Recent Developments on Parton Distributions on a Lattice

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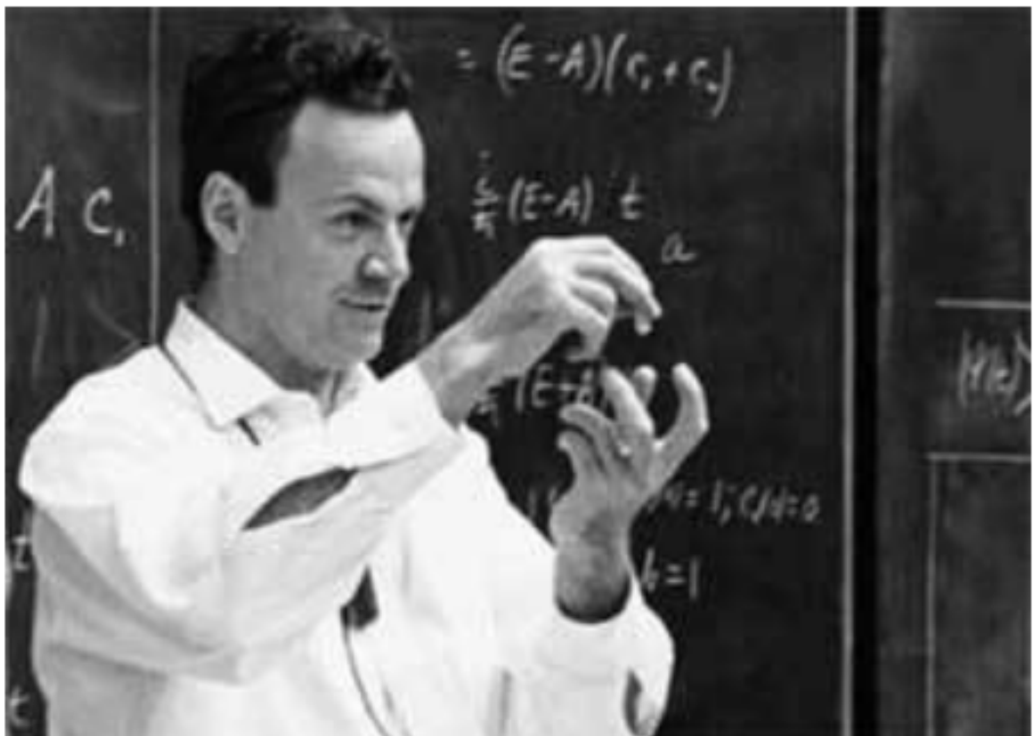
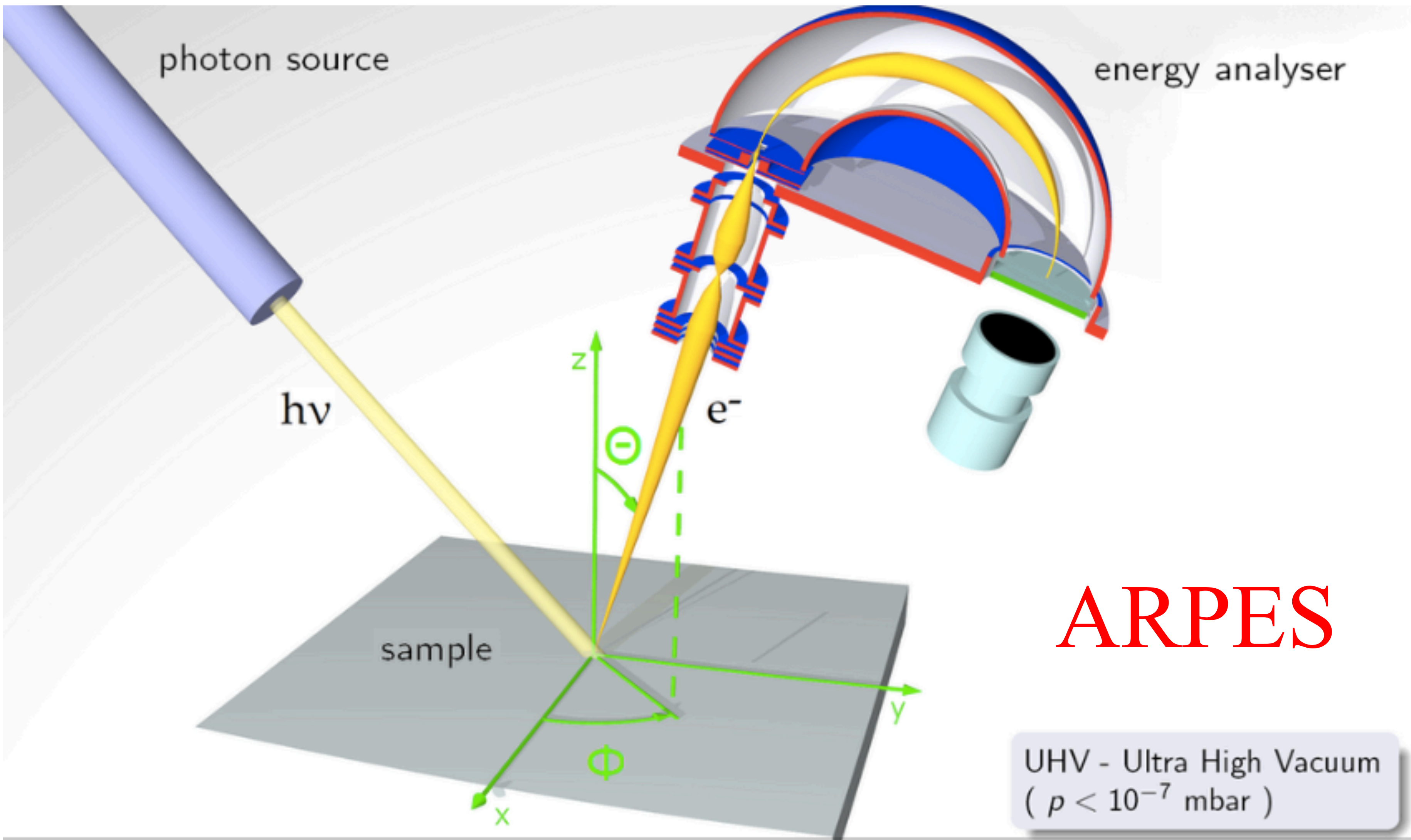
One Sentence to Be Passed on to the Next Generation



All things are made by atoms.

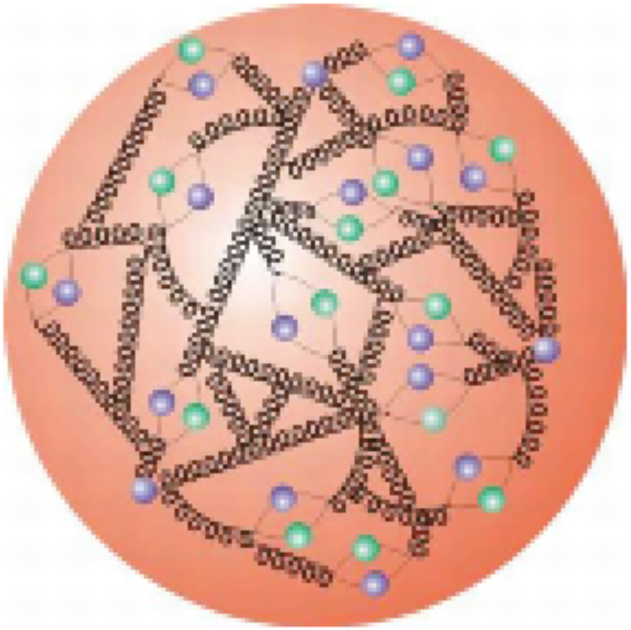
Structures, reactions, applications



ARPES

The Saga Continues

Proton is made by partons (quarks and gluons)



Structures:1d mom+spin PDF to 3d GPD & TMD to Wigner (and beyond?) [BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …] to applications (Higgs, new physics…)

Can we determine these distributions theoretically?

