Recent lattice advances in challenges of K=>$\pi\,\pi$ decays

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2015 The Eleventh Particle Physics Phenomenology Workshop Program

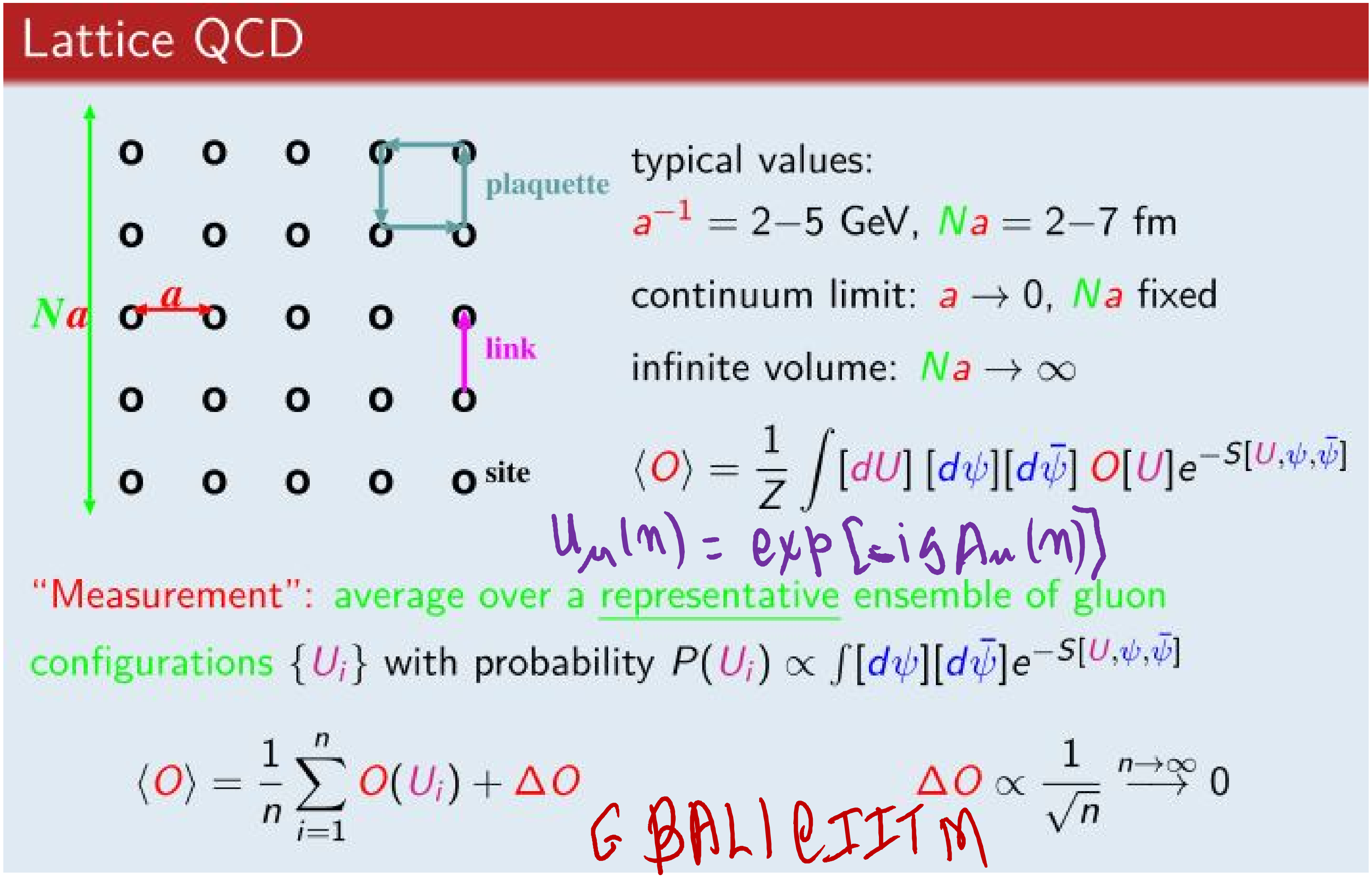
Taipei; 05/12/15

Tribute to Hai-Yang Cheng: Apetite for tackling difficult problems

- Tried to address
- **$\Delta I=1/2$ rule in several papers**

- η’ & glueballs

- Final state interactions
- Proton spin puzzle
- And many more



Lattice QCD

typical values:

$a^{-1} = 2\text{–}5$ GeV, $Na = 2\text{–}7$ fm

continuum limit: $a \to 0$, Na fixed

infinite volume: $Na \to \infty$

$$\langle O \rangle = \frac{1}{Z} \int [dU]\,[d\psi][d\bar{\psi}]\; O[U] e^{-S[U,\psi,\bar{\psi}]}$$

$U_\mu(n) = \exp[-igA_\mu(n)]$

"Measurement": average over a representative ensemble of gluon configurations $\{U_i\}$ with probability $P(U_i) \propto \int [d\psi][d\bar{\psi}] e^{-S[U,\psi,\bar{\psi}]}$

$$\langle O \rangle = \frac{1}{n} \sum_{i=1}^{n} O(U_i) + \Delta O \qquad \Delta O \propto \frac{1}{\sqrt{n}} \xrightarrow{n\to\infty} 0$$

G BALI @IITM

Outline: K=>ππ, ΔI=1/2 Rule and ε'/ε

- Introduction & Motivation
- Obstacles aglore
- Chiral symmetry: era of DWF simulations
- Dramatic Failure of Quenched approximation
- Limitations of ChPT
- Direct K=> ππ a la Lellouch Lucher
- Resolution of the ΔI=1/2 Puzzle
- ε'
- BK ; Kl3; ReA2; ImA2; ε'/ε[EWP]
- Summary & outlook

What's the hooplah

TEXT BOOK PROBLEM

- $\Gamma[K_s \Rightarrow \pi^+ \pi^-] / \Gamma[K^+ \Rightarrow \pi^+ \pi^0] = 670!$

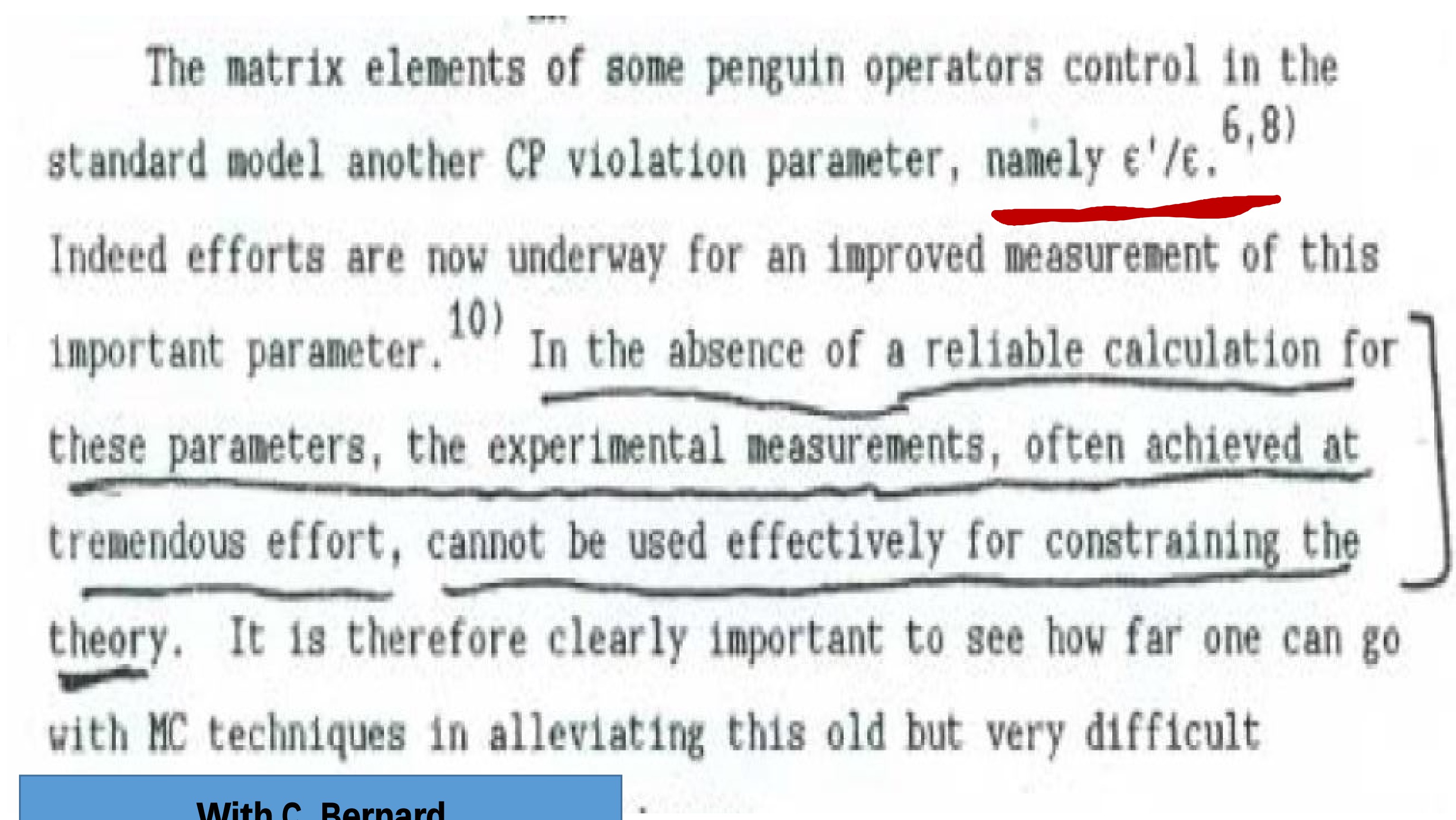
W/O QCD should be O(1) ; QCD and/or BSM causes huge effects O(25) !!

- ε' / ε

$\Longleftarrow$ Real target

Num/Denom $\Rightarrow \Delta I = 1/2 + 3/2 \,/\, \Delta I = 3/2$

A.S. in Proceedings of Lattice '85 (FSU)..1st Lattice meeting ever attended



The matrix elements of some penguin operators control in the standard model another CP violation parameter, namely ϵ'/ϵ.6,8) Indeed efforts are now underway for an improved measurement of this important parameter.10) In the absence of a reliable calculation for these parameters, the experimental measurements, often achieved at tremendous effort, cannot be used effectively for constraining the theory. It is therefore clearly important to see how far one can go with MC techniques in alleviating this old but very difficult

With C. Bernard [UCLA]

Direct CP violation in K=>ππ decays

$\frac{1}{\omega} \sim \frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} \sim 22.5$ $\Delta I = 1/2$ Rule

NA48

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \frac{\omega}{\sqrt{2}|\epsilon|}\left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right], \sim 1.65(26) \times 10^{-3}$$

K TE

Indirect CP $\frac{K_L \to 2\pi}{K_S \to 2\pi} \Rightarrow \epsilon \sim 2.27 \times 10^{-3}$ 1964 BNL expt

Nobel Prize

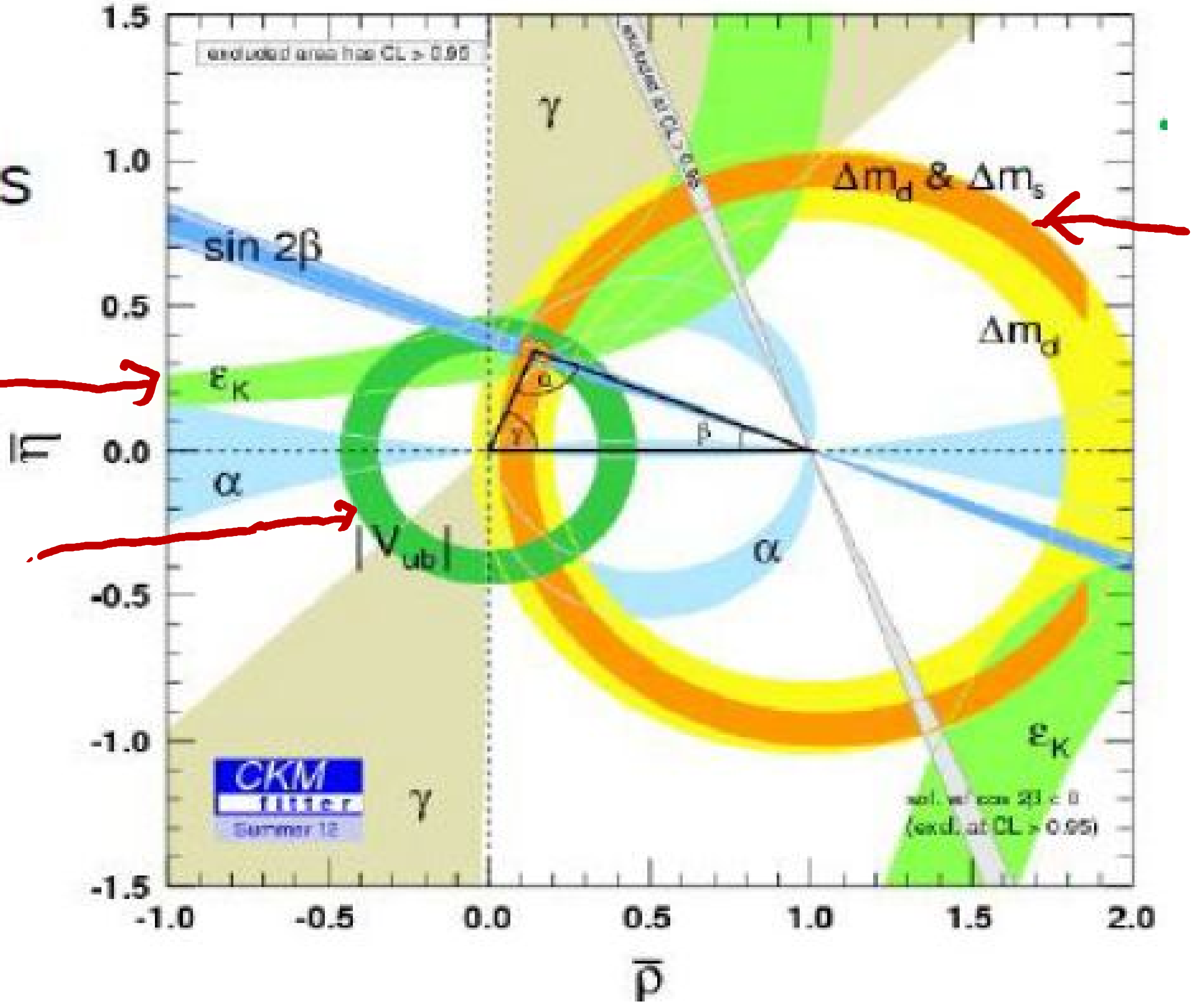
LATTICE INPUTS

as

B_K →

form factors for $B \to \pi$,
$B_s \to K$ →

← ξ

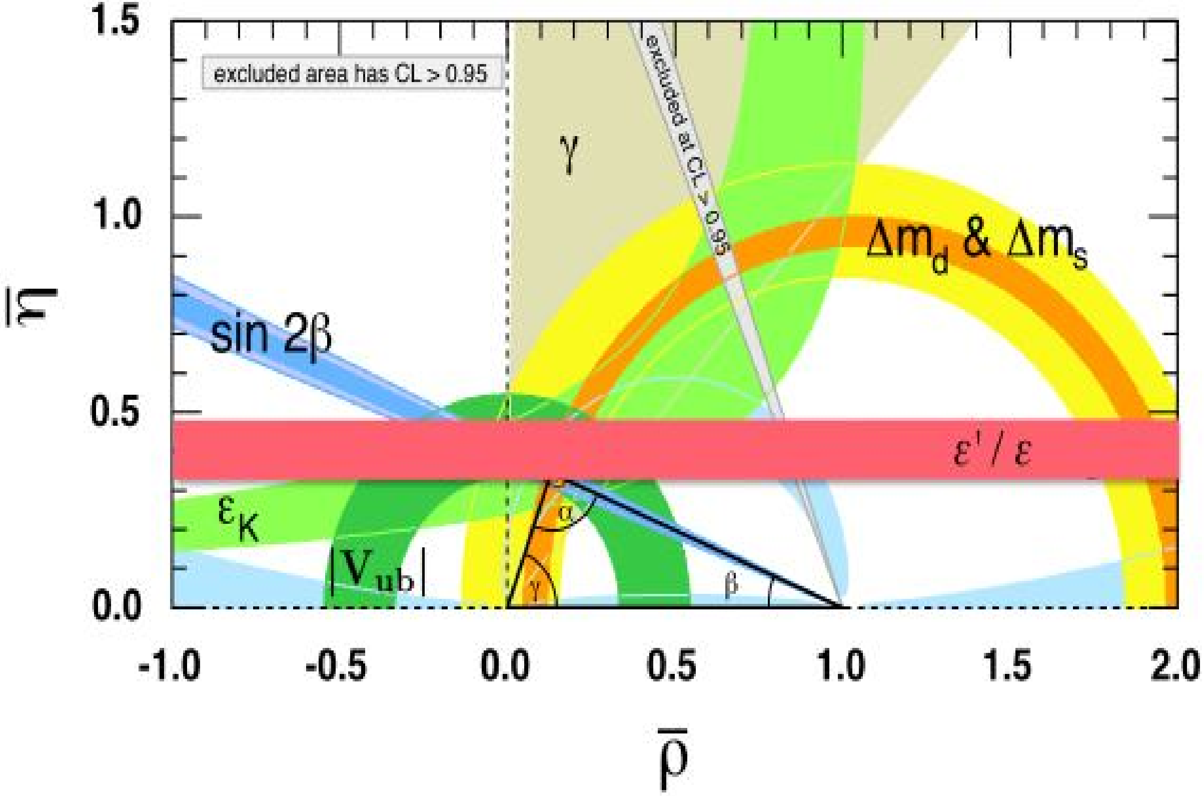


Critical Role of the B factories in the verification of the KM hypothesis was recognized and cited by the Nobel Foundation

A single irreducible phase in the weak interaction matrix accounts for most of the CPV observed in kaons and B's.

