# Observable Nucleon Spin Decomposition :
# Canonical or Mechanical ?

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1. **Introduction to the nucleon spin decomposition problem of QCD**

2. **"Canonical" or "Mechanical" decomposition ?**

3. **Summary and conclusion**

## 1. Introduction to nucleon spin decomposition problem of QCD

Although one might think it a little "academic problem", to get a complete decomposition of nucleon spin is a fundamentally important task of QCD.

In fact, if our research ends up without accomplishing this task, a tremendous efforts since the first discovery of the nucleon spin crisis would go up in smoke.

Unfortunately, this is a very delicate and difficult problem, which has rejected a clear answer for more than 20 years since the first seminal paper by

- R.L. Jaffe and A.V. Manohar, Nucl. Phys. B337, 509 (1990).

Recently, two reviews appeared to overview controversial status of the problem :

- E. Leader and C. Lorcé, Phys. Rept. 541, 163 (2014) [arXiv : 1309.4235].
- M. Wakamatsu, Int. J. Mod. Phys. A29, 1430012 (2014) [arXiv:1402.4193].

**Two remaining issues in the nucleon spin decomposition problem**

1) Are there infinitely many decompositions of the nucleon spin ? If not, what physical principle favors one particular decomposition among many candidates ?

⇕ intimately connected !

1') Can the total gluon angular momentum be gauge-invariantly decomposed into its spin and orbital parts without causing conflict with the textbook negative statement on the similar question on the total photon angular momentum ?

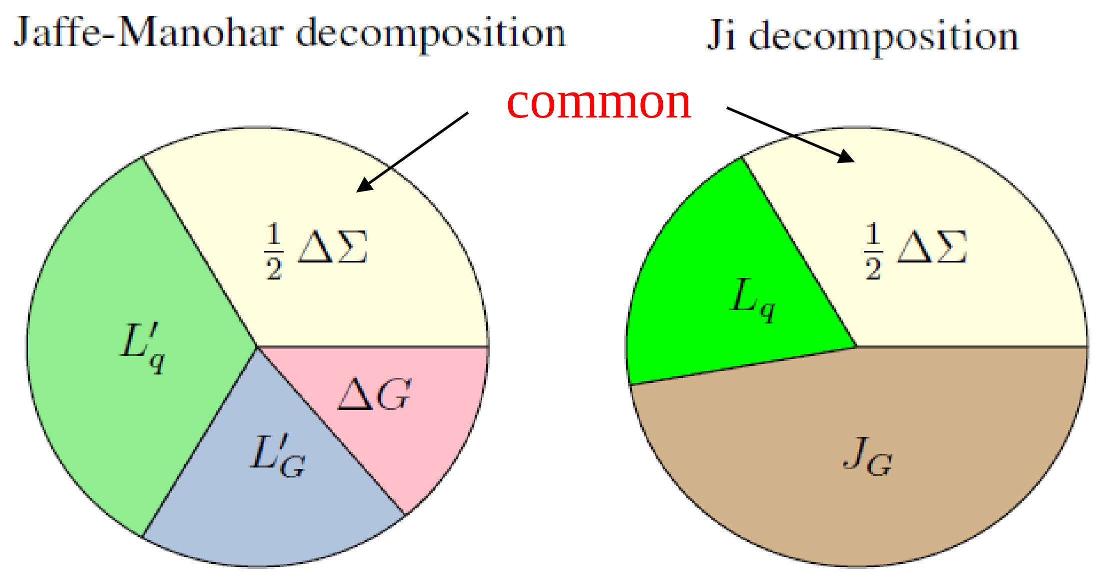
2) Among the two different decompositions, i.e. the "canonical" type and "mechanical" type decompositions, which can we say is more physical ? (More "physical" here means that it is closer to direct observation.)

Regrettably, I had not reached a clear answer when I wrote the previous review. Now, I believe that I have got satisfactory answers to both questions.

- M. Wakamatsu, arXiv : 1409.4474, Eur. Phys. J. A51, 52 (2015).

In the present talk, because of the time limit, let me confine to the 2nd question, which is certainly more important from the practical viewpoint.

## two popular decompositions of the nucleon spin : (1990 and 1997)




$$\boldsymbol{J}_{QCD} = \int \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma} \psi \, d^3x + \int \psi^\dagger \boldsymbol{x} \times \frac{1}{i} \nabla \psi \, d^3x + \int \boldsymbol{E}^a \times \boldsymbol{A}^a \, d^3x + \int E^{ai} \boldsymbol{x} \times \nabla A^{ai} \, d^3x$$

