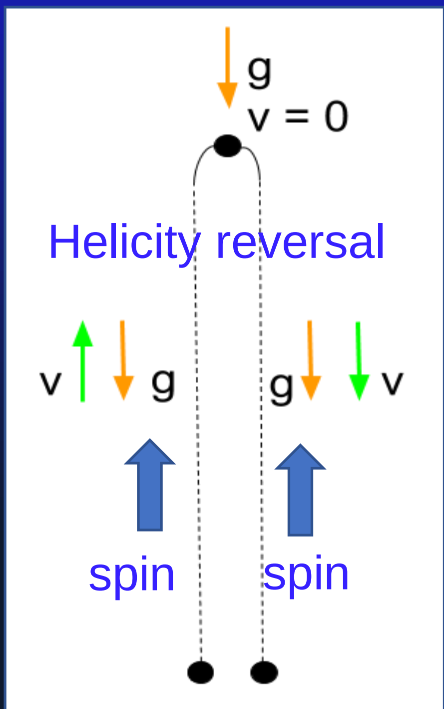
Neutrinos from the big bang: probing cosmic gravitational inhomogeneities & magnetic fields en route to eventual detection



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PRL 126, 191803 (2021) (magnetic)
PRD 103, 123019 (2021)(gravitational)

Institute of Physics, Academia Sinica
04 January 2023

Relic neutrinos – take home messages

Density of neutrinos left over from the Big Bang is about $337/cm^3$
(100 X solar neutrino density)

Some 20,000,000 inside you now – only “unprocessed” relic of Big Bang

At least two of the three neutrino mass states are non-relativistic now:
$v < 1/4\ c$ -- but they all have a relativistic fermion distribution

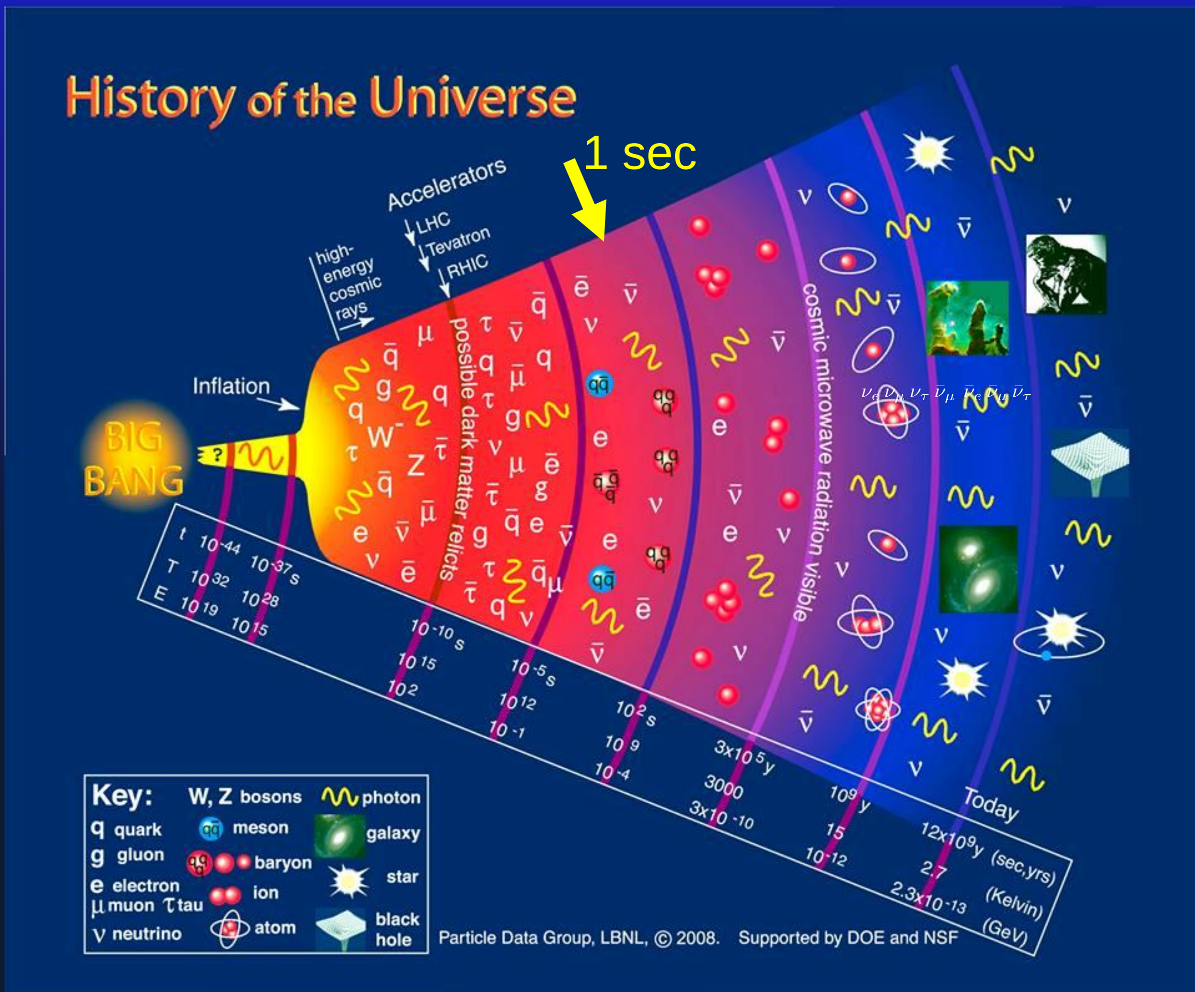
Could be Dirac or Majorana (own antiparticle)

Although they start out essentially with left handed helicity, gravitational inhomogeneities, both scalar and tensor, can change their helicity

Cosmic and galactic magnetic fields can also change helicity of Dirac relic neutrinos

=> Cosmic neutrino background is polarized

Prior to about one second after the Big Bang neutrinos (ν_e, ν_μ, ν_τ) and antineutrinos were in thermal equilibrium with electrons and quarks.



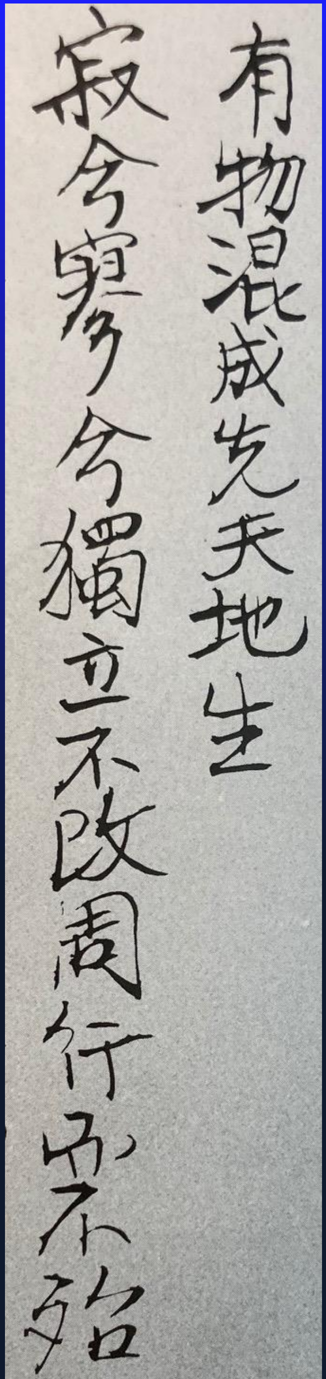
Since decoupling, neutrinos have been "free streaming" through the cosmos.

Present density 56.25 /cm^3 of each

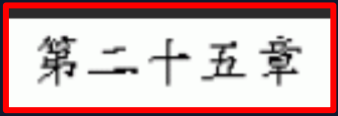
$$\nu_e \, \nu_\mu \, \nu_\tau \;\; \bar{\nu}_e \, \bar{\nu}_\mu \, \bar{\nu}_\tau$$

(~100 X solar ν_e)

Never detected!!



Something mysteriously formed,
Born before heaven and earth.
In the silence and the void,
Standing alone and unchanging,
Ever present and in motion.
: Dàodé Jīng



ChatGPT on relic neutrinos. in the style of Lao Tsu (老子)

In the beginning, the great void was filled with the energy of the universe, and from this energy, neutrinos emerged. They are the smallest of particles, yet they hold great power, born of the primal energy of the universe. Like the wind, they cannot be seen, but they can be felt in their presence.

Two ChatGPT-generated haiku

From the dawn of time,
Neutrinos roam the vast expanse,
Reaching Earth at last.

Neutrinos born in fire
Of the big bang's fiery birth
Dancing through the void

Cosmic neutrino background (CvB) vs cosmic microwave background (CMB)

	CMB	CvB	Relation
Temperature	2.73 K	1.9 K (1.7 x 10^{-4} eV)	$T_\nu/T_\gamma = (4/11)^{1/3} = 0.714$
Decoupling	3.8 x 10^5 years	~ 1 sec	
Density	~ 411 / cm^3	~ 337 / cm^3	$n_\nu = (9/11)\, n_\gamma$

Neutrino temperature < T_{CMB} = 2.725 K: neutrinos were not reheated!

CvB took a snapshot of the Universe at a much earlier epoch than CMB

At least two of the three neutrinos are non-relativistic ($v \ll c$) with masses at least 100 K (>> T_ν =1.9 K)

Processes in equilibrium

$$\nu_\chi(\bar{\nu}_\chi) + e^\pm \leftrightarrow \nu_\chi(\bar{\nu}_\chi) + e^\pm$$

$$\nu_\chi(\bar{\nu}_\chi) + \nu_{\chi'}(\bar{\nu}_{\chi'}) \leftrightarrow \nu_\chi(\bar{\nu}_\chi) + \nu_{\chi'}(\bar{\nu}_{\chi'})$$

Scattering
χ = e, μ, τ

$$\nu_\chi + \bar{\nu}_\chi \leftrightarrow e^- + e^+$$

Annihilation. For T < m_μ= 106 MeV annihilations only to electrons

$$\nu_e + e^- \leftrightarrow e^- + \nu_e, \quad \bar{\nu}_e + e^- \leftrightarrow e^- + \bar{\nu}_e$$

Charge exchange, keeps ν_e in equilibrium longer

Decoupling: as densities decrease in expanding universe mean free paths become too long for further interactions

$$T(\nu_\mu) = T(\nu_\tau) \sim 1.5\,\mathrm{MeV}$$

$$T(\nu_e) \sim 1.3\,\mathrm{MeV}$$

$$t(\mathrm{sec}) \simeq \frac{1}{\sqrt{\mathrm{T(MeV)}}}$$

Evolution of primordial neutrinos from freezeout

Neutrinos produced in flavor eigenstates, linear superpositions of mass eigenstates 1,2,3,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Pontecorvo–Maki–Nakagawa–Sakata
PMNS mixing matrix

and in wave packets of size
~ electron mean free path $1/\alpha^2 T \sim 10^6 - 10^7 \mathrm{fm}$
and velocity $v = p/\sqrt{p^2 + m^2}$

Velocity dispersion of mass components $\delta v = (\Delta m/m) m^2/p^2$
>> velocity dispersion $(\delta p/p) m^2/p^2$ of given mass component

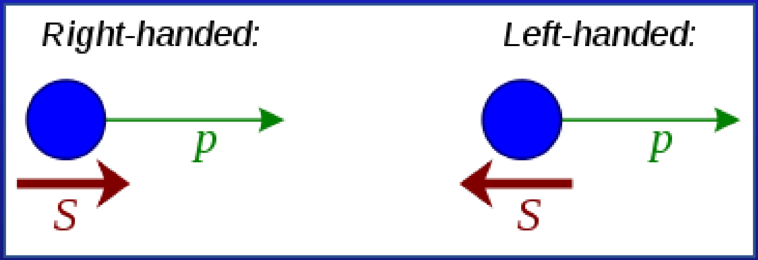
Initial flavor eigenstate arrives at Earth in three well separated mass packets with relativistic thermal distributions:

$$\frac{1}{e^{p/T} + 1}$$

What happens to neutrinos between 1 sec and now, 13.8 billion years later?

Neutrinos have negative chirality: L handed

& antineutrinos positive chirality: R handed



A property of the weak interaction processes,
not an intrinsic property of neutrinos

Both cosmic, and later galactic, magnetic fields as well as gravitational inhomogeneities -- both scalar, and tensor gravitational waves -- can rotate the spins with respect to the momentum, and thus give neutrinos an amplitude to be right handed, and antineutrinos left handed!

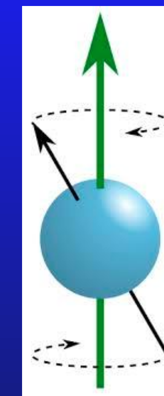
The helicities of relic neutrinos are a new probe of cosmic gravitational and magnetic fields.

Magnetic field B rotates spins, but not momenta:

Since ν have non-zero mass they have magnetic moment

$$\frac{d\vec{S}_\perp}{dt} = 2\mu_\nu \left(\vec{S}_\parallel \times \vec{B}_\perp + \frac{1}{\gamma} \vec{S}_\perp \times \vec{B}_\parallel \right)$$

Bargmann-Michel-Telegdi eqn.

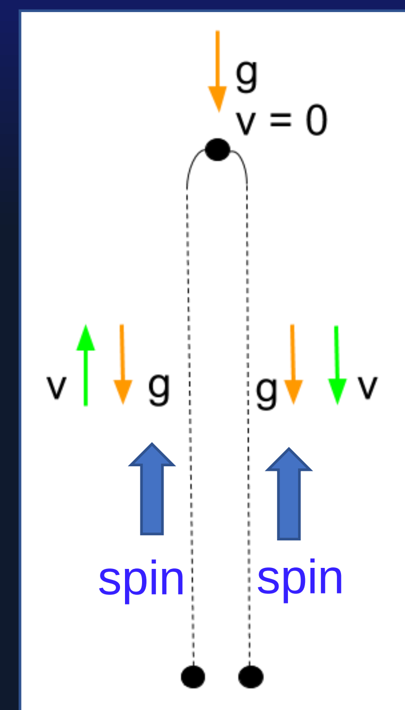


μ_ν = magnetic moment and $\gamma = 1/\sqrt{1-v^2}$ of neutrino

Gravitational potential Φ rotates momentum and spin:

$$\left.\frac{d\hat{p}}{dt}\right|_\perp = -\left(v + \frac{1}{v}\right)\vec{\nabla}_\perp \Phi \quad , \quad \left.\frac{d\vec{S}}{dt}\right|_\perp = -\frac{2\gamma+1}{\gamma+1}\vec{S}\cdot\vec{v}\,\vec{\nabla}_\perp \Phi$$

relativistic effect

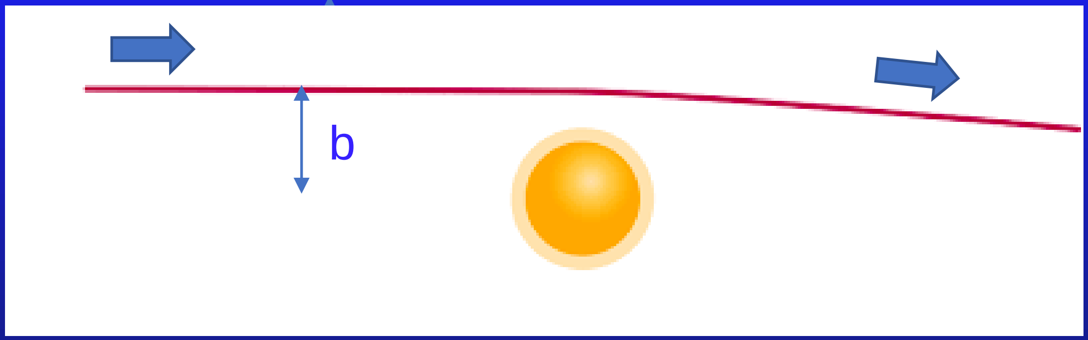


Spin bending lags momentum bending
(helicity = $h = \hat{p}\cdot\hat{S}$)

$$\left(h\frac{d\hat{S}}{dt} - \frac{d\hat{p}}{dt} \right)_\perp = \frac{m}{p}\vec{\nabla}_\perp \Phi$$

Spin and momentum bent equally for massless particle (photon); no spin bending of non-relativistic particle

Ex: particle passing star of mass M at impact parameter b



$$\Delta\theta_p = \frac{2MG}{bv^2}(1+v^2)$$

momentum bending
v → 1 Einstein light bending

$$\Delta\theta_s = \frac{2MG}{b}\frac{2\gamma+1}{\gamma+1}$$

spin bending $\gamma = 1/\sqrt{1-v^2}$

$$\theta \equiv \Delta\theta_s - \Delta\theta_p = -\frac{2MG}{b\gamma v^2}$$

lag of spin with respect to momentum; for photons spin tracks momentum.