CP violating effects in the B sector are O(1) rather than O(10^{-3}) as in the kaon system.

One POTENTIAL Application of ε'/ε from theory



← Near future

Sad story of ε'/ε

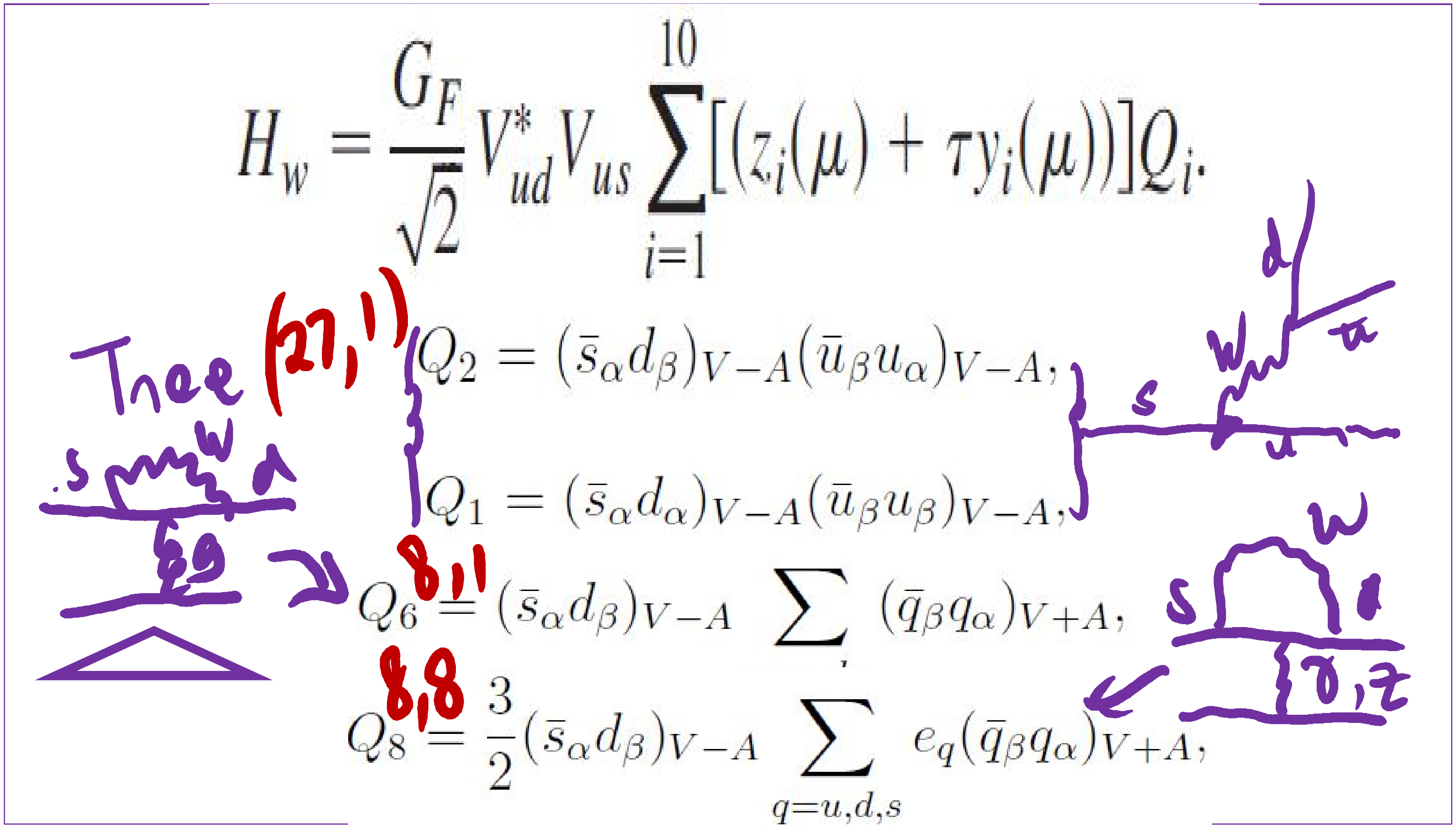
[ACTUALLY all DIRECT CP]

- ~15% measurement obtained with heroic efforts (spanning over 20 years!) on both sides of the Atlantic at a cost very iikely well **over $200M**:

$$\mathrm{Re}(\epsilon'/\epsilon) = 1.65(26) \times 10^{-3}$$

- Its WORTHLESS FOR NOW! It has 0 impact on theory
- ONLY LATTICE METHODS CAN CHANGE THIS FACT
- My entry into lattice methods ~1982 was motivated by wanting to reliably calculate ε'

Theory: Salient features



$$H_w = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{i=1}^{10} [(z_i(\mu) + \tau y_i(\mu))] Q_i.$$

Tree (27,1)

$$Q_2 = (\bar{s}_\alpha d_\beta)_{V-A} (\bar{u}_\beta u_\alpha)_{V-A},$$

$$Q_1 = (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{u}_\beta u_\beta)_{V-A},$$

8,1

$$Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum (\bar{q}_\beta q_\alpha)_{V+A},$$

8,8

$$Q_8 = \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V+A},$$

Four stages

[TARGET ε'/ε]

PhD

TD

- ~1982 – 1994 .with Claude Bernard

WF, ChPT, QA

- ~ 1995-1998 with Tom Blum

DWF, ChPT, QA

- 1998---2008 RBC

DWF, ChPT, QA $\rightarrow N_F=2$

C.C

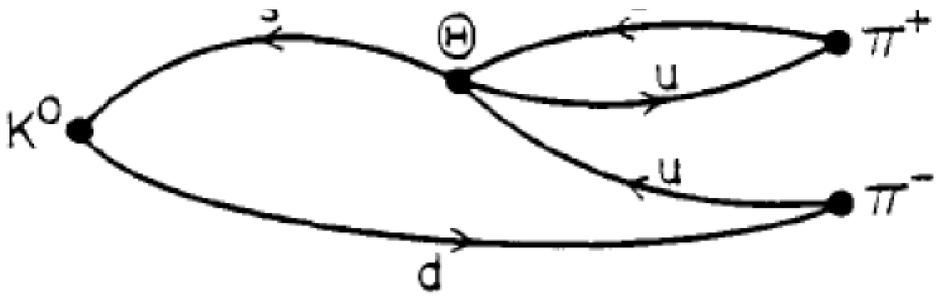
I.L

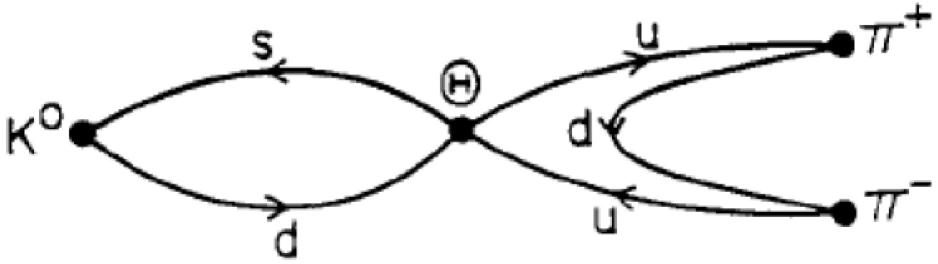
S.L

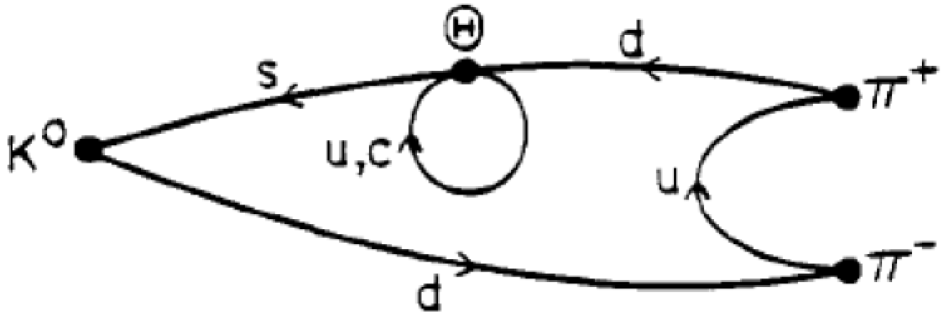
- 2008 -> present RBC-UKQCD

DWF, LL, $N_F=2+1$ (Full QCD)

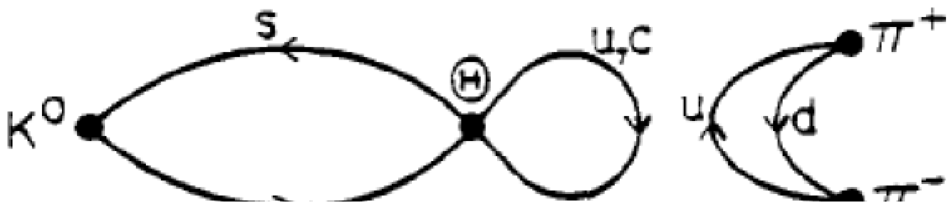
QL → DZ/CK



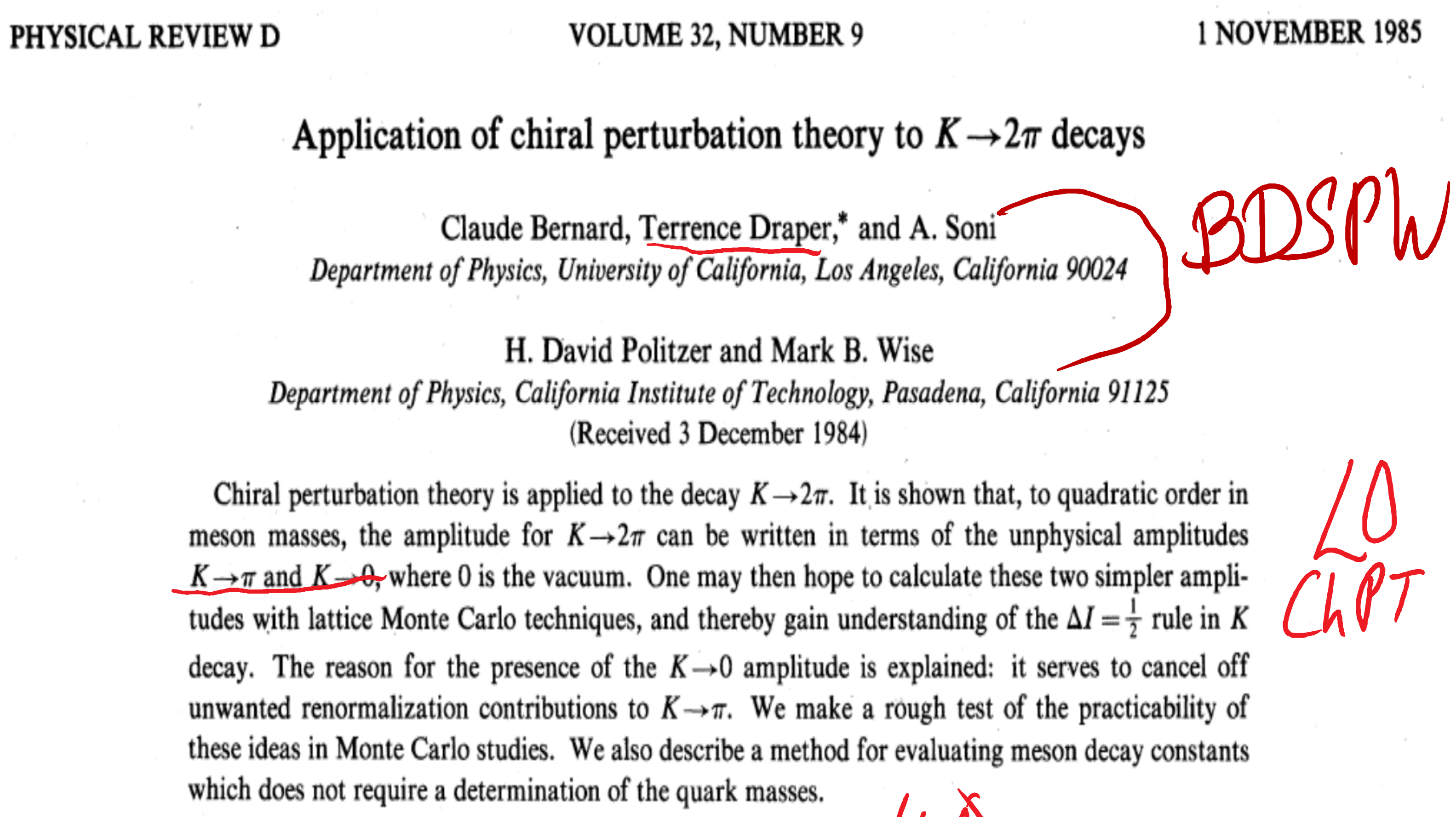




mix



mix + DISCON



PHYSICAL REVIEW D VOLUME 32, NUMBER 9 1 NOVEMBER 1985

Application of chiral perturbation theory to $K \to 2\pi$ decays

Claude Bernard, Terrence Draper,* and A. Soni
Department of Physics, University of California, Los Angeles, California 90024

