Subatomic swirls and beyond : spin transport in ultra-relativistic nuclear collisions

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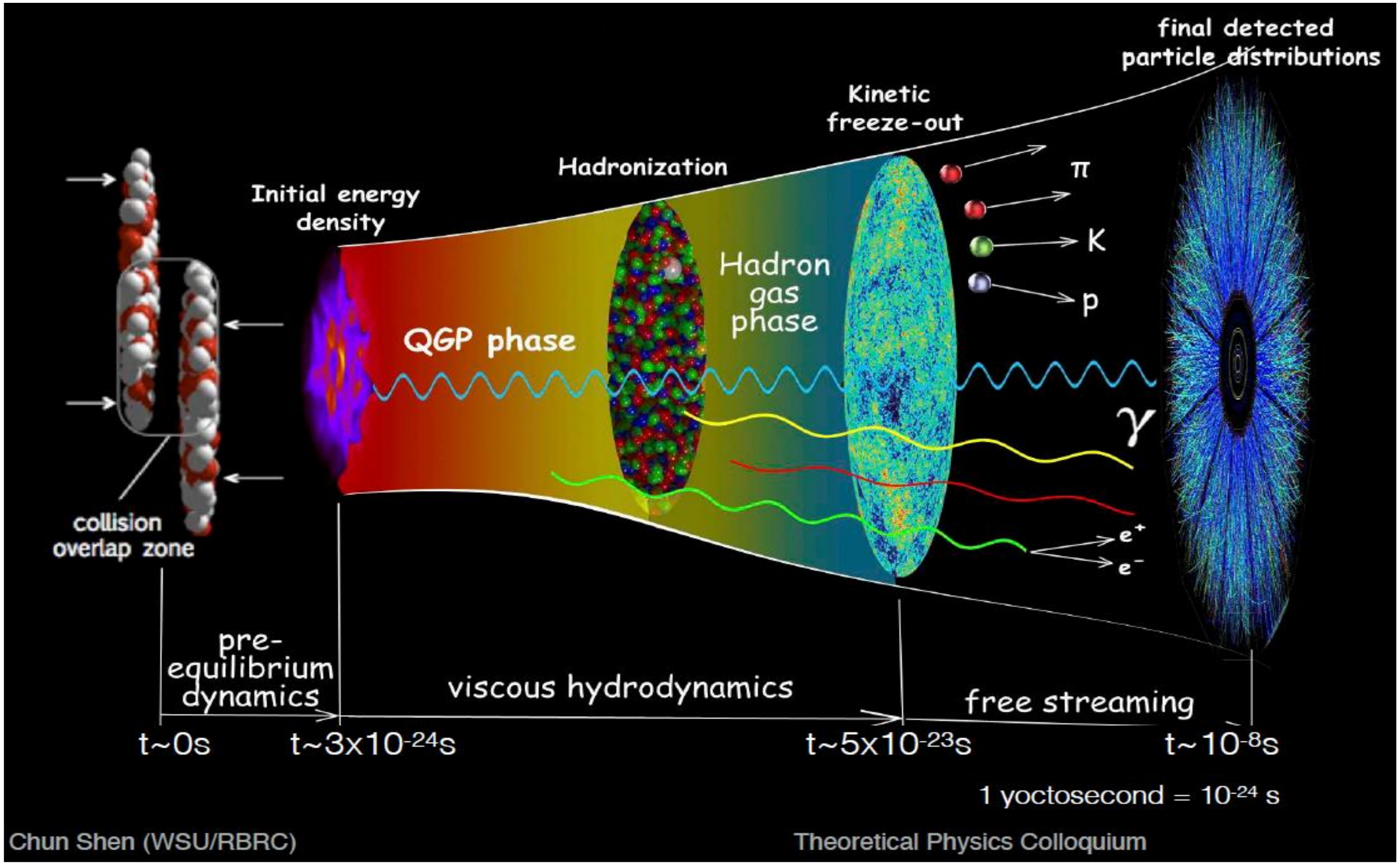
(AS IoP colloquium, Feb. 8th, 2022)

Outline

- A brief introduction to ultra-relativistic heavy ion collisions
- Spin polarization from the subatomic swirls : the puzzle, progress, and opportunity
- Beyond the subatomic swirls : anomalous spin polarization from chromo-electromagnetic fields

Relativistic heavy ion collisions

- Creating a little bang in the laboratory : formation of quark gluon plasmas (QGP)



Experimental facilities

- Heavy ion colliders :
  - Relativistic Heavy Ion Collider (RHIC) : Brookhaven National Lab. (U.S.)
  - Large Hadron Collider (LHC) : CERN (Europe)
  - collisional energy : from few GeV to few TeV

RHIC

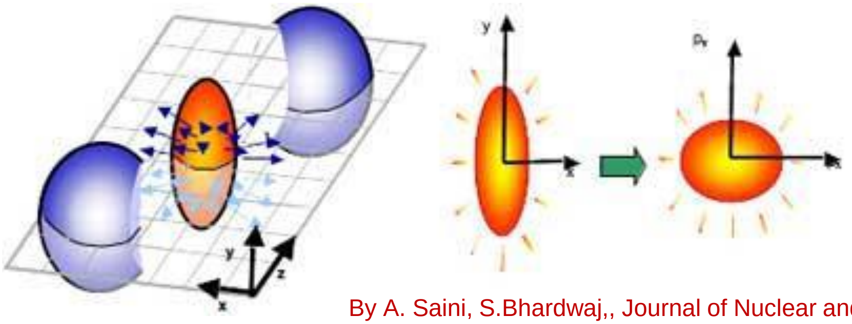
cerncourier.com

LHC

https://www.researchgate.net/publication/308718766_Introduction_to_Particle_Accelerators_and_their_Limitations

Nearly perfect fluid

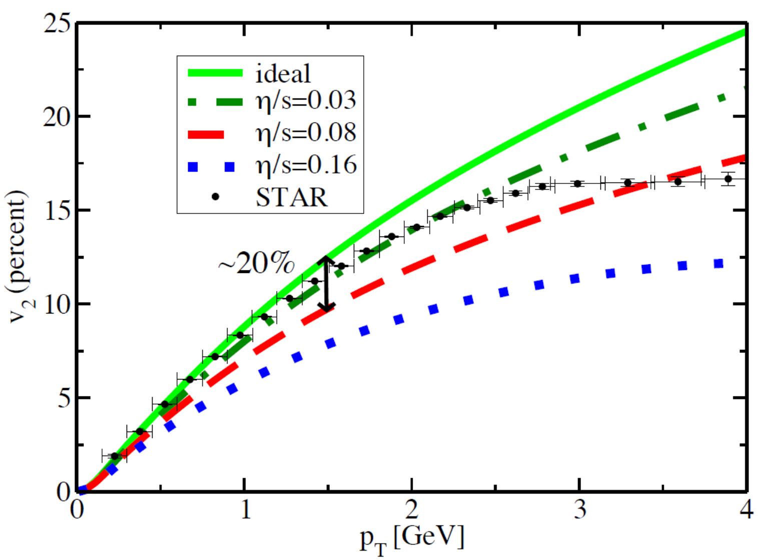
- Anisotropy flow of hadrons : hydrodynamic behaviors of the QGP



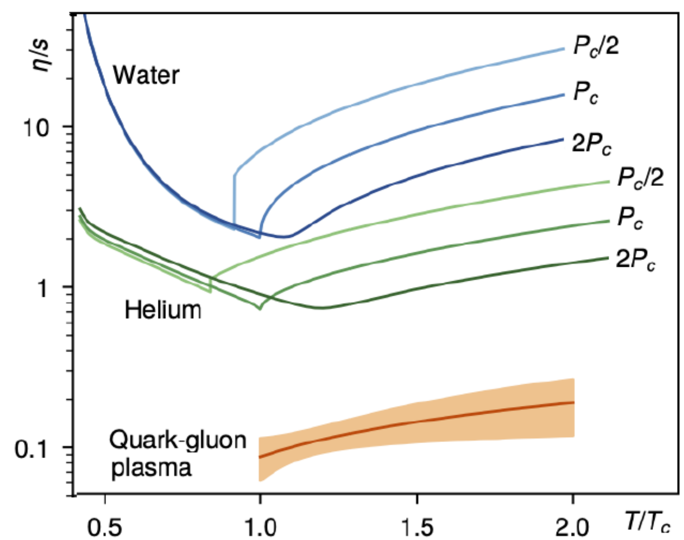
collective flow :
geometric asymmetry
⇒ momentum asymmetry

By A. Saini, S.Bhardwaj,, Journal of Nuclear and Particle Physics, Vol. 4 No. 6, 2014, pp. 164-170.

- The QGP has small ratio of shear viscosity to entropy density, $\eta/s \sim 1/(4\pi)$



P. Romatschke, U. Romatschke, PRL. 99 (2007) 172301

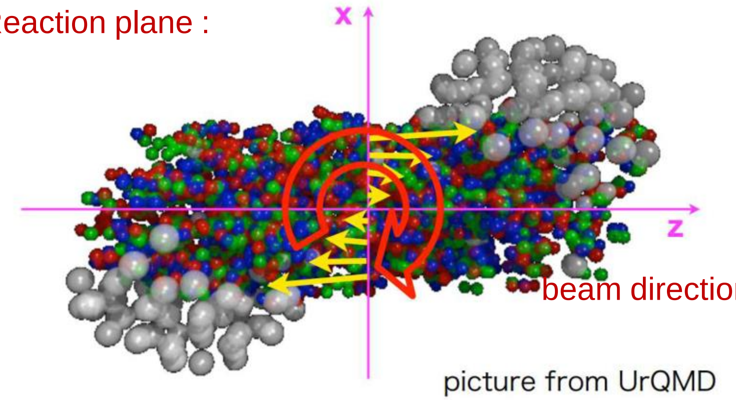


J. E. Bernhard, J. S. Moreland, S. A. Bass, Nature Phys. 15, no.11, 1113-1117 (2019).

Subatomic swirls

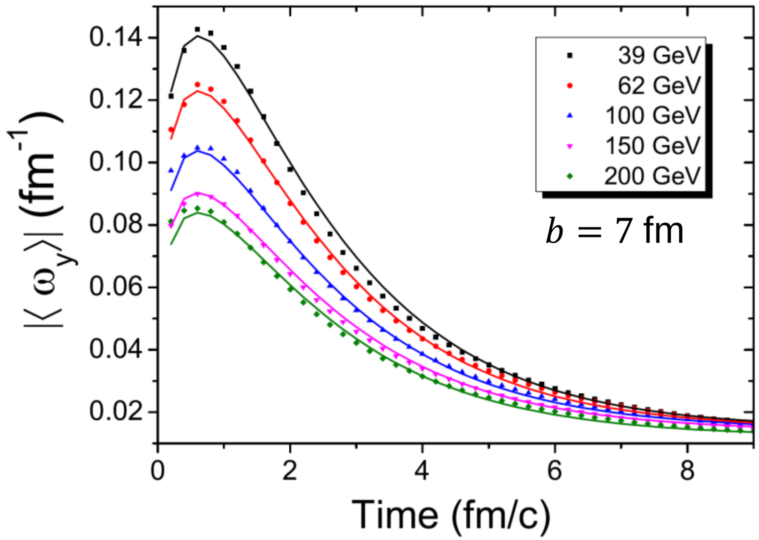
- Strong vortical fields in HIC :

