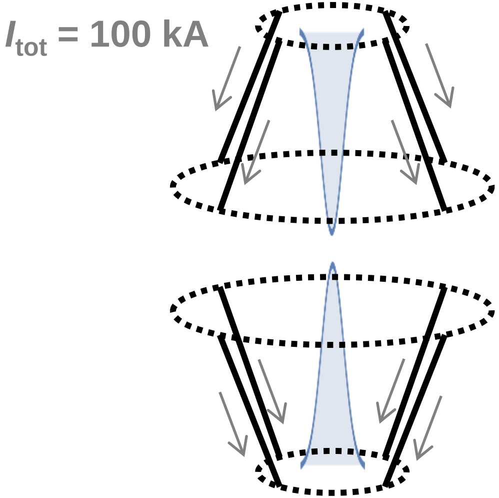
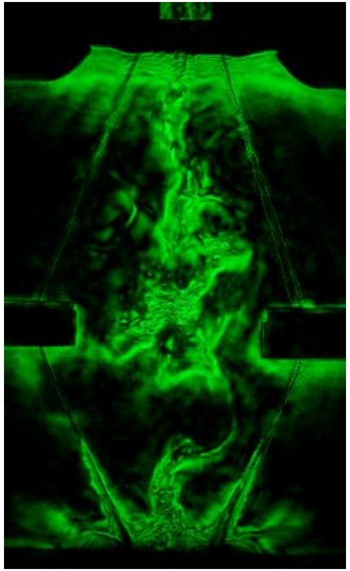
Study of the head-on collision of two plasma jets





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2024 Colloquium
Institute of Physics, Academia Sinica
Taipei, Taiwan
14 May 2024

Summary

Plasma stagnated when two counter-propagating plasma jets colliding with each other

- **A 1-kJ pulsed-power system with a suite of diagnostics was built.**
- **A supersonic plasma jet with a speed of 170 ± 70 km/s was generated using a conical-wire array. The corresponding Mach number was greater than 5.**
- **Plasma stagnated and reached a density of ~ 10^{16} cm^{-3} when two counter-propagating plasma jets colliding with each other.**

- **Technologies of pulsed-power systems are being adapted to the spherical tokamak, FIRST, that is being developed.**

Acknowledgment

Pulsed-Plasma Laboratory (PPL) at ISAPS, NCKU

C.-Y. Chen, J.-Y. Chen, K.-Y. Chen, T.-Y. Chen, Y.-H. Chen, C.-H. Du, C.-J. Hsieh, I.-T. Huang, M.-F. Huang, T.-H. Iang, M.-C. Jheng, S. Kumar, M.-H. Kuo, P.-W. Lai, S.-Y. Lin, W.-Y. Lin, Y.-C. Lin, C.-S. Liu, C.-Y. Liu, J.-K. Liu, Y.-Z. Pan, Y.-W. Pi, C. Royer, C.-J. Tsai, Y.-L. Tsai, S.-H. Yang, and I-L. Yeh, Pulsed-Plasma Laboratory (PPL) at ISAPS, NCKU

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Outlines

- Introduction to pulsed-power systems
- Laboratory astrophysics
- Pulsed-power generator for space sciences (PGS)
- Head-on collision of two counter-propagating plasma jets using a bi-conical-wire array

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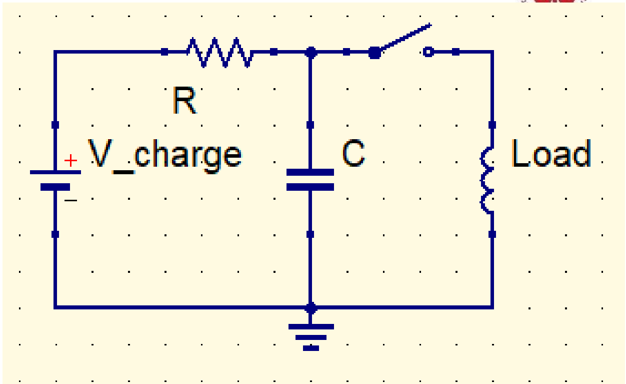
A pulsed-power system provides a high-power output by releasing the stored energy in a short period of time

- **Hammer**

$$E = \frac{1}{2}CV^2$$

$$\omega = \frac{2\pi}{T} \sim \frac{1}{\sqrt{LC}}$$

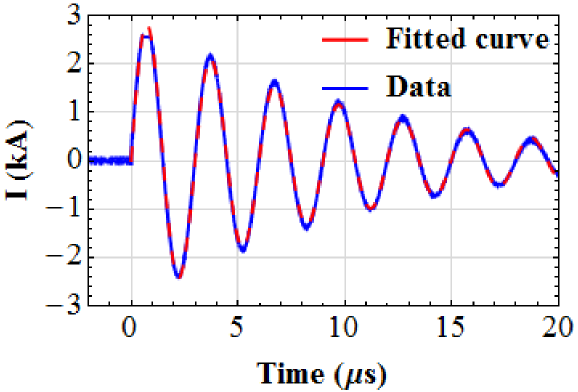
$$T_{rise} = \frac{T}{4} \sim \frac{\pi}{2}\sqrt{LC}$$



$$\text{Power} \sim \frac{\text{Energy}}{T_{rise}}$$

$$= \frac{\frac{1}{2}CV^2}{\frac{\pi}{2}\sqrt{LC}} = \frac{V^2}{\pi}\sqrt{\frac{C}{L}}$$

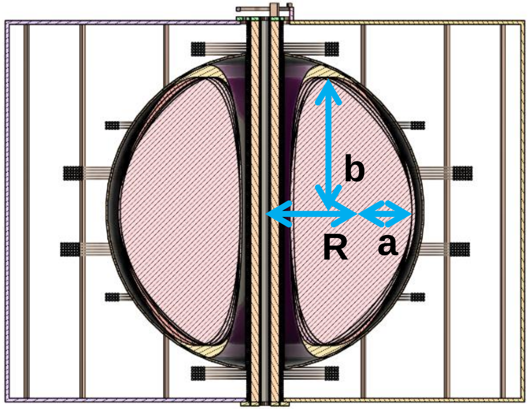
$$\text{Power} = \frac{\text{Energy}}{\Delta t} \qquad \Delta t \downarrow \;=>\; \text{Power} \uparrow$$



- **To provide short pulse and high power, a pulsed-power system with high voltage, low inductance, and low capacitance is commonly used.**

http://learnhowtowritesongs.com/tag/thesaurus/

Appplication of pulsed-power system –
Formosa Integrated Research Spherical Tokamak (FIRST)



- R=47 cm
- a=32 cm
- Aspect ratio (R/a): 1.5
- Elongation (b/a): κ = 2~2.8
- B_T = 0.1 ~ 0.5 T
- T ≥ 100 eV

- **It will be the 1st Tokamak in Taiwan.**
- **The expected first plasma will be generated in 2027.**

- FIRST team:
  - National Atomic Research Institute
  - Prof. Chang @ NCKU
  - Prof. Shaing @ NCKU
  - Prof. Kawamori @ NCKU
  - Prof. Leou @ NTHU
  - National Center for high-performance computing
- FIRST is targeted for
  - Low aspect ratio
  - High beta
  - High bootstrap current
- FIRST is still under designed. Figure here is only to illustrate the idea, NOT the final design.

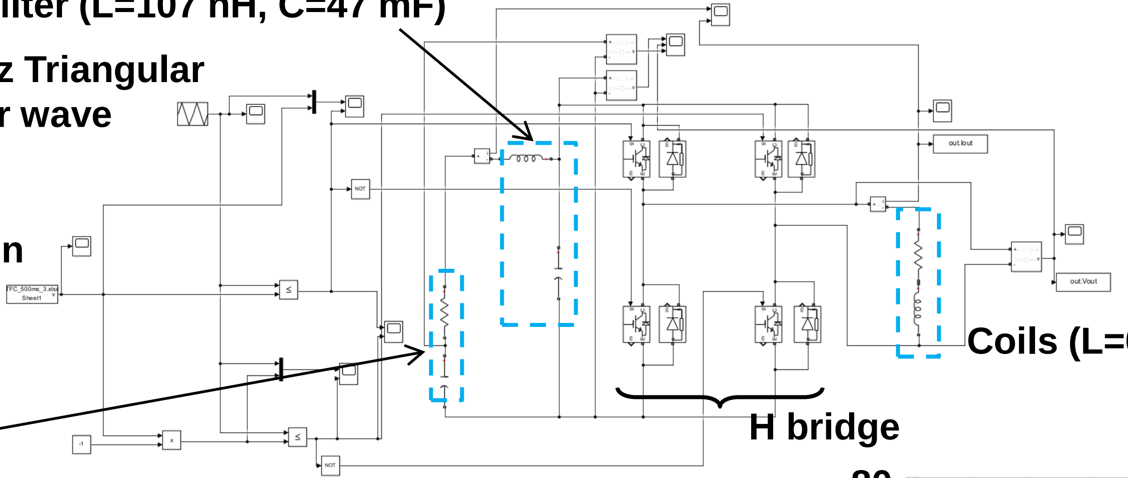
A trapezoid current with a flat top of ~70 kA will be provided for TFC

LC filter (L=107 nH, C=47 mF)

10-kHz Triangular carrier wave

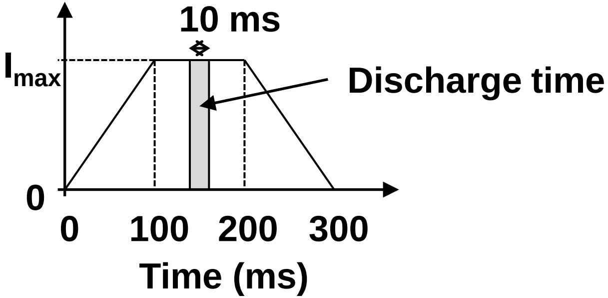
Input function

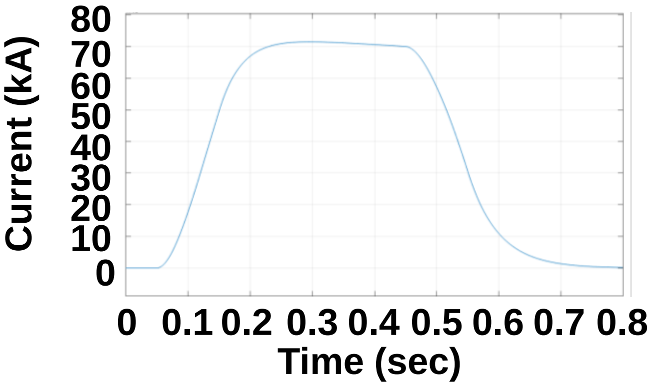
Capacitor bank (C=559.3 F, V=680.4V, ESR=1.79 mΩ, N_p=47, N_s=14, N_{tot}=658)



Coils (L=0.3 mH, R=4.76 mΩ)

H bridge





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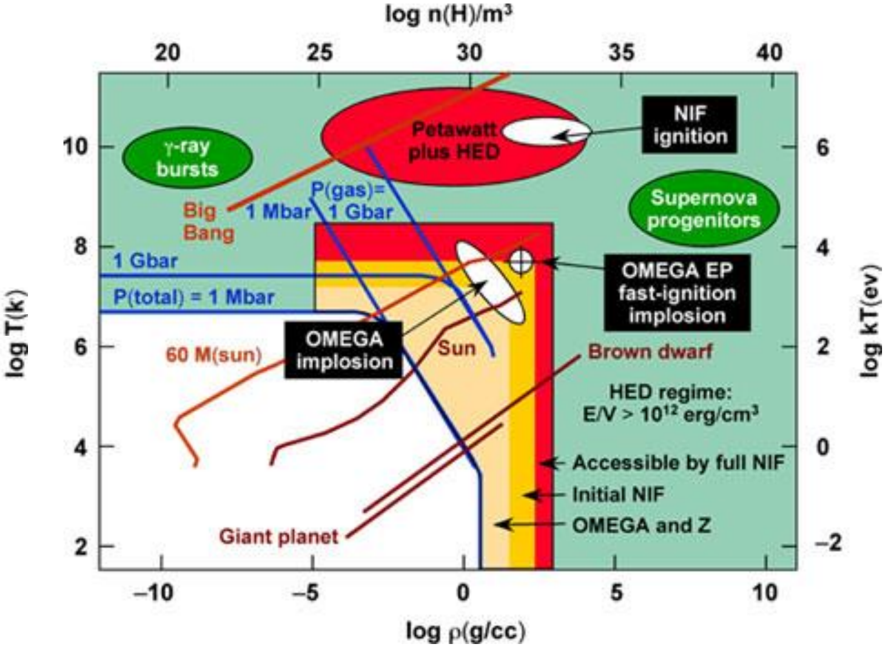
Laboratory astrophysics is to experimentally “simulate” the astrophysical phenomena in the laboratory environment



- High energy density plasma (HEDP) is the regime where the pressure is greater than 1 Mbar.
- The energy density of HEDP regime is higher than 1 kJ of energy per 10 mm^3.

Frontiers in High Energy Density Physics: The X-Games of Contemporary Science © (2003) by the National Academy of Sciences, courtesy of the National Academies Press, Washington, D.C.

