Physical Concepts behind the 2022 Physics Nobel Prize
2022 諾貝爾物理獎背後的物理概念

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November 1, 2022
Colloquium in IOP, AS

Abstract:

A general talk is given to highlight the main ideas and concepts behind the works which won the 2022 Nobel Prize in Physics.

Topics:
Quantum reality, Hidden variables, Entanglement, Bell Inequality, Quantum teleportation, and Quantum secret communication.

2022 Nobel Prize in Physics:

Alain Aspect (France)
John Clauser (USA)
Anton Zeilinger (Austria)

"… for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

Key words: Entangled photons (糾纏態)
Bell inequality (貝爾不等式)
Quantum information science (量子資訊科學)

1. “Entanglement and Bell Inequality” are related to the foundation of quantum mechanics:

   (a) quantum reality 量子真相
   (b) hidden variables 隱變量
   (c) entanglement 糾纏
   (d) Bell Inequality 貝爾不等式

2. “Quantum information science” is the application of quantum mechanics in information science:

   (a) quantum teleportation 量子遙傳
   (b) quantum secret communication 量子秘密通訊

Old Quantum Theory: Planck (1900)

Black-body radiation (黑體輻射)

Einstein (1905)

Photo-electric effect (光電效應)

Bohr (1913)

Model of hydrogen atom (氫原子模型)

Quantum Theory: Heisenberg (1925)

Matrix mechanics (矩陣力學)

Schrodinger (1926)

Schrodinger Equation (薛定諤方程)

Schrodinger Equation:

$$i\frac{\partial}{\partial t}\Psi - \hat{H}\Psi = 0$$

where Ψ is called the wave-function of the microscopic particle, e.g. electron

Ψ gives the state of the particle, but not the trajectory of its motion, as in Newtonian mechanics.

Schrodinger Equation is a cornerstone of quantum mechanics, and also one of the most important equations in physics.

Quantum mechanics has a very peculiar property:

If ψ_1, ψ_2 are solutions of the Schrodinger Equation, then any arbitrary linear combination

$$\Psi = c_1\psi_1 + c_2\psi_2$$

is also a solution! (c_1, c_2 are arbitrary constants.)

This is called the "<u>Principle of Superposition</u>" (疊加原理)

Mathematically, it is simple:

$$(H - i\frac{\partial}{\partial t})\psi_1 = 0, \qquad (H - i\frac{\partial}{\partial t})\psi_2 = 0$$

$$(H - i\frac{\partial}{\partial t})(c_1\psi + c_2\psi_2) = 0$$

The hard part is the interpretation of the solution Ψ !
What does such a state mean or represent?

$$\Psi = c_1\psi_1 + c_2\psi_2$$

Does it mean a microscopic object can simultaneously be in two different states? How can we picture it?

Copenhagen Interpretation(哥本哈根解釋):
(Bohr, Heisenberg, Born, etc.)

1. When measured, the result is either ψ_1 or ψ_2.
2. The probability of obtaining either result is given by $|c_1|^2, |c_2|^2$ respectively, such that $|c_1|^2 + |c_2|^2 = 1$
3. After the measurement, the wave-function Ψ collapses to ψ_1 or ψ_2.

If $|c_1|^2 = |c_2|^2 = 1/2$, then the measurement result is totally random, like throwing a dice!

This is a very radical view of nature, for unlike classical mechanics, ➔ nature is intrinsically indeterministic, or probabilistic!

As if a dice is thrown whenever a measurement is made!

Einstein: **God does not play dice!**
'The theory produces a good deal but hardly brings us closer to the secret of the Old One.
I am at all events convinced that He does not play dice.'
(Letter to Max Born, 1926)

If we accept the Copenhagen Interpretation, then the reality of the microscopic quantum world is very counter-intuitive, and starkly different from our experiences in the everyday macroscopic world.

Macroscopic:
Suppose I tell you I have a ball in my pocket, which could be either red or green, then when I take it out, the ball you see must be either red or green, and never in a mixed (red+green) state!

Yes, it is also probabilistic, red or green. But this kind of randomness is explainable. It was because I had thrown a dice before I prepared it. <u>And if you see a red ball, you can be sure that it has been red all the time.</u>

Einstein believes nothing in nature should be intrinsically probabilistic, all randomness in nature can and must be explained, just like dice or Brownian motion (布朗運動).

Since nothing in the formulation of quantum mechanics can explain the observed probabilistic phenomena,
➔ QM must be incomplete.
Einstein: "The Lord is subtle but not malicious."