Reaction plane :



impact parameter

Y. Jiang, Z.-W. Lin, J. Liao, PRC 94, 044910 (2016)
see also W.-T. Deng and X.-G. Huang, PRC 93, 064907 (2016)



❖ Angular momentum (AM) to vorticity :

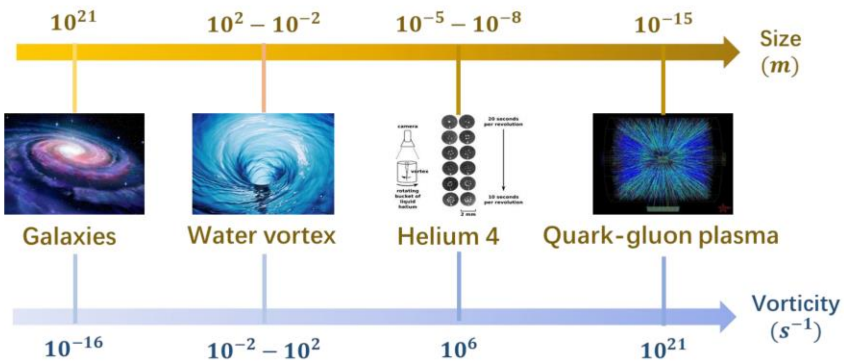
$$\mathbf{L} = \frac{1}{2}\int d^3\mathbf{r}\epsilon|\mathbf{r}|^2(1-\hat{\boldsymbol{\omega}}\cdot\hat{\mathbf{r}})\boldsymbol{\omega},$$

$$\boldsymbol{\omega} = \nabla \times \mathbf{v} = \text{const.}$$

ϵ : energy density

❖ AM to spin polarization?

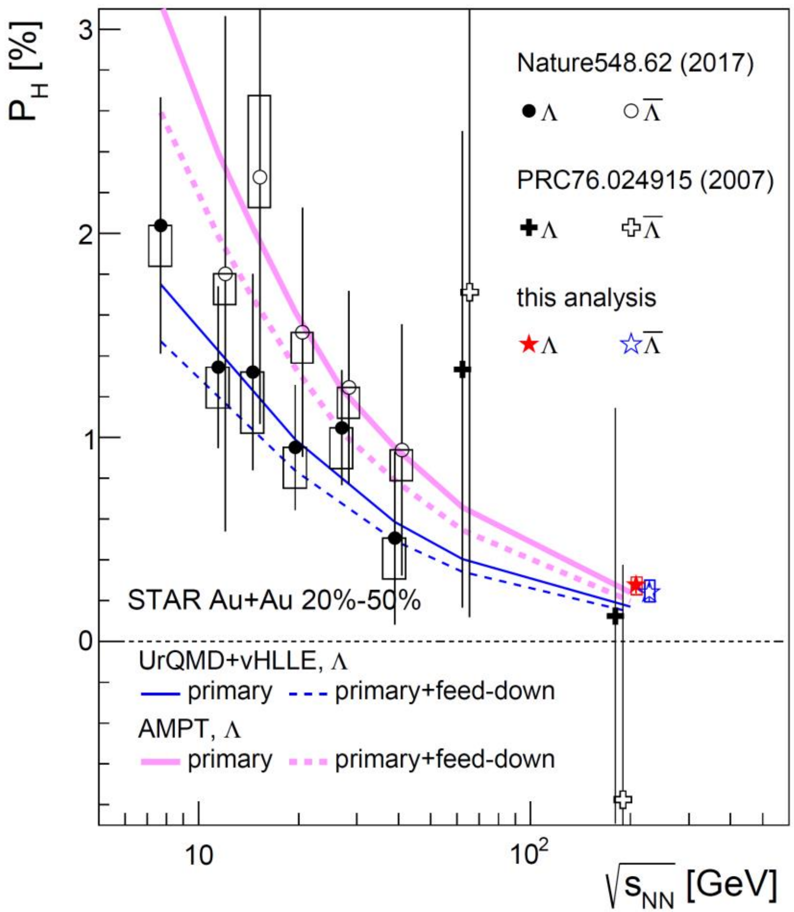
Z.-T. Liang and X.-N. Wang, PRL. 94, 102301 (2005)



X.-G. Huang, QM 19

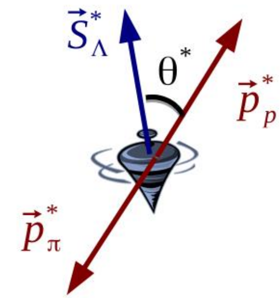
Λ polarization in HIC

- Global polarization of Λ hyperons :



L. Adamczyk et al. (STAR), Nature 548, 62 (2017)

❖ Self-analyzing via the weak decay :



T. Niida, QM18

❖ Statistical model/Wigner-function approach (in global equilibrium):

Killing cond. : $"\partial_\nu(u_\rho/T) + \partial_\rho(u_\nu/T) = 0"$

F. Becattini, et al., Ann. Phys. 338, 32 (2013)

R. Fang, et al., PRC 94, 024904 (2016)

$$\mathcal{P}^\mu(q) \approx \frac{1}{8m}\epsilon^{\sigma\mu\nu\rho}q_\sigma \frac{\int d\Sigma \cdot q\omega_{\nu\rho}f_q^{(0)}(1-f_q^{(0)})}{\int d\Sigma \cdot q f_q^{(0)}},$$

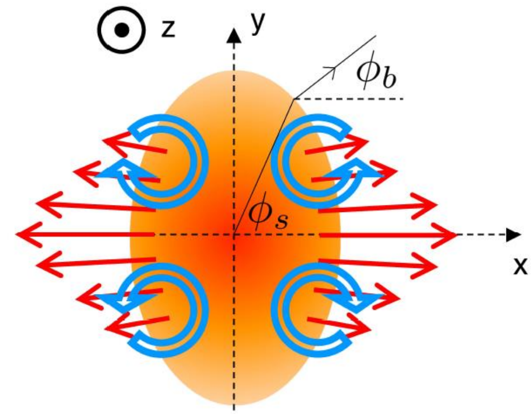
$$\boxed{\omega_{\nu\rho} = \frac{1}{2}\big(\partial_\rho(u_\nu/T) - \partial_\nu(u_\rho/T)\big)}.$$

thermal vorticity $\xrightarrow[\text{T=const.}]{}$ $\omega_{\alpha\beta} = -\epsilon_{\alpha\beta\mu\nu}\omega^\mu u^\nu$

Local (longitudinal) polarization : a sign problem

- Local vorticity :

  transverse expansion :
  longitudinal vorticity & polarization



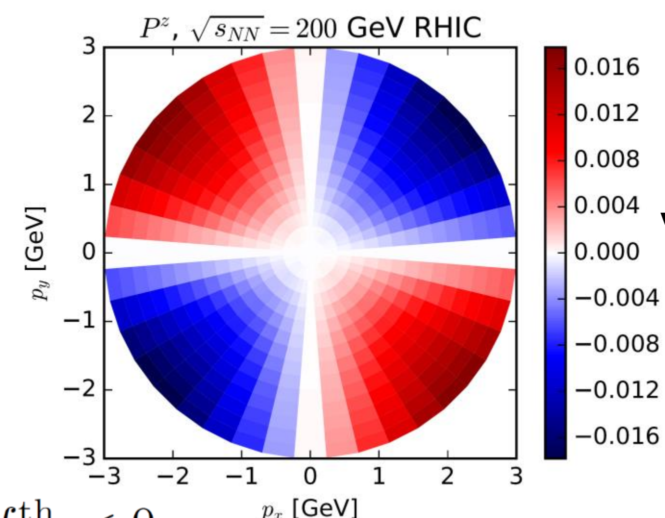
- A “sign problem” for longitudinal polarization

F. Becattini, I. Karpenko, PRL 120, 012302 (2018).

J. Adam et al. (STAR, PRL. 123, 132301 (2019)

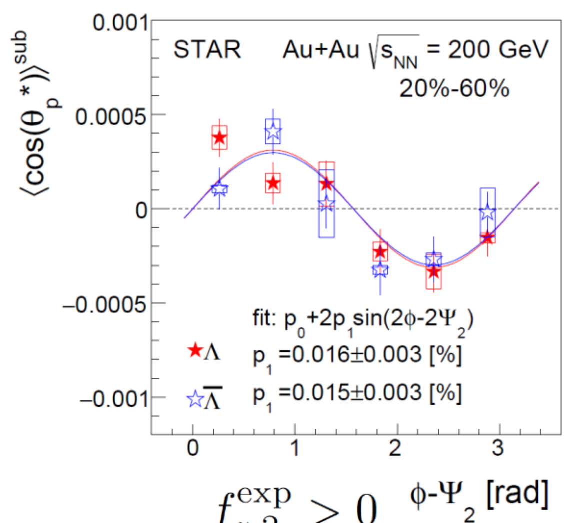
Spin harmonics :

$$\frac{dP^z}{2\pi d\phi} = f_{z,0} + 2 f_{z,2} \sin(2\phi)$$



$f_{z,2}^{\mathrm{th}} < 0$

v.s.



