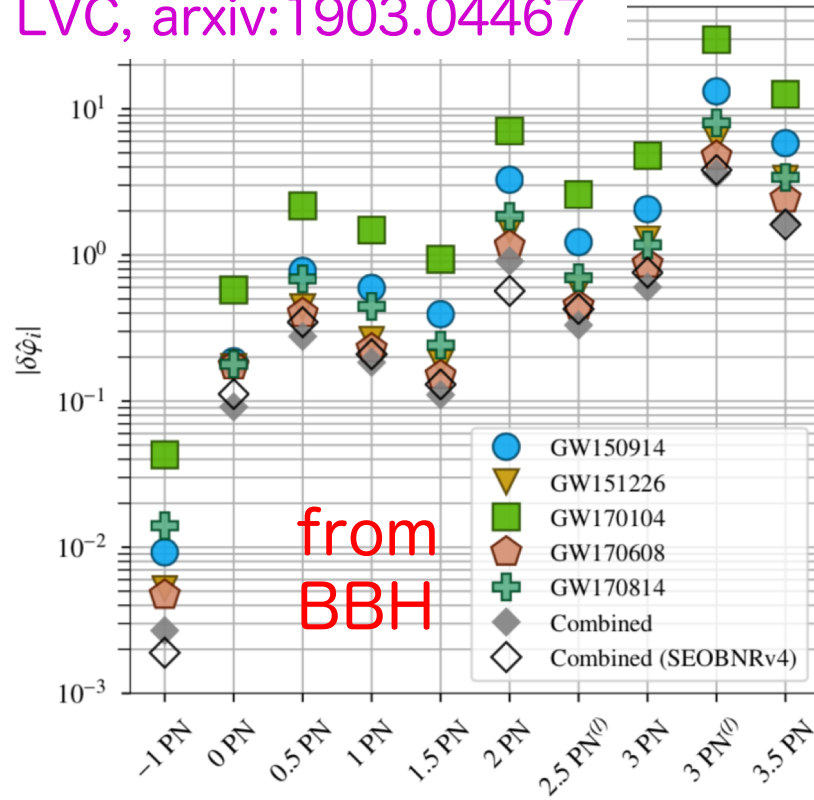
IMR PhenomPv2 waveform

$$\alpha_0, \dots, \alpha_5, \beta_0, \dots, \beta_3$$

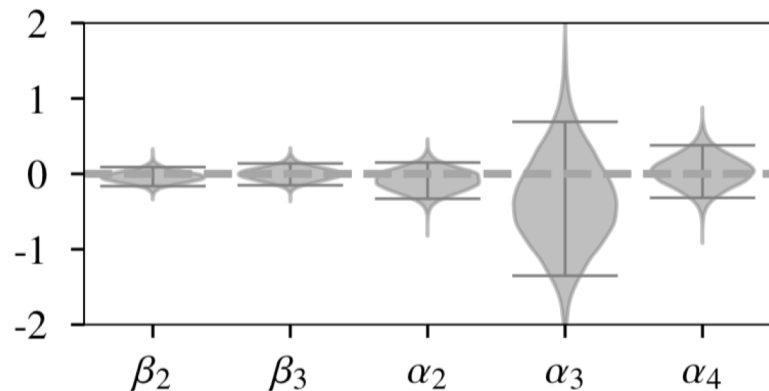
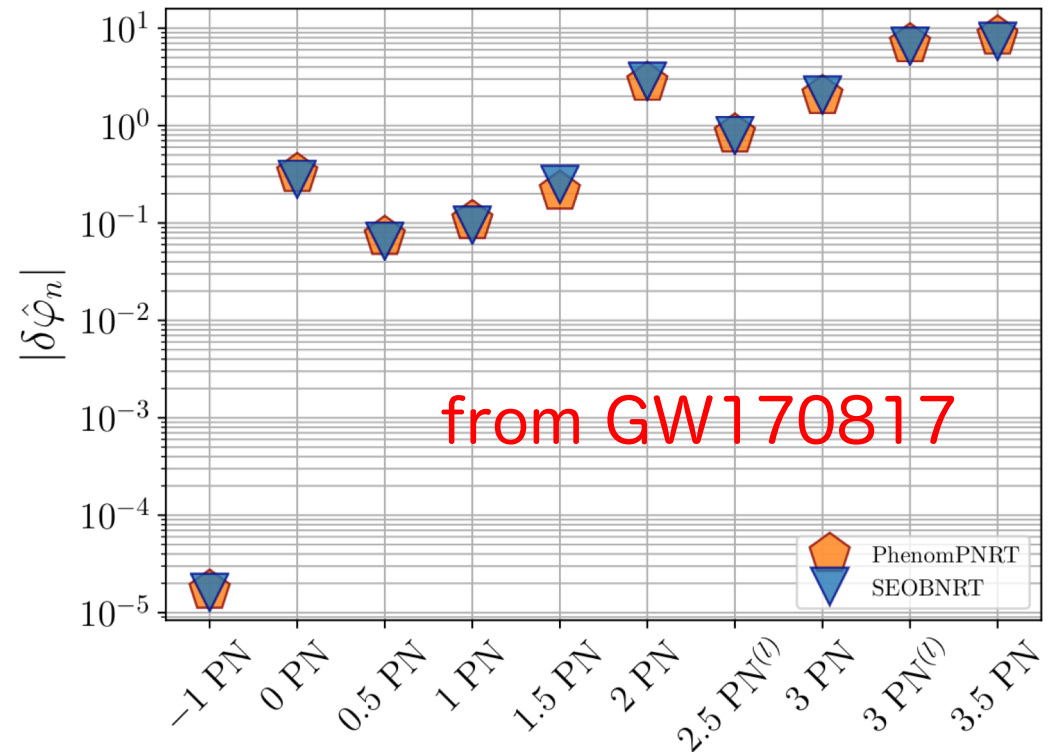
Khan et al., PRD (2016)