H. David Politzer and Mark B. Wise
Department of Physics, California Institute of Technology, Pasadena, California 91125

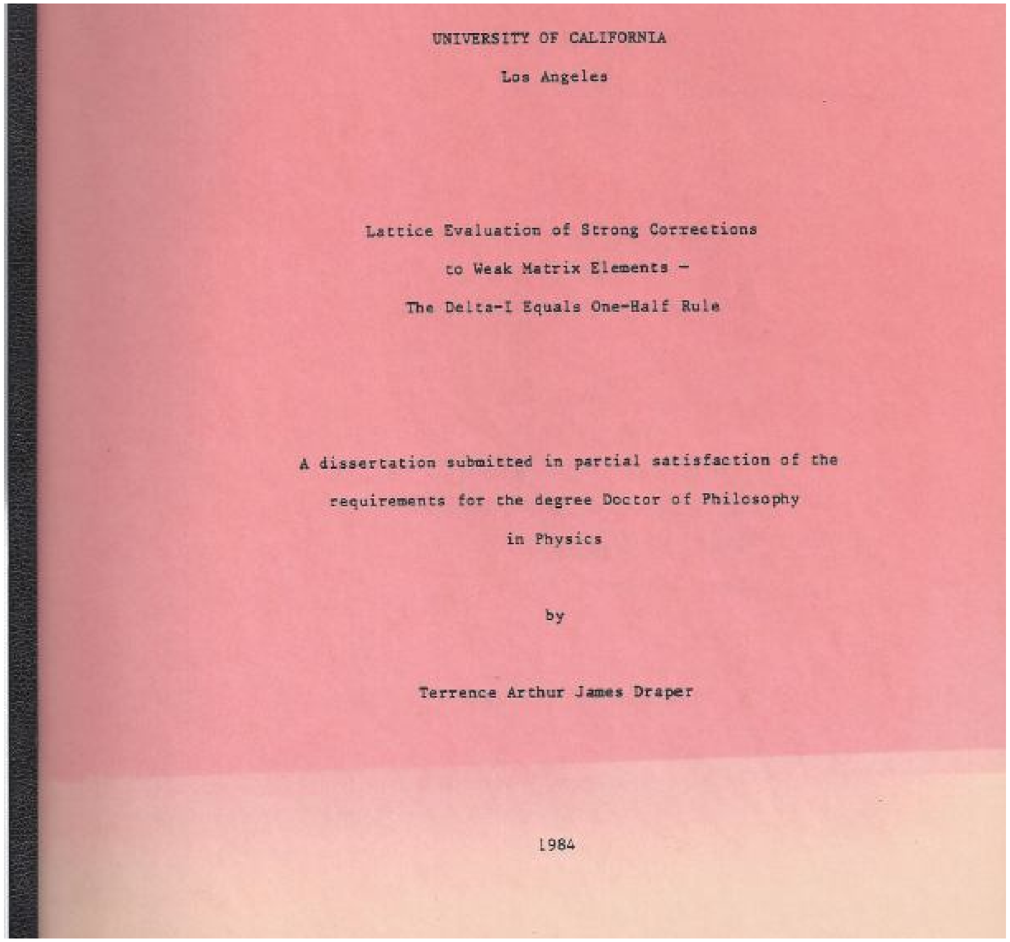
(Received 3 December 1984)

BDSPW

Chiral perturbation theory is applied to the decay $K \to 2\pi$. It is shown that, to quadratic order in meson masses, the amplitude for $K \to 2\pi$ can be written in terms of the unphysical amplitudes $K \to \pi$ and $K \to 0$, where 0 is the vacuum. One may then hope to calculate these two simpler amplitudes with lattice Monte Carlo techniques, and thereby gain understanding of the $\Delta I = \frac{1}{2}$ rule in K decay. The reason for the presence of the $K \to 0$ amplitude is explained: it serves to cancel off unwanted renormalization contributions to $K \to \pi$. We make a rough test of the practicability of these ideas in Monte Carlo studies. We also describe a method for evaluating meson decay constants which does not require a determination of the quark masses.

LO
ChPT

J. LAIHO & AS ~ 2004 NLO



UNIVERSITY OF CALIFORNIA

Los Angeles

Lattice Evaluation of Strong Corrections
to Weak Matrix Elements –
The Delta-I Equals One-Half Rule

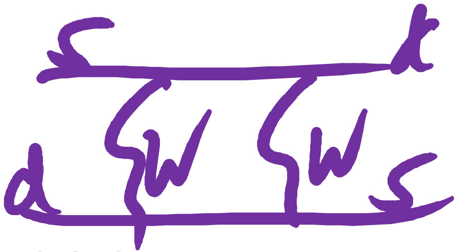
A dissertation submitted in partial satisfaction of the
requirements for the degree Doctor of Philosophy
in Physics

by

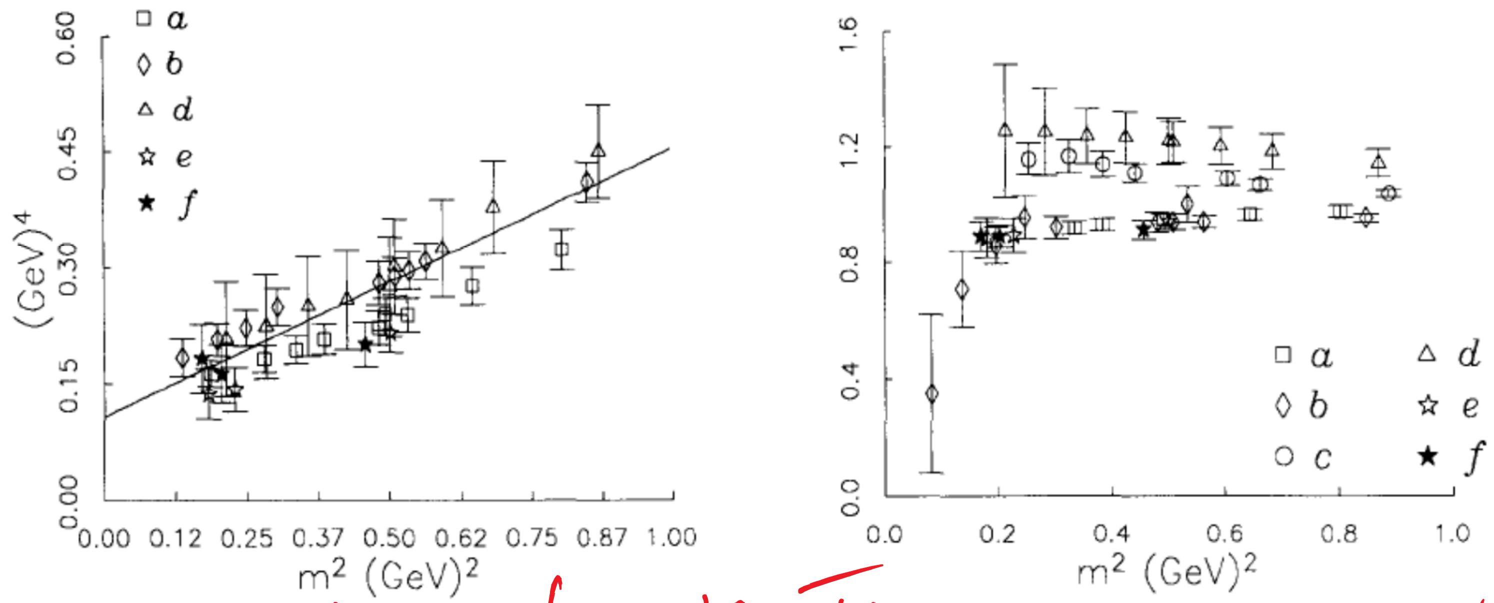
Terrence Arthur James Draper

1984

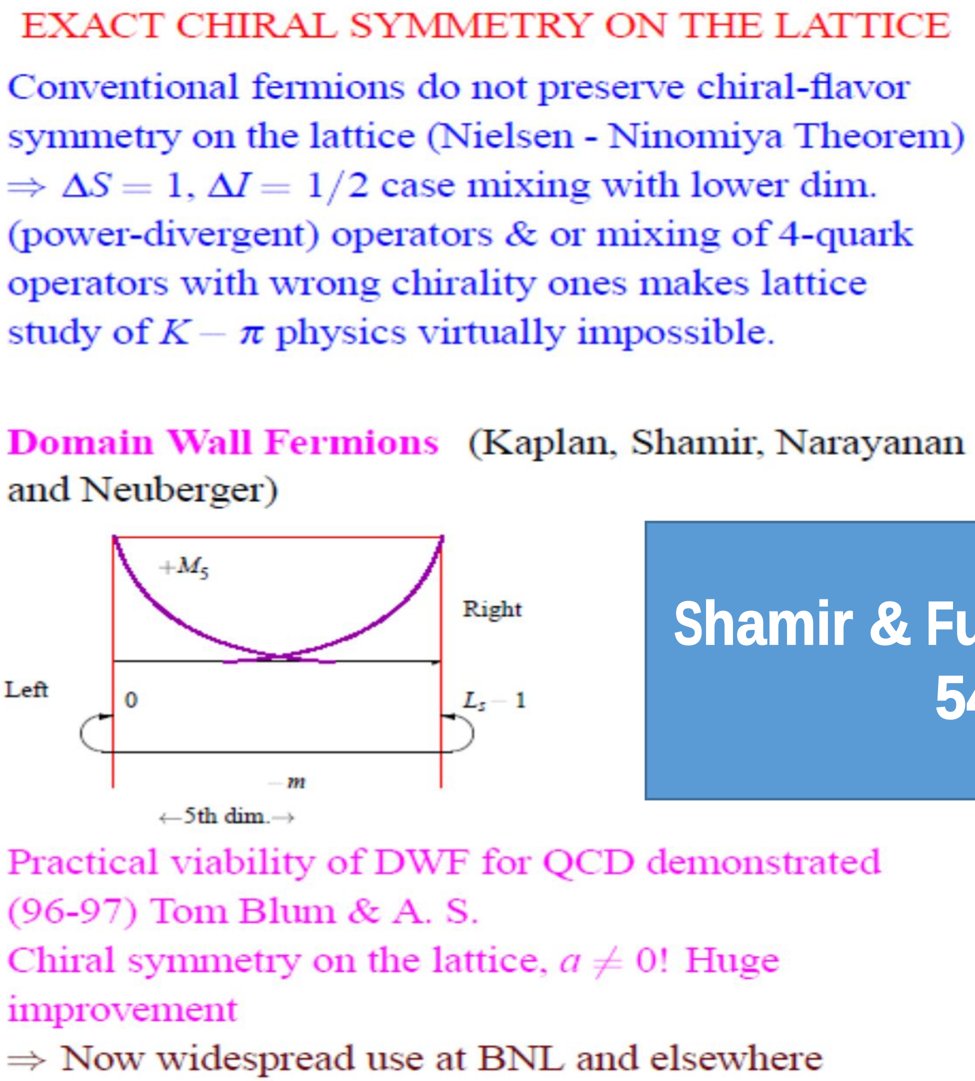
$\langle K|(\bar{s}\gamma_{\mu L} d)^2|\bar{K}\rangle$



162 *C. Bernard, A. Soni / Weak matrix elements on the lattice*



χS Violation by K-$\bar{K}$ ⇒ FINE TUNING PROBLEM



EXACT CHIRAL SYMMETRY ON THE LATTICE

Conventional fermions do not preserve chiral-flavor symmetry on the lattice (Nielsen - Ninomiya Theorem) $\Rightarrow \Delta S = 1, \Delta I = 1/2$ case mixing with lower dim. (power-divergent) operators & or mixing of 4-quark operators with wrong chirality ones makes lattice study of $K - \pi$ physics virtually impossible.

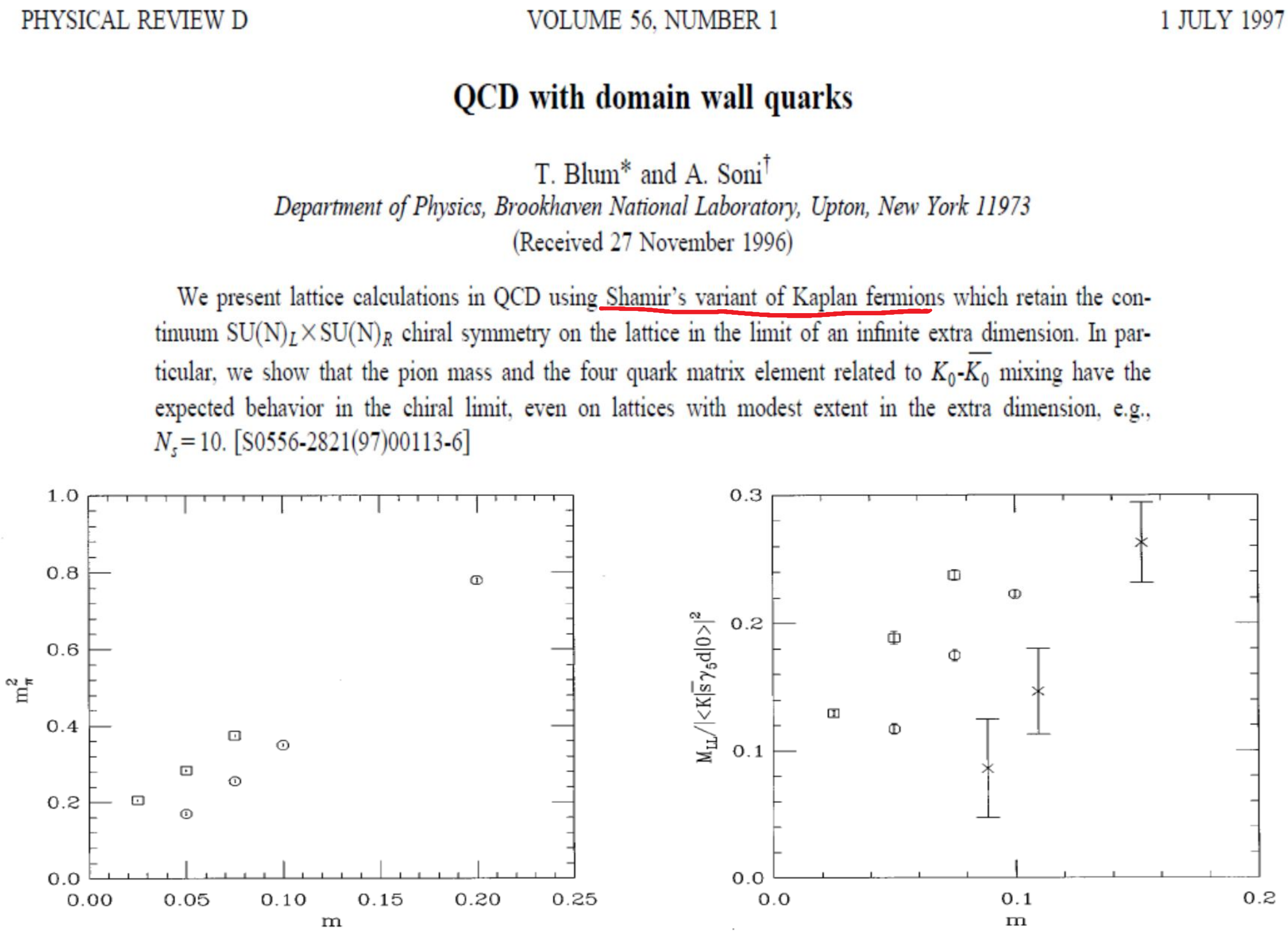
Domain Wall Fermions (Kaplan, Shamir, Narayanan and Neuberger)

Practical viability of DWF for QCD demonstrated (96-97) Tom Blum & A. S.