Studies of laboratory astrophysics/space sciences using ideal magnetohydrodynamic scaling

- **Dimensional form:**

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

$$\rho\left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u}\right) = -\nabla p + \frac{1}{\mu_0}(\nabla \times \vec{B}) \times \vec{B}$$

$$\frac{\partial p}{\partial t} + \vec{u} \cdot \nabla p = -\gamma p \nabla \cdot \vec{u}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B})$$

$$\tilde{\vec{r}} = \frac{\vec{r}}{L^*} \qquad \tilde{p} = \frac{p}{p^*} \qquad \tilde{t} = \frac{t}{L^*}\sqrt{\frac{p^*}{\rho^*}}$$

$$\tilde{\rho} = \frac{\rho}{\rho^*} \qquad \tilde{\vec{B}} = \frac{\vec{B}}{\sqrt{\mu_0 p^*}} \qquad \tilde{\vec{u}} = \vec{u}\sqrt{\frac{p^*}{\rho^*}}$$

- **Dimensionless form:**

$$\frac{\partial \tilde{\rho}}{\partial \tilde{t}} + \nabla \cdot \left(\tilde{\rho}\, \tilde{\vec{u}}\right) = 0$$

$$\tilde{\rho}\left(\frac{\partial \tilde{\vec{u}}}{\partial \tilde{t}} + \tilde{\vec{u}} \cdot \nabla \tilde{\vec{u}}\right) = -\nabla \tilde{p} + \left(\nabla \times \tilde{\vec{B}}\right) \times \tilde{\vec{B}}$$

$$\frac{\partial \tilde{p}}{\partial \tilde{t}} + \tilde{\vec{u}} \cdot \nabla \tilde{p} = -\gamma \tilde{p} \nabla \cdot \tilde{\vec{u}}$$

$$\frac{\partial \tilde{\vec{B}}}{\partial \tilde{t}} = \nabla \times \left(\tilde{\vec{u}} \times \tilde{\vec{B}}\right)$$

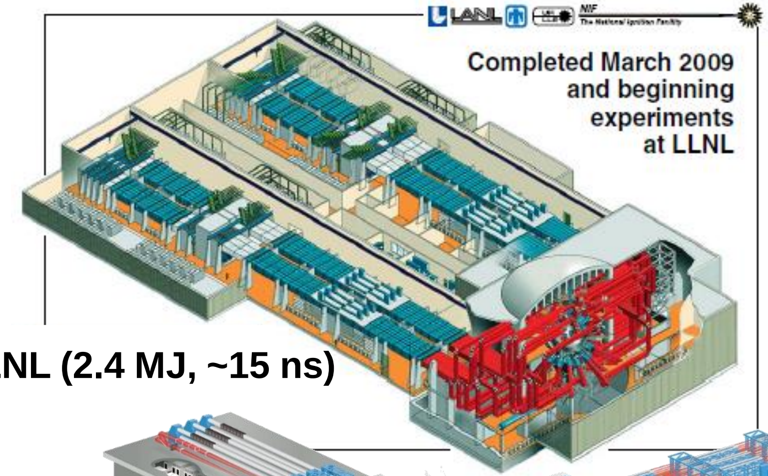
$$\tilde{\rho}_0(\tilde{r}) = f(\tilde{r}) \qquad \tilde{p}_0(\tilde{r}) = g(\tilde{r})$$

$$\tilde{\vec{u}}_0(\tilde{r}) = \sqrt{\frac{\rho^*}{p^*}}\,\vec{h}(\tilde{r}) \qquad \tilde{\vec{B}}_0(\tilde{r}) = \frac{\vec{k}(\tilde{r})}{\sqrt{\mu_0 p^*}}$$

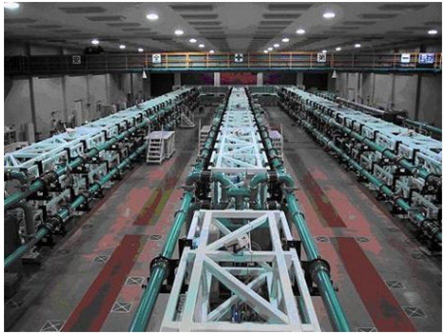
- **Any two ideal MHD systems involve identically in a scaled sense if f, g, h, k, and $u^*(\rho^*/p^*)^{1/2}$ are the same.**

D. D. Ryutov, etc., Phys. Plasmas, 8, 1804 (2001)

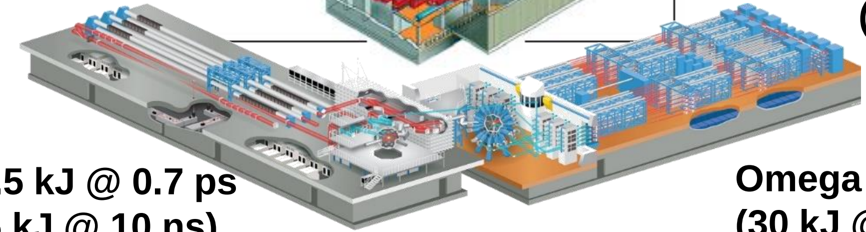
HEDP regime can be obtained using giant lasers, i.e., very expensive!



NIF Laser@LLNL (2.4 MJ, ~15 ns)



Gekko XII Laser@Osaka University (10 kJ @ 2 ns)



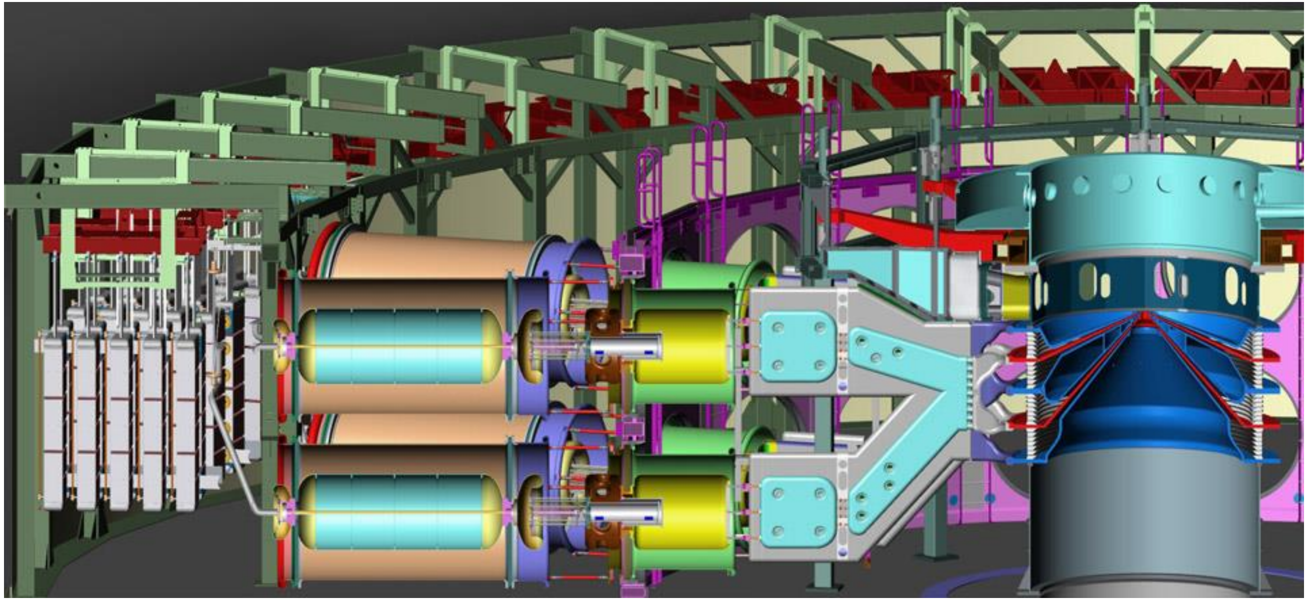
Omega EP Laser (0.5 kJ @ 0.7 ps / 2.3 kJ @ 100 ps / 5 kJ @ 10 ns)

Omega Laser@LLE, U of Rochester (30 kJ @ 4 ns)

High-Field Physics and Ultrafast Technology Laboratory @ NCU (3.3J @ 30 fs)

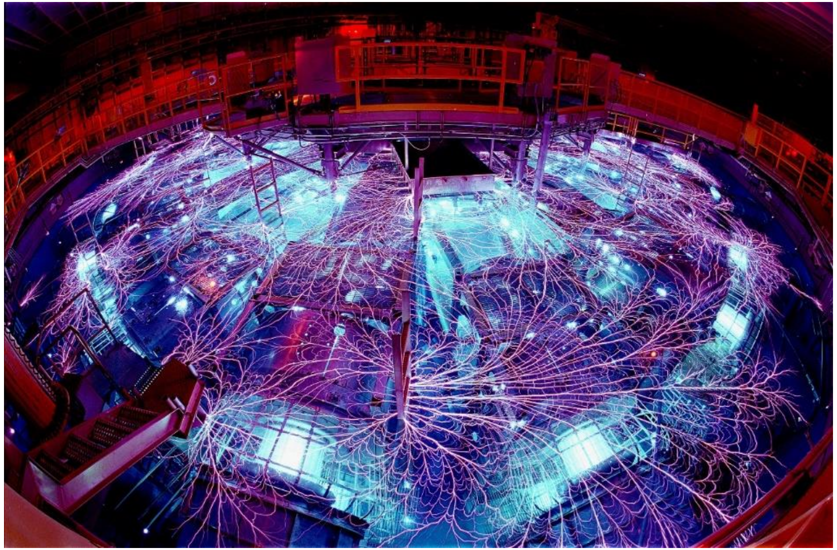
https://www.ile.osaka-u.ac.jp/eng/facilities/gxii/index.html
https://www.lle.rochester.edu/
https://hfp.phy.ncu.edu.tw/

Sandia’s Z machine is the world's most powerful and efficient laboratory radiation source



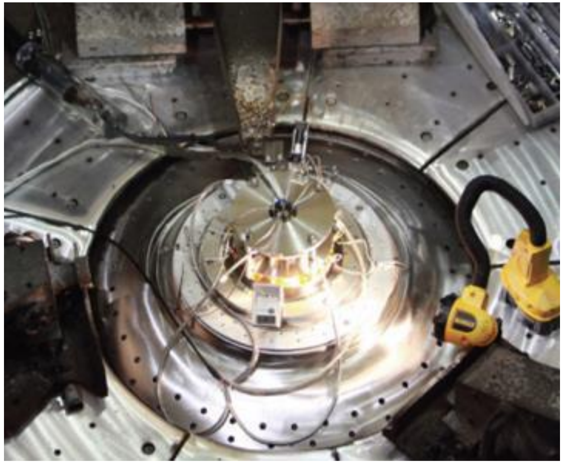
- Stored energy: 20 MJ
- Marx charge voltage: 85 kV
- Peak electrical power: 85 TW
- Peak current: 26 MA
- Rise time: 100 ns
- Peak X-ray emissions: 350 TW
- Peak X-ray output: 2.7 MJ

Z machine discharge

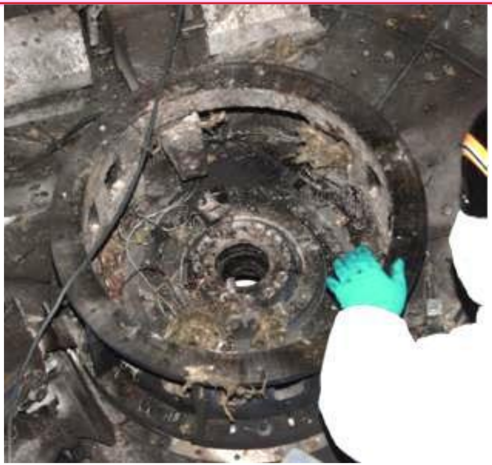


Before and after shots

- **Before shots**



- **After shots**



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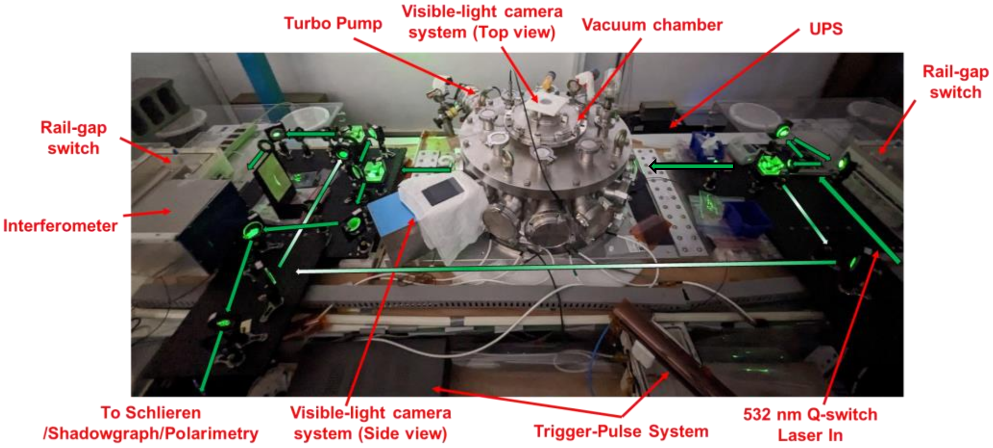
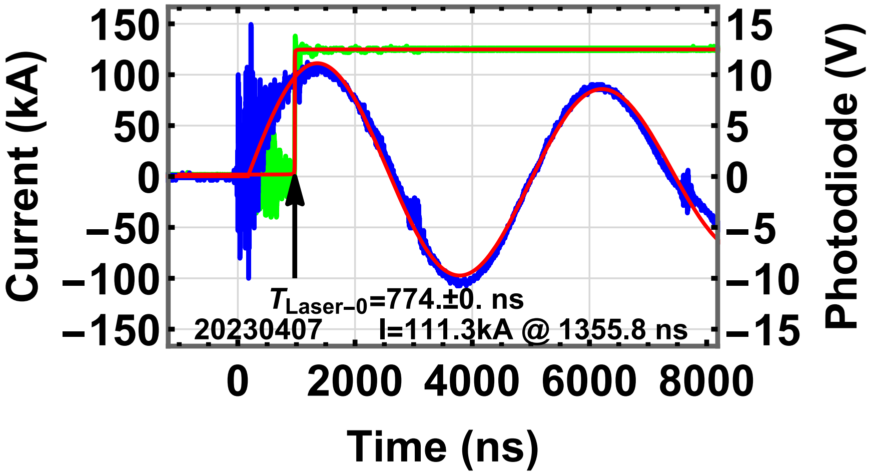
A peak current of ~135 kA with a rise time of ~1.6 us is provided by the Pulsed-Power Generator for Space Science (PGS)





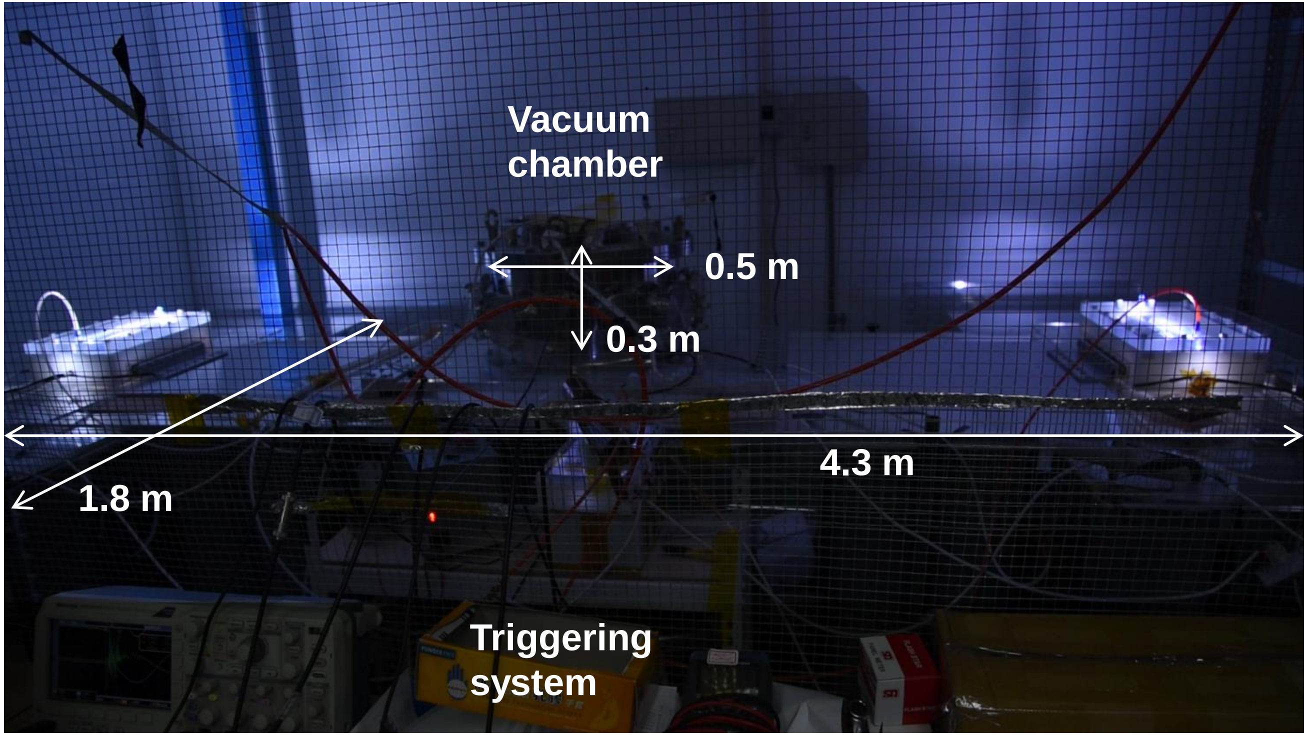
Capacitance (μF)	5
V_{charge} (kV)	20
Energy (kJ)	1
Inductance (nH)	204 ± 4
Rise time (ns)	1592 ± 3
I_{peak} (kA)	135 ± 1