Current constraint

LVC, arxiv:1903.04467



LVC, PRL123, 011102 (2019)



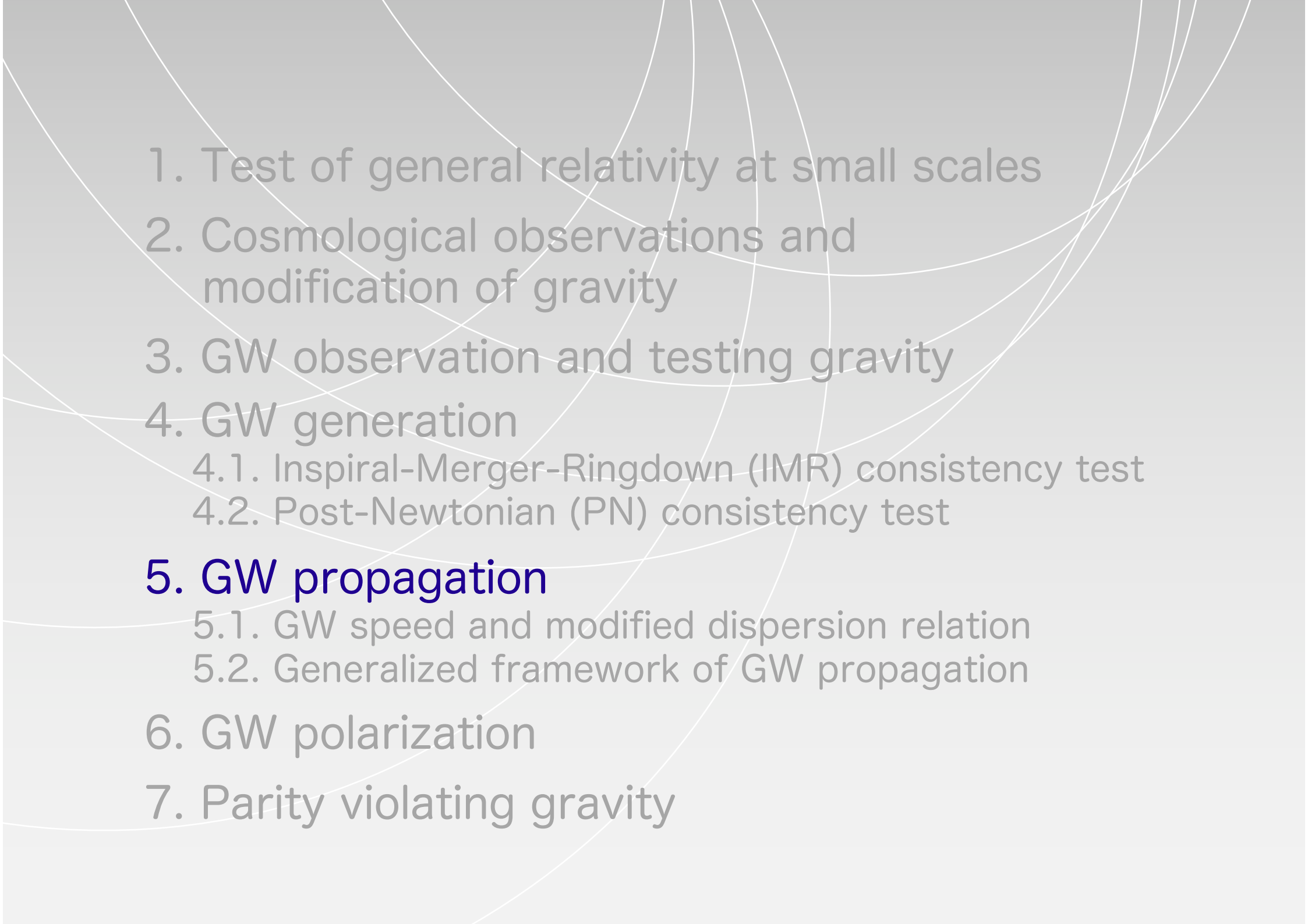
φ_{-2} (-1PN) & φ_1 (0.5PN)
are absolute values of deviation.

