Transformative A_4 Neutrino Mixing

Ernest Ma

Physics and Astronomy Department
University of California
Riverside, CA 92521, USA

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Brief History of A_4

In 1978 (37 years ago), soon after the putative discovery of the third family of leptons and quarks, it was conjectured independently by Cabibbo and Wolfenstein:

$$U_{l\nu} = U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. In the PDG convention, this implies $s_{23} = c_{23} = 1/\sqrt{2}$, $s_{12} = c_{12} = 1/\sqrt{2}$, $s_{13} = 1/\sqrt{3}$, $c_{13} = \sqrt{2/3}$, and $\delta = \pi/2$. If $\omega \leftrightarrow \omega^2$, then $\delta = -\pi/2$.

In 2001 (14 years ago), without knowing about Cabibbo and Wolfenstein, U_ω was discovered by Ma and Rajasekaran in the context of A_4.

This non-Abelian discrete symmetry has 12 elements and 4 irreducible representations: $\underline{1}, \underline{1}', \underline{1}'', \underline{3}$. Using

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}.$$

the following decompositions are obtained:

$$\begin{aligned} \underline{1} &= 11 + 22 + 33, \\ \underline{1}' &= 11 + \omega 22 + \omega^2 33, \\ \underline{1}'' &= 1 + \omega^2 22 + \omega 33. \end{aligned}$$

Let $(\nu, l)_i \sim \underline{3}$, $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, and $\Phi_i \sim \underline{3}$, then

$$\mathcal{M}_l = \begin{pmatrix} f_e v_1^* & f_\mu v_1^* & f_\tau v_1^* \\ f_e v_2^* & f_\mu \omega v_2^* & f_\tau \omega^2 v_2^* \\ f_e v_3^* & f_\mu \omega^2 v_3^* & f_\tau \omega v_3^* \end{pmatrix}$$

$$= \begin{pmatrix} v_1^* & 0 & 0 \\ 0 & v_2^* & 0 \\ 0 & 0 & v_3^* \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} f_e & 0 & 0 \\ 0 & f_\mu & 0 \\ 0 & 0 & f_\tau \end{pmatrix}.$$

For $v_1 = v_2 = v_3$, a residual Z_3 symmetry exists with $U_\omega^\dagger$ as the link between $\mathcal{M}_l$ and $\mathcal{M}_\nu$.

For many years, theoretical effort was focused on obtaining a specific form of $\mathcal{M}_\nu$ so that tribimaximal neutrino mixing is realized:

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} =$$

$$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & i \end{pmatrix}.$$

This means that

$$\mathcal{M}_\nu = \begin{pmatrix} m_2 & 0 & 0 \\ 0 & (m_1 - m_3)/2 & (m_1 + m_3)/2 \\ 0 & (m_1 + m_3)/2 & (m_1 - m_3)/2 \end{pmatrix}.$$

Pioneer A_4 papers: Ma/Rajasekaran(2001), Ma(2002), Babu/Ma/Valle(2003), Ma(2004), Altarelli/Feruglio(2005), Babu/He(2005).

This $\mathcal{M}_\nu$ is very hard to obtain in the context of a four-dimensional renormalizable field theory, because of the basic clash (or misalignment) of the residual symmetries (Z_3 for $\mathcal{M}_l$ and Z_2 for $\mathcal{M}_\nu$) [Lam]

On March 8, 2012, Daya Bay announced that θ_{13} had been measured at 8.8°, thus ending tribimaximal mixing.

The 2014 PDG values are: $\sin^2(2\theta_{12}) = 0.846 \pm 0.021$,
$\Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5}\ \mathrm{eV}^2$,
$\sin^2(2\theta_{23}) = 0.999\ (+0.001/-0.018)$,
$\Delta m^2_{32} = (2.44 \pm 0.06) \times 10^{-3}\ \mathrm{eV}^2$ (normal),
$\sin^2(2\theta_{23}) = 1.000\ (+0.000/-0.017)$,
$\Delta m^2_{32} = (2.52 \pm 0.07) \times 10^{-3}\ \mathrm{eV}^2$ (inverted),
$\sin^2(2\theta_{13}) = (9.3 \pm 0.8) \times 10^{-2}$.

In retrospect, the $Z_3 - Z_2$ clash should have been a warning against tribimaximal mixing.

Special Form of $\mathcal{M}_\nu$

Ma(2002), Babu/Ma/Valle(2003), Grimus/Lavoura(2004):
A special form of the neutrino mass matrix (in the basis where the charged-lepton mass matrix is diagonal) was written down 13 years ago, i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix},$$

where A, B are real.

