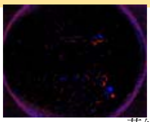
沙石如何“流動” ---- 何謂“流動”?

Rate-dep. transitions from Quicksand, Landslide, to Hourglass



JC Tsai (蔡日強)

with his current research team

蔡承恩(NCU), 范姜泓杰(NTHU), 黃冠皓

and numerous contributing members

黃緗淳, 黃少榆, 黃柏嘉, 周明叡, 費祥霆 ....etc. in the past

Inst. of Physics, Academia Sinica, Taipei, TAIWAN

Special thanks to

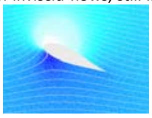
X. Cheng, J. Dijksman, 梁鈞泰, 陳志強, 黃仲仁, 杜其永, 施宏燕

-- AS-IoP Special Colloquium, 2020/12-

Fluid Mechanics (continuum) vs. Granular flow (particles)

- **LAWS of MOTION** in Partial Diff. Eqn.:
  - Euler's equation (est. 1757)
  - ----- for *inviscid* flows, still in use for air foils

(*)



  - Navier-Stokes equation (est. 1845)
  - ---- as complete theory, with *viscosity* included
- **CONSTITUENT RELATIONS**
  - --- Newtonian vs. Non-Newtonian “rheology”(est. 1920+)
  - --- stress determined *locally* by strain rate

- BOTH **in debates** till now
  - -----limited to special cases
  - -----concerns on “*non-locality*”

?

relationship to

Solid Mechanics

*) animation by Thierry Dugnolle, CC0, https://commons.wikimedia.org/w/index.php?curid=18803279

Scenarios of granular flows

Smooth, “rapid”	*Intermittent*	*Smooth*, “slow”
Quicksand	Debris flows / Earthquakes	Hourglass

1962 film《Larence in Arabia》演員 John Dimech 陷在流沙

a block of houses

南投縣集集鎮, 武昌宮

1999/ 09/21 01:47

epi-center

台北市東星大樓 (八德路四段與虎林街口)

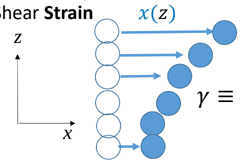
200+ km away (Taipei)

Source: 維基百科

Description of “flows” --- common languages (1)

as opposed to pure translation / rotation

- **Shear Strain**



$\gamma \equiv \frac{\Delta x(z)}{\Delta z}$

Strain rate (Shear rate)

$$\dot{\gamma} \equiv \frac{\Delta \dot{x}\,(z)}{\Delta z} = \frac{\partial V_x(\vec{r})}{\partial z}$$

~ 1/ time

- **Stress** ≡ Force transmitted / Cross-sectional area , in average
  - *e.g.* Shear stress (tangential) : σ_S
  - Normal stress (perpendicular): σ_N ~ Pressure
- both as a *tensor* in 3D, strictly speaking......

Understanding granular flows

Smooth, “rapid”	*Intermittent*	*Smooth*, “slow”
Quicksand	Debris flows / Earthquakes	Hourglass

1962 film《Larence in Arabia》演員 John Dimech 陷在流沙

Q1: How do we define “FAST” versus “SLOW”?

Q2: How do we define “TIGHT” versus “LOOSE”?

Dimensionless parameters --- common languages (2)

Fluid Mechanics [>200 yr-old]

Long list of examples:

- *Reynolds #* = *L V /(η/ρ)* , *V* ~$\dot{\gamma}$
  - -- inertial vs. viscous forces
  - -- turbulent vs. laminar flows
- *Peclet #* = *L V / Diffusion constant*
  - – migration vs. thermal motion
- *Mach #* = *U / Speed of sound*
  - -- importance of compressibility
- *Deborah/Weissenburg #*
  - -- deg. of relaxation (for non-Newtonian fluid)
- *Capillary #*
  - --- viscous force versus surface tension

Granular Flow [in past 20 yrs]

“*How fast* do particles flow”:

- Inertia number [est. 2004+]

$I^2 \equiv \rho\, d^2 \dot{\gamma}^2 / \sigma_N$

Mass/volume — Particle size — Confining pressure

- Viscous number [est. 2011+]

$J \equiv \eta\, \dot{\gamma} / \sigma_N$

Fluid viscosity

→ ARE THEY SUFFICIENT ?