Low PN orders are better
constrained.

Open problems

- Modification on GW is not limited to the “known” waveforms in GR, e.g. PN expansion up to 3.5PN or parameters in the IMR Phenom waveforms.
- The ppE and gIMR parametrizations have a problem when multiple GW events are combined to constrain their deviation parameters.

$$h(f) = (1 + \alpha u^a) e^{i\beta u^b} h_{\text{GR}}(f)$$

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GW speed and modified dispersion relation

Constraints on GW speed before GW170817

From the observations of ultra-high energy cosmic rays (UHECR)

Moore & Nelson 2001

If graviton propagate with subluminal speed, it loses energy due to gravitational Cherenkov radiation.

$$\delta_g \equiv 1 - \frac{v_g}{c} < 2 \times 10^{-15} \quad \text{applied only to subluminal case at } \sim 10^{33} \text{ Hz}$$

From arrival time difference between LIGOs

Cornish et al. 2017

GW150914 6.9 ± 0.30 ms

GW151226 1.1 ± 0.18 ms

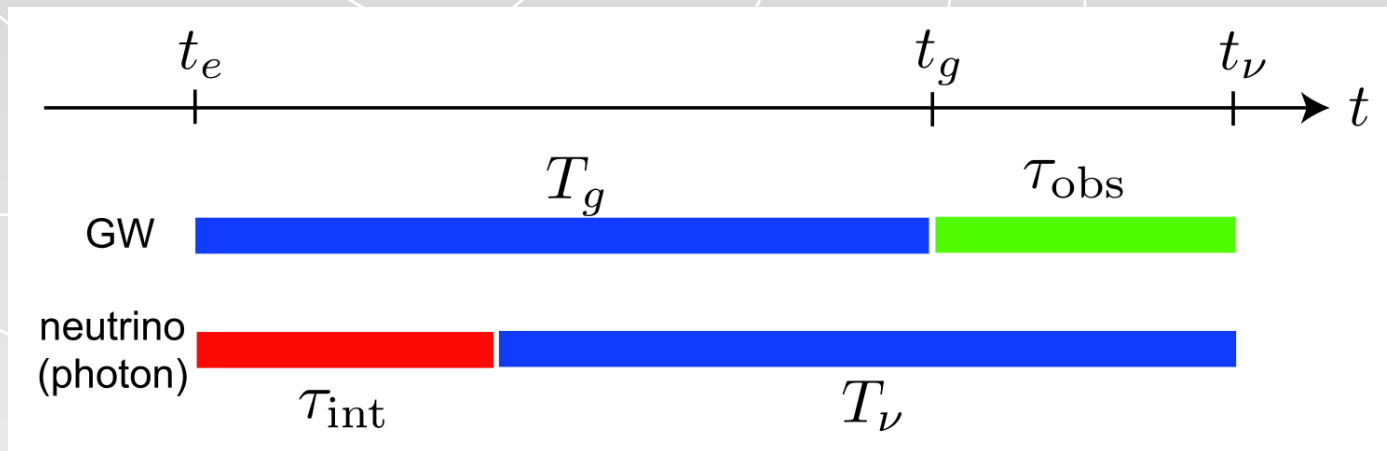
GW170104 3.0 ± 0.30 ms

$$\Rightarrow -0.42 < \delta_g < 0.45$$

Measuring GW speed

Nishizawa & Nakamura (2014)

arrival time difference between GW and short GRB



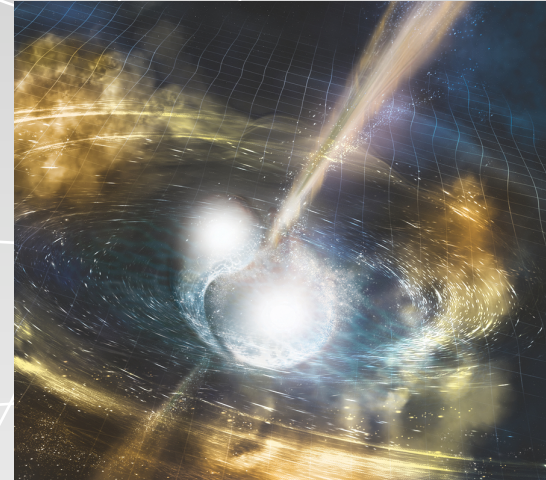
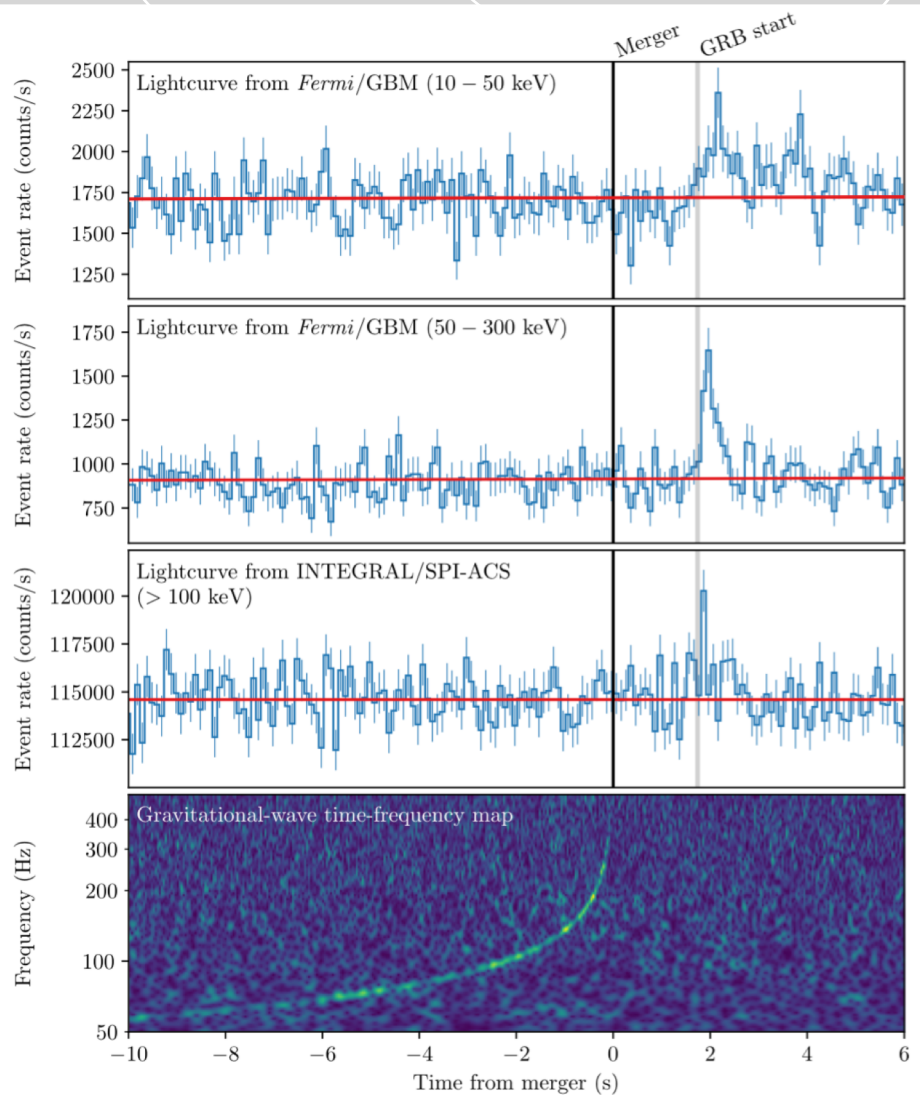
$$|\delta_g| < \frac{c \Delta\tau_{\text{int}}}{D} \quad \delta_g \equiv \frac{c - v_g}{c}$$