An Ultimate Question in Science

An Ultimate Question in Science

Life = known Physical Laws?

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Life = known Physical Laws?

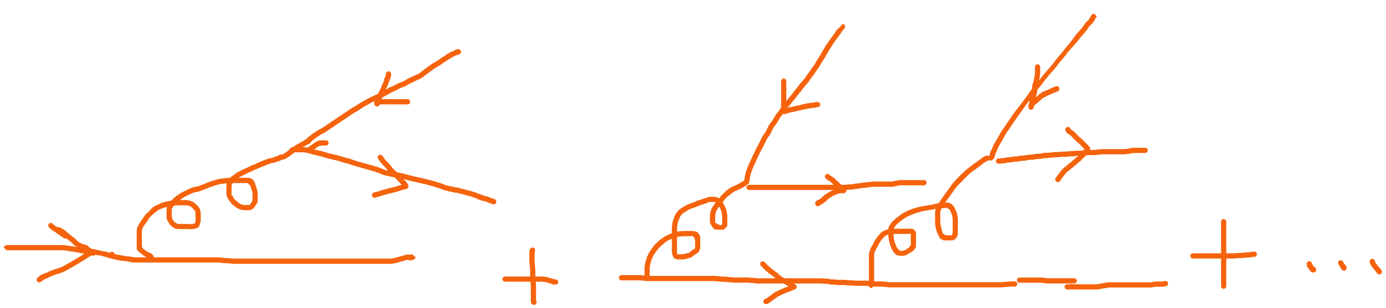
A computational problem!

Why is it so hard?

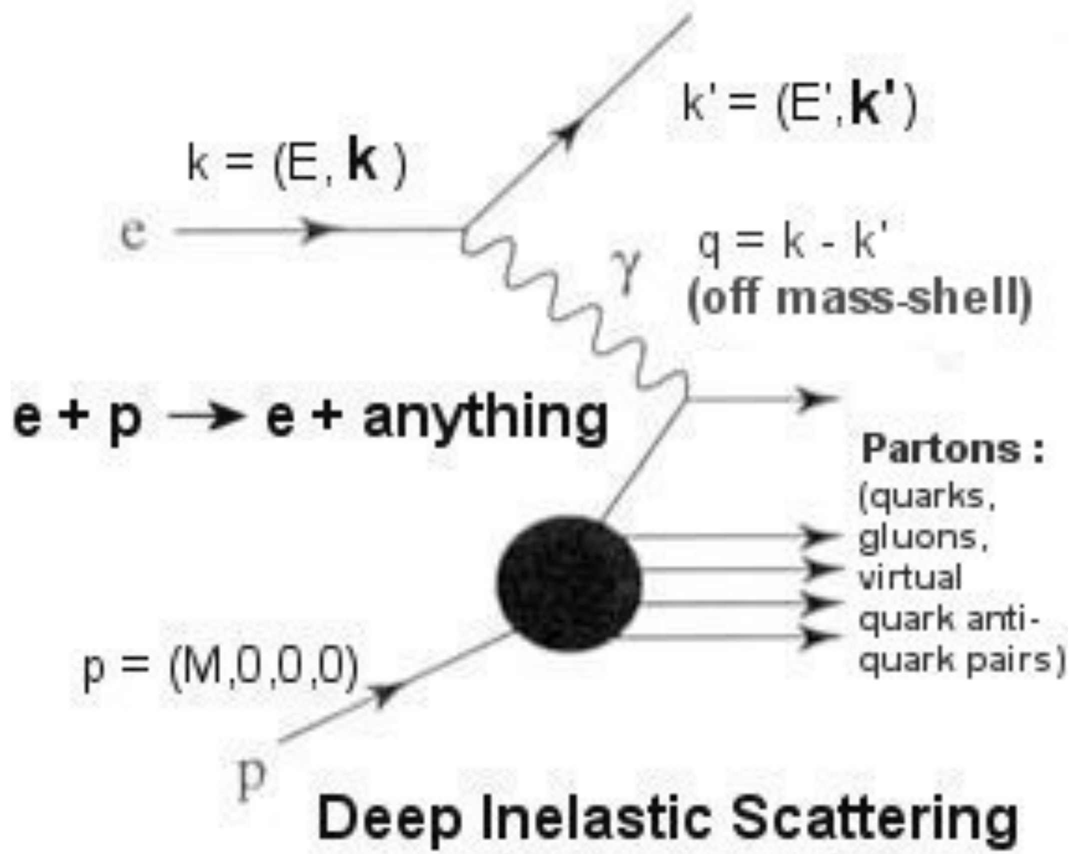
Proton PDFs from QCD

- The number of quark anti-quark pairs diverges (manifestation of non-perturbative nature of the problem): an infinite body problem!
- Lattice QCD
- Euclidean lattice: light cone operators cannot be distinguished from local operators

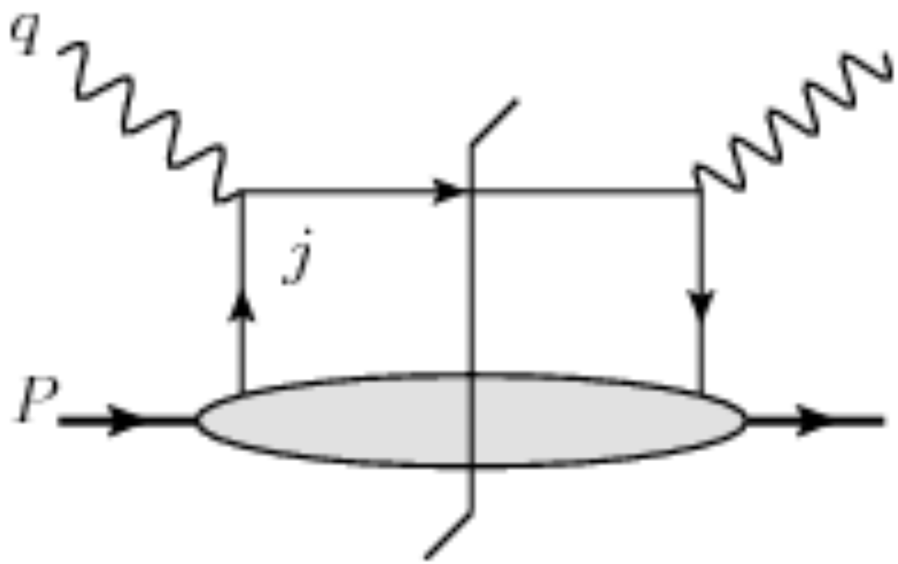
$$t^2 - \mathbf{r}^2 = 0$$
$$-t_E^2 - \mathbf{r}^2 = 0$$



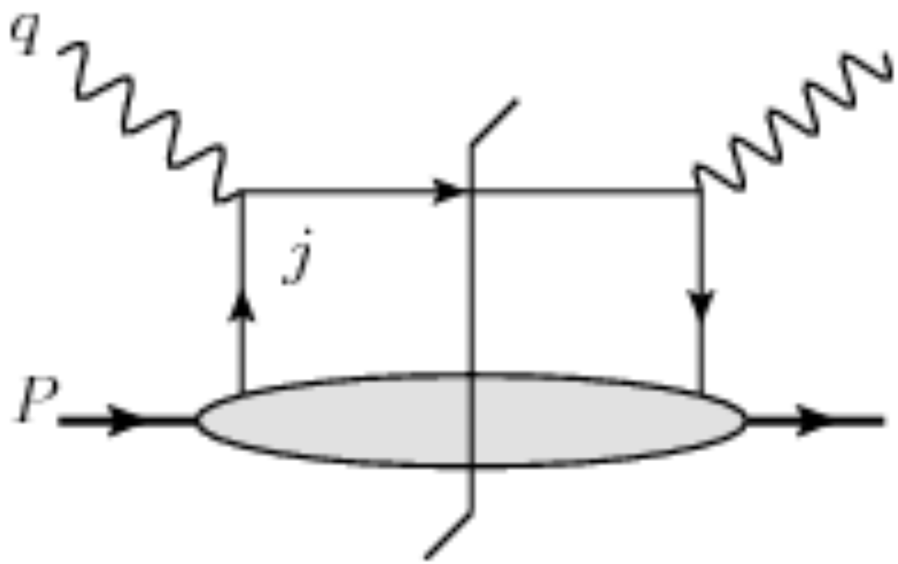
Measuring Parton Distributions Using DIS experiments



Parton Distribution Function (PDF) in QCD



Parton Distribution Function (PDF) in QCD



The struck parton moves on a light cone at the leading order in the twist-expansion.

