CP Violation and Beyond the Standard Model (2)

Matter-anti-Matter-Asymmetry

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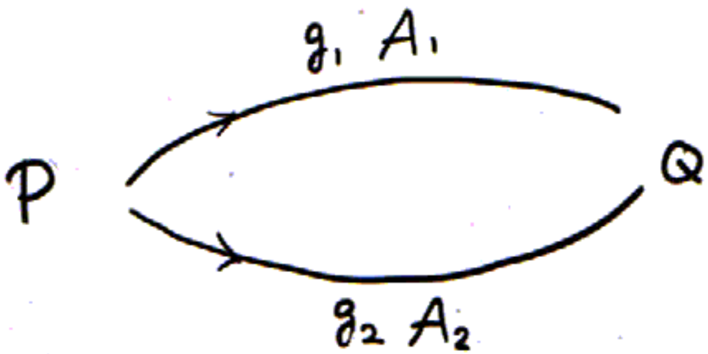
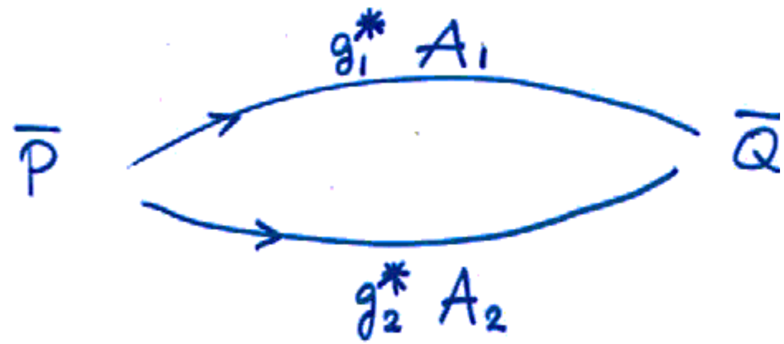
Two phases



$$\mathrm{Amp}(P \to Q) = (\cdots\ g_i\ \cdots) \begin{pmatrix} \vdots \\ A_i \\ \vdots \end{pmatrix}$$



$$\mathrm{Prob}(P \to Q) = (\cdots\ A_j^*\ \cdots) \begin{pmatrix} \vdots \\ g_j^* \\ \vdots \end{pmatrix} (\cdots\ g_i\ \cdots) \begin{pmatrix} \vdots \\ A_i \\ \vdots \end{pmatrix}$$

$$|g_1 A_1| = |g_1^* A_1|$$

$$\mathrm{Prob}(P \to Q) = g_i g_j^* A_i A_j^*$$

$$\mathrm{Prob}(\bar{P} \to \bar{Q}) = g_i^* g_j A_i A_j^*$$

$$\mathrm{diff} = -2\mathrm{Im}(g_i g_j^*)\mathrm{Im}(A_i A_j^*)$$

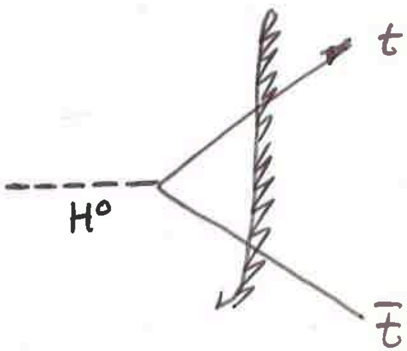
- One complex g not enough for CP

2HDM and Beyond ✓

$$\mathcal{L} = -(\sqrt{2}G_F)^{\frac{1}{2}}\, \bar{t}\left(A\frac{1-\gamma_5}{2} + A^*\frac{1+\gamma_5}{2}\right) t\, H^0$$

phase cannot be rotated away because of the mass term $m\bar{t}t$.

$\sigma(t_L\bar{t}_L) \stackrel{?}{=} \sigma(t_R\bar{t}_R)$



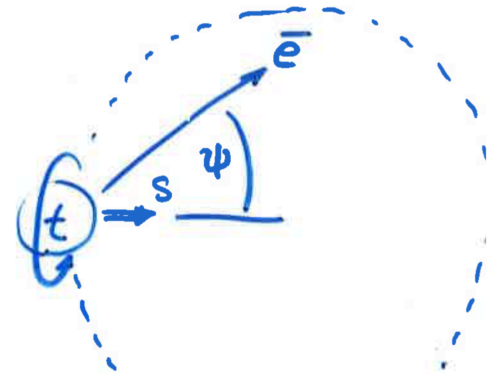
$$\mathrm{Amp}\,(H^0\to t_L\bar{t}_L) \sim A_R\beta + i\,A_I$$

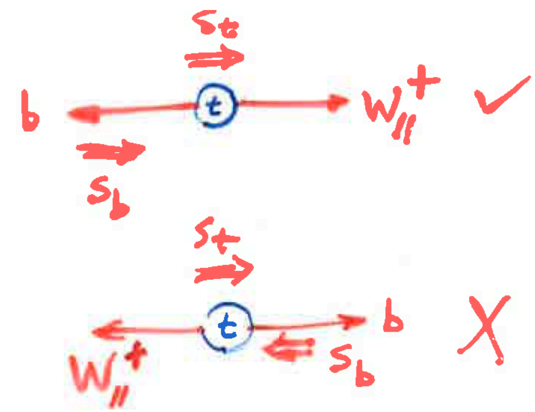
$$\mathrm{Amp}\,(H^0\to t_R\bar{t}_R) \sim A_R\beta - i\,A_I$$

$$\to \quad \begin{array}{l} \mathrm{Amp}\,(t_L\bar{t}_L) \sim A_R\beta \quad + iA_I(1+ia) \\ \mathrm{Amp}\,(t_R\bar{t}_R) \sim A_R\beta \quad - iA_I(1+ia) \end{array}$$

$$P(t_L\bar{t}_L) - P(t_R\bar{t}_R) \sim 4\,a\,A_I A_R\beta$$

$$t \to b\,\bar{e}\,\nu$$





$$\frac{dN}{d\cos\psi} = 1+\cos\psi$$

when t is moving

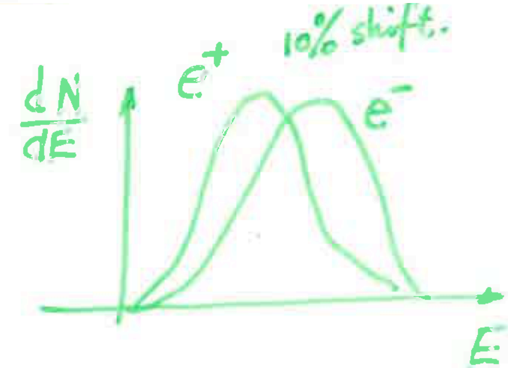
$\bar{e}$ has higher energy profile from t_R than t_L.