The Copenhagen School insists that QM is complete as it is, there is nothing to be added.
Bohr: "**Stop telling God what to play**."

There were long debates between Einstein and Bohr, with Einstein trying to find loopholes or inconsistencies in the theory, but failed to do so.

So the debate is largely philosophical:
➔ It is about our understanding of the physical reality in the microscopic world as described by QM, **rather than the correctness of QM.**

1. If QM is complete, then nature is intrinsically probabilistic, and we can do nothing about it.

2. If QM is incomplete:
   Probabilistic behavior ➔ something (dof, λ) is missing

$$\Psi \rightarrow \Psi(\lambda)$$

   where λ is the "dice" which can account for the probabilistic behavior of nature.
   λ is called "hidden variable (隱變量)".

So that if the value of the hidden variable λ is fixed, then measurement results are also fixed. E.g., for

$$\Psi = c_1\psi_1 + c_2\psi_2$$

$$\Psi(\lambda = 1) \to \psi_1$$

$$\Psi(\lambda = 2) \to \psi_2$$

<u>“Hidden”</u> means we don’t know what it is yet, supposedly it is a physical variable which obeys its own physical laws.

Remark:

The introduction of “hidden variables (隱變量)” is an *ad hoc* solution to the puzzling quantum indeterminacy. It is somewhat like the postulation of “dark matter (暗物質)”. We also don’t know what dark matter is, but we need it to explain the extra attractive force needed to account for the motion of stars around our galaxy.

So everybody agree there is a dice (probability), the Copenhagen School insists that it is in God’s hand, we can only accept it.

Whereas Einstein wants to take the dice from God’s hand and put it into the wave-function (or QM).
It is “hidden” for now, but as part of nature, there is hope that it will be understood in the future.

So, where is the dice?

Not everybody was interested, because …

QM is a very useful tool.
Schrodinger Equation + Copenhagen interpretation
allow us to calculate and explain many phenomena in nature.

Although we don't have a intuitive picture of the physical reality,
it is like a black-box, most people are satisfied.

Much like when an ordinary person is given a cell phone
(plus an user's manual), then most likely he would go about
having fun with it, instead of asking what is inside.

David Mermin (1989): Copenhagen ➔ Shut up and calculate!
He was a graduate student at Harvard in the 1950's,
*'You'll never get a PhD if you allow yourself to be distracted
by such frivolities, so get back to serious business and produce
some results.'*

So calculate most people did, and as a result, much progress was made in atomic physics, nuclear physics, elementary particle physics, solid state physics, etc.

But of course Einstein was no ordinary person,
and he did not have to earn a PhD, so he kept asking.

1935 Einstein, Podolsky, Rosen (EPR):

Entanglement ➔ QM is incomplete!

Superposition:

1-body $\Psi = c\psi + c'\psi'$

2-body $\Psi_{12} = c\psi_1\phi_2 + c'\psi'_1\phi'_2$

Now if $\psi_1 = \psi_1'$ then,

$$\Psi_{12} = \psi_1(c\phi_2 + c'\phi_2')$$

which is a factorized state. For such a state, the states of the two particles can be prepared separately at different sites.

However if $\psi_1 \neq \psi_1', \quad \phi_2 \neq \phi_2'$
Then

$$\Psi_{12} = c\psi_1\phi_2 + c'\psi_1'\phi_2'$$

is not separable. It is an **entangled state,** the states of the particles could not have been prepared separately. They must have been brought together and interact in the past.

(Entanglement is a special case of superposition.)*

For example:

$$\Psi_{1,2} = \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2)$$

This is a typical entangled state of two spin-1/2 particles.

Property: When we measure particle-1, we can predict the state of particle-2, without measuring it, and vice versa.

This statement is independent of the separation between the particles, it is true even if particle-1 is on earth, and particle-2 is on the moon.

That means we can predict the state of particle-2 on the moon by measuring particle-1 on earth, even before the signal of the measurement result have time to reach the moon.

EPR:
"If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

Local Realism (局域真實主義)

That is, if measuring particle-1 produces $\uparrow_1$,
then we can 100% predict particle-2 is in state $\downarrow_2$ without disturbing it, then particle-2 must be in the state $\downarrow_2$ all along.

But QM says no!
QM ➔ Before: we have $\Psi_{1,2} = \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2)$

Immediately after: $\Psi_{12} = \uparrow_1 \downarrow_2$

But how did particle-2 know to jump to $\downarrow_2$ state immediately, before it had a chance to learn about particle-1's result?

