On neutrino and charged lepton masses and mixings: A view from the electroweak-scale right-handed neutrino model

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Outline

Motivation

The discovery of neutrino oscillation

- have revealed many valuable information concerning the mixing matrix U_{PMNS} and the Δm^2 in the neutrino sector.
- first evidence of physics beyond the Standard Model (BSM).

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Many puzzle questions!!!

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- The origin of neutrino masses?
- Why is the mass of neutrino so tiny ($m_\nu < O(eV)$)?
- Can we access experimentally the physics that are responsible for the tininess of the neutrino masses and their mixings?
- Why is the leptonic mixing matrix U_{PMNS} so different from V_{CKM} of the quark sector?

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- For the quark sector we use the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1]

$$|V_{CKM}| = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

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which is really close to a unit matrix.

- For the lepton sector, the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix is used to study the mixings [1]

$$|U_{PMNS}| = \begin{pmatrix} 0.779 \ldots 0.848 & 0.510 \ldots 0.604 & 0.122 \ldots 0.190 \\ 0.183 \ldots 0.568 & 0.385 \ldots 0.728 & 0.613 \ldots 0.794 \\ 0.200 \ldots 0.576 & 0.408 \ldots 0.742 & 0.589 \ldots 0.775 \end{pmatrix}$$

[1] Werner Rodejohann, 2012

Motivation

- Logical understanding of PMNS matrix within the framework of **Electroweak-scale Right-handed Neutrino (EW ν_R) Model**
- Nature of differences between CKM and PMNS matrices
- Extracting $\mathcal{M}_l \mathcal{M}_l^\dagger$ for the charged lepton sector

Neutrino masses

Dirac mass

Dirac neutrino masses are the neutrino analogues of the SM quark and charged lepton masses. They come from Yukawa coupling to the SM Higgs field $\tilde{\Phi}$

$$\underbrace{g_\nu}_{\text{Yukawa coupling}} \bar{\nu}_L \tilde{\Phi} \nu_R + h.c. \Rightarrow g_\nu \langle \tilde{\Phi} \rangle \bar{\nu}_L \nu_R + h.c. \equiv \underbrace{m_\nu^D}_{\text{Dirac Mass}} \left(\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R \right)$$

Dirac neutrino masses do not mix neutrinos and antineutrinos $\rightarrow$ lepton number is conserved

Neutrino masses

Majorana mass of ν_R

$$\underbrace{M_R}_{\text{Majorana mass}}\ \nu_R^T \sigma_2 \nu_R$$

Majorana masses do mix ν and $\bar{\nu}$ $\rightarrow$ lepton number is violated

Seesaw Mechanism

A generic model used to understand the observed neutrino masses ($\sim$ O(eV)), compared to those of quarks and charged leptons which are millions of times heavier.

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With $\chi \equiv \sigma_2 \nu_R^*$ and $\nu \equiv \nu_L$ the mass terms can be written as

$$\begin{pmatrix} \nu^T & \chi^T \end{pmatrix} \underbrace{\begin{pmatrix} 0 & m_\nu^D \\ m_\nu^D & M_R \end{pmatrix}}_{\text{M}} \sigma_2 \begin{pmatrix} \nu \\ \chi \end{pmatrix}$$

Seesaw Mechanism

With the assumption: $m_\nu^D \ll M_R$, diagonalizing the matrix $\mathbf{M}$ gives eigenvalues

$$m_\nu \approx \frac{\left(m_\nu^D\right)^2}{M_R} \text{ and } M_R$$

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Experimental neutrino mass

- Cosmological constraints [2]: $\sum m_\nu < 0.23 eV$
- Neutrino oscillation experiments [3]:
  the largest Δm^2 is $\Delta m^2_{atm} \cong 2.4 \times 10^{-3}\ eV^2 \Rightarrow$ the heaviest $m_\nu \gtrsim 4.9 \times 10^{-2}\ eV$.

Cosmology + Oscillation: $4.9 \times 10^{-2}\ eV \lesssim m_\nu^{heaviest} \lesssim 0.23\ eV$

[2] Planck 2015 results
[3] Particle Data Group

- ν_R is a singlet under $SU(2)_L \times U(1)_Y$.
- $M_R \sim$ Grand Unified (GUT) mass scale of 10^{16} GeV naturally.

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- Keeping the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$?
- No more forces added?