Helicity flipping

Neutrino spin $|\downarrow\rangle \to \cos(\theta/2)|\downarrow\rangle + \sin(\theta/2)|\uparrow\rangle$

Probability of helicity flip = $\frac{1}{2}(1-\cos\theta) \to \frac{1}{4}\theta^2$ for small rotation

Chinese 9 layers of the sky – 1602 print

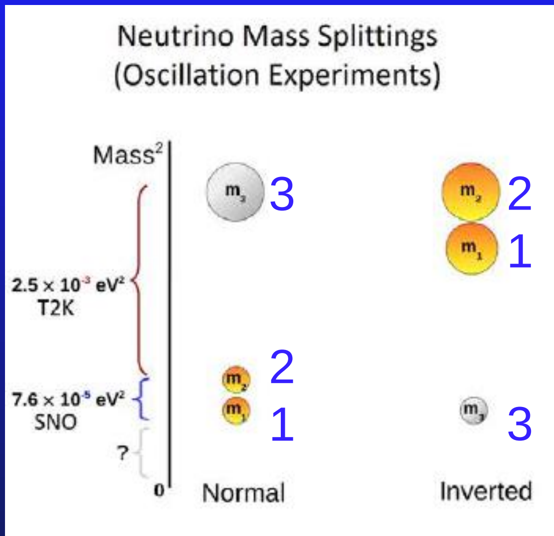
Neutrinos 101

Neutrino magnetic moments & spin precession

Gravitational inhomogeneities & spin precession

Detection of relic neutrinos

Neutrino masses and thermal distributions



N I

$$\Delta m^2_{21} = 7.50 \times 10^{-5} \mathrm{eV}^2$$

$$\Delta m^2_{31,N} = 2.52 \times 10^{-3} \mathrm{eV}^2$$

$$\Delta m^2_{31,I} = -2.51 \times 10^{-3}$$

Higher two masses at least 100 K

$m_i \gg T_{\nu 0}$ = 1.945K = 1.676 x 10^{-4} eV

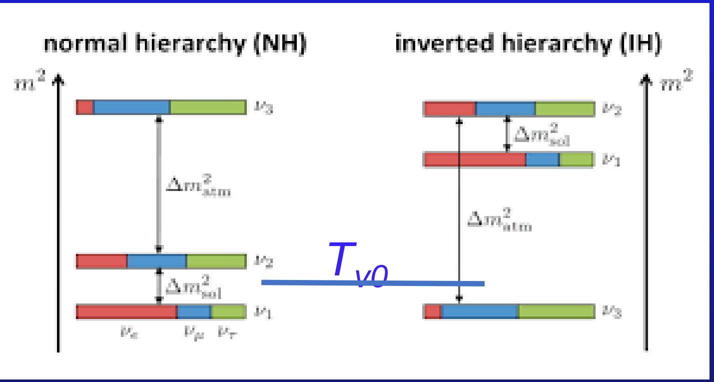
Distributions are fully relativistic even though at least two neutrino states (i=2,3 in N or 1,2 in I) are non-relativistic now

Normal H: $m_1=10^{-5}$ => v_1 ~ 1, v_2 ~1/5, v_3 ~ 1/20

Inverted H: $m_1=10^{-5}$ => v_3 ~ 1 , v_1 ~ v_2 ~ 1/20

Neutrino velocities v/c

Neutrino masses and thermal distributions



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Distributions are fully relativistic even though at least two neutrino states (i=2,3 in N or 1,2 in I) are non-relativistic now

Normal H: $m_1=10^{-5}$ => $v_1 \sim 1$, $v_2 \sim 1/5$, $v_3 \sim 1/20$

Inverted H: $m_1=10^{-5}$ => $v_3 \sim 1$, $v_1 \sim v_2 \sim 1/20$

Neutrino velocities v/c

Neutrino propagation in an expanding universe

Metric of expanding universe with weak gravitational inhomogeneities:

$$ds^2 = a(u)^2\left(-(1+2\Phi)du^2 + \delta_{ij}(1-2\Phi) + h_{ij})dx_i dx_j\right)$$

a = scale factor grows from ~10^{-10} at T =1 MeV to a=1 now

u = conformal time, dt = a du

x = comoving spatial coordinates, h_{ij}= gravitational waves

Φ = weak potential, driven by density fluctuations *δρ*

$$\nabla_x^2 \Phi = 4\pi G \delta\rho(x) a(u)^2$$

Φ(x) independent of *a*, at long wavelengths $\delta\rho \;\; a^2 \propto a^0$

Radiation dominated era ($P = \rho/3$), down to redshift ~ 10^4 :

fluctuation analysis => $\delta\rho/\bar{\rho} \sim a^2, \quad \delta\rho \sim 1/a^2$

Matter dominated era, from redshift ~ 10^4 to now, $\delta\rho/\bar{\rho} \sim a, \quad \delta\rho \sim 1/a^2$

Electron neutrino distribution at decoupling:

$$f_e(p) = \sum_i \frac{|U_{ei}|^2}{e^{E_i/T_e} + 1} \qquad E_i = \sqrt{p^2 + m_i^2} \simeq p$$

Since p >>> m_i have p/T_e => relativistic distribution
Does not change since neutrinos are decoupled!

$$\frac{1}{e^{p/T} + 1}$$

$$\sum_i |U_{ei}|^2 = 1$$

As universe expands:
scale factor a grows from ~10^{-10} at T =1 MeV to a=1 now

$$p \to p_0/a, \quad T \to T_0/a$$ (0 = now)

Were neutrinos to have remained in thermal equilibrium,

$$\frac{E_i}{T} \to \frac{\sqrt{p_0^2/a^2 + m_i^2}}{T_0/a} = \frac{\sqrt{p_0^2 + m_i^2\, a^2}}{T_0}$$

and distribution would be non-relativistic!

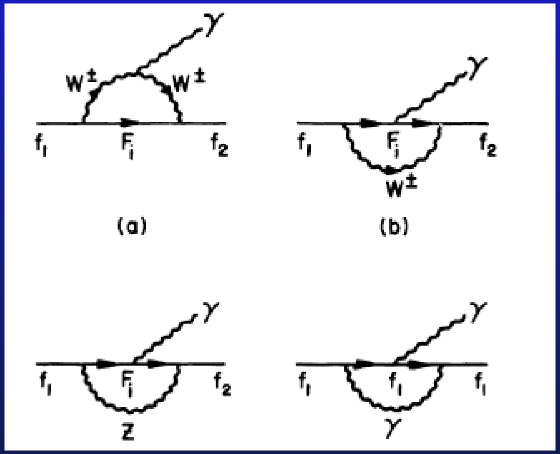
Neutrinos 101

Neutrino magnetic moments & spin precession

Gravitational inhomogeneities & spin precession

Detection of relic neutrinos

Rotation of neutrino spins in magnetic fields via neutrino magnetic moment



Standard model processes lead to a non-zero neutrino magnetic moment

$$\mu_\nu^{\mathrm{SM}} \simeq \frac{3eG_F}{8\sqrt{2}\pi^2} m_\nu \simeq 3 \times 10^{-21} m_{-2}\, \mu_B$$

Fujikawa-Schrock PRL 1980

μ_B = Bohr magneton $= e/2m_e$

$$m_{-2} = m_\nu / 10^{-2}\mathrm{eV}$$

But the magnetic moment could be much larger (Beyond Standard Model)

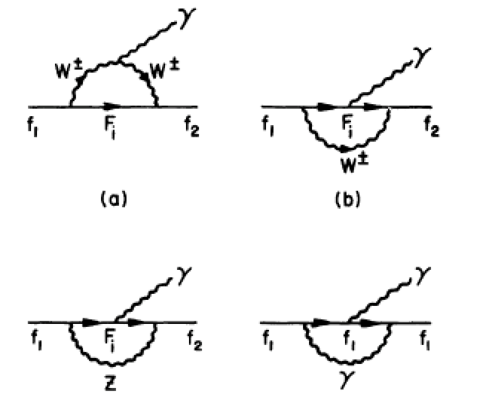
Upper bounds: $\mu_\nu < 2.9 \times 10^{-11} \mu_B$ GEMMA Kalinin reactor expt (2010) $\bar{\nu} + e^-$

$\mu_{\nu_e} < 2.8 \times 10^{-11} \mu_B$ Borexino (2017, solar $\nu + e$)

Theoretical “naturalness” bound: $\mu_\nu \lesssim 10^{-16} m_{-2} \mu_B$

Bell et al. PRL 2005

Diagonal vs. transition magnetic moments



Diagonal: interaction with magnetic field between equal mass states (neutrino $m_1 = m_2$)

Transition: interaction only between different mass states ($m_1 \neq m_2$)

Are neutrinos Dirac or Majorana fermions?

Dirac neutrinos can have both diagonal and transition moments.

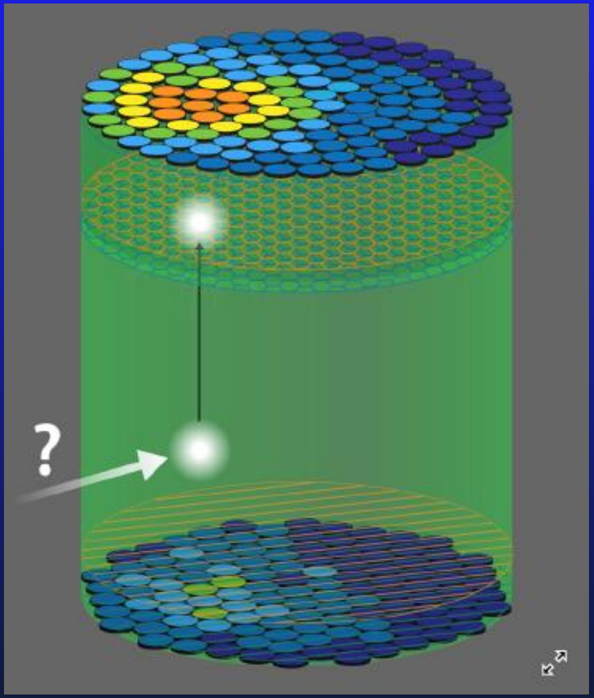
Diagonal moments of Majorana neutrinos identically zero; only transition moments:

$$\mathrm{CPT} => \langle i|\mu_\nu \vec{S}|j\rangle = -\langle \bar{j}\,|\mu_\nu \vec{S}|\bar{i}\,\rangle$$

Propagation through cosmic and galactic magnetic fields cannot change neutrino mass state.

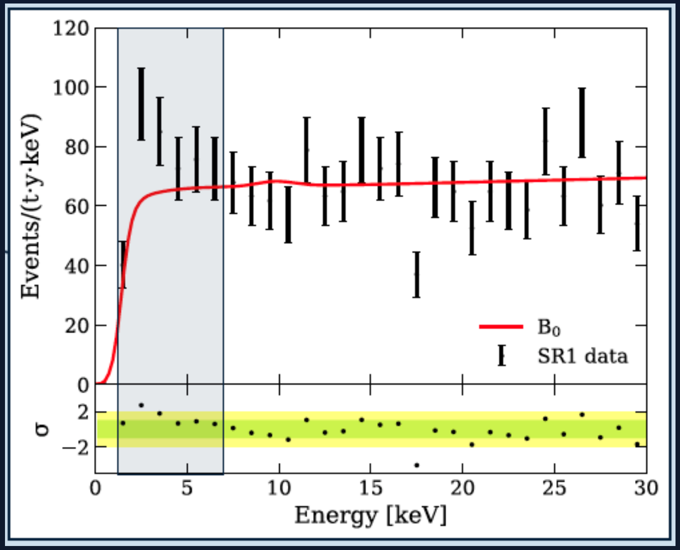
Only Dirac neutrinos can have helicities changed by magnetic fields.

XENON1T/nT experiments (in Gran Sasso)



2 - 8 ton TPCs of liquid Xe.
See both electron & nuclear recoils

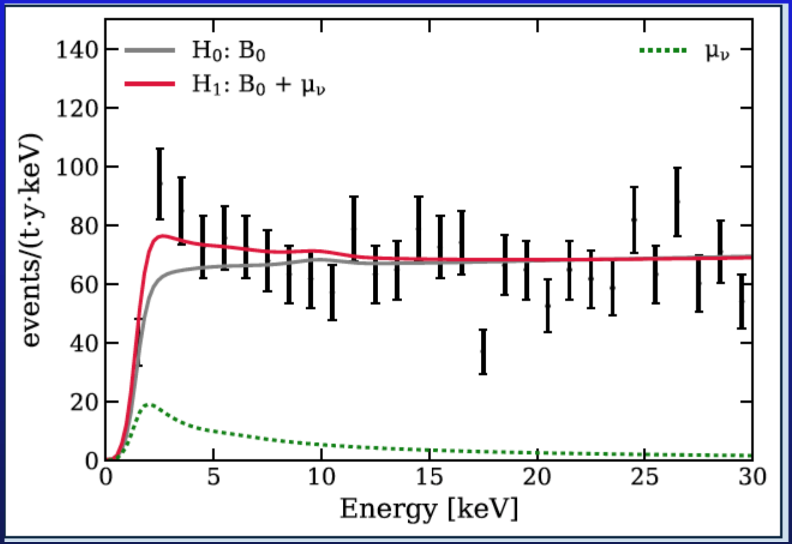
Search for WIMPS (weakly interacting massive particles) & other dark matter.
Sensitive to physics beyond standard model: solar axions bosonic dark matter,
magnetic moments of solar neutrinos?



Excess of low energy electron events 1-7 keV over expected background???

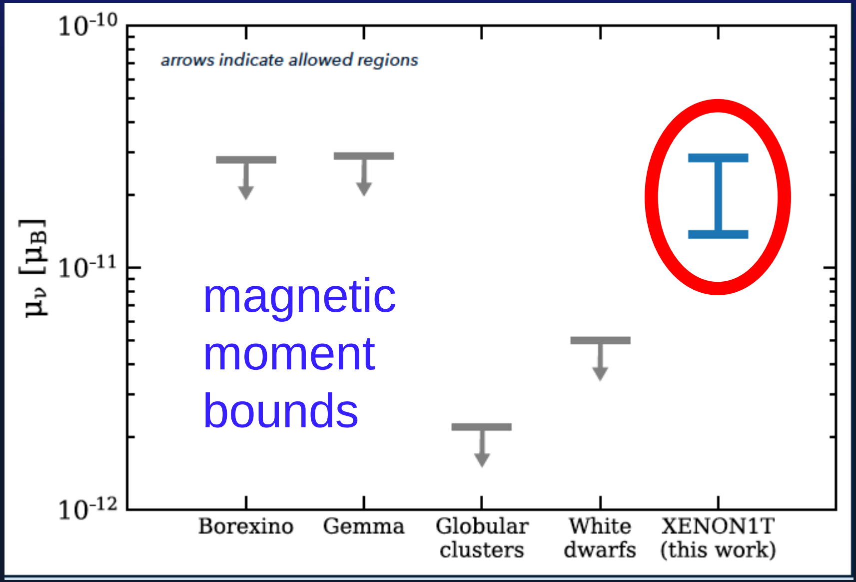
Aprile et al. PR D 102, 072004 (2020)

XENON1T low energy electron event excess



Possible explanations:

Large neutrino mag. moment (3.2σ)
Solar axions (3.5σ)
Tritium (in Xe) beta decays, E_e<18 keV

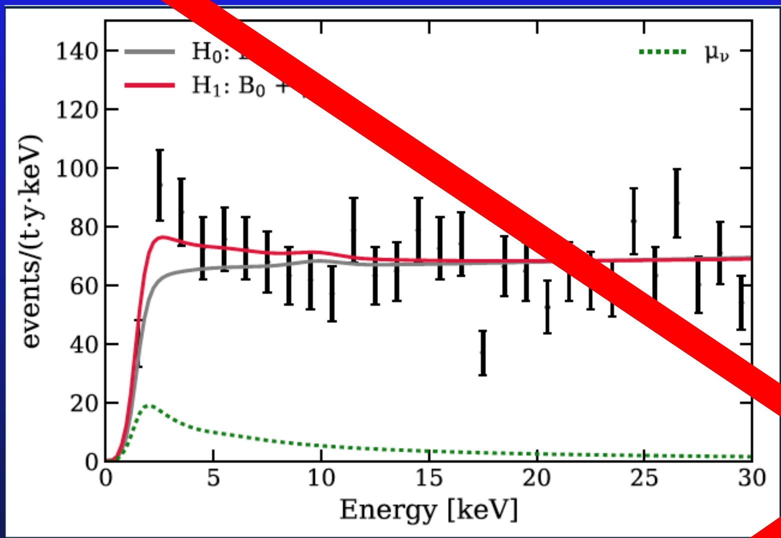


Excess consistent with neutrino magnetic moment
$\mu_{\nu,1T} \sim 1.4 - 2.9 \times 10^{-11} \mu_B$

Beyond Standard Model physics??

No information on whether diagonal or transition moment

XENON1T low energy electron event excess



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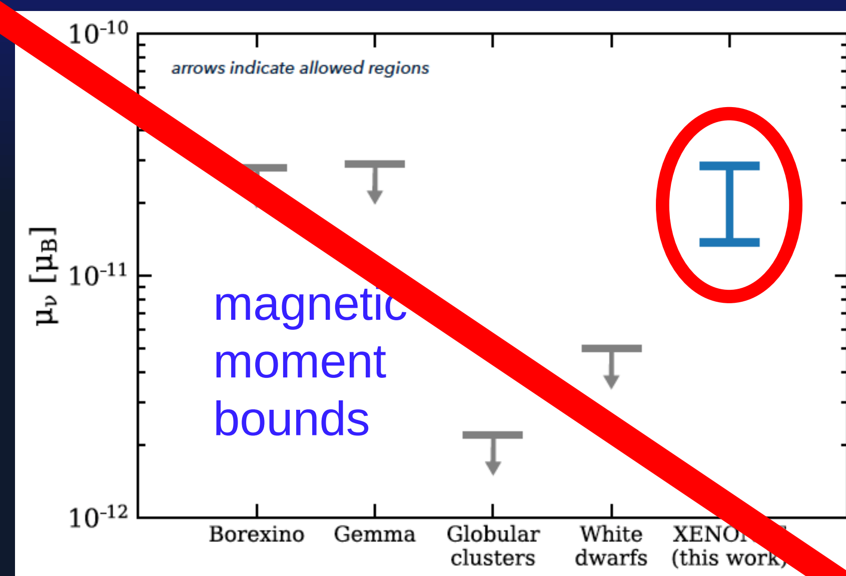


Beyond Standard Model physics??