Chiral symmetry on the lattice, $a \neq 0$! Huge improvement

$\Rightarrow$ Now widespread use at BNL and elsewhere

Shamir & Furman, NPB 439, 54,1995



PHYSICAL REVIEW D VOLUME 56, NUMBER 1 1 JULY 1997

QCD with domain wall quarks

T. Blum* and A. Soni†
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973
(Received 27 November 1996)

We present lattice calculations in QCD using Shamir's variant of Kaplan fermions which retain the continuum $SU(N)_L \times SU(N)_R$ chiral symmetry on the lattice in the limit of an infinite extra dimension. In particular, we show that the pion mass and the four quark matrix element related to K_0-$\overline{K_0}$ mixing have the expected behavior in the chiral limit, even on lattices with modest extent in the extra dimension, e.g., $N_s=10$. [S0556-2821(97)00113-6]

RBC

QA ; ChPT

PRD ~ 02

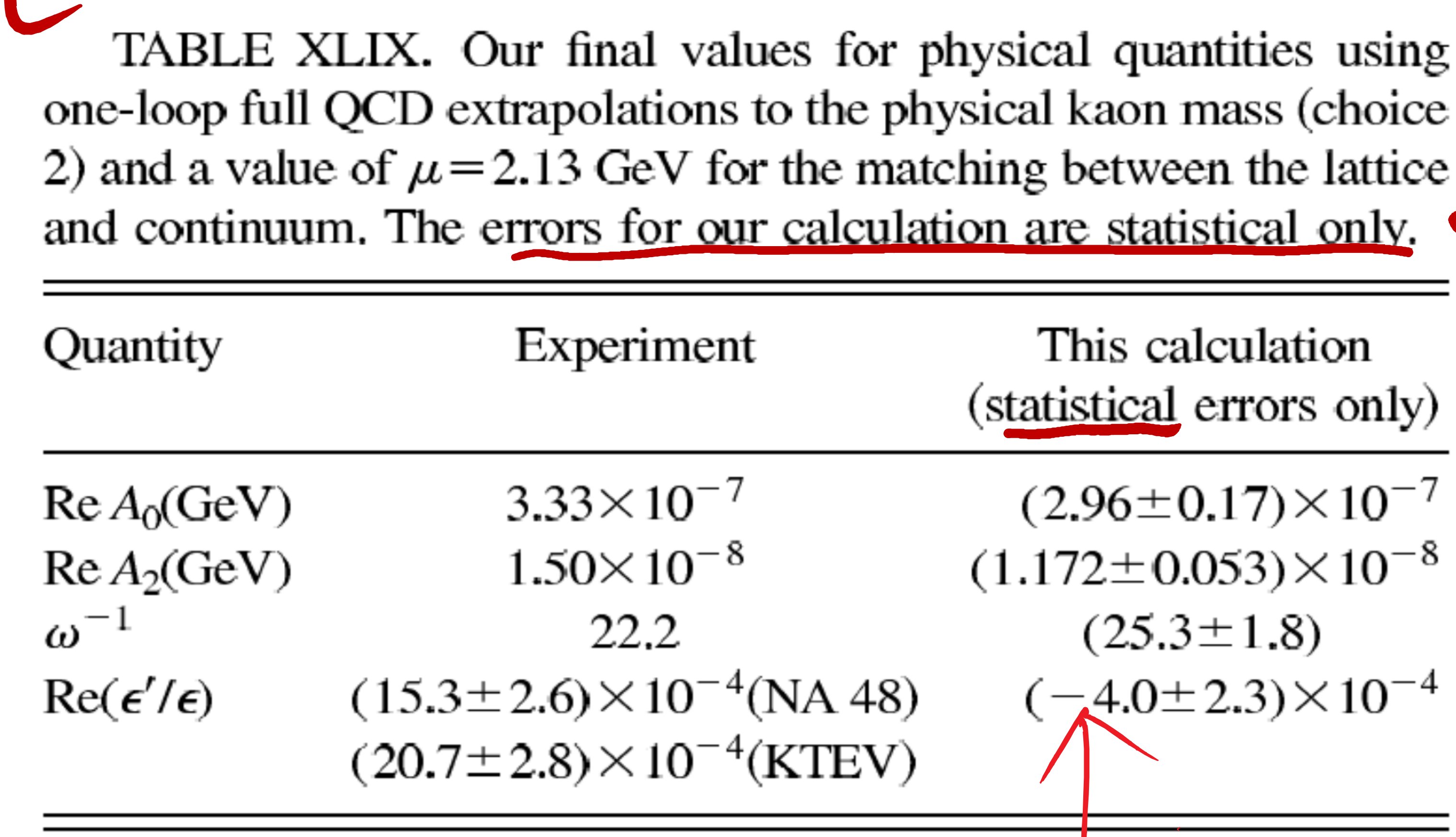


TABLE XLIX. Our final values for physical quantities using one-loop full QCD extrapolations to the physical kaon mass (choice 2) and a value of $\mu = 2.13$ GeV for the matching between the lattice and continuum. The errors for our calculation are statistical only. ←

Quantity	Experiment	This calculation (statistical errors only)
Re A_0(GeV)	3.33×10^{-7}	$(2.96 \pm 0.17) \times 10^{-7}$
Re A_2(GeV)	1.50×10^{-8}	$(1.172 \pm 0.053) \times 10^{-8}$
ω^{-1}	22.2	(25.3 ± 1.8)
Re(ϵ'/ϵ)	$(15.3 \pm 2.6) \times 10^{-4}$(NA 48) $(20.7 \pm 2.8) \times 10^{-4}$(KTEV)	$(-4.0 \pm 2.3) \times 10^{-4}$

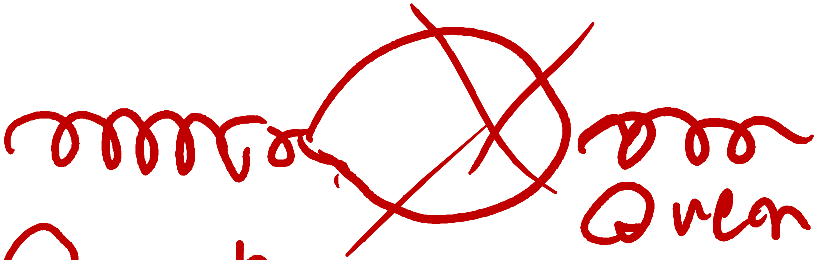
RBC = RBRC + BNL + CU

See Golterman & Pallante '01; '04; Aubin et al (RBC) / 06

Extremely serious quench patholgy

- Most important for Q6 as it LR=> (S+P)(S-P); AND it makes the most important contribution to ε'

Source of problem is that H_eff for ΔS=1 has operators such as Q6 with Quark content $(\bar{s}d)(\bar{u}u) \rightarrow$ quark loop from weak interaction



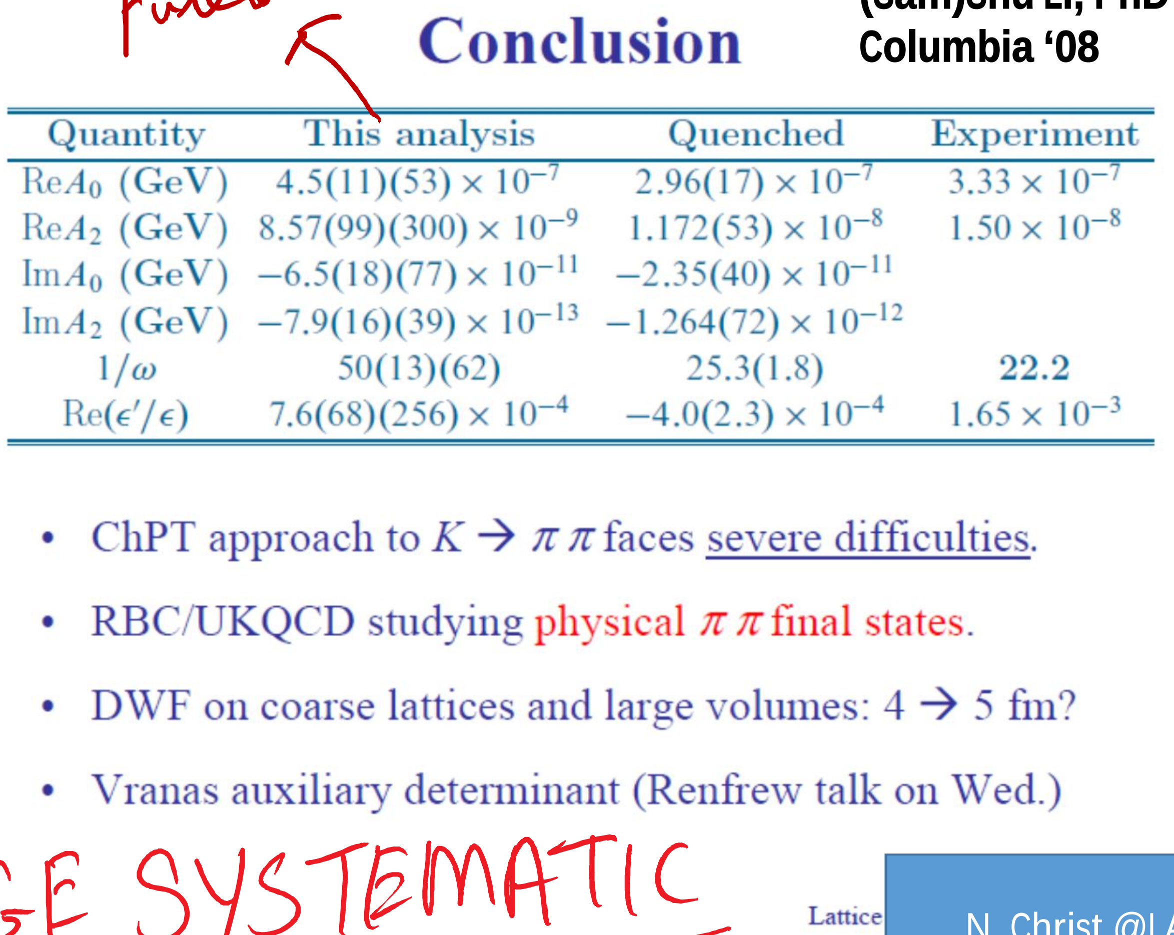
Quench approx

Q_6 gets unphysical contribution fr Q_8

$(8,1)$ $(8,8)$

Full QCD But ChPT a la BDSPW

(Sam)Shu Li, PhD thesis, Columbia '08



Conclusion

Quantity	This analysis	Quenched	Experiment
$\mathrm{Re}A_0$ (GeV)	$4.5(11)(53) \times 10^{-7}$	$2.96(17) \times 10^{-7}$	3.33×10^{-7}
$\mathrm{Re}A_2$ (GeV)	$8.57(99)(300) \times 10^{-9}$	$1.172(53) \times 10^{-8}$	1.50×10^{-8}
$\mathrm{Im}A_0$ (GeV)	$-6.5(18)(77) \times 10^{-11}$	$-2.35(40) \times 10^{-11}$	
$\mathrm{Im}A_2$ (GeV)	$-7.9(16)(39) \times 10^{-13}$	$-1.264(72) \times 10^{-12}$	
$1/\omega$	50(13)(62)	25.3(1.8)	**22.2**
$\mathrm{Re}(\epsilon'/\epsilon)$	$7.6(68)(256) \times 10^{-4}$	$-4.0(2.3) \times 10^{-4}$	1.65×10^{-3}

- ChPT approach to $K \rightarrow \pi\pi$ faces severe difficulties.
- RBC/UKQCD studying physical $\pi\pi$ final states.
- DWF on coarse lattices and large volumes: 4 → 5 fm?
- Vranas auxiliary determinant (Renfrew talk on Wed.)

Lattice

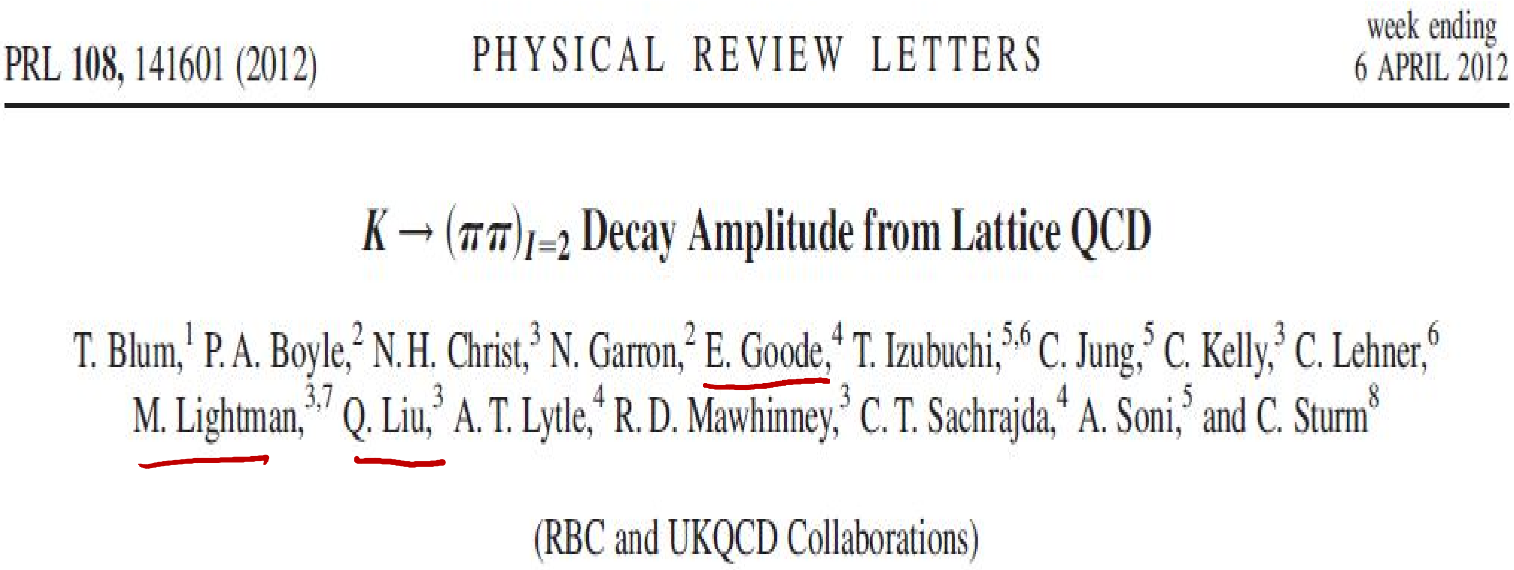
LARGE SYSTEMATIC errors DUE ChPT

N. Christ @LAT08

Direct K-> $\pi\pi$ (a la Lellouch-Luscher), using finite volume correlation* functions, [i.e. w/o ChPT] RBC initiates around 2006

CONTINUED BY RBC-UKQCD{(mostly) Edinburgh–Southampton}

* **Allows to bypass Maini-Testa theorem**



PRL **108,** 141601 (2012) PHYSICAL REVIEW LETTERS week ending
6 APRIL 2012

$K \to (\pi\pi)_{I=2}$ Decay Amplitude from Lattice QCD

T. Blum,[1] P. A. Boyle,[2] N. H. Christ,[3] N. Garron,[2] E. Goode,[4] T. Izubuchi,[5,6] C. Jung,[5] C. Kelly,[3] C. Lehner,[6] M. Lightman,[3,7] Q. Liu,[3] A. T. Lytle,[4] R. D. Mawhinney,[3] C. T. Sachrajda,[4] A. Soni,[5] and C. Sturm[8]

(RBC and UKQCD Collaborations)

DWQ + Full QCD + Physical kinematics!