$\Delta\tau_{\text{int}}$: uncertainty of intrinsic time delay

$$\Delta\tau_{\text{int}} = 10 \text{ sec} , \quad D = 200 \text{ Mpc} \quad \Rightarrow \quad |\delta_g| < 5 \times 10^{-16}$$

GW170817

LSC + Fermi + INTEGRAL, ApJL 848, L13 (2017)



The arrival time delay between a GW and a sGRB was ~ 1.7 sec after propagating the distance of ~ 40 Mpc.

$$-3 \times 10^{-15} < \frac{c_{\text{gw}} - c}{c} < 7 \times 10^{-16}$$

More details

measured values

$$\tau_{\text{obs}} = 1.7 \text{ sec} , \quad D_{\text{min}} = 26 \text{ Mpc}$$

$$\delta_g \equiv \frac{c - v_g}{c}$$

- superluminal

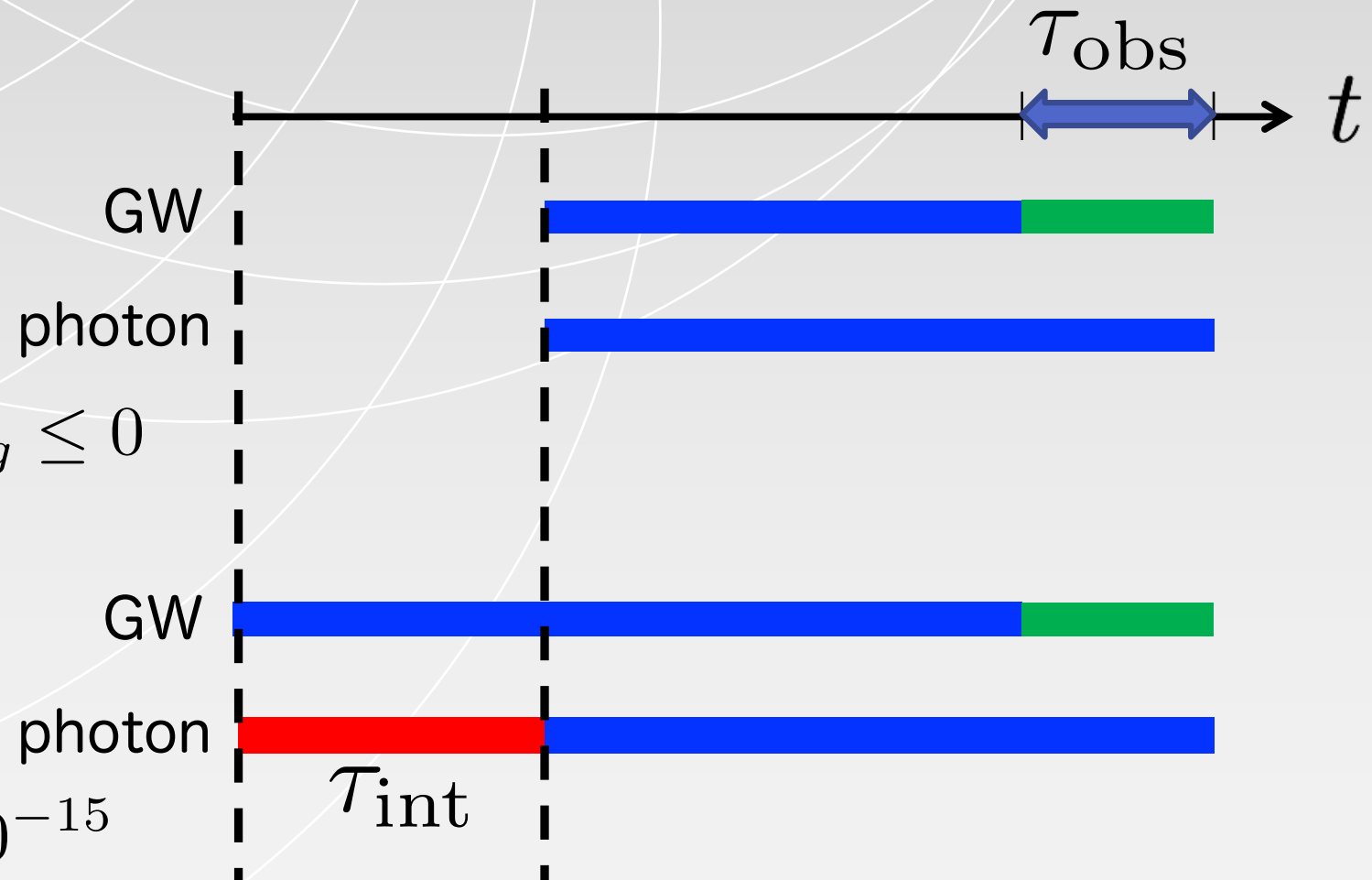
$$\tau_{\text{int}} > 0$$

$$-7 \times 10^{-16} < \delta_g \leq 0$$

- subluminal

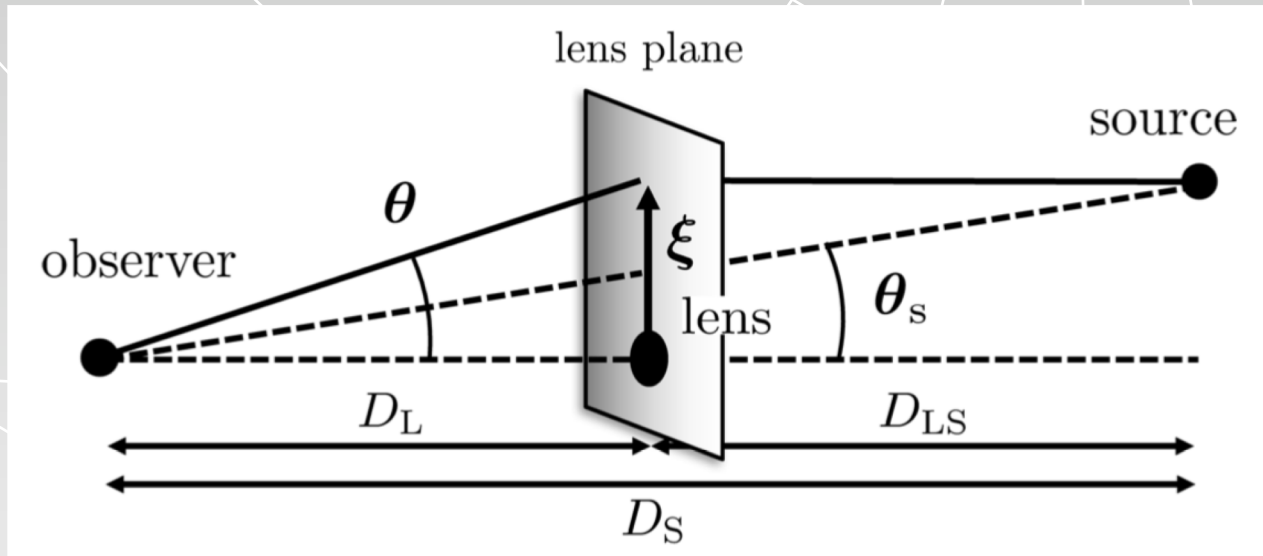
$$\tau_{\text{int}} < 10 \text{ sec}$$

$$0 \leq \delta_g < 3 \times 10^{-15}$$



Wave effect of lensing

Takahashi, ApJ (2017)