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda) \right| P \rangle$$

Why is it so hard?

Proton PDFs from QCD

- Euclidean lattice: light cone operators cannot be distinguished from local operators
- Moments of PDF given by local twist-2 operators (twist = dim - spin); limited to first few moments but carried out successfully

$$\langle x^n \rangle$$

Beyond the first few moments

- Smeared sources: Davoudi & Savage
- Gradient flow: Monahan & Orginos
- Current-current correlators: K.-F. Liu & S.-J. Dong; Braun & Müller; Detmold & Lin; QCDSF; Qiu & Ma
- Xiangdong Ji (Phys. Rev. Lett. 110 (2013) 262002): quasi-PDF: computing the x-dependence directly. (variation: pseudo-PDF, Radyushkin; w/ Karpie, Orginos, Zafeiropoulos)

Ji's idea

- Quark PDF in a proton: $(\lambda^2 = 0)$

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P \left| \overline{\psi}(0) \lambda \cdot \gamma \Gamma \psi(\xi^- \lambda) \right| P \rangle$$

- Boost invariant in the z-direction, rest frame OK
- Quark bilinear op. always on the light cone
- What if the quark bilinear is slightly away from the light cone (space-like) in the proton rest frame?

- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.

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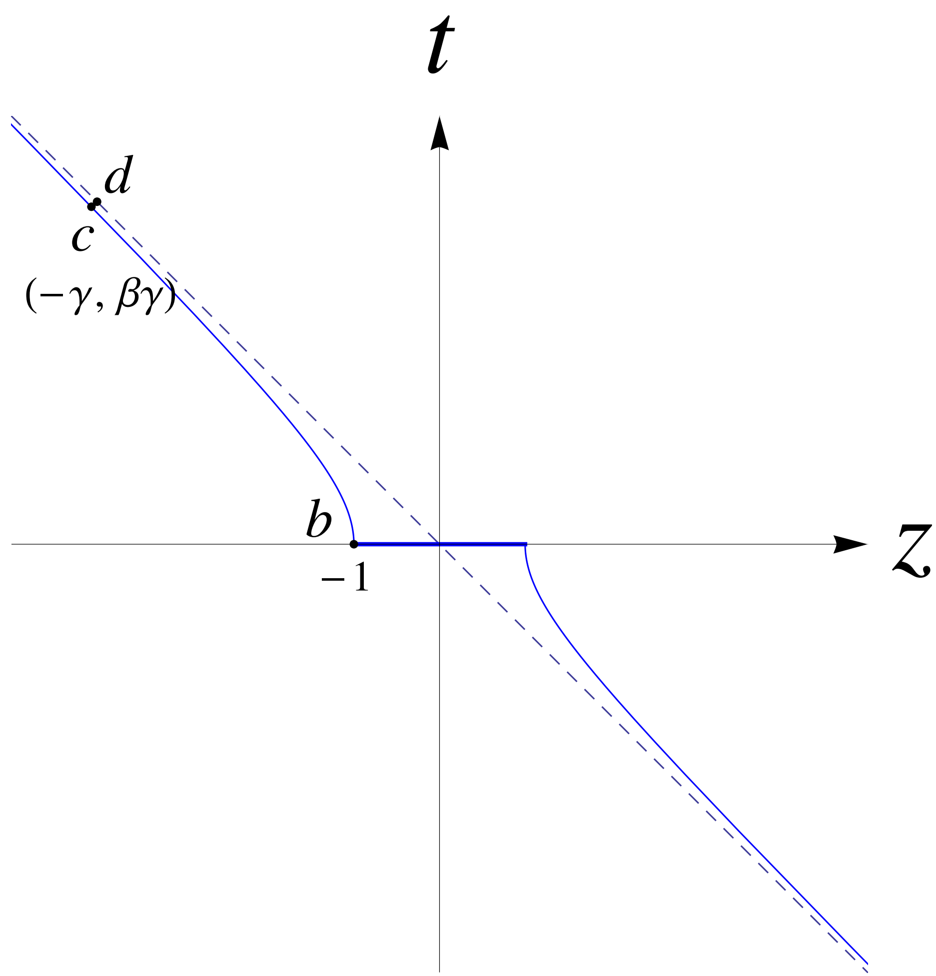

- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.
- Analogous to HQET: need power corrections & matching---LaMET (Large Momentum Effective Theory)

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- Analogous to HQET: need power corrections & matching---LaMET (Large Momentum Effective Theory)

$$\tilde{q}(x, \Lambda, P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{P_z^2}, \frac{M^2}{P_z^2}\right) + \ldots .$$

Matching

$$\tilde{q}(x, \Lambda, P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{P_z^2}, \frac{M^2}{P_z^2}\right) + \ldots .$$

Xiong, Ji, Zhang, Zhao (GPD: Ji, Schafer, Xiong, Zhang; Xiong, Zhang) Factorization (Ma, Qiu; Li; OPE: Izubuchi, Ji, Jin, Stewart, Zhao), Linear divergence, LPT (Ishikawa, Ma, Qiu, Yoshida; JWC, Ji, Zhang; Xiong, Luu, Meissner; Rossi, Testa; Constantinou et al.) Multiplictive Renormalizability (Ji, Zhang, Zhao; Ishikawa, Ma, Qiu, Yoshida; Green, Jansen, Steffens; Zhang, Ji, Schäfer, Wang, Zhao; Li, Ma, Qiu), RI (Monahan & Orginos; Yong & Stewart; Constantinou et al.; LP3), NPR(Constantinou et al.; LP3), E vs. M spaces (Carlson et al.; Briceno et al.), Renormalon (Braun, Vladimirov, Zhang), Modeling (Xing et al.,…),...

Lattice Setup (isovector proton PDF)

- Lattice: $64^3 \times 96$

  $$a = 0.09 \text{ fm} \qquad L \approx 5.8 \text{ fm}$$

- Fermions: MILC highly improved staggered quarks (HISQ) Clover (valence)