This allows $\theta_{13} \neq 0$ and yet $\theta_{23} = \pi/4$ is maintained, together with the prediction that $\delta_{CP} = \pm\pi/2$. This pattern is protected by a symmetry, i.e. $e \to e$ and $\mu \leftrightarrow \tau$ exchange with CP conjugation. Present T2K data with input from reactor data indicate a preference for $\delta_{CP} = -\pi/2$.

Note that this special form predicts that $|U_{\mu i}| = |U_{\tau i}|$. This harkens back to the original U_ω of 1978, where indeed this is satisfied. It is strongly suggestive that U_ω itself must have something to do with the realization of this special form of $\mathcal{M}_\nu$.

Since 2012, many authors have incorporated this generalized CP transformation into non-Abelian discrete symmetries (some rather complicated) to pin down the other angles, i.e. θ_{12} and θ_{13}.

See for example: Mohapatra/Nishi(2012),
Holthausen/Lindner/Schmidt(2013),
Feruglio/Hagedorn/Ziegler(2013, 2014),
Chen/Fallbacher/Mahanthappa/Ratz/Trautner(2014),
Hagedorn/Meroni/Molinaro(2014),
Ding/King/Neder(2014).

Typical result links θ_{12} with θ_{13}.

A_4 Bounces Back

Whereas tribimaximal mixing is dead, A_4 is not.
In fact, two papers appeared just recently:

X.-G. He, arXiv:1504.01560; E. Ma, arXiv:1504.02086;

which use A_4 to obtain $\theta_{23} = \pi/4$ and $\delta_{CP} = -\pi/2$, and in the case of the former, also $\sin^2\theta_{12} = 1/3$.

This talk is on the latter paper which also addresses the issue of dark matter and the apparent one Higgs boson of electroweak symmetry breaking.

The first observation is that if $\mathcal{M}_\nu$ is somehow purely real in the A_4 basis, then

$$\mathcal{M}_\nu^{(e,\mu,\tau)} = U_\omega^\dagger \begin{pmatrix} a & c & e \\ c & d & b \\ e & b & f \end{pmatrix} U_\omega^* = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix},$$

where $A = (a + 2b + 2c + d + 2e + f)/3$,
$B = (a - b - c + d - e + f)/3$,
$C = (a - b - \omega c + \omega^2 d - \omega^2 e + \omega f)/3$,
$D = (a + 2b + 2\omega c + \omega^2 d + 2\omega^2 e + \omega f)/3$.
The special form of $\mathcal{M}_\nu$ is thus automatically obtained.

The Majorana neutrino mass matrix is in general complex, so how does one guarantee it to be real? The answer was already there in a radiative inverse seesaw model of neutrino mass [Fraser/Ma/Popov(2014), Ma/Natale/Popov(2015)], where the origin of the neutrino mass matrix is that of a real scalar mass-squared matrix.

Actually, to obtain $|U_{\mu i}| = |U_{\tau i}|$, all that is required is

$$U_{l\nu} = U_{\omega}^{\dagger} \mathcal{O},$$

where $\mathcal{O}$ is a real orthogonal matrix.