THE ANSWER IS
YES

The Non-sterile Electroweak-scale Right-handed neutrino (EW ν_R) Model

[P. Q. Hung, PLB 649 (2007)]

The EW ν_R Model [4]

[4] P.Q. Hung, 2007

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Model Content

What's new???

[4] P.Q. Hung, 2007

Model Content

Leptons

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$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

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$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \longleftrightarrow \quad l_R^M = \begin{pmatrix} \nu_R \\ e_R^M \end{pmatrix},$$

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Quarks

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$$e_R \longleftrightarrow e_L^M$$

Quarks

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \longleftrightarrow q_R^M = \begin{pmatrix} u_R^M \\ d_R^M \end{pmatrix},$$

$$u_R,\ d_R \longleftrightarrow u_L^M,\ d_L^M$$

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Mirror particles are totally different from the SM particles!

1. **EW precision**
   V. Hoang, P. Q. Hung and A. S. Kamat, Nucl. Phys. B **877**, 190 (2013) [arXiv:1303.0428 [hep-ph]].

2. **Implications of the 125-GeV SM-like scalar:** Dr Jekyll (SM-like) and Mr Hyde (very different from SM)
   V. Hoang, P. Q. Hung and A. S. Kamat, arXiv:1412.0343 [hep-ph] (To appear in Nuclear Physics B).

3. **Signals of mirror fermions** (Paper in preparation)
   P.Q. Hung, Trinh Le (UVA); Nandi, Chakdar, Gosh (Oklahoma State University).

Question

What is the Higgs sector to give Majorana and Dirac mass?

Majorana mass of ν_R

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$$L_M = g_M \left(l_R^{M,T} \; \sigma_2 \right) \; (i \; \tau_2 \; \tilde{\chi}) \; l_R^M \tag{1}$$

Majorana mass of ν_R

$$
\begin{aligned}
L_M &= g_M \left(l_R^{M,T}\ \sigma_2\right)\ (i\ \tau_2\ \tilde{\chi})\ l_R^M \qquad (1)\\
&= g_M\ \nu_R^T\ \sigma_2\ \nu_R\ \chi^0 - \frac{1}{\sqrt{2}}\ \nu_R^T\ \sigma_2\ e_R^M\ \chi^+ + \ldots
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\chi} &= (3,\ Y/2 = 1)\\
\tilde{\chi} &= \frac{1}{\sqrt{2}}\ \vec{\tau}.\vec{\chi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\chi^+ & \chi^{++} \\ \chi^0 & -\frac{1}{\sqrt{2}}\chi^+ \end{pmatrix}
\end{aligned}
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Majorana mass of ν_R

$$L_M = g_M \left(l_R^{M,T}\,\sigma_2\right)\,(i\,\tau_2\,\tilde{\chi})\,l_R^M \qquad (1)$$
$$= g_M\,\nu_R^T\,\sigma_2\,\nu_R\,\chi^0 - \frac{1}{\sqrt{2}}\,\nu_R^T\,\sigma_2\,e_R^M\,\chi^+ + \ldots$$

$$\tilde{\chi} = (3,\ Y/2 = 1)$$
$$\tilde{\chi} = \frac{1}{\sqrt{2}}\,\vec{\tau}.\vec{\chi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\chi^+ & \chi^{++} \\ \chi^0 & -\frac{1}{\sqrt{2}}\chi^+ \end{pmatrix}$$

From (1), the Majorana mass $M_R = g_M v_M$ where $\langle \chi^0 \rangle = v_M \sim \Lambda_{EW}$

Z-boson decay width: $M_R > M_Z/2$

Dirac mass

Dirac mass

The singlet scalar field ϕ_S couples to fermion bilinear.

$$\begin{aligned} L_S &= g_{Sl}\, \bar{l}_L\, \phi_S\, l_R^M + h.c. \\ &= g_{Sl}\bar{\nu}_L\, \phi_S\, \nu_R + ... + h.c. \end{aligned} \tag{2}$$

$$\phi_S\,(1,\ Y/2 = 0)$$

From (2), Dirac mass: $m_\nu^D = g_{Sl} v_S$ where $\langle \phi_S \rangle = v_S$.

Charged fermion mass

We also need a Higgs doublet for charged fermion masses (leptons and quarks)

$$L_{Y_l} = g_l \, \bar{l}_L \, \Phi \, e_R + h.c. \quad (3)$$

$$L_{Y_q} = g_q \, \bar{q}_L \, \Phi \, u_R + h.c. \quad (4)$$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \langle \phi^0 \rangle = \frac{v_2}{\sqrt{2}}$$

ρ parameter at the tree level

In the Standard Model,

$$\rho \equiv \frac{M_W^2}{M_Z^2\ cos^2\theta_W}$$

Experimentally, $\rho = 1$ to a good precision.