$f_{z,2}^{\mathrm{exp}} > 0$

(same structure, opposite signs!)

Go beyond global equilibrium

- The assumption for global equilibrium may be too naïve.

  Killing cond. needs not be satisfied : $"\partial_\nu(u_\rho/T) + \partial_\rho(u_\nu/T) \neq 0"$
- To understand non-equilibrium effects and dynamical spin polarization :
  - Spin hydrodynamics (macroscopic) W. Florkowski, A. Kumar, R. Ryblewski Prog.Part.Nucl.Phys. 108 (2019) 103709
  - Quantum kinetic theory (QKT) (microscopic)

  How a (strange) quark traversing QGP becomes polarized?

$$\mathcal{P}^\mu(\mathbf{p}) = \frac{\int d\Sigma \cdot p \boxed{\mathcal{J}_5^\mu(p,X)}}{2m \int d\Sigma \cdot \mathcal{N}(p,X)}$$

- The key ingredient to generate dynamical spin polarization : quantum corrections in collisions (spin-orbit int.).
- Tracking the non-equilibrium evolution in phase space is challenging : solving multi-dimensional differential equations.
- Near equilibrium : QKT ⇨ spin hydro.

Applying QKT to spin polarization

- Review of QKT : Yoshimasa Hidaka, Shi Pu, Di-Lun Yang, Qun Wang, "Foundations and Applications of Quantum Kinetic Theory", submitted to Prog.Part.Nucl.Phys., (2022) 2201.07644



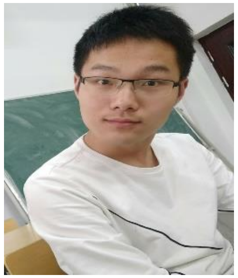
❖ Local equilibrium Wigner function for massless fermions

Y. Hidaka, S. Pu, DY, PRD 97, 016004 (2018)

- Local spin polarization and helicity polarization :

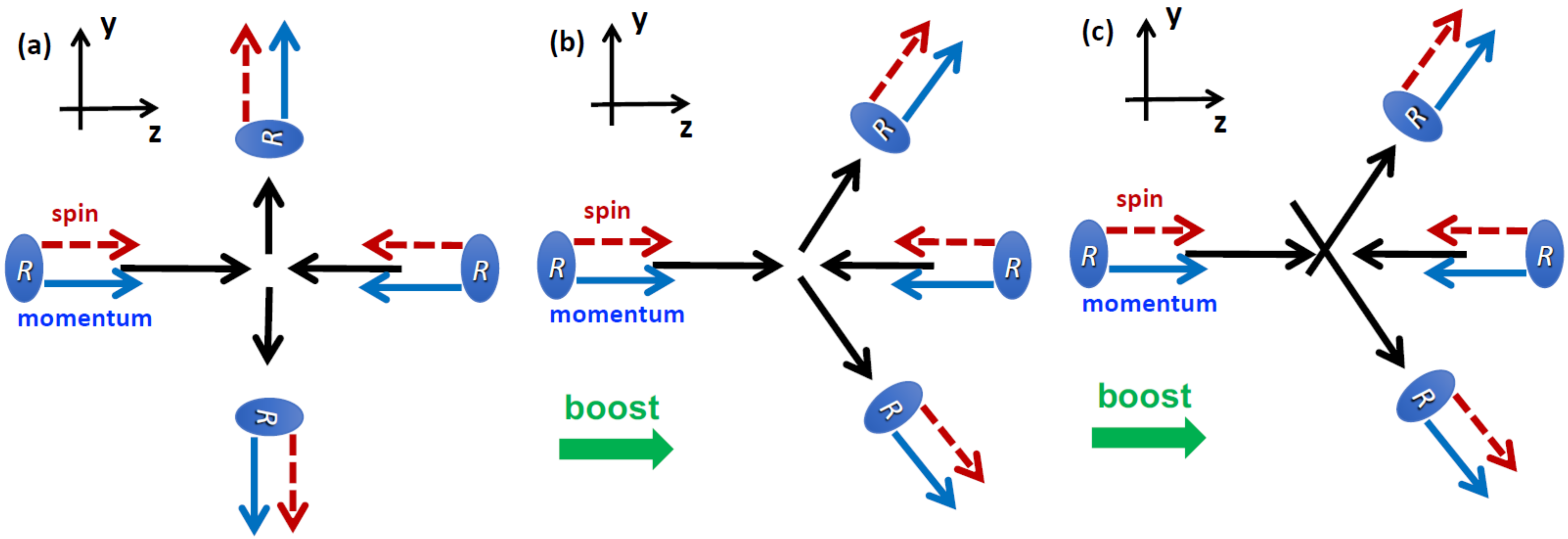
Cong Yi, Shi Pu, DY, PRC 104, 064901(2021)

Cong Yi, Shi Pu, Jian-Hua Gao, DY, (2021) 2112.15531




Spin-orbit interaction in collisions

- Massless particles : spin locking by helicity=chirality
- Side jumps : spin locking + AM conservation J.-Y. Chen, et al., PRL. 113, 182302 (2014)



- Equilibrium distribution functions receive the vortical correction in the fluid rest frame.

$$\Longrightarrow \quad J_5^\mu \equiv J_R^\mu - J_L^\mu = \int \frac{d^4p}{(2\pi)^4} \mathcal{J}_5^\mu(p,X) = \left(\frac{T^2}{6} + \frac{\mu_V^2}{8\pi^2}\right)\hbar\omega^\mu$$

(chiral vortical effect)

A. Vilenkin, PRD 20, 1807 (1979)
D. T. Son, P. Surowka, PRL 103, 191601 (2009)
K. Landsteiner, E. Megias, F. Pena-Benitez, PRL107,021601(2011)

Spin polarization in local equilibrium

- Spin polarization for massless fermions in local equilibrium can be obtained from the CKT with Coulomb scattering. Y. Hidaka, S. Pu, DY, PRD 97, 016004 (2018)

$$\mathcal{J}_5^\mu = \mathcal{J}^\mu_{\text{thermal}} + \mathcal{J}^\mu_{\text{shear}} + \mathcal{J}^\mu_{\text{accT}} + \mathcal{J}^\mu_{\text{chemical}} + \mathcal{J}^\mu_{\text{EB}},$$ (+ dissipative terms)

$$\mathcal{J}^\mu_{\text{thermal}} = a\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}p_\nu\partial_\alpha\frac{u_\beta}{T}, \quad \Rightarrow \quad J_5^\mu = \sigma_{5\omega}\omega^\mu$$

$$\mathcal{J}^\mu_{\text{shear}} = -a\frac{1}{(u\cdot p)T}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta p^\sigma\partial_{\langle\sigma}u_{\nu\rangle},$$

$$\mathcal{J}^\mu_{\text{accT}} = -a\frac{1}{2T}\epsilon^{\mu\nu\alpha\beta}p_\nu u_\alpha\left(Du_\beta - \frac{1}{T}\partial_\beta T\right),$$

$$a = 4\pi\hbar\,\text{sign}(u\cdot p)\delta(p^2)f_V^{(0)}(1-f_V^{(0)}).$$

C. Yi, S. Pu, DY, PRC 104, 064901(2021)
("naïve" generalization to massive fermions)

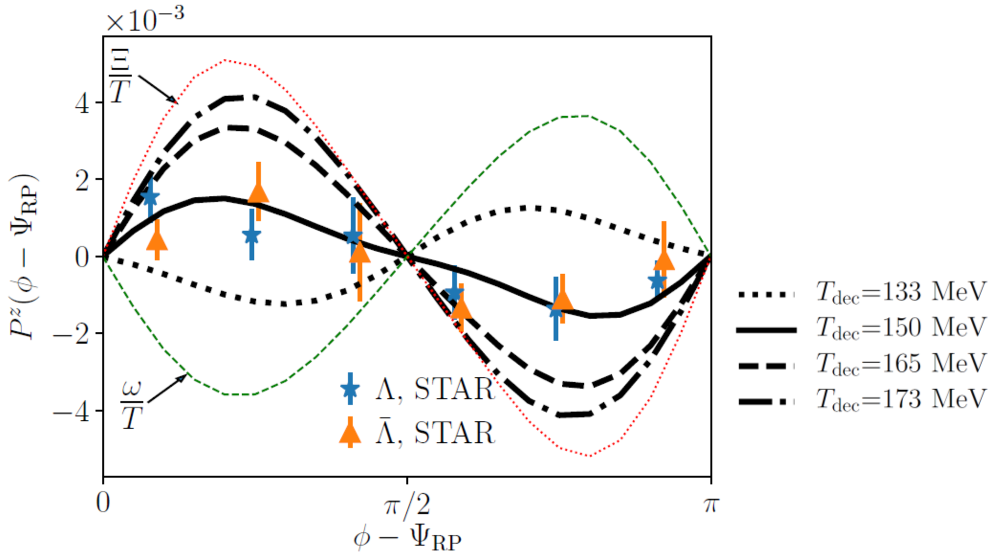
- Generalization to the massive case was also derived from the linear response theory and statistical field theory.
  (The same and similar results are found)

S. Y. F. Liu and Y. Yin, PRD 104, 054043 (2021)
S. Y. F. Liu, Y. Yin, JHEP 07, 188 (2021)
F. Becattini, M. Buzzegoli, A. Palermo, PLB 820,136519 (2021)

Shear corrections on longitudinal polarization

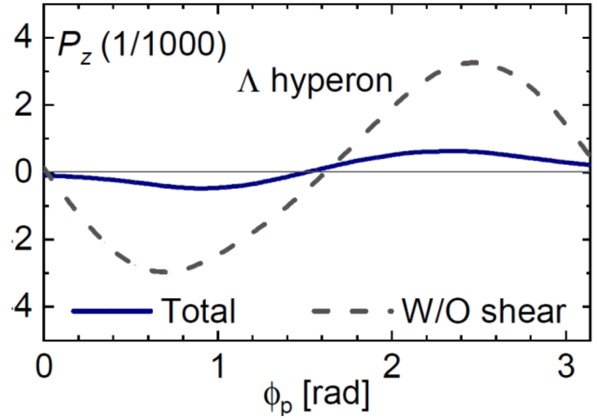
- Isothermal approximation :