The geometrical optics breaks down for GW lensed by mass of $\lesssim 10^5 M_\odot \left(\frac{1 \text{ Hz}}{f} \right)$

GW with frequency f can arrive earlier than EM wave by time lag of $0.1 \text{ sec} \left(\frac{1 \text{ Hz}}{f} \right)$ $\rightarrow$ relevant to space-based detectors (LISA, DECIGO)

Massive graviton

$$E^2 = m_g^2 c^2 + p^2 c^2$$

Will, PRD (1998)

→ propagation speed $\frac{v_g}{c} = \frac{1}{c} \frac{d\omega}{dk} \approx 1 - \frac{m_g^2 c^4}{2E^2}$

→ GW phase shift (at 1.0 PN) $\Delta\Psi(f) = -\frac{\pi^2 D M_z}{\lambda_g^2 (1+z)} (\pi M_z f)^{-1}$

From the observation of BBHs,

$$m_g \lesssim 5.0 \times 10^{-23} \text{ eV}$$

LVC, arxiv:1903.04467

Stronger than recent one from the Solar system

$$m_g \lesssim 6.8 \times 10^{-23} \text{ eV}$$

Bernus et al., PRL (2019)

Other constraints

Galaxy cluster	$m_g \lesssim 10^{-29} \text{ eV}$	Goldhaber & Nieto 1974
Weak lensing	$m_g \lesssim 10^{-31} \text{ eV}$	Choudhury et al. 2004
CMB	$m_g \lesssim 10^{-30} \text{ eV}$	Dubovsky et al. 2010

The above is static bounds based on the modification of Newtonian potential (background level).

Th new bound obtained from BBH is dynamical and for the tensor pol of a GW.

Mass constraints on scalar and vector pol modes of GW are NOT so strong and graviton mass could be quite massive.

Modified dispersion relation

$$E^2 = p^2 c^2 + A_\alpha p^\alpha c^\alpha$$

Mirshekari et al., PRD (2012)