$m_K \sim 511$ MeV, $m_\pi \sim 142$ MeV

RBC-UKQCD, arXiv:1111.1699, PRL 2012

$$\mathrm{Re}\,A_2 = (1.436 \pm 0.062_{\mathrm{stat}} \pm 0.258_{\mathrm{syst}}) \times 10^{-8}\,\mathrm{GeV}$$

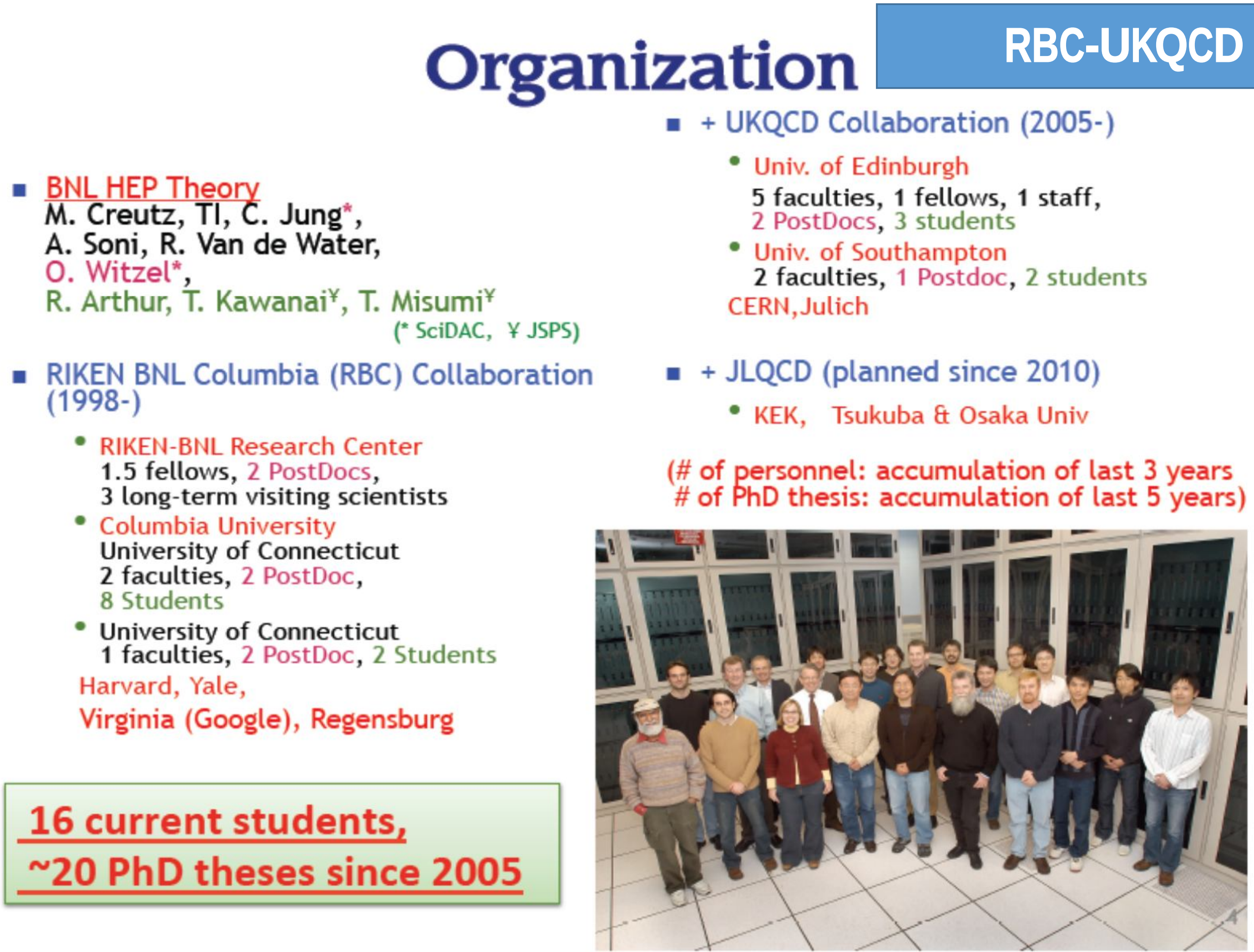
$$\mathrm{Im}\,A_2 = -(6.83 \pm 0.51_{\mathrm{stat}} \pm 1.30_{\mathrm{syst}}) \times 10^{-13}\,\mathrm{GeV}.$$

EXPT $\left.\right\}$ $1.479(4)\,K^+$

Re A_2 $\left\{\right.$ $1.573(57)\,K_S$

ERROR Re, Im $A_2 \sim 20\%$

RBC-UKQCD



Organization

- BNL HEP Theory
  M. Creutz, TI, C. Jung*,
  A. Soni, R. Van de Water,
  O. Witzel*,
  R. Arthur, T. Kawanai¥, T. Misumi¥
  (* SciDAC, ¥ JSPS)
- RIKEN BNL Columbia (RBC) Collaboration (1998-)
  - RIKEN-BNL Research Center
    1.5 fellows, 2 PostDocs,
    3 long-term visiting scientists
  - Columbia University
    University of Connecticut
    2 faculties, 2 PostDoc,
    8 Students
  - University of Connecticut
    1 faculties, 2 PostDoc, 2 Students

  Harvard, Yale,
  Virginia (Google), Regensburg

16 current students,
~20 PhD theses since 2005

- + UKQCD Collaboration (2005-)
  - Univ. of Edinburgh
    5 faculties, 1 fellows, 1 staff,
    2 PostDocs, 3 students
  - Univ. of Southampton
    2 faculties, 1 Postdoc, 2 students

  CERN,Julich
- + JLQCD (planned since 2010)
  - KEK, Tsukuba & Osaka Univ

(# of personnel: accumulation of last 3 years
# of PhD thesis: accumulation of last 5 years)

T Izubuchi

RBC-UKQCD
PRD 2015

Re + Im A_2 in the Continuum Limit

ReA_2 systematic errors	48^3	64^3	cont.
NPR (nonperturbative)	0.1%	0.7%	0.7%
NPR (perturbative)	2.7%	2.3%	2.7%
Finite volume corrections	2.2%	2.4%	2.4%
Unphysical kinematics	1.1%	4.5%	4.5%
Wilson coefficients	6.8%	6.8%	6.8%
Derivative of the phase shift	1.6%	1.0%	1.6%
Total	8%	9%	9%

TABLE IX: Systematic error breakdown for Re

ImA_2 systematic errors	48^3	64^3	cont
NPR (nonperturbative)	0.2%	0.7%	0.7%
NPR (perturbative)	6.15%	5.48%	6.15%
Finite volume corrections	2.4%	2.6%	2.6%
Unphysical kinematics	0.1%	1%	1%
Wilson coefficients	10%	8%	10%
Derivative of the phase shift	1.6%	1.0%	1.6%
Total	12%	10%	12%

TABLE X: Systematic error breakdown for Im A_2

	a^{-1}	m_K	m_π
$48^3 \times 96 \times 24$	1.728(4) GeV	498.81(14) MeV	139.2(1) MeV
$64^3 \times 128 \times 10$	2.357(7) GeV	507.4 (4) MeV	139.1(3) MeV

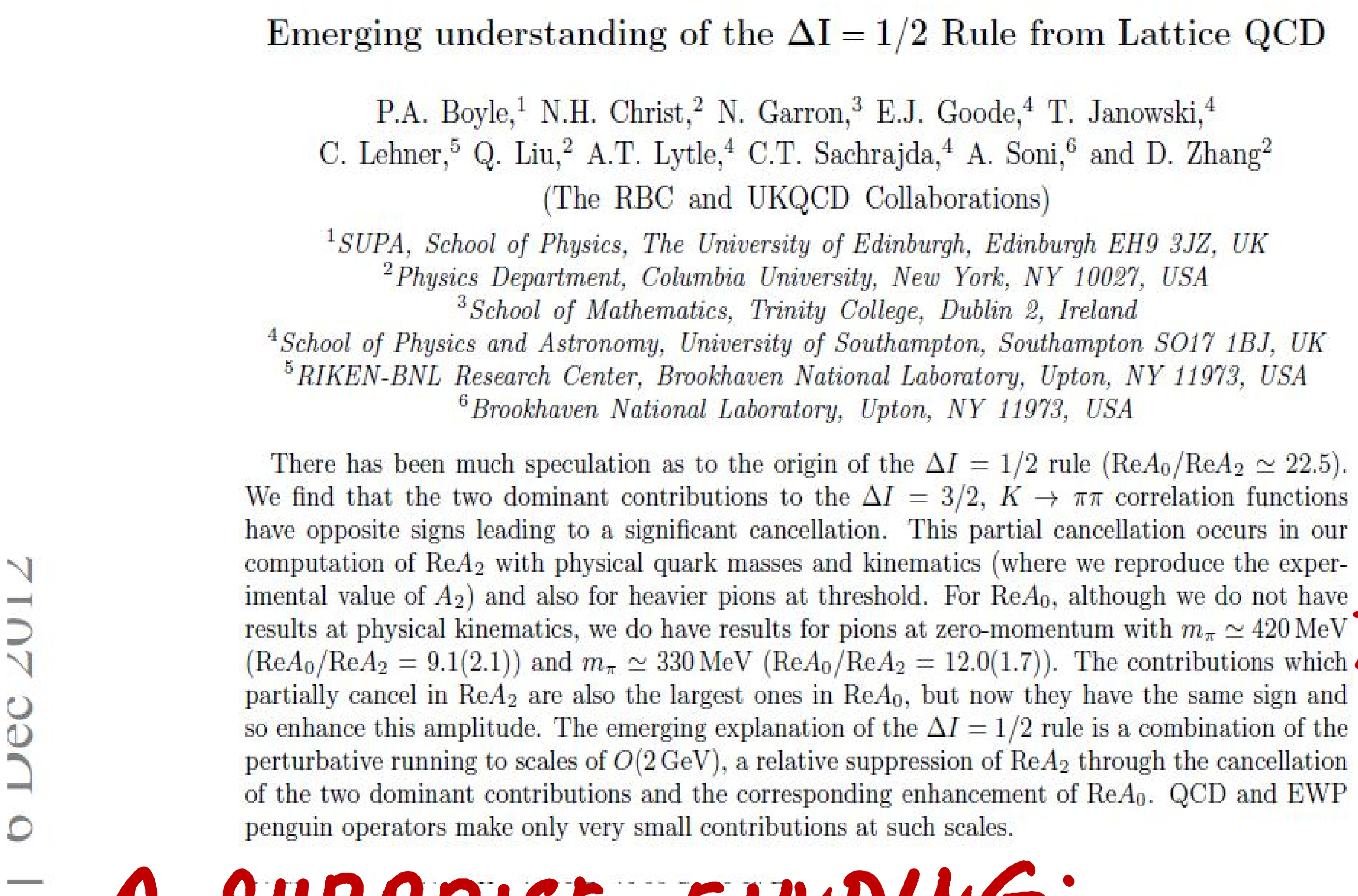
$$\left(\frac{\epsilon'}{\epsilon}\right)_{\mathrm{EWP}}$$

From our lattice calculated $\mathrm{Im}A_2$ and experimental $\mathrm{Re}A_2$ we get:

$$\left(\frac{\epsilon'}{\epsilon}\right)_{\mathrm{EWP}} \equiv \frac{\omega}{\sqrt{2}|\epsilon|}\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} = -6.6(10)\times 10^{-4},$$

$\sim$15% error

Next challenge is to compute complex A_0 which has also been underway for past few years



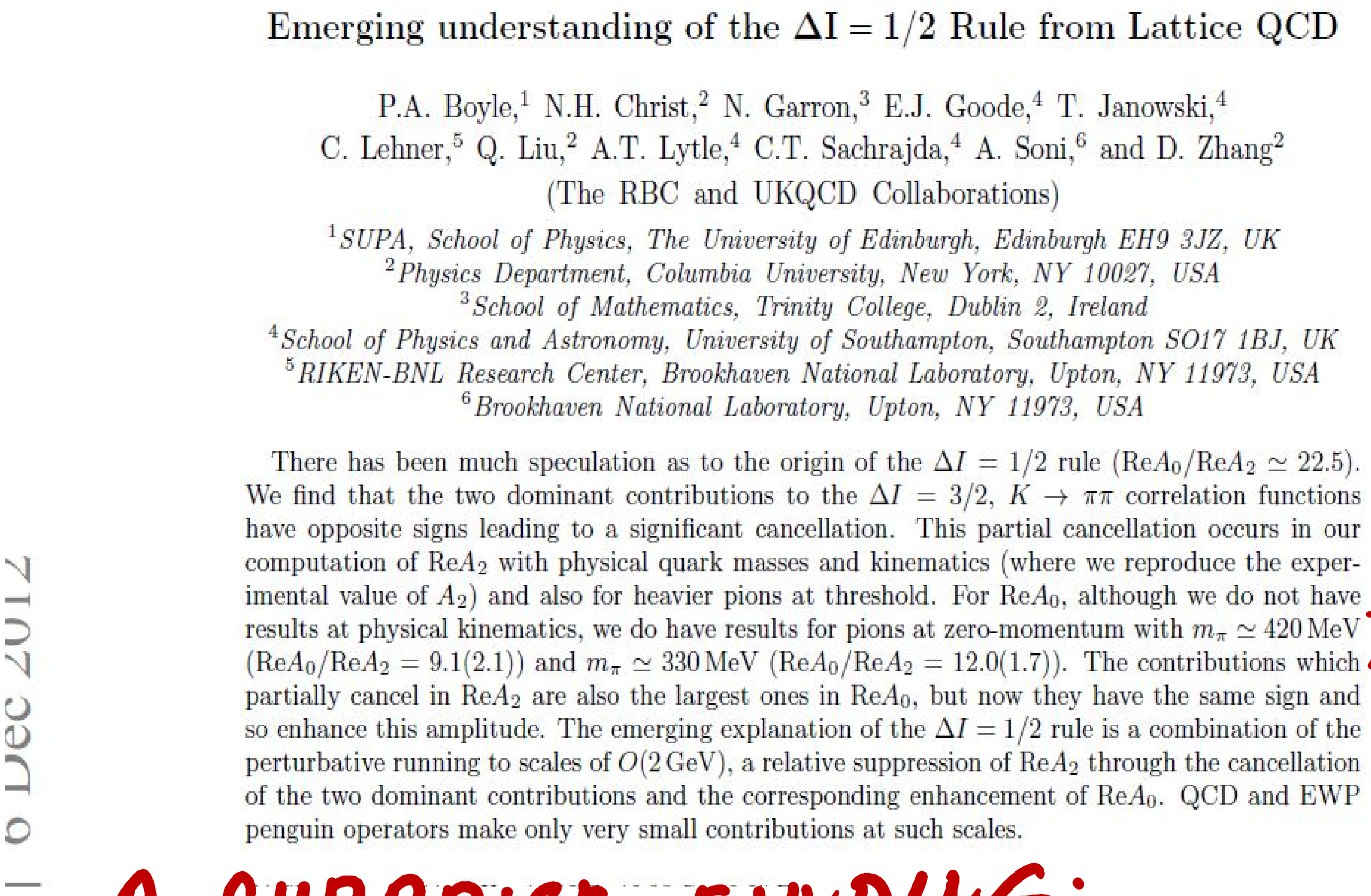
Emerging understanding of the $\Delta I = 1/2$ Rule from Lattice QCD

P.A. Boyle,[1] N.H. Christ,[2] N. Garron,[3] E.J. Goode,[4] T. Janowski,[4]
C. Lehner,[5] Q. Liu,[2] A.T. Lytle,[4] C.T. Sachrajda,[4] A. Soni,[6] and D. Zhang[2]
(The RBC and UKQCD Collaborations)

[1] *SUPA, School of Physics, The University of Edinburgh, Edinburgh EH9 3JZ, UK*
[2] *Physics Department, Columbia University, New York, NY 10027, USA*
[3] *School of Mathematics, Trinity College, Dublin 2, Ireland*
[4] *School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, UK*
[5] *RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA*
[6] *Brookhaven National Laboratory, Upton, NY 11973, USA*

There has been much speculation as to the origin of the $\Delta I = 1/2$ rule ($\mathrm{Re}A_0/\mathrm{Re}A_2 \simeq 22.5$). We find that the two dominant contributions to the $\Delta I = 3/2$, $K \to \pi\pi$ correlation functions have opposite signs leading to a significant cancellation. This partial cancellation occurs in our computation of $\mathrm{Re}A_2$ with physical quark masses and kinematics (where we reproduce the experimental value of A_2) and also for heavier pions at threshold. For $\mathrm{Re}A_0$, although we do not have results at physical kinematics, we do have results for pions at zero-momentum with $m_\pi \simeq 420\,\mathrm{MeV}$ ($\mathrm{Re}A_0/\mathrm{Re}A_2 = 9.1(2.1)$) and $m_\pi \simeq 330\,\mathrm{MeV}$ ($\mathrm{Re}A_0/\mathrm{Re}A_2 = 12.0(1.7)$). The contributions which partially cancel in $\mathrm{Re}A_2$ are also the largest ones in $\mathrm{Re}A_0$, but now they have the same sign and so enhance this amplitude. The emerging explanation of the $\Delta I = 1/2$ rule is a combination of the perturbative running to scales of $O(2\,\mathrm{GeV})$, a relative suppression of $\mathrm{Re}A_2$ through the cancellation of the two dominant contributions and the corresponding enhancement of $\mathrm{Re}A_0$. QCD and EWP penguin operators make only very small contributions at such scales.