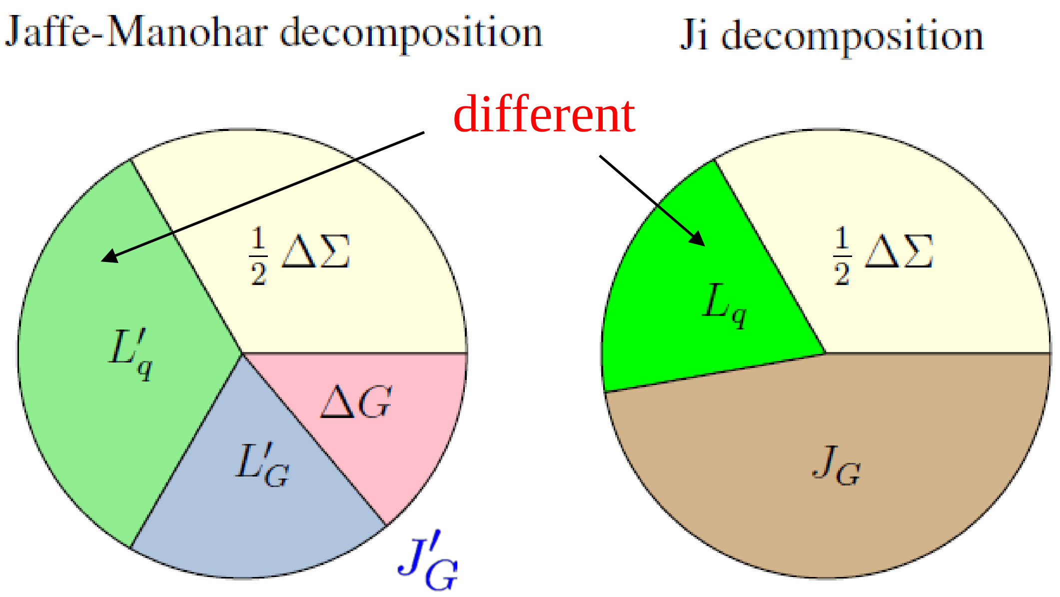
$$\boldsymbol{J}_{QCD} = \int \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma} \psi \, d^3x + \int \psi^\dagger \boldsymbol{x} \times \frac{1}{i} \boldsymbol{D} \psi \, d^3x + \int \boldsymbol{x} \times (\boldsymbol{E}^a \times \boldsymbol{B}^a) \, d^3x$$

$J_G$

Each term is not separately gauge-invariant !

No further GI decomposition !

## two popular decompositions of the nucleon spin - continued -




An especially annoying observation here was that, since

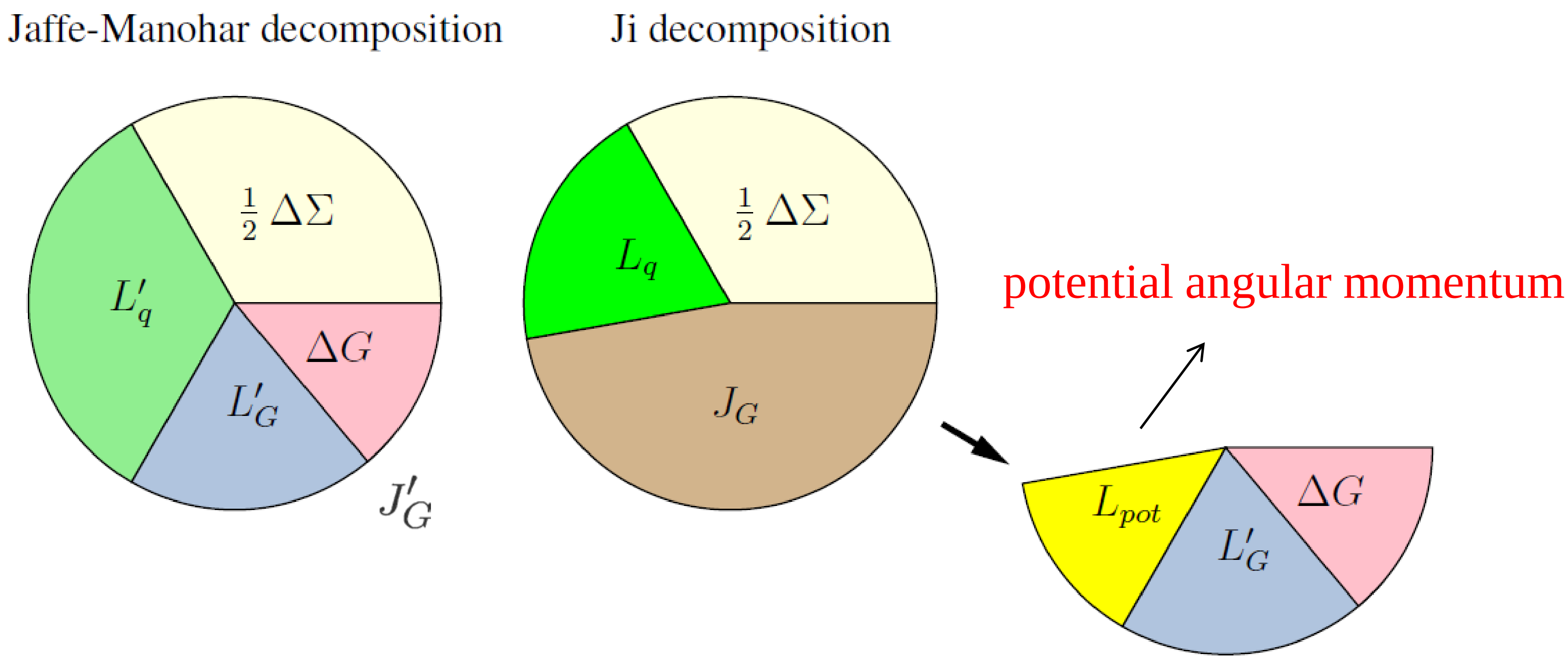
$$L'_q \neq L_q$$

one must inevitably conclude that

$$J'_G = \Delta G + L'_G \neq J_G \; !$$

Now we know the answer of this puzzle.

- M.W. , Phys. Rev. D81 (2010) 114010 ; Phys. Rev. D83 (2012) 014012.




$L_{pot}$ characterizes the difference between $J_G(\texttt{Ji})$ and $J'_G(\texttt{J-M})$ .

$$J_G(\texttt{Ji}) \ - \ J'_G(\texttt{J-M}) \ = \ L_{pot}$$

Pay attention to the **difference** of **quark OAMs** in the two decompositions.

$$L_Q(\mathrm{JM}) \;\sim\; \psi^\dagger \, \boldsymbol{x} \times \boldsymbol{p} \, \psi \qquad\qquad L_Q(\mathrm{Ji}) \;\sim\; \psi^\dagger \, \boldsymbol{x} \times (\boldsymbol{p} - g\,\boldsymbol{A}) \, \psi$$

**canonical OAM** | **mechanical OAM** (or **kinetic OAM**)

**not gauge invariant** ! | **gauge invariant** !

**gauge principle**

**observables must be gauge-invariant** !

- Observability of canonical OAM has long been questioned ?