Radiative Neutrino Mass with Dark Matter

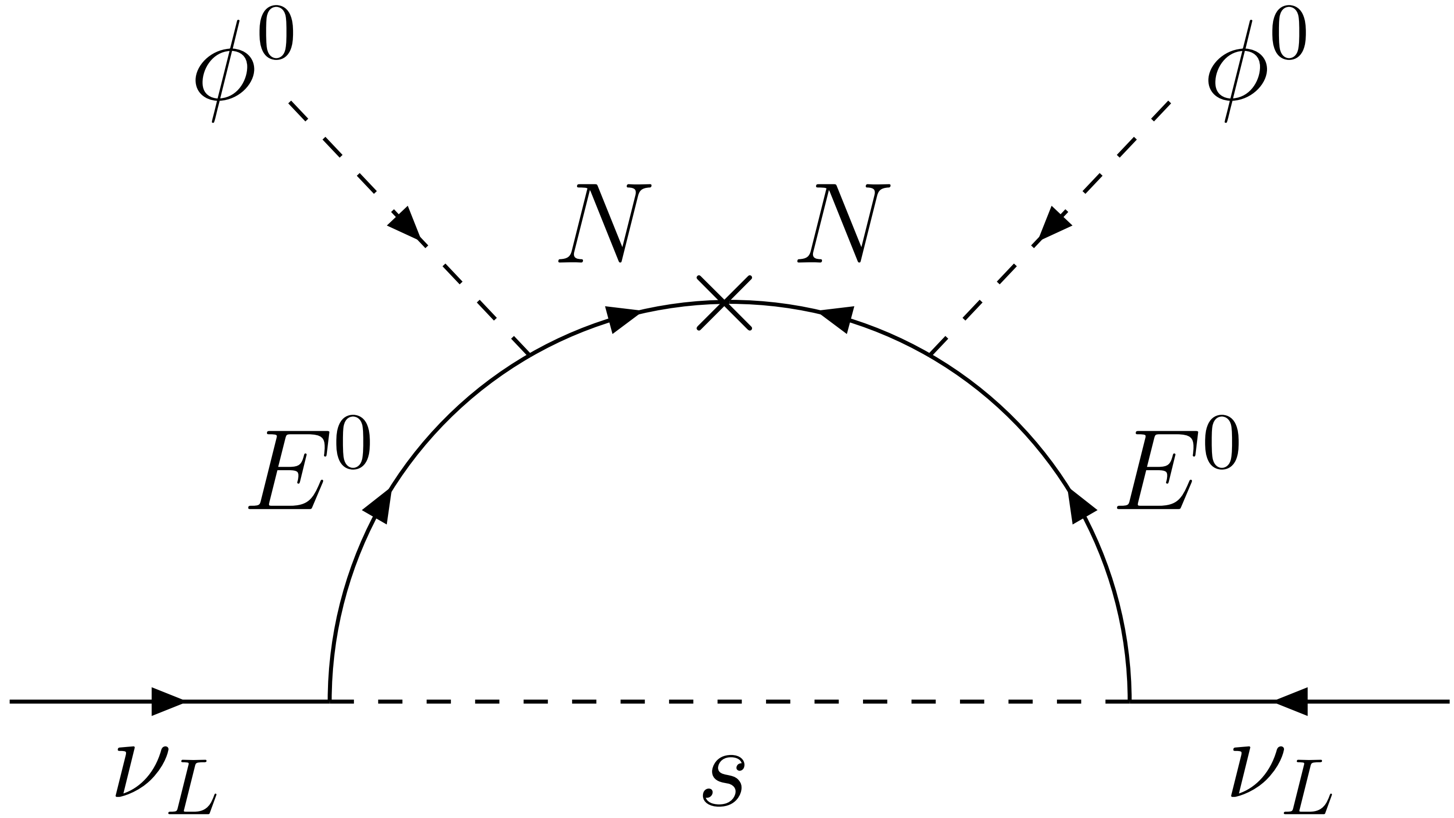
Under A_4, let the three families of leptons transform as

$$(\nu_i, l_i)_L \sim \underline{3}, \quad l_{iR} \sim \underline{1}, \underline{1}', \underline{1}''.$$

Add the following new particles, all assumed odd under an exactly conserved discrete Z_2 (dark) symmetry, whereas all SM particles are even:

$$(E^0, E^-)_{L,R} \sim \underline{1}, \quad N_{L,R} \sim \underline{1}, \quad s_i \sim \underline{3},$$

where (E^0, E^-) is a fermion doublet, N a neutral fermion singlet, and $s_{1,2,3}$ are real neutral scalar singlets.


ϕ^0
ϕ^0
N
N
E^0
E^0
ν_L
s
ν_L

The linkage of neutrino mass to dark matter provides an important clue to the scale of new physics. It is a possible answer to the Question: Is the new physics responsible for neutrino mass also responsible for some other phenomenon in particle physics and astrophysics? Here the Answer is yes, and it is dark matter. Since dark matter is mostly assumed to be a Weakly Interacting Massive Particle (WIMP), its mass scale is reasonably set at 1 TeV. This is the crucial missing piece of information which allows us to expect observable new physics related to both dark matter and neutrino mass at the LHC.

The mass matrix linking $(\bar{N}_L, \bar{E}_L^0)$ to (N_R, E_R^0) is given by

$$\mathcal{M}_{N,E} = \begin{pmatrix} m_N & m_D \\ m_F & m_E \end{pmatrix},$$

where m_N, m_E are invariant mass terms, and m_D, m_F come from the respective Higgs couplings with $\langle \phi^0 \rangle = v/\sqrt{2}$. As a result, N and E^0 mix to form two Dirac fermions of masses $m_{1,2}$ with mixing angles

$m_D m_E + m_F m_N = \sin\theta_L \cos\theta_L (m_1^2 - m_2^2)$,

$m_D m_N + m_F m_E = \sin\theta_R \cos\theta_R (m_1^2 - m_2^2)$.