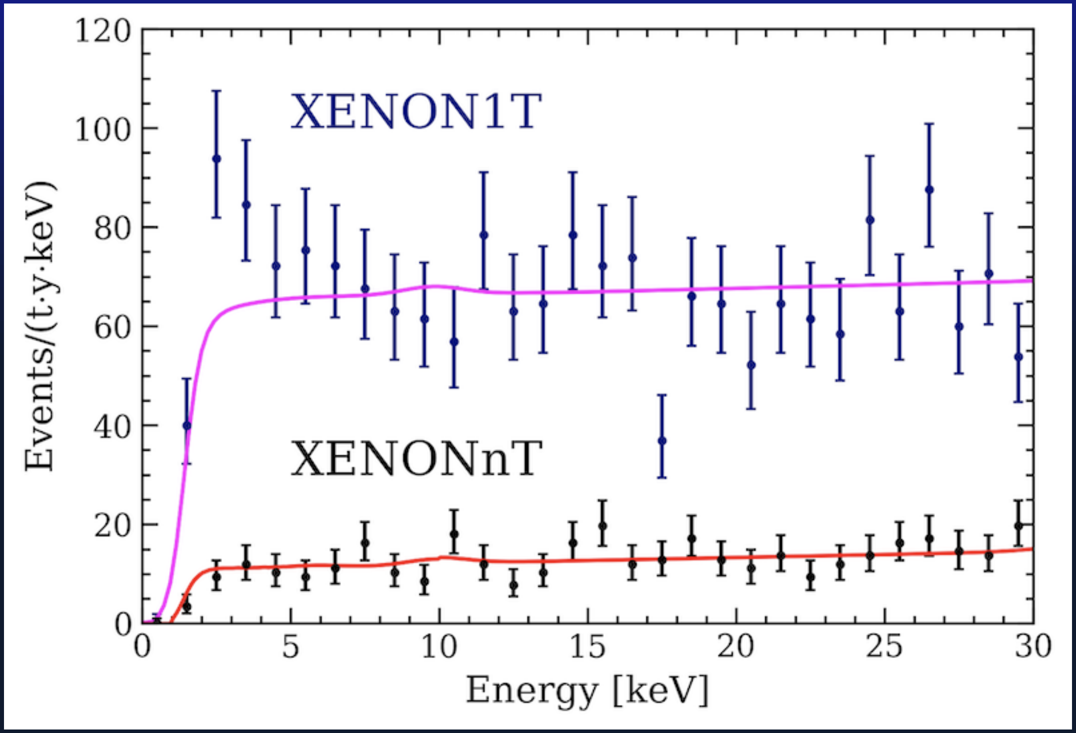
No information on whether diagonal or transition moment

Excess now tracked to tritium contamination

E. Aprile et al, PR: 129, 161805 (2022)

XENONnT = 6 tons of Xe.

No indication of beyond-the-standard-model neutrino magnetic moment



No such signal in PandaX-4T either

Spin precesses in magnetic field, but momentum does not

(neutrinos electrically neutral)

Thus magnetic fields change neutrino helicity: $h = \hat{S} \cdot \hat{p}$

Spin rotation by angle θ => helicity reversal probability = $\sin^2(\theta/2)$

Measure spin in rest frame of neutrino, B in ``lab” frame

$$\frac{d\vec{S}_\perp}{dt} \simeq 2\mu_\nu \vec{S}_\| \times \vec{B}_\perp$$ (Bargmann-Michel-Telegdi)

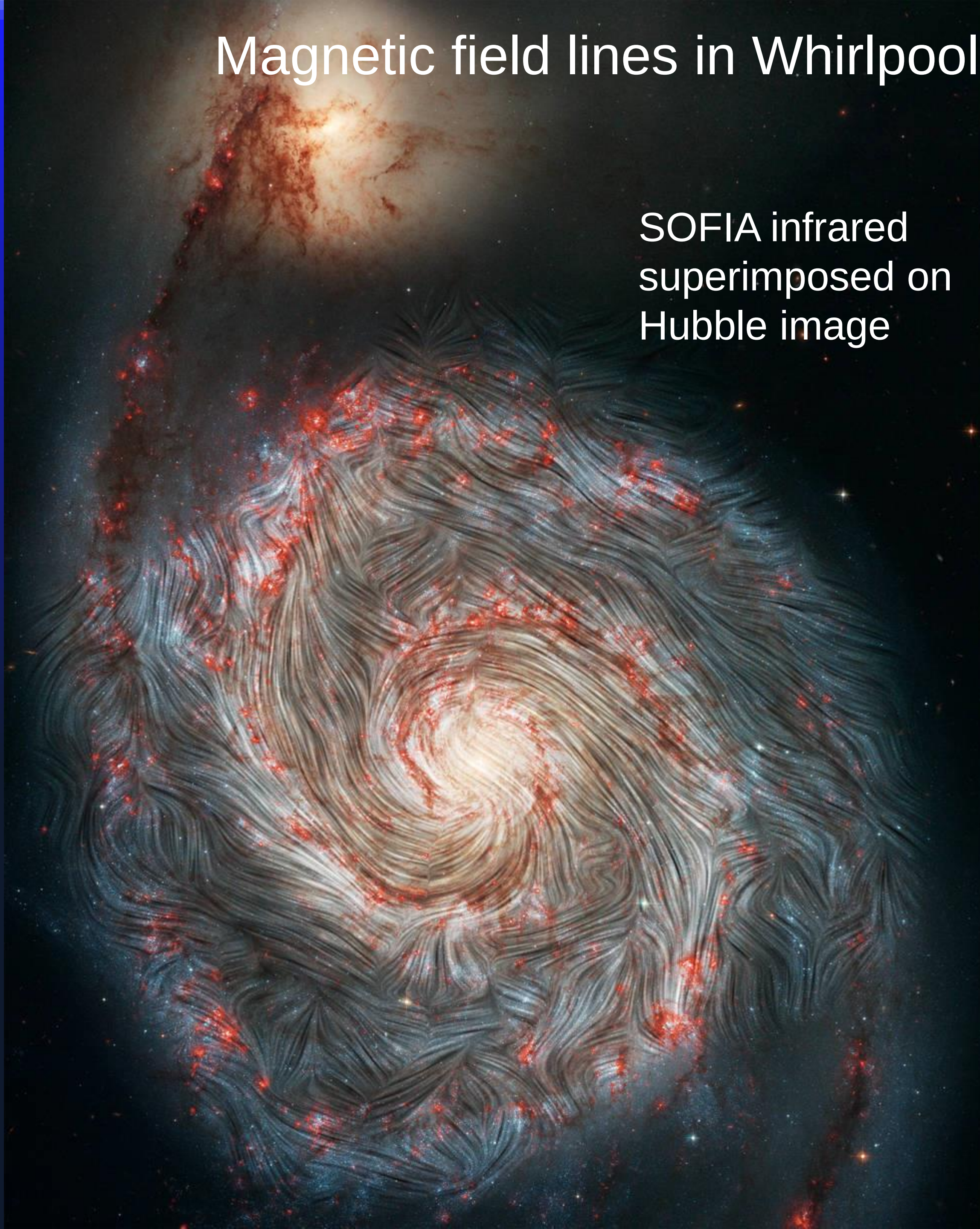
Cumulative spin rotation along ν trajectory:

$$\frac{\vec{S}_\perp}{|\vec{S}\,|} = \pm 2\mu_\nu \int dt\, \hat{v} \times \vec{B}(t)$$

for small angular changes.

Apply to galaxies, and to cosmic magnetic fields

Magnetic field lines in Whirlpool Galaxy M51



van Gogh 1889

Stratospheric Observatory for Infrared Astronomy

Last flight Sept 2022

Neutrino spin rotation by galactic magnetic field

For uniform galactic magnetic field: $\theta_g \sim 2\mu_\nu B_g \frac{\ell_g}{v}$

ℓ_g = mean crossing distance of the galaxy

But galactic fields are uniform only over coherence length Λ_g~ kpc
so spin direction does a random walk in magnetic field.

$$\langle\theta^2\rangle_g \simeq \left(2\mu_\nu B_g \frac{\Lambda_g}{v}\right)^2 \frac{\ell_g}{\Lambda_g}$$

ex., Milky Way with characteristic parameters (spherical cow approx):

$$B_g \sim 10\,\mu\mathrm{G}, \ell_g \sim 16\mathrm{kpc}, \Lambda_g \sim \mathrm{kpc}$$

$$\langle\theta^2\rangle_{MW} \sim 0.4 m^2_{-2} \left(\frac{\Lambda_g}{1\mathrm{kpc}}\right) \left(\frac{B_g}{10\mu G}\right)^2 \left(\frac{10^{15}\mu_\nu}{\mu_B}\right)^2$$

$\mu_\nu \sim 1.5 \times 10^{-15}\mu_B \sim 10^{-4}\mu_{1T} => \sqrt{\langle\theta^2\rangle} \sim 1$: helicity randomizes

Neutrino spin rotation by cosmic magnetic fields

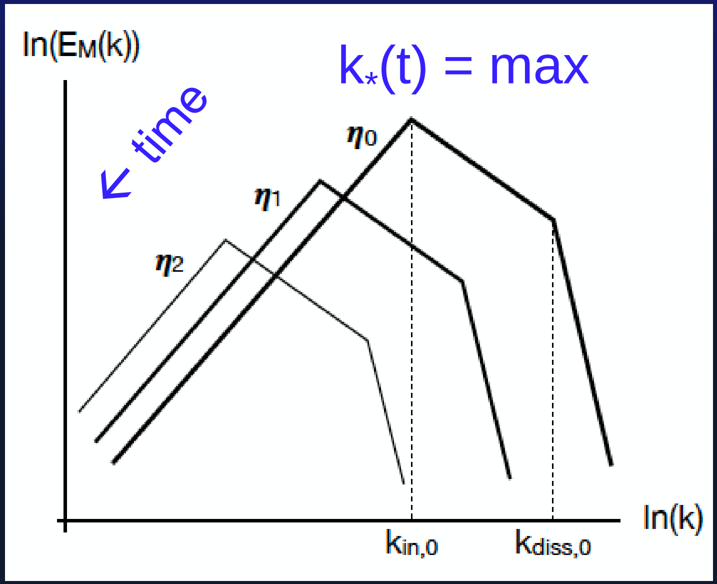
$$\frac{\vec{S}_\perp}{|\vec{S}|} = \pm 2\mu_\nu \int dt\, \hat{v} \times \vec{B}(t) \quad \Rightarrow \quad \langle \theta^2 \rangle_c = 4\mu_\nu^2 \langle \left(\int dt \vec{B}_\perp(t) \right)^2 \rangle$$

perp to v

Magnetic field correlation function:

$$\langle B_i(\vec{x}) B_j(\vec{x}\,') \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{2} P_B(k) e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}$$

+ helical part (no role here)



sum rule: $$\int \frac{d^3k}{(2\pi)^3} P_B(k) = \langle \vec{B}^{\,2} \rangle$$

Schematic of $P_B(k) = (2\pi)^2 E_M(k)/k^2$ with increasing conformal time ($\eta = u$): $\eta_0 < \eta_1 < \eta_2$ *T. Vachaspati, Rep. Prog. Phys.* **84** 074901 *(2021)*

Conservation of flux: $B \sim 1/a^2$ => $\langle \vec{B}^2(u) \rangle \simeq B_0^2/a(u)^4$ (0= now)

$$k_*(u) \sim \frac{2\pi}{\Lambda_0 a(u)^{1/2}}$$

(Λ_0 = coherence length of cosmic B field)

$$\langle \theta^2 \rangle_c = \frac{1}{2}\mu_\nu^2 B_0^2 \Lambda_0 \int_{u_d}^{u_0} \frac{du}{a(u)^{3/2}}$$

Main contribution is from radiation-dominated era (a ~ u):
from neutrino decoupling, u_d ($a_d \sim 10^{-10}$)
to matter-radiation equality, u_{eq} ($a_{eq} \sim 0.8 \times 10^{-4}$)

$R_u = cu_0$ = radius of universe
$u_0 = 3t_0$

$$\langle \theta^2 \rangle_C \sim 0.2 m_{-2}^2 \left(\frac{\Lambda_g}{1\mathrm{Mpc}}\right)\left(\frac{B_g}{10^{-12}G}\right)^2\left(\frac{10^{14}\mu_\nu}{\mu_B}\right)^2$$

Cosmic magnetic field rotation of neutrino spin

$$\langle \theta^2 \rangle_C \sim 0.2 m^2_{-2} \left(\frac{\Lambda_g}{1\mathrm{Mpc}} \right) \left(\frac{B_g}{10^{-12} G} \right)^2 \left(\frac{10^{14} \mu_\nu}{\mu_B} \right)^2$$

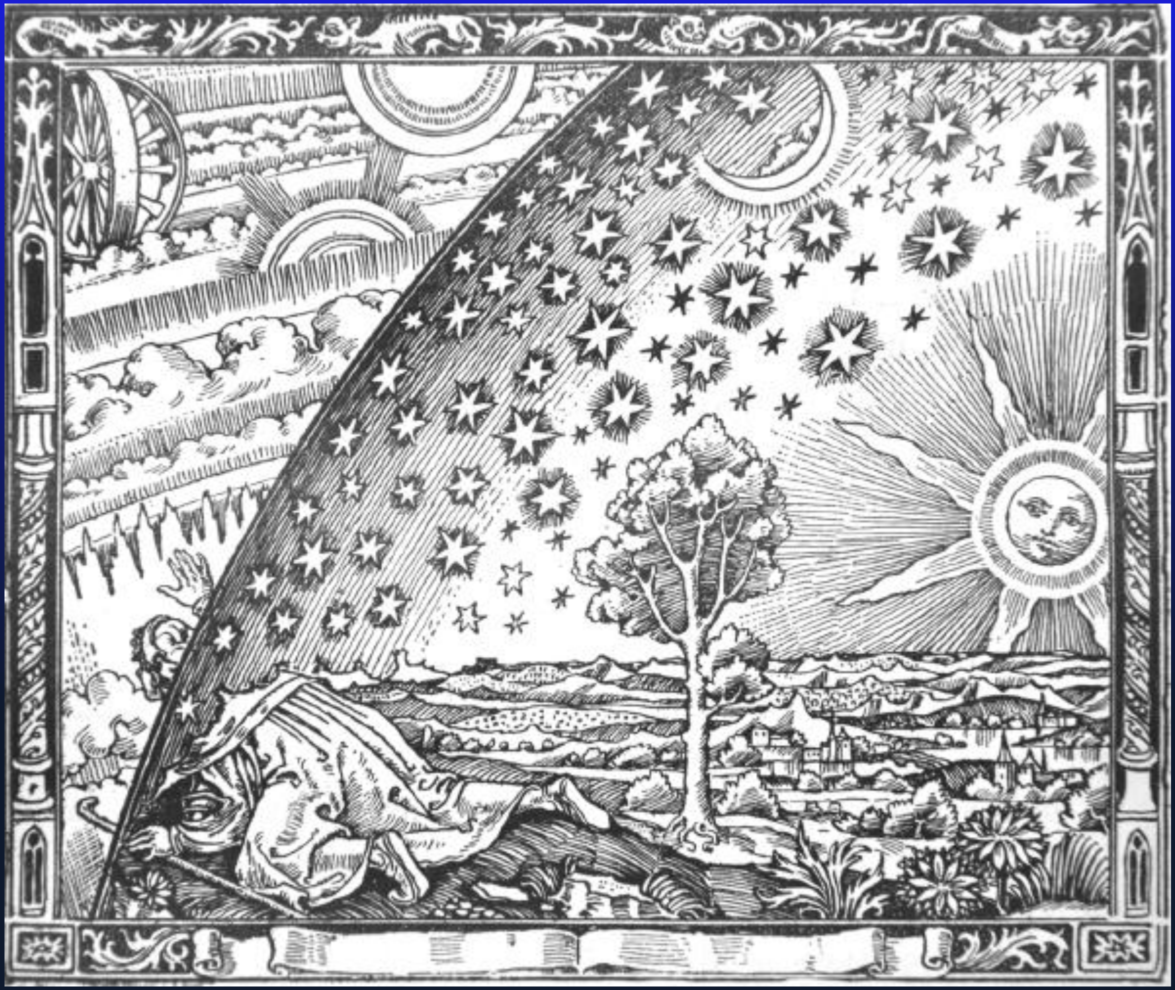
Λ_0 = coherence length of magnetic field

Magnetic moment $\mu_\nu \sim 10^{-14} \mu_B$ would be experimentally significant:
~1% of neutrinos could flip helicity

Effects of standard model magnetic moment insignificant.

If the neutrino is Majorana, no helicity changes from magnetic fields.