The recent intensive dispute began with Chen et al.'s papers.

- X.-S. Chen et al., Phys. Rev. Lett. 103, 062001 (2009) ; 100, 232002 (2008).

**basic idea**

$$A^{\mu} \;=\; A^{\mu}_{phys} \;+\; A^{\mu}_{pure}$$

which is a sort of generalization of the familiar decomposition of photon field in QED into the transverse and longitudinal components :

$$\boldsymbol{A}_{phys} \;\Leftrightarrow\; \boldsymbol{A}_{\perp}\,(\texttt{gauge-invariant}), \qquad \boldsymbol{A}_{pure} \;\Leftrightarrow\; \boldsymbol{A}_{||}$$

Their decomposition is given in the following form :

$$\begin{aligned} \boldsymbol{J}_{QCD} &= \boldsymbol{S}'_q \;+\; \boldsymbol{L}'_q \;+\; \boldsymbol{S}'_G \;+\; \boldsymbol{L}'_G \\ &= \int \psi^{\dagger}\,\frac{1}{2}\,\Sigma\,\psi\,d^3x \;+\; \int \psi^{\dagger}\,\boldsymbol{x} \times \left(\frac{1}{i}\nabla \,-\, g\,\boldsymbol{A}_{pure}\right)\,\psi\,d^3x \\ &+\; \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys}\,d^3x \;+\; \int E^{aj}\,(\boldsymbol{x} \times \boldsymbol{\mathcal{D}}_{pure})\,A^{aj}_{phys}\,d^3x \end{aligned}$$

It can be shown that each term is separately gauge-invariant !

\- GI version of Jaffe-Manohar decomposition ? -

Soon after, we noticed that the way of gauge-invariant decomposition of nucleon spin is not necessarily unique, and proposed another G.I. decomposition :

- M.W. , Phys. Rev. D81 (2010) 114010 ; Phys. Rev. D83 (2012) 014012.

$$\boldsymbol{J}_{QCD} = \boldsymbol{S}_q + \boldsymbol{L}_q + \boldsymbol{S}_G + \boldsymbol{L}_G$$

where

$$\boldsymbol{S}_q = \boldsymbol{S}'_q$$
$$\boldsymbol{S}_G = \boldsymbol{S}'_G$$
$$\boldsymbol{L}_q = \int \psi\, \boldsymbol{x} \times \left(\frac{1}{i}\nabla - g\,\boldsymbol{A}\right) \psi^\dagger\, d^3x = \boldsymbol{L}_q\,(\texttt{Ji})$$
$$\boldsymbol{L}_G = \boldsymbol{L}'_G + \boxed{\int \rho^a\,(\boldsymbol{x} \times \boldsymbol{A}^a_{phys})\, d^3x} \longleftrightarrow \boldsymbol{L}_{pot}$$

"**potential angular momentum**"

The QED correspondent of $\boldsymbol{L}_{pot}$ is the orbital angular momentum carried by electromagnetic potential, appearing in the famous Feynman paradox.

An **arbitrariness** of the spin decomposition arises, because this potential angular momentum term is solely gauge-invariant ! Shifting it to the quark OAM part

$$\left.\begin{array}{l} \boldsymbol{L}_q + \boldsymbol{L}_{pot} = \boldsymbol{L}'_q\ (\text{Chen}) \\ \boldsymbol{L}_G - \boldsymbol{L}_{pot} = \boldsymbol{L}'_G\ (\text{Chen}) \end{array}\right\} \Rightarrow \boldsymbol{J}_G = \boldsymbol{J}'_G + \boldsymbol{L}_{pot}$$

Ji     J-M or Chen

We are thus left with two gauge-invariant decompositions of the nucleon spin :

"canonical" decomposition

$$J_{QCD} = S'_q + L'_q + S'_G + L'_G$$

with

$$\begin{aligned} S'_q &= \int \psi^\dagger \frac{1}{2} \Sigma \psi \, d^3x \\ L'_q &= \int \psi^\dagger \boldsymbol{x} \times \left( \frac{1}{i} \nabla - g \boldsymbol{A}_{pure} \right) \psi \, d^3x \\ S'_G &= \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys} \, d^3x \\ L'_G &= \int E^{aj} \left( \boldsymbol{x} \times \boldsymbol{D}_{pure} A^{aj}_{phys} \right) d^3x \end{aligned}$$

"mechanical" decomposition

$$J_{QCD} = S_q + L_q + S_G + L_G$$

with

$$\begin{aligned} S_q &= S'_q \\ L_q &= \int \psi^\dagger \left( \frac{1}{i} \nabla - g \boldsymbol{A} \right) \psi \, d^3x \\ S_G &= S'_G \\ L_G &= L'_G + \boldsymbol{L}_{pot} \end{aligned}$$

**[A word of caution]** - related to question (1) or (1') -

- These decompositions are based on the familiar transverse-longitudinal decomposition of the gauge field. (- Helmholz decomposition -)
- However, the transverse-longitudinal decomposition is given only after fixing the Lorentz-frame of reference. ( - breaks **Lorentz-covariance** - )

The central question here is which is physically favorable decomposition,

"canonical" or "mechanical" ?