Einstein: There exists spooky action at a distance?
Since QM cannot explain this effect, it must be **incomplete**. Again, something (λ) must be missing!

Actually, measuring one object can reveal the state of another distance object is not unique to QM. For example, I have two identical envelopes (1, 2), envelope-1 contains a red card, and envelope-2 a green card. Then I throw a dice, if odd (1,3,5), I send the envelope-1 to the moon, and keep envelope-2 here. Whereas if even (2,4,6), I send envelope-2 to the moon, and keep envelope-1 here.

Then opening the envelope on the earth would immediately reveal the color of the card on the moon, and vice versa.
Is this spooky action at a distance?

The answer is no, because this is just a case of perfect classical correlations, and there exist a physical dice. Moreover, when we see a red card, we know it has been red all along, there is an element of physical reality. In the language of QM, such a state is separable or factorized, as the envelopes can be prepared separately on earth and on the moon. I can throw a dice on earth and then make a phone call to the moon. The envelopes are not entangled. Quantum entanglement means more than this.

Question remains: **Complete or not complete?**

Nothing much happened for 29 years, until 1964.

In 1964 John Bell derived an inequality which can be used to experimentally test local hidden variable models.

Question: What's the difference? $\Psi \leftrightarrow \Psi(\lambda)$

Do they give different predictions in experiments?

Bell considered correlations of quantum measurements on two-level systems, such as spin-1/2 particles.

Eigenstates: $\uparrow, \downarrow$ in some direction (**z**)
According to QM, when we measure these states in any other direction **a**, one would also get $\uparrow, \downarrow$, but along the measurement direction **a**, the outcomes are probabilistic.

Consider the entangled state

$$\Psi_{12} = \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2)$$

If there exist hidden variables λ , then the above expression is incomplete,

$$\Psi_{12} \rightarrow \Psi_{12}(\lambda)$$

Such that when λ is fixed, then all measurements are deterministic.

Let A(**a**, λ) = (+1, -1) for ($\uparrow, \downarrow$) along **a** for particle-1

Let B(**b**, λ) = (+1, -1) for ($\uparrow, \downarrow$) along **b** for particle-2

The correlation function between the two measurements is given by

P(**a**,**b**,λ) = A(**a**,λ)B(**b**,λ) = +1, -1
= +1 (for all λ, perfectly correlated)
= -1 (for all λ, perfectly anti-correlated)

Suppose the hidden variable λ has a distribution $\rho(\lambda)$ such that

$$\int d\lambda\rho(\lambda) = 1$$

Then the averaged correlation can be written as

$$\mathbf{P}(\mathbf{a}, \mathbf{b}) = \int \mathbf{d}\lambda\rho(\lambda)\mathbf{A}(\mathbf{a}, \lambda)\mathbf{B}(\mathbf{b}, \lambda)$$

$$-1 \leq \mathbf{P}(\mathbf{a}, \mathbf{b}) \leq \mathbf{1}$$

Assumption:
The result of measuring particle-1 does not depend on how particle-2 is measured, and vice versa. That is,

A(**a**,λ) does not depend on **b,** and **B**(**b**,λ) not on **a**

Now, let particle-1 be measured in two possible directions (**a, a'**), and particle-2 in two directions (**b, b'**)

Then one can derive the inequality

$$|\mathrm{P(a,b)+P(a,b')}|+|\mathrm{P(a',b)-P(a',b')}|\leq 2$$

It is called **Bell Inequality**, **generalized Bell Inequality**, or **Bell-CHSH Inequality**.
Proved by CHSH (Clauser, Horne, Shimony, Holt [1969])
It is a generalization of the original inequality Bell proved in 1964.

For the state

$$\Psi_{12} = \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2)$$

QM ➔ $$\mathbf{P}(\mathbf{a}, \mathbf{b}) = -\mathbf{a} \cdot \mathbf{b} = -\cos(\theta_{\mathbf{ab}})$$

Then choosing

$$(\mathbf{a}', \mathbf{a}) = (0^o, 90^o) \qquad (\mathbf{b}, \mathbf{b}') = (45^o, 135^o)$$

