Classical Mechanics, Quantum Mechanics, Quantum Cosmology: new view on the universe

Ue-Li Pen (ASIAA and CITA) Dylan Jow (CITA) and Job Feldbrugge (Edinburgh)

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Waves, Particles, Geodesics

- Everything is a wave
- Short wavelength limit has classical particle interpretation: Eikonal
- Picard-Lefschetz: no waves, everything is a particle!
- New tool for QM Cosmology, FRBs, pulsars
- alternative picture for quantization: quantum gravity?

Fermat's Principle

- light takes shortest path. Why?
- Huygen's principle: light takes all paths
- Kirchoff/Feynmann path integral
- stationary phase dominates
- classical equation of motion: extremal paths, independent of wavelength

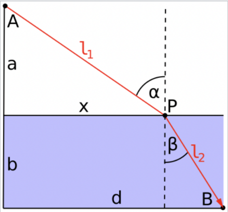


Fig. 1: Fermat's principle in the case of refraction of light at a flat surface between (say) air and water. Given an object-point *A* in the air, and an observation point *B* in the water, the refraction point *P* is that which

History

- Huygens, Fermat, Feynmann
- Picard-Lefschetz (19th century), Witten (2010)
- concept: Oscillatory (Kirchoff) path integral
- sum over all possible paths, weighted by phase
- new phenomena: imaginary images, diffraction
- Eikonal: Dual description of waves by particles

Optics: Geometric, Eikonal, Wave, P-L

- Consider 1-D lens
- lensing potential $\Psi(\theta)$
- deflection Ψ'
- simplify for $D_{\rm ds} = \infty$

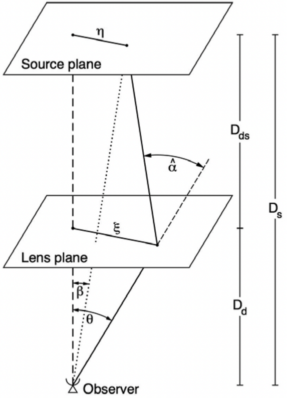


Figure 1. Geometry of a lensing event (reproduced from Schneider et al. 1992). See text for details.

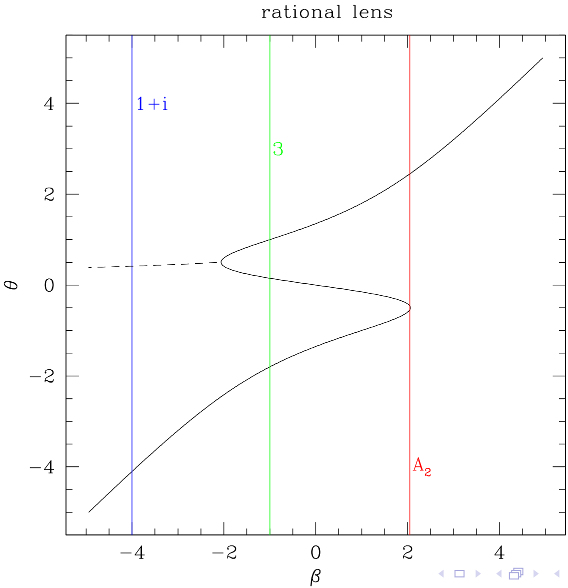
Huygen's Principle: Path Integral

- $A(\mu) = \int e^{iS(\theta,\mu)} d\theta$
- $S = \nu[(\theta-\mu)^2 + \Psi(\theta)]$
- Highly oscillatory integral, even for $\Psi = 0$
- Stationary phase points: $\partial_\theta S = 0$ leads to (complex) Eikonal images θ_i.
- flux/phase through curvature expansion (known as *steepest descent*): exact as $\nu \to \infty$
- Geometric limit considers only *Real* solutions θ_i and gives up phase information (length of trajectory)
- Geometric optics applicable at short wavelengths for extended sources (e.g. optical gravitational lensing of finite size sources, stars)
- roughly 1/4 of the degrees of freedom of Eikonal