- Optical diagnostics:
  - Q-switch Nd:Yag Laser @ 532 nm, 5-ns pulse width
  - Shadowgraph
  - Schlieren
  - Interferometer

P.-Y. Chang, etc., Rev. Sci. Instrum. 93, 043505 (2022)

First shot with two synchronized rail-gap switches in 2019

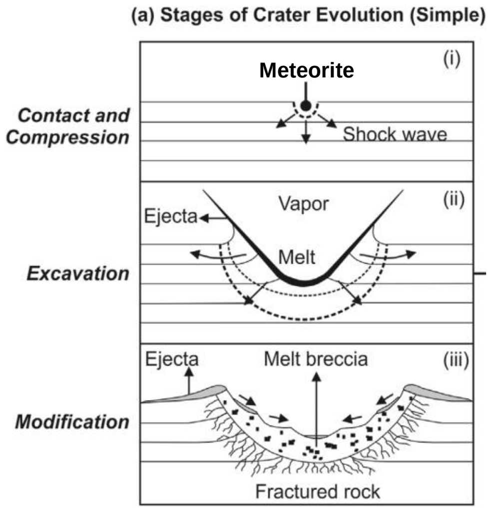
Rail-gap switch

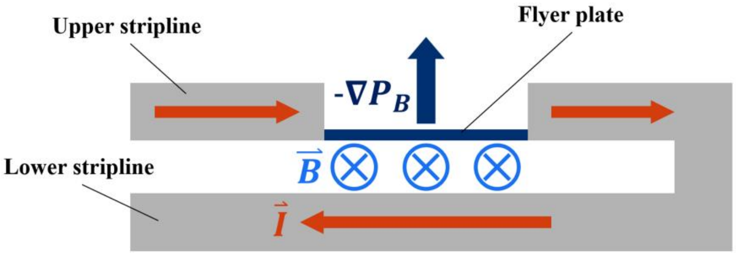


Rail-gap switch

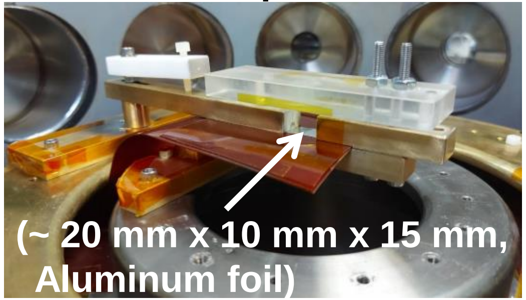
A shock in a target can be generated by being collided by a flyer plate

- Cooperation with Prof. Jennifer Kung's group in the Dept. Earth Science, NCKU

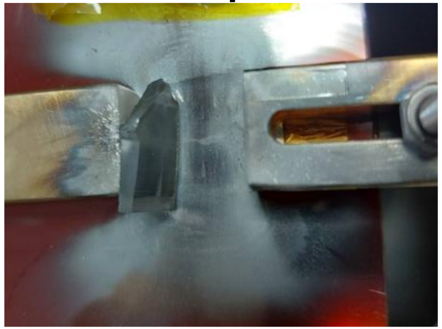




- Before the experiment:

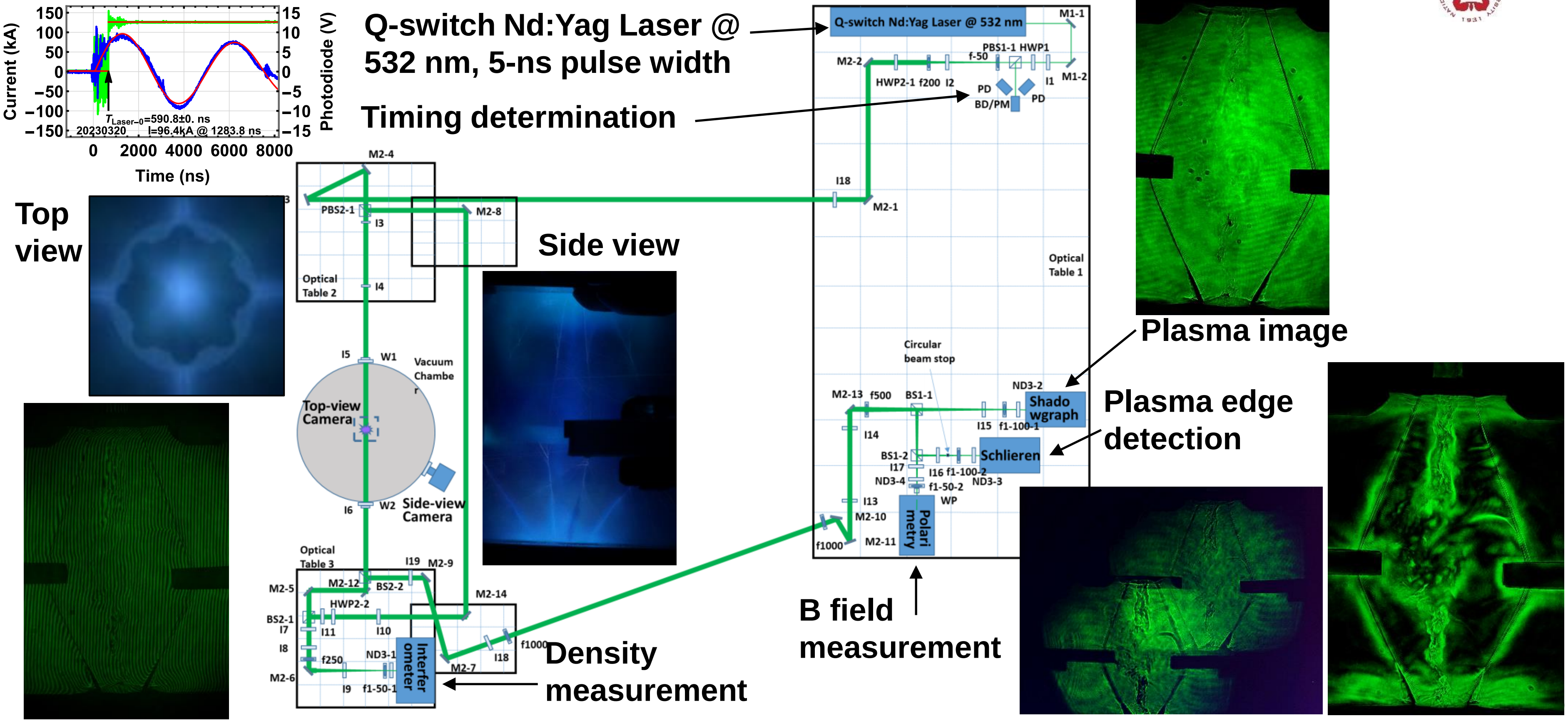


- After the experiment:

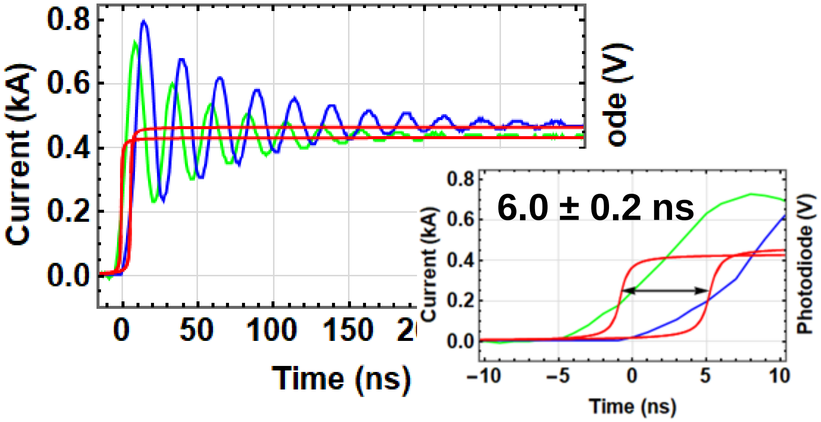


- Y.-Z. Pan, NCKU Master Thesis 2024
- S. James, etc., Energy Geoscience **3**, 136 (2022)

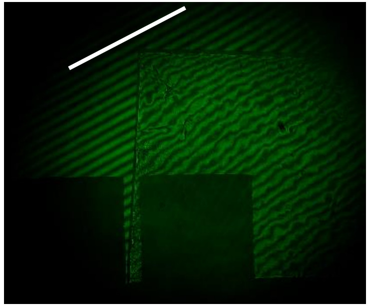
Time-resolved imaging system with temporal resolution in the order of nanoseconds was implemented



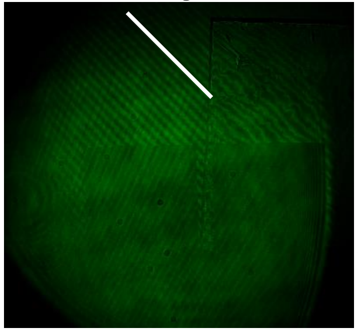
Two interferometer images with 6 ns in temporal separation will be taken in the same shot

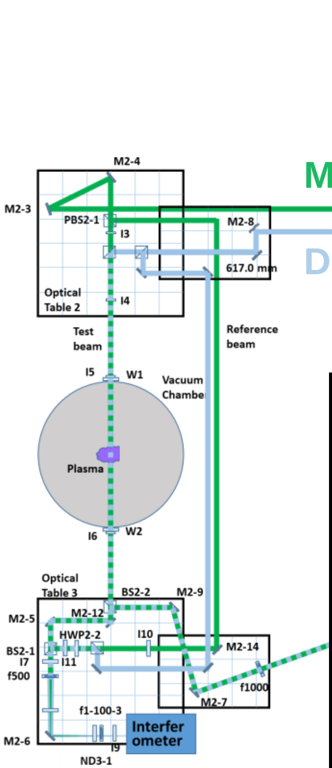
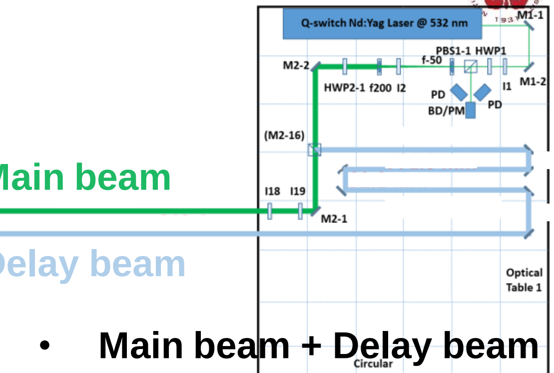


- Main beam

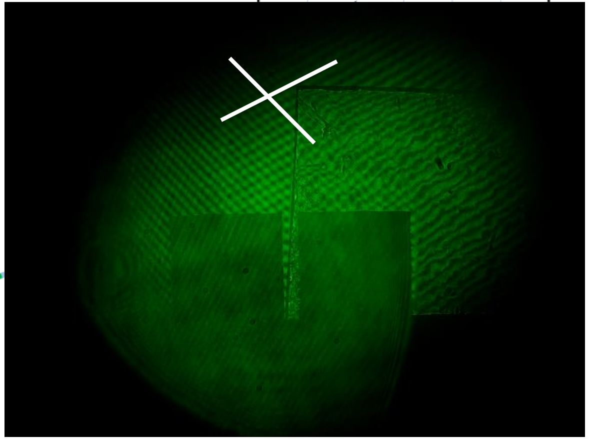


- Delay beam



- Main beam + Delay beam



- White lines: pattern directions
- Z. Liu, etc., Optics letters, 27, 22 (2002)

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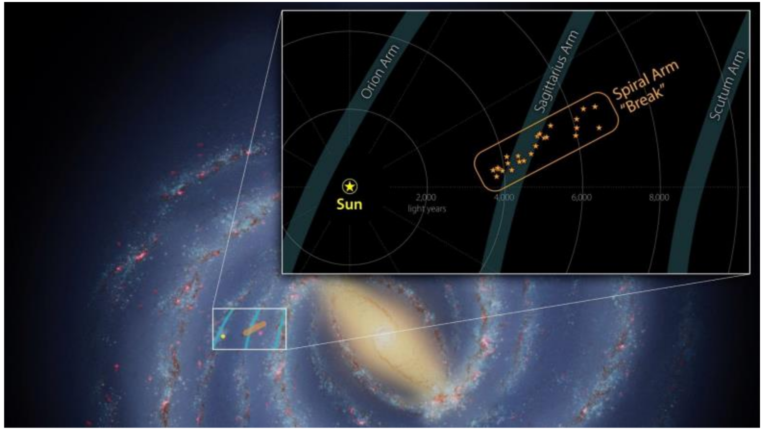
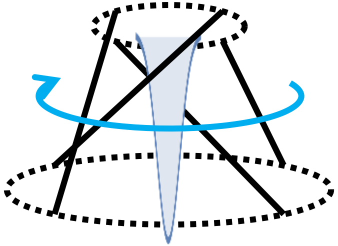
Plasma disks are commonly observed in the universe

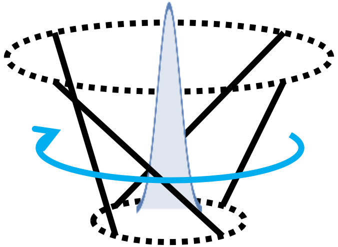
- Milky way

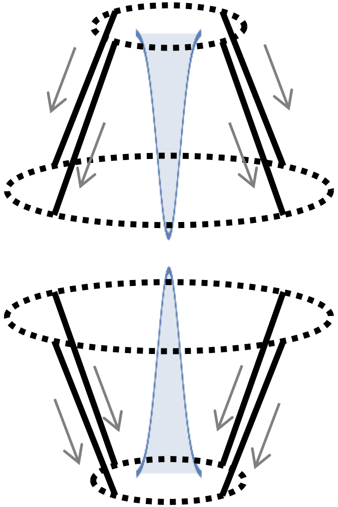


- CW



- CW

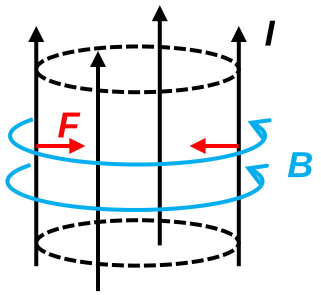


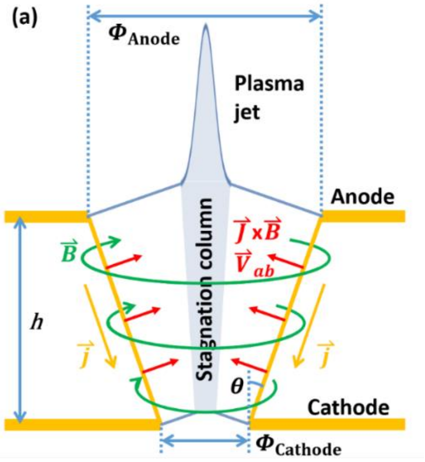


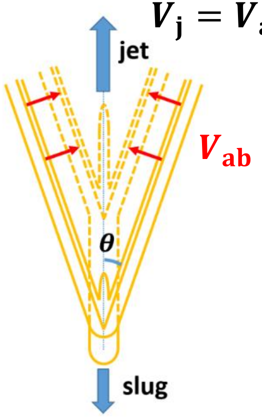
- No rotation

Jet Propulsion Laboratory [NASA/JPL] Astronomers Find a 'Break' in One of the Milky Way's Spiral Arms (Aug 17, 2021)