ρ parameter at the tree level

In Higgs sector [5]: a number of Higgs multiplets ϕ_k of isospin T_k and hypercharge Y_k

$$\rho = \frac{\sum_k [T_k(T_k+1) - \frac{1}{4}Y_k^2] v_k^2 c_k}{\sum_k \frac{1}{2} Y_k^2 v_k^2}$$

where $v_k \equiv$ VEV of the neutral component of the Higgs multiplet
$c_k = 1/2$ (1) for real (complex) multiplet

[5] Phys. Lett. B, 568 (2003)

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One would have $\rho \neq 1$ when both a triplet and a doublet are present.

[5] Phys. Lett. B, 568 (2003)

ρ parameter at the tree level

In order to restore Custodial global SU(2) symmetry ($\rho = 1$) at three level (Chanowitz, Golden and Georgi, Machacek), we add

$$\xi = (3, Y/2 = 0)$$

and group it with $\tilde{\chi} = (3, \; Y/2 = 1)$ in

$$\chi = \begin{pmatrix} \chi^0 & \xi^+ & \chi^{++} \\ \chi^- & \xi^0 & \xi^+ \\ \chi^{--} & \xi^- & \chi^{0*} \end{pmatrix} \qquad (5)$$

The doublet Higgs can be written as

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix} \qquad (6)$$

Proper vacuum alignment for custodial symmetry

$$\langle \chi^0 \rangle = \langle \xi^0 \rangle = v_M$$

$$\langle \chi \rangle = \begin{pmatrix} v_M & 0 & 0 \\ 0 & v_M & 0 \\ 0 & 0 & v_M \end{pmatrix}, \ and \ \langle \Phi \rangle = \begin{pmatrix} \frac{v_2}{\sqrt{2}} & 0 \\ 0 & \frac{v_2}{\sqrt{2}} \end{pmatrix}$$

With this vacuum alignment $SU(2)_L \times U(1)_Y \to U(1)_{em}$ and the global $SU(2)_D$ custodial symmetry is preserved.

Model of neutrino masses

It was conjectured by Cabibbo[6] and Wolfenstein[7] independently that

$$U_{CW} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \tag{7}$$

[6] N. Cabibbo, 1978
[7] L. Wolfenstein, 1978

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What's kind of symmetry that give rise to U_{CW}?

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What's kind of symmetry that give rise to U_{CW}?

A_4 Symmetry

---

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A_4 Symmetry

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Why A_4?

A_4 Symmetry

Why A_4?

With 3 families, we need a group containing a $\underline{3}$ representation.

The simplest one is A_4.

A_4 Symmetry

What is A_4?

A_4 Symmetry

What is A_4?

- Non-Abelian discrete group
- Four irreducible representations: **Three** 1-dimension representations called $\underline{1}$, $\underline{1}$', $\underline{1}$" and **One** 3-dimension representation called $\underline{3}$

A_4 Symmetry

If denoting $\underline{3}$ as (1, 2, 3) then

Multiplication rule[8]

$$\begin{aligned}\underline{3}\times\underline{3} &= \underline{1}(11+22+33)+\underline{1}'(11+\omega^2 22+\omega 33)+\underline{1}''(11+\omega 22+\omega^2 33)\\ &+ \underline{3}(23,31,12)+\underline{3}(32,13,21)\end{aligned}$$

where $\omega = e^{i2\pi/3}$

[8] Ernest Ma, 2007

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Assignments of the model's content

Field	$(\nu, l)_L$	$(\nu, l^M)_R$	e_R	e_L^M	ϕ_{0S}	$\tilde{\phi}_S$	Φ_2
A_4	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{3}$	$\underline{1}$

Neutrino Dirac mass

The Yukawa interactions

$$L_S = \bar{l}_L \left(g_{0S}\phi_{0S} + g_{1S}\tilde{\phi}_S + g_{2S}\tilde{\phi}_S\right) l_R^M + h.c. \tag{8}$$

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where g_{1S} and g_{2S} reflect the two different ways that $\tilde{\phi}_S$ couples to the product of $\bar{l}_L$ and l_R^M.

