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粒子物理和核物理研究所

# S-Wave Contributions in Semi-Leptonic B decays

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11<sup>th</sup> Particle Physics Phenomenology Workshop

- Introduction
- Semi-Leptonic B decays into a vector  
 $B \rightarrow \rho l \nu; B \rightarrow K^* l^+ l^-$
- S-wave contributions
- Results based on  $\chi$ PT
- Summary and Outlook

# SM is complete but...



- Matter–AntiMatter Asymmetry
- CP violation
- Dark Matter
- New Physics
- ...

Mass Eigenstates  $\neq$  Weak Eigenstates  $\Rightarrow$  Quark Mixing

$$V_{\text{CKM}} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

## CKM Matrix

Complex matrix described by 4 independent real parameters

### Wolfenstein parametrization:

$$V_{\text{CKM}} \approx \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

phase  $\rightarrow$

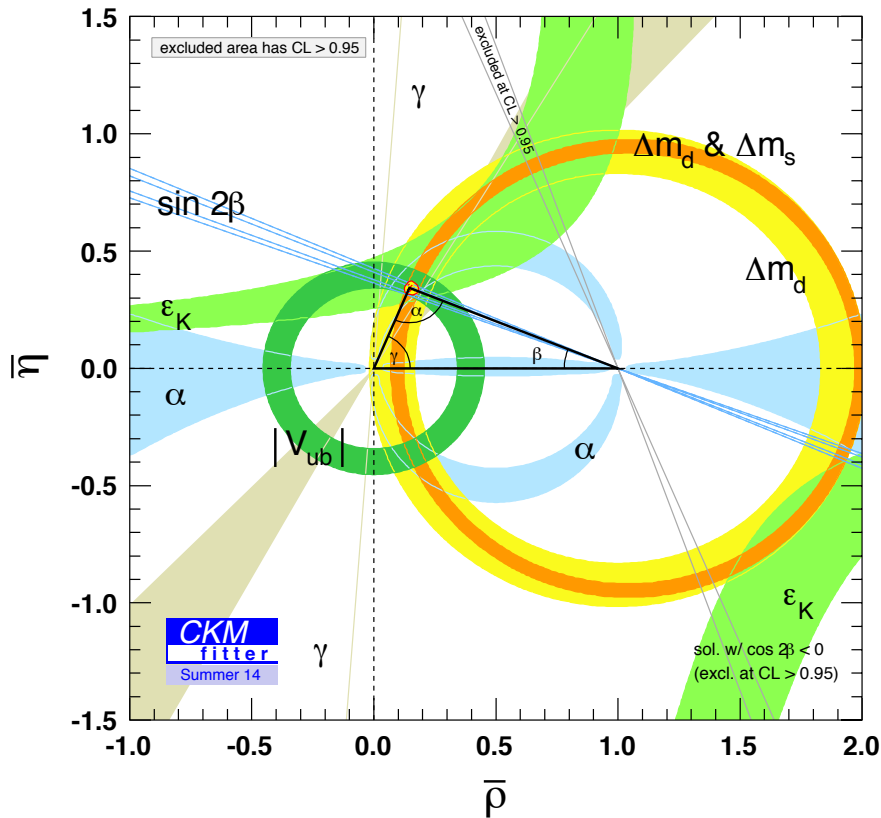
### CP Violation:

$$J = \text{Im} \left( V_{ik} V_{jk}^* V_{j\ell} V_{i\ell}^* \right) \neq 0$$

$$J \approx A^2 \lambda^6 \eta$$

$\eta = 0 \Rightarrow$  no CPV from SM

# 3σ deviation in $|V_{ub}|$



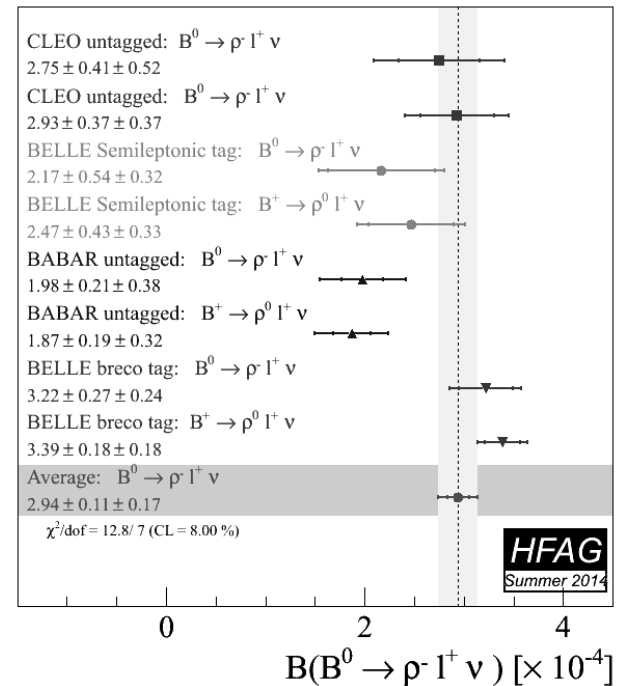
$$|V_{ub}| = (4.41 \pm 0.15 \pm_{-0.17}^{+0.15}) \times 10^{-3} \quad (\text{inclusive}),$$

$$|V_{ub}| = (3.23 \pm 0.31) \times 10^{-3} \quad (\text{exclusive}).$$

$B \rightarrow \pi l \nu$

More channels:

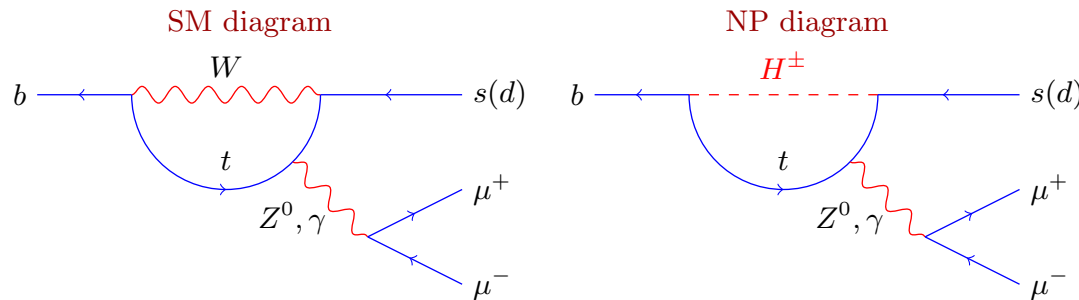
$B_s \rightarrow K l \nu$   
 $B \rightarrow \rho l \nu$



# Indirect Search for NP



- Flavor Changing Neutral Currents (FCNC) is forbidden at tree level in SM, but can proceed via the loop diagrams.
- New particles may change the amplitudes



- Rare decays FCNC can probe NP virtual particles

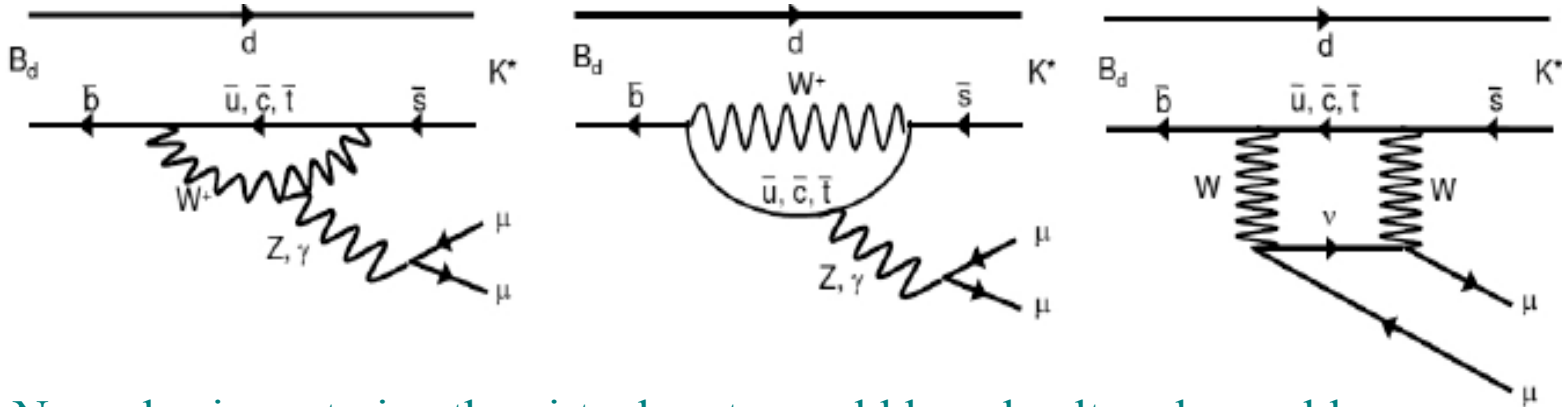
- NP phases
- Masses, Couplings
- Helicity structure