A plasma jet can be generated by a conical-wire array due to the nonuniform z-pinch effect



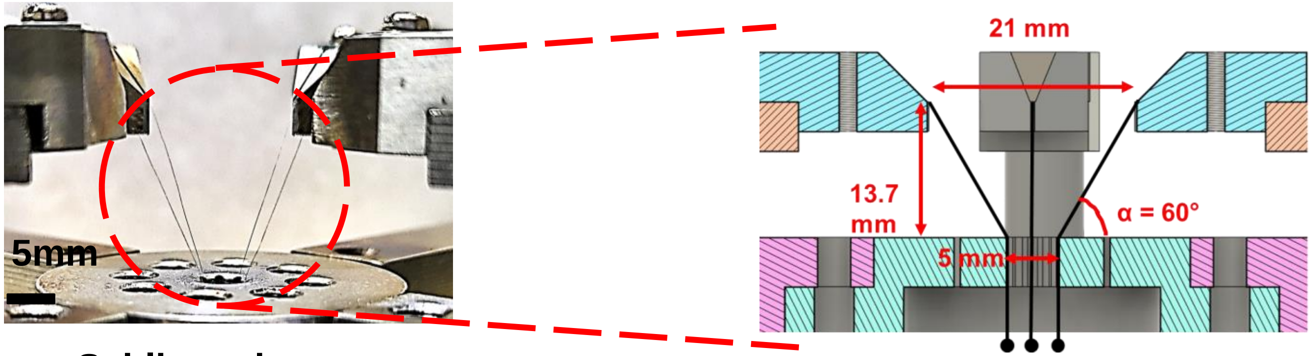




1. **Wire ablation : corona plasma is generated by wire ablations.**
2. **Precursor : corona plasma is pushed by the $\vec{J}\times\vec{B}$ force and accumulated on the axis forming a precursor.**
3. **Plasma jet is formed by the nonuniform z-pinch effect due to the radius difference between the top and the bottom of the array.**

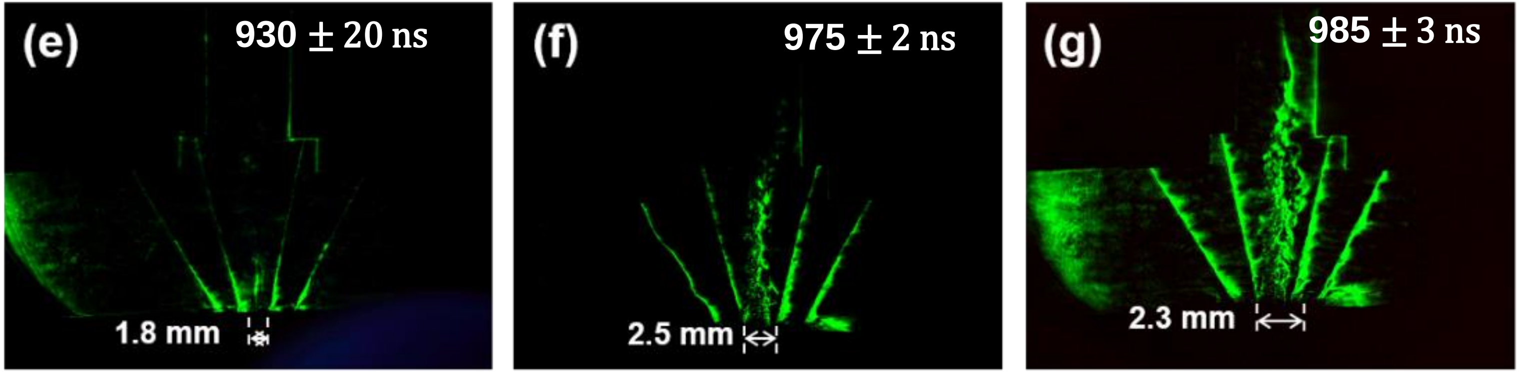
D. J. Ampleforda, et al., Phys. Plasmas 14, 102704 (2007)
G. Birkhoff, et al., J. Applied Physics 19, 563 (1948)

The plasma jet was generated using the conical-wire array



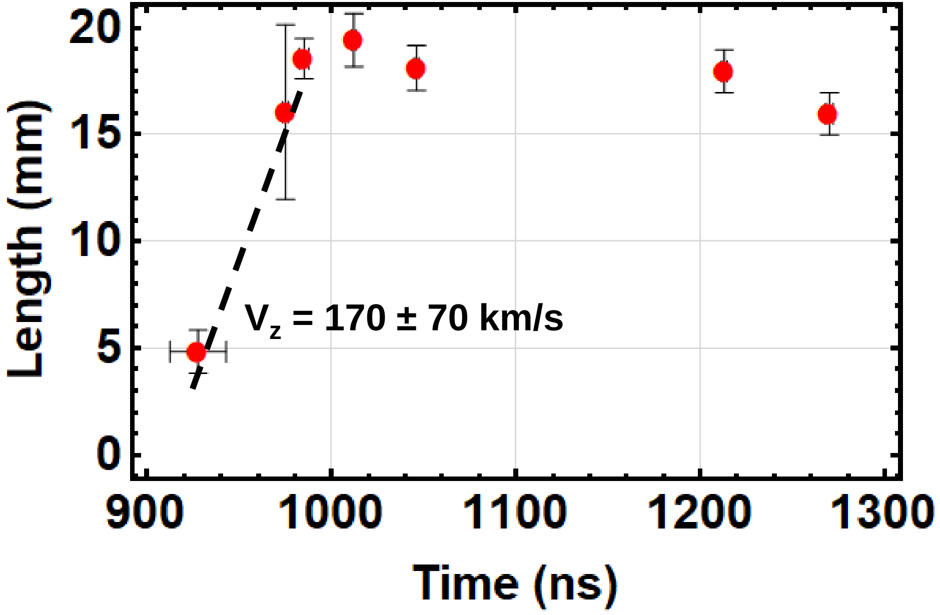
- Material : Tungsten
- Diameter : 20 μm
- Number of wires: 4

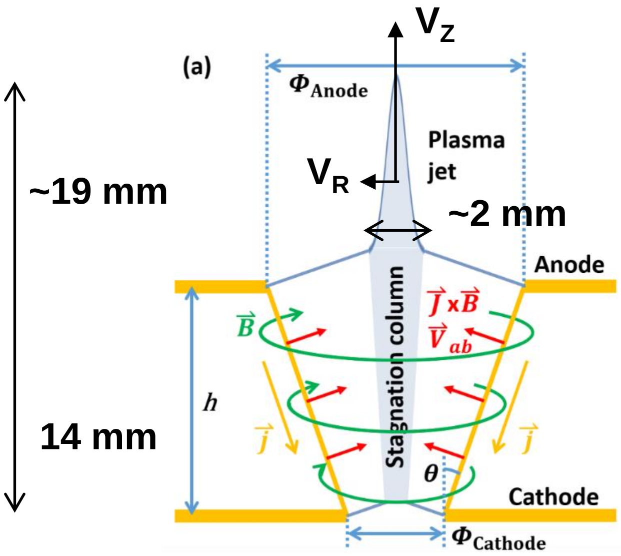
- Schlieren images:



P.-Y. Chang, etc., Rev. Sci. Instrum. **93**, 043505 (2022)

The measured plasma jet speed was 170 ± 70 km/s with the corresponding Mach number greater than 5



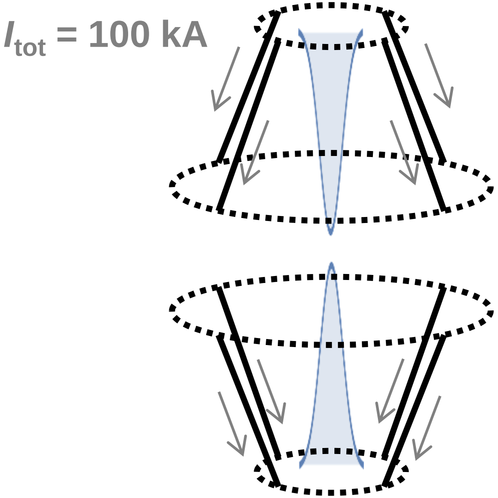


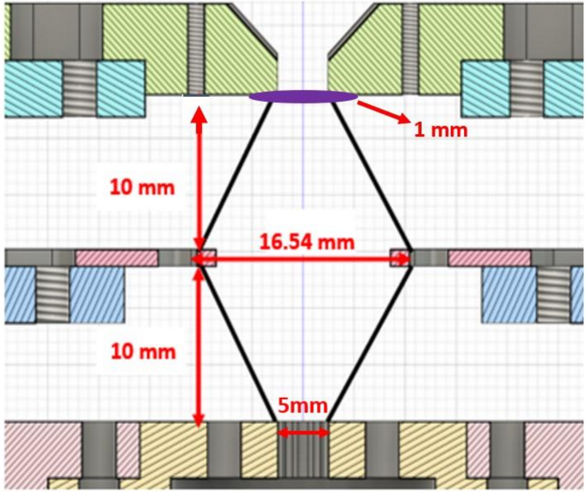
$$M = \frac{V_Z}{V_R} \geq \frac{Z}{r} \approx \frac{(19-14)\ \text{mm}}{\frac{2\ \text{mm}}{2}} = 5$$

$$V_{ab} = V_j \frac{\sin\theta}{1+\cos\theta} = 50\ \pm 20\ \text{km/s}$$

P.-Y. Chang, etc, Rev. Sci. Instrum., 93, 043505 (2022)

Two head-on plasma jets can be generated by using two conical-wire arrays facing against each other



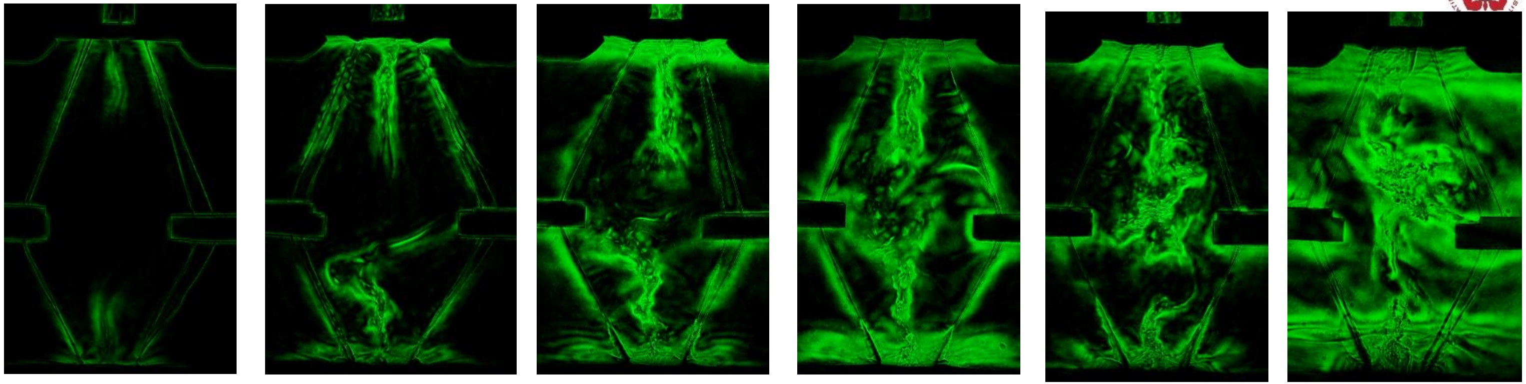


- Material : Tungsten
- Diameter : 20 μm
- Number of wires: 4 for each conical-wire array

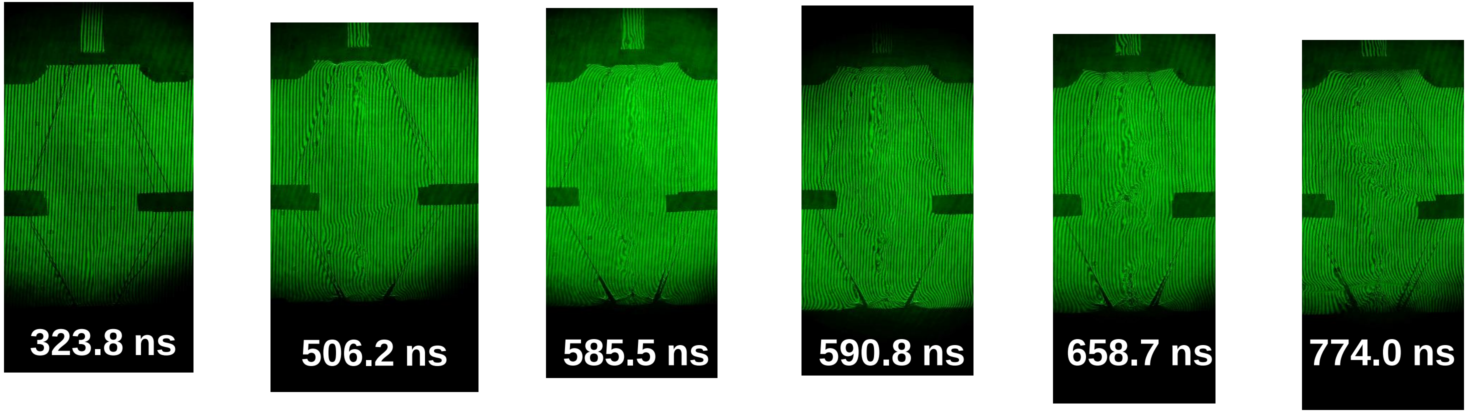
- **A disk is needed at the bottom of each conical-wire array to accumulate plasma.**

Images of two head-on plasma jets colliding with each other were taken using multiple shots