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Neutrino Dirac mass

Neutrino Dirac mass matrix:

$$M_\nu^D = \begin{pmatrix} g_{0S}v_0 & g_{1S}v_3 & g_{2S}v_2 \\ g_{2S}v_3 & g_{0S}v_0 & g_{1S}v_1 \\ g_{1S}v_2 & g_{2S}v_1 & g_{0S}v_0 \end{pmatrix} \tag{9}$$

where $v_0 = \langle \phi_{0S} \rangle$ and $v_i = \langle \phi_{iS} \rangle$ with $\imath = 1, 2, 3$.

Neutrino Dirac mass

If $v_1 = v_2 = v_3 = v \sim O(10^5\ eV)$ [10], M_ν^D can be diagonalized as follows

$$U_{\nu_L}^\dagger M_\nu^D U_{\nu_R} = U_\nu^\dagger M_\nu^D U_\nu = \begin{pmatrix} m_{1D} & 0 & 0 \\ 0 & m_{2D} & 0 \\ 0 & 0 & m_{3D} \end{pmatrix} \tag{10}$$

where $U_\nu = U_{CW}^\dagger = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$

Notice that $U_{\nu_L} = U_{\nu_R} = U_\nu$.

[10] P.Q. Hung, 2007

Charged-lepton mass

[11] P.Q. Hung, 2007

Charged-lepton mass

- Charged leptons can couple with singlet Higgs field which give rise to mass mixing between charged SM and mirror leptons. However, the mixing is very small so its contribution to the charged-lepton mass matrix can be negligible [11].

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- The Yukawa couplings (with Higgs doublet)

$$\begin{aligned} L_{Y_l} &= g_l \, \bar{l}_L \;\; \Phi_2 \;\; e_R + h.c. \\ &= \quad \underline{3} \otimes \underline{1} \otimes \underline{3} \end{aligned} \tag{11}$$

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Charged lepton mass

The charged-lepton mass matrix is

$$\mathcal{M}_l = g_l \frac{v_2}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{12}$$

Diagonalizing $\mathcal{M}_l$ by $U_{lL}^\dagger\ \mathcal{M}_l\ U_{lR}$ gives rise to

$$U_{lL} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{13}$$

The PMNS Matrix

$$U_{\nu_L} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} ; \; U_{lL} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The PMNS Matrix

$$U_{PMNS} = U_{\nu_L}^{\dagger} \; U_{lL} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \tag{14}$$

which mainly comes from neutrino mixing matrix.

Why is the U_{PMNS} different from the V_{CKM}?

- It has known that $V_{CKM} = U^{\dagger}_{U,L} U_{D,L}$ comes <u>totally</u> from couplings between quarks and Higgs doublet.
- We are showing that the $U_{PMNS} = U^{\dagger}_{\nu_L} U_{lL}$ comes from
  - $U_{lL} \Longleftarrow$ couplings between leptons and Higgs doublet
  - $U_{\nu_L} \Longleftarrow$ couplings between leptons and Higgs singlets

Why is the U_{PMNS} different from the V_{CKM}?

In a nutshell

There are two different sources of PMNS matrix whereas the CKM matrix comes totally from one source.

One expects a natural difference between V_{CKM} and U_{PMNS}.

Ansätz for U_{IL}

Ansätz for U_{lL}

A_4 requires degenerate charged leptons e, μ, $\tau \Rightarrow \; U_{lL} = \mathbb{I}$.

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Breaking A_4 by making some deviations from U_{lL}

Ansätz for U_{lL}

A_4 requires degenerate charged leptons e, μ, τ $\Rightarrow$ $U_{lL} = \mathbb{I}$.

Breaking A_4 by making some deviations from U_{lL}

We can use Wolfenstein parameters to construct U_{lL}.

$$U_{lL} \to U_{lL} = \begin{pmatrix} 1 - \frac{\lambda_l^2}{2} & \lambda_l & A_l \lambda_l^3(\rho_l - i\eta_l) \\ -\lambda_l & 1 - \frac{\lambda_l^2}{2} & A_l \lambda_l^2 \\ A_l \lambda_l^3(1 - \rho_l - i\eta_l) & -A_l \lambda_l^2 & 1 \end{pmatrix} \qquad (15)$$

where $A_l, \ \rho_l, \ \eta_l$ are real parameters of O(1).