*See Prof. W.S. Hou's talk*

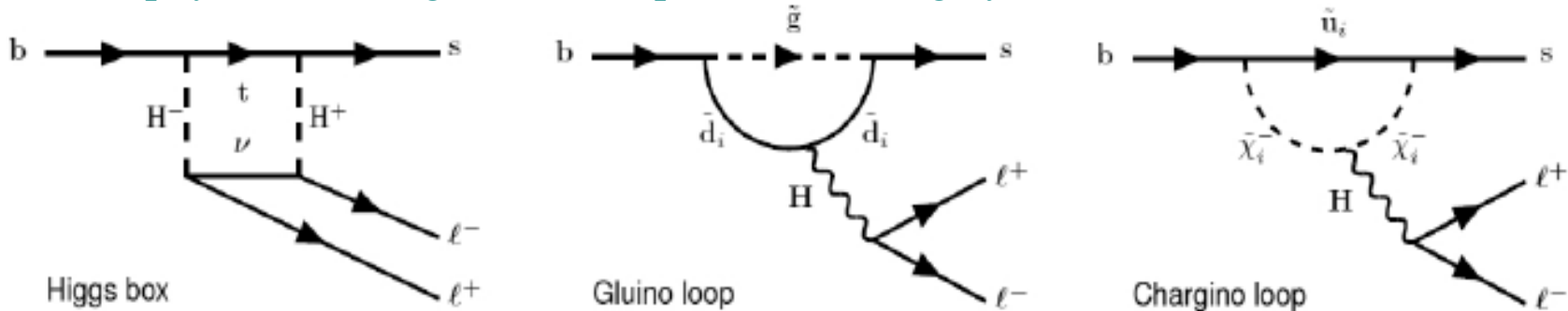
# B → K\* |+-



- Within the SM, these processes proceed via loop diagrams like



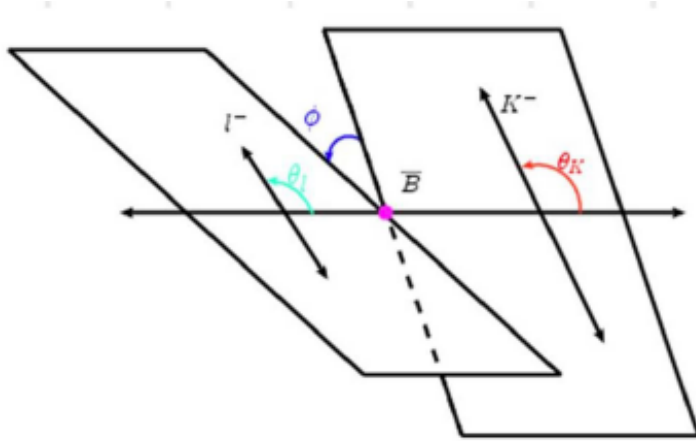
- New physics entering the virtual parts, could largely alter observables



- Effective Hamiltonian: 
$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} (C_i^{SM} + \Delta C_i^{NP}) O_i$$

Wilson coeffs. (short-dist. interactions)     Operators (long-dist. interactions)

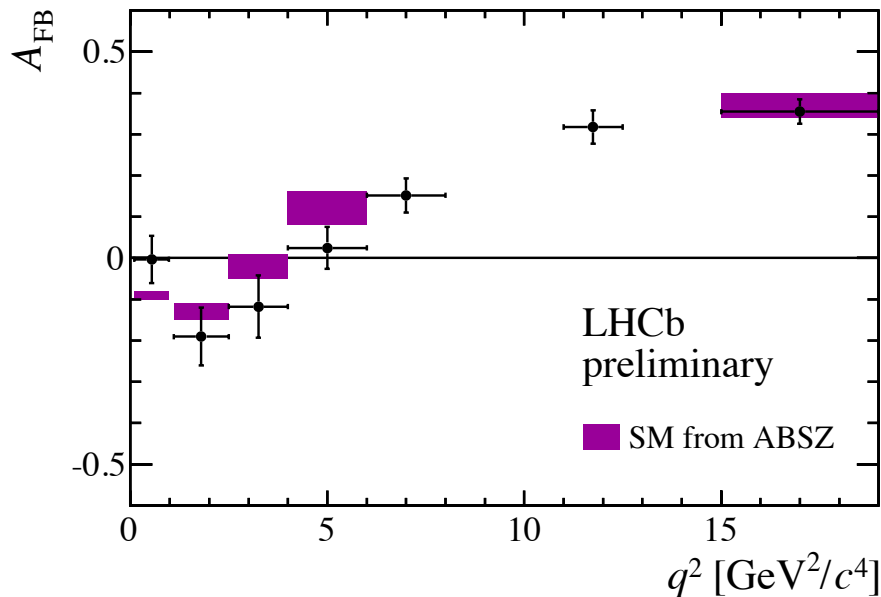
# Forward-backward asymmetry



- $\theta_l$ : angle of emission between  $K^{*0}$  and  $\mu^-$  in di-lepton rest frame
- $\theta_{K^*}$ : angle of emission between  $K^{*0}$  and  $K^-$  in di-meson rest frame.
- $\phi$ : angle between the two planes
- $q^2$ : dilepton invariant mass square

$$A_{\text{FB}}(q^2) = \frac{P_{\text{F}}(q^2) - P_{\text{B}}(q^2)}{P_{\text{F}}(q^2) + P_{\text{B}}(q^2)}$$

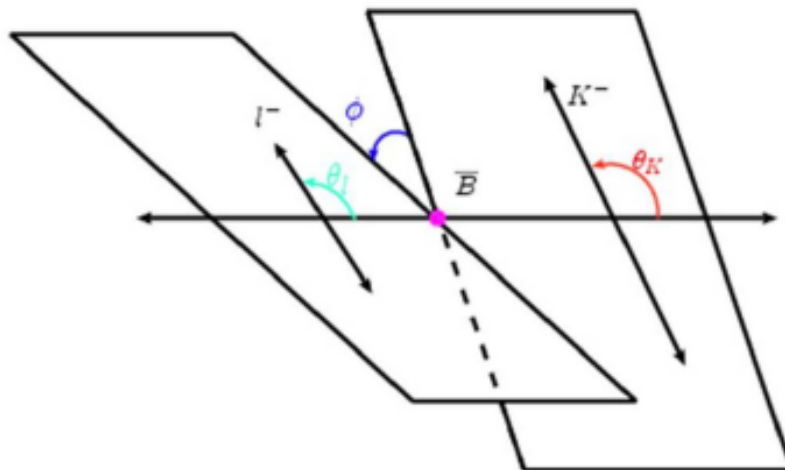
*A.Ali, et. al, hep-ph/9910221*



LHCb-CONF-2015-002 3fb<sup>-1</sup>  
ABSZ: 1503.05534

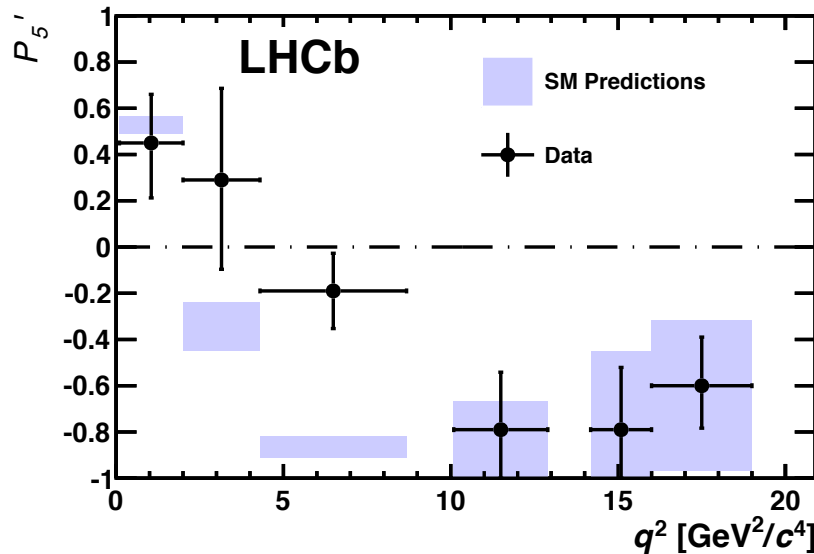


# Angular distributions



$$\begin{aligned} \frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \frac{d^3(\Gamma + \bar{\Gamma})}{d\vec{\Omega}} &= \frac{9}{32\pi} \left[ \frac{3}{4}(1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K + \frac{1}{4}(1 - F_L) \sin^2 \theta_K \cos 2\theta_\ell \right. \\ &\quad - F_L \cos^2 \theta_K \cos 2\theta_\ell + S_3 \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi \\ &\quad + S_4 \sin 2\theta_K \sin 2\theta_\ell \cos \phi + S_5 \sin 2\theta_K \sin \theta_\ell \cos \phi \\ &\quad + \frac{4}{3} A_{FB} \sin^2 \theta_K \cos \theta_\ell + S_7 \sin 2\theta_K \sin \theta_\ell \sin \phi \\ &\quad \left. + S_8 \sin 2\theta_K \sin 2\theta_\ell \sin \phi + S_9 \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi \right] \end{aligned}$$