A SURPRISE FINDING:
SIGNFICANT SUPPRESSED Re A2!



Emerging understanding of the ΔI = 1/2 Rule from Lattice QCD

PRL, 2013

P.A. Boyle,[1] N.H. Christ,[2] N. Garron,[3] E.J. Goode,[4] T. Janowski,[4]
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(The RBC and UKQCD Collaborations)

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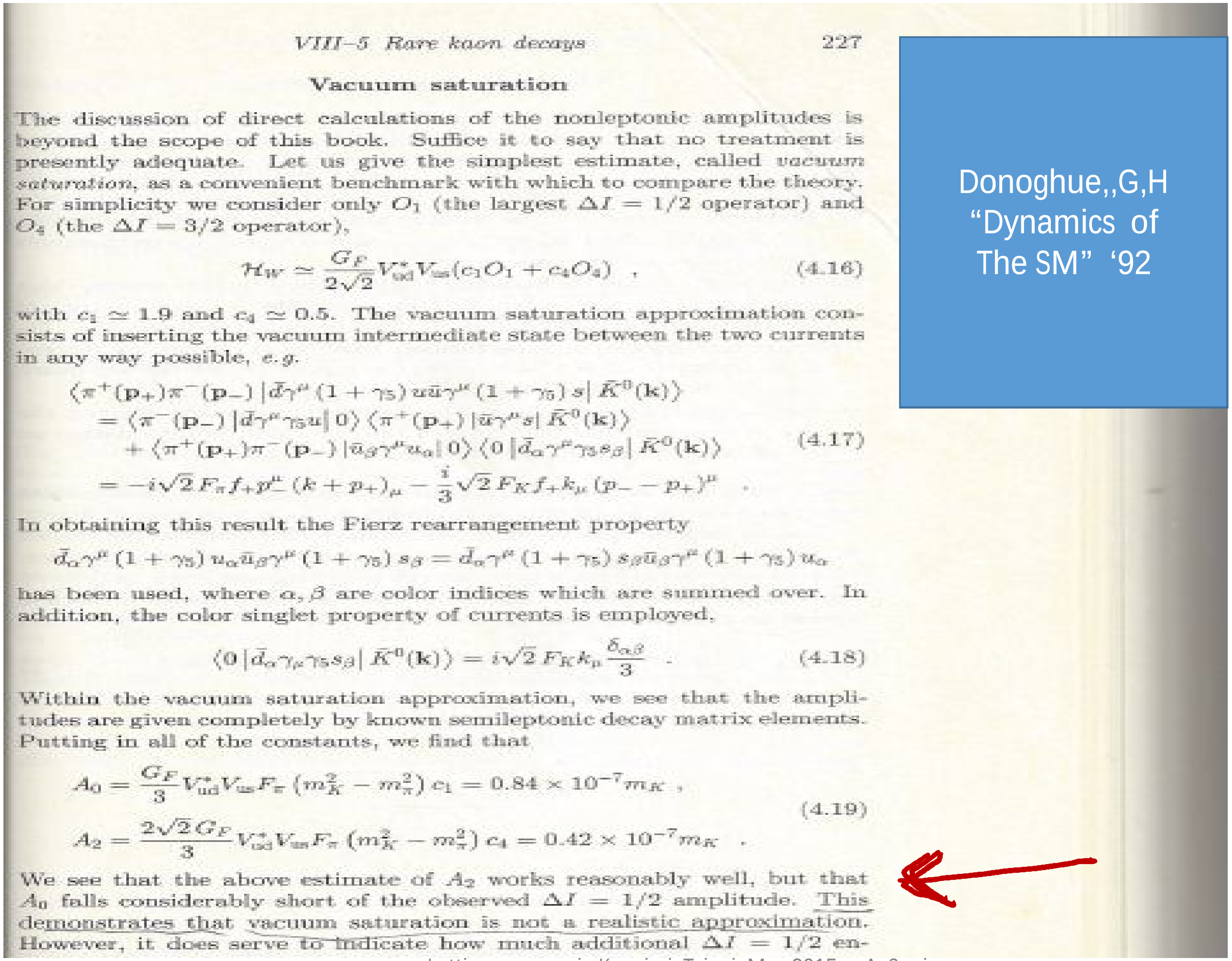
There has been much speculation as to the origin of the $\Delta I = 1/2$ rule ($\mathrm{Re}A_0/\mathrm{Re}A_2 \simeq 22.5$). We find that the two dominant contributions to the $\Delta I = 3/2$, $K \to \pi\pi$ correlation functions have opposite signs leading to a significant cancellation. This partial cancellation occurs in our computation of $\mathrm{Re}A_2$ with physical quark masses and kinematics (where we reproduce the experimental value of A_2) and also for heavier pions at threshold. For $\mathrm{Re}A_0$, although we do not have results at physical kinematics, we do have results for pions at zero-momentum with $m_\pi \simeq 420\,\mathrm{MeV}$ ($\mathrm{Re}A_0/\mathrm{Re}A_2 = 9.1(2.1)$) and $m_\pi \simeq 330\,\mathrm{MeV}$ ($\mathrm{Re}A_0/\mathrm{Re}A_2 = 12.0(1.7)$). The contributions which partially cancel in $\mathrm{Re}A_2$ are also the largest ones in $\mathrm{Re}A_0$, but now they have the same sign and so enhance this amplitude. The emerging explanation of the $\Delta I = 1/2$ rule is a combination of the perturbative running to scales of $O(2\,\mathrm{GeV})$, a relative suppression of $\mathrm{Re}A_2$ through the cancellation of the two dominant contributions and the corresponding enhancement of $\mathrm{Re}A_0$. QCD and EWP penguin operators make only very small contributions at such scales.

A SURPRISE FINDING:
SIGNFICANT SUPPRESSED Re A2!



VIII–5 Rare kaon decays 227

Vacuum saturation

The discussion of direct calculations of the nonleptonic amplitudes is beyond the scope of this book. Suffice it to say that no treatment is presently adequate. Let us give the simplest estimate, called *vacuum saturation*, as a convenient benchmark with which to compare the theory. For simplicity we consider only O_1 (the largest $\Delta I = 1/2$ operator) and O_4 (the $\Delta I = 3/2$ operator),

$$\mathcal{H}_W \simeq \frac{G_F}{2\sqrt{2}} V_{ud}^* V_{us} (c_1 O_1 + c_4 O_4) \quad , \tag{4.16}$$

with $c_1 \simeq 1.9$ and $c_4 \simeq 0.5$. The vacuum saturation approximation consists of inserting the vacuum intermediate state between the two currents in any way possible, *e.g.*

$$\begin{aligned} &\langle \pi^+(\mathbf{p}_+)\pi^-(\mathbf{p}_-)\,|\bar{d}\gamma^\mu(1+\gamma_5)\,u\bar{u}\gamma^\mu(1+\gamma_5)\,s|\,\bar{K}^0(\mathbf{k})\rangle \\ &\quad = \langle \pi^-(\mathbf{p}_-)\,|\bar{d}\gamma^\mu\gamma_5 u|\,0\rangle\,\langle \pi^+(\mathbf{p}_+)\,|\bar{u}\gamma^\mu s|\,\bar{K}^0(\mathbf{k})\rangle \\ &\qquad + \langle \pi^+(\mathbf{p}_+)\pi^-(\mathbf{p}_-)\,|\bar{u}_\beta\gamma^\mu u_\alpha|\,0\rangle\,\langle 0\,|\bar{d}_\alpha\gamma^\mu\gamma_5 s_\beta|\,\bar{K}^0(\mathbf{k})\rangle \\ &\quad = -i\sqrt{2}\,F_\pi f_+ p_-^\mu\,(k+p_+)_\mu - \frac{i}{3}\sqrt{2}\,F_K f_+ k_\mu\,(p_- - p_+)^\mu \quad . \end{aligned} \tag{4.17}$$

In obtaining this result the Fierz rearrangement property

$$\bar{d}_\alpha\gamma^\mu(1+\gamma_5)\,u_\alpha\bar{u}_\beta\gamma^\mu(1+\gamma_5)\,s_\beta = \bar{d}_\alpha\gamma^\mu(1+\gamma_5)\,s_\beta\bar{u}_\beta\gamma^\mu(1+\gamma_5)\,u_\alpha$$

has been used, where α, β are color indices which are summed over. In addition, the color singlet property of currents is employed,

$$\langle 0\,|\bar{d}_\alpha\gamma_\mu\gamma_5 s_\beta|\,\bar{K}^0(\mathbf{k})\rangle = i\sqrt{2}\,F_K k_\mu \frac{\delta_{\alpha\beta}}{3} \quad . \tag{4.18}$$

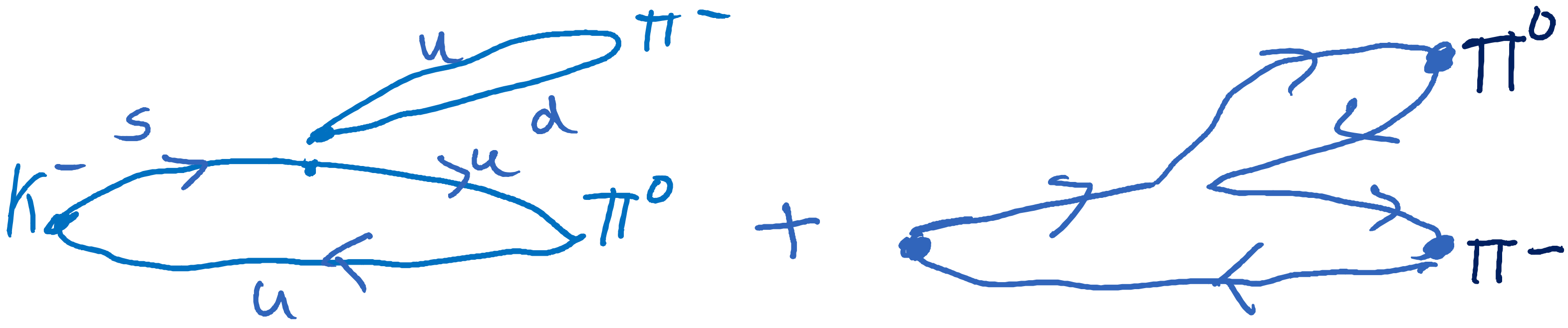
Within the vacuum saturation approximation, we see that the amplitudes are given completely by known semileptonic decay matrix elements. Putting in all of the constants, we find that

$$\begin{aligned} A_0 &= \frac{G_F}{3} V_{ud}^* V_{us} F_\pi\,(m_K^2 - m_\pi^2)\,c_1 = 0.84 \times 10^{-7} m_K \;, \\ A_2 &= \frac{2\sqrt{2}\,G_F}{3} V_{ud}^* V_{us} F_\pi\,(m_K^2 - m_\pi^2)\,c_4 = 0.42 \times 10^{-7} m_K \quad . \end{aligned} \tag{4.19}$$

We see that the above estimate of A_2 works reasonably well, but that A_0 falls considerably short of the observed $\Delta I = 1/2$ amplitude. This demonstrates that vacuum saturation is not a realistic approximation. However, it does serve to indicate how much additional $\Delta I = 1/2$ en-

Donoghue,,G,H
“Dynamics of
The SM” ‘92

Dissecting 3/2 Amp on the lattice

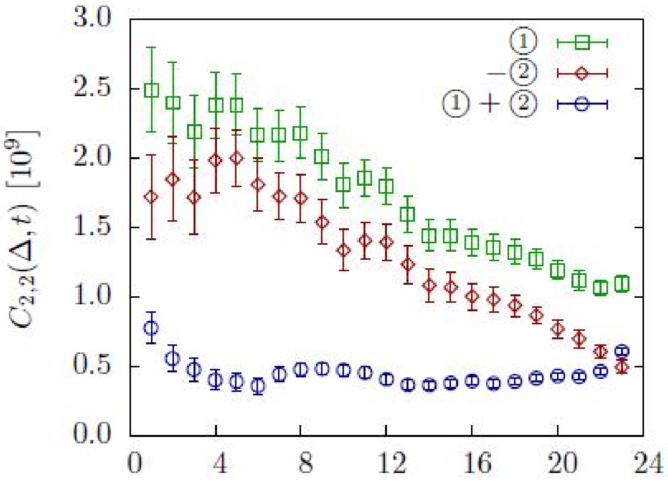


Tr × Tr

(N^2)

Tr (N)

Simplest basic step is Significantly different from phenomenological expectations



DRAMATIC CANCELLATION!