Understanding granular flows

Smooth, "rapid" — **Quicksand**

1962 film《Larence in Arabia》
演員 John Dimech 陷在流沙

Intermittent — **Debris flows / Earthquakes**

Smooth, "slow" — **Hourglass**

Q1: How do we define "FAST" versus "SLOW"?
Q2: How do we define "TIGHT" versus "LOOSE"?

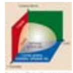
Land-mark concepts & simulations: role of *friction* X_μ

$\phi_{hcp} = \frac{\pi}{\sqrt{18}} \approx 0.74$

Packing Fraction ϕ

FRICTIONLESS ("X_μ= 0") — **FRICTIONAL ("X_μ~ O(1)")**

- "Ideal jamming" =



- Packing of *realistic* particles

Random Close Packing (RCP*) or Max. Random Jamming (MRJ)**
$\phi_c \approx 0.64$

But, what if X_μ also depends on driving rate $\dot{\gamma}$ ......?

$\phi_c \approx 0.55$
Random Loose Packing (RCP)

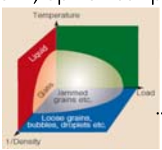
*) Scott & Kilgour (1969)
**) Proposal on MRJ [Torquato et al, 2000-PRL]

Land-mark concepts
--- "jamming" of random, spherical particles

$\phi_{hcp} = \frac{\pi}{\sqrt{18}} \approx 0.74$
Hard sphere, ordered)

Packing Fraction ϕ

- Jamming diagram
(amorphous)



....[Liu & Nagel, 1998-Nature]

Random Close Packing (RCP*) or Maximally Random Jamming (MRJ)**
$\phi_c \approx 0.64$

*) Scott & Kilgour (1969)
**) Proposal on MRJ [Torquato et al, 2000-PRL]

Short (incomplete) answers:
Q1: "FAST" vs. "SLOW" → Dimensionless #
Q2: "TIGHT" vs. "LOOSE" → Dep. on *friction*

Q3: What about **particles** immersed **in fluid**...

(a) Are *frictions* "eliminated" with things "simplified"?

(b) Are their behaviors covered by physics of ***suspension***
[>60yr-old in *Fluid Mechanics*] ?

Land-mark concepts & simulations: role of *friction* X_μ

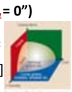
"Friction coeff."

$\phi_{hcp} = \frac{\pi}{\sqrt{18}} \approx 0.74$

Packing Fraction ϕ

FRICTIONLESS ("X_μ= 0") — **FRICTIONAL ("X_μ~ O(1)")**

- "Ideal jamming" =
[Liu & Nagel, 1998]



- Packing of *realistic* particles

Random Close Packing (RCP*) or Max. Random Jamming (MRJ)**
$\phi_c \approx 0.64$

$\phi_c \approx 0.55$
Random Loose Packing (RLP)
MD on *isotropic compression* of dry particles with varying values of X_μ
..............[Silbert, 2010-SoftMatter]

*) Scott & Kilgour (1969)
**) Proposal on MRJ [Torquato et al, 2000-PRL]

Empirical FACTS versus existing Concepts

$\phi_{hcp} = \frac{\pi}{\sqrt{18}} \approx 0.74$ (HS, ordered)

Solid Volume Fraction ϕ

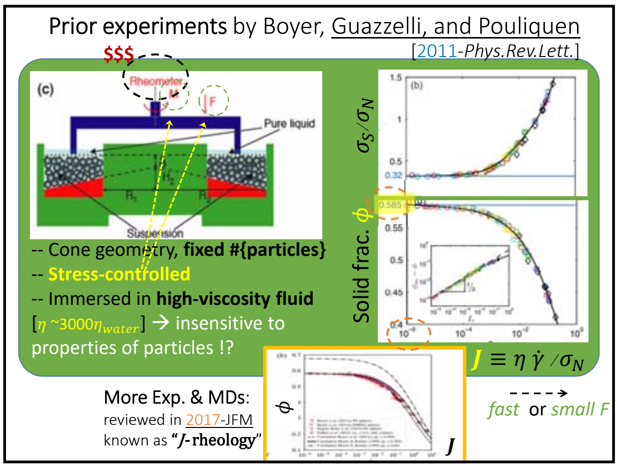
FRICTIONLESS ("X_μ = 0") — FRICTIONAL ("X_μ~ O(1)")