From the slide of my talk at "Transversity **2011**", Veli Losinj, Croatia

**canonical OAM party**

- Jaffe-Manohar
- Bashinsky-Jaffe
- Chen et al.
- Cho et al.
- Leader

**mechanical OAM party**

- Ji
- Wakamatsu

**Neutral party**

- Burkardt-BC

From the slide of my talk at ECT* **2015** Workshop

**canonical OAM party**

- Jaffe-Manohar
- Bashinsky-Jaffe
- Chen et al.
- Leader
- Lorce
- Hatta
- Ji (?)

…

**mechanical OAM party**

- Wakamatsu

**Neutral party**

- Burkardt
- Tiwari (?)

Often-claimed **advantages** of "canonical" decomposition.

(1) Each piece of the decomposition satisfies the angular momentum algebra

$$[L^i_{can}, L^j_{can}] \;=\; i\,\epsilon^{ijk}\,L^k_{can}$$

This advantage was already denied for the massless particle.

- M.W., Int. J. Mod. Phys. A29, 1430012 (2014).
- W.-M. Sun, arXiv : 1407.2035 [quant-ph].

(2) $L_{can}$ seems compatible with free partonic picture of constituent orbital motion.

$$\boldsymbol{L}_{mech} \;=\; \int \psi^\dagger\, \boldsymbol{r} \times \frac{1}{i}\,(\nabla - i\,g\,\boldsymbol{A})\,\psi\,d^3r \;\xrightarrow{G.F.}\; \int \psi^\dagger\, \boldsymbol{r} \times \frac{1}{i}\left(\nabla - i\,g\,\boldsymbol{A}_{phys}\right)\psi\,d^3r$$

$$\boldsymbol{L}_{can} \;=\; \int \psi^\dagger\, \boldsymbol{r} \times \frac{1}{i}\,(\nabla - i\,g\,\boldsymbol{A}_{pure})\,\psi\,d^3r \;\xrightarrow{G.F.}\; \int \psi^\dagger\, \boldsymbol{r} \times \frac{1}{i}\,\nabla\,\psi\,d^3r$$

- The "mechanical" OAM appears to contain quark-gluon interaction.
- The "canonical" OAM does not seem to contain quark-gluon interaction,

Does this really mean that the "canonical" OAM is easier to measure in the same sense as twist-2 parton model ?

## 2. "Canonical" or "Mechanical" decomposition ?

Historically, it was a common belief that the canonical OAM appearing in the Jaffe-Manohar decomposition would not correspond to observables, because they are not gauge-invariant quantities.

This nebulous impression did not change even after a gauge-invariant version of the Jaffe-Manohar decomposition a la Bashinsky and Jaffe appeared in 1999.

However, the impression has changed drastically after Lorcé and Pasquini showed that the canonical quark OAM can be related to a certain moment of a quark distribution function in a phase space called the Wigner distribution.

$$\rho^q(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp ; \mathcal{L}) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i \Delta_\perp \cdot \boldsymbol{b}_\perp} \frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i (x \bar{P}^+ z^- - \boldsymbol{k}_\perp \cdot \boldsymbol{z}_\perp)} \times \langle P'^+, \frac{\boldsymbol{\Delta}_\perp}{2}, S \,|\, \bar{\psi}\left(-\frac{z}{2}\right) \gamma^+ \mathcal{L}\left[-\frac{z}{2}, \frac{z}{2}\right] \psi\left(\frac{z}{2}\right) |\, P^+, -\frac{\boldsymbol{\Delta}_\perp}{2}, S \rangle \,|_{z^+=0}$$

$x = k^+ / \bar{P}^+$, $\boldsymbol{k}_\perp$ : transverse momentum

$\mathcal{L}$ : gauge-link, $\boldsymbol{b}_\perp$ : impact parameter

According to them, a natural definition of quark OAM density in the phase-space

$$L^3(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp \,;\, \mathcal{L}) \;=\; (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)^3 \; \rho^q(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp \,;\, \mathcal{L})$$

After integrating over $x$, $\boldsymbol{k}_\perp$, and $\boldsymbol{b}_\perp$ , they found a remarkable relation

$$\langle L^3 \rangle^{\mathcal{L}} \;=\; \int dx\, d^2k_\perp\, d^2b_\perp\, L^3(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp \,;\, \mathcal{L}) \;=\; - \int dx\, d^2k_\perp\, \frac{\boldsymbol{k}_\perp^2}{M^2}\, F^q_{1,4}(x, 0, \boldsymbol{k}_\perp^2, 0, 0, \mathcal{L})$$

where

$$\begin{aligned} \rho^q(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp \,;\, \mathcal{L}) \;&=\; F^q_{1,1}(x, \boldsymbol{k}_\perp^2, \boldsymbol{k}_\perp \cdot \boldsymbol{b}_\perp, \boldsymbol{b}_\perp^2 \,;\, \mathcal{L}) \\ &\quad -\; \frac{1}{M^2} (\boldsymbol{k}_\perp \times \nabla_{\boldsymbol{b}_\perp})_z\, F^q_{1,4}(x, \boldsymbol{k}_\perp^2, \boldsymbol{k}_\perp \cdot \boldsymbol{b}_\perp, \boldsymbol{b}_\perp^2 \,;\, \mathcal{L}) \end{aligned}$$

A delicacy here is that the Wigner distribution $\rho^q$ generally depends on the chosen path of the gauge-link $\mathcal{L}$ connecting the points $z/2$ and $-z/2$.

As shown by a careful study by Hatta, with the choice of a staple-like gauge-link in the light-front direction, corresponding to the kinematics of the semi-inclusive reactions or the Drell-Yan processes, the above quark OAM turns out to coincide with the (GI) canonical quark OAM not the mechanical OAM :

$$\langle L^3 \rangle^{\mathcal{L} = LC} \;=\; L_{can}$$

This observation holds out a hope that the canonical quark OAM in the nucleon would also be a measurable quantity, at least in principle.