The mass terms $(m_{L,R}/2) N_{L,R} N_{L,R}$ also exist.

$$m_\nu = f^2 m_R s_R^2 c_R^2 (m_1^2 - m_2^2)^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_1^2)^2} \frac{1}{(k^2 - m_2^2)^2}$$

$$+ f^2 m_L m_1^2 s_R^2 c_L^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_1^2)^2}$$
$$+ f^2 m_L m_2^2 s_L^2 c_R^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_2^2)^2}$$
$$- 2 f^2 m_L m_1 m_2 s_L s_R c_L c_R \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_1^2)} \frac{1}{(k^2 - m_2^2)},$$

where s is a mass eigenstate. If A_4 is unbroken, then $\mathcal{M}_\nu$ is proportional to the identity matrix. However, if A_4 is softly broken by the necessarily real $s_i s_j$ terms, then

$$\mathcal{M}_\nu = \mathcal{O} \begin{pmatrix} m_{\nu 1} & 0 & 0 \\ 0 & m_{\nu 2} & 0 \\ 0 & 0 & m_{\nu 3} \end{pmatrix} \mathcal{O}^T,$$

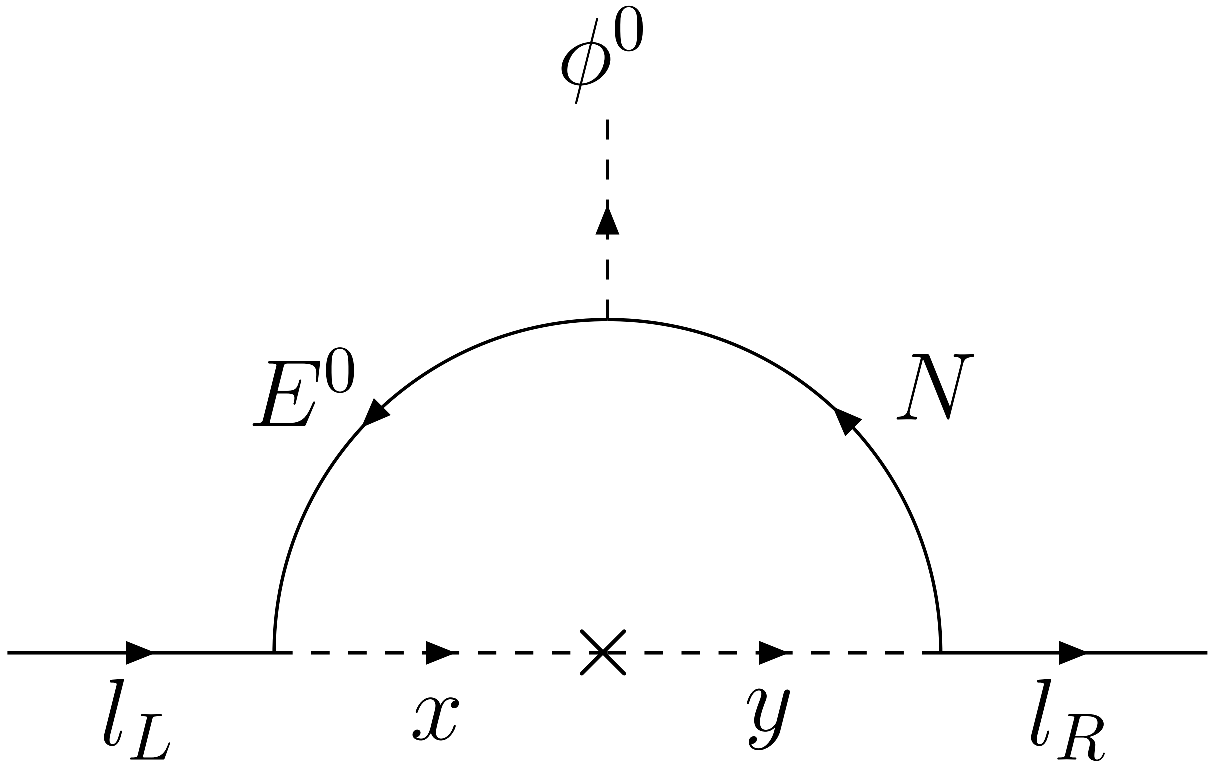
where $\mathcal{O}$ is an orthogonal matrix. Whereas f, m_L, m_R may be complex, only the relative phase between m_L and m_R appears in the two relative intrinsic Majorana phases of the neutrino mass eigenstates from the different s masses. Thus the desired form of $U_{l\nu}$ is obtained with $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$, once U_ω is applied.