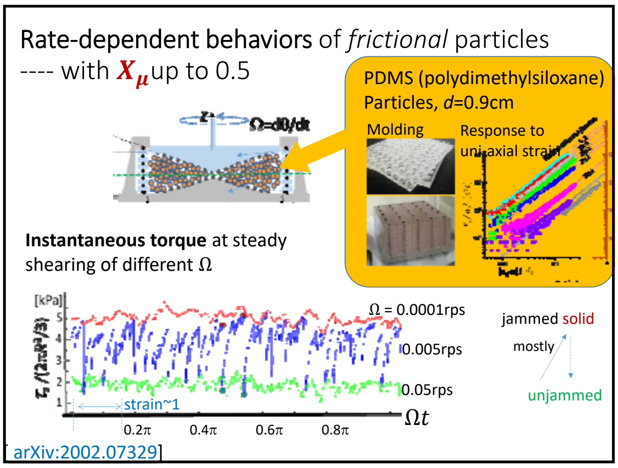
Random Close Packing (RCP*)
$\phi_c \approx 0.64$

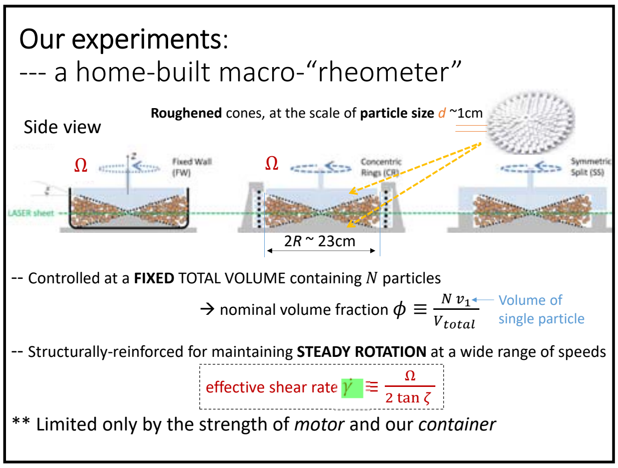
1) Upper limit observed from stress-controlled *shear flow* of particles immersed in *viscous* fluids ......[Pouliquen, 2011-PRL]
$\phi_c \approx 0.58$

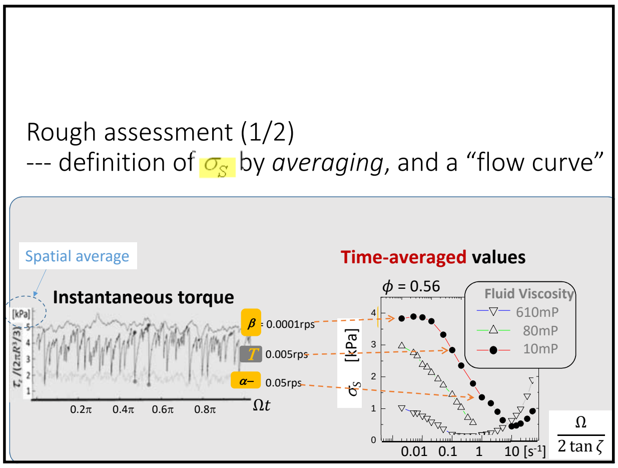
$\phi_c \approx 0.55$
Random Loose Packing (RCP)

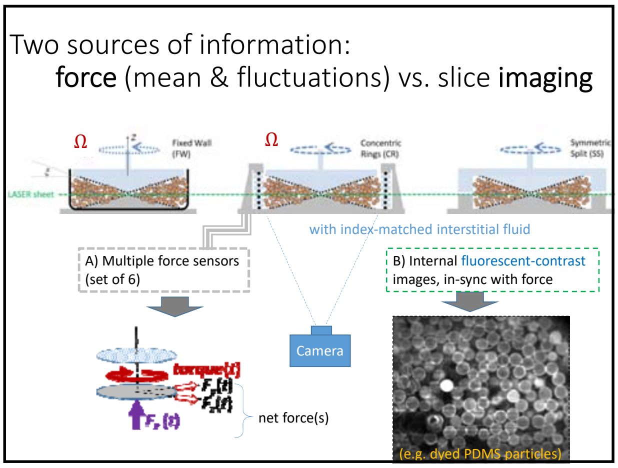
$\phi^{(exp.)} \geq 0.48$
2) Lower limit known for non-cohesive particles to exhibit a "yield stress"..