To within uncertainties in magnetic fields, correlation lengths, and neutrino masses, spin rotation in cosmic magnetic fields ~ galactic



Flammarion 1888

Neutrinos 101

Neutrino magnetic moments & spin precession

Gravitational inhomogeneities and waves & spin precession

Detection of relic neutrinos

Rotation of neutrino spins by scalar inhomogeneities

Gravitational potential Φ rotates momentum and spin:

$$\left.\frac{d\hat{p}}{dt}\right|_\perp = -\left(v + \frac{1}{v}\right)\vec{\nabla}_\perp \Phi \quad , \quad \left.\frac{d\vec{S}}{dt}\right|_\perp = -\frac{2\gamma+1}{\gamma+1}\vec{S}\cdot\vec{v}\,\vec{\nabla}_\perp \Phi$$

Spin bending lags momentum bending $\left(h\frac{d\hat{S}}{dt} - \frac{d\hat{p}}{dt}\right)_\perp = \frac{m}{p}\vec{\nabla}_\perp \Phi$

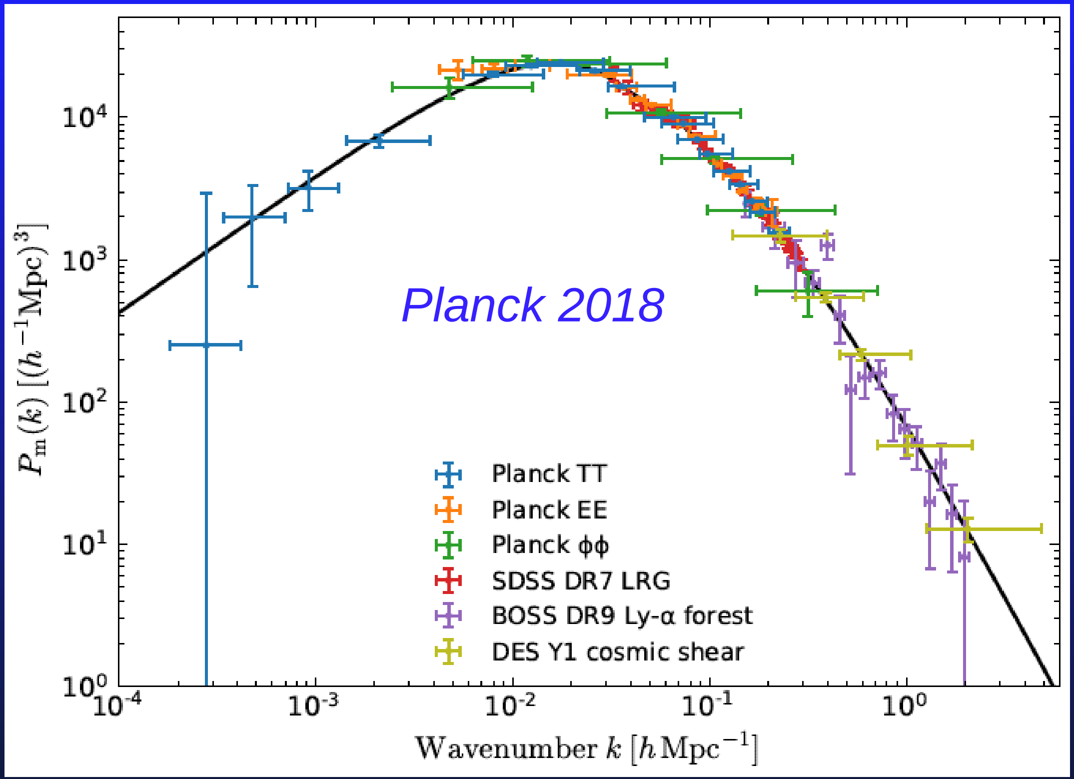
Again, neutrino undergoes a random walk through the inhomogeneities.

Relate field fluctuations to density fluctuations $\delta(\vec{x}\,) \equiv \delta\rho(\vec{x}\,)/\bar{\rho}$

$$\langle \delta(\vec{x}\,)\delta(\vec{x}\,')\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')} P(k)$$

$$\langle (\Delta\theta_p)^2 \rangle = \frac{2}{\pi}\int du (4\pi G\bar{\rho}a^2)^2 \int dk \frac{P(k)}{k}$$

Density fluctuation spectrum



P(k) ~ k for k <k_{max}

(Harrison-Zel'dovich)

P(k) ~ k^{-v} for k > k_{max}

Scales as

a^2 in matter dom. era

a^4 in rad. dom. era, k <k_{max}

$$\langle(\Delta\theta_p)^2\rangle = \frac{2}{\pi}\int du (4\pi G\bar{\rho}a^2)^2 \int dk \frac{P(k)}{k}$$

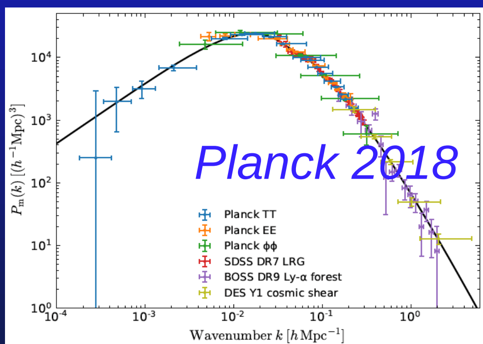
At present $$\int (dk/k)P_0(k) \simeq 7.25\times 10^4 (\mathrm{Mpc/h})^3 \quad \equiv \mathcal{P}$$

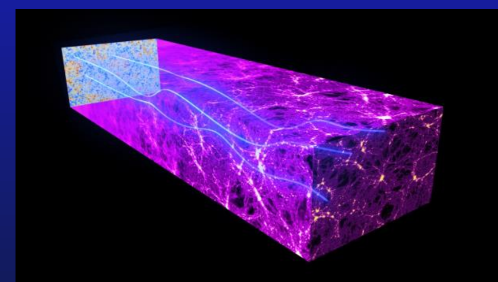
h = Hubble parameter ~ 0.7

Gravitational lensing of cosmic neutrino background

(G. Holder)

$$\langle (\Delta\theta_p)^2 \rangle = \frac{9}{2\pi}\mathcal{P}H_0^3 \int_0^1 \frac{da}{a^2}\left(\Omega_M a + \Omega_V a^4\right)^{3/2} \simeq 2.2 \times 10^{-6}$$





RMS momentum bending = lensing of cosmic neutrino background ~ 5.1 arcmin

Lensing of CMB ~ 2.7 arcmin. Most efficient at smaller z (≲10). Reionization of intergalactic H => photon-e scattering.

(Weak electron-neutrino scattering after reionization insignificant)

Gravitational spin rotation with respect to momentum, Θ

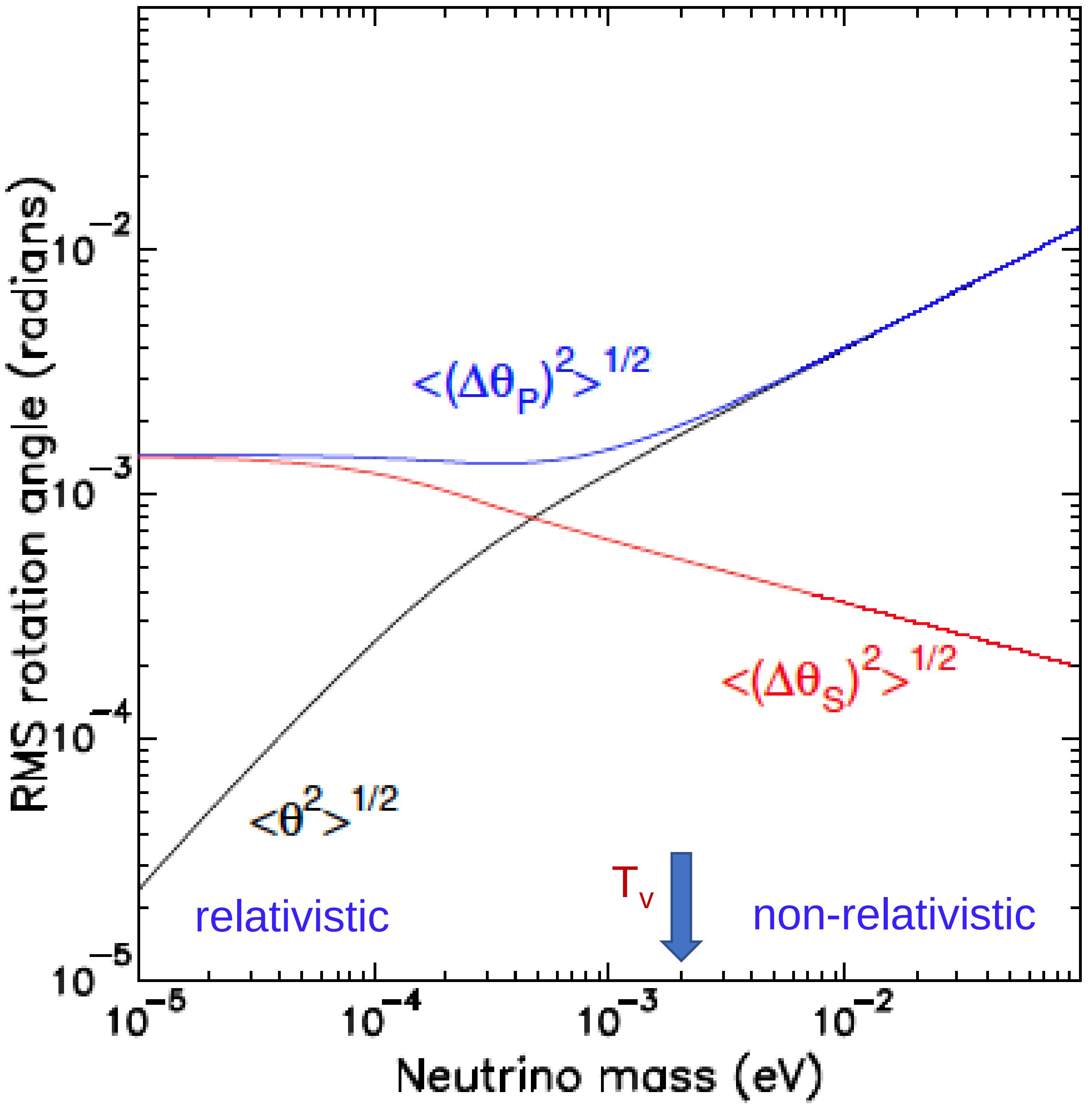
Main effect in matter dominated era from redshift ~ 10^4 to now.
Slower neutrinos have greater rotation of momentum vs spin

Including matter and dark energy:

$$\langle \theta^2 \rangle = \frac{9}{8\pi} \mathcal{P} H_0^3 \int_0^1 \frac{da}{a^2} \left(\Omega_M a + \Omega_V a^4 \right)^{3/2} \left(\frac{1}{v} - v \right)$$

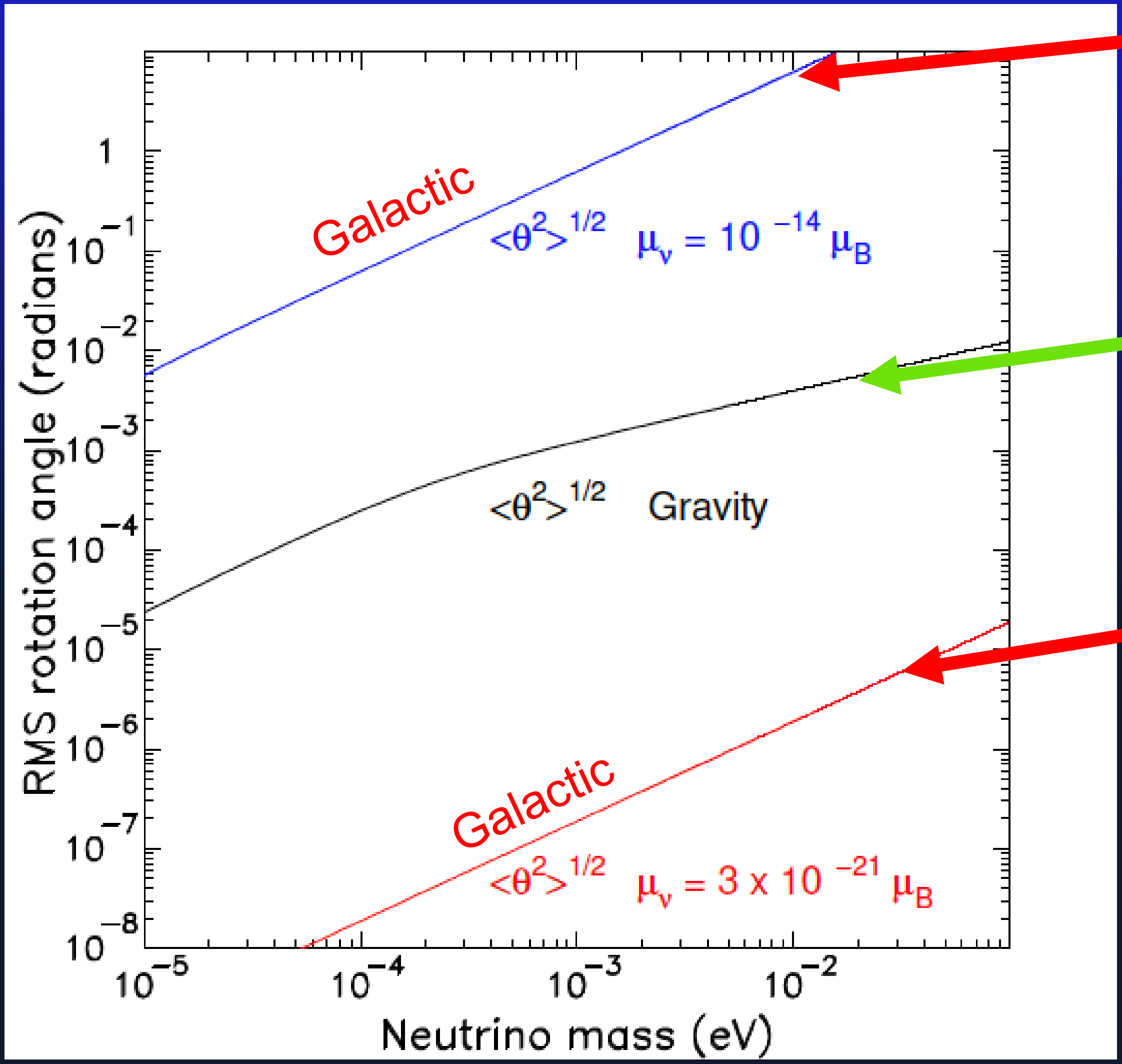
Ω_M = matter fraction, Ω_V = dark energy fraction

Neutrino velocity $v(a) = p_0 / \sqrt{p_0^2 + (m_\nu a)^2}.$ p_0 = present momentum



RMS rotation angle (radians)
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$<(\Delta\theta_P)^2>^{1/2}$
$<(\Delta\theta_S)^2>^{1/2}$
$<\theta^2>^{1/2}$
T_ν
relativistic
non-relativistic
10^{-5} 10^{-4} 10^{-3} 10^{-2}
Neutrino mass (eV)

Spin rotation from gravitational vs. magnetic fields



Rotation in Milky Way

$$B_g = 10\,\mu\mathrm{G},\ \Lambda_g = 1\,\mathrm{kpc}$$

$$\mu_\nu = 10^{-14}\mu_B$$

Gravitational rotation
GB+JCP PRD

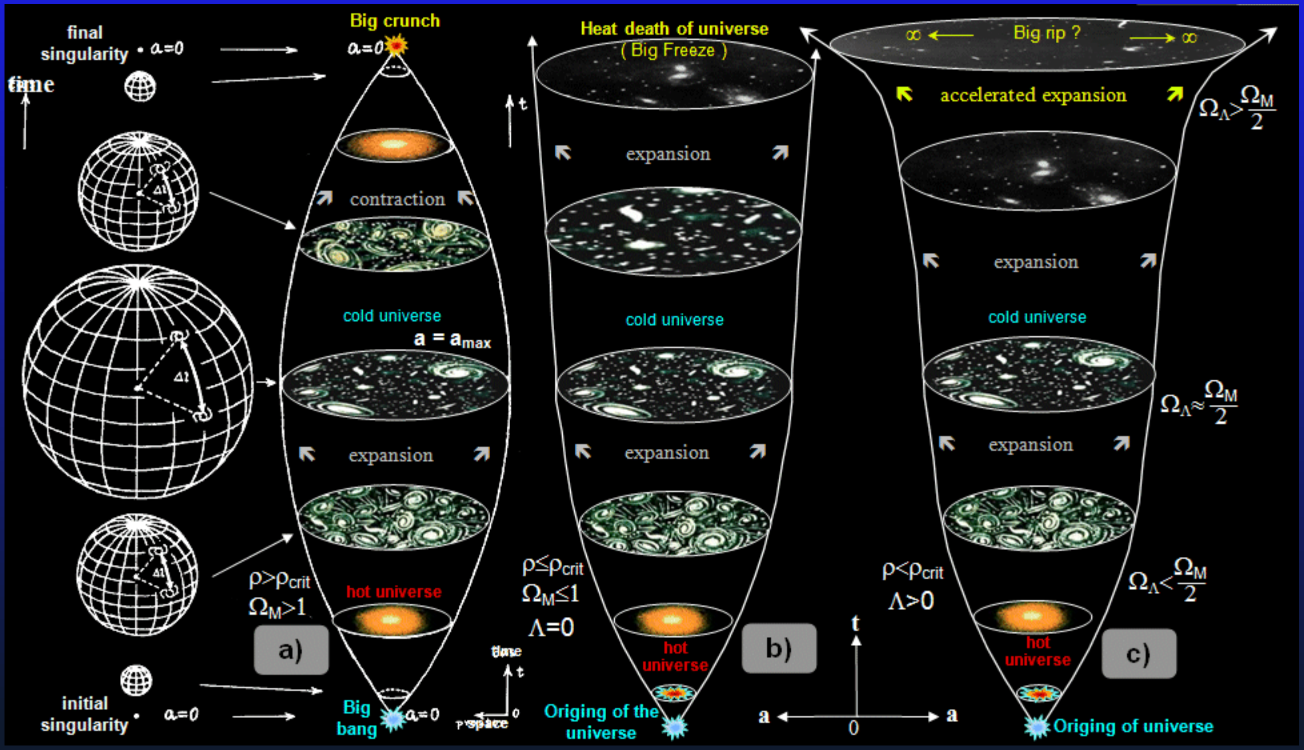
Rotation in Milky Way with standard model magnetic moment

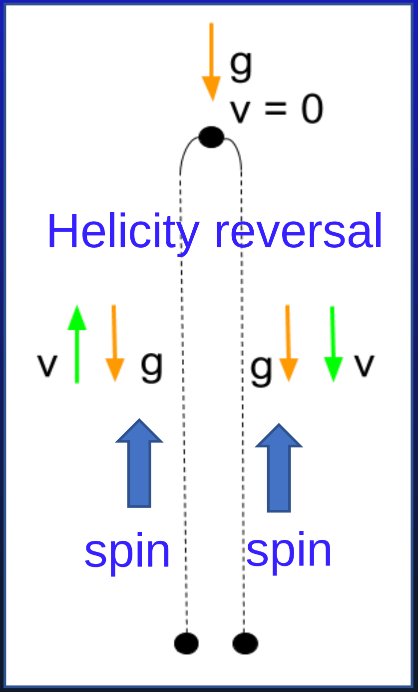
$$\mu_\nu^{\mathrm{SM}} \simeq 3\times 10^{-19} m_{\mathrm{eV}}\,\mu_B$$