However, in a recent paper

- A. Courtoy et al., Phys. Lett. B731 (2014) 141.

Courtoy et al. throws a serious doubt on the practical observability of the Wigner function $F^q_{14}$ appearing in the above intriguing sum rule.

According to them, even though $F^q_{14}$ may be nonzero in particular models and also in real QCD, it would not correspond to direct observables, because

it drops out in both factorization schemes of TMDs and GPDs.

It appears to us that this takes a discussion on the observability of the canonical OAM back to its starting point ?

"canonical" OAM or "mechanical" OAM ?

Now, an interesting question is the physical implication of the relation

**Why** ? $\langle L^3 \rangle^{\mathcal{L} = LC} \;=\; L_{can} \quad \neq \quad L_{mech}$

$\Updownarrow$

Wigner-distribution-based average OAM

Comparative analysis of

average transverse momentum & longitudinal OAM of quarks

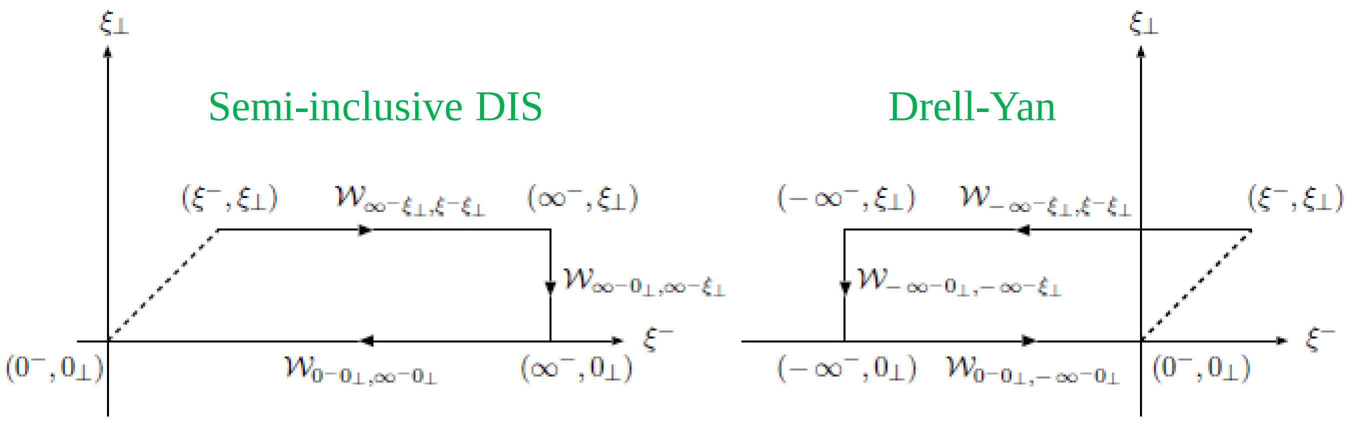
$$\langle \boldsymbol{k}_\perp^i \rangle^{\mathcal{L}} = \int dx \int d^2\boldsymbol{b}_\perp \int d^2\boldsymbol{k}_\perp \; \boldsymbol{k}_\perp^i \; \rho(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp \, ; \, \mathcal{L}) \qquad (i = 1, 2)$$

$$\langle L^3 \rangle^{\mathcal{L}} = \int dx \int d^2 b_\perp \int d^2 k_\perp \; (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)^3 \; \rho(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp \, ; \, \mathcal{L})$$

with

$\rho(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp \, ; \, \mathcal{L})$ = generally gauge-link-path dep. Wigner distribution

2 paths with physical interest




(1) future-pointing staple-like LC path $\mathcal{W}^{+LC}$ (2) past-pointing staple-like LC path $\mathcal{W}^{-LC}$

Burkardt showed the relation

FSI or ISI

$$\langle k_\perp^i \rangle^{\pm LC} = \langle k_\perp^i \rangle_{mech} + \langle k_\perp^i \rangle_{int}^{\pm LC}.$$

where

$$\langle k_\perp^i \rangle_{mech} = \frac{1}{2p^+} \langle p, s \,|\, \bar{\psi}(0) \frac{1}{i} D_\perp^i(0) \psi(0) \,|\, p, s \rangle.$$

while

$$\langle k_\perp^i \rangle_{int}^{\pm LC} = -\frac{1}{2p^+} \int_0^{\pm\infty} d\eta^- \times \langle p, s \,|\, \bar{\psi}(0) \mathcal{L}[0^- \mathbf{0}_\perp, \eta^- \mathbf{0}_\perp] \, g \, F^{+i}(\eta^-, \mathbf{0}_\perp) \mathcal{L}[\eta^- \mathbf{0}_\perp, 0^- \mathbf{0}_\perp] \psi(0) \,|\, p, s \rangle.$$

In the LC gauge, $\mathcal{L} \rightarrow 1$, and

$$-\sqrt{2} \, g \, F^{+y} = g \, (E^y - B^x) = g \, [\boldsymbol{E} + (\boldsymbol{v} \times \boldsymbol{B})]^y$$

Then, $\langle k_\perp^i \rangle_{int}^{+LC}$ can be interpreted as the change of transverse momentum for the struck quark by color Lorentz force when it leaves the target after ejected by the virtual photon in the semi-inclusive DIS processes.