$\alpha = 2$: constant change of propagation speed

$\alpha = 0$: massive graviton

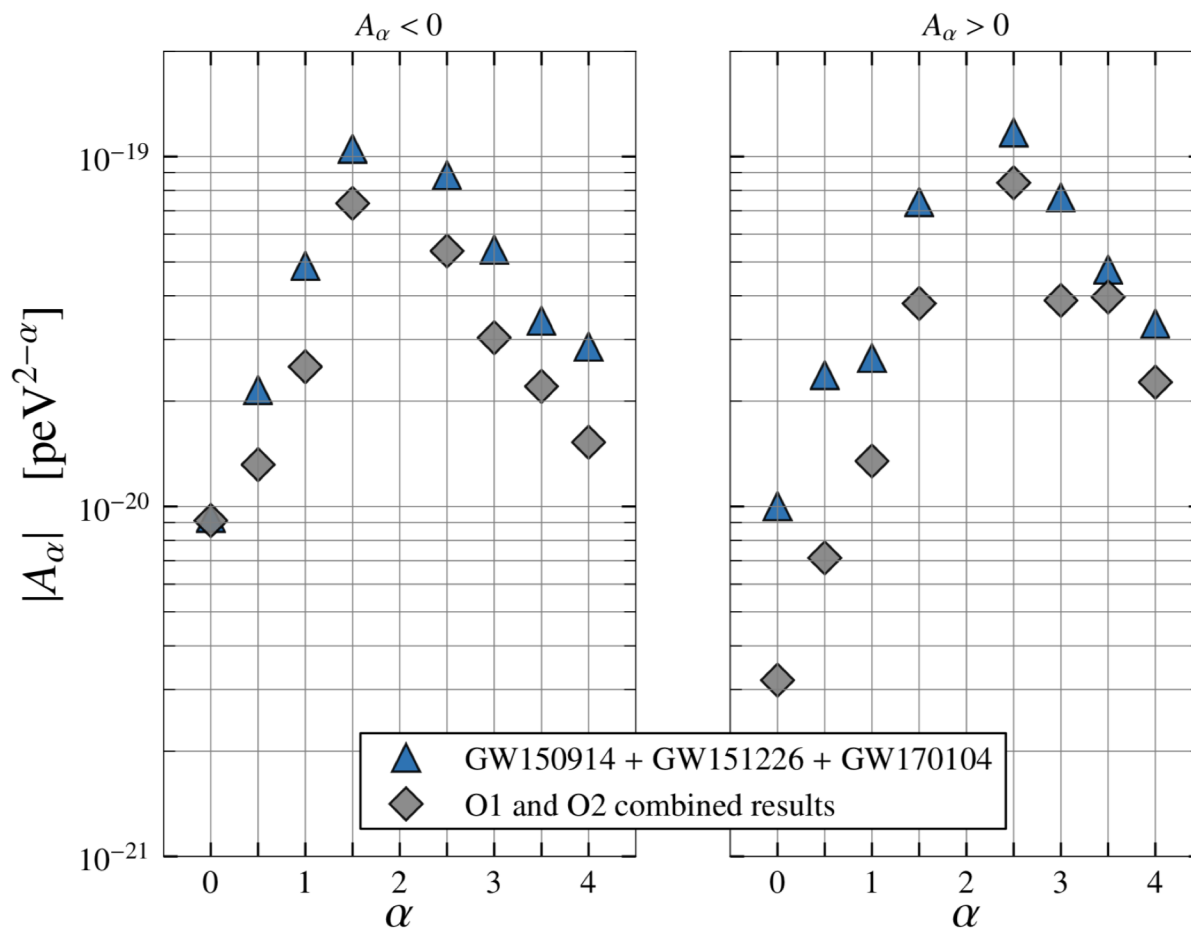
$$\Rightarrow \delta\Phi_\alpha(f) = \text{sign}(A_\alpha) \begin{cases} \frac{\pi D_L}{\alpha - 1} \lambda_{A,\text{eff}}^{\alpha-2} \left(\frac{f}{c}\right)^{\alpha-1} & (\alpha \neq 1) \\ \frac{\pi D_L}{\lambda_{A,\text{eff}}} \ln \left[\frac{\pi G \mathcal{M} f}{(1+z)c^3} \right] & (\alpha = 1) \end{cases}$$

$$\lambda_{A,\text{eff}} \equiv \left[(1+z)^{1-\alpha} \frac{D_L}{D_\alpha} \right] hc |A_\alpha|^{1/(\alpha-2)}$$

$$D_\alpha \equiv \frac{(1+z)^{1-\alpha}}{H_0} \int_0^z \frac{(1+z')^{\alpha-2}}{\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}} dz'$$

Current constraint

LVC, arxiv:1903.04467



from BBH

No constraint on $\alpha = 2$ has been obtained because of no freq dependence. (constant GW speed)

upper limit

$$|A_\alpha| \lesssim 10^{-19} \text{ peV}^{2-\alpha}$$

Interpretation of the result

$$E^2 = p^2 c^2 + A_\alpha p^\alpha c^\alpha$$

$$\longleftrightarrow E^2 = p^2 c^2 \left[1 + \sum_{n=1}^{\infty} \frac{\xi_n}{n!} \left(\frac{E}{E_{\text{QG}}} \right)^n \right]$$

Amelino-Camelia,
LRR (2013)

At the order of $n = \alpha - 2$, $E_{\text{QG}} \sim |A_\alpha|^{1/(2-\alpha)}$

For $\alpha = 3$ or $n = 1$, $E_{\text{QG}} \gtrsim 10^{19} \text{ peV} = 10^7 \text{ eV}$

For $\alpha = 4$ or $n = 2$, $E_{\text{QG}} \gtrsim 10^{9.5} \text{ peV} = 10^{-2.5} \text{ eV}$

Constraint from GWs is so weak because of low freq of GWs.

For some special theories, the constraint may be useful.