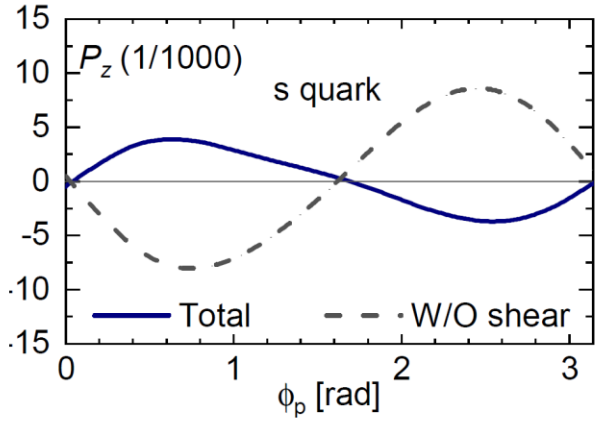
F. Becattini, M. Buzzegoli, A. Palermo, G. Inghirami, and I. Karpenko, PRL. 127 (2021) 27, 272302



- Λ and s equilibrium scenarios:

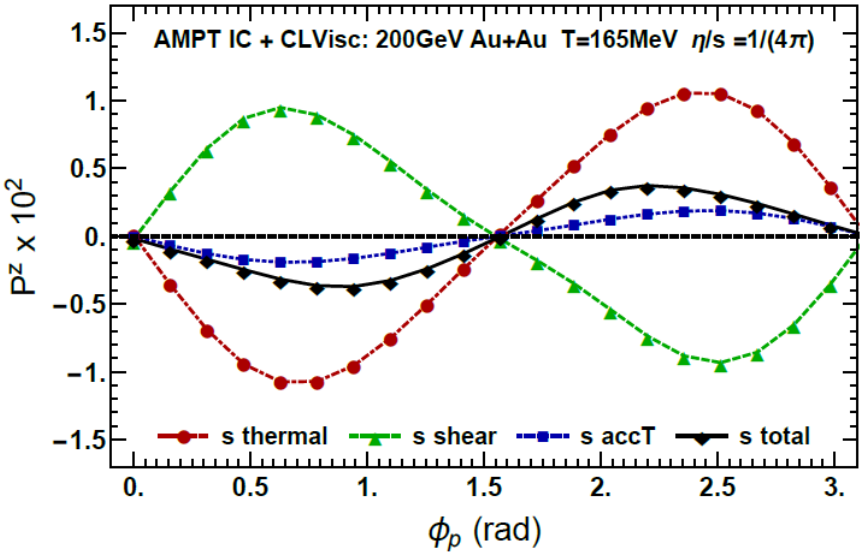
B. Fu, S. Y. F. Liu, L. Pang, H. Song, and Y. Yin, PRL 127, 142301 (2021)

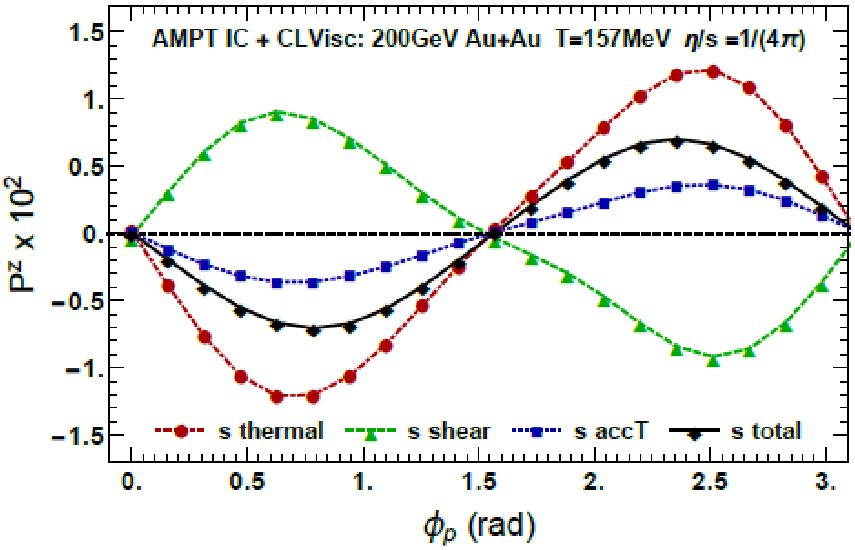




Is the spin sign problem solved?

- Sensitive to the equation of state and freeze-out T : out-off equilibrium corrections depending on interaction should be still considered.





C. Yi, S. Pu, DY, PRC 104 (2021) 6, 064901

Helicity polarization

- A better observable to probe the strength of local vorticity?
- Helicity polarization : $S^h = \widehat{\mathbf{p}}\cdot\boldsymbol{S}(\mathbf{p}) = \widehat{p}^x\mathcal{S}^x + \widehat{p}^y\mathcal{S}^y + \widehat{p}^z\mathcal{S}^z,$

F. Becattini et al., Phys.Lett.B 822 (2021) 136706
J.-H. Gao, Phys. Rev. D 104, 076016
C. Yi, S. Pu, J.-H. Gao, DY, (2021) 2112.15531

$$S^h_{\text{thermal}}(\mathbf{p}) = \int d\Sigma^\sigma F_\sigma p_0 \epsilon^{0ijk}\widehat{p}_i \nabla_j \left(\frac{u_k}{T}\right),$$

$$S^h_{\text{shear}}(\mathbf{p}) = -\int d\Sigma^\sigma F_\sigma \frac{\epsilon^{0ijk}\widehat{p}^i p_0}{(u\cdot p)T}\{p^\sigma(\partial_\sigma u_j + \partial_j u_\sigma - u_\sigma D u_j)u_k\},$$

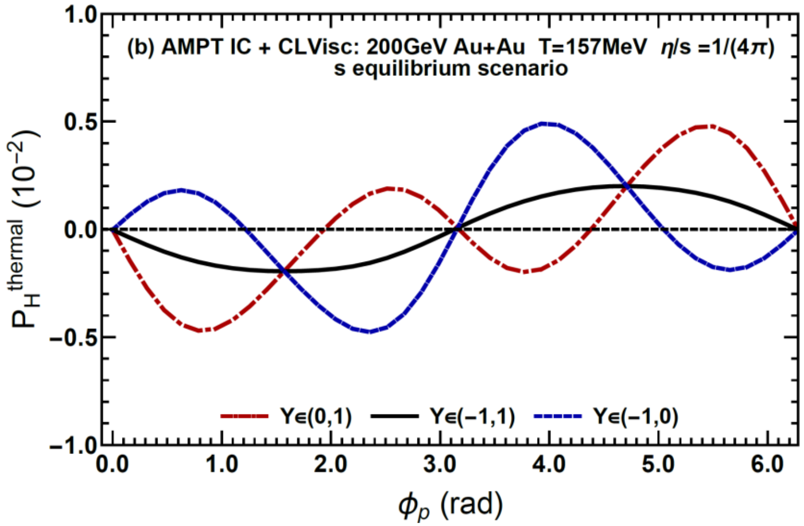
$$S^h_{\text{accT}}(\mathbf{p}) = \int d\Sigma^\sigma F_\sigma \frac{1}{T}\epsilon^{0ijk}\widehat{p}^i p_0 u_j (D u_k - \frac{1}{T}\partial_k T),$$

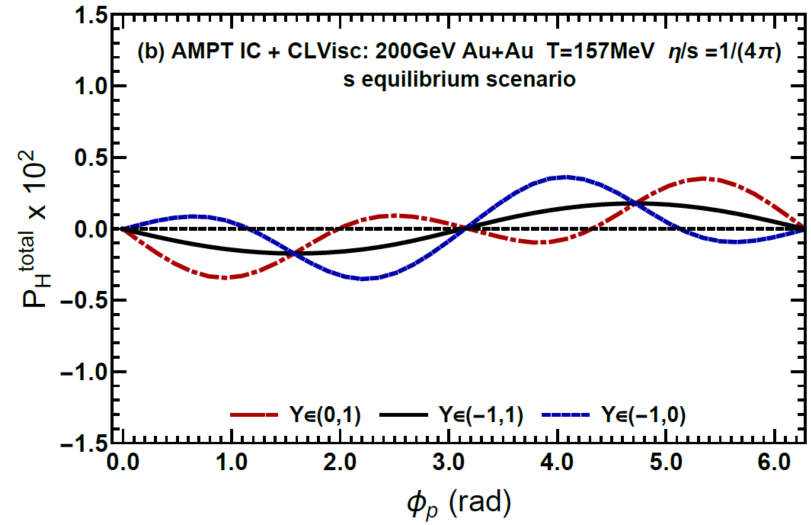
$$F^\mu = \frac{\hbar}{8m_\Lambda N} p^\mu f_V^{(0)}(1 - f_V^{(0)})$$

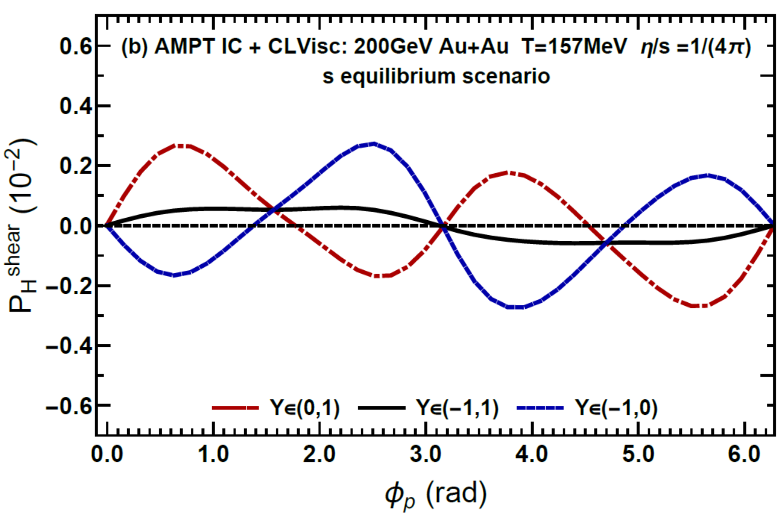
- When the fluid velocity is small, (fluid) vorticity contribution becomes dominant.