Probability of helicity flip

$$\sim \langle\theta^2\rangle/4$$

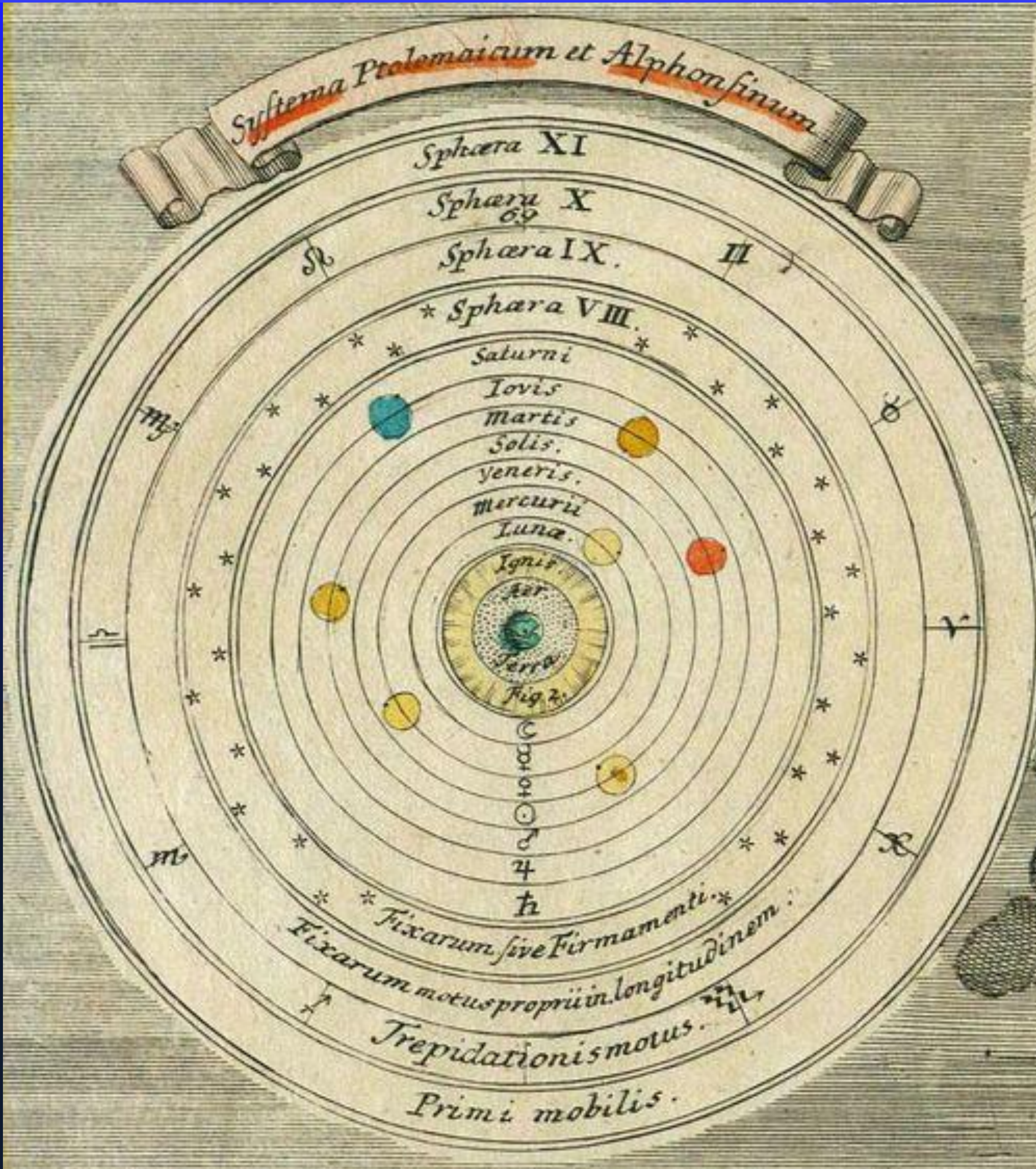
Neutrino helicity in a closed universe





What happens to neutrino helicities if the universe is closed and begins to implode? Momenta reverse in non-relativistic gravity.

Will have lots of R handed neutrinos on the infall, which will not partake in weak interactions. (Weak interaction theory does not change.} Very different nuclear physics – not time reversed!



Neutrinos 101

Neutrino magnetic moments & spin precession

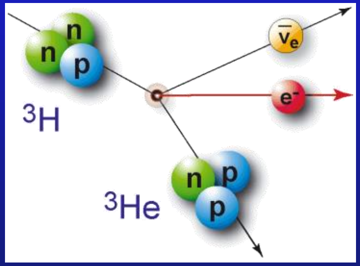
Gravitational inhomogeneities & spin precession

Detection of relic neutrinos

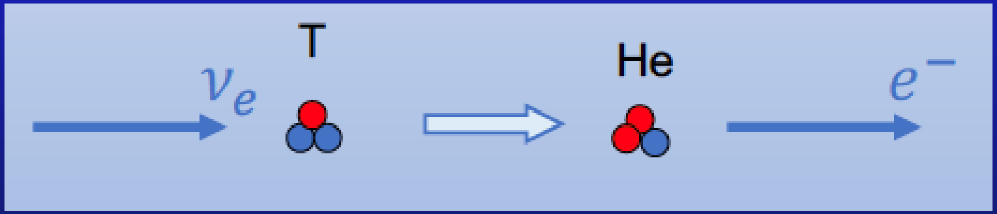
Ptolemy’s Treatise Almagest
ca. 150 AD

Detection of relic neutrinos

Tritium beta decay



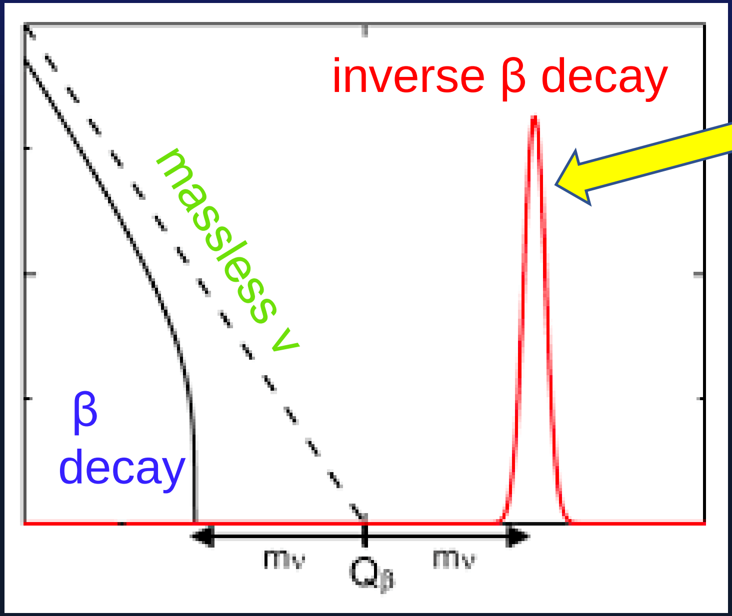
Detect electron neutrinos via
inverse tritium beta decay (never observed)



KATRIN

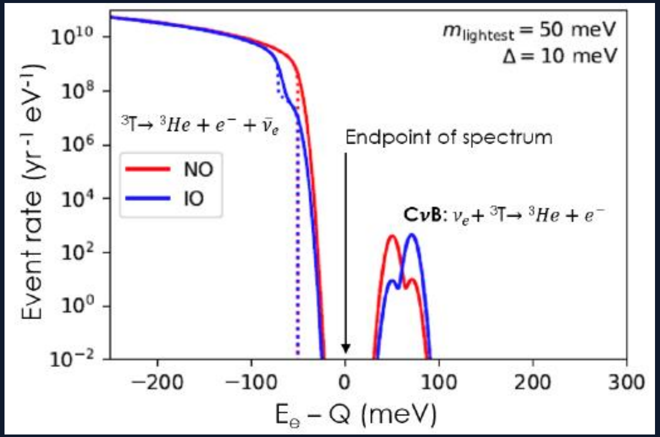
Weinberg PR 1962

PTOLEMY experiment
➔ 100 g ^3H



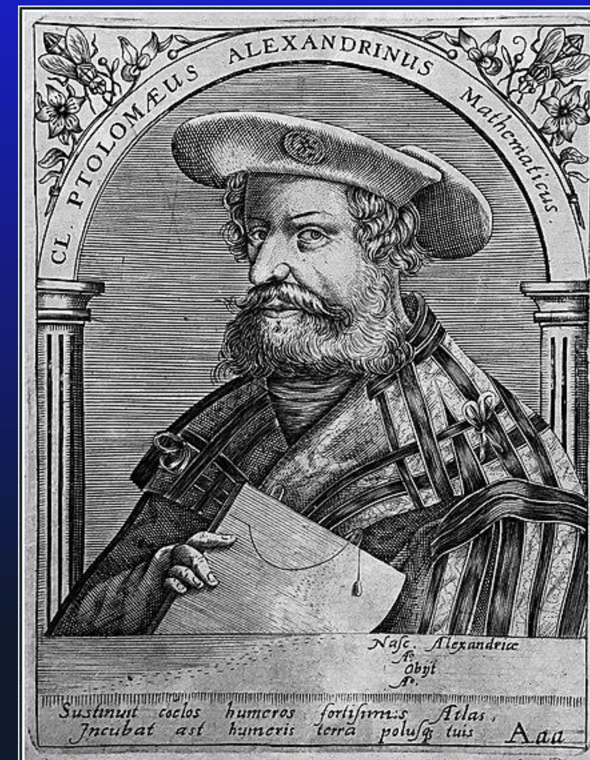
events vs. electron energy

in principle, have 3 peaks at the
3 neutrino masses (i), weighted by $|U_{ei}|^2$



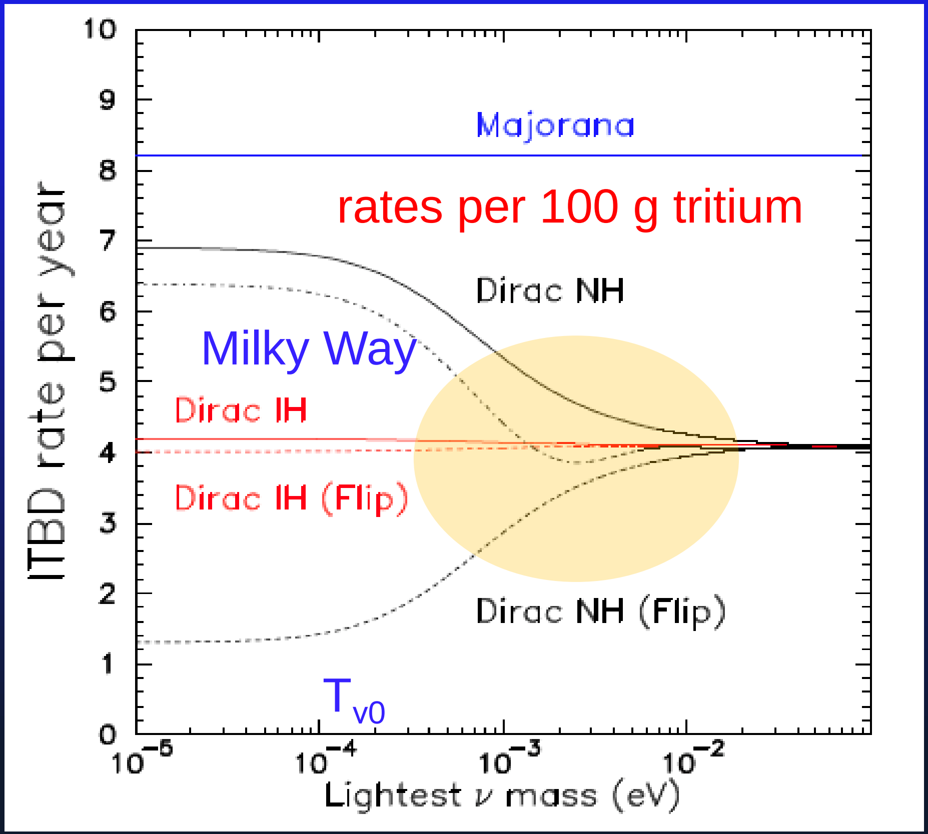
PTOLEMY, JCAP 2019

PTOLEMY = Princeton Tritium Observatory for Light, Early-Universe, Massive-Neutrino Yield



Claudius Ptolemy
Alexandria, Egypt
~ 100-170 AD

Expected event rates with and without helicity flipping



Relativistic neutrino dominates

Bounded by Dirac NH (no flip) and Dirac NH (flip) curves

Dash-dot curve

Helicity rotation in Milky Way

$$B_g = 10\,\mu\mathrm{G},\ \Lambda_g = 1\,\mathrm{kpc}$$

$$\mu_\nu = 5 \times 10^{-14} \mu_B$$

Distinguishes Dirac from Majorana, and hierarchies !!

Inverted Hierarchy: spin rotation makes tiny difference

Normal Hierarchy: spin rotation makes noticeable difference for $m_1 \lesssim 10^{-2}$ eV

Peaks from small mass neutrinos hard to resolve with present technology

PTOLEMY energy resolution problems

PTOLEMY planned energy resolution ~ 50 meV = 5 X 10^{-2} eV.
need to separate inverse beta decay electrons, $E_e > Q_0 + m_v$
from simple tritium beta decay electrons: $E_e < Q_0 – m_v$.

Q_{0v} = total energy available ~ 18.6 keV. $m_v > 10^{-2}$ *eV*

Y. Cheipesh, V. Cheianov, and A. Boyarsky, Phys. Rev. D 104, 116004 (2021):
Must worry about zero-point motion of tritium in estimating energy resolution => resolution closer to 0.3-0.7 eV.

A. Apponi et al., arXiv:2203.11228v2
Further discussion of tritium-embedded graphene substrate.
Perhaps use nanotubes of tritium-embedded graphene.

But not only have relic neutrinos never been observed, neither has the ITBD!

To detect the ITBD, use known source of electron neutrinos

Jen-Chieh Peng and GB, PRD 106, 063018 (2022)

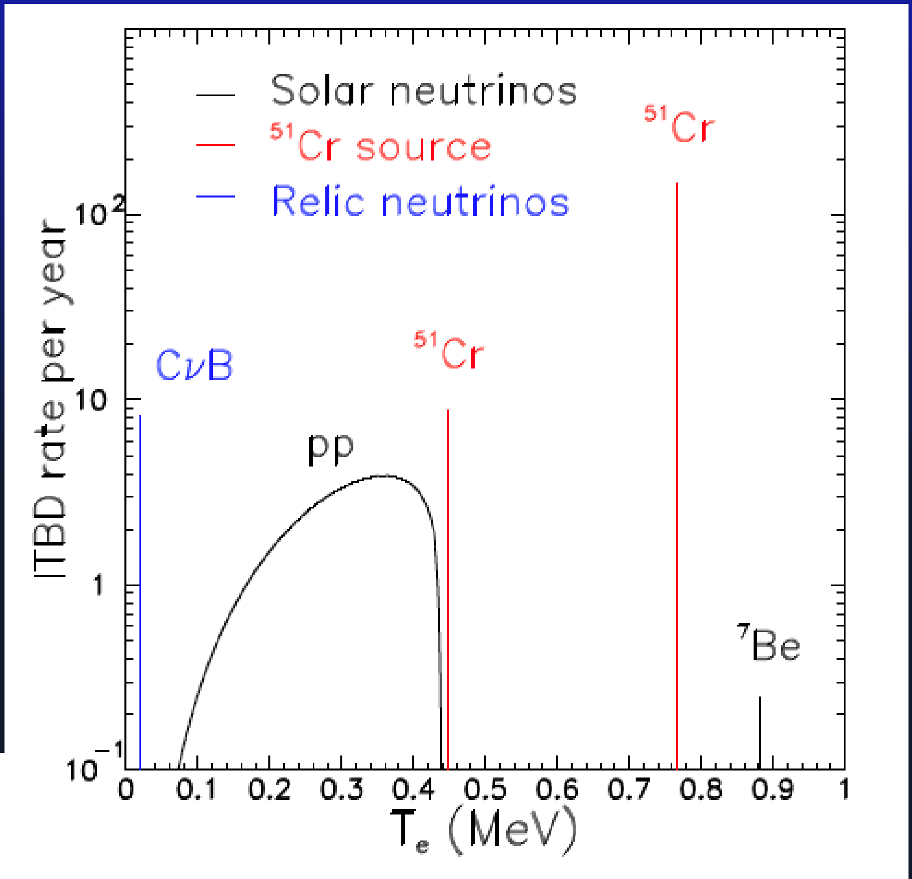
Solar neutrinos and ^{51}Cr

$^{51}Cr \rightarrow {}^{51}V + e^+ + \nu_e$

$\nu_e + {}^3H \rightarrow e^- + {}^3He$

3.4-MCi ^{51}Cr at 50 cm

(double GALLEX)



Coloma et al. (Snowmass 2020)

Experiment	Isotope	Strength	Production Process
GALLEX [3]	^{51}Cr	1.69 MCi	Thermal neutron capture on ^{50}Cr
SAGE [2]	^{51}Cr	0.517 MCi	Epithermal neutron capture on ^{50}Cr
GALLEX [1]	^{51}Cr	1.87 MCi	Thermal neutron capture on ^{50}Cr
SAGE [4]	^{37}Ar	0.409 MCi	Fast neutron $^{40}Ca(n,\alpha)^{37}Ar$
BEST [5]	^{51}Cr	3.4 MCi	Thermal neutron capture on ^{50}Cr

Table 1: Mega-Curie-scale electron capture neutrino sources that have been produced.