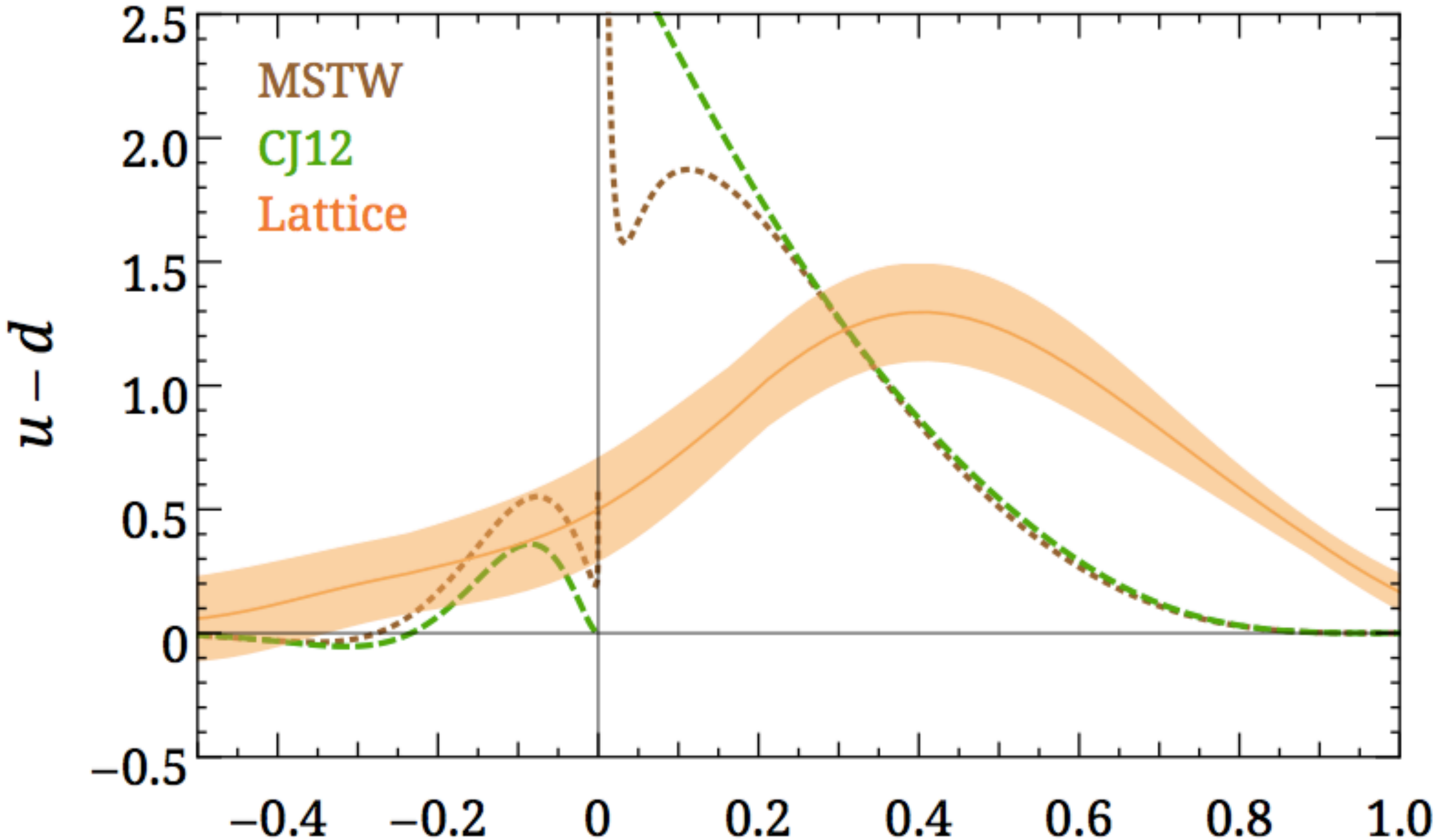
  $$N_f = 2 + 1 + 1 \qquad M_\pi \approx 135 \text{ MeV}$$

- Gauge fields/links: hypercubic (HYP) smearing (one step), 884 config.

- $P^z = n\frac{2\pi}{L}$ = 2.2, 2.4, 3.0 GeV (n = 10,12,14)

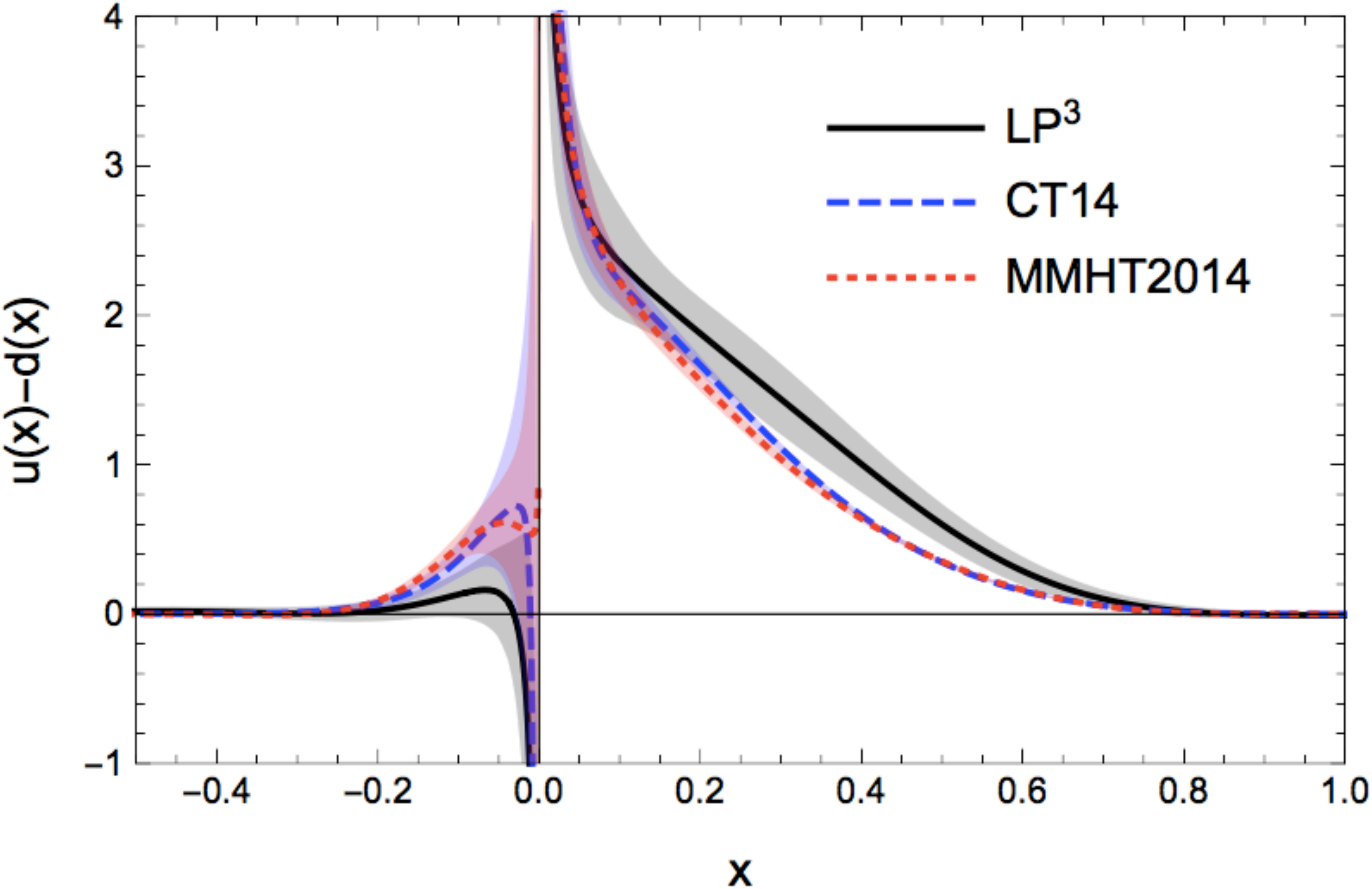
(hıgn momentum smearing: Bali, Lang, Musch, Schafer; smaller energy gap)

LP3 (1402.1462)