- **Schlieren**

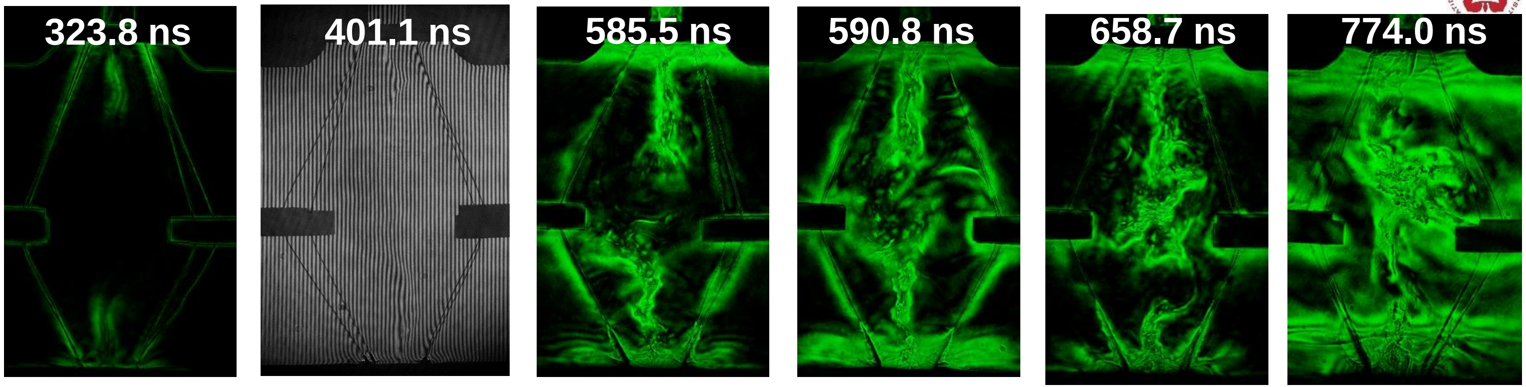


- **Interferometer**

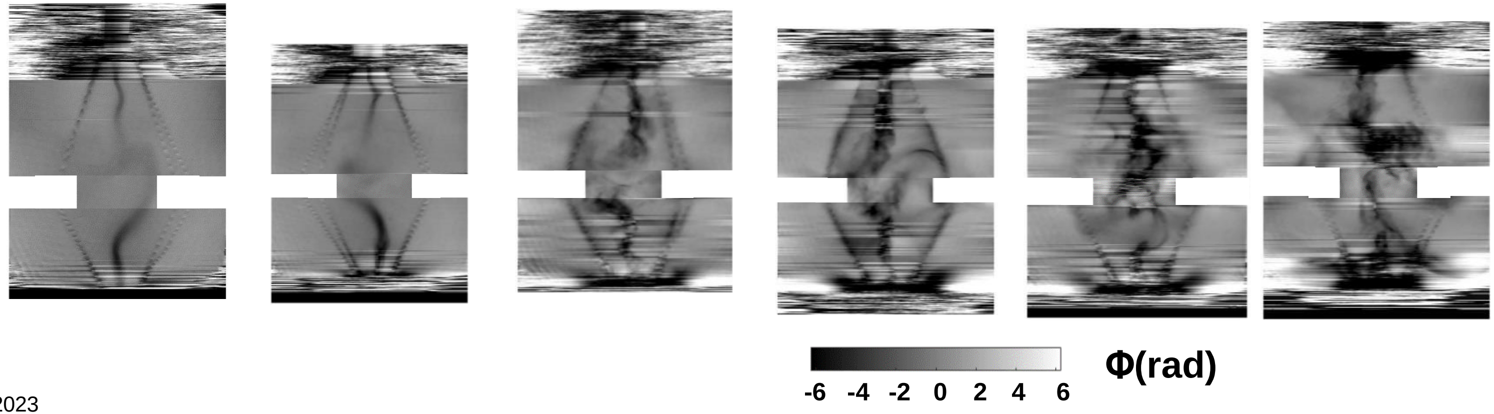


Densities are obtained from the interferometer images

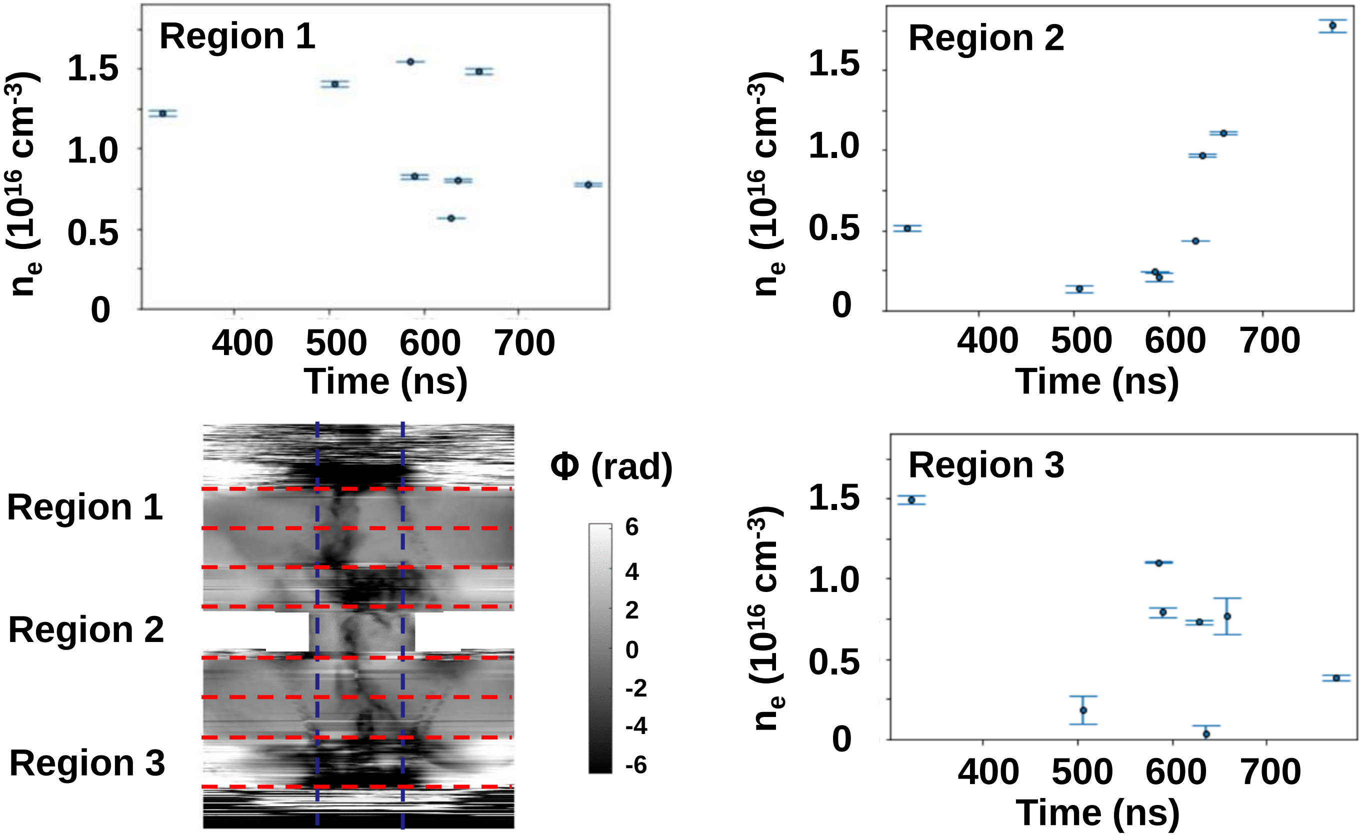
- **Schlieren**



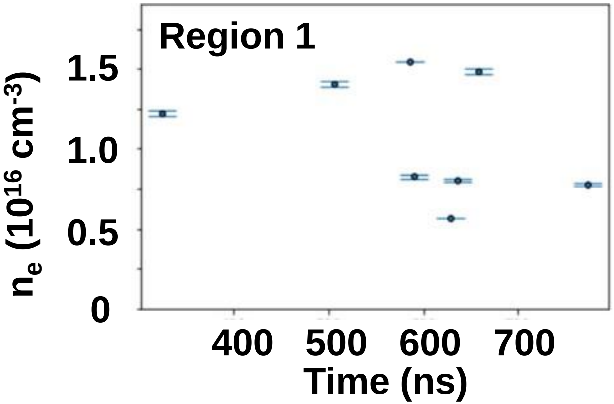
- **Interferometer**

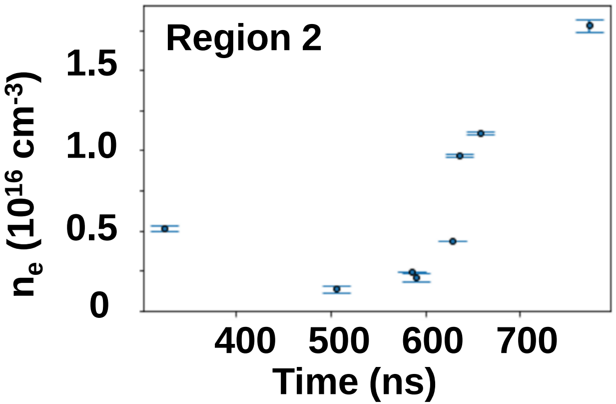


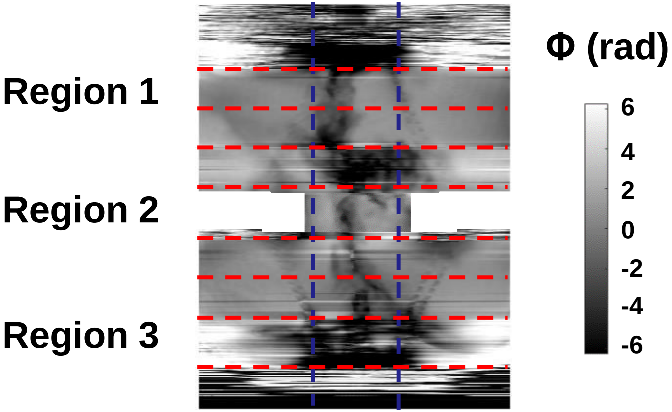
Plasma started to stagnate at the center after ~ 600 ns and reached a density of ~ 10^{16} cm^{-3}

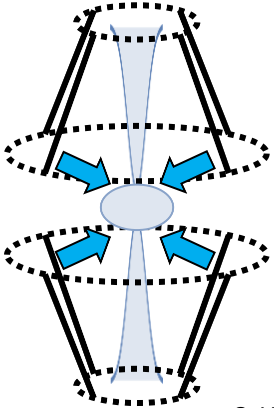


Plasma started to stagnate at the center after ~ 600 ns and reached a density of ~ 10^{16} cm^{-3}

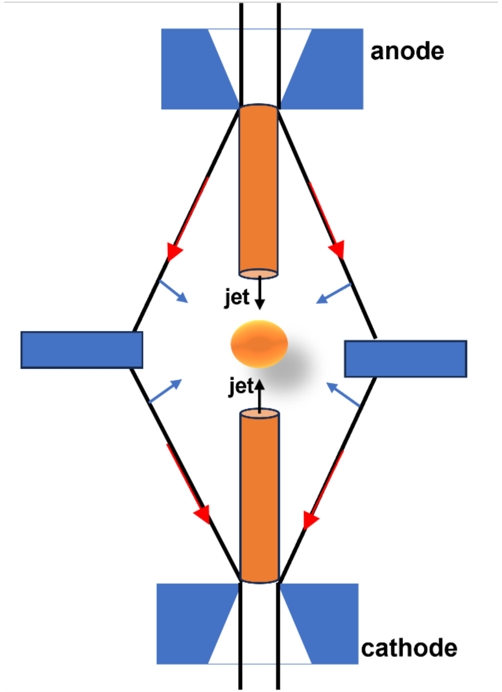


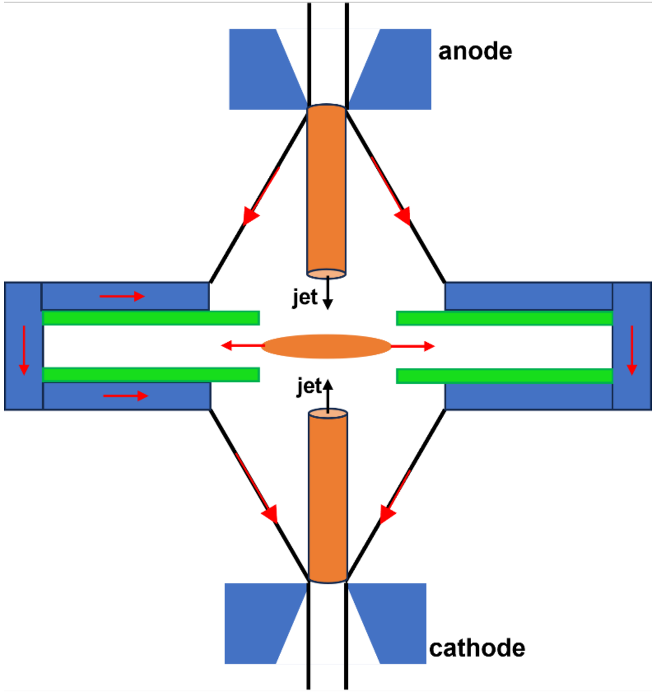


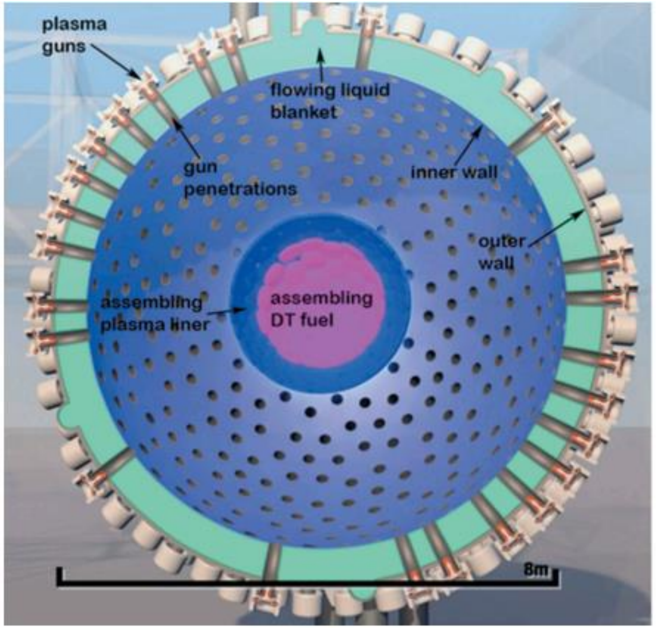




Our result was similar to that of the Plasma Jet Driven Magneto-Inertial Fusion (PJMIF) project

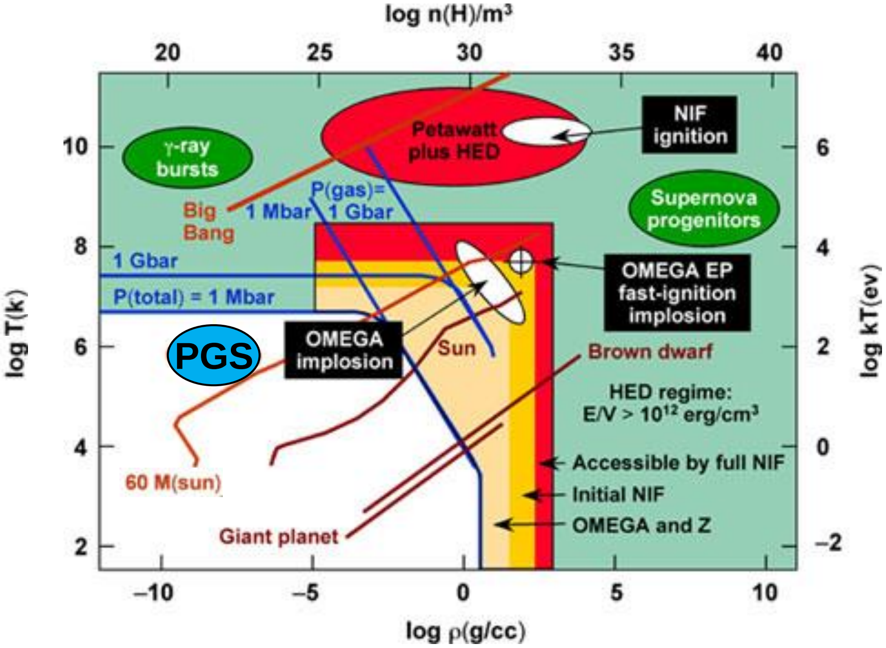






C.-Y. Liu, NCKU Master Thesis 2023
J. C. Valenzuela, et al, High energy density physics **17**, 140 (2015)
Y. C. Francis Thio, et al, Fusion Science and Technology **75**, 581 (2019)