In parallel,

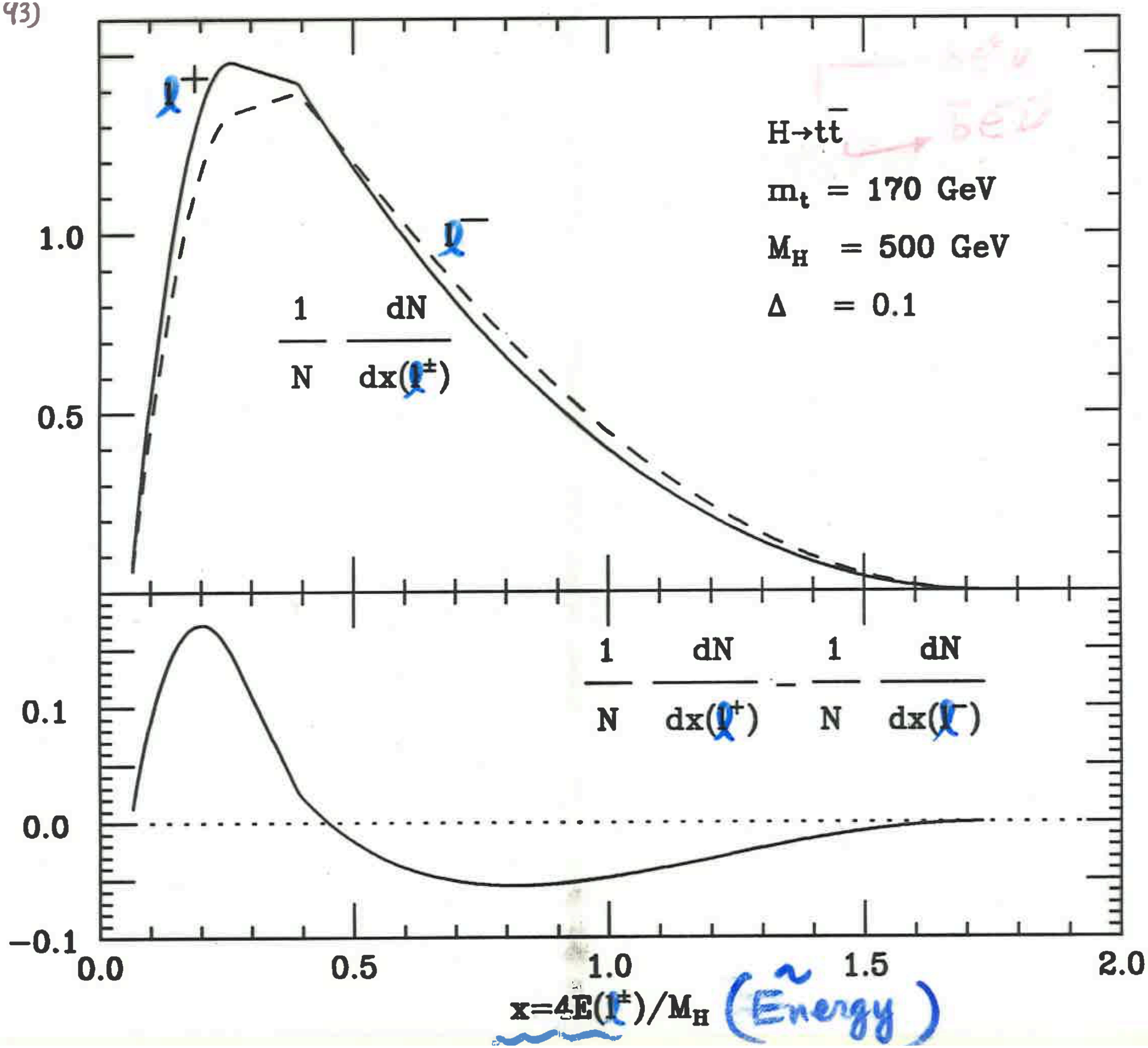
e has higher energy profile from $\bar{t}_L$ than $\bar{t}_R$.

If $t_L\bar{t}_L$ and $t_R\bar{t}_R$ are produced equally, difference is even out. But $N(t_L\bar{t}_L) \neq N(t_R\bar{t}_R)$;

⇒ Asymmetry in the energy of the secondary leptons.



PL, B305, 261(93)



Angular correlation in Z' to ZZ

With Low and Shu, arXiv:0806.2864

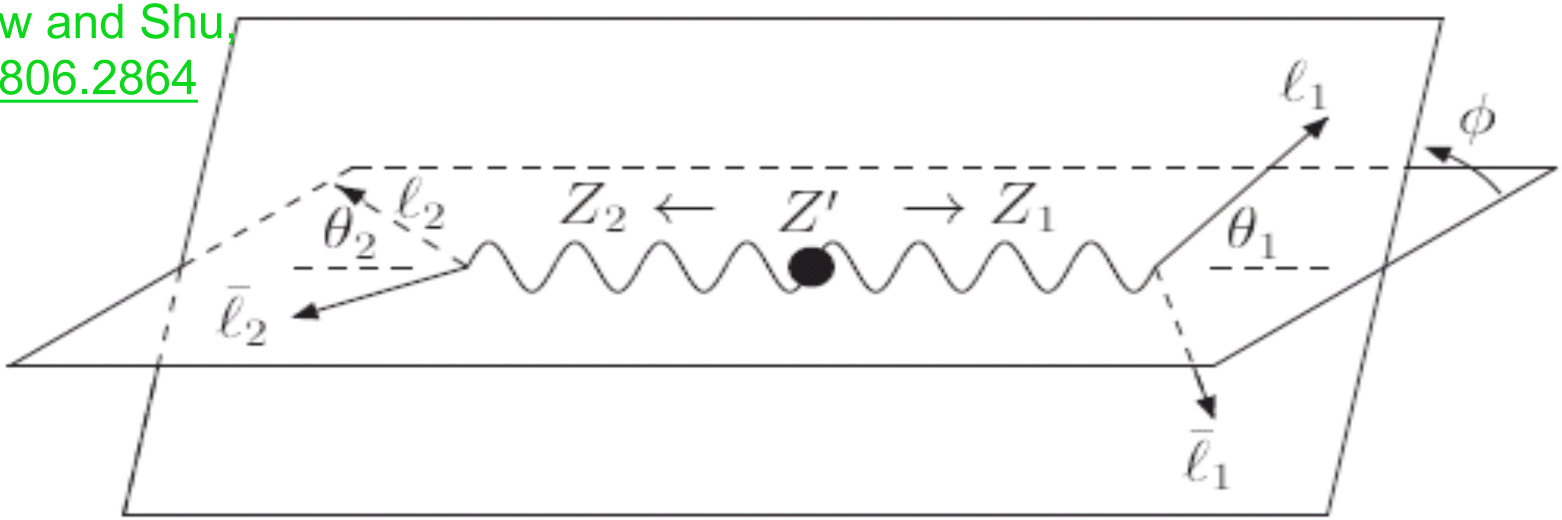


FIG. 1: *Two decay planes of* $Z_1 \to \ell_1\bar{\ell}_1$ *and* $Z_2 \to \ell_2\bar{\ell}_2$ *define the azimuthal angle* $\phi \in [0, 2\pi]$ *which rotates* ℓ_2 *to* ℓ_1 *in the transverse view. The polar angles* θ_1 *and* θ_2 *shown are defined in the rest frame of* Z_1 *and* Z_2*, respectively.*