Imaginary Images

- consider "rational lens" potential $\psi(\theta) = \alpha/(1+\theta^2)$
- Geometric/eikonal images at $\psi' = \theta$
- 5 roots. 1 or 3 real roots, rest imaginary
- P-L: at most one imaginary image contributes!
- imaginary image can be brighter than unlensed real image

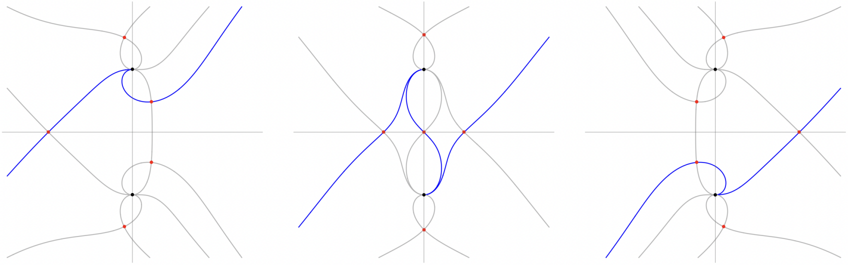
Rational 1-D lens



Picard-Lefschetz Theory

- descend integral along real line along Morse function Im(S)
- contour deforms into finite number of Thimbles of constant phase with maximum at saddle point (extrema $dS = 0$)
- correctly identifies relevant saddle points
- resolves numerical challenges of oscillatory integral
- complex analysis works in multiple variables
- elevates concept of “image” deep into wave optics
- multiple public implementations (Feldbrugge+, Jow+)

Picard-Lefschetz Theory



(a) $\alpha = 2, \mu < -\mu_c$ (b) $\alpha = 2, -\mu_c < \mu < \mu_c$ (c) $\alpha = 2, \mu > \mu_c$

Feldbrugge+2019

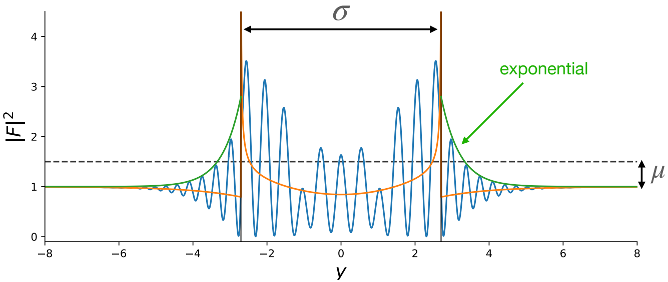
Imaginary classical paths

- PL interpretation: oscillatory integral is sum over thimbles
- each thimble has exactly one stationary point (maxima, classical path)
- path integral is sum over thimbles
- eikonal limit is quadratic expansion at maxima (WKB for imaginary images)
- alternative interpretation of quantum mechanics from classical mechanics:
- sum over real and imaginary classical paths
- Turok 2014, Cherman+ 2014, Mou+ 2019

New Observables

- for coherent sources: FRBs, pulsars
- weak lensing: imaginary image allows time delay measurement (Jow+21)
- strong lensing: delay measurements enable measurement of co-linearity (Jow++21)
- microlensing: instant time delay, planets (Jow+20)
- macrolensing: potentially nano-second delay – universe expands! Dark energy, etc (Wucknitz+21)
- dimensionless strain cm/Gigalightyears $h \sim \Delta t/t \sim 10^{-26}$: competitive with LIGO, etc