Laboratory astrophysics is to experimentally “simulate” the astrophysical phenomena in the laboratory environment

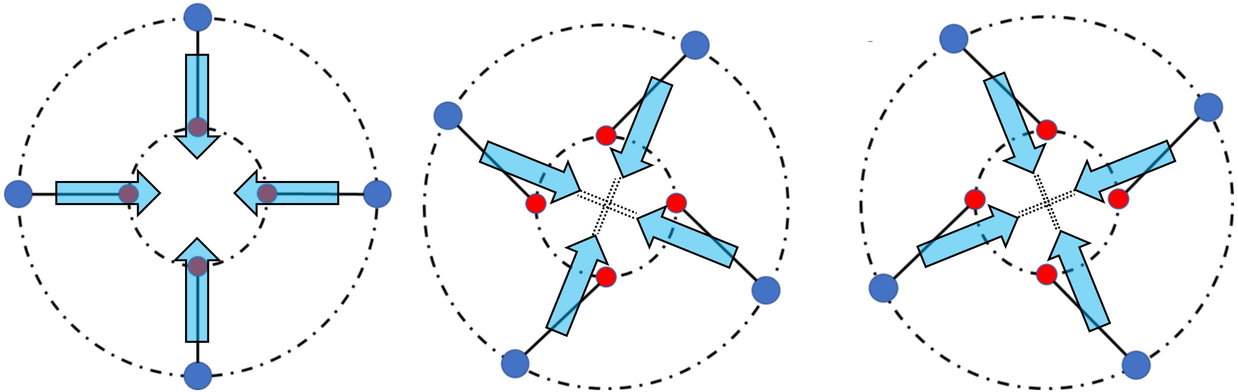


- High energy density plasma (HEDP) is the regime where the pressure is greater than 1 Mbar.
- The energy density of HEDP regime is higher than 1 kJ of energy per 10 mm³.

Frontiers in High Energy Density Physics: The X-Games of Contemporary Science © (2003) by the National Academy of Sciences, courtesy of the National Academies Press, Washington, D.C.

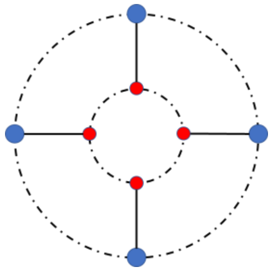
What if we twist the conical-wire array?

- **Non-rotation**
- **Clockwise 45°**
- **CCW 45°**

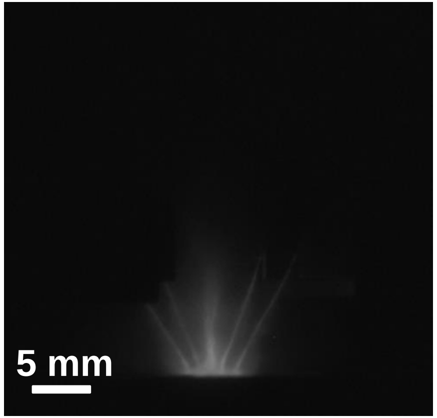


The plasma jet is a bright spot from the top view

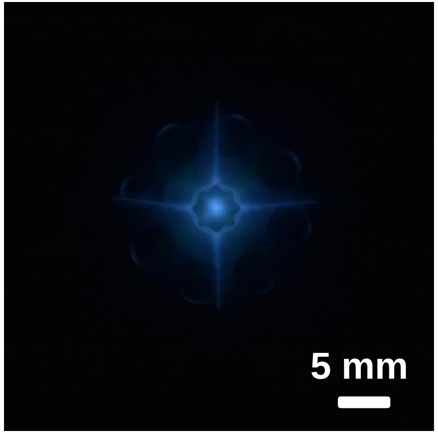
- **Non-rotation**



- **Side view**

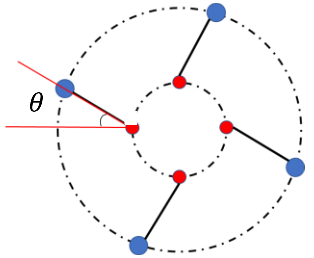


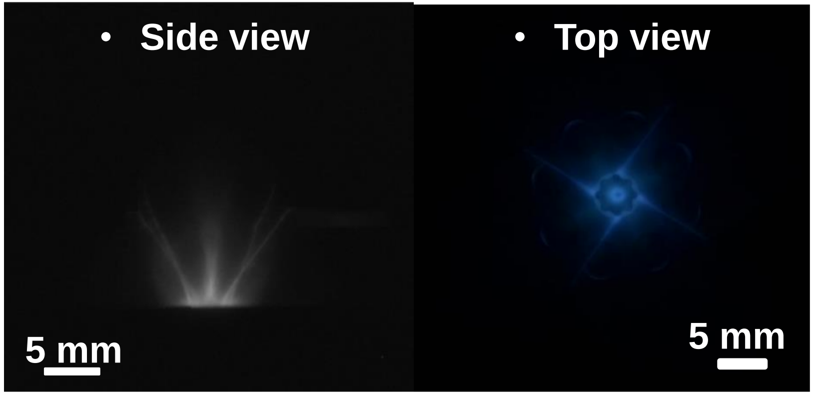
- **Top view**



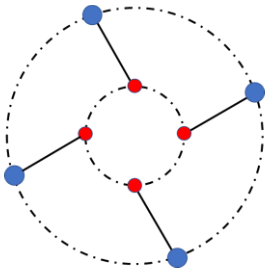
Hollow plasma jets were generated when the conical-wire arrays were twisted

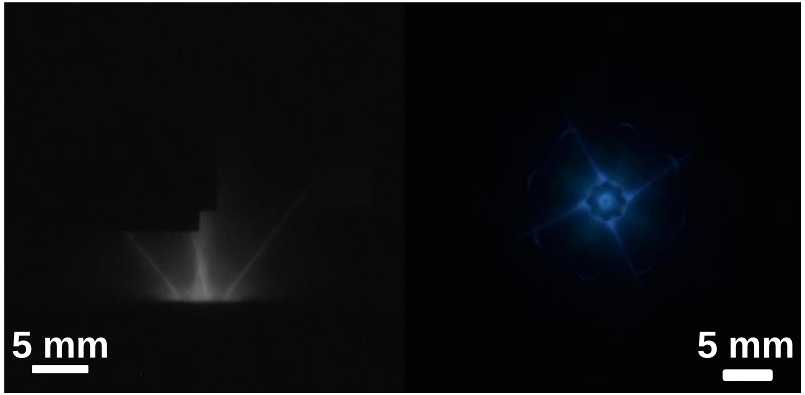
- **Clockwise 30 °**





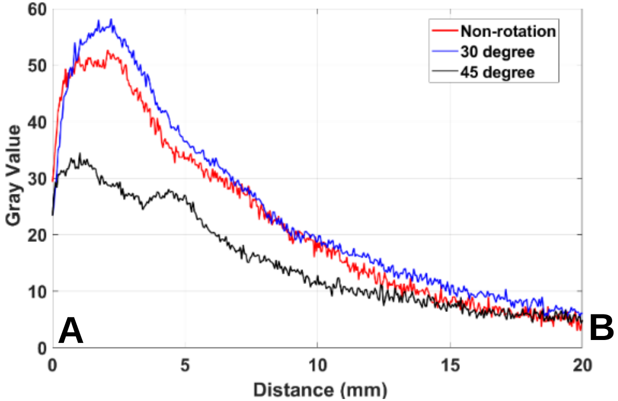
- **Counterclockwise 30 °**

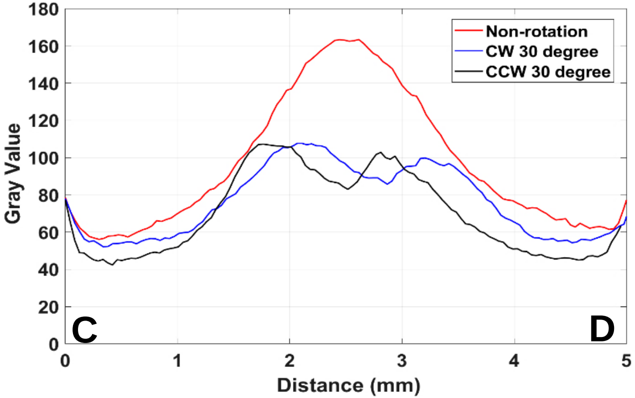


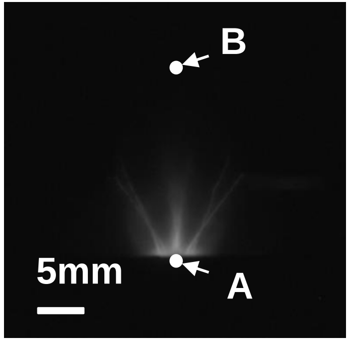


Y.-C. Lin, NCKU Master Thesis 2021

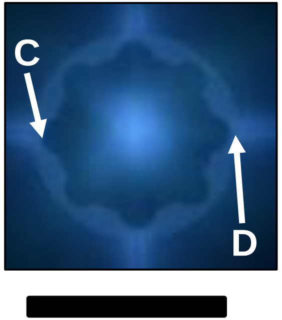
The hollow region at the center was due to angular momentum conservation of the in-coming plasma flow



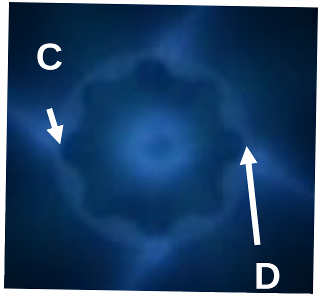




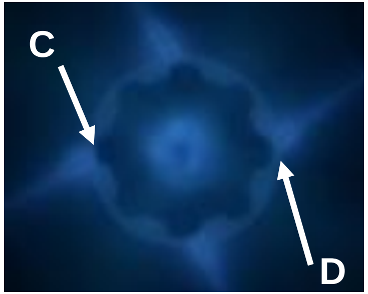
- 0 °



- CW 30 °

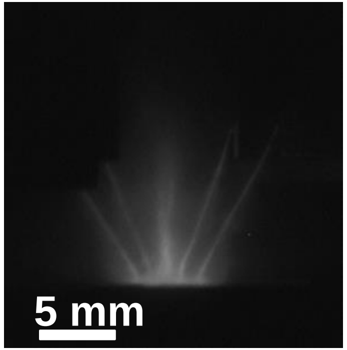


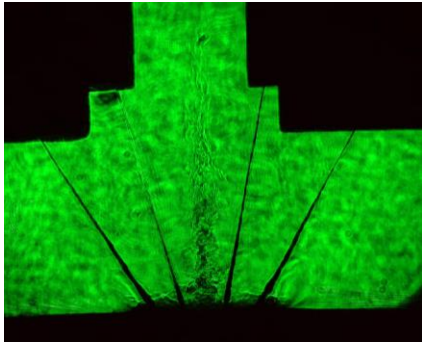
- CCW 30 °

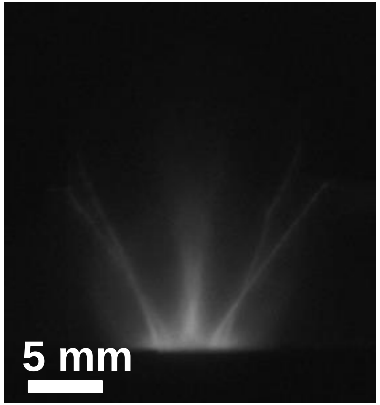


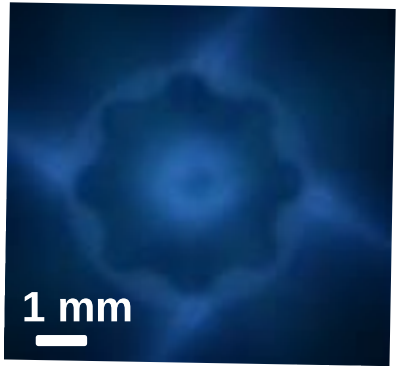
D. J. Ampleforda, et al., PRL 100, 035001 (2008)
Y.-C. Lin, NCKU Master Thesis 2021

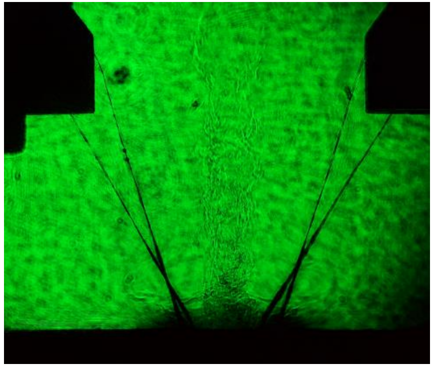
A rotating plasma jet was generated by the twisted conical-wire array