Ansätz for U_{lL}

$$U_{PMNS} = U^{\dagger}_{\nu_L} U_{lL} =$$

$$\frac{1}{\sqrt{3}}\begin{pmatrix} A_l\lambda_l^3(1-\rho_l-i\eta_l)-\frac{\lambda_l^2}{2}-\lambda_l+1 & -\left(A_l+\frac{1}{2}\right)\lambda_l^2+\lambda_l+1 & A_l\lambda_l^3(\rho_l-i\eta_l)+A_l\lambda_l^2+1 \\ \omega^2A_l\lambda_l^3(1-\rho_l-i\eta_l)-\frac{\lambda_l^2}{2}-\omega\lambda_l+1 & -\left(\omega^2A_l+\frac{\omega}{2}\right)\lambda_l^2+\lambda_l+\omega & A_l\lambda_l^3(\rho_l-i\eta_l)+\omega A_l\lambda_l^2+\omega^2 \\ \omega A_l\lambda_l^3(1-\rho_l-i\eta_l)-\frac{\lambda_l^2}{2}-\omega^2\lambda_l+1 & -\left(\omega A_l+\frac{\omega^2}{2}\right)\lambda_l^2+\lambda_l+\omega^2 & A_l\lambda_l^3(\rho_l+i\eta_l)+\omega^2A_l\lambda_l^2+\omega \end{pmatrix}$$

Toward $\mathcal{M}_l\mathcal{M}_l{}^\dagger$

Diagonalizing mass matrices $\mathcal{M}_l$ and $\mathcal{M}_l{}^\dagger$ as follows.

$$U_{lL}^\dagger \mathcal{M}_l U_{lR} \quad ; \quad U_{lR}^\dagger \mathcal{M}_l^\dagger U_{lL}$$

Therefore,

$$U_{lL}^\dagger \mathcal{M}_l\mathcal{M}_l{}^\dagger U_{lL} = \begin{pmatrix} {m_e}^2 & 0 & 0 \\ 0 & {m_\mu}^2 & 0 \\ 0 & 0 & {m_\tau}^2 \end{pmatrix}$$

$$\mathcal{M}_l\mathcal{M}_l{}^\dagger = U_{lL} \cdot \begin{pmatrix} {m_e}^2 & 0 & 0 \\ 0 & {m_\mu}^2 & 0 \\ 0 & 0 & {m_\tau}^2 \end{pmatrix} \cdot \; U_{lL}^\dagger \qquad (16)$$

Toward $\mathcal{M}_l \mathcal{M}_l^\dagger$

$\Rightarrow$ Up to O(λ_l^2)

$$\begin{pmatrix} (1-\lambda_l^2)\ {m_e}^2 + \lambda_l\ {m_\mu}^2 & \lambda_l({m_\mu}^2 - {m_e}^2) & 0 \\ \lambda_l({m_\mu}^2 - {m_e}^2) & (1-\lambda_l^2)\ {m_\mu}^2 + \lambda_l\ {m_e}^2 & A\lambda_l^2({m_\tau}^2 - {m_\mu}^2) \\ 0 & A_l\lambda_l^2({m_\tau}^2 - {m_\mu}^2) & {m_\tau}^2 \end{pmatrix} \tag{17}$$

A_l, λ_l are extracted from U_{PMNS} and experimental values m_e, m_μ, m_τ.

Summary

- The differences between CKM and PMNS matrices come from the fact that U_{PMNS} is constructed by couplings with Higgs singlets and mainly comes from neutrinos.

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Summary

- The differences between CKM and PMNS matrices come from the fact that U_{PMNS} is constructed by couplings with Higgs singlets and mainly comes from neutrinos.
- The simplicity of our approach as compared with previous works is due to the source of the neutrino Dirac masses which comes from the Higgs singlets as opposed to Higgs doublets.
- By slightly breaking A_4 symmetry, we avoided the case of degenerate charged-lepton mass and were able to extract $\mathcal{M}_l \mathcal{M}_l{}^\dagger$ for the charged-lepton sector (as well as the quark sector).

Thank you!

Appendix

1. Characters of A_4 reperesentations

A_4	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3
C_1	1	1	1	1	3
C_3	2	1	1	1	-1
C_4	3	1	ω	ω^2	0
$C_{4'}$	3	1	ω^2	ω	0

where $\omega = e^{i2\pi/3}$ which is the cube root of unity.