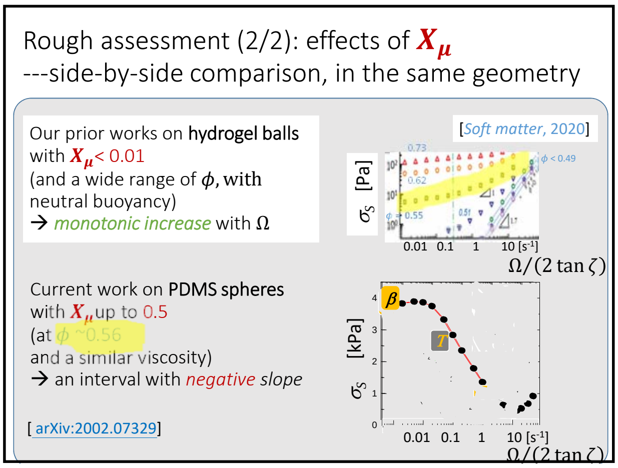

Prior experiments by Boyer, Guazzelli, and Pouliquen
[2011-Phys.Rev.Lett.]
$$$
Rheometer
Pure liquid
Suspension
σ_S/σ_N
Solid frac. ϕ
-- Cone geometry, fixed #{particles}
-- Stress-controlled
-- Immersed in high-viscosity fluid
[η ~3000η_{water}] → insensitive to properties of particles !?
$J \equiv \eta\,\dot{\gamma}\,/\sigma_N$
More Exp. & MDs:
reviewed in 2017-JFM
known as "J-rheology"
fast or small F


Rate-dependent behaviors of frictional particles
---- with X_μ up to 0.5
PDMS (polydimethylsiloxane) Particles, d=0.9cm
Molding
Response to uni-axial strain
Ω=dθ/dt
Instantaneous torque at steady shearing of different Ω
[kPa]
$T_s/(2\pi R^3/3)$
Ω = 0.0001rps
0.005rps
0.05rps
strain~1
0.2π 0.4π 0.6π 0.8π
Ωt
jammed solid
mostly
unjammed
[arXiv:2002.07329]

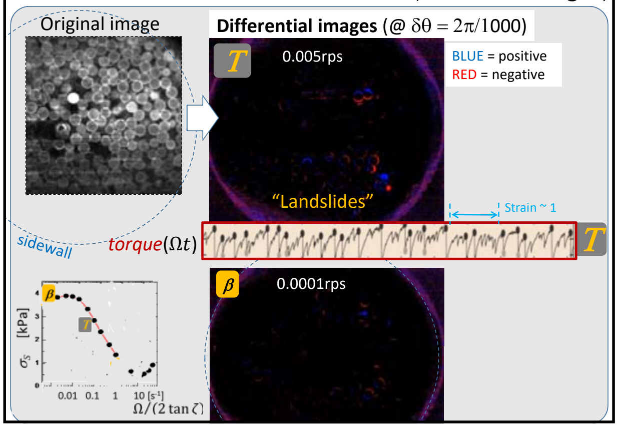

Our experiments:
--- a home-built macro-"rheometer"
Side view
Roughened cones, at the scale of particle size d ~1cm
Ω
Fixed Wall (FW)
Concentric Rings (CR)
Symmetric Split (SS)
LASER sheet
2R ~ 23cm
-- Controlled at a FIXED TOTAL VOLUME containing N particles
→ nominal volume fraction $\phi \equiv \frac{N\,v_1}{V_{total}}$ ← Volume of single particle
-- Structurally-reinforced for maintaining STEADY ROTATION at a wide range of speeds
effective shear rate $\dot{\gamma} \equiv \frac{\Omega}{2\tan\zeta}$
** Limited only by the strength of motor and our container