# 3.7 $\sigma$ deviations

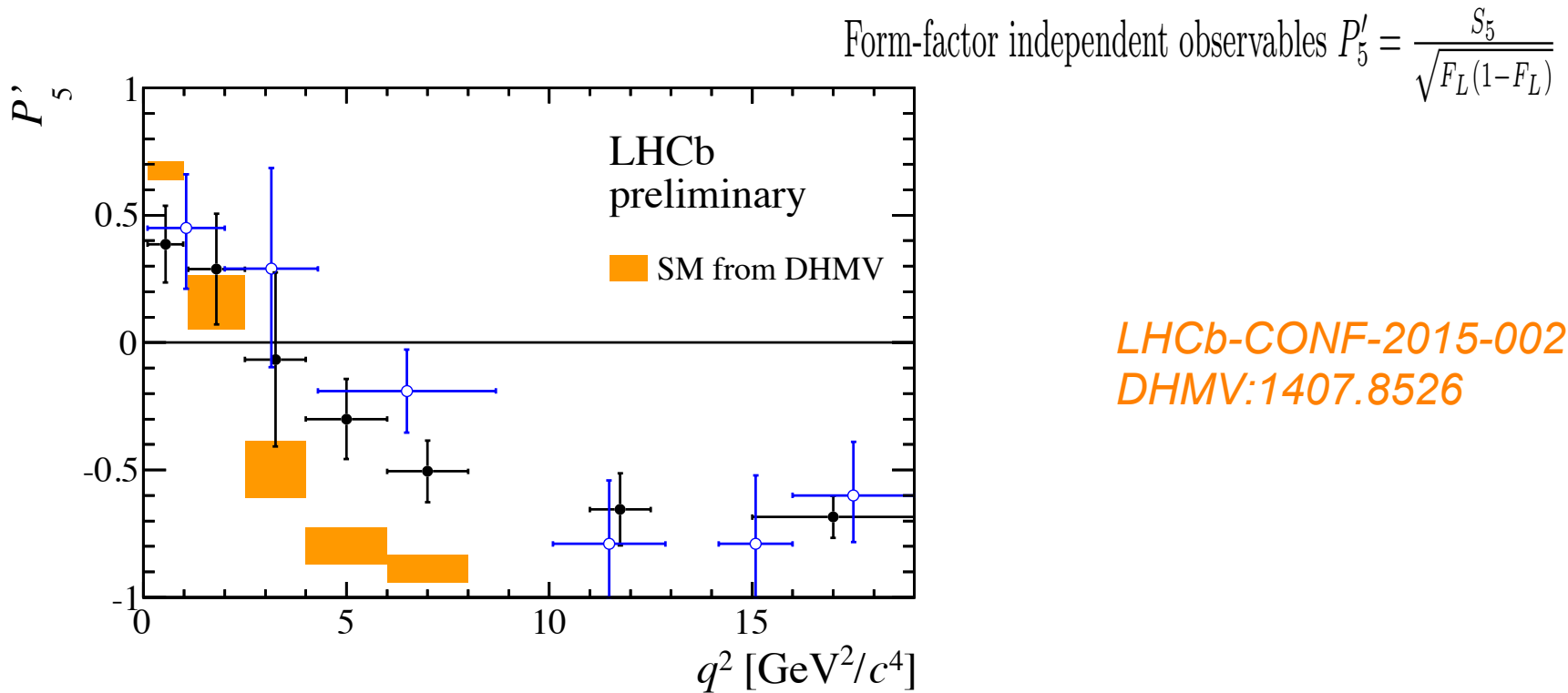


$$\text{Form-factor independent observables } P'_5 = \frac{S_5}{\sqrt{F_L(1-F_L)}}$$

LHCb: 1308.1707  
SM: 1303.5794

We present a measurement of form-factor independent angular observables in the decay  $B^0 \rightarrow K^*(892)^0 \mu^+ \mu^-$ . The analysis is based on a data sample corresponding to an integrated luminosity of  $1.0 \text{ fb}^{-1}$ , collected by the LHCb experiment in  $pp$  collisions at a center-of-mass energy of 7 TeV. Four observables are measured in six bins of the dimuon invariant mass squared,  $q^2$ , in the range  $0.1 < q^2 < 19.0 \text{ GeV}^2/c^4$ . Agreement with Standard Model predictions is found for 23 of the 24 measurements. A local discrepancy, corresponding to 3.7 Gaussian standard deviations, is observed in one  $q^2$  bin for one of the observables. Considering the 24 measurements as independent, the probability to observe such a discrepancy, or larger, in one is 0.5%.

# 3.7 $\sigma$ deviations



See Rahul Sinha's talk

Neglecting the correlations between the observables, the measurements are largely in agreement with the Standard Model predictions. However, the observable  $P'_5$  exhibits a local tension with respect to the Standard Model prediction at a level of 3.7 $\sigma$ .

# S-wave contributions



Due to limited life-time, vector mesons are reconstructed from two-pseudo-scalar mesons:  $K^*$  (50 MeV):  $K\pi$   
 $B \rightarrow K^* l^+ l^-$  is a four-body process.

Experimental cuts by LHCb:

*LHCb-CONF-2015-002*

$$m_{K^*} - \delta_m < m_{K\pi} < m_{K^*} + \delta_m \quad \delta_m = 100\text{MeV}$$

$$\int_{(m_{K^*} - \delta_m)^2}^{(m_{K^*} + \delta_m)^2} dm_{K\pi}^2 |L_{K^*}(m_{K\pi}^2)|^2 = 0.56$$

$L$  denotes the distribution function of  $K\pi$  system from  $K^*$

Narrow width limit (theoretical results):

$$\int dm_{K\pi}^2 |L_{K^*}(m_{K\pi}^2)|^2 = \mathcal{B}(K^{*+} \rightarrow K^0 \pi^+) = \frac{2}{3}$$

# S-wave contributions



Experimental cuts by LHCb:

$$m_{K^*} - \delta_m < m_{K\pi} < m_{K^*} + \delta_m \quad \delta_m = 100\text{MeV}$$

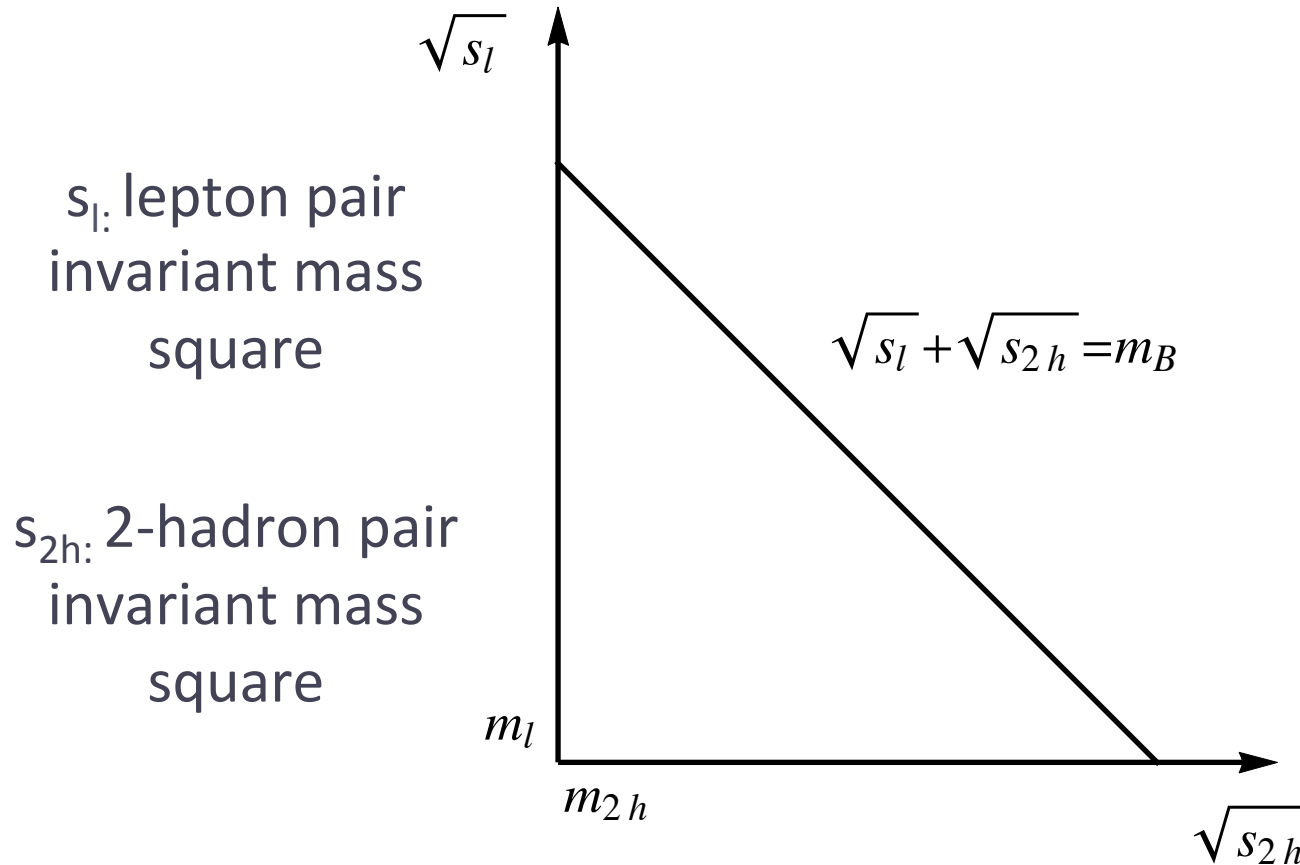
We expect the S-wave:

*Doring, Meissner, WW, 1307.0947*

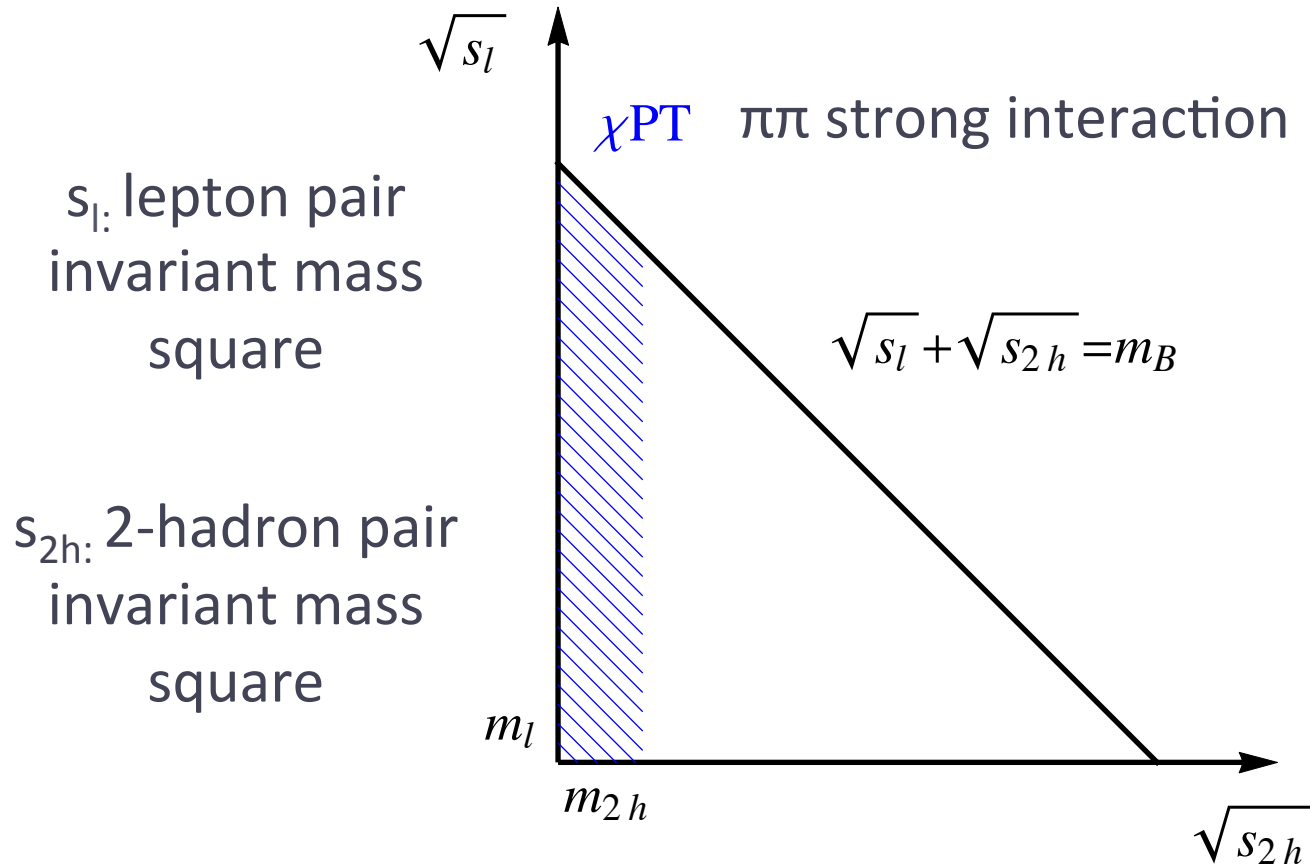
$$\int_{(m_{K^*} - \delta_m)^2}^{(m_{K^*} + \delta_m)^2} dm_{K\pi}^2 |L_S(m_{K\pi}^2)|^2 = 0.17$$

It is mandatory to include the S-wave.

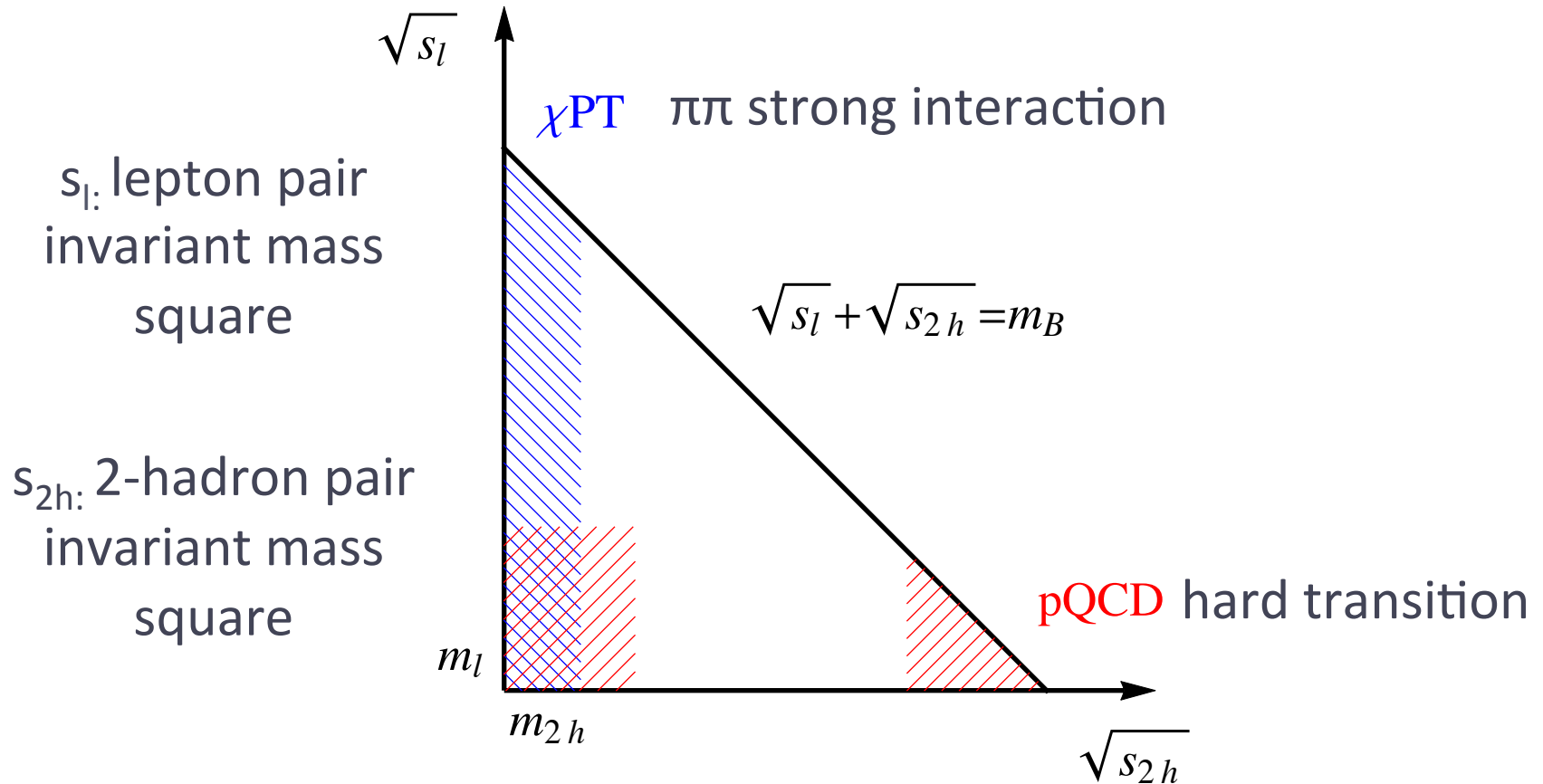
# Phase-space for $B \rightarrow M_1 M_2 \ell(\nu)$