$$O_{CPV} = f_4 Z'_\mu (\partial_\nu Z^\mu) Z^\nu, O_A = f_5 \epsilon^{\mu\nu\rho\sigma} Z'_\mu Z_\nu (\partial_\rho Z_\sigma)$$

Amplitudes $Z'(q_1+q_2,\mu) \to Z(q_1,\alpha)Z(q_2,\beta)$

$$\Gamma^{\mu\alpha\beta}_{Z'\to Z_1 Z_2} = i f_4 (q_2^\alpha g^{\mu\beta} + q_1^\beta g^{\mu\alpha}) + i f_5 \epsilon^{\mu\alpha\beta\rho}(q_1 - q_2)_\rho .$$

$$\beta^2 = 1 - 4m_Z^2/m_{Z'}^2$$

$$\begin{aligned}\mathcal{M}_{+,+0} &= -\mathcal{M}_{-,0+} = R(-f_5\beta + i f_4)\\ \mathcal{M}_{+,0-} &= -\mathcal{M}_{-,-0} = R(-f_5\beta - i f_4)\end{aligned}$$

$$R = \frac{\beta m_{Z'}^2}{2m_Z}$$

$$\sum_{\kappa,h_1,h_2}\left|\sum_{\lambda_1,\lambda_2}\mathcal{M}_{\kappa,\lambda_1\lambda_2}\, g_{h_1} f^{h_1}_{\lambda_1}(\theta_1,\phi)\, g_{h_2} f^{h_2}_{\lambda_2}(\theta_2,0)\right|^2$$

$$e^{2i\delta} = \frac{-f_5\beta + i f_4}{-f_5\beta - i f_4}$$

$$f^h_m(\bar\theta,\bar\phi) = (1 + mh\cos\bar\theta)\frac{e^{im\bar\phi}}{2},\; f^h_0(\bar\theta,\bar\phi) = \frac{h}{\sqrt{2}}\sin\bar\theta.$$

Universal Angular dependence

$$\frac{8\pi dN}{N\,d\cos\theta_1 d\cos\theta_2 d\phi} = \frac{9}{8}\Big[1 - \cos^2\theta_1\cos^2\theta_2$$

$$- \cos\theta_1\cos\theta_2\sin\theta_2\sin\theta_1\cos(\phi + 2\delta)$$

$$+ \frac{(g_L^2 - g_R^2)^2}{(g_L^2 + g_R^2)^2}\sin\theta_1\sin\theta_2\cos(\phi + 2\delta)\Big]$$

Amp. Squared sum

$$\mathcal{M}[+\to(+,0)\text{ or }(0,-)]_{RR} = +g_R^2[(1+\cos\theta)e^{i\phi}\sin\theta' + (1-\cos\theta')\sin\theta]$$

$$\mathcal{M}[+\to(+,0)\text{ or }(0,-)]_{LL} = -g_L^2[(1-\cos\theta)e^{i\phi}\sin\theta' + (1+\cos\theta')\sin\theta]$$

$$\mathcal{M}[-\to(-,0)\text{ or }(0,+)]_{RR} = +g_R^2[(1-\cos\theta)e^{-i\phi}\sin\theta' + (1+\cos\theta')\sin\theta]$$

$$\mathcal{M}[-\to(-,0)\text{ or }(0,+)]_{LL} = -g_L^2[(1+\cos\theta)e^{-i\phi}\sin\theta' + (1-\cos\theta')\sin\theta]$$

$$\mathcal{M}[+\to(+,0)\text{ or }(0,-)]_{RL} = -g_R g_L[(1+\cos\theta)e^{i\phi}\sin\theta' - \sin\theta(1+\cos\theta')]$$

$$\mathcal{M}[+\to(+,0)\text{ or }(0,-)]_{LR} = +g_L g_R[(1-\cos\theta)e^{i\phi}\sin\theta' - \sin\theta(1-\cos\theta')]$$

$$\mathcal{M}[-\to(-,0)\text{ or }(0,+)]_{RL} = -g_R g_L[(1-\cos\theta)e^{-i\phi}\sin\theta' - \sin\theta(1-\cos\theta')]$$

$$\mathcal{M}[-\to(-,0)\text{ or }(0,+)]_{LR} = +g_L g_R[(1+\cos\theta)e^{-i\phi}\sin\theta' - \sin\theta(1+\cos\theta')]$$

$$4(g_L^2+g_R^2)^2[1-\cos^2\theta\cos^2\theta' - \cos\theta\cos\theta'\sin\theta'\sin\theta\cos\phi] + 4(g_L^2-g_R^2)^2\sin\theta\sin\theta'\cos\phi$$

Ang. Integrated Oscillation

$$\frac{2\pi dN_\pm}{N d\phi} = \frac{1}{2}\left[1 \mp \frac{1}{8}\cos(\phi+2\delta) + \frac{9\pi^2}{128}\frac{(g_L^2-g_R^2)^2}{(g_L^2+g_R^2)^2}\cos(\phi+2\delta)\right]$$

$$\frac{(g_L^2-g_R^2)^2}{(g_L^2+g_R^2)^2} \rightarrow \frac{(g_L^2-g_R^2)(g'^2_L-g'^2_R)}{(g_L^2+g_R^2)(g'^2_L+g'^2_R)}$$

SM ZZ background 79 fb

For 100 fb^{-1} luminosity at the LHC, if we require the ratio of the signal S to the statistical error in the background $\sqrt{B}$ to be 5 we need a σ(ZZ) about 70 fb for a 240 GeV Z'.

In the Littlest Higgs Model with T-parity, the predicted total cross section for T-odd particles, will be 1.3 pb.