We have

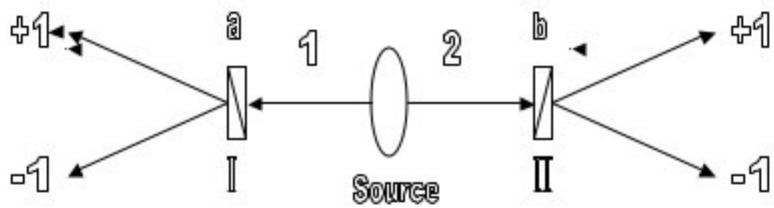
$$\cos(\theta_{\mathbf{ab}}) = \cos(\theta_{\mathbf{ab}'}) = \cos(\theta_{\mathbf{a}'\mathbf{b}}) = -\cos(\theta_{\mathbf{a}'\mathbf{b}'}) = 1/\sqrt{2}$$

$$|\mathbf{P}(\mathrm{a}, \mathrm{b}) + \mathbf{P}(\mathrm{a}, \mathrm{b}')| + |\mathbf{P}(\mathrm{a}', \mathrm{b}) - \mathbf{P}(\mathrm{a}', \mathrm{b}')| = \mathbf{2}/\sqrt{\mathbf{2}}$$

which is greater than 2 ➔ violation of Bell Inequality.

That means local hidden variable model gives a prediction which is different from QM. So they can be tested experimentally.

First experimental demonstration:
1972 Freedman and Clauser



They used entangled photon pairs from cascade decays of calcium atoms. One could equivalently translate their result to the spin-1/2 case, giving

$$|\mathrm{P(a,b)} + \mathrm{P(a,b')}| + |\mathrm{P(a',b)} - \mathrm{P(a',b')}| = 2.71 \pm 0.11$$

which is >2 by six standard deviations

➔ Clearly violates Bell Inequality!

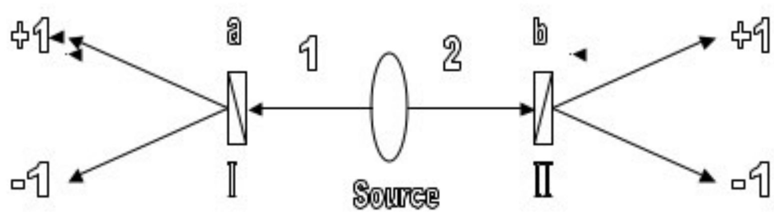
Loopholes?

1. Locality loophole
   To guarantee the requirement that how particle-1 is measured does not affect the outcome on particle-2, information on the choice of **a** or **a’** must not have time to reach particle-2 when it is measured, and vice versa. This is violated in Freedman-Clauser.

2. Detection loophole
   Due to detection efficiency, not all produced photons are detected. The quoted detection efficiency was only ~1/1000, so there was the worry that the detected photons might not be a fair sample of the whole set.

In the 1980's, loophole (2) was not technically possible to overcome, so Aspect *et al.* set out to close loophole (1).

To do so, the orientation of one of measuring device was randomly chosen when the photons were in flight,
So that this information has not enough time to reach the other photon before it is measured.



1. The detectors are placed **6** meters from the source, and the orientation of one of the detectors (I or II) is chosen only after the photons has left the source.
2. Two-channel photo-polarizers are used. (Freedman and Clauser used one-channel ones).

The result they get is

$$|\mathrm{P(a,b)} + \mathrm{P(a,b')}| + |\mathrm{P(a',b)} - \mathrm{P(a',b')}| = 2.75 \pm 0.15$$

which is >2 by five standard deviations
➔Violation of Bell Inequality!

This is the first experiment which claims to have closed the locality loophole, however, the experimental setup is not ideal:

1. Only one polarizer is rotated in each run
2. The "random" rotation is not really random, it is sinusoidal.
3. The measuring devices were linked by physical electronic circuits, not really independent.

These deficiencies were closed finally by Zeilinger *et al.* in 1998. In this experiment, the two detectors were separated by **400m**, so that there is enough time to do truly independent random switching of both detectors. Results:

$$|\mathrm{P(a,b)} + \mathrm{P(a,b')}| + |\mathrm{P(a',b)} - \mathrm{P(a',b')}| = 2.73 \pm 0.02$$

which is >2 by 30 standard deviations.
➔ Violation of Bell Inequality.

Detection efficiency ~5%, so detection loophole remains.
If we trust the data are fair samples of reality, then:

Conclusion: Bell inequality is violated in experiments
➔Local hidden variable models are ruled out.

Four experiments which are free of both the locality and detection loopholes were performed in 2015 – 2017.
All confirmed violation of Bell inequality.

See for example, Rosenfeld *et al*. PRL 119, 010402(2017)

1. Entanglement of atomic spins
2. Detector separation = 398 m

Remarks:
One might have the impression that HV/~~HV~~ is only a matter of interpretation of QM, it's about whether Copenhagen is correct or Einstein is, and Bell inequality can distinguish the two different points of view.