# Phase-space for $B \rightarrow M_1 M_2 \ell(\nu)$

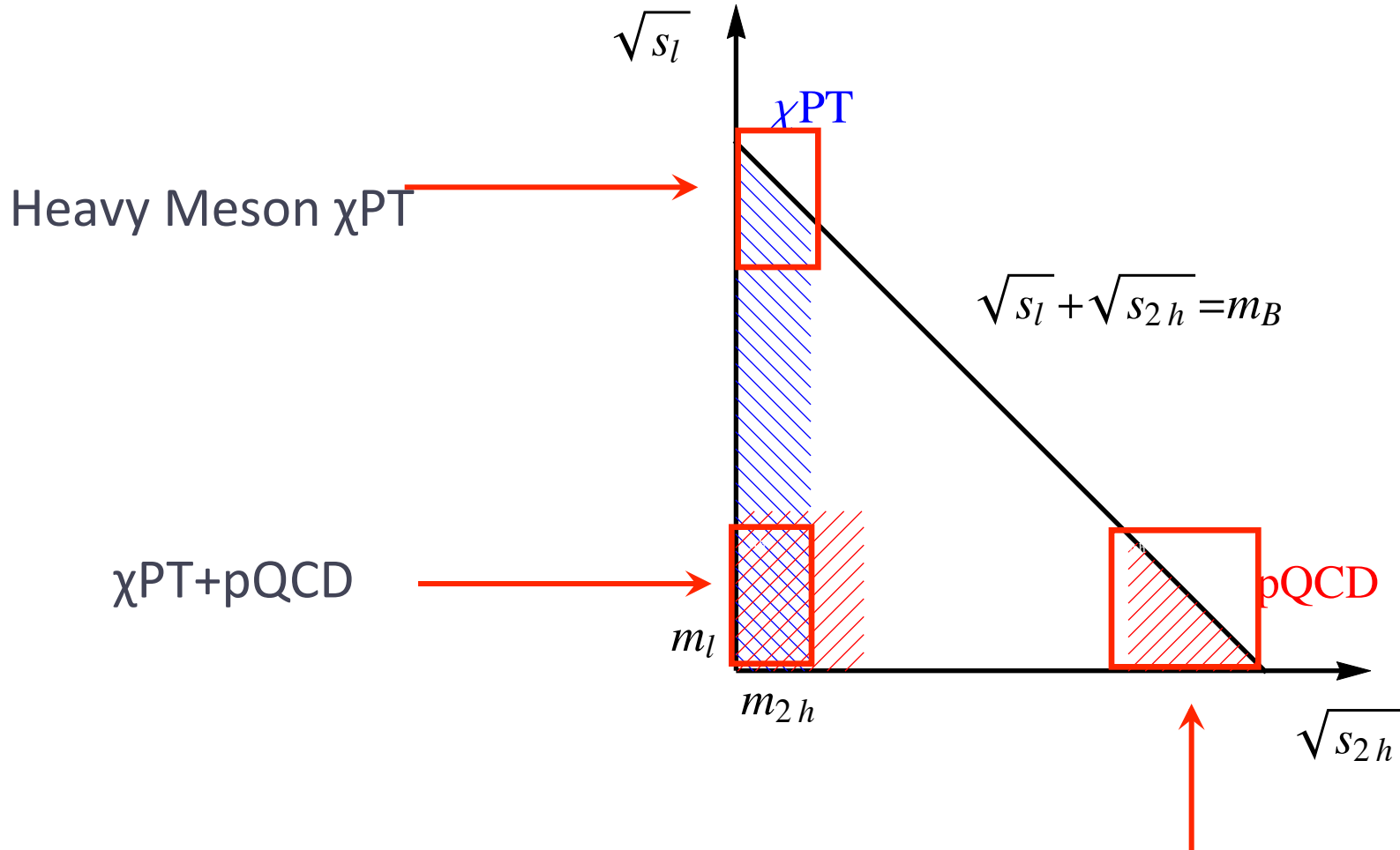


# Phase-space for $B \rightarrow M_1 M_2 \ell(\nu)$





# Phase-space for $B \rightarrow M_1 M_2 \ell(\nu)$



# S-wave contributions



- To be more specific, consider the generalized transition form factors:

$$\langle (K\pi)_0(p_{K\pi}) | \bar{s} \gamma_\mu \gamma_5 b | \bar{B}(p_B) \rangle = -i \frac{1}{m_{K\pi}} \left\{ \left[ P_\mu - \frac{m_B^2 - m_{K\pi}^2}{q^2} q_\mu \right] \mathcal{F}_1^{B \rightarrow K\pi}(m_{K\pi}^2, q^2) + \frac{m_B^2 - m_{K\pi}^2}{q^2} q_\mu \mathcal{F}_0^{B \rightarrow K\pi}(m_{K\pi}^2, q^2) \right\},$$

$$\langle (K\pi)_0(p_{K\pi}) | \bar{s} \sigma_{\mu\nu} q^\nu \gamma_5 b | \bar{B}(p_B) \rangle = -\frac{\mathcal{F}_T^{B \rightarrow K\pi}(m_{K\pi}^2, q^2)}{m_{K\pi}(m_B + m_{K\pi})} [q^2 P_\mu - (m_B^2 - m_{K\pi}^2) q_\mu],$$

# Generalized Form factors in LCSR



$$\langle (K\pi)_0(p_{K\pi}) | \bar{s} \gamma_\mu \gamma_5 b | \bar{B}(p_B) \rangle = -i \frac{1}{m_{K\pi}} \left\{ \left[ P_\mu - \frac{m_B^2 - m_{K\pi}^2}{q^2} q_\mu \right] \mathcal{F}_1^{B \rightarrow K\pi}(m_{K\pi}^2, q^2) + \frac{m_B^2 - m_{K\pi}^2}{q^2} q_\mu \mathcal{F}_0^{B \rightarrow K\pi}(m_{K\pi}^2, q^2) \right\},$$

$$\langle (K\pi)_0(p_{K\pi}) | \bar{s} \sigma_{\mu\nu} q^\nu \gamma_5 b | \bar{B}(p_B) \rangle = -\frac{\mathcal{F}_T^{B \rightarrow K\pi}(m_{K\pi}^2, q^2)}{m_{K\pi}(m_B + m_{K\pi})} [q^2 P_\mu - (m_B^2 - m_{K\pi}^2) q_\mu],$$

Consider a generic correlation function

$$\Pi(p_{K\pi}, q) = i \int d^4x e^{iq \cdot x} \langle (K\pi)_0(p_{K\pi}) | T \{ j_{\Gamma_1}(x), j_{\Gamma_2}(0) \} | 0 \rangle$$

Hadron level:

$$\frac{\langle (K\pi)_0(p_{K\pi}) | j_{\Gamma_1} | \bar{B}(p_{K\pi} + q) \rangle \langle \bar{B}(p_{K\pi} + q) | j_{\Gamma_2} | 0 \rangle}{m_B^2 - (p_{K\pi} + q)^2} + \int_{s_0}^{\infty} ds \frac{\rho^h(s, q^2)}{s - (p_{K\pi} + q)^2},$$

Quark level: Light cone OPE

$$\langle (K\pi)_0 | \bar{s}(x) \gamma_\mu d(0) | 0 \rangle$$

$$\langle (K\pi)_0 | \bar{s}(x) d(0) | 0 \rangle$$

$$\langle (K\pi)_0 | \bar{s}(x) \sigma_{\mu\nu} d(0) | 0 \rangle$$

Quark  
Hadron  
Duality

*U.G. Meißner, WW, arXiv:1312.3087*

# Chiral perturbation theory



$\chi PT$  effective field theory based on the two assumptions

- $\pi$ 's are the Goldstone boson of  $SU(3)_L \otimes SU(3)_R \rightarrow SU(3)_V$
- (chiral) power counting i.e. the theory has a small expansion parameter:  $p^2 / \Lambda_{\chi SB}^2$ :  
 $\Lambda_{\chi SB} \sim 4\pi F_\pi \sim 1.2 \text{ GeV}$

$$\mathcal{L}_{\Delta S=0} = \mathcal{L}_{\Delta S=0}^2 + \mathcal{L}_{\Delta S=0}^4 + \dots = \frac{F_\pi^2}{4} \overbrace{\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \rangle}^{\pi \rightarrow l\nu, \pi\pi \rightarrow \pi\pi, K \rightarrow \pi..} + \sum_i \overbrace{L_i O_i}^{K \rightarrow \pi..} + \dots$$

Fantastic chiral prediction  $A_{\pi\pi} \sim (s - m_\pi^2) / F_\pi^2$

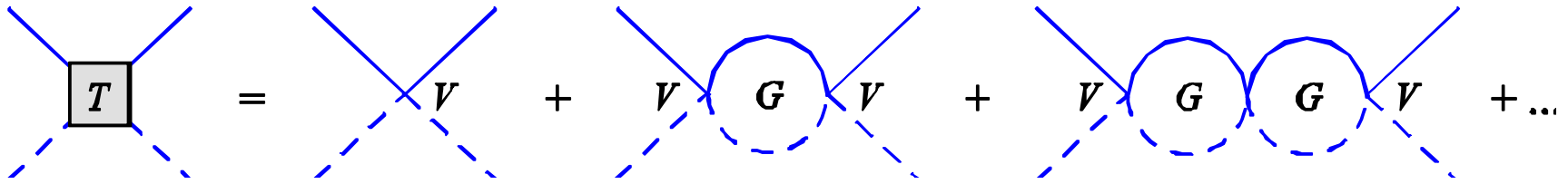
Weinberg, Colangelo *et al*

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}_{\Delta S=1}^2 + \mathcal{L}_{\Delta S=1}^4 + \dots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K \rightarrow 2\pi/3\pi} + G_8 F^2 \sum_i \underbrace{N_i W_i}_{K^+ \rightarrow \pi^+ \gamma\gamma, K \rightarrow \pi l^+ l^-} + \dots$$



ChiPT limited to low energies

# Unitarized Approach

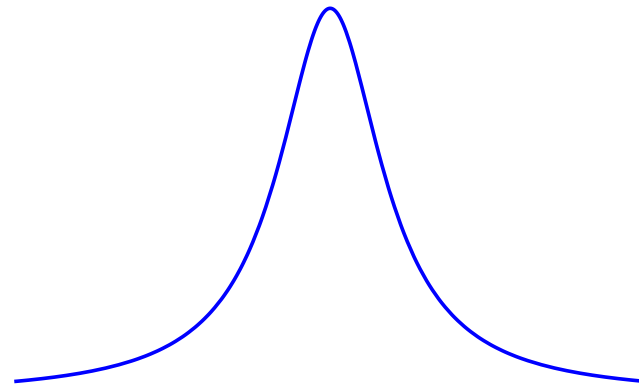
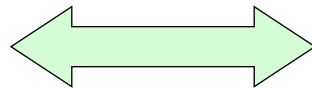