Models of CP violation

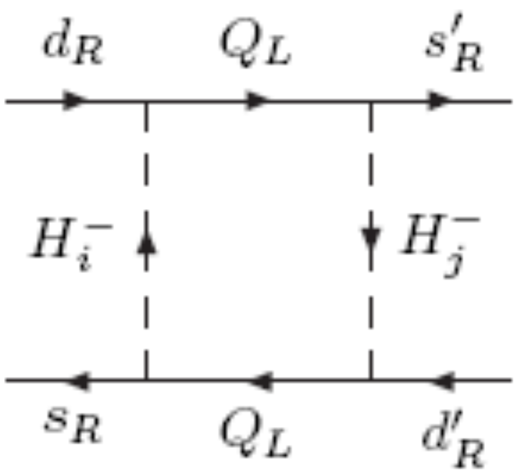
$$\theta = \theta_0 - \mathrm{Arg}[\mathrm{Det}(\mathcal{M}_u \mathcal{M}_d)]$$

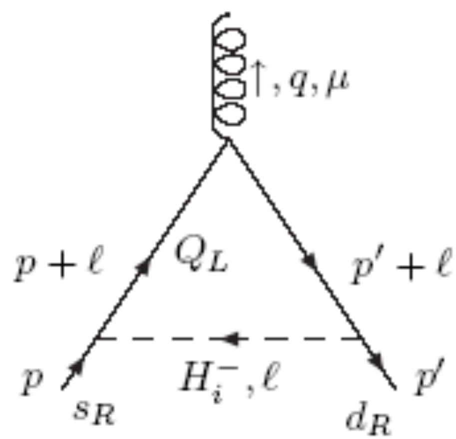
- Extreme fine tune at tree
- finite start at 3-loop
- infinite at 7-loop

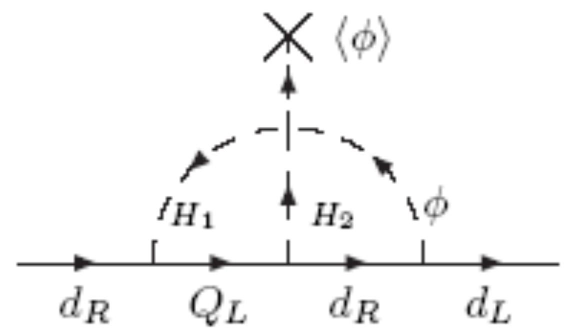
VOLUME 81, NUMBER 10 PHYSICAL REVIEW LETTERS 7 SEPTEMBER 1998

Simple Charged Higgs Model of Soft or Spontaneous *CP* Violation without Flavor Changing Neutral Currents

David Bowser-Chao,[1] Darwin Chang,[2,3] and Wai-Yee Keung[1]







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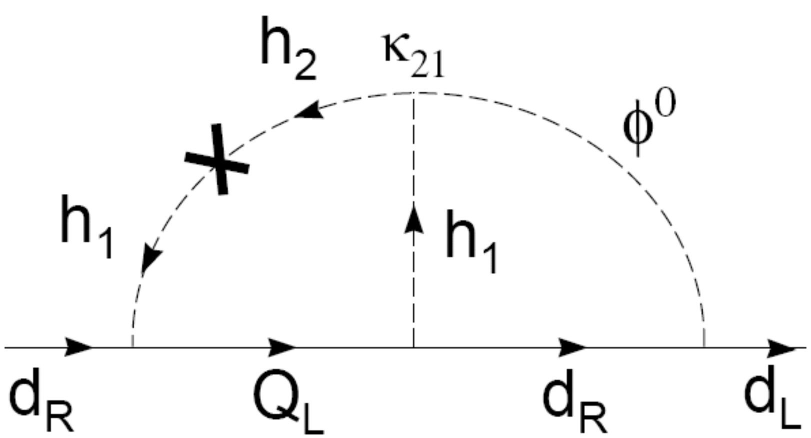
H-CP

Simple Charged Higgs Model of Soft or Spontaneous *CP* Violation without Flavor Changing Neutral Currents

David Bowser-Chao,[1] Darwin Chang,[2,3] and Wai-Yee Keung[1]

$$\mathcal{L}_{h_i} = \left[(g\lambda_{i\alpha}\bar{Q}_L d_{iR} h_\alpha + M_Q \bar{Q}_L Q_R) + \text{h.c.}\right] - (m^2)_{\alpha\beta} {h_\alpha}^\dagger h_\beta - \kappa_{\alpha\beta}(\phi^\dagger\phi - |\langle\phi\rangle|^2)\, h_\alpha^\dagger h_\beta$$

$\mathrm{Im}(m^2)_{12}$ $\quad h_\alpha = U_{\alpha i} H_i$ $\quad \mathcal{L}_{QqH} = g \sum_{q=d,s,b} \xi_{qj} (\bar{Q}_L q_R) H_j^- \; + \; \text{h.c.}$



$$\xi_{qj} \equiv \lambda_{q\alpha} U_{\alpha j}\,.$$

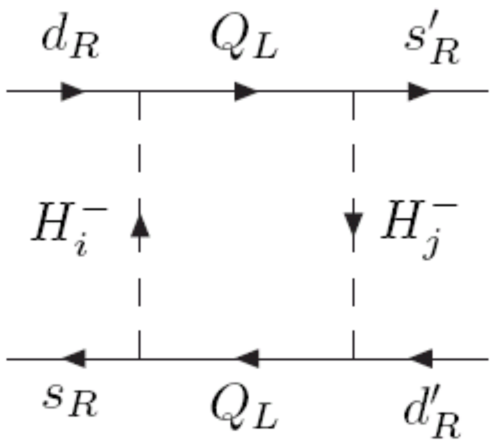
H. Georgi and S. Glashow

S. Barr and A. Zee

2-loop θ

ε And box

$$\mathcal{H}^{\Delta S=2} = \frac{G_F^2 m_W^2}{16\pi^2} \sum_{I=R,L} C^I_{\Delta S=2}(\mu) O^I_{\Delta S=2}(\mu) \ ,$$



with $\quad O^{R,L}_{\Delta S=2} = \bar{s}\gamma_\mu(1 \pm \gamma_5)d\,\bar{s}\gamma^\mu(1 \pm \gamma_5)d$.