But what if experiments had validated Bell inequality?
Does it merely mean Copenhagen interpretation is wrong?
The answer is NO, for it actually means QM is wrong!

Since QM has never been shown to be wrong, so some people thought checking Bell inequality was a waste of time. As Clauser said in a telephone interview after the prize: "I thought it was important at the time, even though I was going to ruin my career by doing it, and in some sense I did: I've never been a professor." "I had a short conversation with Feynman, and he threw me out of his office." (Clauser was a post-doc at Berkeley.)

Quantum Entanglement as a resource

While some people were puzzled by entanglement's counter-intuitive properties and tried to understand why, some other people realized that quantum entanglement is actually a very useful resource, with many novel applications in quantum information science.

Two examples:

1. Quantum teleportation (量子遙傳)
   (Th: 1993 Bennett et al.; Ex: 1997 Zeilinger et al.)

2. Quantum secret communication (量子秘密通訊)
   (Th: 1991 Ekert; Ex: 2006 Zeilinger et al.)

Quantum Teleportation:

Suppose one has an **unknown** state $\chi_c = d \uparrow_c + d' \downarrow_c$ at site-A and there exist an entanglement pair shared between two remote sites A and B,

$$\psi^{-}_{AB} = \frac{1}{\sqrt{2}}[\uparrow_A \downarrow_B - \downarrow_A \uparrow_B]$$

Then one can teleport the quantum state χ from A to B.

➔ One can recreate the state χ at site-B, without having to physically transport any particles from A to B; with the original state destroyed at A (No Cloning Theorem, 1982).

Protocol: The wave function of the 3-particle state is

$$\Psi_{cAB} = \chi_c \psi_{AB}$$

It can be rewritten as

$$\Psi_{cAB} = \frac{1}{2}\{\psi^-_{cA}[\chi_B] + \psi^+_{cA}[\sigma_z \chi_B] - \phi^-_{cA}[\sigma_x \chi_B] - \phi^+_{cA}[\sigma_z \sigma_x \chi_B]\}$$

where $\{\psi^{\pm}, \phi^{\pm}\}$ are the four orthogonal Bell states:

$$\psi^{\pm} = \frac{1}{\sqrt{2}}[\uparrow\downarrow \pm \downarrow\uparrow]$$

$$\phi^{\pm} = \frac{1}{\sqrt{2}}[\uparrow\uparrow \pm \downarrow\downarrow]$$

which can be used as a 2-body measurement basis, just like $\{\uparrow\uparrow, \quad \downarrow\downarrow, \quad \uparrow\downarrow, \quad \downarrow\uparrow\}$.

After the Bell measurement, depending on the outcomes, particle-B is now transformed into one of 4 different state,

$$\psi_{cA}^{-} \rightarrow \chi_B = (d \uparrow_B + d' \downarrow_B)$$

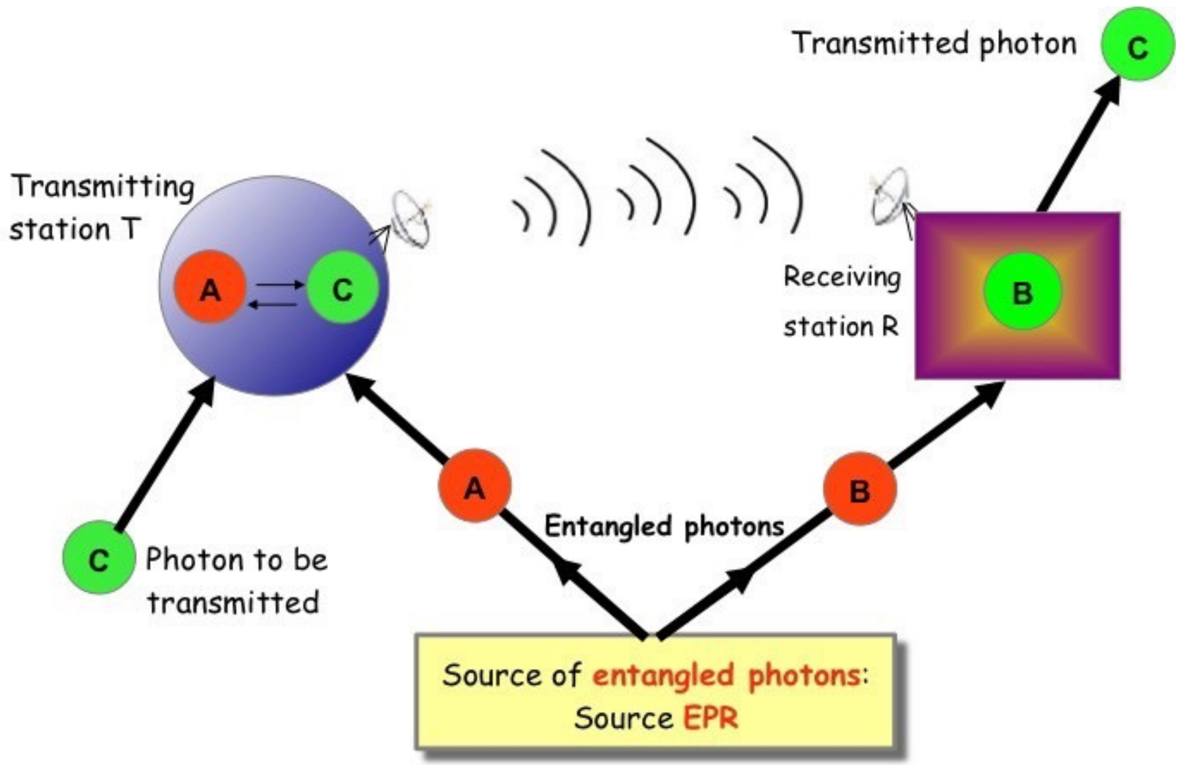
$$\psi_{cA}^{+} \rightarrow \sigma_z \chi_B$$