Similarly, for the average longitudinal OAM

FSI or ISI

$$\langle L^3 \rangle^{\pm LC} = \langle L^3 \rangle_{mech} + \langle L^3 \rangle^{\pm LC}_{int},$$

where

$$\langle L^3 \rangle_{mech} = \mathcal{N} \int d^2 r_\perp \times \langle p, s \,|\, \bar{\psi}(0^-, \boldsymbol{r}_\perp) \, \gamma^+ \, \epsilon^{ij}_\perp \, r^i_\perp \, \frac{1}{i} \, D^j_\perp(\boldsymbol{r}_\perp)(0^-, \boldsymbol{r}_\perp) \, \psi(0^-, \boldsymbol{r}_\perp) \,|\, p, s \rangle$$

while

$$\langle L^3 \rangle^{\pm LC}_{int} = -\mathcal{N} \int d^2 r_\perp \int_0^{\pm\infty} d\eta^- \; \epsilon^{ij}_\perp \, r^i_\perp \, \langle p, s \,|\, \bar{\psi}(0^-, \boldsymbol{r}_\perp) \, \gamma^+ \times \mathcal{L}[0^- \, \boldsymbol{r}_\perp, \eta^- \, \boldsymbol{r}_\perp] \, g \, F^{+j}[\eta^-, \boldsymbol{r}_\perp] \, \mathcal{L}[\eta^- \, \boldsymbol{r}_\perp, 0^- \, \boldsymbol{r}_\perp] \, \psi(0^-, \boldsymbol{r}_\perp) \,|\, p, s \rangle.$$

Change

Lorentz force ⇨ torque by Lorentz force

$$T^z(r^-, \boldsymbol{r}_\perp) \equiv -g \left( x \, F^{+y}(r^-, \boldsymbol{r}_\perp) - y \, F^{+x}(r^-, \boldsymbol{r}_\perp) \right)$$

Hatta showed that, due to the parity and time-reversal (PT) symmetry

$$\langle L^3 \rangle^{-LC} = \langle L^3 \rangle^{+LC} = \langle L^3 \rangle_{can}$$

That is, the average longitudinal OAM defined through the Wigner distribution coincide with the GI canonical OAM (not the mechanical one) and it is process-independent.

One might expect that a similar relation holds also for the average trans. mom.

$$\langle k_\perp^i \rangle^{\pm LC} \stackrel{?}{=} \langle k_\perp^i \rangle_{can}$$

where the r.h.s. is the GI canonical transverse momentum defined by

$$\langle k_\perp^i \rangle_{can} = \frac{1}{2\,p^+} \langle p, s \,|\, \bar{\psi}(0)\, \gamma^+ \frac{1}{i} D_{\perp, pure}^i(0)\, \psi(0) \,|\, p, s \rangle$$

In fact, Lorce claims in a recent paper that the momentum variable in the Wigner distribution refers to the canonical momentum not the mechanical momentum.

In the following, we show that this statement is not always true and instead give universally correct physical interpretation of the average transverse momentum as well as the average longitudinal OAM defined through the Wigner distribution.

To this end, we first recall the fact that, according to Hatta, there are plural choices to define the physical component of the gluon in the decomposition

$$A^\mu = A^\mu_{phys} + A^\mu_{pure}$$

Choice (I)

$$A^i_{phys}(0) \equiv - \int_{-\infty}^{+\infty} d\eta^- \, (\pm \theta(\pm \eta^-)) \times \mathcal{L}[0^- \mathbf{0}_\perp, \eta^- \mathbf{0}_\perp] \, g \, F^{+i}(\eta^-, \mathbf{0}_\perp) \, \mathcal{L}[\eta^- \mathbf{0}_\perp, 0^- \mathbf{0}_\perp],$$

Choice (II)

$$A^i_{phys}(0) \equiv - \frac{1}{2} \int_{-\infty}^{+\infty} d\eta^- \, \epsilon(\eta^-) \times \mathcal{L}[0^- \mathbf{0}_\perp, \eta^- \mathbf{0}_\perp] \, g \, F^{+i}(\eta^-, \mathbf{0}_\perp) \, \mathcal{L}[\eta^- \mathbf{0}_\perp, 0^- \mathbf{0}_\perp],$$

Remarkably, in the case of average longitudinal OAM, any of the above choices for $A^i_{phys}$ gives the same answer for $\langle L^3 \rangle^{\pm LC}$, which coincides with the canonical OAM of quarks.

This is related to the PT-even nature of the quantity $\langle L^3 \rangle$.

However, it is not necessarily true for $\langle k^i_\perp \rangle^{\pm LC}$.

For choice (I), we certainly have

$$\begin{aligned}\langle k_\perp^i \rangle^{\pm LC} &= \frac{1}{2\,p^+} \langle p, s\,|\,\bar{\psi}(0)\,\gamma^+ \frac{1}{i} D_\perp^i(0)\,\psi(0)\,|\,p, s\rangle \\ &+ \frac{1}{2\,p^+} \langle p, s\,|\,\bar{\psi}(0)\,\gamma^+\, g\, A_{phys}^i(0)\,\psi(0)\,|\,p, s\rangle \\ &= \frac{1}{2\,p^+} \langle p, s\,|\,\bar{\psi}(0)\,\gamma^+ \frac{1}{i} D_{\perp,pure}^i(0)\,\psi(0)\,|\,p, s\rangle \;=\; \langle k_\perp^i \rangle_{can},\end{aligned}$$

However, it is a well known fact that the average transverse momentum corresponding to the future-pointing staple-like LC path and the past-pointing staple-like LC path have different signs as

$$\langle k_\perp^i \rangle^{-LC} \;=\; -\,\langle k_\perp^i \rangle^{+LC} \qquad \text{(Collins, 2002)}$$

This means that the canonical transverse momentum defined as above is not a universal quantity, i.e. it is a process-dependent quantity.