Rough assessment (1/2)
--- definition of σ_S by averaging, and a "flow curve"
Spatial average
Time-averaged values
Instantaneous torque
ϕ = 0.56
Fluid Viscosity
610mP
80mP
10mP
0.0001rps
0.005rps
0.05rps
0.2π 0.4π 0.6π 0.8π
Ωt
σ_S [kPa]
0.01 0.1 1 10 [s⁻¹]
$\frac{\Omega}{2\tan\zeta}$


Two sources of information:
force (mean & fluctuations) vs. slice imaging
Ω
Fixed Wall (FW)
Concentric Rings (CR)
Symmetric Split (SS)
LASER sheet
with index-matched interstitial fluid
A) Multiple force sensors (set of 6)
B) Internal fluorescent-contrast images, in-sync with force
Camera
torque(t)
net force(s)
(e.g. dyed PDMS particles)

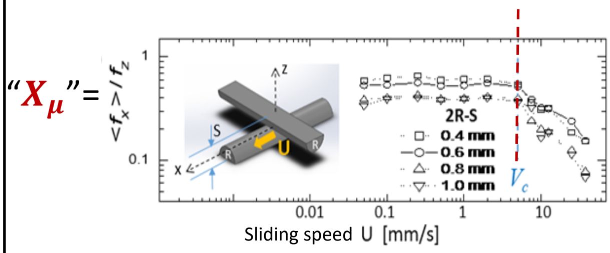

Rough assessment (2/2): effects of X_μ
---side-by-side comparison, in the same geometry
Our prior works on hydrogel balls with X_μ < 0.01
(and a wide range of ϕ, with neutral buoyancy)
→ monotonic increase with Ω
[Soft matter, 2020]
σ_S [Pa]
0.01 0.1 1 10 [s⁻¹]
Ω/(2 tan ζ)
Current work on PDMS spheres with X_μ up to 0.5
(at ϕ ~0.56
and a similar viscosity)
→ an interval with negative slope
[arXiv:2002.07329]
σ_S [kPa]
0.01 0.1 1 10 [s⁻¹]
Ω/(2 tan ζ)

Observations of *internal avalanches* (at the midheight)



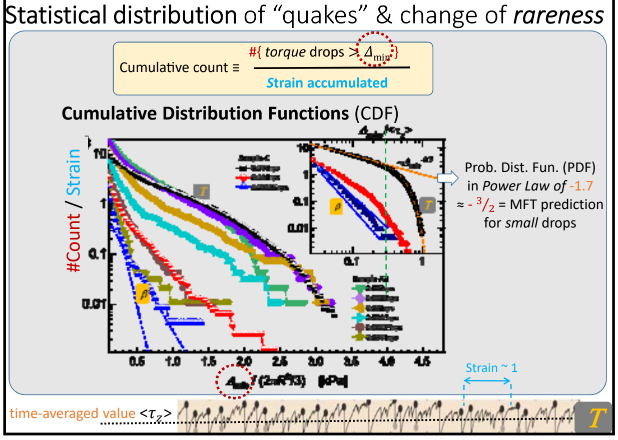
Empirically measured *velocity-weakening* tribology

1) Preliminary experiments with fluid-immersed PDMS surfaces



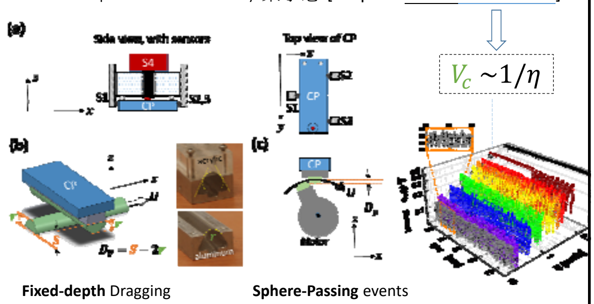
[arXiv:2002.07329]