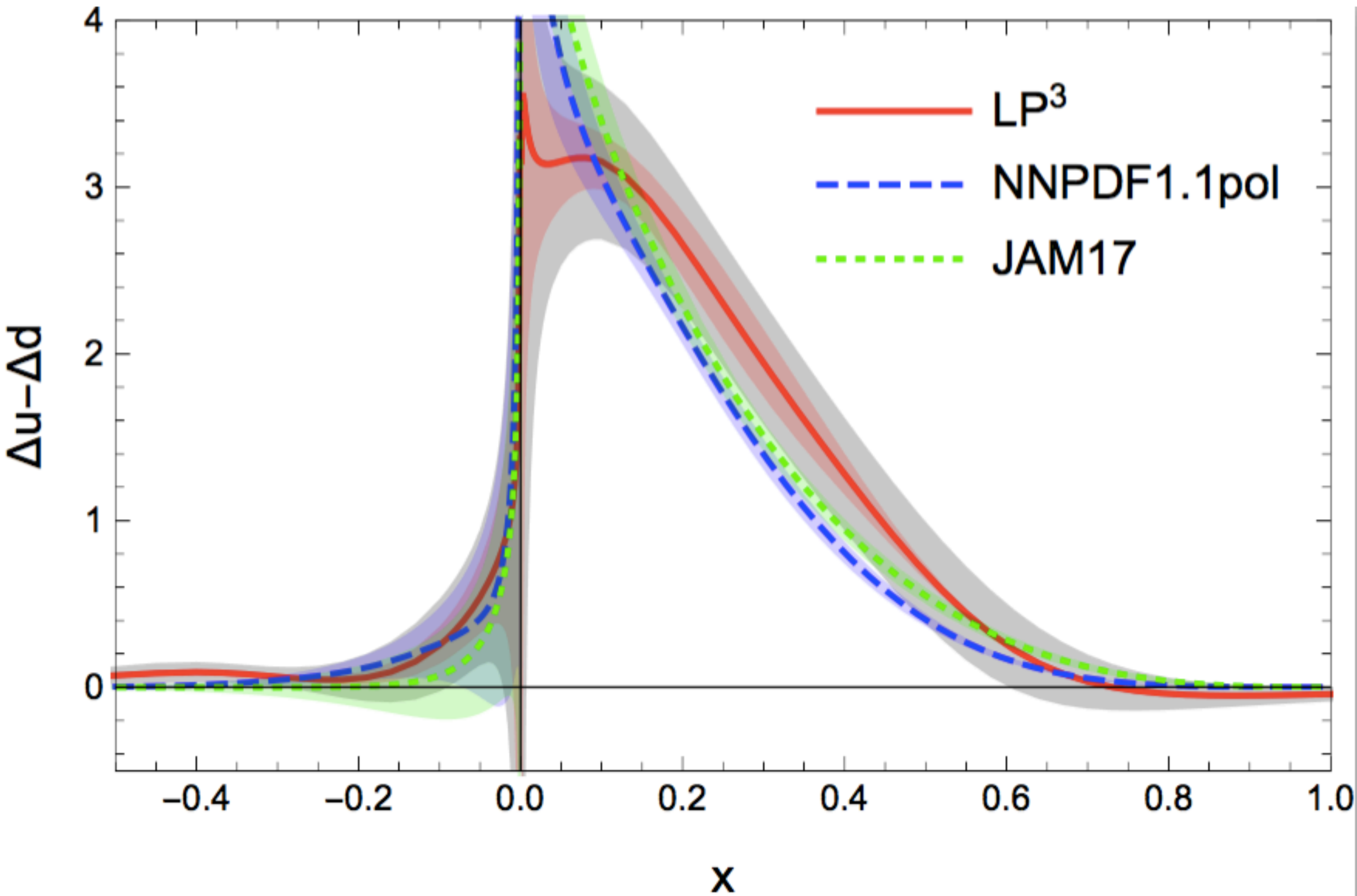


$q(-x) = -\bar{q}(x)$ x

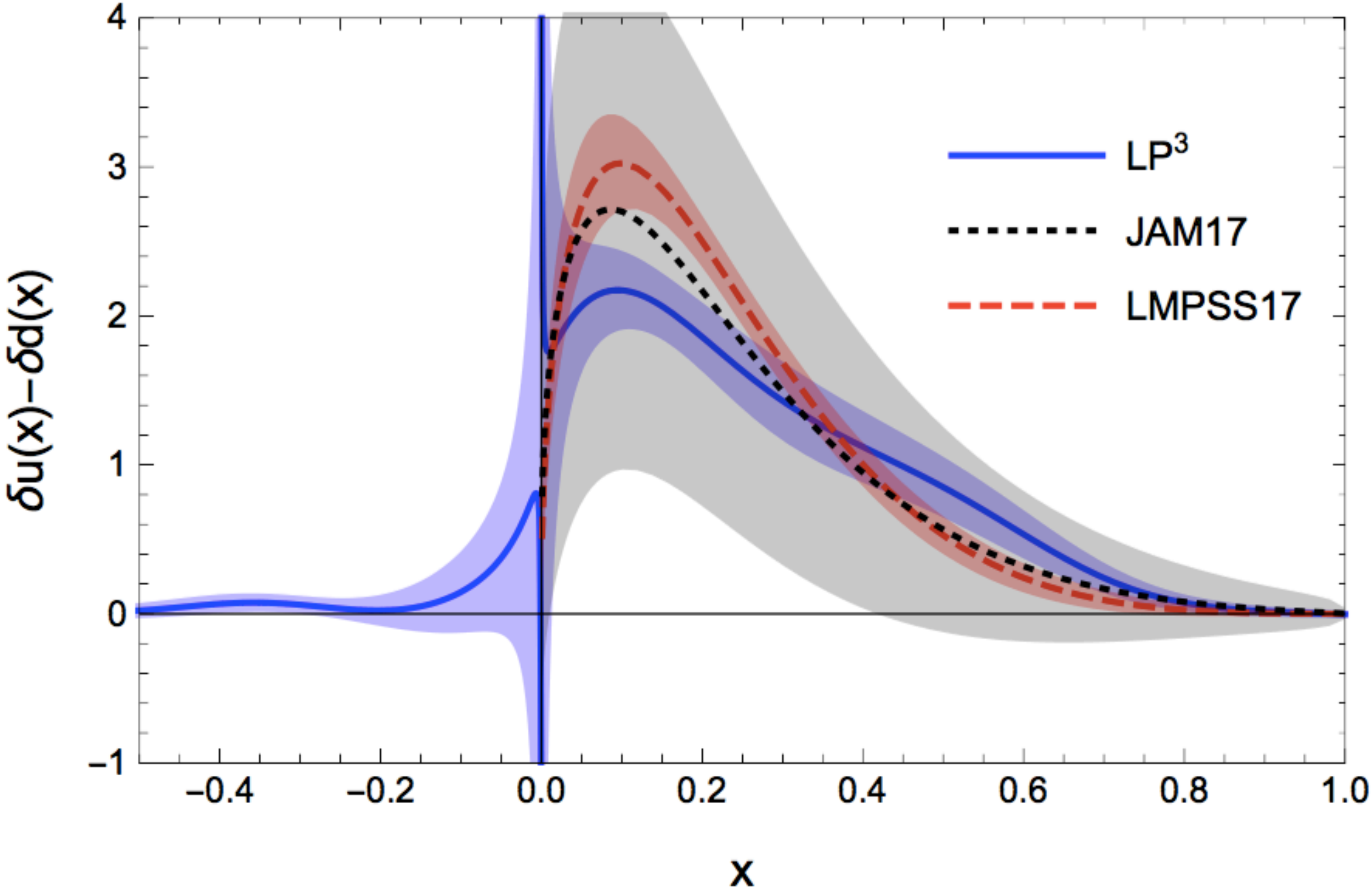
LP3 (1803.04393 v2)



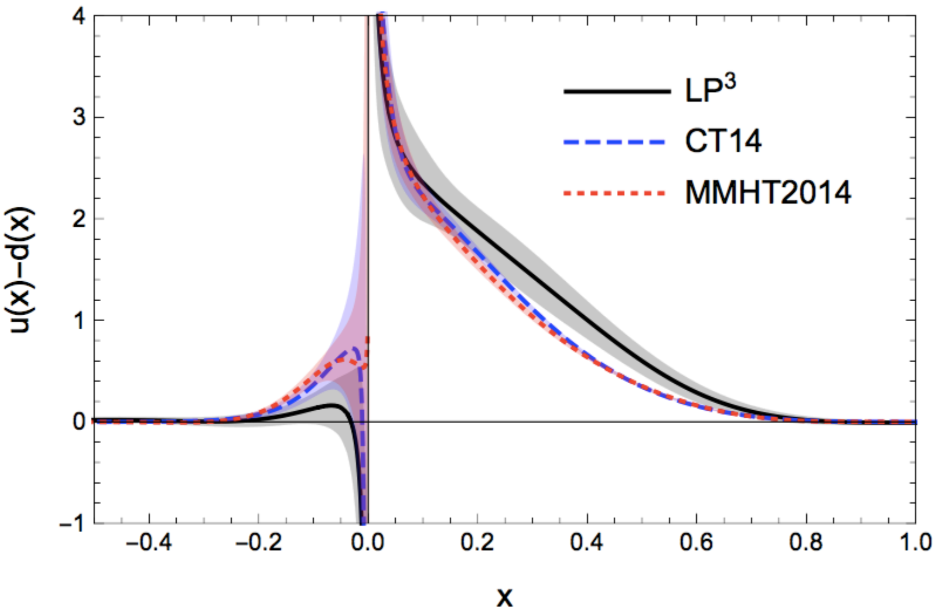
LP3 (1807.07431,PRL)