The $W^\pm$ diagrams yield a purely real Wilson coefficient $C^L_{\Delta S=2}(\mu)$; CP violation is due solely to the operator $O^R_{\Delta S=2}$ rather than $O^L_{\Delta S=2}$, in contrast to the KM model, because the complex coefficient $C^R_{\Delta S=2}(\mu)$ is generated by the charged Higgs. At the scale $\mu = M_Q$, we have

$$C^R_{\Delta S=2}(M_Q) = 2\xi_{d1}\xi^*_{s1}\xi_{d2}\xi^*_{s2}\frac{m_W^2}{M_Q^2}\frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$$+ \sum_{i=1,2}(\xi_{di}\xi^*_{si})^2\frac{m_W^2}{M_Q^2}\frac{df}{dx}(x_i)$$

with $f(h) = (1-h)^{-2}(1+2h+h^2+h^2\ln h)$.

$$\mathcal{A}^{qq'}_{ij} = \lambda_{q\alpha}\lambda_{q'\beta}U_{\beta i}U^*_{\alpha j} = \xi^*_{qi}\xi_{q'j}$$

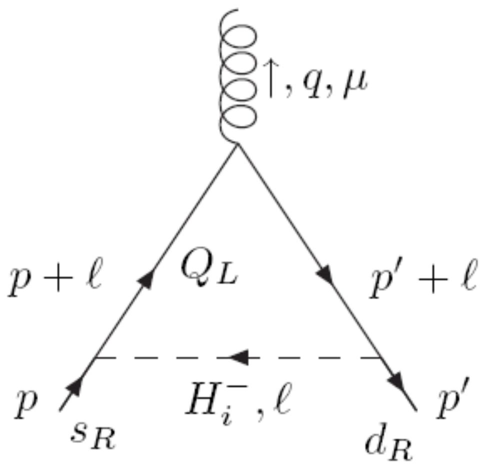
$$R_Q = 300 \text{ GeV}/M_Q$$

$$\text{Im}\left(\mathcal{A}_{sd}/(0.058)^2\right)^2 R_Q^2 = 1 \ ,$$

$$\text{Re}\left(\mathcal{A}_{sd}/(0.058)^2\right)^2 R_Q^2 = 156\mathcal{F}$$

ε'

$$\mathcal{H}^{\Delta S=1} = (G_F/\sqrt{2})\tilde{C}(\bar{s}T^a\gamma_\mu(1+\gamma_5)d) \times \sum_q (\bar{q}T^a\gamma^\mu q)$$



At the electroweak scale, the Wilson coefficient is

$$\tilde{C} = -\alpha_s \sum_i \frac{\xi_{di}\xi^*_{si}}{6\pi}\frac{m_W^2}{M_Q^2} F\left(\frac{m_{H_i}}{M_Q^2}\right) ,$$

$$F(h) = \left[\frac{(2h-3)h^2\ln h}{(1-h)^4} + \frac{7-29h+16h^2}{6(1-h)^3}\right]$$

$$\epsilon'/\epsilon = -1.9\times 10^{-5}\,\mathrm{Im}(\mathcal{A}_{sd}/(0.058)^2)R_Q^2$$

$$= \pm 1.9\times 10^{-5}\ (\sqrt{(156\mathcal{F})^2+1} - 156\mathcal{F})^{\frac{1}{2}} R_Q/\sqrt{2}$$

Therefore the present experimental value of ϵ'/ϵ does not favor the above simplified scenario of lightest-Higgs-dominance ($m_2 \gg m_1$). On the other hand, if we let the two charged Higgs $H^\pm_{1,2}$ have comparable but *not* degenerate masses, it is possible that the three amplitudes in Eq.(9) cancel each other (even completely) for certain large complex couplings ξ_{si} and ξ_{di} ($\gg 0.058$). If such modest fine tuning of parameters is allowed, one can easily boost up the value of ϵ'/ϵ to the recently measured value as will be discussed in more details elsewhere[8].

B Physics

$$|1-\rho-i\eta| = 1.01 \pm 0.21$$

B_d^0–$\bar{B}_d^0$ oscillation $\quad |(1-\rho)^2 + C_{RR}^{RH}/C_{LL}^{SM}| = (1.01 \pm 0.21)^2$

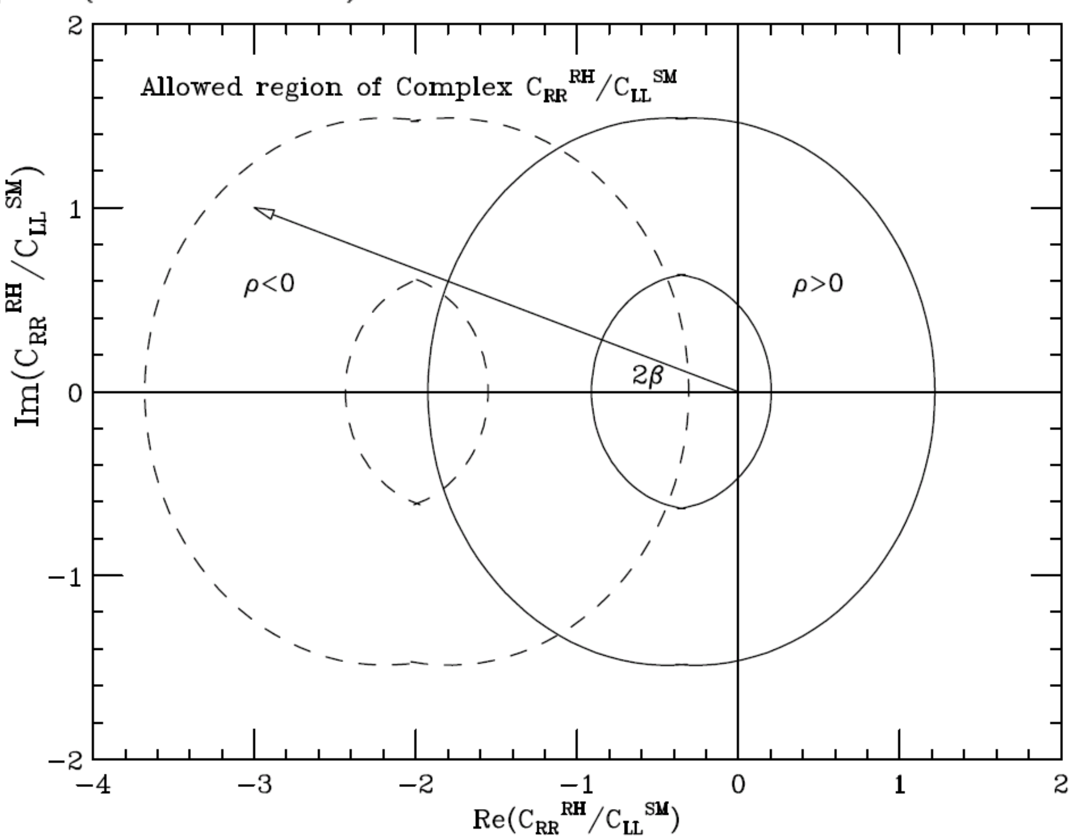
$|V_{ub}/V_{cb}|$ constraint $\quad \rho^2 + \eta^2 = (0.41 \pm 0.07)^2 \qquad \rho = \pm(0.41 \pm 0.07)$

$$|(1-0.41 \pm 0.07)^2 + C_{RR}^{RH}/C_{LL}^{SM}| = (1.01 \pm 0.21)^2$$

$$C_{RR}^{RH} \simeq C_{LL}^{SM}$$

when

$$\mathcal{A}_{bd} = 0.3^2$$



CKM-like and 2-loop θ

$SU_L(2)$ singlets vector-like $U_{Li}, U_{Ri}, D_{Li}, D_{Ri}$

in an analogous fashion with the known quarks, $q_{Li} \equiv \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}, u_{Ri}, d_{Ri}$