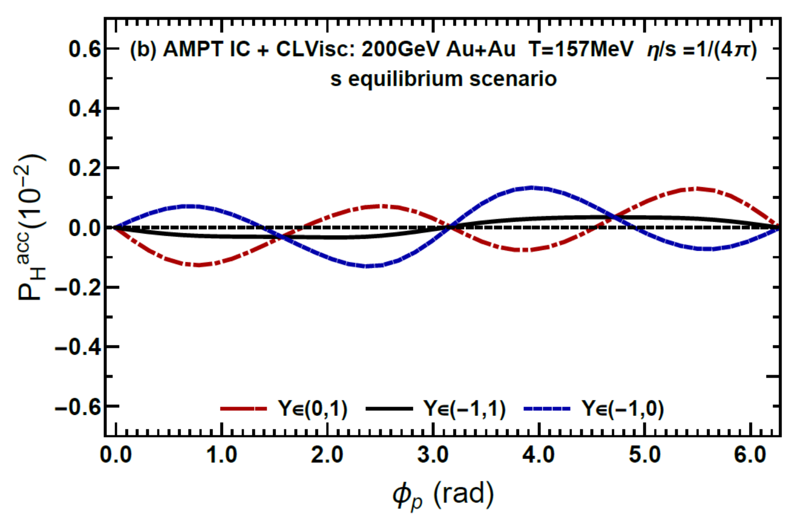
Hydrodynamic helicity polarization

- The dominant contribution from thermal vorticity :





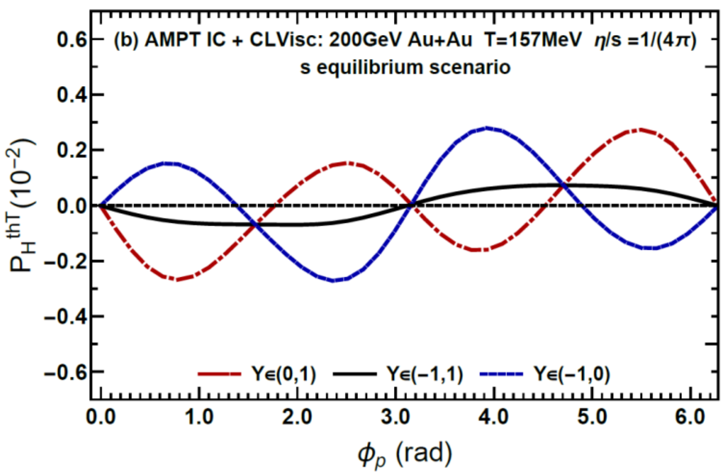


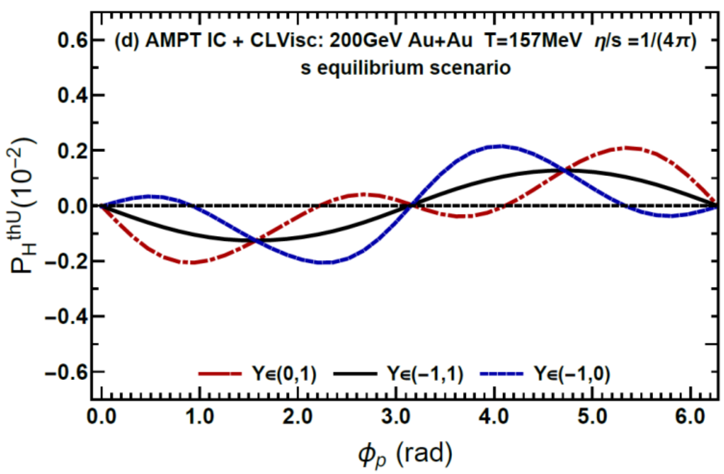


Helicity polarization from fluid vorticity

- Decomposition of the polarization from thermal vorticity:

$$S^h_{\mathrm{thT}}(\mathbf{p}) = \int d\Sigma^\sigma F_\sigma \frac{p_0}{T^2}\widehat{\mathbf{p}}\cdot(\mathbf{u}\times\nabla T), \qquad S^h_{\mathrm{thU}}(\mathbf{p}) = \int d\Sigma^\sigma F_\sigma \frac{p_0}{T}\widehat{\mathbf{p}}\cdot\boldsymbol{\omega}$$





- For low-energy collisions, fluid vorticity increases and the velocity decreases.

  ⇒ Helicity polarization from fluid-vorticity becomes more dominant

- Probing the strongest local fluid vorticity from helicity polarization with the beam energy scan?

Beyond the subatomic swirls

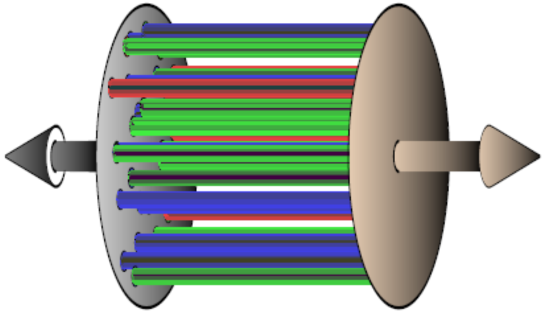
- Is the spin polarization of QGP only related to the fluid properties?

- The role of gluons : anomalous spin polarization from turbulent color fields in QGP

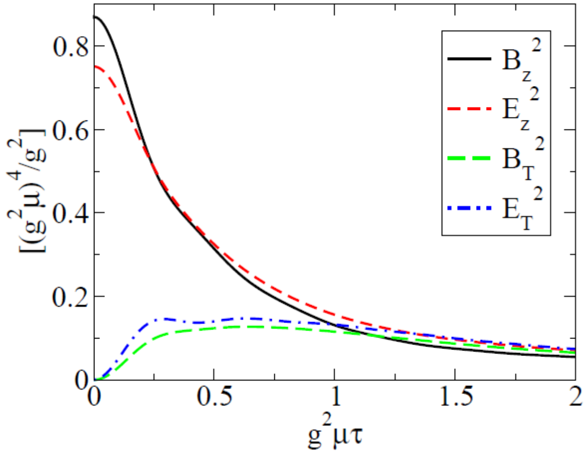
Berndt Müller, DY, PRD 105, L011901 (2022)
DY, (2021), 2112.14392

Chromo-electromagnetic fields in HIC

- Color flux tubes in the glasma phase : longitudinal chromo-EM fields in early times.



review: F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan, Ann.Rev.Nucl.Part.Sci.60:463-489,2010



T. Lappi, PLB 643 (2006) 11-16

- Plasma instability could enhance the color fields.

S. Mrowczynski, PLB 214, 587 (1988), PLB314,118 (1993)

P. Romatschke and M. Strickland, PRD 68, 036004 (2003)

- No local parity violation :

$$\langle B^a_\mu(X)B^a_\nu(X')\rangle \neq 0, \quad \langle E^a_\mu(X)E^a_\nu(X')\rangle \neq 0, \quad \langle B^a_\mu(X)E^a_\nu(X')\rangle = 0.$$

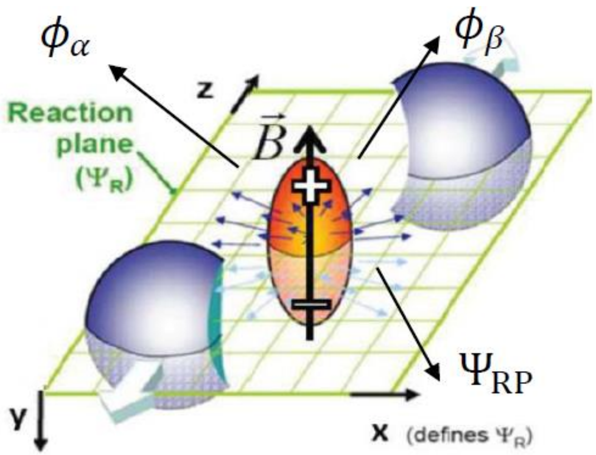
- Local-parity violation : $\langle B^a_\mu(X)E^a_\nu(X')\rangle \neq 0$

e.g. N. Tanji, N. Mueller, and J. Berges, PRD 93, 074507 (2016)

- Nonzero (parity-odd) correlators could lead to spin pol. of quarks although their forms in QGP are unknown.

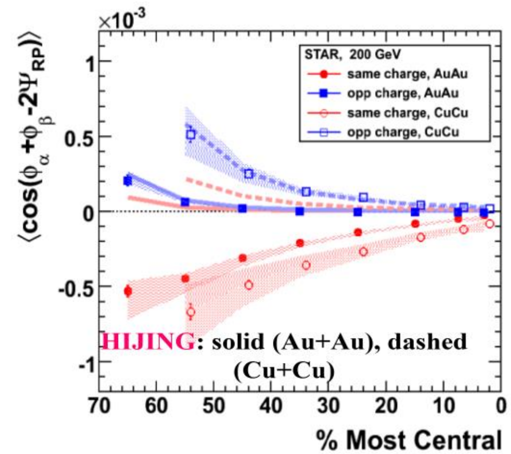
Probing local parity violation in QCD matter

- Probing the local parity violation via the chiral magnetic effect (CME) :



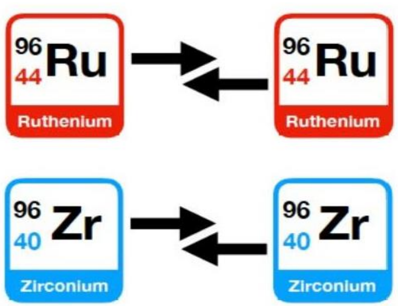
$$\mathbf{J}_V = \frac{\mu_5}{2\pi^2}\mathbf{B}$$

D. E. Kharzeev et al.,
Prog. Part. Nucl. Phys. 88, 1 (2016)



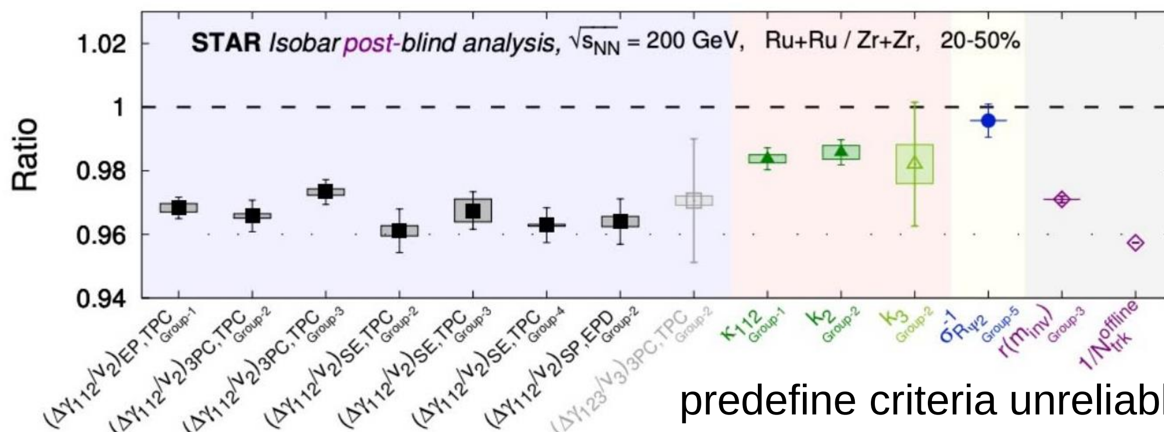
B. I. Abelev, et al (STAR), PRL 103 (2009) 251601

- Separating the signal from background : isobar collisions



(same shape=background?)