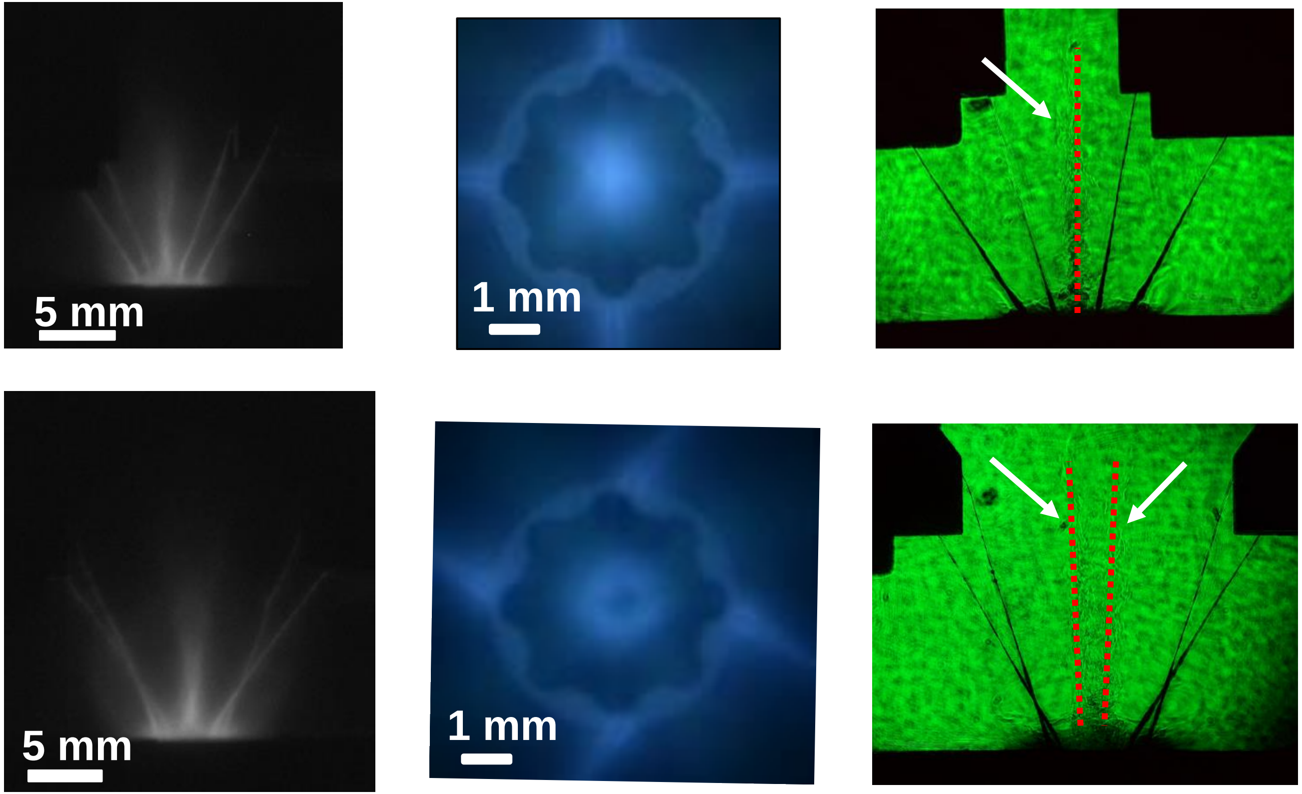




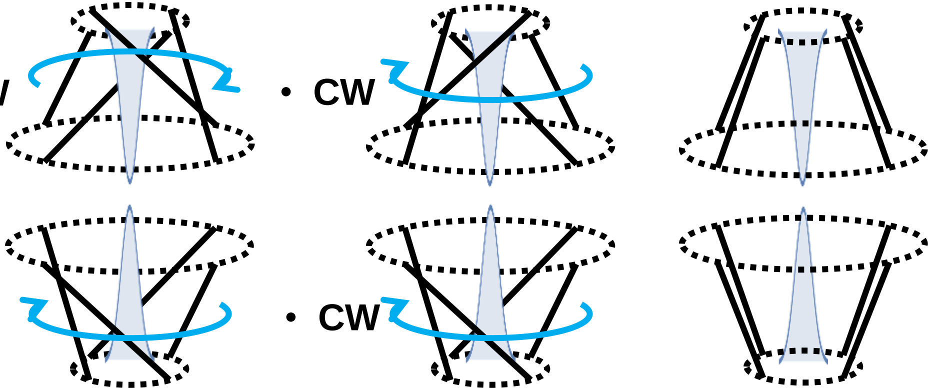




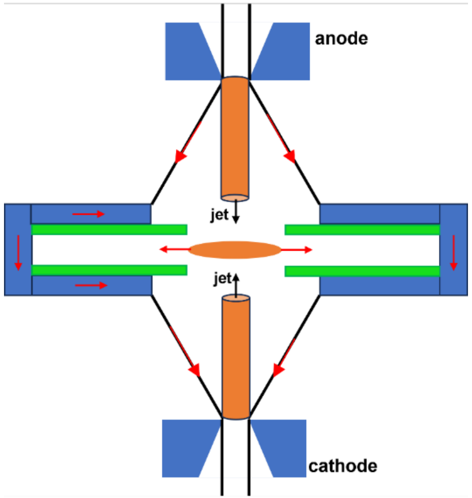
A rotating plasma jet was generated by the twisted conical-wire array

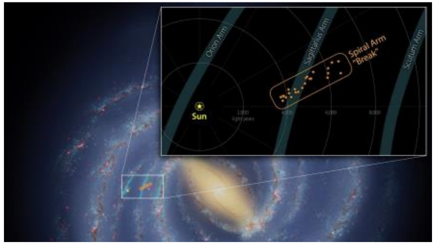


Can a rotating plasma disk be formed? To be continue…



- CCW
- CW
- CW
- CW
- No rotation





- **Astronomers Find a 'Break' in One of the Milky Way's Spiral Arms.**

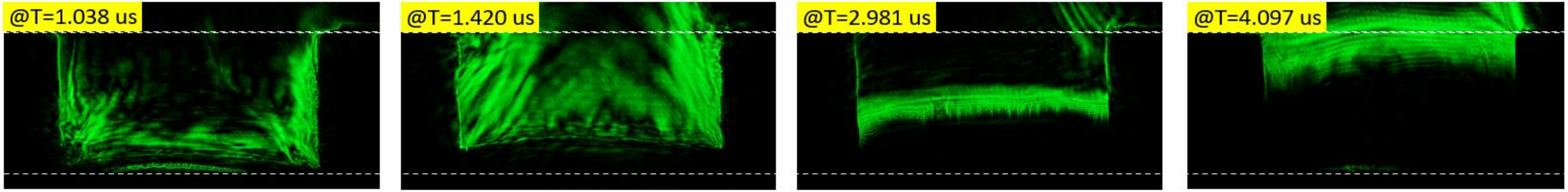
https://www.nasa.gov/feature/jpl/astronomers-find-a-break-in-one-of-the-milky-way-s-spiral-arms
C.-Y. Liu, NCKU Master Thesis 2023
J. C. Valenzuela, et al, High energy density physics **17**, 140 (2015)

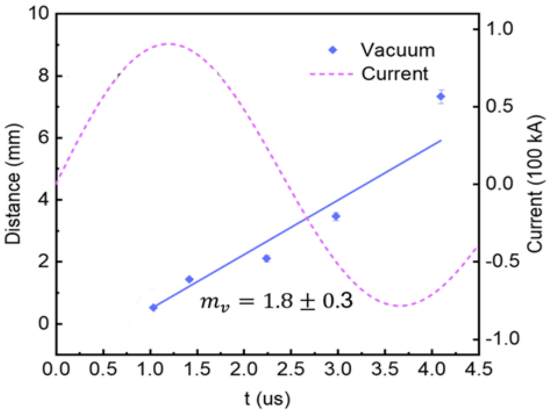
Plasma stagnated when two counter-propagating plasma jets colliding with each other

- A 1-kJ pulsed-power system with a suite of diagnostics was built.
- A supersonic plasma jet with a speed of 170 ± 70 km/s was generated using a conical-wire array. The corresponding Mach number was greater than 5.
- Plasma stagnated and reached a density of ~ 10^{16} cm^{-3} when two counter-propagating plasma jets colliding with each other.
- Technologies of pulsed-power systems are being adapted to the spherical tokamak, FIRST, that is being developed.

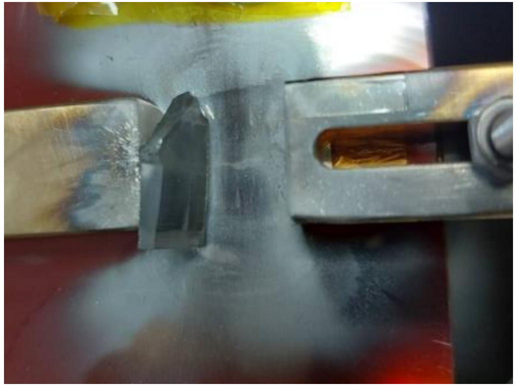
The flyer plate was accelerated to a velocity of 1.8 ± 0.3 km/s

- Schlieren images of the flyer plate taken at different times:
- @ ~10^{-5} torr

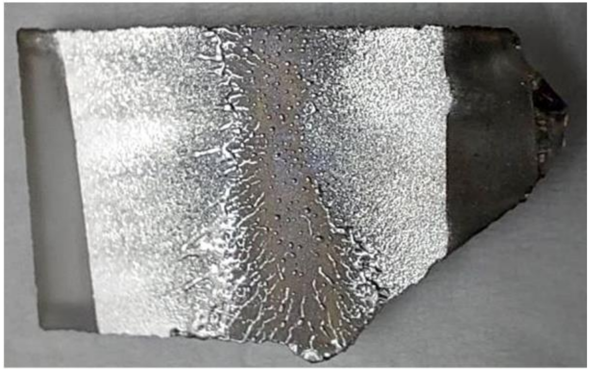




- After the experiment:



- Impacted surface:

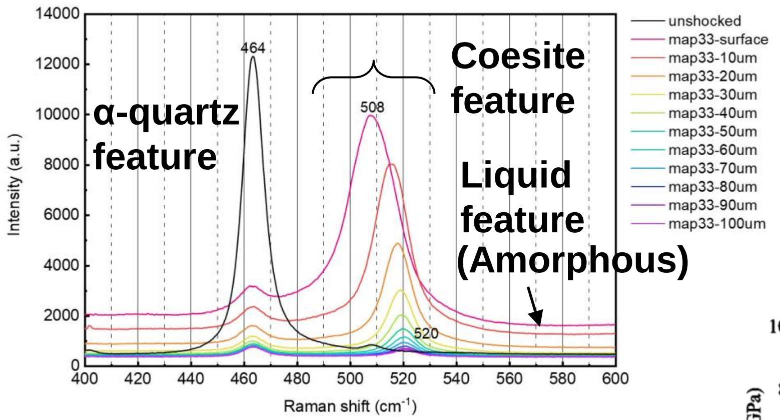


- Poster section: P2-PP-015

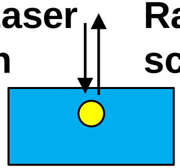
Y.-Z. Pan, NCKU Master Thesis 2024

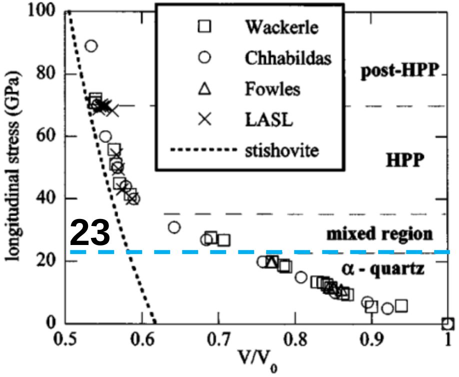
A shock with the pressure above 4 GPa and the temperature above 2400 ℃ was generated when the flyer plate collides the target

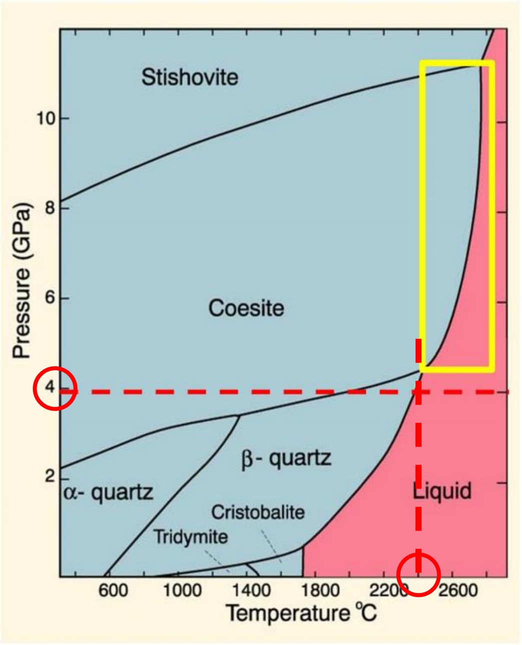
- **Raman shift at different depth:**



- **Raman scattering measurement:**







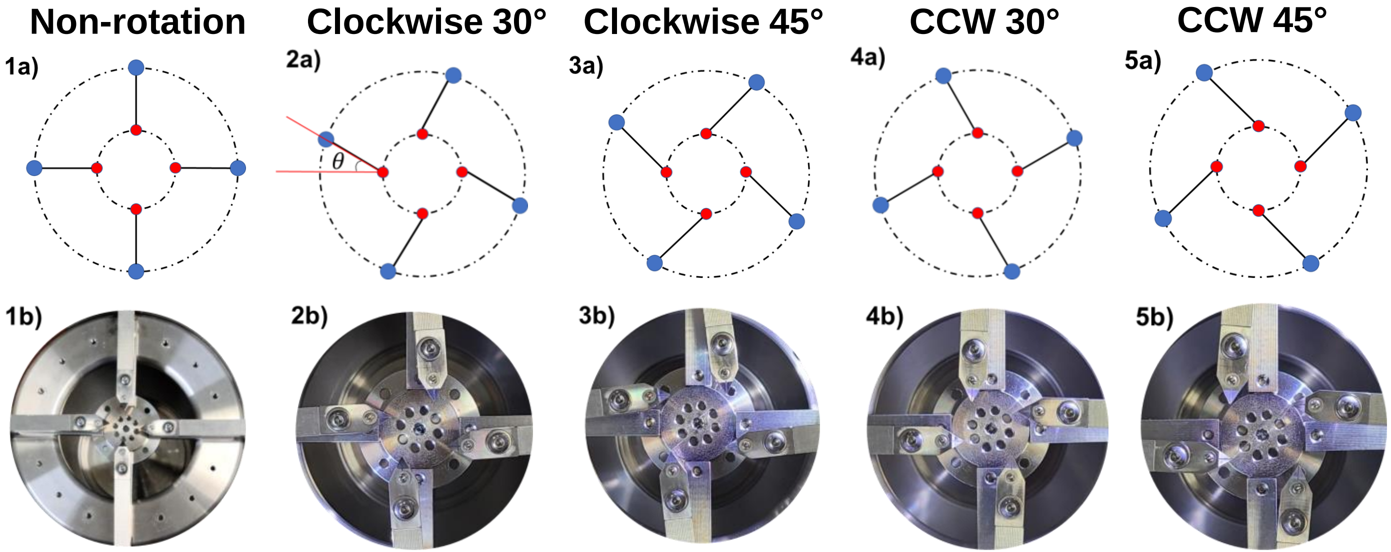
- **Since the Coesite feature remained after the target was shocked, the target should have entered the mixed region, i.e., P > 23 GPa is possible.**
- **We suspect that the pressure at the surface is lower than that in the target. More studies are needed.**
- **Poster section: P2-PP-015**

Y.-Z. Pan, NCKU Master Thesis 2024
T. De Reséguier, etc., J. Appl. Phys. 94, 2123 (2003)

Outlines

- Introduction to pulsed-power systems
- Application of pulsed-power systems at NCKU
  - Plasma jet generation using a bi-conical-wire array
  - High-pressure generation by generating a shock through colliding a flyer plate on a target
  - Development of the spherical tokamak, FIRST

Conical-wire arrays were twisted with different angles and in different directions