Summing all order contributions:

$$V + VGV + VGVG + \dots = \frac{V}{1 - GV}$$

$$1 - GV = 0$$

$$S = S_0$$



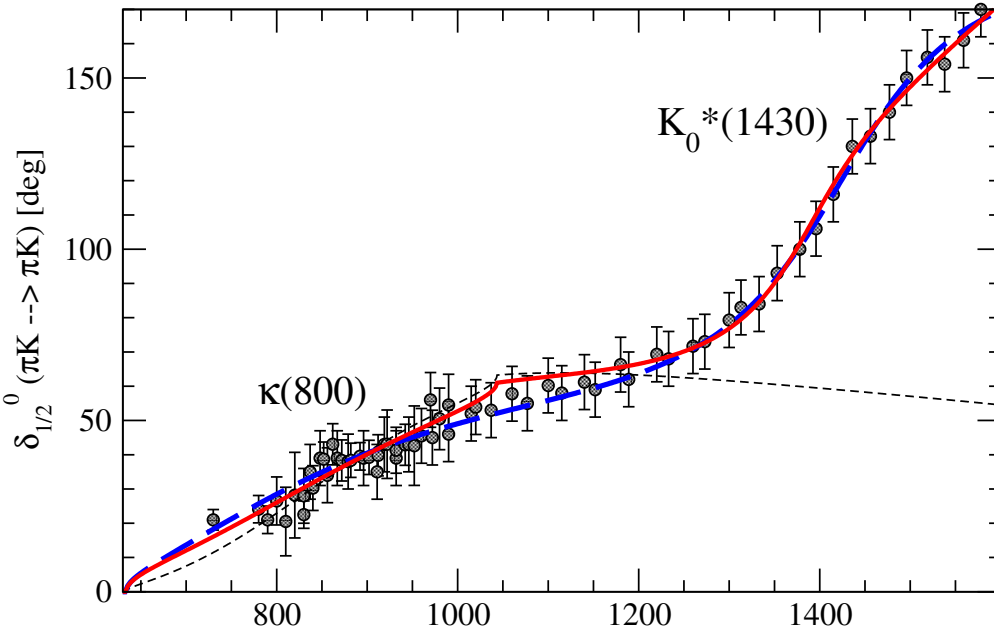
Above Threshold: pole corresponds to resonance

→ Hadron Molecule

# Unitarized $\chi$ PT and phase shift



M.Döring, U.-G.Meißner, WW, arXiv:1307.0947



Phase Shift

		$z_0$ [MeV]	$a_{-1}(K\eta)$ [ $M_\pi$ ]	$a_{-1}(K\pi)$ [ $M_\pi$ ]
$\kappa(800)$	this work (2-ch.)	$792 - i 279$		$-29 - i 57$
	this work (1-ch.)	$715 - i 283$		$-45 - i 62$
	Ref. [32] ( $\chi$ U)	$815 - i 226$		$-30 - i 57$
	Ref. [65] (Roy-S.)	$658 - i 279$		

Mass Pole

# Scalar form factors in $\chi$ PT



Scalar form factor:  $\langle 0 | \bar{s}d | K \pi \rangle = C_X B_0 F_{K\pi}(m_{K\pi}^2)$

QCD:  $\mathcal{L} = \bar{q}i\not{D}q - m_q\bar{q}q$

$$\bar{q}q = -\frac{\partial\mathcal{L}}{\partial m_q}$$

# Scalar form factors in $\chi$ PT



Scalar form factor:  $\langle 0 | \bar{s}d | K\pi \rangle = C_X B_0 F_{K\pi}(m_{K\pi}^2)$

Chiral Lagrangian:  $\mathcal{L} = \frac{f^2}{4} \langle U^\dagger \chi_s + \chi_s^\dagger U \rangle$

$$\chi_s = 2B_0 \begin{pmatrix} 0 & 0 & m_{\bar{u}s} \\ 0 & 0 & m_{\bar{d}s} \\ m_{\bar{s}u} & m_{\bar{s}d} & 0 \end{pmatrix}$$

$$U = \exp\left(\frac{i\sqrt{2}}{f}\Phi\right)$$

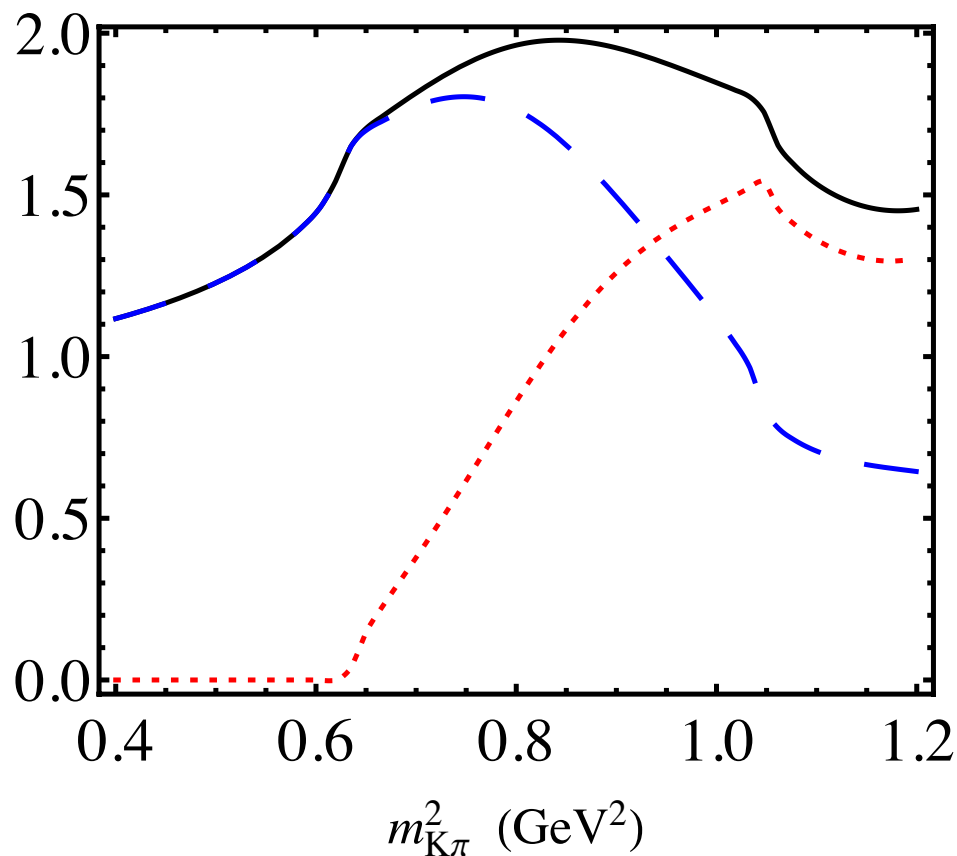
$$\Phi = \begin{bmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{bmatrix}$$

Tree Level current:  $\bar{s}u = -\partial\mathcal{L}/\partial m_{\bar{s}u}$ .

$$\begin{aligned} \bar{q}q' = \bar{u}s + \bar{d}s + \bar{s}u + \bar{s}d &= \frac{B_0}{6} \left[ 6(\pi^- K^+ + K^- \pi^+ + \pi^+ K^0 + \pi^- \bar{K}^0) \right. \\ &\quad - \sqrt{2}(K^- + K^+) (\sqrt{3}\eta_8 - 3\pi^0) \\ &\quad \left. - \sqrt{2}(K^0 + \bar{K}^0) (\sqrt{3}\eta_8 + 3\pi^0) \right]. \end{aligned}$$



# Scalar form factors in $\chi$ PT

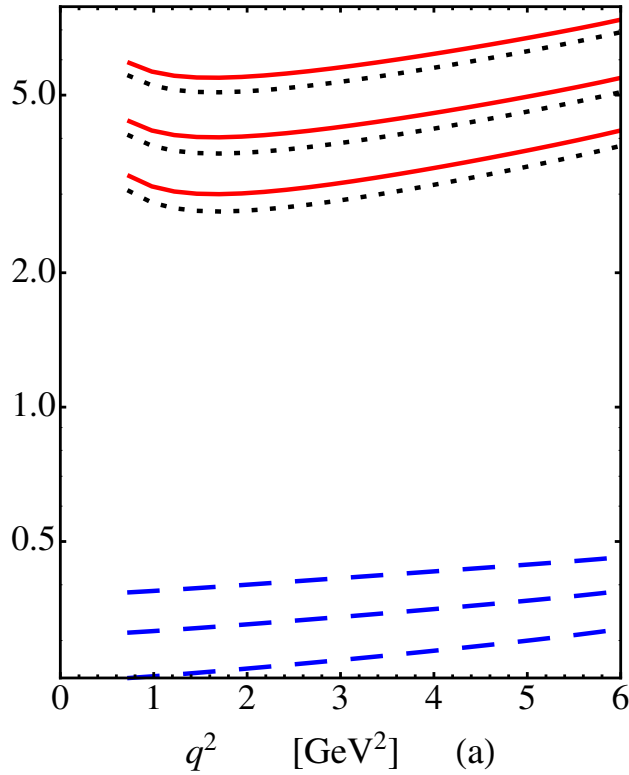


$$\langle 0 | \bar{s}d | K\pi \rangle = C_X B_0 F_{K\pi}(m_{K\pi}^2)$$

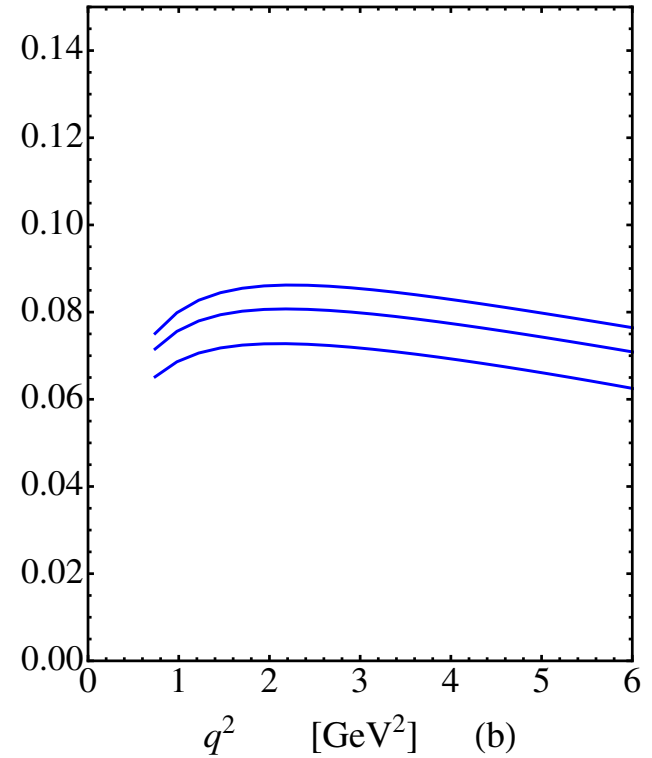
twice-subtracted Omnes  
solution matched onto  $\chi$ PT