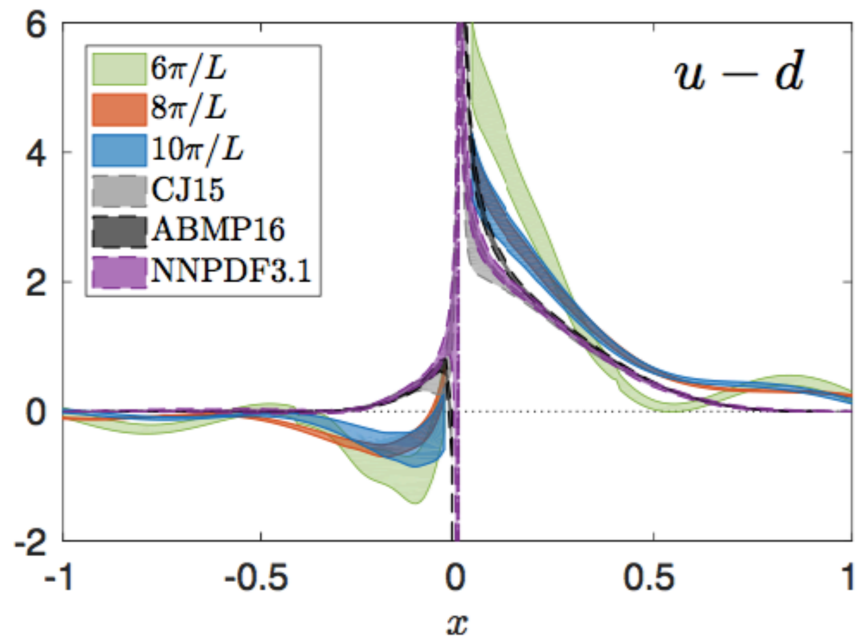


LP3 (1810.05043)



Compared with ETMC

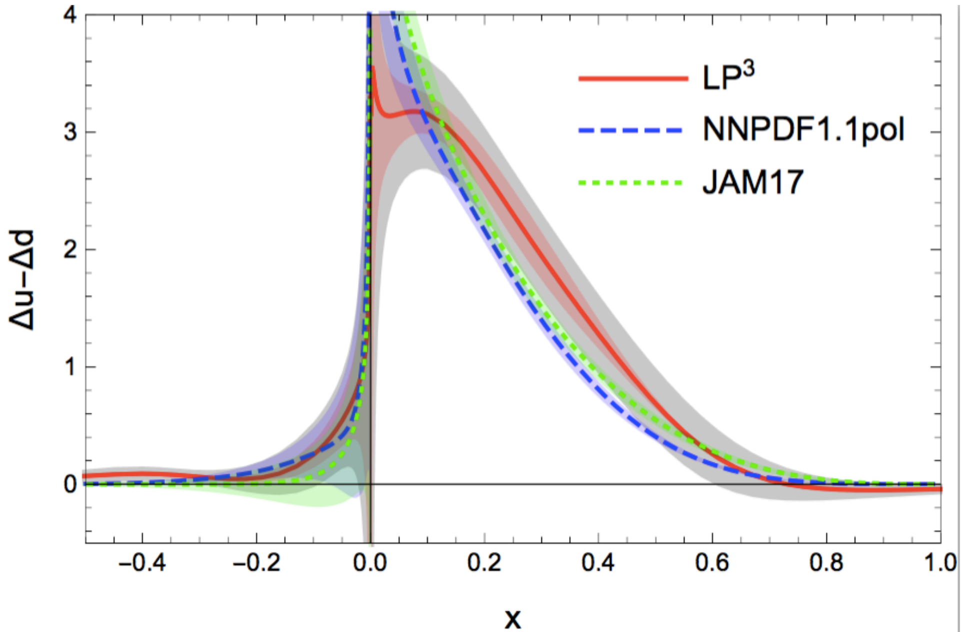


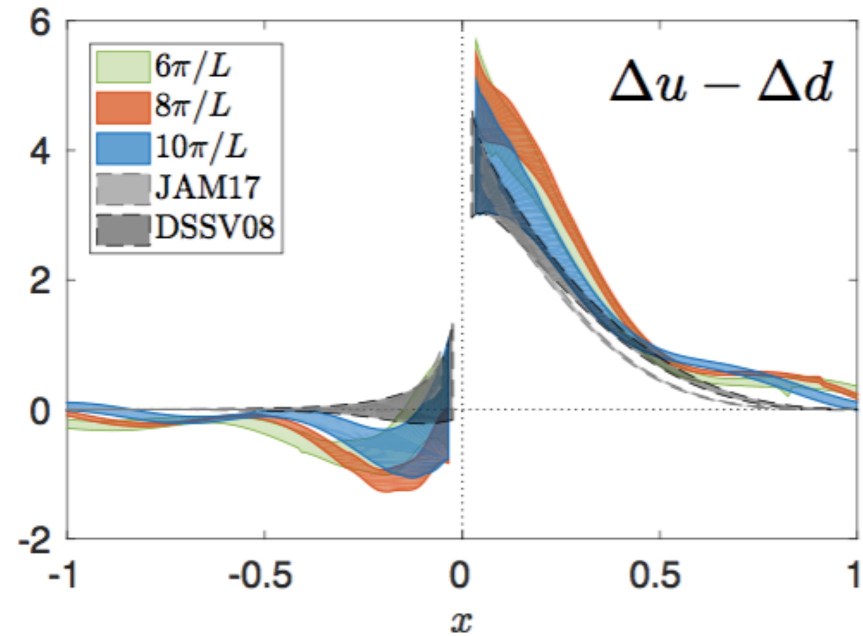


LP3(1803.04393)

ETMC(1803.02685)