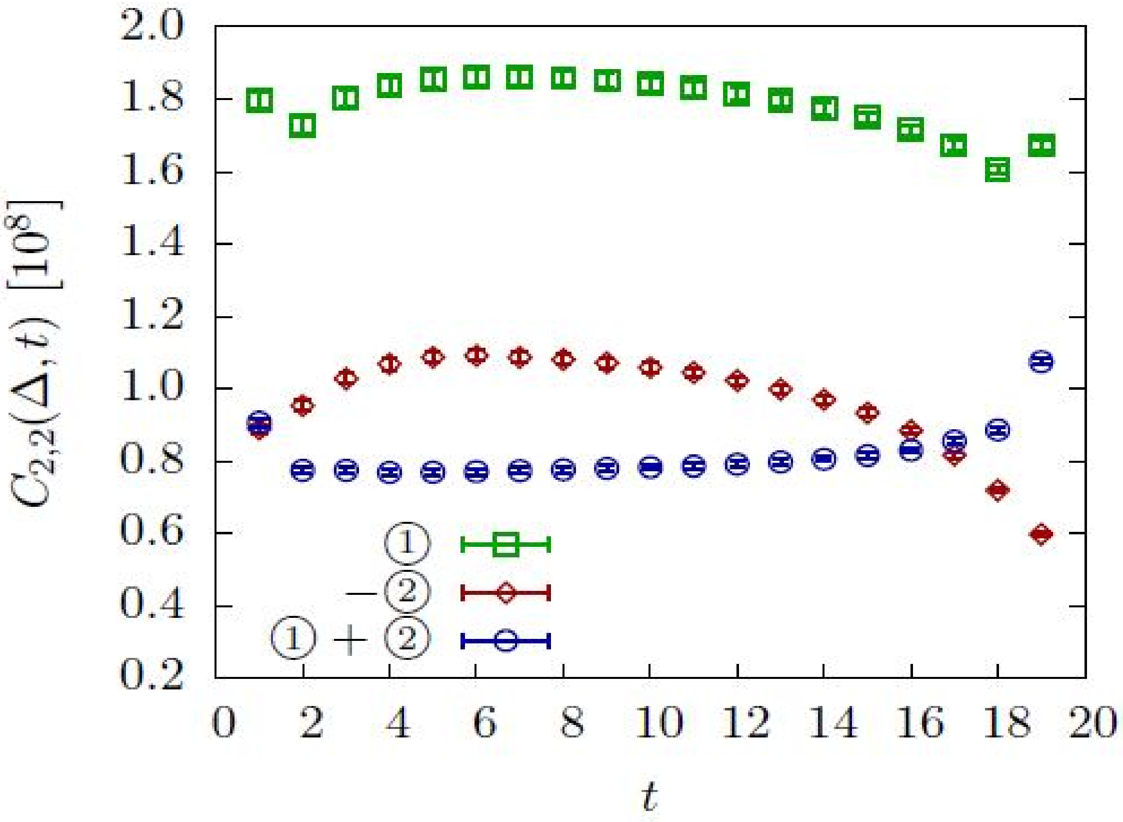
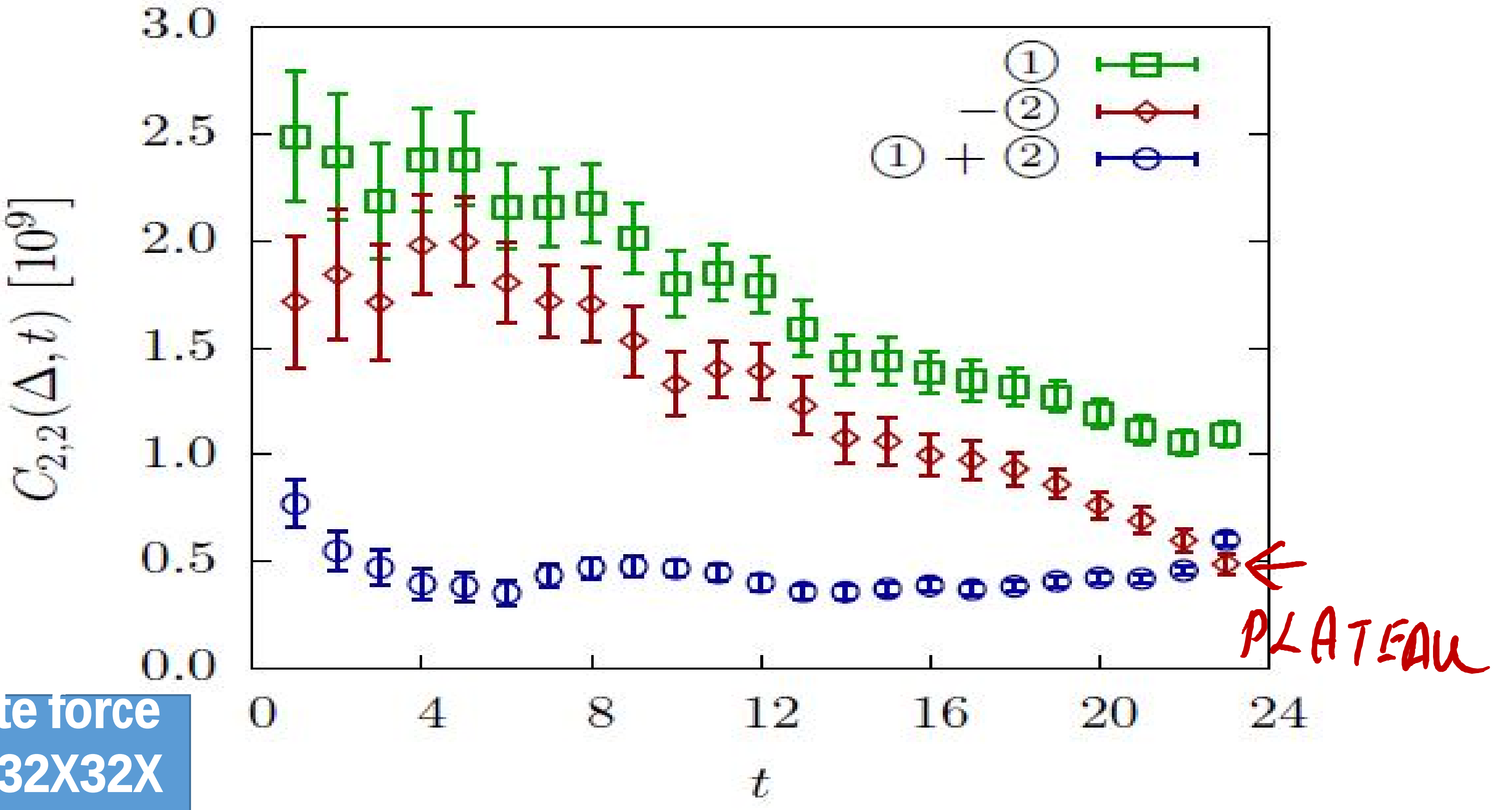


FIG. 3: Contractions ①, -② and ① + ② as functions of t from the simulation at threshold with $m_\pi \simeq 330\,\mathrm{MeV}$ and $\Delta = 20$.



PLATEAU

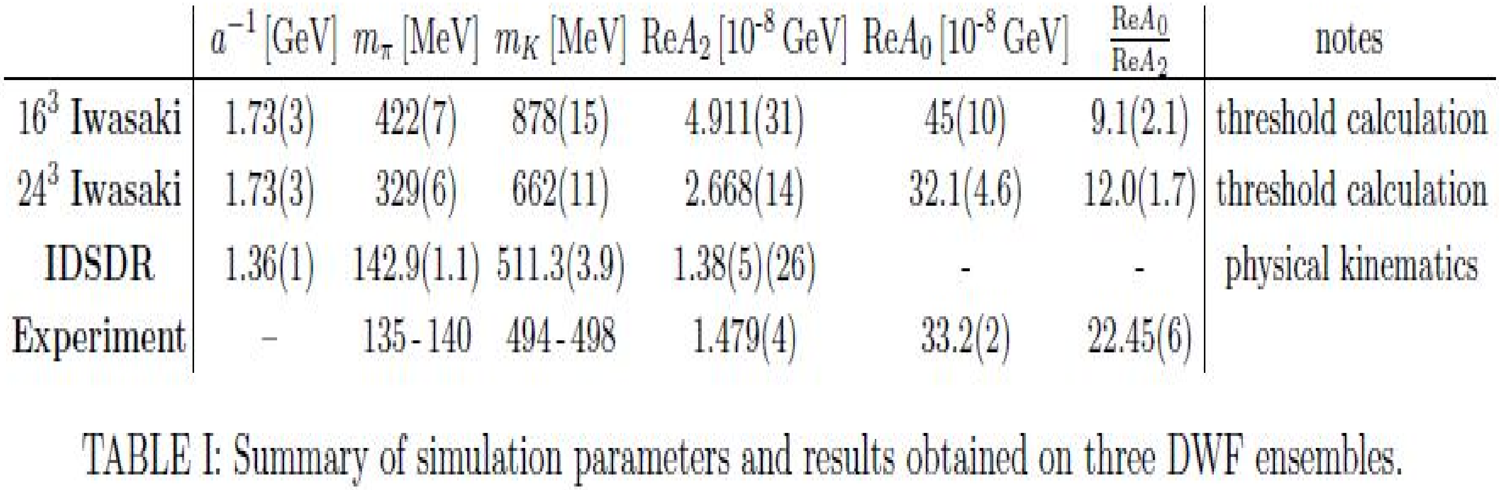
Brute force 32X32X32X 64X16

FIG. 2: Contractions ①, -② and ① + ② as functions of t from the simulation at physical kinematics and with $\Delta = 24$.

QCDOC 10 Tf

Mass depends of ReA2, A0

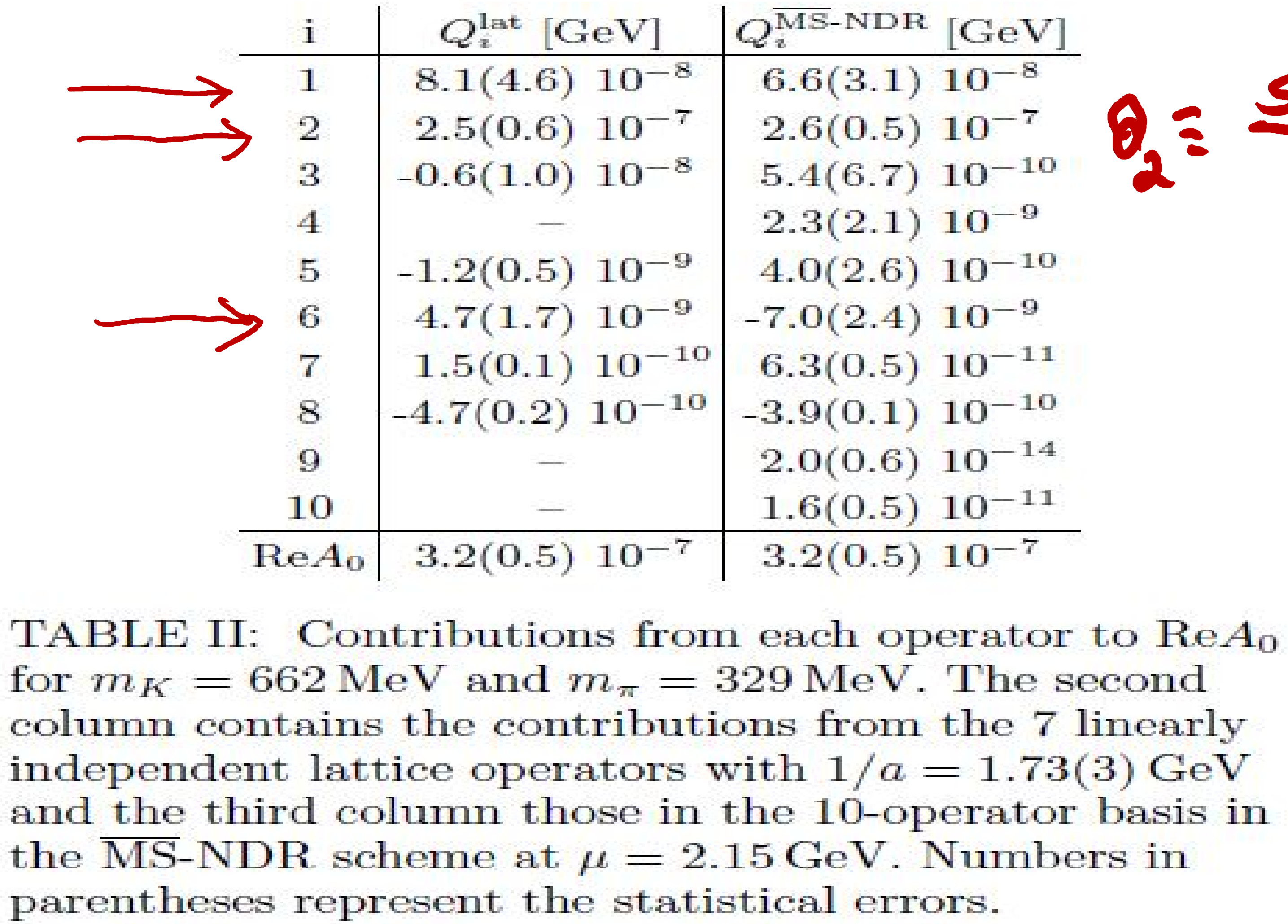
PRL
2013



	a^{-1} [GeV]	m_π [MeV]	m_K [MeV]	ReA_2 [10^{-8} GeV]	ReA_0 [10^{-8} GeV]	$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}$	notes
16^3 Iwasaki	1.73(3)	422(7)	878(15)	4.911(31)	45(10)	9.1(2.1)	threshold calculation
24^3 Iwasaki	1.73(3)	329(6)	662(11)	2.668(14)	32.1(4.6)	12.0(1.7)	threshold calculation
IDSDR	1.36(1)	142.9(1.1)	511.3(3.9)	1.38(5)(26)	-	-	physical kinematics
Experiment	–	135 - 140	494 - 498	1.479(4)	33.2(2)	22.45(6)	

TABLE I: Summary of simulation parameters and results obtained on three DWF ensembles.

Due to the cancellation, 3/2 amplitude decreases significantly as the pion mass is lowered towards its physical value



i	Q_i^{lat} [GeV]	$Q_i^{\overline{\text{MS}}\text{-NDR}}$ [GeV]
1	8.1(4.6) 10^{-8}	6.6(3.1) 10^{-8}
2	2.5(0.6) 10^{-7}	2.6(0.5) 10^{-7}
3	-0.6(1.0) 10^{-8}	5.4(6.7) 10^{-10}
4	—	2.3(2.1) 10^{-9}
5	-1.2(0.5) 10^{-9}	4.0(2.6) 10^{-10}
6	4.7(1.7) 10^{-9}	-7.0(2.4) 10^{-9}
7	1.5(0.1) 10^{-10}	6.3(0.5) 10^{-11}
8	-4.7(0.2) 10^{-10}	-3.9(0.1) 10^{-10}
9	—	2.0(0.6) 10^{-14}
10	—	1.6(0.5) 10^{-11}
ReA_0	3.2(0.5) 10^{-7}	3.2(0.5) 10^{-7}

TABLE II: Contributions from each operator to ReA_0 for $m_K = 662$ MeV and $m_\pi = 329$ MeV. The second column contains the contributions from the 7 linearly independent lattice operators with $1/a = 1.73(3)$ GeV and the third column those in the 10-operator basis in the $\overline{\text{MS}}$-NDR scheme at $\mu = 2.15$ GeV. Numbers in parentheses represent the statistical errors.

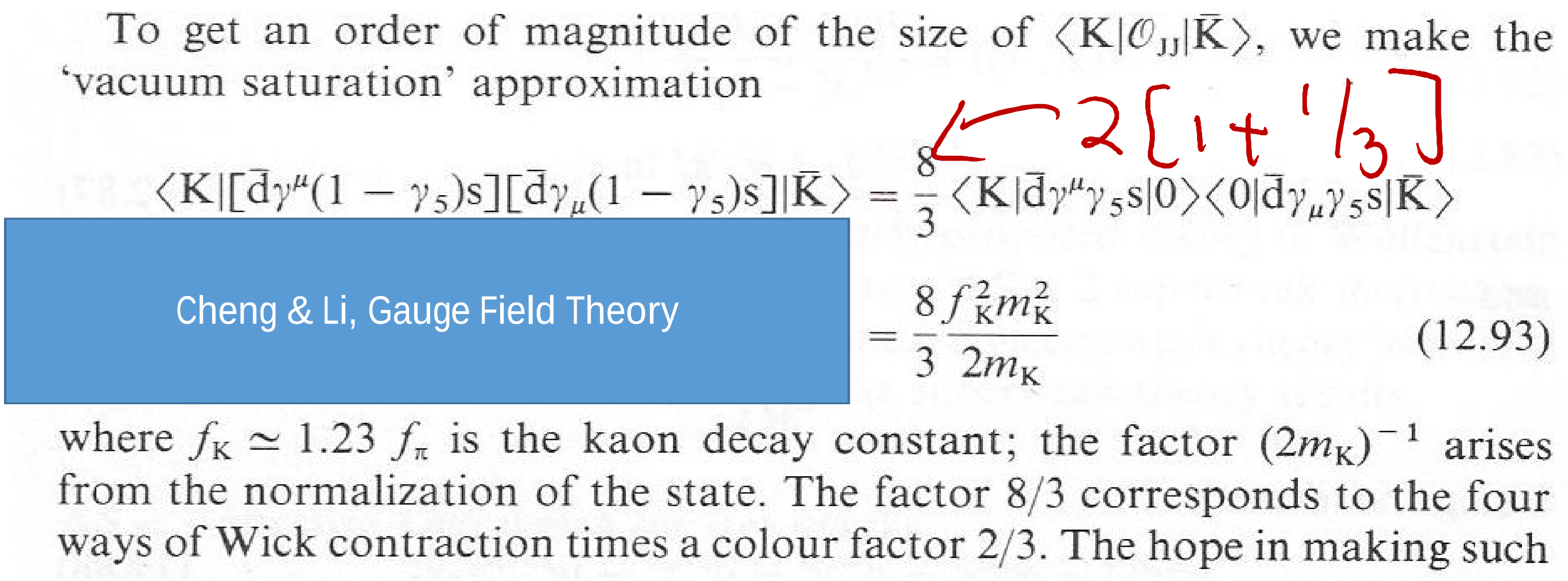
$Q_2 =$

TXT cancellation with T

- At physical kinematics we find

T ~ - 0.7 T X T in contrast to expectation from common folklore (large N): T ~ T X T/N ~ 0.3