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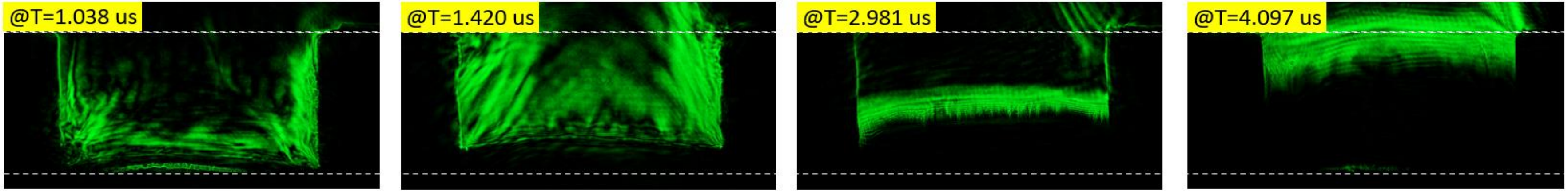
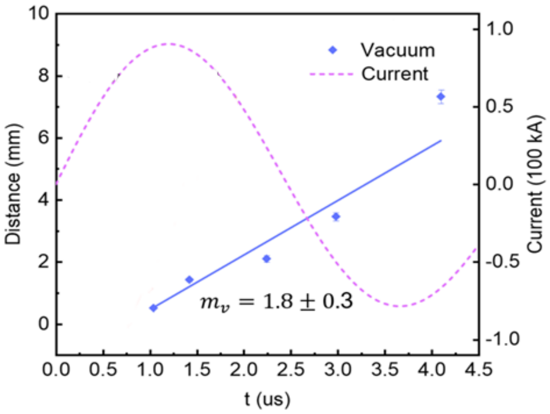
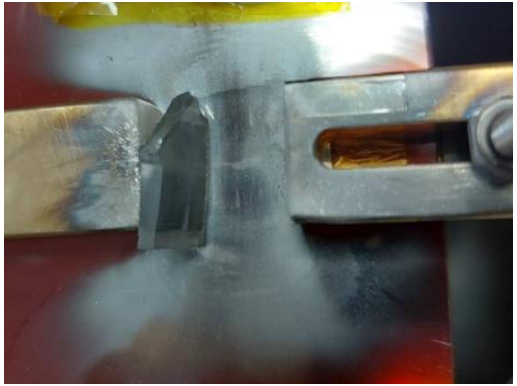
- @ ~10^{-5} torr



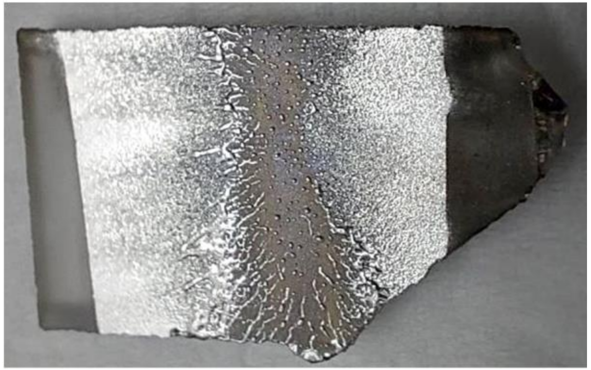


- Poster section: P2-PP-015

- After the experiment:



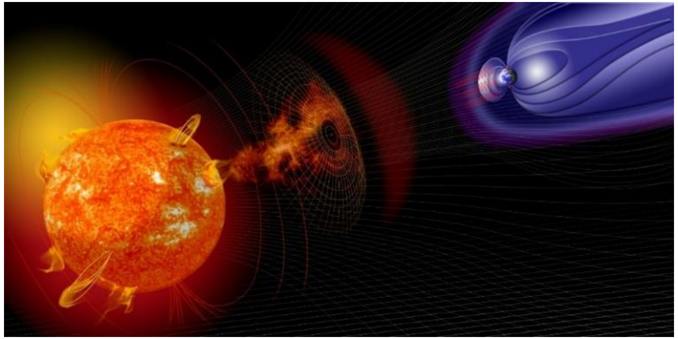
- Impacted surface:



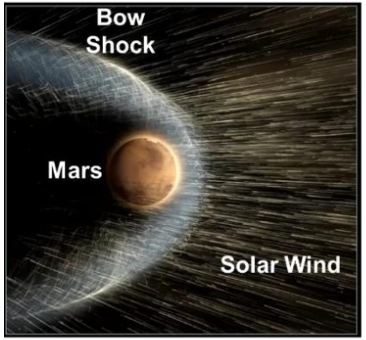
Y.-Z. Pan, NCKU Master Thesis 2024

Astrophysics and space sciences can be studied in the laboratory called "laboratory astrophysics"

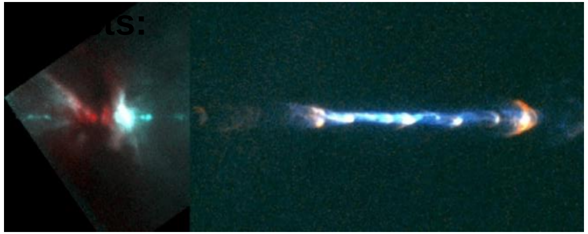
- **Solar wind:**



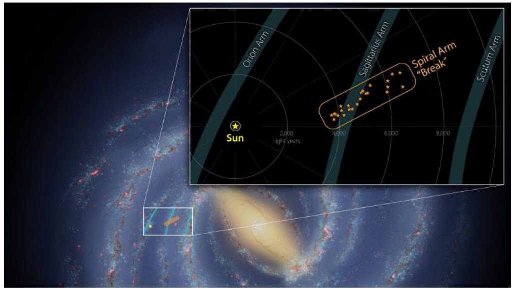
- **Bow shock:**



- **Supersonic jets in Herbig-Haro ts:**



- **Milky way's Spiral Arms:**



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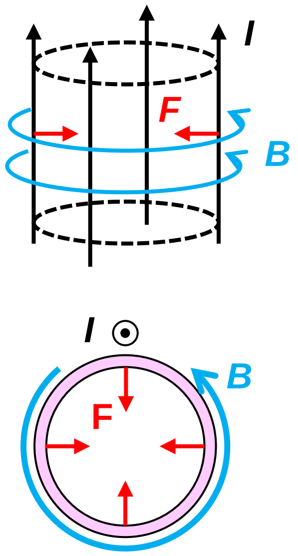
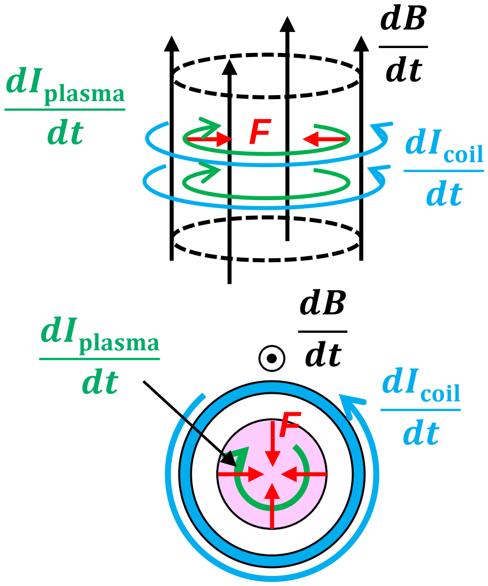
https://www.nasa.gov/mission_pages/sunearth/spaceweather/index.html
Chih-Jui Hsieh, NCKU Master Thesis 2020
B. Reipurth and J. BallyAnnu. Rev. Astron. Astrophys., 39:403-455, September 2001.
Jet Propulsion Laboratory [NASA/JPL] Astronomers Find a 'Break' in One of the Milky Way's Spiral Arms (Aug 17, 2021)

High pressure plasma can be achieved via pinch compression

- Z pinches



- Theta pinches



2D Fourier transform can be used to separate two phase images from two interferometer images overlapping on top of each other

$$\mathrm{I}(x,y) = I_0(x,y) + m_1(x,y)\cos[\nu_1 x + \phi_1(x,y)] + m_2(x,y)\cos[\nu_2 y + \phi_2(x,y)]$$
$$= I_0(x,y) + c_1(x,y)e^{\mathrm{i}\nu_1 x} + c_1^*(x,y)e^{-\mathrm{i}\nu_1 x} + c_2(x,y)e^{\mathrm{i}\nu_2 y} + c_2^*(x,y)e^{-\mathrm{i}\nu_2 y}$$

$$c_{1,2}(x,y) \equiv \frac{1}{2} m_{1,2}(x,y)e^{\mathrm{i}\phi_{1,2}(x,y)} \qquad \phi_{1,2}(x,y) = \tan^{-1}\left(\frac{\mathrm{Im}[c_{1,2}(x,y)]}{\mathrm{Re}[c_{1,2}(x,y)]}\right)$$

- **Two interferometer images need to be independent to each other.**
- **Fourier transform in x:**

$$\hat{I}(f_x,y) = \hat{I}_0(f_x,y) + \hat{c}_1(f_x-\nu_1,y) + \hat{c}_1^*(f_x+\nu_1,y) + \hat{c}_2(f_x,y)e^{\mathrm{i}\nu_2 y} + \hat{c}_2^*(f_x,y)e^{-\mathrm{i}\nu_2 y}$$

- **Fourier transform in y:**

$$\tilde{I}(x,f_y) = \tilde{I}_0(x,f_y) + \tilde{c}_1(x,f_y)e^{\mathrm{i}\nu_1 x} + \tilde{c}_1^*(x,f_y)e^{-\mathrm{i}\nu_1 x} + \tilde{c}_2(x,f_y-\nu_2) + \tilde{c}_2^*(x,f_y+\nu_2)$$

- **Fourier transform in x&y:**

$$\hat{\tilde{I}}(f_x,f_y) = \hat{\tilde{I}}_0(f_x,f_y) + \hat{\tilde{c}}_1(f_x-\nu_1,f_y) + \hat{\tilde{c}}_1^*(f_x+\nu_1,f_y) + \hat{\tilde{c}}_2(f_x,f_y-\nu_2) + \hat{\tilde{c}}_2^*(f_x,f_y+\nu_2)$$

Fourier transform in different directions can also be used to separate two interferometer images overlapping on top of each other

- **Fourier transform in x:**

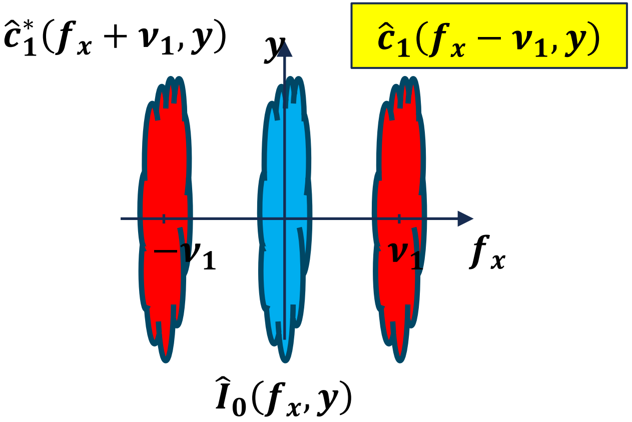
$$\hat{I}(f_x,y)=\hat{I}_0(f_x,y)+\hat{c}_1(f_x-\nu_1,y)+\hat{c}_1^*(f_x+\nu_1,y)+\hat{c}_2(f_x,y)e^{\mathrm{i}\nu_2 y}+\hat{c}_2^*(f_x,y)e^{-\mathrm{i}\nu_2 y}$$

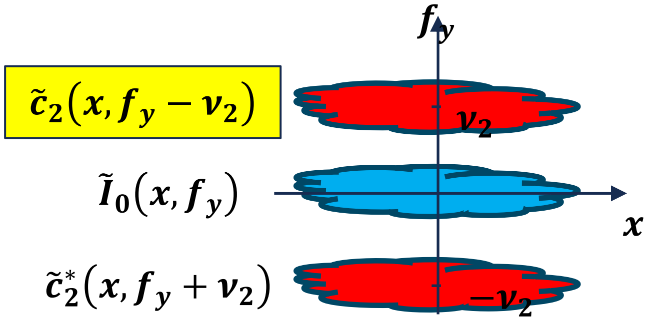
- **Fourier transform in y:**

$$\tilde{I}(x,f_y)=\tilde{I}_0(x,f_y)+\tilde{c}_1(x,f_y)e^{\mathrm{i}\nu_1 x}+\tilde{c}_1^*(x,f_y)e^{-\mathrm{i}\nu_1 x}+\tilde{c}_2(x,f_y-\nu_2)+\tilde{c}_2^*(x,f_y+\nu_2)$$

- **Fourier transform in x&y:**

$$\hat{\tilde{I}}(f_x,f_y)=\hat{\tilde{I}}_0(f_x,f_y)+\hat{\tilde{c}}_1(f_x-\nu_1,f_y)+\hat{\tilde{c}}_1^*(f_x+\nu_1,f_y)+\hat{\tilde{c}}_2(f_x,f_y-\nu_2)+\hat{\tilde{c}}_2^*(f_x,f_y+\nu_2)$$





Fringes need to be fine enough to separate two overlapping phase images

- **Fourier transform in x:**

$$\hat{I}(f_x, y) = \hat{I}_0(f_x, y) + \hat{c}_1(f_x - \nu_1, y) + \hat{c}_1^*(f_x + \nu_1, y) + \hat{c}_2(f_x, y)e^{i\nu_2 y} + \hat{c}_2^*(f_x, y)e^{-i\nu_2 y}$$

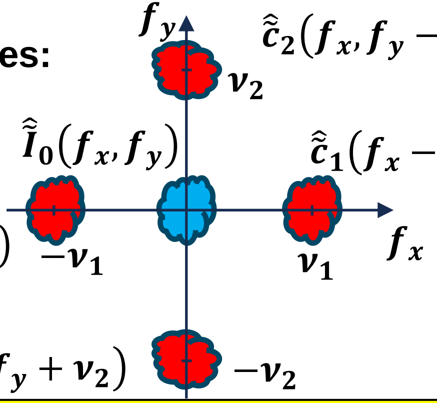
- **Fourier transform in y:**

$$\tilde{I}(x, f_y) = \tilde{I}_0(x, f_y) + \tilde{c}_1(x, f_y)e^{i\nu_1 x} + \tilde{c}_1^*(x, f_y)e^{-i\nu_1 x} + \tilde{c}_2(x, f_y - \nu_2) + \tilde{c}_2^*(x, f_y + \nu_2)$$

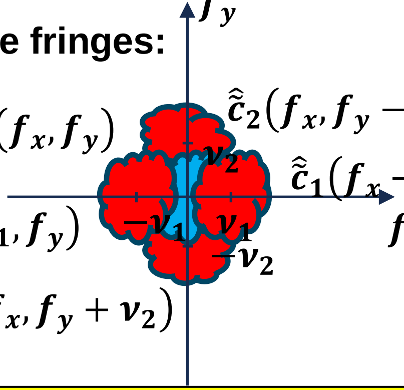
- **Fourier transform in x&y:**

$$\hat{\tilde{I}}(f_x, f_y) = \hat{\tilde{I}}_0(f_x, f_y) + \hat{\tilde{c}}_1(f_x - \nu_1, f_y) + \hat{\tilde{c}}_1^*(f_x + \nu_1, f_y) + \hat{\tilde{c}}_2(f_x, f_y - \nu_2) + \hat{\tilde{c}}_2^*(f_x, f_y + \nu_2)$$

- **Fine fringes:**



- **Coarse fringes:**



- **Spatial frequency of fringes need to be larger than the spatial bandwidth of two signals.**