$$\psi_{cA}^{-} \rightarrow \sigma_x \chi_B$$

$$\psi_{cA}^{+} \rightarrow \sigma_z \sigma_x \chi_B$$

So after the measurement, the sender at site-A must broadcast a two-bit message (00, 11, 01, 10) to inform the receiver at site-B of the outcome. Then the receiver can apply a simple unitary transformation on particle-B to recover the correct state χ_B.

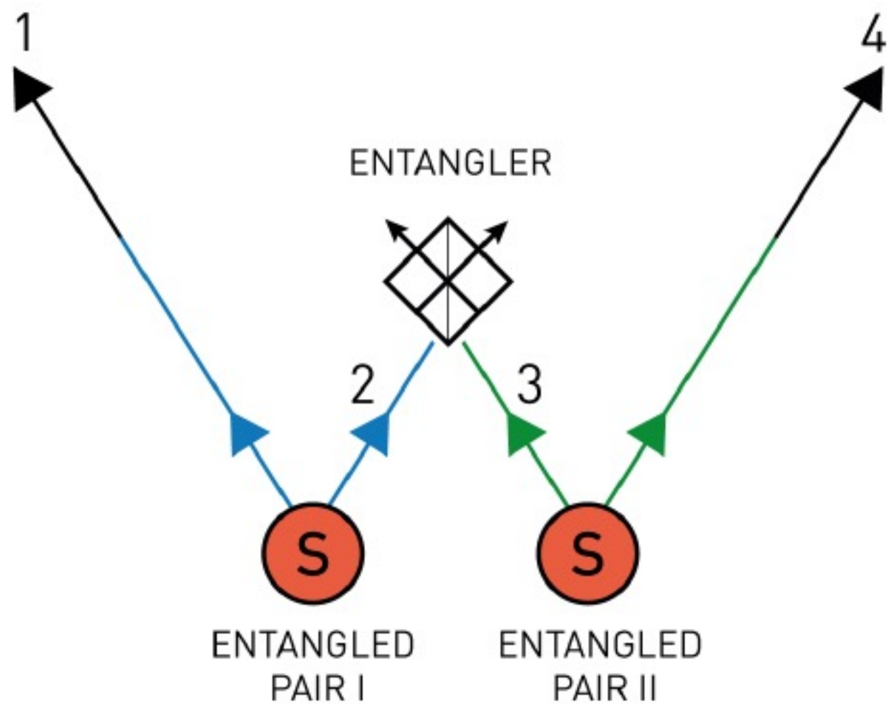
Note that the original state χ is now carried by particle-B, particle-c remains at site-A, it is now entangled with particle-A, and the original quantum state χ has been destroyed at site A.



One particle teleportation was first demonstrated in 1997 by the Zeilinger group using photons.