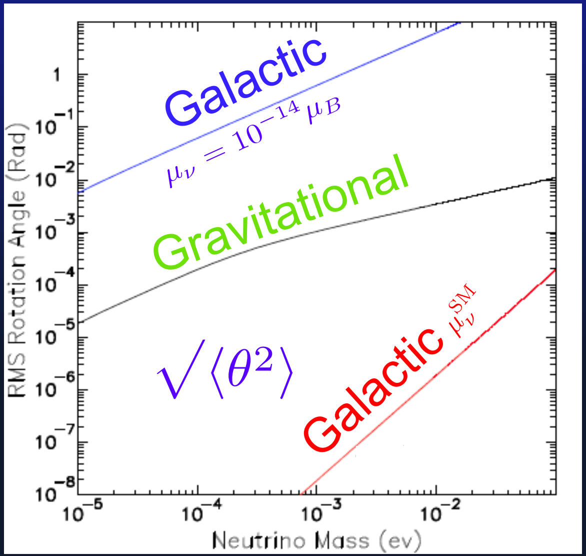
Conclusions

Relic neutrino helicities new probe of cosmic gravitational and magnetic fields

Significant helicity changes of relic neutrinos for neutrino magnetic moment $\mu_\nu \sim 10^{-14\text{-}15}\,\mu_B$

Scalar gravitational helicity changes few orders of magnitude smaller cf. large μ_ν rotations, but much larger than for μ_ν in Standard Model.

Tensor gravitational helicity changes << scalar.



Need significant improvement in electron energy resolution in ITBD to resolve helicity modifications.

Nine layers of the sky

谢谢

Cross section for ITBD (p = neutrino and p_e = electron momenta)

$$\sigma_i^h(p,p_e) = \frac{G_F^2}{2\pi v_i}|V_{ud}|^2|U_{ei}|^2 F(\eta)\frac{m_{^3\mathrm{He}}}{m_{^3\mathrm{H}}} E_e p_e A_i^h(\bar{f}^2 + 3\bar{g}^2)$$

(F = f = Fermi)

V_{ud} = CKM matrix element, U_{ei} = PMNS matrix element

$F = 2\pi\eta/(1-e^{-2\pi\eta})$ = e - ^{3}He Coulomb correction, $\eta = e^2/v_e$

$\bar{f}^2$ = Fermi and $3\bar{g}^2$ = Gamow-Teller nuclear form factors

$A_i^{helicity=\pm} = 1 \mp v_i$: only helicity dependence in cross section

Heuristically, tritium accepts neutrino in state:

$$|\nu\rangle = \sqrt{\frac{1-v}{2}}\,|h=1\rangle + \sqrt{\frac{1+v}{2}}\,|h=-1\rangle$$

Total ITBD rate:

$$\Gamma_{ITBD} = \sum_{masses\,i, h=\pm}\int \frac{d^3p}{(2\pi)^3}\frac{1}{e^{p/T_{\nu 0}}+1}\sigma_i^h v_i$$

$$\sigma_i^h(p,p_e) = \frac{G_F^2}{2\pi v_i}|V_{ud}|^2|U_{ei}|^2 F(\eta)\frac{m_{^3\mathrm{He}}}{m_{^3\mathrm{H}}} E_e p_e A_i^h(\bar f^2 + 3\bar g^2)$$

Neutrino helicity dependent part of rate = A_{eff} :

Dirac: neutrinos only, no antineutrinos

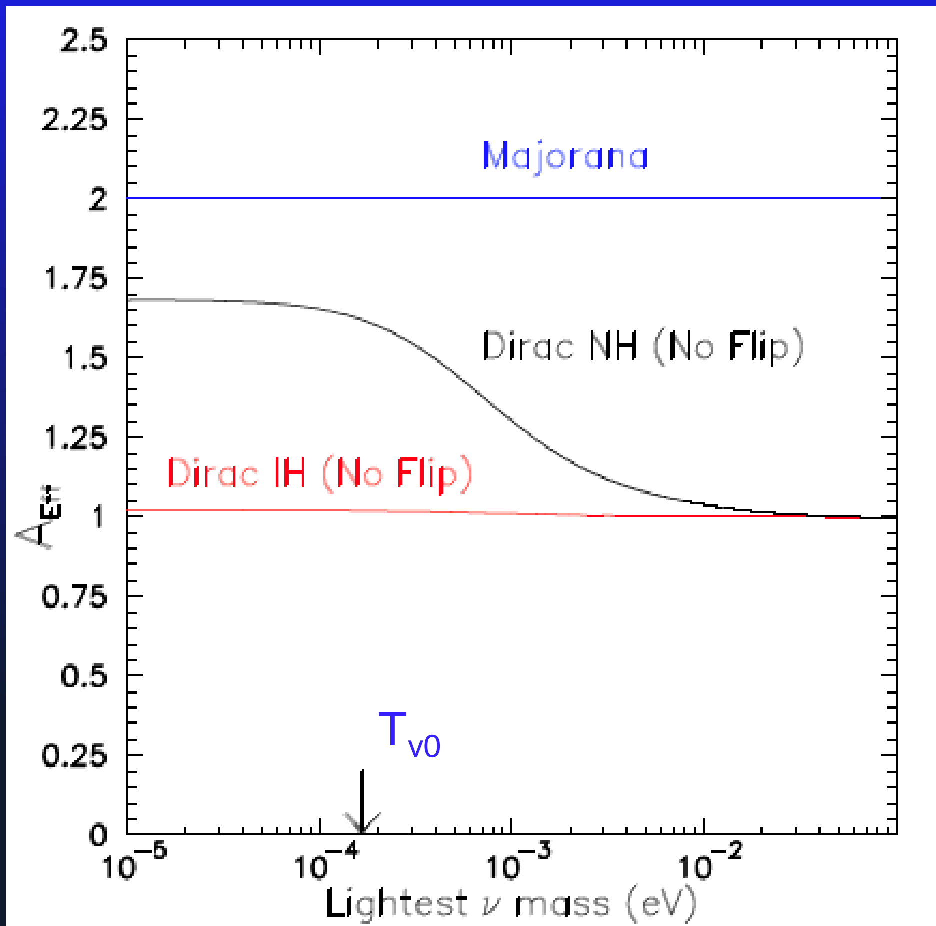
$$A_{\mathrm{eff,D}} = \sum_{i,h=\pm} |U_{ei}|^2 \Big\langle (1 \mp v_i)\langle \frac{1}{2}(1 \mp \cos\theta_i)\Big\rangle_T = \sum_i \big(|U_{ei}|^2(1 + \langle v_i \cos\theta_i\rangle_T\big)$$

T = thermal average plus average of spin rotation in neutrino's history.

Majorana: both neutrinos and antineutrinos contribute:

$$A_{\mathrm{eff,M}} = \big(1 + \sum_i |U_{ei}|^2 \langle v_i\rangle_T\big) + \big(1 - \sum_i |U_{ei}|^2 \langle v_i\rangle_T\big) \equiv 2$$

Neutrino mass and hierarchy dependence in ITBD capture

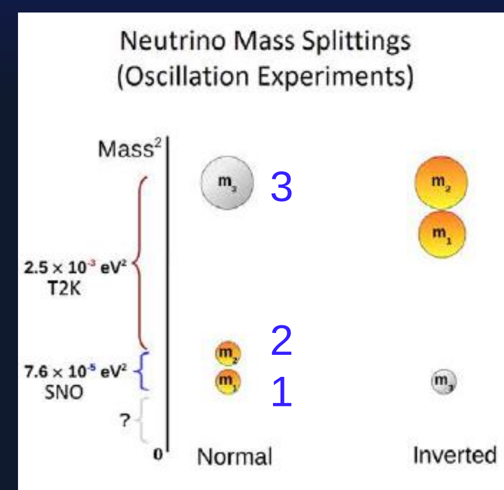


no helicity flipping

$$A_{\mathrm{eff,M}} = 2$$

$$A_{\mathrm{eff,D}} = \left(1 + \sum_i |U_{ei}|^2 \langle v_i \rangle_T \right)$$

Normal	Inverted
$\lvert U_{e1}\rvert^2 = 0.6794$	$\lvert U_{e1}\rvert^2 = 0.6793$
$\lvert U_{e2}\rvert^2 = 0.2990$	$\lvert U_{e2}\rvert^2 = 0.2989$
$\lvert U_{e3}\rvert^2 = 0.0216$	$\lvert U_{e3}\rvert^2 = 0.0218$



NH: m_1=10^{-5} => v_1 ~ 1, v_2 ~1/5, v_3 ~ 1/20

IH: m_1=10^{-5} => v_3 ~ 1 , v_1 ~ v_2 ~ 1/20

But 3 couples most weakly => small mass dependence in IH

Neutrinos from neutron stars and supernovae

Magnetic rotation comparable to that in galaxies:

$$\theta \sim \mu_\nu BR/c \sim 5 \times 10^{13}\,(R/10\,\mathrm{km})\left(B/10^{12}\mathrm{G}\right)(\mu_\nu/\mu_B) \quad \propto 1/R$$

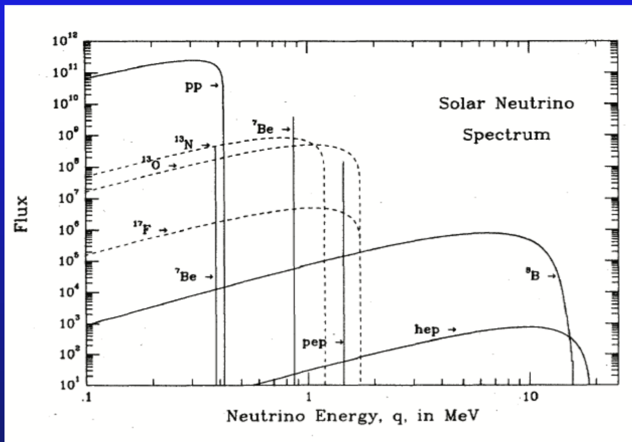
$B \propto 1/R^2 =>$ neutron stars rotate spins more than SN

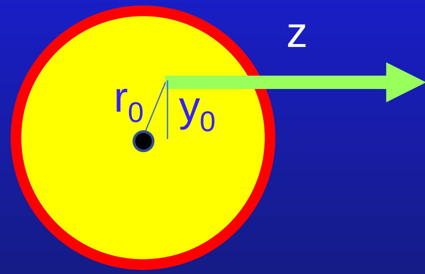
Galaxies: $\sqrt{\langle\theta_g^2\rangle} \sim 5 \times 10^{14}(\mu_\nu/\mu_B)$ for $B_g \sim 10\,\mu\mathrm{G}, \ell_g \sim 16\mathrm{kpc}, \Lambda_g \sim \mathrm{kpc}$

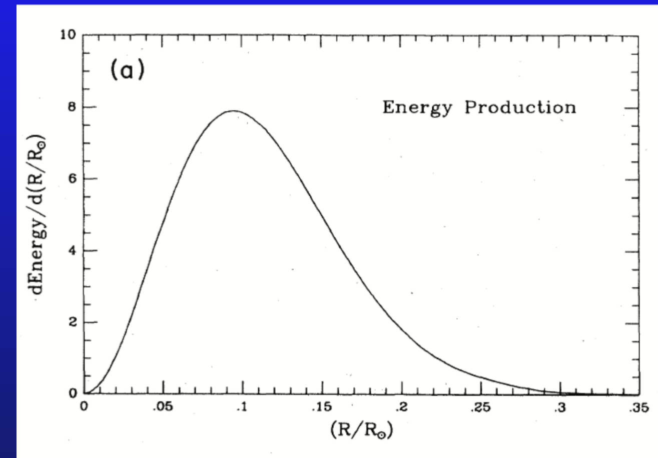
Gravitational rotation of spins $\sim GM/\gamma R$, negligible since $\gamma \sim 10^{8-9}$

Spin rotation of MeV neutrinos from the diffuse supernova background and neutron stars potentially detectable. Ex., via $\bar\nu + p \to e^+ + n$ using Gd-doped Super-K detector to detect n.

Gravitational helicity modification of solar neutrinos







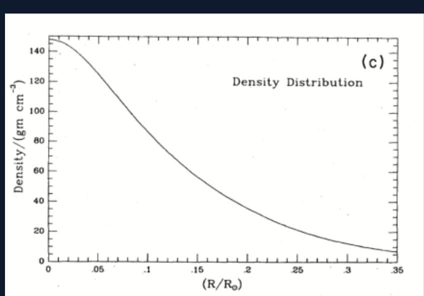
dominated by pp neutrinos

$$p + p \to {}^2\,\mathrm{H} + e^+ + \nu_e$$

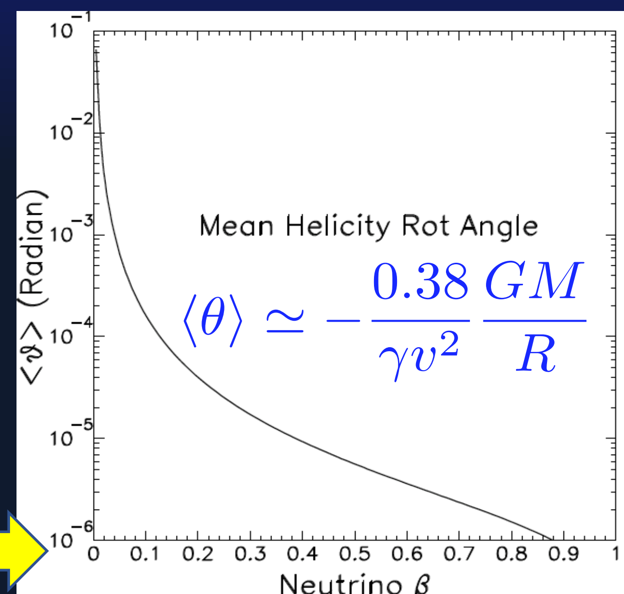
emitted at about $R_{sun}/10$.

$$\theta(y_0, r_0)) = -\frac{1}{\gamma v^2}\int_{z_0}^{\infty} dz\, \nabla_y \Phi(r)$$

$$= -\frac{1}{\gamma v^2}\int_{z_0}^{\infty} dz\, \frac{GM(r) y_0}{r^3}$$



Average over spatial emission and density distributions in Sun



Significant helicity modification of heavy particles with spin, e.g., dark photons, from Sun

Neutrino spin rotation by galactic magnetic field

For uniform galactic magnetic field: $\theta_g \sim 2\mu_\nu B_g \frac{\ell_g}{v}$

ℓ_g = mean crossing distance of the galaxy

But galactic fields are uniform only over coherence length Λ_g~ kpc so spin direction does a random walk in magnetic field.

$$\langle\theta^2\rangle_g \simeq \left(2\mu_\nu B_g \frac{\Lambda_g}{v}\right)^2 \frac{\ell_g}{\Lambda_g}$$

ex., Milky Way with characteristic parameters (spherical cow approx):

$$B_g \sim 10\,\mu\mathrm{G}, \ell_g \sim 16\mathrm{kpc}, \Lambda_g \sim \mathrm{kpc}$$

$$\langle\theta^2\rangle_{\mathrm{MW}} \sim 4 \times 10^{29} m^2_{-2} \left(\frac{\Lambda_g}{1\,\mathrm{kpc}}\right)\left(\frac{B_g}{10\mu\mathrm{G}}\right)^2\left(\frac{\mu_\nu}{\mu_B}\right)^2$$

$\mu_\nu \sim 1.5 \times 10^{-15}\mu_B \sim 10^{-4}\mu_{1T} => \sqrt{\langle\theta^2\rangle} \sim 1$: helicity randomizes

Neutrino spin rotation by cosmic magnetic fields

$$\frac{\vec{S}_\perp}{|\vec{S}\,|} = \pm 2\mu_\nu \int dt\, \hat{v} \times \vec{B}(t)$$ =>

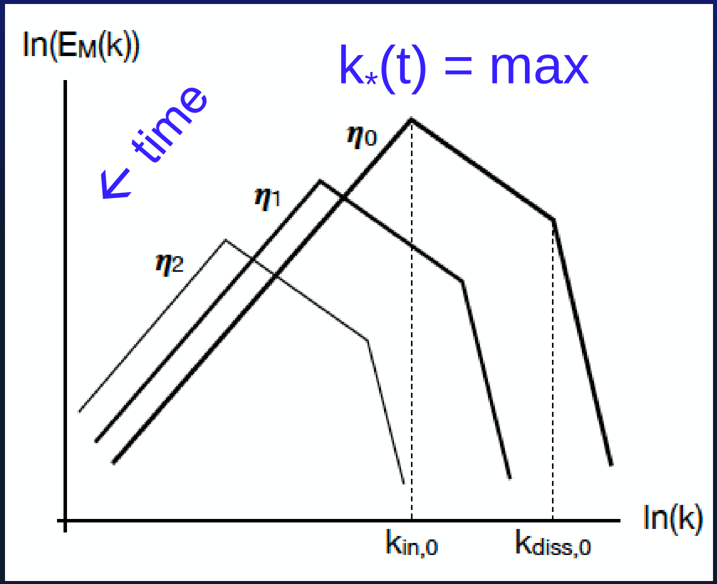
$$\langle \theta^2 \rangle_c = 4\mu_\nu^2 \langle \left(\int dt \vec{B}_\perp(t) \right)^2 \rangle$$

perp to v

Magnetic field correlation function:

$$\langle B_i(\vec{x}) B_j(\vec{x}\,') \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{2} P_B(k) e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}$$

+ helical part (no role here)



Schematic of $P_B(k)$ with increasing conformal time (η =u): $\eta_0 < \eta_1 < \eta_2$