For choice (II), using the identity,

$$\pm\theta(\pm\eta^-) = \frac{1}{2}[\epsilon(\eta^-) \pm 1]$$

we can show

$$\langle k_\perp^i \rangle^{\pm LC} = \frac{1}{2p^+}\langle p,s|\bar{\psi}(0)\gamma^+ \frac{1}{i} D^i_{\perp,pure}(0)\psi(0)|p,s\rangle \rightarrow \langle k_\perp^i\rangle_{can}$$

$$\mp \frac{1}{4p^+}\int_{-\infty}^{+\infty} d\eta^- \langle p,s|\bar{\psi}(0)\gamma^+ \mathcal{L}[0^-,\eta^-]\, g\, F^{+i}(\eta^-)\,\mathcal{L}[\eta^-,0^-]\,\psi(0)\,|p,s\rangle,$$

which means that

$$\langle k_\perp^i \rangle^{\pm LC} \neq \langle k_\perp^i \rangle_{can}$$

The above argument confirms non-universal nature of the statement by Lorce that the momentum variable in the Wigner distribution refers to the canonical momentum not the mechanical momentum.

In our opinion, the above-mentioned arbitrariness in the definition of the canonical transverse momentum is an indication of its mathematical or theoretical (rather than physical) nature in contrast to the mechanical transverse momentum with more physical nature.

What is universally correct physical interpretation of Wigner-distribution-based definitions of the average transverse momentum and longitudinal OAM, then ?

Taking the semi-inclusive DIS case as a concrete example, one can say that the average transverse momentum of quarks defined by the Wigner distribution represents the asymptotic momentum of a quark after it leaves the target.

Note that this interpretation is physical so that it holds independently of the arbitrariness of the definitions of the canonical transverse momentum.

Naturally, how to relate this asymptotic momentum of quarks to observables is a highly nontrivial question, because of the color confinement of QCD, which does not allow the existence of free quarks.

Nevertheless, to grasp the physical meaning of the average transverse momentum defined by the Wigner distribution, it may be instructive to imagine a very hard quark jet produced in the above-mentioned semi-inclusive DIS.

The produced parent quark is supposed to be fragmented into several hadrons running very fast.

If one can measure the transverse momenta of all these hadrons, one can in principle reconstruct the transverse momentum of the original quark.

Needless to say, the fact is not so simple, because the fragmentation process occurs through the interaction with the residual spectator. The detail may not be unrelated to the definition of the quark jet algorithm.

Nonetheless, it seems clear that this gedankenexperiment clarifies the physical interpretation of the average transverse momentum defined through the Wigner distribution.

Exactly the same interpretation holds also for the average longitudinal OAM.

Namely, we can interpret the average longitudinal OAM defined by the Wigner distribution represents the OAM of the quark in the asymptotic distance after leaving the spectator.

If it were not for the color confinement, this quark would be free.

This naturally explains the reason why the canonical OAM not the mechanical OAM appears in the Wigner-function-motivated definition of OAM.

However, this does not mean that the canonical OAM is easier to measure within the standard factorization scheme of DIS scattering cross sections.

**Non-observability of the OAM of a constituent in a composite particle ?**

( - in the absence of factorization theorem - )

[Example] **deuteron** as the **simplest composite system**

deuteron w.f. and S- and D-state probabilities

$$\psi_d(\boldsymbol{r}) \;=\; \left[ u(r) \;+\; \frac{S_{12}(\hat{\boldsymbol{r}})}{\sqrt{8}}\, w(r) \right] \frac{\chi}{\sqrt{4\,\pi}}$$

$$P_S \;=\; \int_0^\infty u^2(r)\, r^2\, dr, \quad P_D \;=\; \int_0^\infty w^2(r)\, r^2\, dr$$

**angular momentum decomposition of deuteron spin**

$$\begin{aligned} \langle J_3 \rangle \;&=\; \langle (\boldsymbol{r} \times \boldsymbol{p})_3 \rangle \;+\; \langle S_3 \rangle \\ &=\; \langle L_3 \rangle \;+\; \langle S_3 \rangle \;=\; \frac{3}{2} P_D \;+\; \left( P_S - \frac{1}{2} P_D \right) \;=\; 1 \end{aligned}$$

The OAM contribution to the net deuteron spin is determined by $P_D$ !

However, we know that the D-state probability is not a direct observable !

♣ The point is that bound state w.f.'s are not direct observables.

- R.D. Amado, Phys. Rev. C19 (1979) 1473.

♣ 2-body unitary transformation arising in the theory of meson-exchange currents can change the D-state probability, while keeping the deuteron observables intact.

- J.L. Friar, Phys. Rev. C20 (1979) 325.

♣ The D-state probability, for instance, depends on the cutoff $\Lambda$ of short range physics in an effective theory of 2-nucleon system.

- S.K. Bogner et al., Nucl. Phys. A784 (2007) 79.

⇩

See the figure in the next page !