(diff. B fields=signal)



M. Abdallah et al. (STAR), PRC 105, 014901 (2022)

<1
no
CME?

predefine criteria unreliable :
different backgrounds!

Anomalous spin polarization & spin alignments of vector mesons

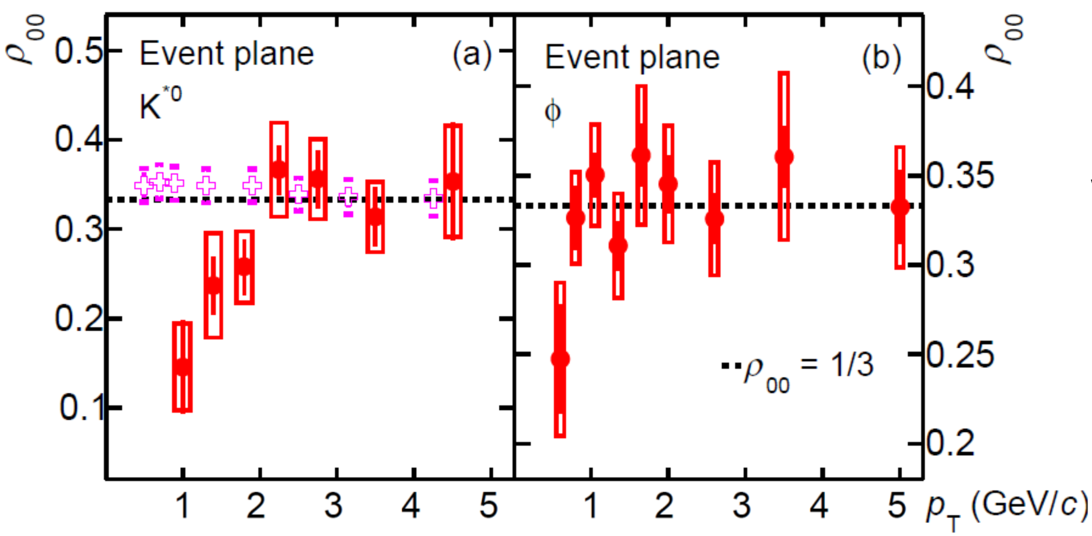
- From the QKT with background color fields : B. Müller, DY, PRD 105, L011901 (2022)

$$\mathcal{P}^{\mu}(\mathbf{p}) \approx \frac{-\pi^{3/2}\tau_c\beta\int d\Sigma\cdot p\big(\langle E^a\cdot B^a\rangle u^{\mu}+\langle B^{a\mu}E^{a\nu}\rangle p_{\nu}\beta(1-2f_{\rm eq}(p\cdot u))\big)f_{\rm eq}(p\cdot u)\big(1-f_{\rm eq}(p\cdot u)\big)}{4mp\cdot u\int d\Sigma\cdot p f_{\rm eq}(p\cdot u)}$$

- Decay daughter : $\frac{dN}{d\cos\theta^*} \propto [1-\rho_{00}+\cos^2\theta^*(3\rho_{00}-1)]$

$$\rho_{00}=\frac{1-\mathcal{P}^i_q\mathcal{P}^i_{\bar{q}}}{3+\mathcal{P}^i_q\mathcal{P}^i_{\bar{q}}}$$

Z.-T. Liang and X.-N. Wang, PLB 629, 20 (2005)



S. Acharya et al. (ALICE), PRL.125, 012301 (2020)

$\rho_{00} \neq 1/3$: spin polarization

What is the source for the large deviations ??

- Could spin alignments complement the CME study to probe the local parity violation?

The axial charge currents and Ward identity

- A constant axial charge current (finite τ_c):

$$J_5^\mu = 4N_c \int \frac{d^4p}{(2\pi)^4}\mathrm{sign}(p_0)\mathcal{A}^{s\mu}(p,X) = \boxed{-\frac{\hbar u^\mu}{8\pi^2}\sqrt{\pi}\tau_c\langle E^a\cdot B^a\rangle\mathcal{I}} \quad \mathcal{I}=1 \text{ for } m=0$$

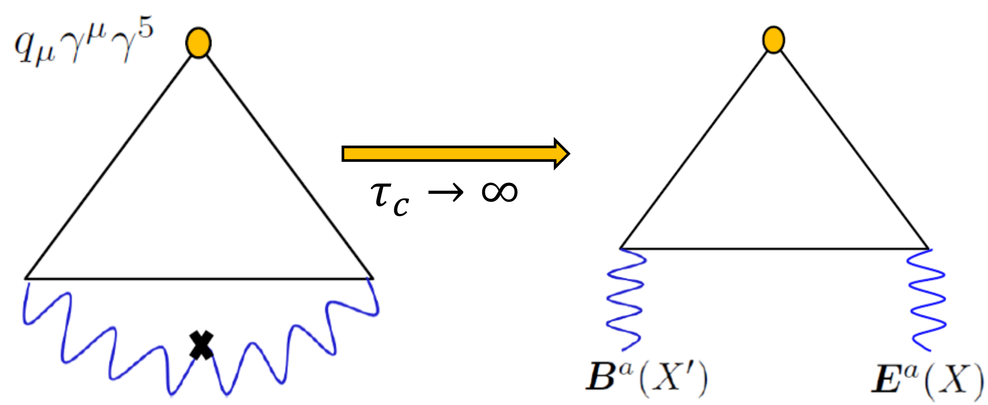
(similar to the steady state in Weyl semimetals $n_5 \sim \tau_R E\cdot B$)

- Vanishing axial-Ward identity : $\partial\cdot J_5 = 0$

- Constant-field limit ($\tau_c \to \infty$) : $\partial_\mu J_5^\mu(X) = -\hbar\frac{\langle B^a\cdot E^a\rangle}{4\pi^2} + 2m\langle\bar\psi i\gamma_5\psi\rangle,$

DY, (2021), 2112.14392

$$\langle\bar\psi i\gamma_5\psi\rangle = -\frac{\hbar\langle B^a\cdot E^a\rangle}{8m\pi^2}\int_0^\infty d|\boldsymbol{p}|\left(1-\frac{|\boldsymbol{p}|}{\epsilon_{\boldsymbol{p}}}\right)\frac{d}{d|\boldsymbol{p}|}\left[f_{\mathrm{eq}}(\epsilon_{\boldsymbol{p}})-f_{\mathrm{eq}}(-\epsilon_{\boldsymbol{p}})\right]$$



Summary

- ✓ Relativistic heavy ion collisions provides an opportunity to study the spin transport of the most ideal and vortical fluid.
- ✓ Although local-equilibrium corrections (in particular the shear) yield substantial contributions to local spin polarization, non-equilibrium effects should be involved.
- ✓ Helicity polarization may be a better probe for local vorticity.
- ✓ Not only the collisions, but also dynamically generated color fields may affect the spin polarization of quarks in QGP.
- ✓ Anomalous spin polarization could be triggered by parity-odd correlators of color fields, which may possibly explain the spin alignments in high-energy nuclear collisions.

Outlook

- ❖ Phenomenology :
- ➢ Investigating helicity polarization at low-energy collisions
- ➢ Local polarization with p_T dependence

Hydro. simulations : with Yi Cong, Shi Pu et al.

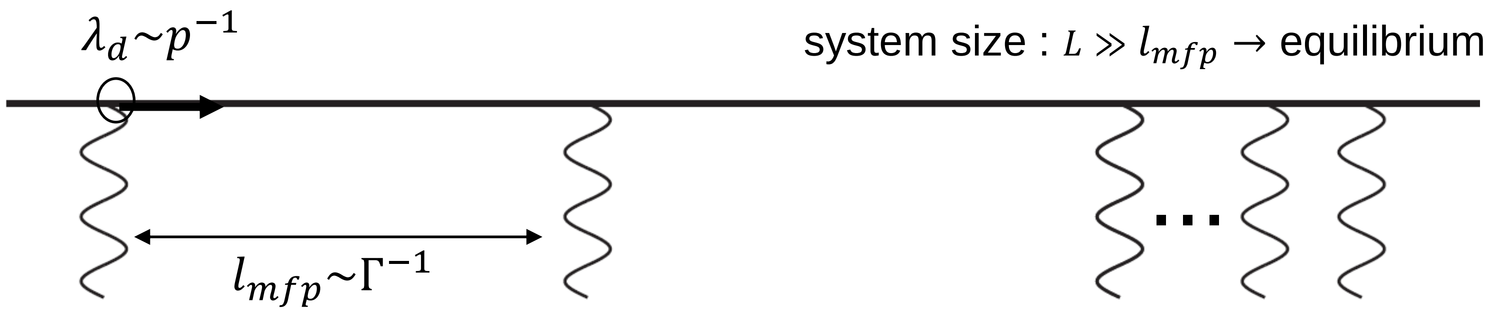
Thermal model : with Avdhesh Kumar

- ❖ Theory :
- ➢ Non-equilibrium corrections from QKT with Shuo Fang, Shi Pu
- ➢ More realistic estimations for spin alignments from color fields with Berndt Müller
- ➢ Scattering with polarized photons (gluons)
  with Koichi Hattori, Yoshimasa Hidaka, Naoki Yamamoto
- ➢ Transport and particle production with strong EM fields and vorticity (or strain)
  with Koichi Hattori, Patrick Copinger

Thank you!