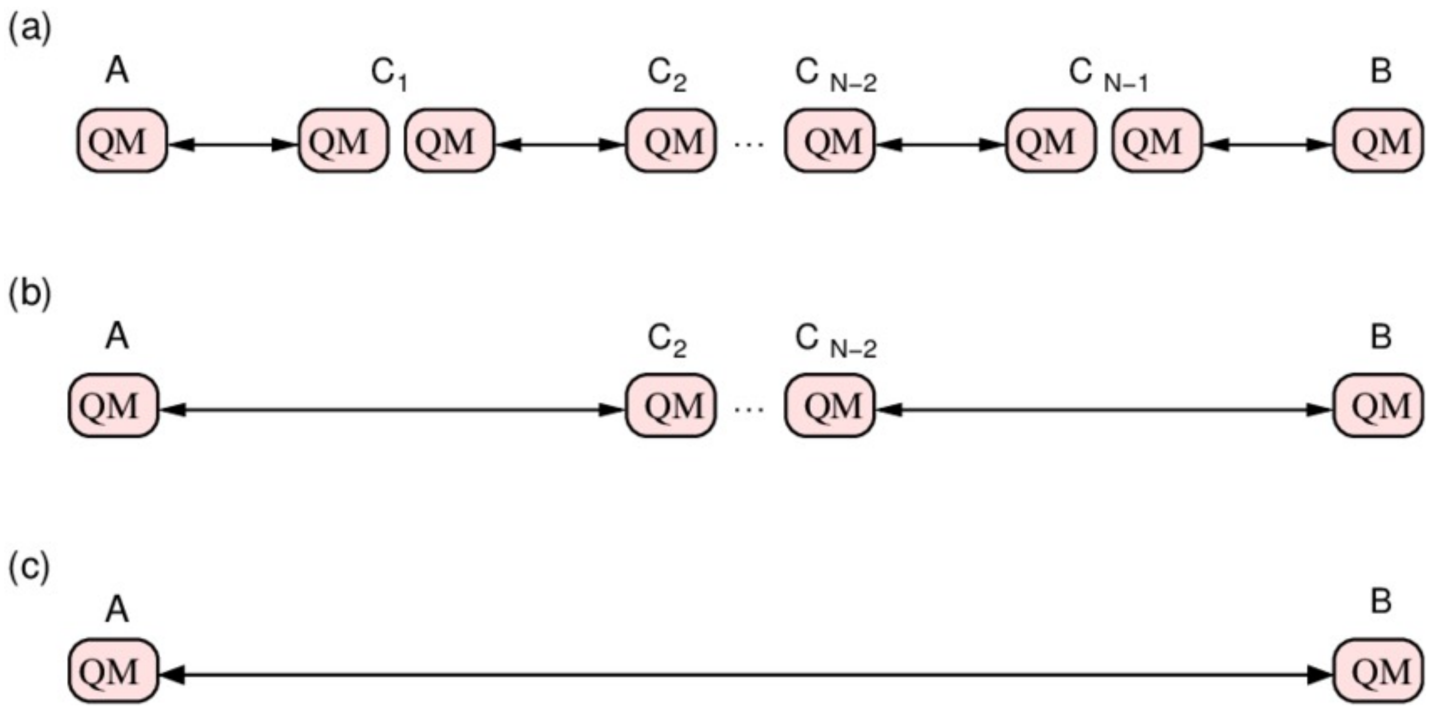
The same procedure can be repeated on multi-particle states, so that with N entangled pairs, one can teleport an N-particle state.

For 2-particle entangled states ➔ entanglement swapping



After teleportation: (1,4) are entangled, and (2,3) are entangled.
(Monogamy of entanglement.)

Quantum repeater:
Entanglement swapping can be used to establish entanglement over long distances, which is required for building quantum networks.



Quantum Secret Communication：
(Quantum Key Distribution, QKD 量子鑰匙分發)

To safely transmit a piece of information, say, **0110101**, we can encrypt it with a secret “key”, say, **1101001**

Encryption:		**0110101**	Message
(sender **A**)	XOR	**1101001**	Secret key
		1011100	Code sent

Decryption:		**1011100**	Code received
(receiver **B**)	XOR	**1101001**	Secret key
		0110101	Message

To succeed:

1. Both the sender and the receiver must have the same secret key. (**How to distribute a key safely?)**
2. The key must not be used more than once, otherwise information might be leaked to eavesdroppers.

Quantum entanglement can help solve the key distribution problem.