T. Vachaspati, Rep. Prog. Phys. **84** 074901 *(2021)*

$$k < k_* : P_B \sim k^s, s \sim 2$$

$$k > k_* : P_B \sim k^{-q}, q \sim 2 + 5/3$$

sum rule: $$\int \frac{d^3k}{(2\pi)^3} P_B(k) = \langle \vec{B}^{\,2} \rangle$$

$$P_B(k) = (2\pi)^2 E_M(k)/k^2$$

Neutrino propagation in an expanding universe

Metric of expanding universe with weak gravitational inhomogeneities:

$$ds^2 = a(u)^2 \left(-(1+2\Phi)du^2 + (\delta_{ij}(1-2\Phi) + h_{ij})_d x_i dx_j\right)$$

a = scale factor grows from ~10^{-10} at T =1 MeV to a=1 now
u = conformal time, dt = a du
x = comoving spatial coordinates, h_{ij}= gravitational waves

Φ = weak potential, driven by density fluctuations

$$\nabla_x^2 \Phi = 4\pi G \delta\rho(x) a(u)^2$$

Φ(x) independent of *a*, at long wavelengths $\delta\rho \;\; a^2 \propto a^0$

Radiation dominated era ($P = \rho/3$) , down to redshift ~ 10^4 :

fluctuation analysis => $\delta\rho/\bar{\rho} \sim a^2, \quad \delta\rho \sim 1/a^2$

Matter dominated era, from redshift ~ 10^4 to now, $\delta\rho/\bar{\rho} \sim a, \quad \delta\rho \sim 1/a^2$

$$\langle\theta^2\rangle_c \simeq \mu_\nu^2 \pi \int_{u_d}^{u_0} du\, a(u)^2 \frac{\langle\vec{B}^2\rangle(u)}{k_*(u)}$$

u = conformal time
a = scale factor of universe
dt = a du

Conservation of flux: $a^2B \sim$ const. => $\langle\vec{B}^2(u)\rangle \simeq B_0^2/a(u)^4$ (0= now)

$$k_*(u) \sim \frac{2\pi}{\Lambda_0 a(u)^{1/2}}$$

(Λ_0 = coherence length of cosmic B field)

$$\langle\theta^2\rangle_c = \frac{1}{2}\mu_\nu^2 B_0^2 \Lambda_0 \int_{u_d}^{u_0} \frac{du}{a(u)^{3/2}}$$

Main contribution is from radiation-dominated era (a ~ u):
from neutrino decoupling, u_d ($a_d \sim 10^{-10}$)
to matter-radiation equality, u_{eq} ($a_{eq} \sim 0.8 \times 10^{-4}$)

$$\langle\theta^2\rangle_c \simeq 9\left(\frac{\Lambda_0}{R_u}\right)\frac{(\mu_\nu t_0 B_0)^2}{(a_{eq}a_d)^{1/2}}$$

$R_u = cu_0$ = radius of universe
$u_0 = 3t_0$

$$\simeq 2\times 10^{27}\left(\frac{\Lambda_0}{1\,\mathrm{Mpc}}\right)\left(\frac{B_0}{10^{-12}\mathrm{G}}\right)^2\left(\frac{\mu_\nu}{\mu_B}\right)^2$$

Cosmic magnetic field rotation of neutrino spin

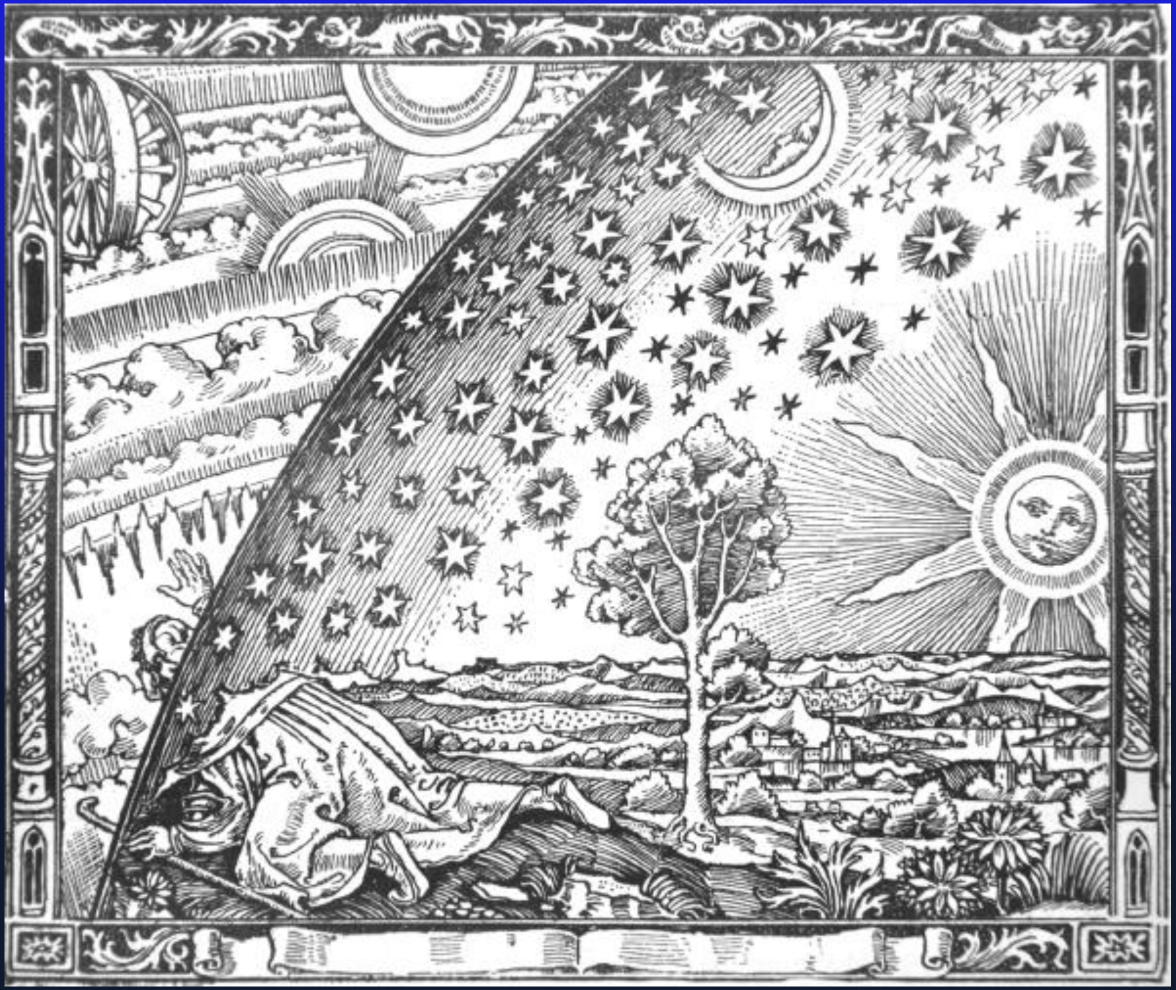
$$\langle \theta^2 \rangle_c \simeq 2 \times 10^{27} \left(\frac{\Lambda_0}{1\,\mathrm{Mpc}}\right) \left(\frac{B_0}{10^{-12}\mathrm{G}}\right)^2 \left(\frac{\mu_\nu}{\mu_B}\right)^2$$

A magnetic moment $\mu_\nu \sim 10^{-3} \mu_{1T} \sim 10^{-14} \mu_B$
would be experimentally significant :
~1% of neutrinos could flip helicity

Effects of standard model magnetic moment insignificant.

If the neutrino is Majorana, no helicity changes from magnetic fields.

To within uncertainties in magnetic fields, correlation lengths, and neutrino masses, spin rotation in cosmic magnetic fields ~ galactic



Flammarion 1888

Neutrinos 101

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Matter dominated era, from redshift ~ 10^4 to now, $\delta\rho/\bar{\rho} \sim a, \quad \delta\rho \sim 1/a^2$

Spin rotation from gravitational perturbations

With linearized metric perturbations

$$ds^2 = a(u)^2 \left(-(1+2\Phi)du^2 + (\delta_{ij}(1-2\Phi) + h_{ij})_d x_i dx_j \right)$$

$$h_{00} = -(1+2\Phi) \qquad\qquad h_{0j} = 0$$

measure neutrino spin in Lorentz rest frame of the neutrino. Use geodesic equations for spin and velocity to derive the evolution of the angle between spin and velocity vectors:

$$\frac{d\theta^i}{du} = \frac{1}{2\gamma} P_{ij} \left(\partial_u h_{jk} \hat{v}^k - \frac{1}{v} \partial_j h_{00} \right)$$

P projects transverse to the velocity: $P_{ij} = \delta_{ij} - \hat{v}_i \hat{v}_j$

h_{00} gives effects of scalar perturbations and
h_{jk} effects of tensor (gravitational wave) perturbations

Rotation of neutrino spins by scalar inhomogeneities

Gravitational potential Φ rotates momentum and spin:

$$\frac{d\hat{p}}{dt}\Big|_\perp = -\left(v+\frac{1}{v}\right)\vec{\nabla}_\perp \Phi \quad , \quad \frac{d\vec{S}}{dt}\Big|_\perp = -\frac{2\gamma+1}{\gamma+1}\vec{S}\cdot\vec{v}\,\vec{\nabla}_\perp \Phi$$

Spin bending lags momentum bending $\left(h\frac{d\hat{S}}{dt} - \frac{d\hat{p}}{dt}\right)_\perp = \frac{m}{p}\vec{\nabla}_\perp \Phi$

Again, neutrino undergoes a random walk through the inhomogeneities.

For massless neutrino, momentum bending angle is:

$$\langle(\Delta\theta_p)^2\rangle = 4\int dx_3 dx_3' \nabla_{x\perp}\cdot\nabla_{x'\perp}\langle\Phi(x_3)\Phi(x_3')\rangle$$

In terms of gravitational fluctuation power spectrum,

$$\langle\Phi(x)\Phi(x')\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}\Psi(k)$$

$$\langle(\Delta\theta_p)^2\rangle = 4\int dx_3 dx_3' \nabla_{x\perp}\cdot\nabla_{x'\perp}\langle\Phi(x_3)\Phi(x_3')\rangle \quad , \quad \langle\Phi(x)\Phi(x')\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}\Psi(k)$$

Thus $\langle(\Delta\theta_p)^2\rangle = 4\int dx_3 dx_3' \int \frac{d^3k}{(2\pi)^3} e^{ik_3(x_3-x_3')} k_\perp^2 \Psi(k)$

x_3' integral => $2\pi\delta(k_z)$

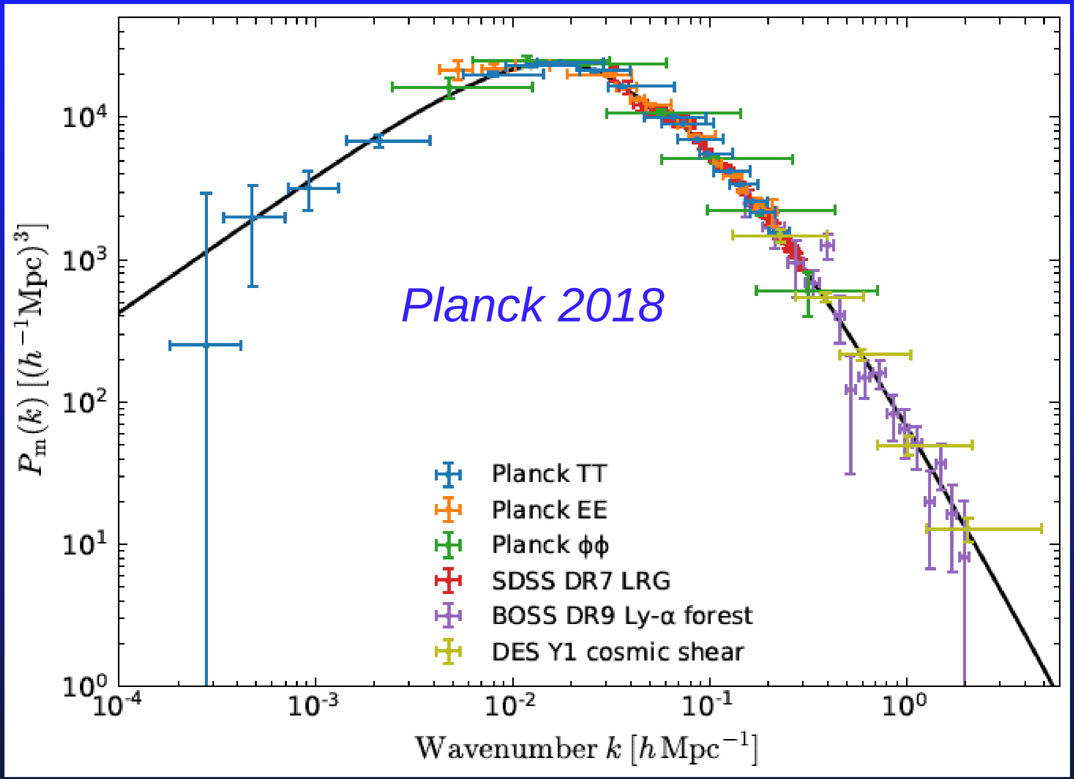
$$\langle(\Delta\theta_p)^2\rangle = \frac{2}{\pi}\int du \int \frac{dk_\perp}{k_\perp}\Psi(k_\perp)$$

Relate field fluctuations to density fluctuations $\delta(\vec{x}\,) \equiv \delta\rho(\vec{x}\,)/\bar{\rho}$

$$\langle\delta(\vec{x}\,)\delta(\vec{x}\,')\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')} P(k)$$

$$\nabla_x^2\Phi = 4\pi G\delta\rho(\vec{x}\,)a(u)^2 \qquad \Psi(k) = (4\pi G\bar{\rho}a^2)^2\frac{P(k)}{k^4}$$

=> $$\langle(\Delta\theta_p)^2\rangle = \frac{2}{\pi}\int du(4\pi G\bar{\rho}a^2)^2 \int dk \frac{P(k)}{k}$$



Density fluctuation spectrum

P(k) ~ k for k <k_{max}
(Harrison-Zel'dovich)
P(k) ~ k^{-v} for k > k_{max}
Scales as
a^2 in matter dom. era
a^4 in rad. dom. era, k <k_{max}

$$\langle(\Delta\theta_p)^2\rangle = \frac{2}{\pi}\int du(4\pi G\bar{\rho}a^2)^2 \int dk \frac{P(k)}{k}$$

At present $$\int (dk/k)P_0(k) \simeq 7.25\times10^4 (\mathrm{Mpc/h})^3 \equiv \mathcal{P}$$

h = Hubble parameter ~ 0.7

Include dark energy in matter dominated era

(noticeable for redshifts below 0.5)

$$\frac{da}{du} = \sqrt{\frac{8\pi G\bar{\rho}(a)a^4}{3}} = H_0\sqrt{\Omega_M a + \Omega_V a^4}$$

$$\bar{\rho}(a) = \rho_M/a^3 + \rho_V$$

$\rho_M/\rho_c \equiv \Omega_M \simeq 0.32$ Matter w. dark matter fraction

$\rho_V/\rho_c \equiv \Omega_V \simeq 0.68$ Dark energy fraction

$H_0 = \sqrt{8\pi G\rho_c/3}$ Present Hubble constant = h/3000 Mpc

ρ_c = Critical density for closure

h = Hubble parameter ~ 0.7

$$\langle(\Delta\theta_p)^2\rangle = \frac{9}{2\pi}\mathcal{P}H_0^3\int_0^1 \frac{da}{a^2}\left(\Omega_M a + \Omega_V a^4\right)^{3/2} \simeq 2.2\times 10^{-6}$$

$\mathcal{P}H_0^3 \simeq 2.69\times 10^{-6}$ Independent of h

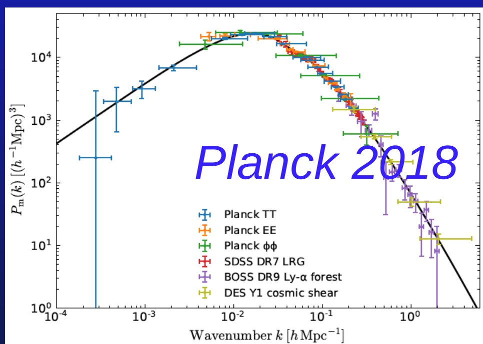
With finite m_ν neutrino velocity $v(a) = p_0/\sqrt{p_0^2 + (m_\nu a)^2}$. p_0 = present momentum

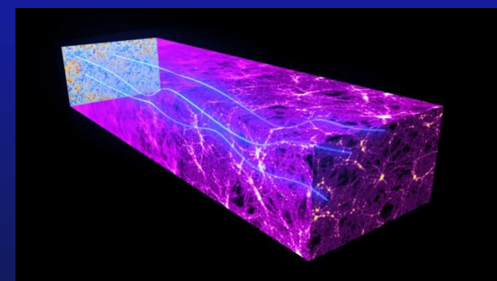
$$\langle(\Delta\theta_p)^2\rangle = \frac{9}{8\pi}\mathcal{P}H_0^3\int_{a_{eq}}^1 \frac{da}{a^2}\left(\Omega_M a + \Omega_V a^4\right)^{3/2} v(a)\left(v(a) + \frac{1}{v(a)}\right)^2$$

Gravitational lensing of cosmic neutrino background

(G. Holder)

$$\langle(\Delta\theta_p)^2\rangle = \frac{9}{2\pi}\mathcal{P}H_0^3\int_0^1 \frac{da}{a^2}\left(\Omega_M a + \Omega_V a^4\right)^{3/2} \simeq 2.2\times 10^{-6}$$





RMS momentum bending = lensing of cosmic neutrino background ~ 5.1 arcmin

Lensing of CMB ~ 2.7 arcmin. Most efficient at smaller z (≲10). Reionization of intergalactic H => photon-e scattering.