A horizontal flavor symmetry $SO(3)_{||}$

$$\begin{aligned} &\bar{d}_R(\mu_d + g_{dS}\phi_S + ig_{dA}\phi_A)D_L \; + \bar{D}_R(\mu_D + g_{DS}\phi_S + ig_{DA}\phi_A)D_L \\ &+\bar{u}_R(\mu_u + g_{uS}\phi_S + ig_{uA}\phi_A)U_L \; + \bar{U}_R(\mu_U + g_{US}\phi_S + ig_{UA}\phi_A)U_L \\ &+(h_d\bar{d}_R + h'_d\bar{D}_R)H^\dagger q_L + (h_u\bar{u}_R + h'_u\bar{U}_R)\tilde{H}^\dagger q_L + \text{H.c.} \end{aligned}$$

$$M_6 = \begin{pmatrix} \mathbf{0} & \mu_d + g_{dS}\langle\phi_S\rangle + ig_{dA}\langle\phi_A\rangle \\ h'_d\langle H^\dagger\rangle\mathbf{1} & \mu_D + g_{DS}\langle\phi_S\rangle + ig_{DA}\langle\phi_A\rangle \end{pmatrix}$$

Mixing

In the limit of $\langle H \rangle = 0$, d_L quarks decouple

the reduced mass matrix of a size 6×3

$$(\bar{d}_R \ \bar{D}_R) \begin{pmatrix} M_d \\ M_D \end{pmatrix} (D_L) \ ,$$

find a 6×6 unitary matrix V

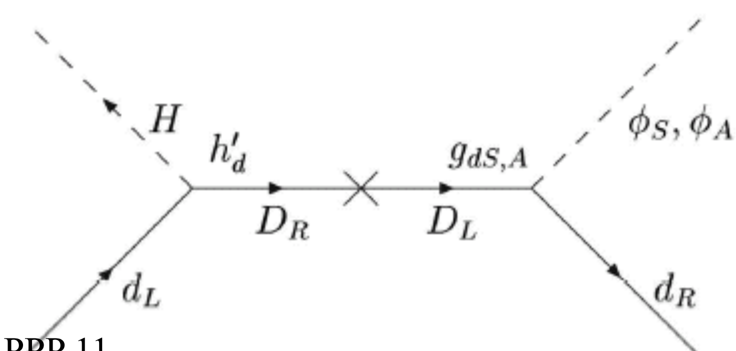
$$V \begin{pmatrix} M_d \\ M_D \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ M'_D \end{pmatrix}$$

top three row vectors of V $\perp$ the 3 column vectors in the mass matrix.

$$(\bar{d}'_R \ \bar{D}'_R) V \begin{pmatrix} \mathbf{0} \\ h'_d \langle H^\dagger \rangle \mathbf{1} \end{pmatrix} d_L = (\bar{d}'_R \ \bar{D}'_R) \begin{pmatrix} \hat{m}_d \\ \hat{m}'_d \end{pmatrix} d_L$$

Including D'_L, we have

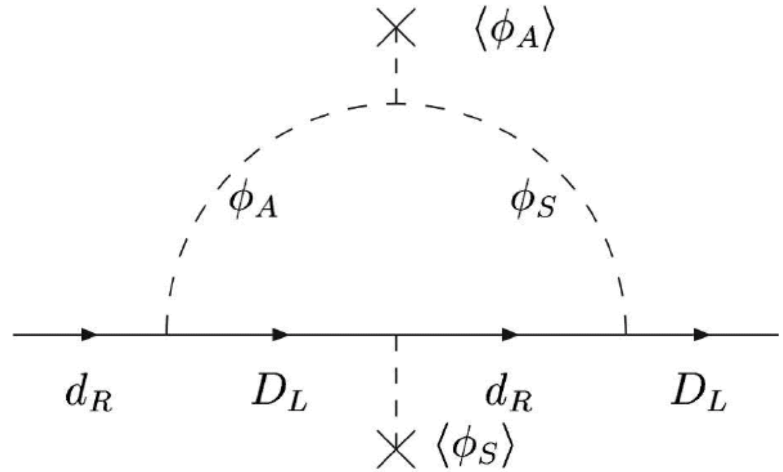
$$(\bar{d}'_R \ \bar{D}'_R) \begin{pmatrix} \hat{m}_d & \mathbf{0} \\ \hat{m}'_d & M'_D \end{pmatrix} \begin{pmatrix} d_L \\ D'_L \end{pmatrix} \ .$$



Loop, hermiticity

$$i\bar{D}_L(f_{SA}\phi_S\phi_A + f_{AS}\phi_A\phi_S)d_R$$

$$-i\bar{d}_R(f^*_{AS}\phi_S\phi_A + f^*_{SA}\phi_A\phi_S)D_L$$



$$\phi_S \to \phi_S \ , \quad \phi_A \to -\phi_A \ , \quad \bar{D}_L(\cdots)d_R \to \bar{d}_R(\cdots)D_L$$

This require $f_{SA} = f^*_{AS}$.

$$i(f_{SA}\langle\phi_S\rangle\langle\phi_A\rangle + f_{AS}\langle\phi_A\rangle\langle\phi_S\rangle) \quad \text{remains hermitian.}$$

Texture

If we impose a discrete symmetry under which u, d and all $\phi_{A,S}$, H fields are odd, while U and D are even, we have

$$h_d = 0 \ , \quad g_{DS} = 0 \ , \quad g_{DA} = 0 \ .$$

$$M_6 = \begin{pmatrix} \mathbf{0} & \mu_d + g_{dS}\langle\phi_S\rangle + i g_{dA}\langle\phi_A\rangle \\ h'_d\langle H^\dagger\rangle\mathbf{1} & \mu_D\mathbf{1} \end{pmatrix}$$

$$\begin{vmatrix} \mathbf{C} & \mathbf{A} \\ \mathbf{D} & \mathbf{B} \end{vmatrix} = |\mathbf{CB} - \mathbf{DA}| \ , \text{ provided } \mathbf{CD} = \mathbf{DC}$$