Appendix

2. Constraints on A, λ, ρ and η

$$
\begin{aligned}
&(1)\quad 0.779 < \frac{1}{\sqrt{3}}\left|A\lambda^3(1-\rho-i\eta) - \frac{\lambda^2}{2} - \lambda + 1\right| < 0.848\\
&(2)\quad 0.510 < \frac{1}{\sqrt{3}}\left|-\left(A+\frac{1}{2}\right)\lambda^2 + \lambda + 1\right| < 0.604\\
&(3)\quad 0.122 < \frac{1}{\sqrt{3}}\left|A\lambda^3(\rho-i\eta) + A\lambda^2 + 1\right| < 0.190\\
&(4)\quad 0.183 < \frac{1}{\sqrt{3}}\left|\omega^2 A\lambda^3(1-\rho-i\eta) - \frac{\lambda^2}{2} - \omega\lambda + 1\right| < 0.568\\
&(5)\quad 0.385 < \frac{1}{\sqrt{3}}\left|-\left(\omega^2 A+\frac{\omega}{2}\right)\lambda^2 + \lambda + \omega\right| < 0.728\\
&(6)\quad 0.613 < \frac{1}{\sqrt{3}}\left|A\lambda^3(\rho-i\eta) + \omega A\lambda^2 + \omega^2\right| < 0.794\\
&(7)\quad 0.200 < \frac{1}{\sqrt{3}}\left|\omega A\lambda^3(1-\rho-i\eta) - \frac{\lambda^2}{2} - \omega^2\lambda + 1\right| < 0.576\\
&(8)\quad 0.408 < \frac{1}{\sqrt{3}}\left|-\left(\omega A+\frac{\omega^2}{2}\right)\lambda^2 + \lambda + \omega^2\right| < 0.742\\
&(9)\quad 0.589 < \frac{1}{\sqrt{3}}\left|A\lambda^3(\rho-i\eta) + \omega^2 A\lambda^2 + \omega\right| < 0.775
\end{aligned}
$$

$$-4.8517 < A < -4.4580, \quad -0.2404 < \lambda < -0.1882,$$

Appendix

3. Sample numerical results

Taking upper limit values of $A = -4.4580$, $\lambda = -0.1882$, $\rho = -5.5712$ and $\eta = 4.8912$

$$U_l = \begin{pmatrix} 0.9823 & -0.1882 & -0.1656 - 0.1454i \\ 0.1882 & 0.9823 & -0.1579 \\ 0.1953 - 0.1454i & 0.1579 & 1 \end{pmatrix}$$

$$U_l U_l^{\dagger} = \begin{pmatrix} 1.0489 & 0.0261 + 0.0230i & -0.0035 - 0.0026i \\ 0.0261 - 0.0230i & 1.0253 & 0.0340 + 0.0274i \\ -0.0035 + 0.0026i & 0.0340 - 0.0274i & 1.0842 \end{pmatrix}$$

$$\simeq \mathbb{I}$$

Appendix

Using the above numerical U_l and putting in the values of $m_e = 0.51 \times 10^{-3}$ GeV, $m_\mu = 0.1057$ GeV and $m_\tau = 1.7768$ GeV we get

$$\mathcal{M}_l \mathcal{M}_l^\dagger \simeq \begin{pmatrix} 0.1537 & 0.0805 + 0.0725i & -0.5231 - 0.4590i \\ 0.0805 - 0.0725i & 0.0895 & -0.4968 \\ -0.5231 + 0.4590i & -0.4968 & 3.1573 \end{pmatrix}$$

Appendix

4. Possible signature of EW ν_R model

The fact

1. ν_R interacts with the W and Z (part of a doublet)
2. Both ν_R and e_R^M interact with ν_L and e_L through the singlet scalar field ϕ_S

Since $m_{\phi_S} \sim O(10^5\ eV)$, it's possible

$$\begin{aligned} \nu_R &\to \nu_L + \phi_S \\ e_R^M &\to e_L + \phi_S \end{aligned}$$

If $m_{\nu_R} \lesssim m_{e_R^M}$:

$$\begin{aligned} e_M^R \to \quad & \nu_R + e_L + \bar{\nu}_L \\ & \nu_R \to \nu_L + \phi_S \end{aligned}$$

Possible signature of EW ν_R model

The heaviest ν_R could be pair produced

$$\begin{aligned} q + \bar{q} &\rightarrow Z \rightarrow \nu_R + \nu_R \\ \nu_R &\rightarrow e_R^M + W^*(W) \\ e_R^M &\rightarrow e_L + \phi_S \end{aligned}$$

at a 'displaced' vertex.
If ν_R is Majorana

$$e_R^{M,-} + W^+ + e_R^{M,-} + W^+ \rightarrow e_L + e_L + W^+ + W^+ + 2\phi_S$$

same-sign dilepton event which is distinctively different from the Dirac case!