(Weak electron-neutrino scattering after reionization insignificant)

Gravitational spin rotation with respect to momentum, Θ

Main effect in matter dominated era from redshift ~ 10^4 to now.
Slower neutrinos have greater rotation of momentum vs spin

Momentum rotation with finite neutrino mass:

Neutrino velocity $v(a) = p_0/\sqrt{p_0^2 + (m_\nu a)^2}$. p_0 = present momentum

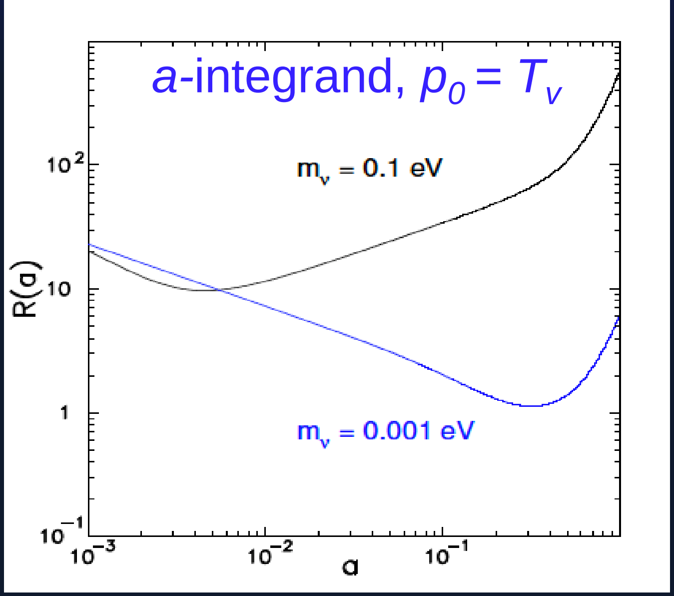
$$\langle(\Delta\theta_p)^2\rangle = \frac{9}{8\pi}\mathcal{P}H_0^3 \int_{a_{eq}}^{1} \frac{da}{a^2}\left(\Omega_M a + \Omega_V a^4\right)^{3/2} v(a)\left(v(a) + \frac{1}{v(a)}\right)^2$$

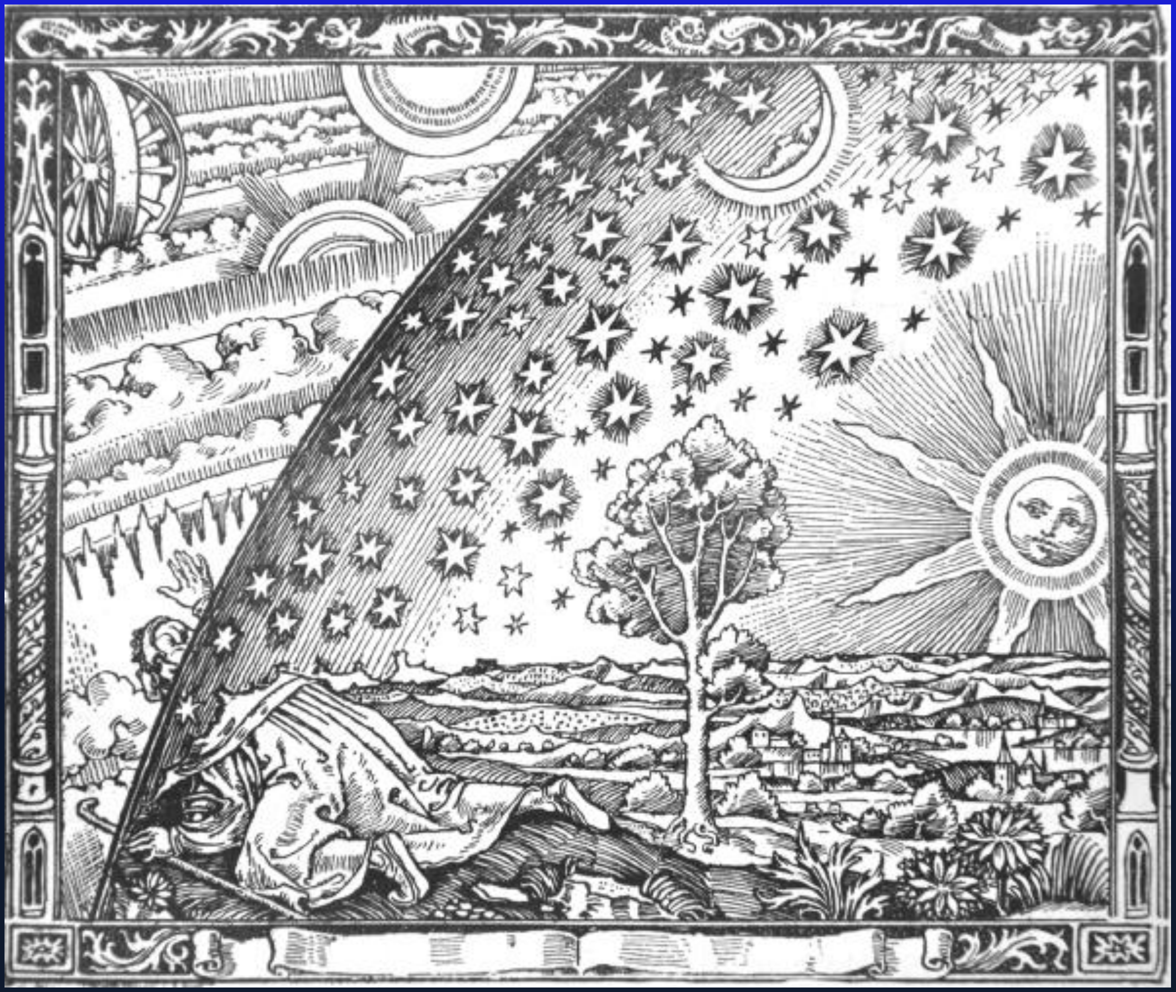
Ω_M = matter fraction, Ω_V = dark energy fraction

Spin rotation with respect to momentum.

$$\langle\theta^2\rangle = \frac{9}{8\pi}\mathcal{P}H_0^3 \int_0^1 \frac{da}{a^2}\left(\Omega_M a + \Omega_V a^4\right)^{3/2}\left(\frac{1}{v} - v\right)$$

Measure of helicity changes





Flammarion 1888

Neutrinos 101

Neutrino magnetic moments & spin precession

Gravitational inhomogeneities & spin precession

Detection of relic neutrinos

Spin precesses in magnetic field, but momentum does not

(neutrinos electrically neutral)

Thus magnetic fields change neutrino helicity: $h = \hat{S} \cdot \hat{p}$

Spin rotation by angle θ => helicity reversal probability = $\sin^2(\theta/2)$

Define spin in rest frame of neutrino.

Rest frame precession

$$\frac{d\vec{S}}{d\tau} = 2\mu_\nu \vec{S} \times \vec{B}_R$$

B_R = magnetic field in rest frame

In terms of ”lab” frame magnetic field $B_{\parallel R} = B_{\parallel}, \quad B_{\perp R} = \gamma B_{\perp}$

$$\gamma = 1/\sqrt{1-v^2}$$

Bargmann-Michel-Telegdi (BMT) equations of motion:

$$\frac{d\vec{S}_\perp}{dt} = 2\mu_\nu \left(\vec{S}_\parallel \times \vec{B}_\perp + \frac{1}{\gamma} \vec{S}_\perp \times \vec{B}_\parallel \right), \quad \frac{dS_\parallel}{dt} = 2\mu_\nu (\vec{S} \times \vec{B})_\parallel$$

negligible for small rotation from longitudinal

Cumulative spin rotation along ν trajectory:

$$\frac{\vec{S}_\perp}{|\vec{S}\,|} = \pm 2\mu_\nu \int dt\, \hat{v} \times \vec{B}(t)$$

for small angular changes.

Apply to galaxies, and to cosmic magnetic fields

Neutrino spin rotation by cosmic magnetic fields

$$\frac{\vec{S}_\perp}{|\vec{S}\,|} = \pm 2\mu_\nu \int dt\, \hat{v} \times \vec{B}(t)$$ =>

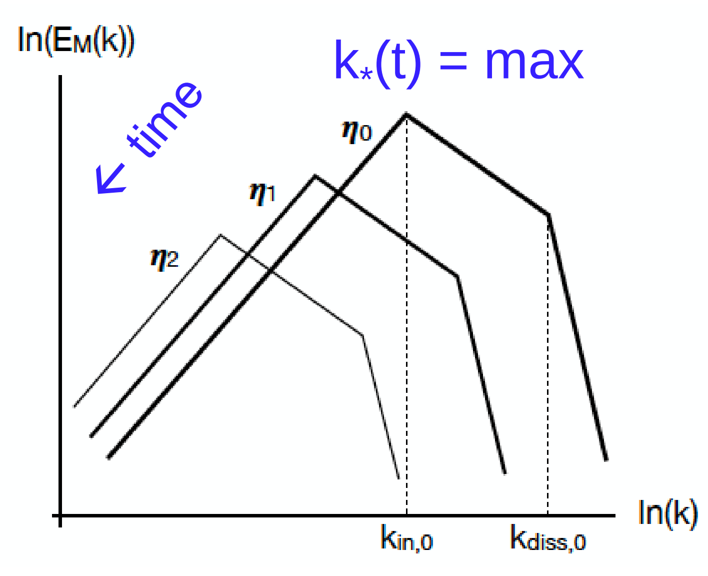
$$\langle \theta^2 \rangle_c = 4\mu_\nu^2 \langle \left(\int dt \vec{B}_\perp(t) \right)^2 \rangle$$

perp to v

Magnetic field correlation function:

$$\langle B_i(\vec{x}) B_j(\vec{x}\,') \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{2} P_B(k) e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}$$

+ helical part (no role here)



Schematic of $P_B(k)$ with increasing conformal time (η =u): $\eta_0 < \eta_1 < \eta_2$

T. Vachaspati, Rep. Prog. Phys. **84** 074901 *(2021)*

$$k < k_* : P_B \sim k^s, s \sim 2$$

$$k > k_* : P_B \sim k^{-q}, q \sim 2 + 5/3$$

sum rule: $$\int \frac{d^3k}{(2\pi)^3} P_B(k) = \langle \vec{B}^{\,2} \rangle$$

$$P_B(k) = (2\pi)^2 E_M(k)/k^2$$

$$\langle\theta^2\rangle_c \simeq \mu_\nu^2 \pi \int_{u_d}^{u_0} du\, a(u)^2 \frac{\langle\vec{B}^2\rangle(u)}{k_*(u)}$$

Conservation of flux: $a^2B\sim$ const. => $\langle\vec{B}^2(u)\rangle \simeq B_0^2/a(u)^4$ (0= now)

$$k_*(u) \sim \frac{2\pi}{\Lambda_0 a(u)^{1/2}}$$

(Λ_0 = coherence length of cosmic B field)

$$\langle\theta^2\rangle_c = \frac{1}{2}\mu_\nu^2 B_0^2 \Lambda_0 \int_{u_d}^{u_0} \frac{du}{a(u)^{3/2}}$$

Main contribution is from radiation-dominated era (a ~ u):
from neutrino decoupling, u_d ($a_d \sim 10^{-10}$)
to matter-radiation equality, u_{eq} ($a_{eq} \sim 0.8 \times 10^{-4}$)

$$\langle\theta^2\rangle_c \simeq 9\left(\frac{\Lambda_0}{R_u}\right)\frac{(\mu_\nu t_0 B_0)^2}{(a_{eq}a_d)^{1/2}}$$

$R_u = cu_0$ = radius of universe
$u_0 = 3t_0$

$$\simeq 2\times 10^{27}\left(\frac{\Lambda_0}{1\,\mathrm{Mpc}}\right)\left(\frac{B_0}{10^{-12}\mathrm{G}}\right)^2\left(\frac{\mu_\nu}{\mu_B}\right)^2$$

Effects of primordial gravitational waves on neutrino helicities

GB, J-C Peng and C.J. Pethick (in the works)

Angle between spin and velocity evolves by

$$\frac{d\theta^i}{du} = \frac{P_{ij}}{2\gamma}\partial_u h_{jk}\hat{v}^k = \frac{1}{2\gamma}\frac{\partial W_i}{\partial u}$$

with $W_i \equiv P_{ij}h_{jk}\hat{v}^k$

P_{ij} projects transverse to v

Angular averages: $$\langle \vec{W}(x)\cdot\vec{W}(\tilde{x})\rangle = \frac{1}{5}\langle\sum_{ij} h_{ij}(x)h^*_{ij}(\tilde{x})\rangle$$

Tensor correlation function:

$$\mathcal{T}(\vec{x}-\vec{\tilde{x}}\,', u-\tilde{u}) \equiv \langle\sum_{ij} a(u)h_{ij}(\vec{x},u)a(\tilde{u})h_{ij}(\vec{\tilde{x}},\tilde{u})\rangle$$

$$\langle(\Delta\vec{\theta}\,)^2\rangle_T = \frac{1}{80\pi H_0}\int_{a_d}^1 \frac{da}{(\gamma a)^2 v}\left(\frac{\rho_c}{\bar{\rho}}\right)^{1/2}\int q^3 dq\,\mathcal{T}(q)$$

tensor bending

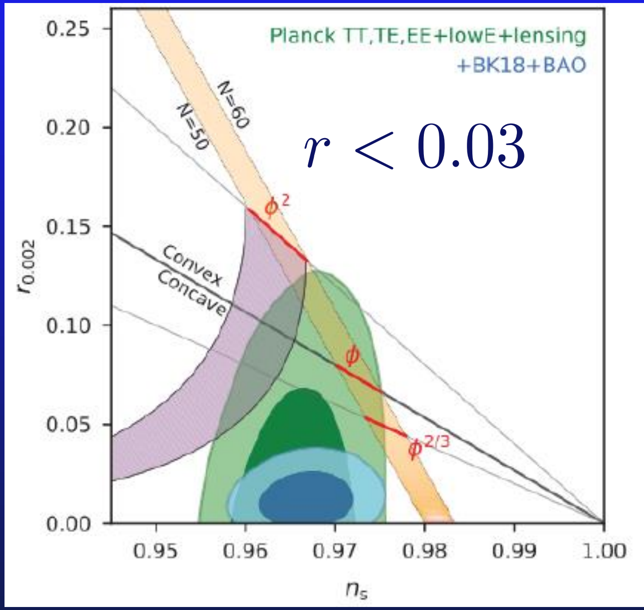
$$\langle(\Delta\vec{\theta}\,)^2\rangle_S = \frac{9}{8\pi}H_0^3\int_{a_d}^1 \frac{da\,a^8}{(\gamma a)^2 v}\left(\frac{\bar{\rho}}{\rho_c}\right)^{3/2}\int \frac{dq}{q}P_0(q)$$

scalar bending

Only have bound on $\mathcal{T}(k_P \sim 0.05 \text{ Mpc}^{-1})$ at time of last photon scattering, $z \sim 1100 = 1/a_R$

$$\frac{k_R^3}{2\pi^2 a_R^2}\mathcal{T}(k_R) = r\mathcal{A}_s\left(\frac{k_R}{k_P}\right)^{n_s-1}$$

$\mathcal{A}_s(k/k_P)^{n_s-1}$ = curvature fluctuations



P. A. R. Ade et al., (Planck, WMAP, & BICEP/Keck) PRL 127, 151301 (2021)

$$\frac{\langle(\Delta\vec{\theta}\,^2\rangle_T}{\langle(\Delta\vec{\theta}\,)^2\rangle_S} \sim \frac{r}{90\pi}\left(\frac{k_P a_R}{H_0}\right)^4 \sim 10^{-5} r$$

Additional neutrino helicity modification from primordial gravitational radiation quite negligible.

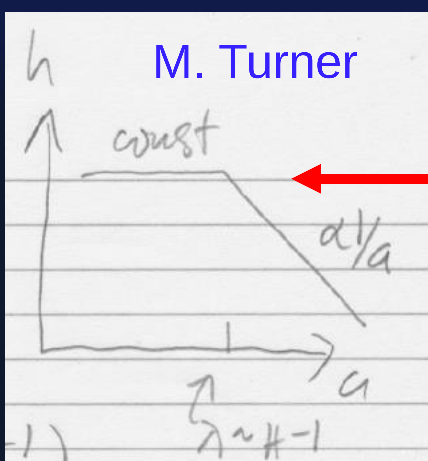
Present scale of tensor fluctuations

$r \sim 10^{-2}$ => $h/(\delta\rho/\rho) \sim r^{1/2}$ ~0.1 at z ~ 1100

$\delta\rho/\rho \sim 10^2$ now, and $\delta\rho/\rho \sim a$ =>

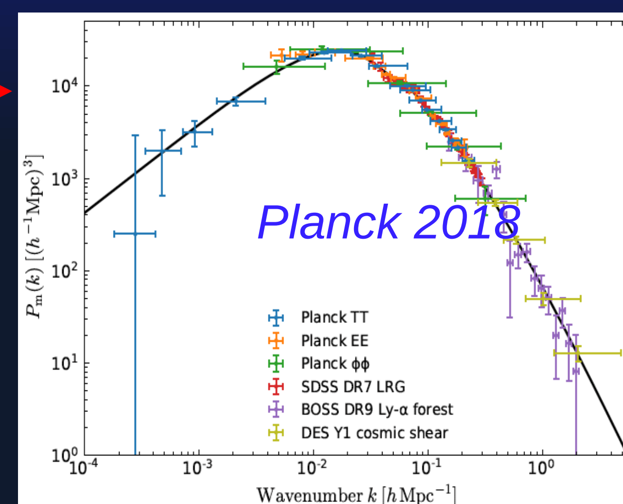
$\delta\rho/\rho \sim 0.1$
$h \sim 0.01$ at z ~ 1100

$h \sim 1/a$ => $h \sim 10^{-5}$ now



h(k) now ~ 10^{-5}

$\delta\rho/\rho \sim 10^2$



Challenge to measure *T*(k) inside horizon