A2 fold

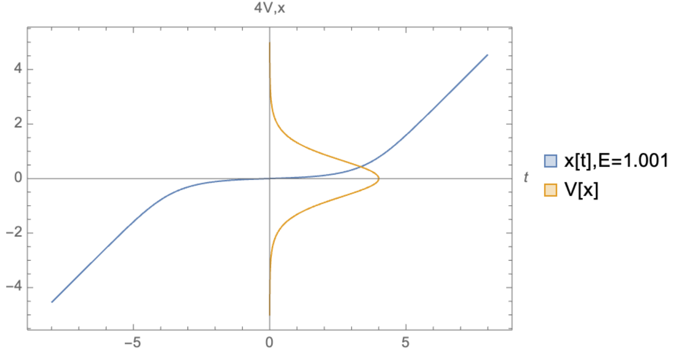


Feynman Path Integrals

- local description of wave function, equivalent to solving Schroedinger equation
- $\Psi(x_f, t_f) = \mathcal{N} \int Dx \int dx_i e^{\frac{i}{\hbar} S(x_f, t_f, x_i, t_i)} \Psi(x_i, t_i)$
- full solution, not perturbative
- more general for unbounded Hamiltonians
- PL: maps to discrete set of complex paths

Pöschl-Teller Potential

- $V(x) = \mathrm{sech}^2(x)$
- exactly solvable: $x(t) = -i \sin^{-1}\left[\frac{\sqrt{E-1}\tanh(\sqrt{E}t)}{\sqrt{E}(\tanh(\sqrt{E}t)-1)}\right]$
- for $\lim_{E\to 1+\epsilon^2} = \sinh^{-1}[\epsilon \sinh(t)]$



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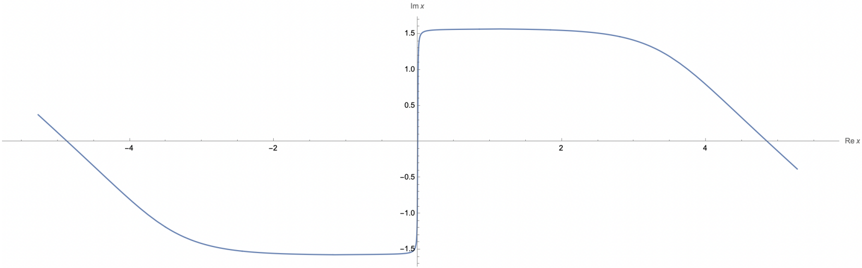

Lorenzian Quantum Cosmology

- Feldbrugge, Lehners & Turok 2017
- Quantize Feynmann path integral instead of wave equation
- PL, Cauchy's theorem allow deformation of paths into complex plane
- minisuperspace: allow imaginary paths (scale factor)
- PL Semi-classical limit: dominant paths
- CPT universe, bounce, etc (Boyle, Finn, Turok 2018)

Discussion

- Eikonal effects applicable to compact radio sources, e.g. FRBs, pulsars
- full wave effect dominates for long wavelengths as Fresnel scale is bigger then Einstein radius
- down to planet size
- gravitational waves: LIGO, LISA, PTA

BURSTT

Bustling Universe Radio Telescope Taiwan: collaboration of ASIAA, NTU, NTHU, NCHU for all sky FRB radio array

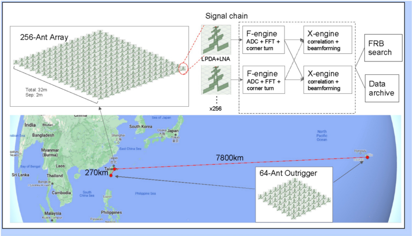

Figure 9: Overview of the final BURSTT instrument. A core of 256 antennas is located in Taiwan. This core is supplemented by a pair of outrigger stations, each equipped with 64 antennas. Signals are processed by a digital back-end which substantially builds on the designs used in CHIME, leveraging new technologies from the telecommunications and artificial intelligence industries.

Figure 16. Rendering of BURSTT main array at prospective Wu Ming Creek site

Conclusions

- Picard-Lefschetz theory provides alternative interpretation of optics, quantum mechanics: imaginary positions and trajectories
- wave optics changes nature of astrophysical observables: Coherent FRB radiation one of the potentially most precise measurements in physics. Imaginary images extends classical tools to include full wave optics: time delay measurements in weak lensing
- new tool to compute quantum gravity, bypassing historical conceptual barriers.