Statistical distribution of "quakes" & change of *rareness*



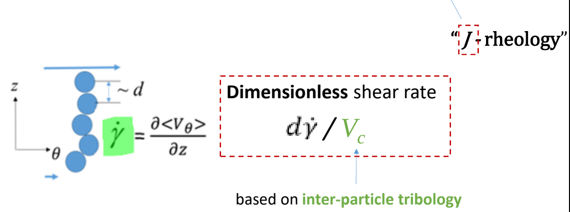
Empirically measured *velocity-weakening* tribology

2) Consistency checks: "Mean force and *fluctuations* btw. PDMS surfaces"
----independent Studies by 蔡承恩 [Preprint **arXiv:** 2010.07907]

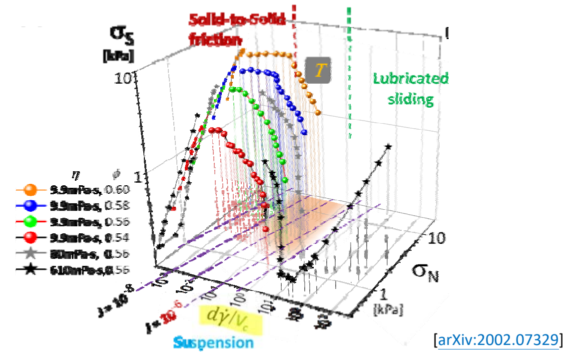


Source of rate dependence:
the velocity-weakening tribology

Understanding the transitions
– connecting regimes of granular "solid", "liquid", and "suspension"



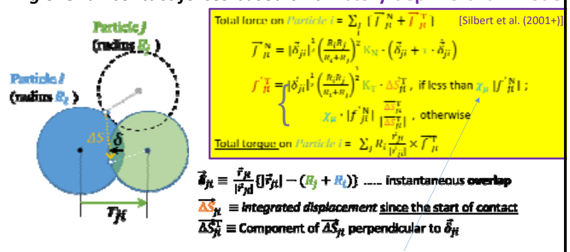
State Diagram:
time-averaged stress (σ_S and σ_N) versus $d\dot{\gamma}/V_c$



[arXiv:2002.07329]

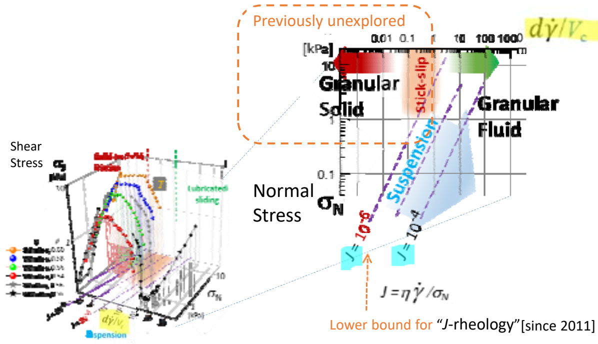
(a) Numerical experiments with **Simple Shear**

* Use of well-established LAMMPS (http://lammps.sandia.gov/) ,
summing over all contact forces based on a **history-dep. Hertzian Model**



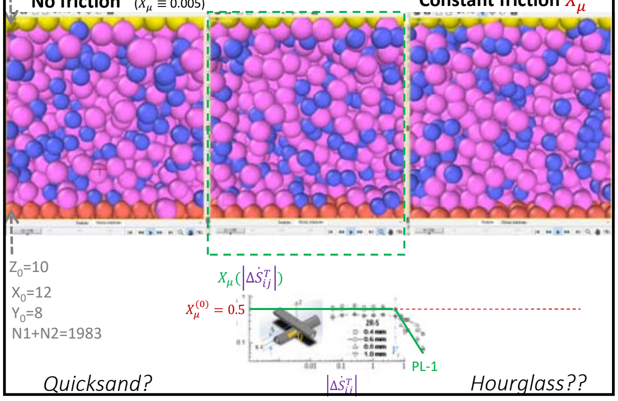
** Our amendment: making its Columb coefficient X_μ dependent on the sliding speed $|\Delta\dot{S}^T_{ij}|$, with parameters determined by lab measurements.

---- Granular "Solid", "Liquid" & "Suspension":
connected by a *stick-slip zone*



[arXiv:2002.07329]

RESULT: Speed-dep. friction induces the stick-slips !