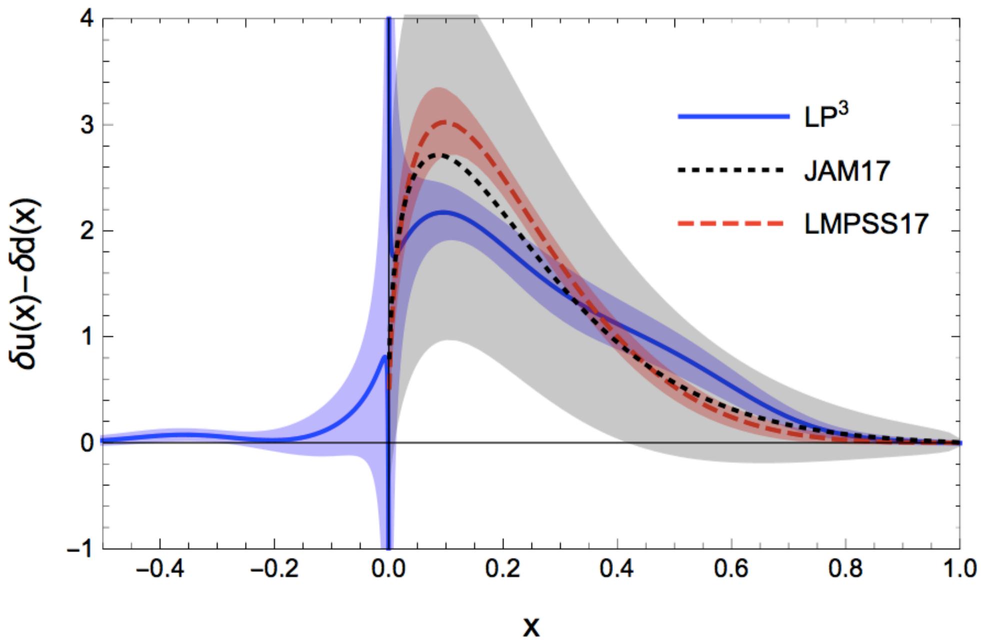
Compared with ETMC

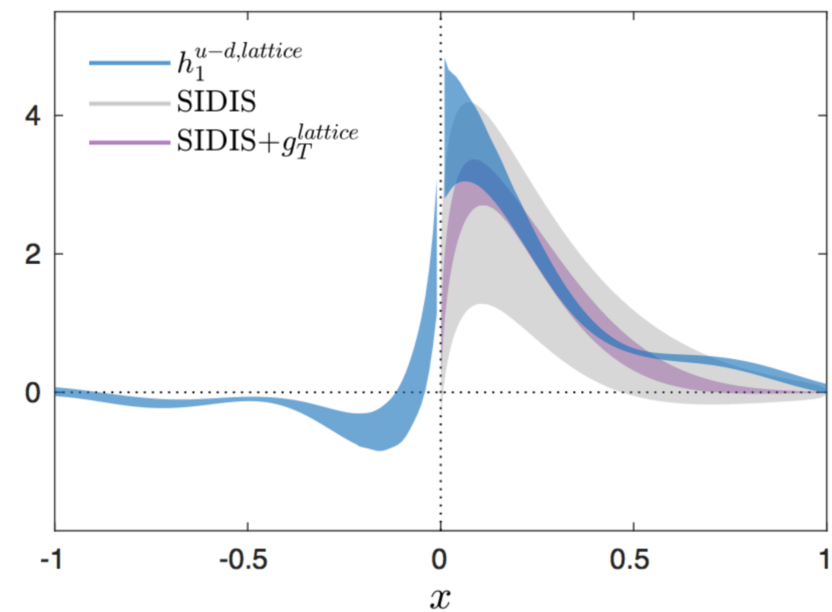




LP3(1807.07431,PRL) ETMC(1803.02685,PRL)

Compared with ETMC

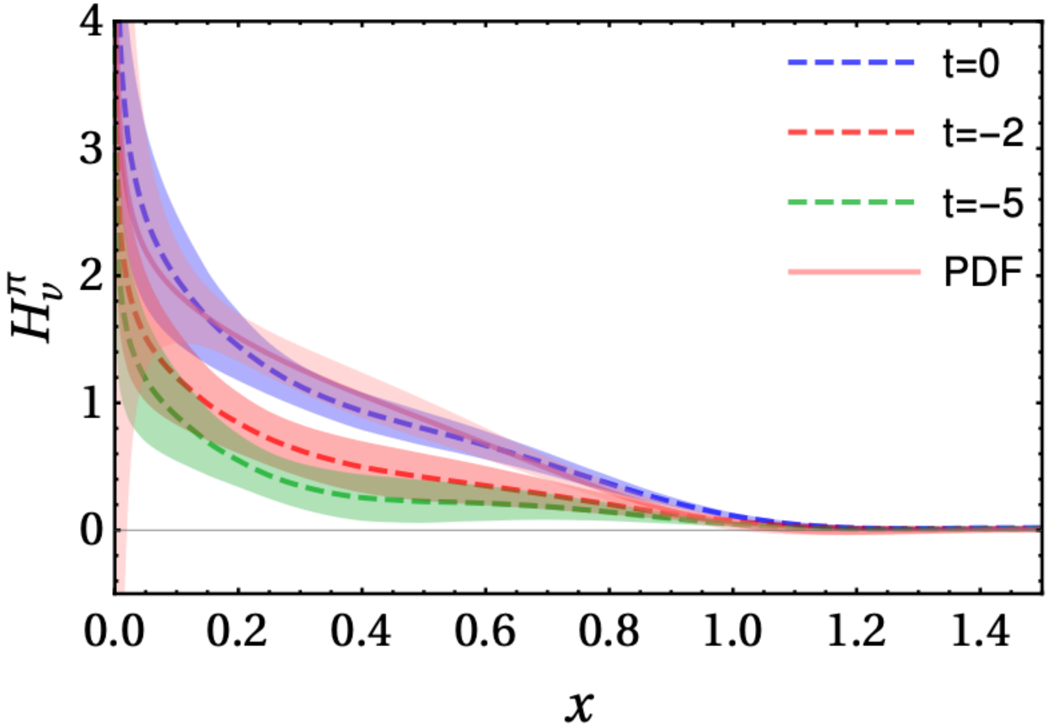




LP3 (1810.05043)

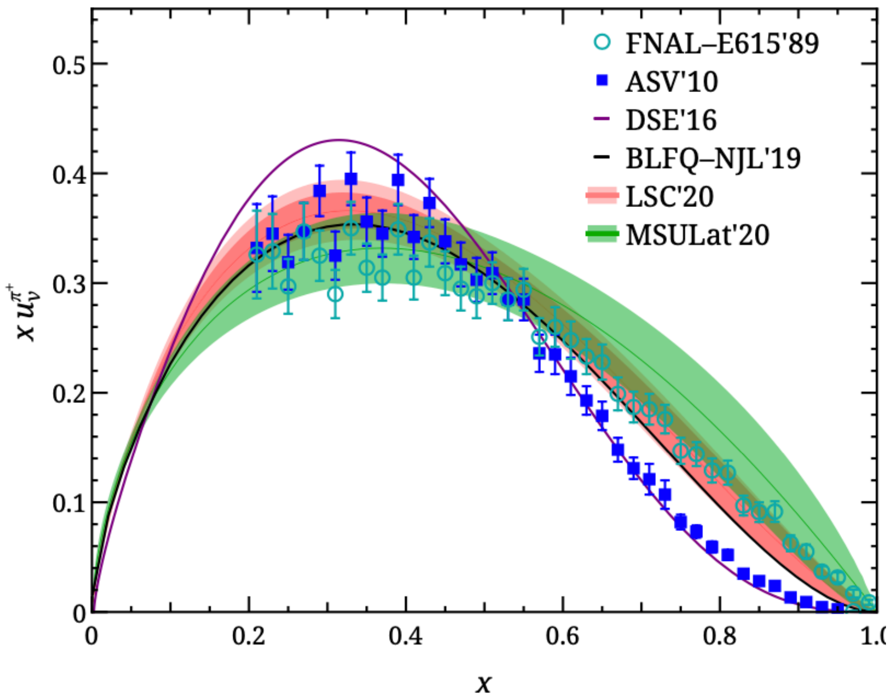
ETMC(1803.02685)

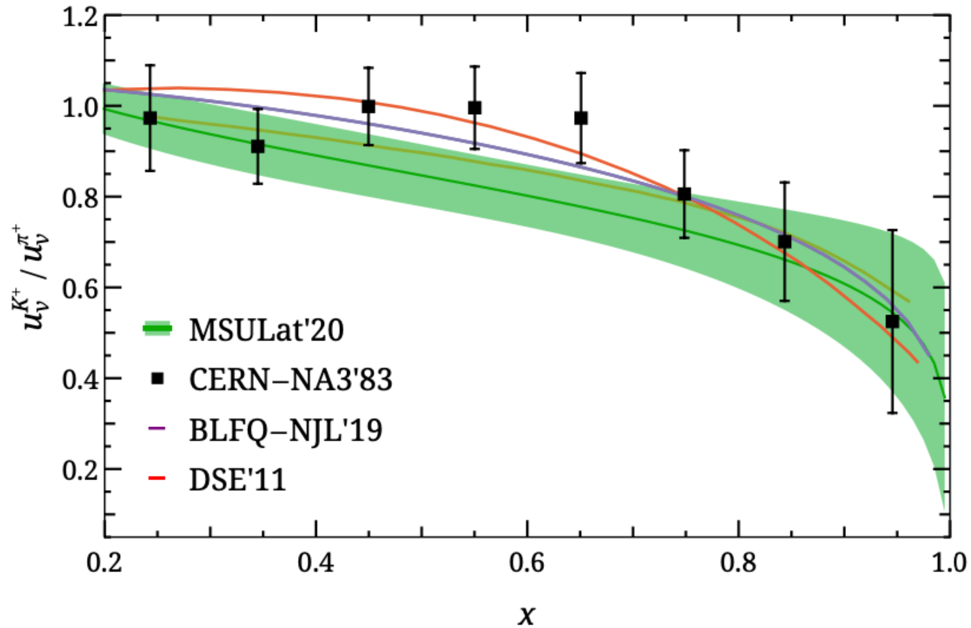
Generalized Parton Distributions



JWC, HW Lin, JH Zhang (1904.12376)