1991 Eckert proposed a method with which a secret key can be distributed safely among **A** and **B**.

Protocol:

1. **A** and **B** share N pairs of entangled states

$$\psi^-_{AB} = \frac{1}{\sqrt{2}}[\uparrow_A \downarrow_B - \downarrow_A \uparrow_B]$$

A measures the spin of particle-A along one of the three directions $\mathrm{a} = (0^o, 45^o, 90^o)$, and **B** measures particle-B along one of the three directions $\mathrm{b} = (45^o, 90^o, 135^o)$

2. After the measurements, they publicly announce the orientations used. Two possibilities:

(a) Events with same orientations

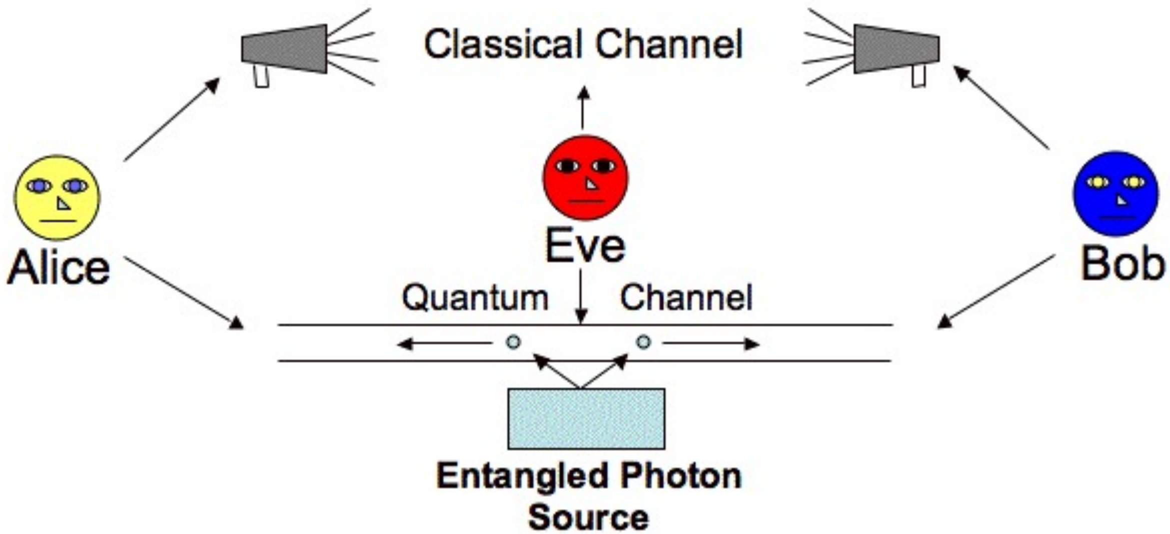
(b) Events with different orientations

For events in group (b), they reveal their measurement outcomes, with which they can calculate the Bell correlation function to check eavesdropping

$$S = |P(a,b) + P(a,b')| + |P(a',b) - P(a',b')|$$

by choosing $(a', a) = (0^o, 90^o)$ $(b, b') = (45^o, 135^o)$

QM requires $S = 2\sqrt{2}$, if so then the quantum channel is safe.



The data in group (a) are kept secret. These data are obtained when **A** and **B** used the same measuring directions, e.g., $(\mathrm{a}, \mathrm{b}) = (45^o, 45^o)$ or $(\mathrm{a}, \mathrm{b}) = (90^o, 90^o)$

For the state $\psi^-_{AB} = \frac{1}{\sqrt{2}}[\uparrow_A \downarrow_B - \downarrow_A \uparrow_B]$,

Their data are perfectly anti-correlated $\uparrow\downarrow, \downarrow\uparrow$.
So the data sets look like

A: (0 1 0 0 1 1 0 1 0 1 ……) where 1 = $\uparrow$, 0 = $\downarrow$.
B: (1 0 1 1 0 0 1 0 1 0 ……)

By inverting his bits, **B** can easily convert his data set to be identical to **A**'s, which can be used as the key for secret communication.

The first experimental realization of this scheme was done in 2006, again by Zeilinger's group, the separation between **A** and **B** was 144 km! They obtained S >2,

$$S = 2.508 \pm 0.037$$

which is not bad for a such a large separation.

Quantum Experiments at Space Scale (QESS)
In 2016, satellite Mozi (墨子號) was launched by China, and a group led by Pan Jian-wei (潘建偉)*et al.* used it to achieve Quantum entanglement over 1203 km. They used it to perform QKD and teleportation over great distances.

They also succeeded in establishing entanglement links between China and Austria, over a distance of ~7500 km.

Summary:

1. We have seen that quantum superposition gives rise to QM's most peculiar feature: quantum entanglement, which prompted Einstein *et al. (1935)* to claim that QM is not complete.

2. Bell (1964) pointed out a way to distinguish between QM and local hidden-variable models. Experiments ('72, '82, '98, ...) showed that the latter is ruled out.

3. Quantum entanglement is an essential resource in quantum information science, giving rise to many novel applications.