Radiative Lepton Mass with Dark Matter

Instead of using three Higgs doublets $\Phi_i \sim \underline{3}$ to obtain U_ω in the charged-lepton sector as in the original A_4 model of 2001, a radiative model of lepton mass is proposed. Again the fermion doublet (E^0, E^-) and singlet N are used, but now in conjunction of two sets of charged scalars which are also odd under dark Z_2, i.e.

$$x_i^- \sim \underline{3}, \quad y_i^- \sim \underline{1}, \underline{1}', \underline{1}''.$$

To connect x with y, the trilinear scalar term $x_i y_j^* \chi_k$ is added, where $\chi \sim \underline{3}$ is even under Z_2 with equal $\langle \chi_i \rangle$.


ϕ^0
E^0
N
l_L
x
y
l_R

Each lepton mass is then given by

$$m_l = f' f_l \mu_l u \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_{1l}^2)(k^2 - m_{2l}^2)} \left[\frac{m_1 c_R s_L}{k^2 - m_1^2} - \frac{m_2 c_L s_R}{k^2 - m_2^2} \right],$$

where f' is the $E_L^0 l_L x^*$ Yukawa coupling, f_l is the $N_R l_R y^*$ Yukawa coupling, μ_l is the scalar $xy^*\chi$ coupling, u is the vacuum expectation value of χ_i, and $m_{1l,2l}$ are the mass eigenvalues of the 2×2 mass-squared matrix

$$\mathcal{M}_{xy}^2 = \begin{pmatrix} m_x^2 & \mu_l u \\ \mu_l u & m_y^2 \end{pmatrix},$$

with $\mu_l u = \sin\theta_l \cos\theta_l (m_{1l}^2 - m_{2l}^2)$.

There is a one-to-one correlation of the neutrino mass eigenstates to the $s_{1,2,3}$ mass eigenstates, the lightest of which is dark matter.

It is also clear that all three neutrino masses are expected to be of the same order of magnitude and their mass-squared differences are related to the scalar mass differences. Using the most recent cosmological data

$$\sum m_\nu < 0.23 \text{ eV},$$

the effective neutrino mass m_{ee} in neutrinoless double beta decay is bounded below 0.07 eV for normal ordering and 0.08 eV for inverted ordering.

Ma(2015):

The dark matter parity of this model is also derivable from lepton parity.

Under lepton parity, let the new particles $(E^0, E^-), N, \chi$ be even and s, x, y be odd, then the same Lagrangian is obtained. As a result, dark parity is simply given by $(-1)^{L+2j}$, which is odd for all the new particles and even for all the SM particles (and χ).

Note that the tree-level Yukawa coupling $\bar{l}_L l_R \phi^0$ would be allowed by lepton parity alone, but is forbidden here because of the A_4 symmetry.

The radiative lepton mass matrix is diagonal because of the Z_3 residual symmetry. This means that the muon anomalous magnetic moment Δa_μ gets a significant contribution from xy exchange, but not $\mu \to e\gamma$. Because Δa_μ is now of order m_μ^2/m_E^2 instead of the usual $(16\pi^2)^{-1} m_\mu^2/m_E^2$, a large $m_E \sim 1$ TeV is possible for the explanation of the experimental-theoretical discrepancy instead of the usual $m_E \sim 200$ GeV.

As for $\mu \to e\gamma$, it will come from s exchange in the analog diagram to radiative neutrino mass. For $m_E \sim 1$ TeV, it will be suitably suppressed.

Conclusion

The notion that neutrino mass is radiatively induced by dark matter is a powerful indication of the scale of new physics, i.e. 1 TeV.

In this framework, the A_4 transformation U_ω has been used to obtain a desirable form of $U_{l\nu}$, i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$, automatically if the origin of the neutrino mass matrix is a set of real scalars $s_i \sim \underline{3}$ under A_4.

Only one Higgs doublet with $\langle\phi^0\rangle$ accounting for all of electroweak symmetry breaking is required. New particles $(E^0, E^-), N, x, y, s, \chi$ are predicted.