*Imaginary part*  
*Real part*  
*Magnitude*

# S-wave contributions in $B \rightarrow K\pi^+\pi^-$



Decay widths:  
Red: total  
Black: P-wave  
Blue: S-wave

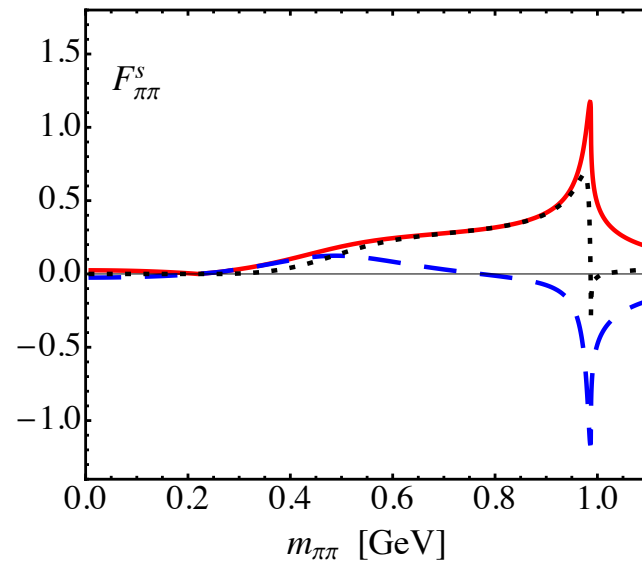
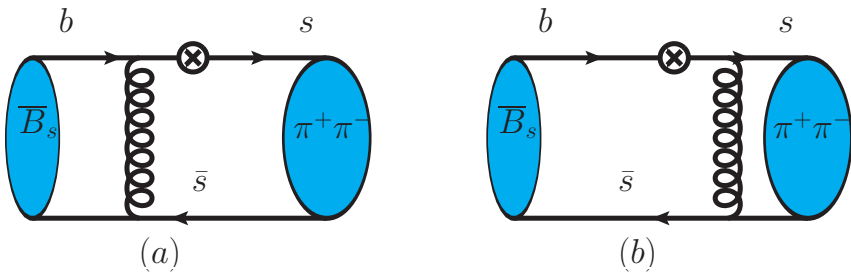


S-wave fraction

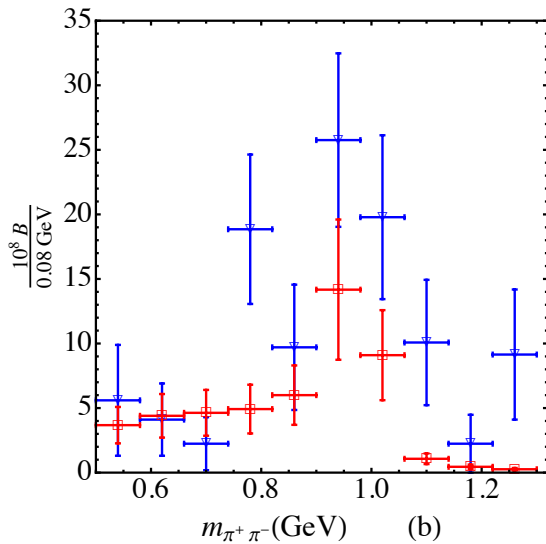
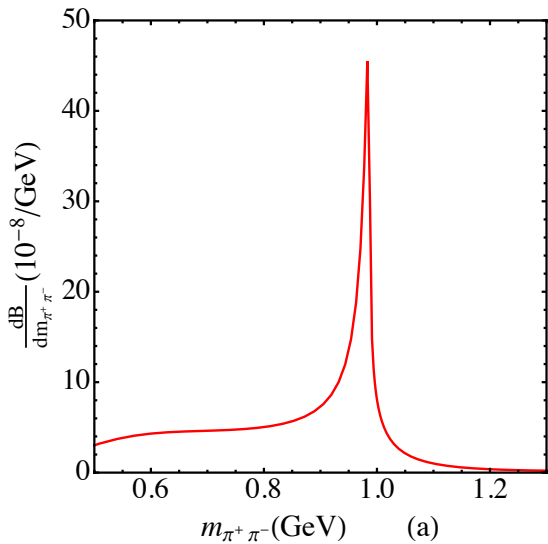
LHCb:  $F_S = 0.04 \pm 0.04$

1304.6325

# B<sub>s</sub> → π<sup>+</sup>π<sup>-</sup>μ<sup>+</sup>μ<sup>-</sup>



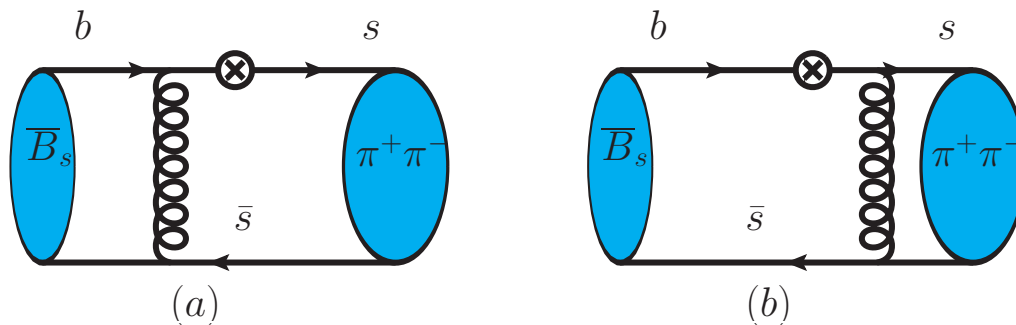
$$\sqrt{2}B_0 F_{\pi\pi}^s(s) = \langle 0 | \bar{s}s | \pi\pi \rangle_{I=0}$$



LHCb:1412.6433

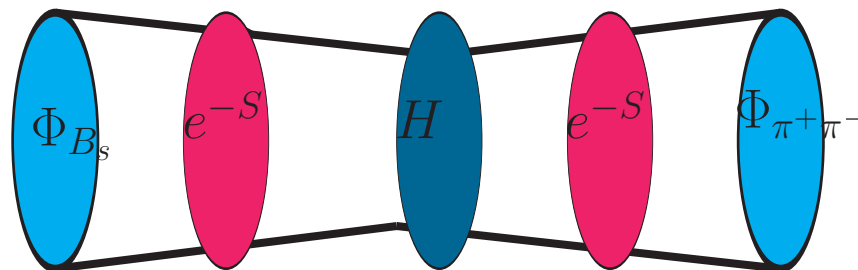
LCSR+χPT: WW,R.Zhu,1502.15104

# $B_s \rightarrow \pi^+ \pi^- \mu^+ \mu^-$ in PQCD



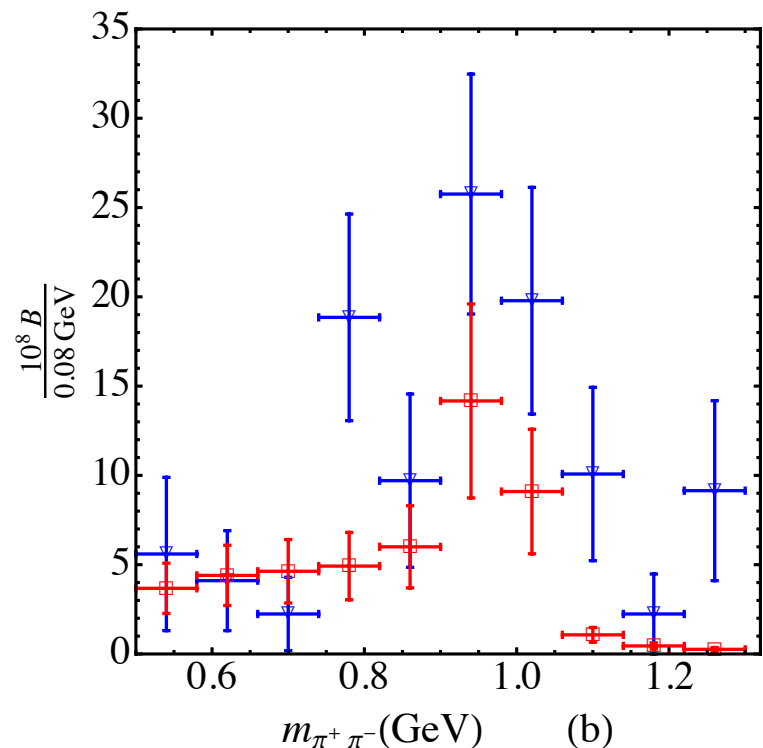
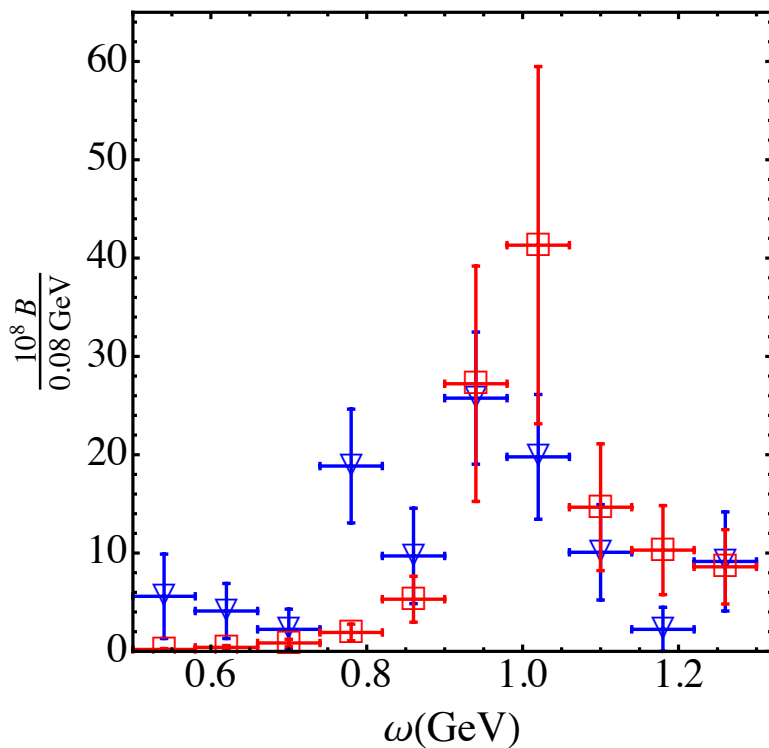
PQCD:

Y.Y.Keum, H.N.Li, A.I.Sanda  
 hep-ph/0004004  
 hep-ph/0004173



$$F \sim \int d^4k_1 d^4k_2 \text{Tr} [ C(t) \Phi_B(k_1) \Phi_1(k_2) H(k_1, k_2, t) ] \exp\{-S(t)\}$$

# $B_s \rightarrow \pi^+ \pi^- \mu^+ \mu^-$ in PQCD



LHCb:1412.6433

LCSR+ $\chi$ PT: WW,R.Zhu,1502.15104

PQCD: Wang, Li, Wang, Lu,1502.15104

## ➤ Heavy Flavor Physics

➤  $B \rightarrow \rho l \nu$  and  $B \rightarrow K^* l^+ l^-$  are valuable

➤ S-wave contributions

➤ In large recoil & small invariant mass region:

*$\chi$ PT & pQCD*

## Final state interactions in hadronic $B$ decays

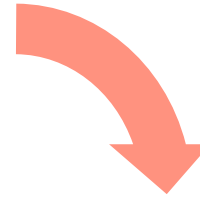
Hai-Yang Cheng,<sup>1</sup> Chun-Khiang Chua,<sup>1</sup> and Amarjit Soni<sup>2</sup>

<sup>1</sup>*Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China*

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There exist many experimental indications that final-state interactions (FSIs) may play a prominent role not only in charmful  $B$  decays but also in charmless  $B$  ones. We examine the final-state rescattering effects on the hadronic  $B$  decay rates and their impact on direct CP violation. The color-suppressed neutral modes such as  $B^0 \rightarrow D^0 \pi^0$ ,  $\pi^0 \pi^0$ ,  $\rho^0 \pi^0$ ,  $K^0 \pi^0$  can be substantially enhanced by long-distance rescattering effects. The direct CP-violating partial rate asymmetries in charmless  $B$  decays to  $\pi\pi/\pi K$  and  $\rho\pi$  are significantly affected by final-state rescattering, and their signs are generally different from that predicted by the short-distance (SD) approach. For example, direct CP asymmetry in  $B^0 \rightarrow \rho^0 \pi^0$  is increased to around 60% due to final-state rescattering effects whereas the short-distance picture gives about 1%. Evidence of direct CP violation in the decay  $\bar{B}^0 \rightarrow K^- \pi^+$  is now established, while the combined *BABAR* and *Belle* measurements of  $\bar{B}^0 \rightarrow \rho^+ \pi^-$  imply a  $3.6\sigma$  direct CP asymmetry in the  $\rho^+ \pi^-$  mode. Our predictions for CP violation agree with experiment in both magnitude and sign, whereas the QCD factorization predictions (especially for  $\rho^+ \pi^-$ ) seem to have some difficulty with the data. Direct CP violation in the decay  $B^- \rightarrow \pi^- \pi^0$  is very small ( $\lesssim 1\%$ ) in the standard model even after the inclusion of FSIs. Its measurement will provide a nice way to search for new physics as in the standard model QCD penguins cannot contribute (except by isospin violation). Current data on  $\pi K$  modes seem to violate the isospin sum-rule relation, suggesting the presence of electroweak penguin contributions. We have also investigated whether a large transverse polarization in  $B \rightarrow \phi K^*$  can arise from the final-state rescattering of  $D^{(*)} \bar{D}_s^{(*)}$  into  $\phi K^*$ . While the longitudinal polarization fraction can be reduced significantly from short-distance predictions due to such FSI effects, no sizable perpendicular polarization is found owing mainly to the large cancellations occurring in the processes  $\bar{B} \rightarrow D_s^* \bar{D} \rightarrow \phi \bar{K}^*$  and  $\bar{B} \rightarrow D_s \bar{D}^* \rightarrow \phi \bar{K}^*$ , and this can be understood as a consequence of CP and SU(3) [CPS] symmetry. To fully account for the polarization anomaly (especially the perpendicular polarization) observed in  $B \rightarrow \phi K^*$ , FSI from other states or other mechanism, e.g., the penguin-induced annihilation, may have to be invoked. Our conclusion is that the small value of the longitudinal polarization in  $VV$  modes cannot be regarded as a clean signal for new physics.



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## Final state interaction in $B \rightarrow KK$ decays

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We study the final state interaction effects in  $B \rightarrow KK$  decays. We find that the  $t$  channel one-particle-exchange diagrams cannot enhance the branching ratios of  $\bar{B}^0 \rightarrow K^0 \bar{K}^0$  and  $B^- \rightarrow K^0 K^-$  very sizably. For the pure annihilation process  $\bar{B}^0 \rightarrow K^+ K^-$ , the obtained branching ratio by the final state interaction is at  $\mathcal{O}(10^{-8})$ .



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*Thank you for your attention!*

第十一屆粒子現象學研討會

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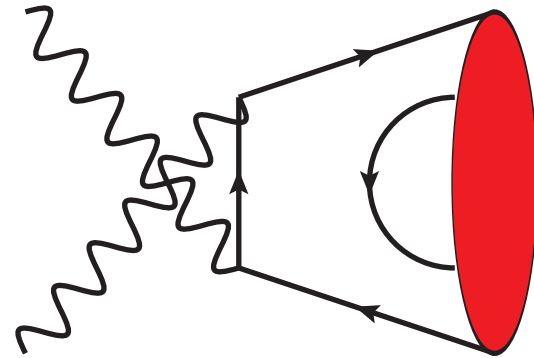
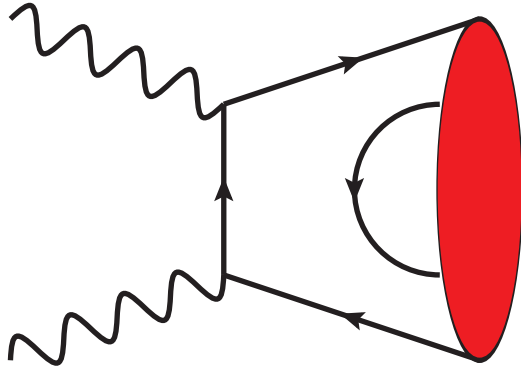
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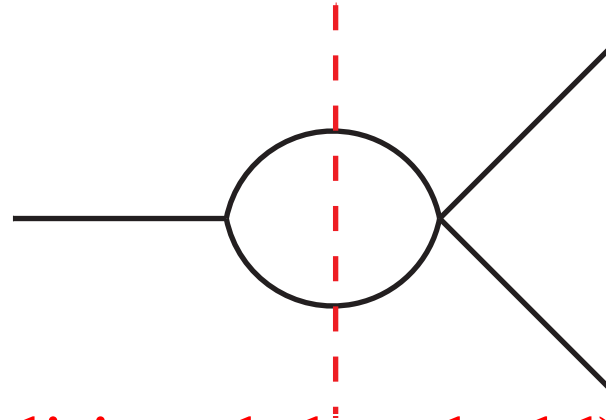
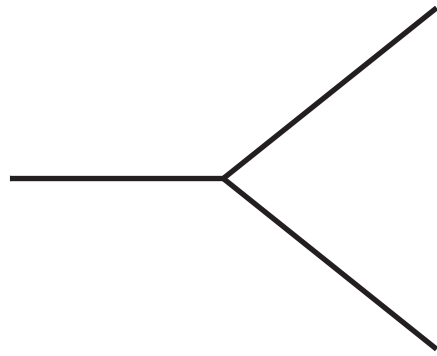
$\gamma\gamma^* \rightarrow \pi\pi$



$$T^{\mu\nu} = i \int d^4x e^{-iq \cdot x} \langle \pi(p)\pi(p') | T J_{\text{em}}^\mu(x) J_{\text{em}}^\nu(0) | 0 \rangle = -g_T^{\mu\nu} \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2),$$

S-wave can be projected out!

# Watson's theorem (1954)



In elastic region (below additional threshold):

$$\text{Im}[F] = F \sigma T^*$$

Form factor and scattering amplitude carry the same phase!