Some properties of kinetic theory

- Kinetic theory : microscopic theory for quasi-particles in phase space
  - ❖ Boltzmann (Vlasov) Eq. : $q^\mu \Delta_\mu f(q,X) = q^\mu \mathcal{C}_\mu[f], \quad \Delta_\mu = \partial_\mu + F_{\nu\mu}\frac{\partial}{\partial q^\nu}.$
  - ❖ Physical quantities : $J^\mu(X) = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\frac{q^\mu}{E_q} f(q,X), \quad T^{\mu\nu}(X) = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\frac{q^\mu q^\nu}{E_q} f(q,X).$
  - ❖ Valid for weak coupling : mean free path ≫ de Broglie wavelength



  - ❖ Near equilibrium : kinetic theory ⇨ hydrodynamics
- Classical kinetic theory ⇨ $\partial_\mu J^\mu = 0, \quad \partial_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho.$
- CKT: $(q\cdot\Delta + \hbar\tilde{\Delta}) f_R = q\cdot C[f_R] + \hbar\tilde{C}[f_R]$ ⇨ $\partial_\mu J^\mu_R = -\frac{\hbar}{4\pi^2} E\cdot B$

(for right-handed fermions) (chiral anomaly)

CKT with collisions

- WF up to $\mathcal{O}(\hbar)$: (for right-handed fermions)

quantum corrections

$$\grave{S}^{<}(q,X)=\bar{\sigma}_{\mu}2\pi\bar{\epsilon}(q\cdot n)\left(q^{\mu}\delta(q^2)f_q^{(n)}+\boxed{\hbar\delta(q^2)S^{\mu\nu}_{(n)}\boxed{\mathcal{D}_{\nu}}f_q^{(n)}+\hbar\epsilon^{\mu\nu\alpha\beta}q_{\nu}F_{\alpha\beta}\frac{\partial\delta(q^2)}{2\partial q^2}f_q^{(n)}}\right)$$

$$\mathcal{D}_{\beta}f_q^{(n)}=\Delta_{\beta}f_q^{(n)}-\mathcal{C}_{\beta}$$

- CKT with collisions $(\partial_{\rho}n^{\mu}=0)$:

Y. Hidaka, S. Pu, DY, PRD 95, 091901 (2017), PRD 97, 016004 (2018)

$$\boxed{\delta\left(q^2-\hbar\frac{B\cdot q}{q\cdot n}\right)}\left\{\left[q\cdot\Delta+\boxed{\hbar\frac{S^{\mu\nu}_{(n)}E_{\mu}}{(q\cdot n)}\Delta_{\nu}+\hbar S^{\mu\nu}_{(n)}(\partial_{\mu}F_{\rho\nu})\partial^{\rho}_{q}}\right]f_q^{(n)}-\tilde{\mathcal{C}}\right\}=0,$$

magnetic-moment coupling

spin tensor : $S^{\mu\nu}_{(n)}=\dfrac{\epsilon^{\mu\nu\alpha\beta}}{2(q\cdot n)}q_{\alpha}n_{\beta}$

($F^{\mu\nu}=0$: the quantum corrections only appear in collisions)

- Quantum corrections on the collision term :

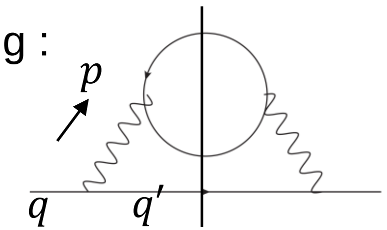
$$\tilde{\mathcal{C}}=q\cdot\mathcal{C}+\hbar\frac{S^{\mu\nu}_{(n)}E_{\mu}}{(q\cdot n)}\mathcal{C}_{\nu}+\hbar S^{\alpha\beta}_{(n)}\boxed{\left((1-f_q^{(n)})\Delta_{\alpha}\Sigma^{<}_{\beta}-f_q^{(n)}\Delta_{\alpha}\Sigma^{>}_{\beta}\right)},$$

induced by inhomogeneity of the medium

$$\mathcal{C}_{\beta}=\boxed{\Sigma^{<}_{\beta}}(1-f_q^{(n)})-\boxed{\Sigma^{>}_{\beta}}f_q^{(n)}.$$

also include hbar corrections

2-2 scattering :



Applications : Chiral fluids

- Anomalous hydrodynamics (R-handed) : $\partial_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho, \quad \partial_\mu J^\mu = \frac{\hbar}{4\pi^2}(\mathbf{E}\cdot\mathbf{B})$
- Constitutive equations :

$$T^{\mu\nu} = \underbrace{u^\mu u^\nu \epsilon - p\Theta^{\mu\nu}}_{\mathcal{O}(1)} + \underbrace{\Pi^{\mu\nu}_{\rm non}}_{\mathcal{O}(\hbar)} + \underbrace{\Pi^{\mu\nu}_{\rm dis}}_{\mathcal{O}(1)+\mathcal{O}(\hbar)}, \quad J^\mu = \underbrace{N_0 u^\mu}_{\mathcal{O}(1)} + \underbrace{\nu^\mu_{\rm non}}_{\mathcal{O}(\hbar)} + \underbrace{\nu^\mu_{\rm dis}}_{\mathcal{O}(1)+\boxed{\mathcal{O}(\hbar)}}, \quad \Theta^{\mu\nu} = \eta^{\mu\nu} - u^\mu u^\nu$$

Son & Surowka, 09

equilibrium : CME/CVE

our focus

- In local equilibrium : (in the co-moving frame $n^\mu = u^\mu$)

$$f_q^{\mathrm{eq}(u)} = \left(\exp\left[\beta(q\cdot u - \mu) + \boxed{\frac{\hbar q\cdot\omega}{2q\cdot u}}\right] + 1\right)^{-1}, \quad \omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu(\partial_\alpha u_\beta).$$

spin-vorticity coupling

J.-Y. Chen, et.al. 15

Hidaka, Pu, DY, 17

- Equilibrium anomalous transport :

$$\nu^\mu_{\rm non} = \hbar\sigma_B B^\mu + \hbar\sigma_\omega\omega^\mu \qquad \Pi^{\mu\nu}_{\rm non} = \hbar\xi_\omega\left(\omega^\mu u^\nu + \omega^\nu u^\mu\right) + \hbar\xi_B\left(B^\mu u^\nu + B^\nu u^\mu\right)$$

$$\sigma_\omega = \frac{T^2}{12}\left(1 + \frac{3\bar\mu^2}{\pi^2}\right), \quad \sigma_B = \frac{\mu}{4\pi^2}, \quad \xi_\omega = \frac{T^3}{6}\left(\bar\mu + \frac{\bar\mu^3}{\pi^2}\right), \quad \xi_B = \frac{T^2}{24}\left(1 + \frac{3\bar\mu^2}{\pi^2}\right).$$

CVE CME Hidaka, Pu, DY, 17

(agree with different approaches, e.g. Son & Surowka, 09. K. Landsteiner, et.al. Lect. Notes, 13.)

Axial kinetic theory

- QKT for massive fermions :
- Wigner functions : $S^{<}(p,X)=\int d^4Y e^{ip\cdot Y/\hbar}\langle\bar{\psi}(y)U(y,x)\psi(x)\rangle$

  vector/axial-vector components : $\mathcal{V}^{\mu}(p,X)=\frac{1}{4}\mathrm{tr}\left(\gamma^{\mu}S^{<}(p,X)\right),\ \mathcal{A}^{\mu}(p,X)=\frac{1}{4}\mathrm{tr}\left(\gamma^{\mu}\gamma^{5}S^{<}(p,X)\right)$

- Dynamical variables in $\mathcal{V}^{\mu}/\mathcal{A}^{\mu}$: $f_{V/A}(q,X)$ & $\boxed{a^{\mu}(q,X)}$ spin 4-vector $(\tilde{a}^{\mu}=a^{\mu}f_A)$

  K. Hattori, Y. Hidaka, D.-L. Y, PRD100, 096011 (2019)

  $\xrightarrow{m=0}\ a^{\mu}=q^{\mu},\ f_V=(f_R+f_L)/2,\ f_A=f_R-f_L.$

- Axial kinetic theory : scalar/axial-vector kinetic eqs. (SKE/AKE)

  with quantum corrections

- Collisions for AKE : $\boxed{\square^{(n)}\mathcal{A}^{\mu}}=\boxed{\hat{\mathcal{C}}^{\mu}_{\mathrm{cl}}}+\boxed{\hbar\hat{\mathcal{C}}^{(n)\mu}_{\mathrm{Q}}}$

  D.-L. Y, K. Hattori, Y. Hidaka, JHEP 20, 070 (2020)

  spin diffusion $\propto L^{\mu\nu}\tilde{a}_{\nu}$

  spin polarization coupled to vector charge $\propto H^{\mu\nu}\partial_{\nu}f_V$

- E.g. diffusion for massive quarks in weakly coupled QGP : S. Li, H.-U. Yee, PRD100, 056022 (2019)
- The quantum correction is only studied for purely fermionic interactions.

  Z. Wang, X. Guo, P. Zhuang, Eur. Phys. J. C 81, 799 (2021)
  N. Weickgenannt, et al., PRL 127, 052301 (2021)

  ⇨ global-equilibrium sol. is reproduced from detailed balance (local equilibrium?)
- The role of gluons and color dof. for spin polarization is unknown.