Deuteron **D-state probability** in an effective theory

Bogner et al, 2007

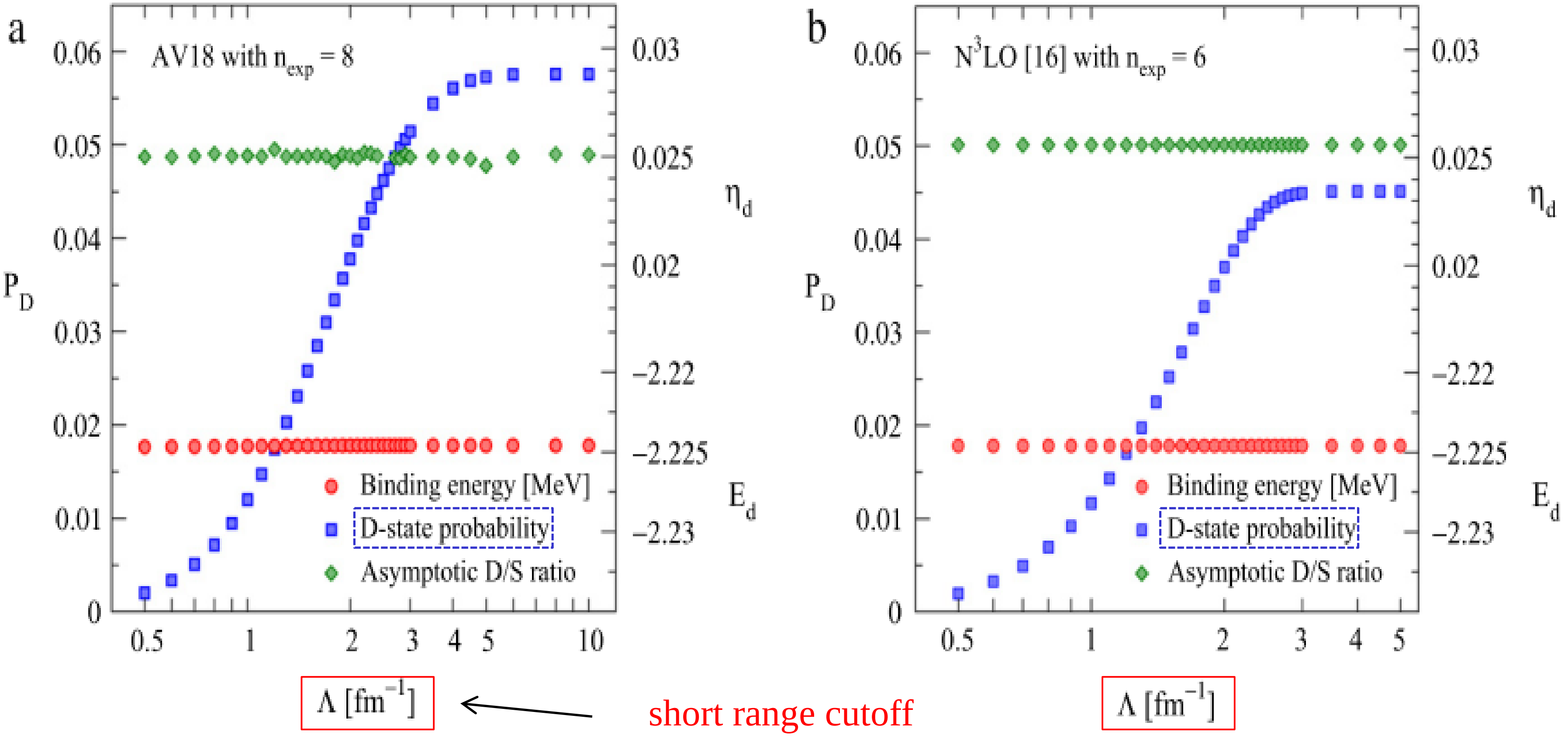



**Fig. 57.** D-state probability $P_D$ (left axis), binding energy $E_d$ (lower right axis), and asymptotic $D/S$-state ratio $\eta_d$ (upper right axis) of the deuteron as a function of the cutoff [6], starting from (a) the Argonne $v_{18}$ [18] and (b) the N$^3$LO NN potential of Ref. [20] using different smooth $V_{\text{low }k}$ regulators. Similar results are found with SRG evolution.

Why can the quark and gluon OAMs in the nucleon be observed, then ?

Owing to the **factorization theorem**

parton distribution functions (momentum distributions) are (quasi-) observable !

Compare the known sum rules for two nonequivalent quark OAMs

$$L^q_{mech} \;=\; - \int dx\, x\, G_2(x, \xi = 0, t = 0) \qquad \Leftrightarrow \quad \texttt{twist-3 GPD}$$

$$L^q_{can} \;=\; - \int dx\, d^2 k_\perp \frac{\boldsymbol{k}_\perp^2}{M^2} F^q_{1,4}(x, 0, \boldsymbol{k}_\perp^2, 0, 0) \qquad \Leftrightarrow \quad \texttt{Wigner distribution}$$

- The GPD $G_2$ can in principle be extracted from GPD analyses.
- The Wigner distribution $F_{1,4}$ drops out in both TMD and GPD factorizations !

The situation for canonical OAM is similar to the deuteron problem !

After all, what would be the crucial factor which discriminates the two OAMs ?

Now that both satisfy the gauge-invariance, the gauge-principle cannot say anything about the superiority and inferiority of these two OAMs.

In our opinion, a vital physical difference between these two OAMs is that the mechanical OAM not the canonical OAM appears in the equation of motion with Lorentz force.

$$\frac{d}{dt} \boldsymbol{L}_{mech} = q \, \boldsymbol{r} \times [\, \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} \,]$$

In fact, the GPD $G_2$ sum rule, which is related to the mechanical OAM, is derived from the following identity based on QCD equation of motion :

$$0 = \langle \bar{\psi}(0) \, \gamma^i \, \not{n} \, \mathcal{L}[0, \lambda] \, \not{D}(\lambda) \, \psi(\lambda) \rangle$$

with

$$\langle \cdots \rangle = \langle p', s' \,|\, \cdots \,|\, p, s \rangle$$

## 3. Summary and conclusion

We have carried out a comparative analysis of two types of nucleon spin decomposition, which are characterized by two types of OAMs, i.e.

"canonical" OAMs & "mechanical" OAMs

We emphasized that the gauge-symmetry cannot say anything about the relative merits of these two OAMs, because both are gauge-invariant now.

Physics lies in the fact that the latter not the former appears in the eq. of motion.

In fact, owing to the QCD equation of motion, the mechanical quark OAM can be related to a measurable GPD $G_2$ .

On the other hand, the direct relation between the canonical quark OAM and the measurable GPDs or TMDs is not known yet.