Quicksand? *Hourglass??*

NEXT:

--Towards a full understanding

(a) Numerical simulations
[with 范姜泓杰 @NTHU]

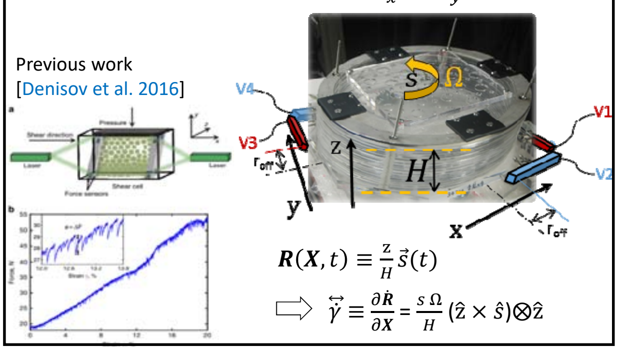
(b) New experimental setup

--Beyond granular flows

(b) Innovated setup with **Circular Shearing**
---- in construction

$\tilde{s} \equiv s_x + i\, s_y \equiv e^{i\Omega t}$

Previous work
[Denisov et al. 2016]

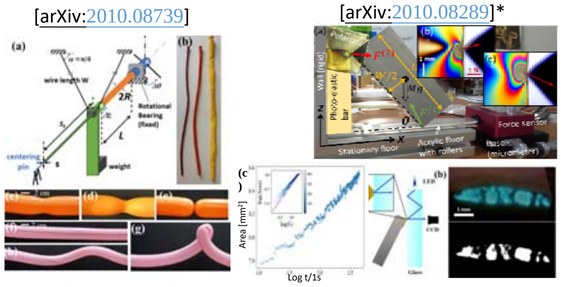


$\boldsymbol{R}(\boldsymbol{X},t) \equiv \frac{z}{H}\vec{s}(t)$

$\Rightarrow \overleftrightarrow{\gamma} \equiv \frac{\partial \dot{\boldsymbol{R}}}{\partial \boldsymbol{X}} = \frac{s\,\Omega}{H}(\hat{z}\times\hat{s})\otimes\hat{z}$

Collaborations with theorists[Prof.洪在明@NTHU]

[arXiv:2010.08739] [arXiv:2010.08289]*

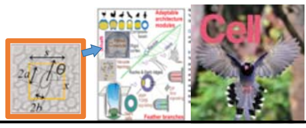


Other collaborations

--with Dr. 阮文滔@CMU

[*Cell*, 2019] , Patent pending

--with Prof.楊馥菱@NTU-ME



Take-home messages

- (1) Inter-particle tribology: essential ingredient for changes behind

  Hourglass – Intermittent flows – Quicksand

  [solid-to-solid] [*mixed* dyn.] [fully-lubricated]

  with a new "dimensionless shear rate" identified, while

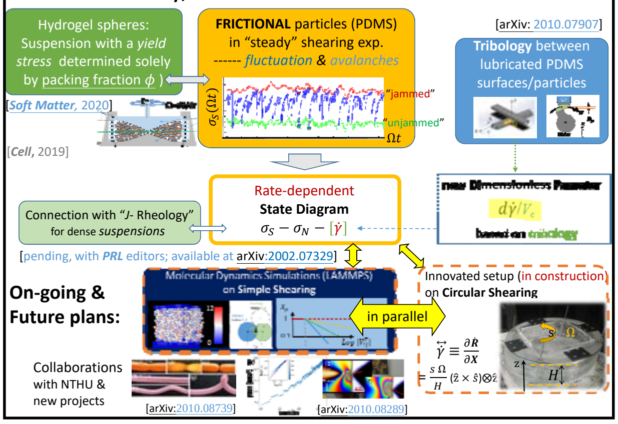
  (2) the *"loose vs. tight"* for packed grains is *coupled to* the driving rate, NOT being determined by just the solid volume fraction alone.

- Force *fluctuations* & images: complimentary in probing the *dynamics*

→ We need further basis to move on dialogues between communities of *Fluid* vs. *Solid* Mechanics.

Recap: What have we learned for *flows* of *packed grains*?

Summary, Outlooks & Thanks to AS-IoP



Studies on densely-packed granular particles