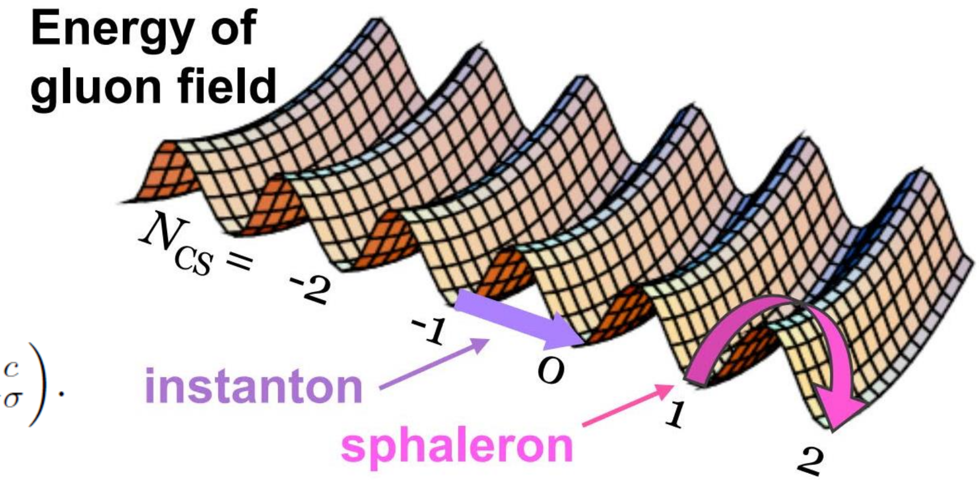
Probing local parity violation in QCD matter

- Transition between topological sectors in QCD vacuum yields parity violation and chirality production.

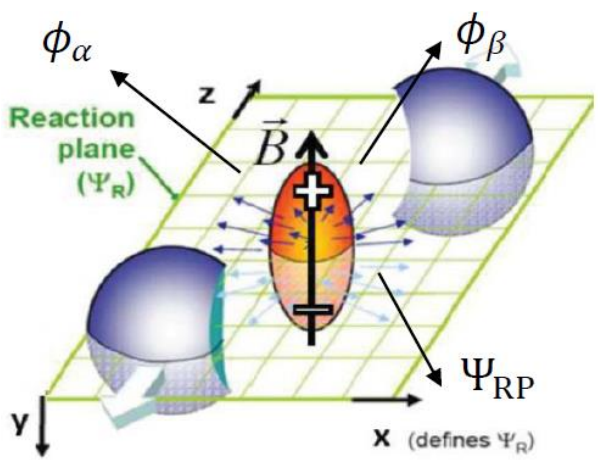
$$\partial_t(N_L - N_R) = 2g^2\partial_t N_{\mathrm{CS}},$$

$$N_{\mathrm{CS}} \equiv \int d^3x K_0,$$

$$K^\mu = \frac{\epsilon^{\mu\nu\rho\sigma}}{16\pi^2}\left(A^a_\nu \partial_\rho A^a_\sigma + \frac{1}{3} f^{abc} A^a_\nu A^b_\rho A^c_\sigma\right).$$



- Probing the local parity violation via the chiral magnetic effect (CME) :

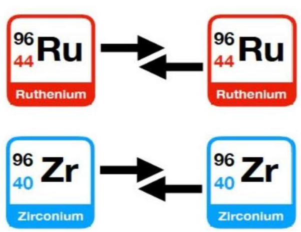


$$\mathbf{J}_V = \frac{\mu_5}{2\pi^2}\mathbf{B}$$

D. E. Kharzeev et al.,
Prog. Part. Nucl. Phys. 88, 1 (2016)

Isobar collisions : so far, negative

STAR, M. Abdallah et al., (2021), 2109.00131



(observed in Weyl semimetals)

Qiang Li, et.al., Nature Phys. 12 (2016) 550-554

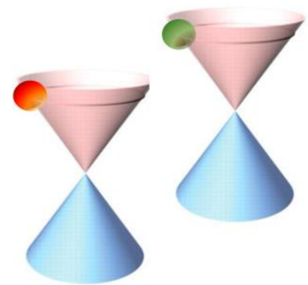
CME in Weyl Semimetals

- Chiral matter in the condensed matter system.

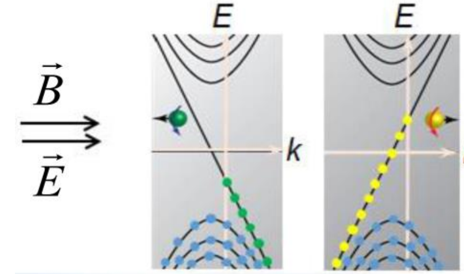
Weyl semimetals :

$$\partial_\mu J_5^\mu = \frac{\mathbf{E}\cdot\mathbf{B}}{2\pi^2} - \frac{n_5}{\tau_R}$$

relaxation time



TaAs
NbAs
NbP
TaP



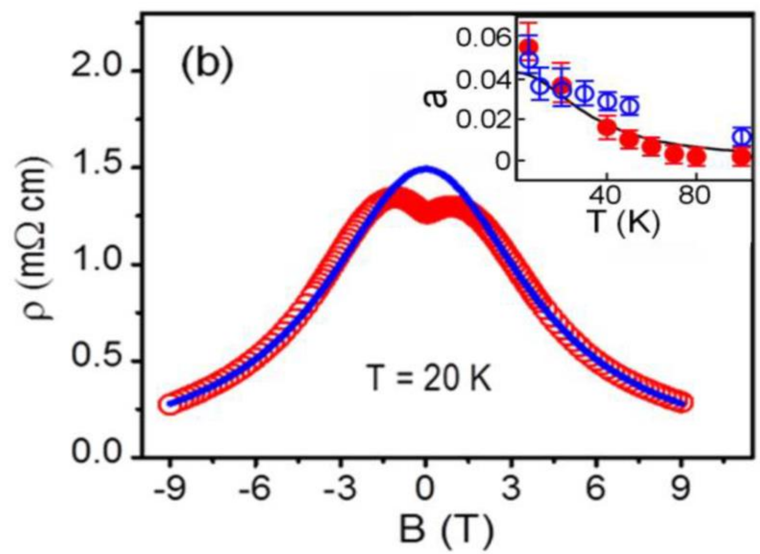
charge pumping via parallel E & B : generate $\mu_5 \sim n_5 \sim E \cdot B$

steady-state approxtion ($\mu_5 \sim \tau_R \boldsymbol{E}\cdot\boldsymbol{B}$):

$$\mathbf{J}_V = \frac{\mu_5}{2\pi^2}\mathbf{B} = \frac{3v_f^3\tau_R|\mathbf{B}|^2}{2(\pi^2T^2+4\mu_V^2)}\mathbf{E}$$

$$\rho = \frac{1}{\sigma} \sim \frac{1}{B^2}$$

"negative magnetoresistance"
(the signal of CME)



Qiang Li, et.al., Nature Phys. 12 (2016) 550-554

WFs and AKE with source terms

- Incorporation of background color fields into WFs and kinetic theory.
- Color decomposition : $O = \boxed{O^s} I + O^a t^a$

U. W. Heinz, Phys. Rev. Lett. 51, 351 (1983)

-> physical observable

e.g. $J_5^\mu = 4\int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}_c\,\mathcal{A}^\mu(p,X)$

$$O^s : \mathcal{O}(g^0),\ O^a : \mathcal{O}(g).$$

- SKE, AKE, WFs are decomposed into color-singlet & octet components.
- Perturbatively, we may rewrite $f_V^a,\ \tilde{a}^{a\mu}$ in terms of $f_V^s,\ \tilde{a}^{s\mu}$.
- Modified Cooper-Frye formula :

$$\mathcal{P}^\mu(\mathbf{p}) = \frac{\int d\Sigma\cdot p\mathcal{J}_5^\mu(p,X)}{2m\int d\Sigma\cdot\mathcal{N}(p,X)}$$

$$\mathcal{N}^\mu(p,X) = 4N_c(p^\mu f_V^s),$$
$$\mathcal{J}_5^\mu(p,X) = 4N_c\big(\tilde{a}^{s\mu} + \hbar\bar{C}_2\mathcal{A}_Q^\mu\big),$$

- Source term in WFs : $\mathcal{A}_Q^\mu = \frac{\partial_{p\kappa}}{2}\int_{k,X'}^{p} p^\beta\langle\tilde{F}^{a\mu\kappa}(X)F^a_{\alpha\beta}(X')\rangle\partial_p^\alpha f_V^s(p,X')$

B. Müller and D.-L. Y, (2021), 2110.15630

M. Asakawa, S. A. Bass, B. Muller, PRL. 96, 252301 (2006)

- SKE & AKE :

$$0 = p\cdot\partial f_V^s(p,X) - \boxed{\partial_p^\kappa\mathscr{D}_\kappa[f_V^s]}$$

$\sim g^2 >$ collosions$\sim g^4$: anomalous viscosity

$$0 = p\cdot\partial\tilde{a}^{s\mu}(p,X) - \underbrace{\boxed{\partial_p^\kappa\mathscr{D}_\kappa[\tilde{a}^{s\mu}]}}_{\text{diffusion}} + \underbrace{\boxed{\hbar\partial_p^\kappa\big(\mathscr{A}_\kappa^\mu[f_V^s]\big)}}_{\text{source}}$$

Spin polarization from color fields

- The real color- field correlators can only be obtained from real-time simulations.
- Physical assumptions : $\langle F^a_{\kappa\lambda}(X)F^a_{\alpha\rho}(X')\rangle = \langle F^a_{\kappa\lambda}F^a_{\alpha\rho}\rangle e^{-(t-t')^2/\tau_c^2}$

$$|\langle B^a_\mu B^a_\nu\rangle| \gg |\langle E^a_\mu B^a_\nu\rangle| \gg |\langle E^a_\mu E^a_\nu\rangle|$$

- f_V reaches thermal equilibrium : $\mathcal{J}_5^\mu(p,X) = 4N_c\big(\tilde{a}^{s\mu} + \hbar\bar{C}_2\mathcal{A}^\mu_Q\big),$

$$\tilde{a}^\mu(t,p) = -\frac{\hbar\bar{C}_2(t-t_0)}{2p_0^2}\big(\partial_{p0}f_{\rm eq}(p_0)\big)\Big(\boxed{\langle B^{a\mu}E^{a\nu}\rangle}p_\nu - \boxed{\langle B^a\cdot E^a\rangle}p_\perp^\mu\Big)$$

$$(\mathcal{A}^\mu_Q)_{\rm eq} = \frac{\pi^{3/2}\tau_c}{2p_0}\delta(p^2-m^2)\Big(\boxed{\langle E^a\cdot B^a\rangle}u^\mu - \boxed{\langle B^{a\mu}E^{a\beta}\rangle}p_\beta\partial_{p0}\Big)\partial_{p0}f_{\rm eq}(p_0)$$

- Origin of the source terms: correlation of the Lorentz force & anomalous force from quantum corrections