Matrix property

$$\begin{vmatrix} \mathbf{1} & \boldsymbol{A} \\ \mathbf{1} & \boldsymbol{B} \end{vmatrix} = |\boldsymbol{B} - \boldsymbol{A}| \neq |\boldsymbol{B}| - |\boldsymbol{A}|$$

$$\begin{vmatrix} c\mathbf{1} & \boldsymbol{A} \\ d\mathbf{1} & \boldsymbol{B} \end{vmatrix} = |c\boldsymbol{B} - d\boldsymbol{A}| \neq |c\boldsymbol{B}| - d|\boldsymbol{A}|$$

$$\begin{vmatrix} \boldsymbol{C} & \boldsymbol{A} \\ \boldsymbol{D} & \boldsymbol{B} \end{vmatrix} = |\boldsymbol{C}\boldsymbol{B} - \boldsymbol{D}\boldsymbol{A}| \,, \quad \text{provided } \boldsymbol{C} \text{ and } \boldsymbol{D} \text{ are diagonal}$$

$$\begin{vmatrix} \boldsymbol{C} & \boldsymbol{A} \\ \boldsymbol{D} & \boldsymbol{B} \end{vmatrix} = |\boldsymbol{C}\boldsymbol{B} - \boldsymbol{D}\boldsymbol{A}| \,, \qquad \text{provided } \boldsymbol{C}\boldsymbol{D} = \boldsymbol{D}\boldsymbol{C}$$

$$\begin{vmatrix} \boldsymbol{C} & \boldsymbol{A} \\ \boldsymbol{D} & \boldsymbol{B} \end{vmatrix} = \begin{vmatrix} \boldsymbol{C} & \boldsymbol{A} \\ \boldsymbol{D} & \boldsymbol{B} \end{vmatrix} \begin{vmatrix} \mathbf{1} & -\boldsymbol{C}^{-1}\boldsymbol{A} \\ 0 & \mathbf{1} \end{vmatrix} = \begin{vmatrix} \boldsymbol{C} & \mathbf{0} \\ \boldsymbol{D} & \boldsymbol{B} - \boldsymbol{D}\boldsymbol{C}^{-1}\boldsymbol{A} \end{vmatrix} = |\boldsymbol{C}\boldsymbol{B} - \boldsymbol{C}\boldsymbol{D}\boldsymbol{C}^{-1}\boldsymbol{A}|$$

Glashow's model on triangular matrix

three Higgs doublets, $H_{(0)}$, $H_{(1)}$ and $H_{(2)}$

$$F(H_{(k)}) = k \ , \qquad F(u_R^{(i)}) = F(d_R^{(i)}) = F(q_L^{(i)}) = f(i)$$

$$f(i) \equiv \begin{cases} +1 \ , & \text{for } i = 1 \\ 0 \ , & i = 2 \\ -1 \ , & i = 3 \end{cases} .$$

$$(\bar{d}_L \ \bar{s}_L \ \bar{b}_L \ ) \begin{pmatrix} y_d^{(1,1)} H_0 & y_d^{(1,2)} H_1 & y_d^{(1,3)} H_2 \\ 0 & y_d^{(2,2)} H_0 & y_d^{(2,3)} H_1 \\ 0 & 0 & y_d^{(3,3)} H_0 \end{pmatrix} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}$$

$$+(\bar{u}_L \ \bar{c}_L \ \bar{t}_L \ ) \begin{pmatrix} y_u^{(1,1)} H_0^* & 0 & 0 \\ y_u^{(1,2)} H_1^* & y_u^{(2,2)} H_0^* & 0 \\ y_u^{(1,3)} H_2^* & y_u^{(2,3)} H_1^* & y_u^{(3,3)} H_0^* \end{pmatrix} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}$$

$$(\bar{d}_L^{(i)})(M_D)_{ij}(d_R)^{(j)} \ , \qquad (\bar{u}_L^{(i)})(M_U)_{ij}(u_R)^{(j)} \ .$$

$$(M_D) = \begin{pmatrix} y_d^{(1,1)} \langle H_0 \rangle & y_d^{(1,2)} \langle H_1 \rangle & y_d^{(1,3)} \langle H_2 \rangle \\ 0 & y_d^{(2,2)} \langle H_0 \rangle & y_d^{(2,3)} \langle H_1 \rangle \\ 0 & 0 & y_d^{(3,3)} \langle H_0 \rangle \end{pmatrix}$$

$$\equiv \begin{pmatrix} m_d^{(0)} & \epsilon_{12} & \epsilon_{13} \\ 0 & m_s^{(0)} & \epsilon_{23} \\ 0 & 0 & m_b^{(0)} \end{pmatrix} ,$$

$$(M_U^{(0)}) = \begin{pmatrix} y_u^{(1,1)} \langle H_0^* \rangle & 0 & 0 \\ y_u^{(1,2)} \langle H_1^* \rangle & y_u^{(2,2)} \langle H_0^* \rangle & 0 \\ y_u^{(1,3)} \langle H_2^* \rangle & y_u^{(2,3)} \langle H_1^* \rangle & y_u^{(3,3)} \langle H_0^* \rangle \end{pmatrix}$$

$$\equiv \begin{pmatrix} m_u^{(0)} & 0 & 0 \\ \epsilon_{21}^* & m_c^{(0)} & 0 \\ \epsilon_{31}^* & \epsilon_{32}^* & m_t^{(0)} \end{pmatrix} ,$$

Quark Mixing

$$\begin{pmatrix} \mathbf{0} & \begin{pmatrix} m & \Delta \\ 0 & M \end{pmatrix} \\ \begin{pmatrix} m & 0 \\ \Delta^* & M \end{pmatrix} & \mathbf{0} \end{pmatrix}$$

$$V_{12} = \frac{\epsilon_{12} m_s}{m_s^2 - m_d^2} - \frac{\epsilon_{21}^* m_u}{m_c^2 - m_u^2} \approx \frac{\epsilon_{12}}{m_s} - \frac{\epsilon_{21}^* m_u}{m_c^2}$$

$$V_{23} = \frac{\epsilon_{23} m_b}{m_b^2 - m_s^2} - \frac{\epsilon_{32}^* m_c}{m_c^2 - m_t^2} \approx \frac{\epsilon_{23}}{m_b} - \frac{\epsilon_{32}^* m_c}{m_t^2}$$

$$V_{13} = \frac{\epsilon_{13} m_b}{m_b^2 - m_d^2} - \frac{\epsilon_{31}^* m_u}{m_t^2 - m_u^2} \approx \frac{\epsilon_{13}}{m_b} - \frac{\epsilon_{31}^* m_u}{m_t^2}$$

It is clear that the down-quark mass matrix provides the dominant contribution to the mixing angles, with $\epsilon_{12} \approx 25MeV$, $\epsilon_{13} \approx 13MeV$ and $\epsilon_{23} \approx 150MeV$. We can choose the convention that ϵ_{12}, ϵ_{23} are real, and only ϵ_{13} complex in the way consistent to the Wolfenstein (Chau–Keung) parameterization.