Meson Valence Quark Distributions





HW Lin, JWC, Z Fan, JH Zhang, R Zhang (2003.14128)

More Systematics Studies

- We need

$$\frac{\pi}{a} \gg P_z \gg \frac{1}{z_{max}} \gg \Lambda_{QCD}, m_\pi \gg \frac{\pi}{L}$$

  Now we have

  $$6.8 > 3 >> 0.15 \sim 0.2, 0.14 > 0.1 \text{ (GeV)}$$

- Finite volume effect: ChPT (w/ Wei-Yang Liu)
- long tail and lattice spacing: Hybrid or self-renormalization (LPC)
- Renormalon: (w/ Wei-Yang Liu)

Outlook

- Rapid progress made since 2013
- Further error study (non-singlet)

  Know whether it works within 3 years (~20%)?
- Singlet PDF's: s, c, b and gluons

  Additional 3-5 yrs?
- If it works, complimentary to exp.: PDF (sea asymmetry, small and large x's, non-valence partons), DA, GPD, TMD, Wigner distributions …

Backup slides

First (isovector) LPDF Computation

- Lattice: $24^3 \times 64$

  $a \approx 0.12 \ \text{fm} \quad L \approx 3 \ \text{fm}$
- Fermions: MILC highly improved staggered quarks (HISQ) Clover (valence)

  $N_f = 2+1+1 \qquad M_\pi \approx 310 \ \text{MeV}$
- Gauge fields/links: hypercubic (HYP) smearing, 461 config.

- $P^z = \frac{2\pi}{L} n = n \times 0.43 \ GeV$ n = 1,2,3…

Review: Ji's LPDF (LaMET)

$$\begin{aligned}\widetilde{q}(x,\mu^2,P^z) &= \int \frac{dz}{4\pi} e^{-ixzP^z} \left\langle P \left| \overline{\psi}(0) \lambda\cdot\gamma\Gamma\psi(z\lambda) \right| P \right\rangle \\ &\equiv \int \frac{dz}{2\pi} e^{-ixzP^z} h(zP^z) P^z\end{aligned}$$

$$\lambda^\mu = (0,0,0,1)$$

- Taylor expansion yields

$$\overline{\psi}\lambda\cdot\gamma\Gamma\left(\lambda\cdot D\right)^n\psi = \lambda_{\mu_1}\lambda_{\mu_2}\cdots\lambda_{\mu_n}O^{\mu_1\cdots\mu_n}$$

op. symmetric but not traceless

$$\left(\lambda_{\mu_1}\lambda_{\mu_2} - g_{\mu_1\mu_2}\lambda^2/4\right)$$

Review: Ji's LPDF (LaMET)

$$\left\langle P \left| O^{(\mu_1 \cdots \mu_n)} \right| P \right\rangle = 2a_n P^{(\mu_1} \cdots P^{\mu_n)}$$

- LHS: trace, twist-4 $\mathcal{O}(\Lambda^2_{QCD}/(P^z)^2)$ corrections, parametrized in this work
- RHS: trace $\mathcal{O}(M^2/(P^z)^2)$
- One loop matching $\alpha_s \ln P^z$, OPE

$$\tilde{q}(x, \Lambda, P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2}, \frac{M^2}{P_z^2}\right) + \ldots .$$

Non-Perturbative Renormalization + Matching

$$\tilde{q}(x,\Lambda,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{P_z^2},\frac{M^2}{P_z^2}\right) + \ldots$$

- NPR (RI/MOM scheme), γ_t $\quad p^2 = -\mu_R^2$
  Landau gauge $\quad p_z = p_z^R$
- RI/MOM to $\overline{\mathrm{MS}}$ performed at one loop

Sensitivity c p_z^R

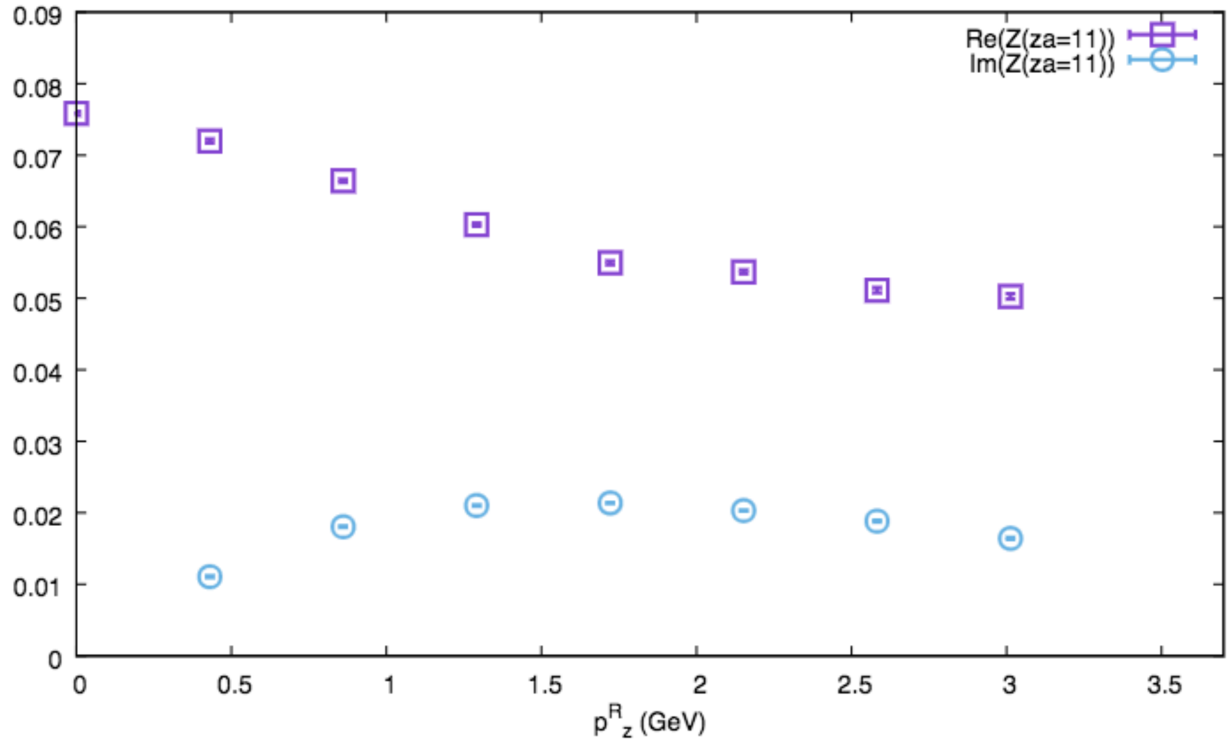


FIG. 1. The values of $Z(z)$ (the inverse of the renormalization constant) at $z = 11a \approx 1.0$ fm as a function of p_z^R. Note that $Z(z)$ becomes stable at large p_z^R.

insensitive to $\mu_R = 2.3$ and 3.7 GeV

Rossi & Testa's criticism

(1706.04428,1806.00808)

- Criticism: The twist-4 effect is $\mathcal{O}(1/(aP_z)^2)$ from dimensional analysis instead of $\mathcal{O}(\Lambda_{QCD}^2/P_z^2)$
- This can be avoided by renormalizing the quark bilinear operators non-perturbatively such that one can go to continuum limit where the lattice spacing dependence disappears.
- The matching formula should be between the renormalized quasi-PDF and PDF, not between bare quasi-PDF and PDF as in earlier versions.

Advantages of RI/MOM

- RI/MOM: Quasi-PDF is renormalized non-perturbatively by performing an off-shell subtraction. Continuum limit can be taken afterwards to recover rotation symmetry, s.t.
- (1) power divergent mixing to lower moments removed
- (2) power divergent mixing with higher twist (same dim. different spin) also removed (Rossi and Testa problem)