One-loop: $\Delta\bar{\theta} = \left(\frac{1}{4\pi}\right)^2 \frac{\epsilon_{13}\,\epsilon_{23}^*\,\epsilon_{21}^*}{(\text{vev})^2\, m_u} K$

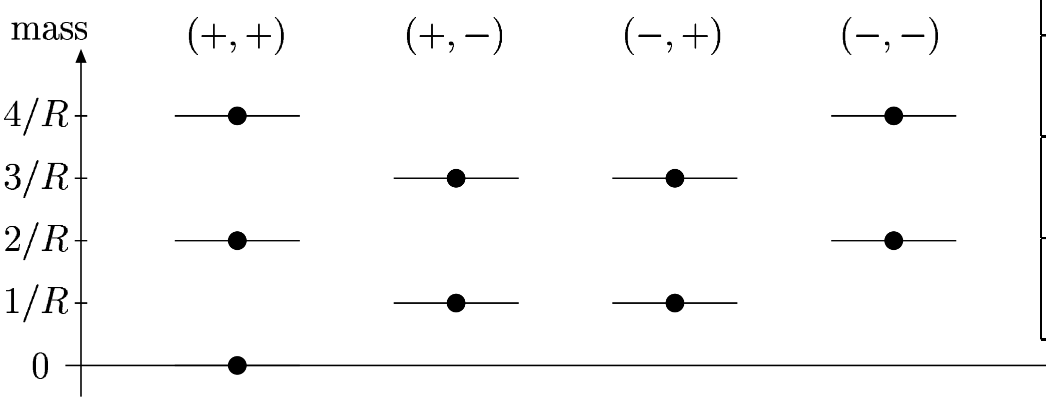
Geometric CP with Chang + Mohapatra, Phys. Lett. **B515** 431

The Fermion field in 5-D flat space time is decomposed into

$$i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + (\bar{\psi}_L\partial_y\psi_R - \bar{\psi}_R\partial_y\psi_L)$$

ψ_L and ψ_R have opposite Z_2

$S_1/(Z_2 \times Z_2')$



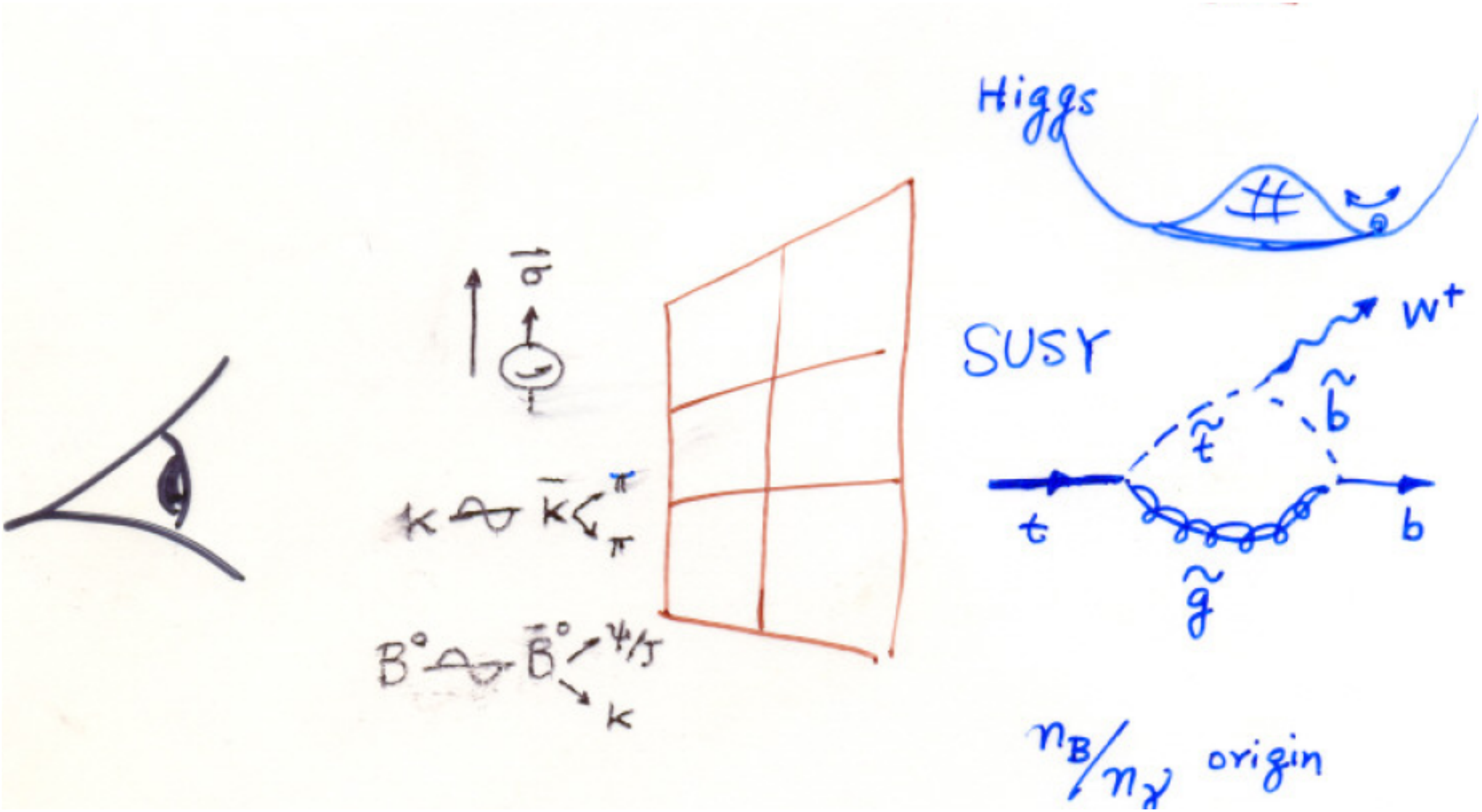
Fields	$Z_2 \times Z_2'$
$q_L, u_R, d_R, \psi_L, e_R, H_A$	$(+,+)$
$q_R, u_L, d_L, \psi_R, e_L, H_B$	$(-,-)$
$Q_L, U_L, D_L, \Psi_L, E_L$	$(+,-)$
$Q_R, U_R, D_R, \Psi_R, E_R$	$(-,+)$

CP invariant Yukawa coupling of the quarks in 5-dim

$$\mathcal{L}'_Y = \; h_d \bar{q} H_A d + h_u \bar{q} \tilde{H}_A u + h_d^* \bar{Q} H_B D + h_u^* \bar{Q} \tilde{H}_B U + H.c.$$

This leads to the familiar CKM Model.

Conclusion



It is like: we already know the answer. We just have to figure out what the question is. *Wolfenstein*