- i.e. the very 1st step in large N and in vacuum saturation (factorization) is not adhered in QCD
- Using in addition (at scale of ~1.7 GeV), z1 = -0.30, z2= 1.14 one finds
- Re A0 / Re A2 ~ 10.9 ; thus a significant fraction of the observed enhancement originates from simple tree operators



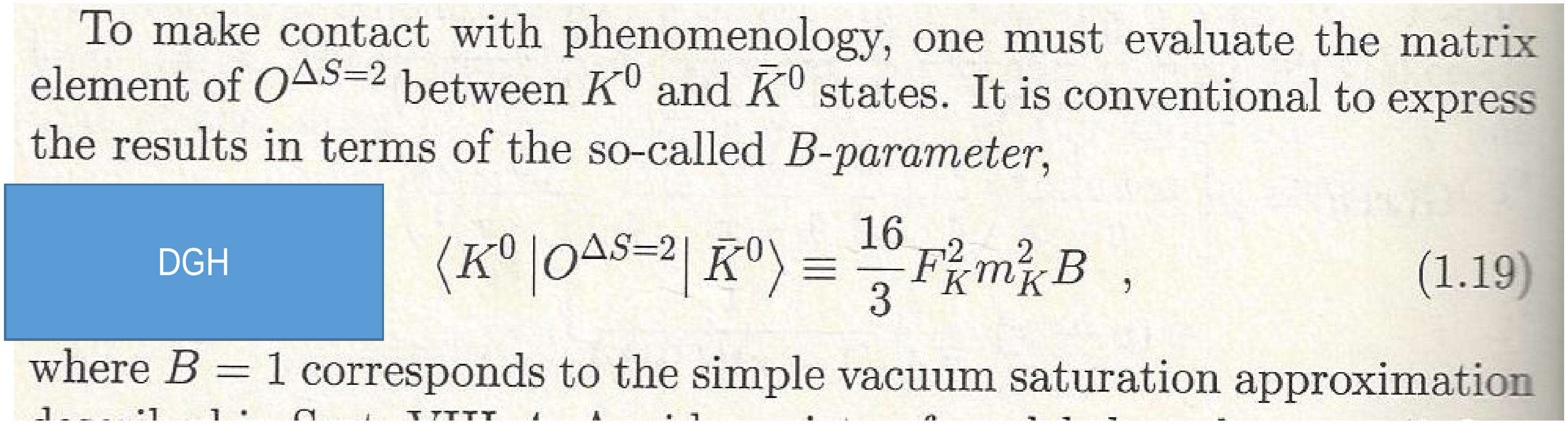
To get an order of magnitude of the size of $\langle \mathrm{K}|\mathcal{O}_{\mathrm{JJ}}|\bar{\mathrm{K}}\rangle$, we make the 'vacuum saturation' approximation

$$\langle \mathrm{K}|[\bar{\mathrm{d}}\gamma^\mu(1-\gamma_5)\mathrm{s}][\bar{\mathrm{d}}\gamma_\mu(1-\gamma_5)\mathrm{s}]|\bar{\mathrm{K}}\rangle = \overset{\leftarrow 2[1+1/3]}{\frac{8}{3}}\langle \mathrm{K}|\bar{\mathrm{d}}\gamma^\mu\gamma_5\mathrm{s}|0\rangle\langle 0|\bar{\mathrm{d}}\gamma_\mu\gamma_5\mathrm{s}|\bar{\mathrm{K}}\rangle$$

$$= \frac{8}{3}\frac{f_\mathrm{K}^2 m_\mathrm{K}^2}{2m_\mathrm{K}} \qquad (12.93)$$

Cheng & Li, Gauge Field Theory

where $f_\mathrm{K} \simeq 1.23\, f_\pi$ is the kaon decay constant; the factor $(2m_\mathrm{K})^{-1}$ arises from the normalization of the state. The factor 8/3 corresponds to the four ways of Wick contraction times a colour factor 2/3. The hope in making such

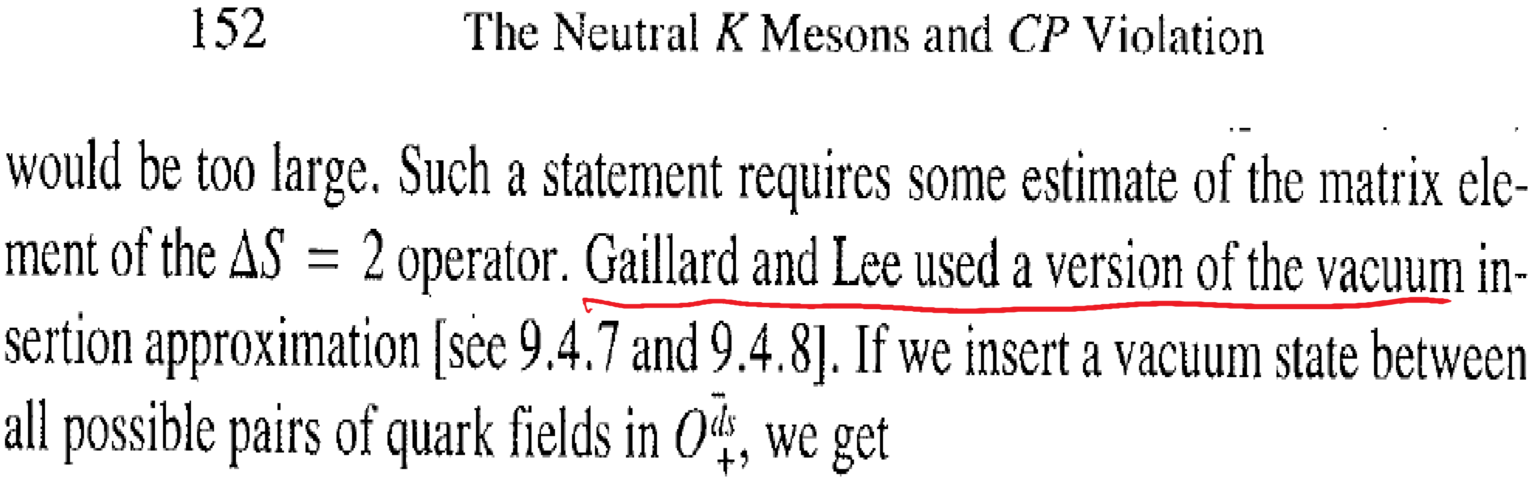


To make contact with phenomenology, one must evaluate the matrix element of $O^{\Delta S=2}$ between K^0 and $\bar{K}^0$ states. It is conventional to express the results in terms of the so-called *B-parameter*,

$$\langle K^0 | O^{\Delta S=2} | \bar{K}^0\rangle \equiv \frac{16}{3} F_K^2 m_K^2 B \ , \qquad (1.19)$$

DGH

where $B = 1$ corresponds to the simple vacuum saturation approximation



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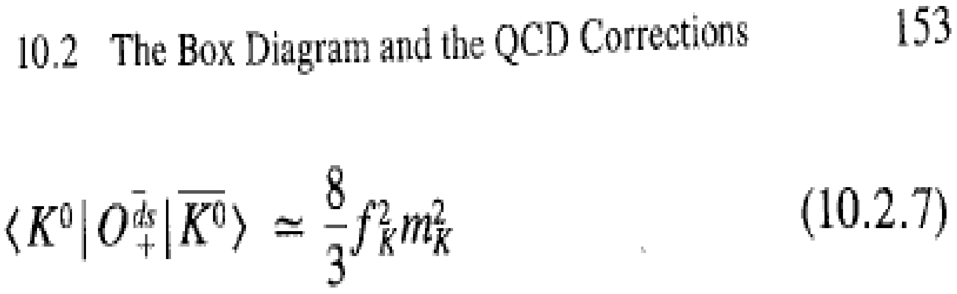
would be too large. Such a statement requires some estimate of the matrix element of the $\Delta S = 2$ operator. Gaillard and Lee used a version of the vacuum insertion approximation [see 9.4.7 and 9.4.8]. If we insert a vacuum state between all possible pairs of quark fields in $O_+^{\bar{d}s}$, we get



10.2 The Box Diagram and the QCD Corrections 153

$$\langle K^0|O_+^{\bar{d}s}|\overline{K^0}\rangle \simeq \frac{8}{3} f_K^2 m_K^2 \qquad (10.2.7)$$

Weak Interactions & Modern Particle Theory
Howard Georgi

Progress on complex amplitude A_0

- This is much much harder than A2 because of
- Disconnected diagrams
- Mixing with lower dimensional operators
- G-parity boundary conditions

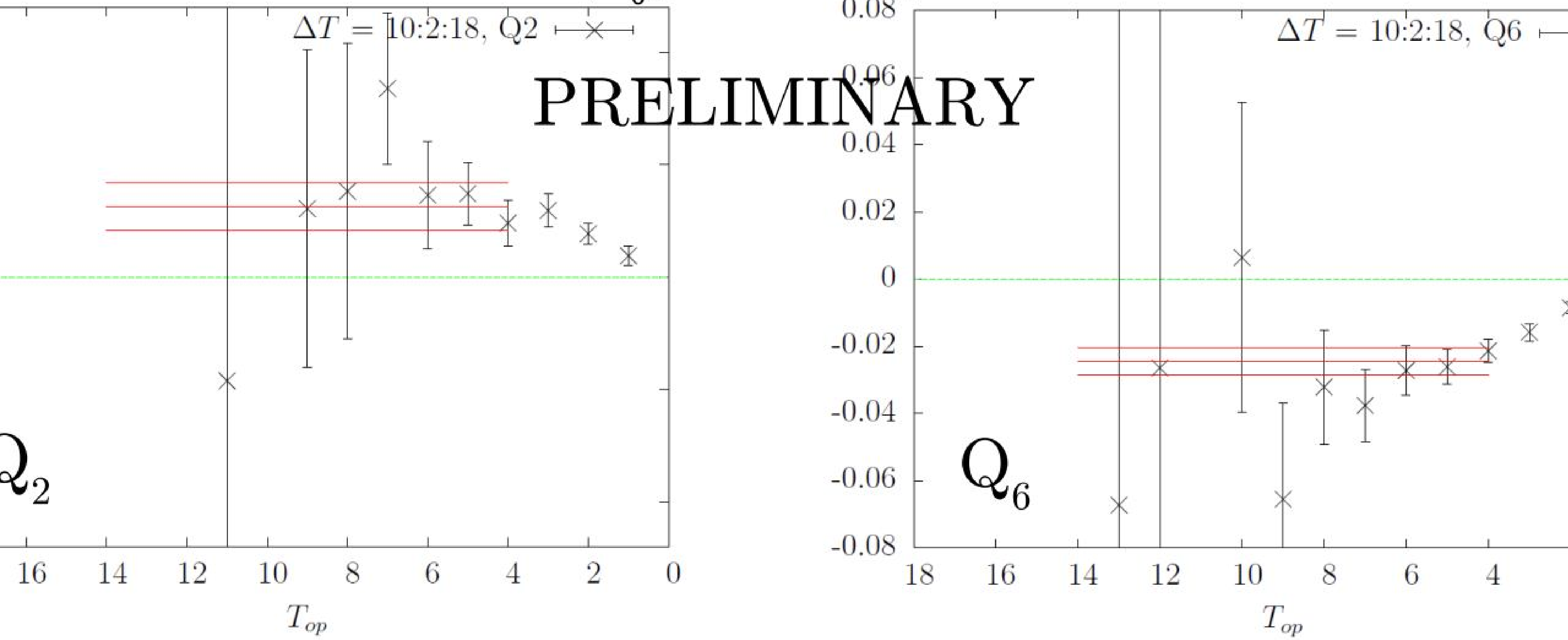
Status of A_0 calculation as of 03/25/15

- ~170 measurements (about 45 more now)
- Statistical errors on Re and ImA0 ~30% ; systematic errors around same
- ReA0 in reasonably good agreement with the experimental number
- For calculating ε'/ε plan to use lattice calculated ImA0 and experimental value of ReA0
- Because of appreciable cancellation with EWP contribution,

statistical error on ε'/ε around 40% ; systematic 30%

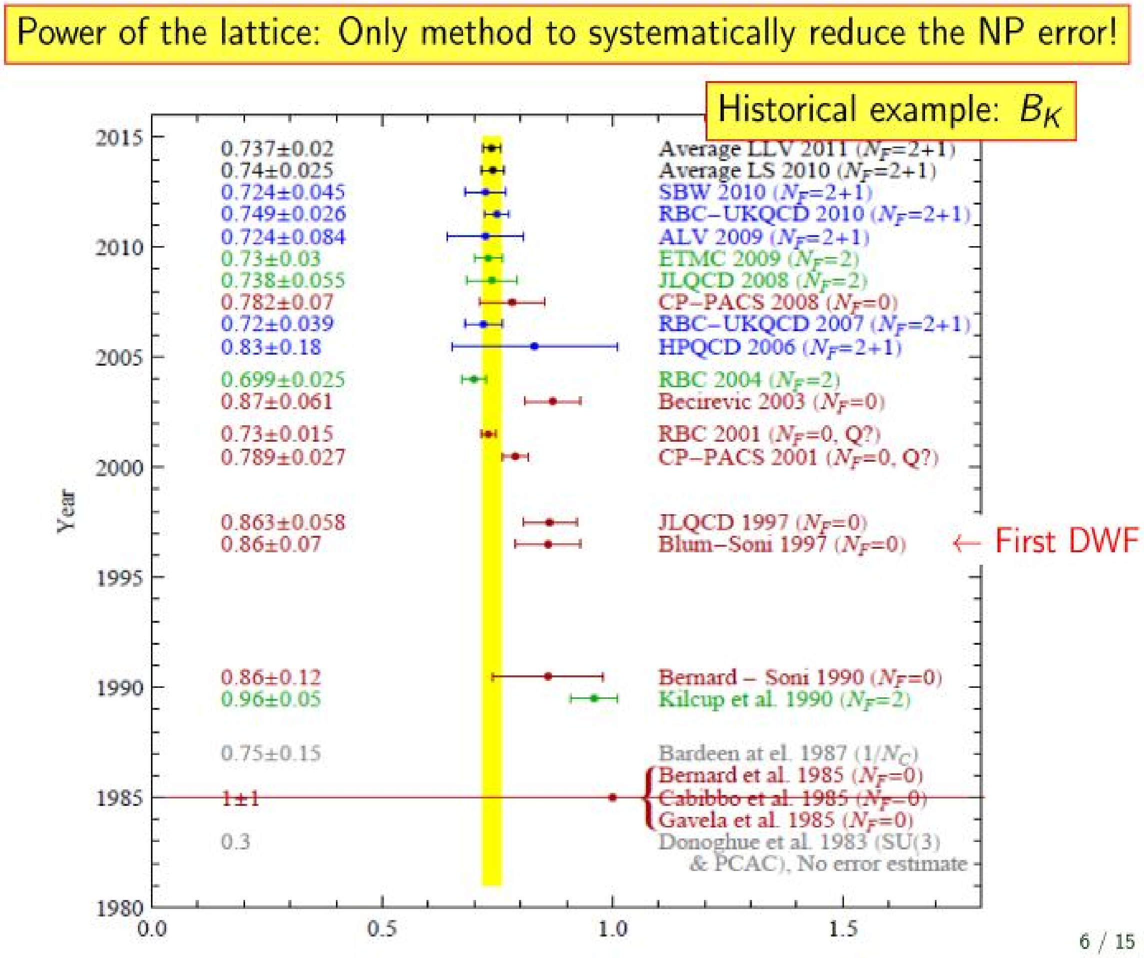
- 1st publication soon with controllable errors ninant contribution to Re(A_0)]

[Dominant contribution to In



$\mathrm{Re}(A_0) = 3.36(92) \times 10^{-7}$ Lattice (PRELIMINARY, STAT ERRS ON

$\mathrm{Re}(A_0) = 3.3201(18) \times 10^{-7}$ Experiment ; C.Kelly ~ 3/25/15



Power of the lattice: Only method to systematically reduce the NP error!

Historical example: B_K

← First DWF

6 / 15

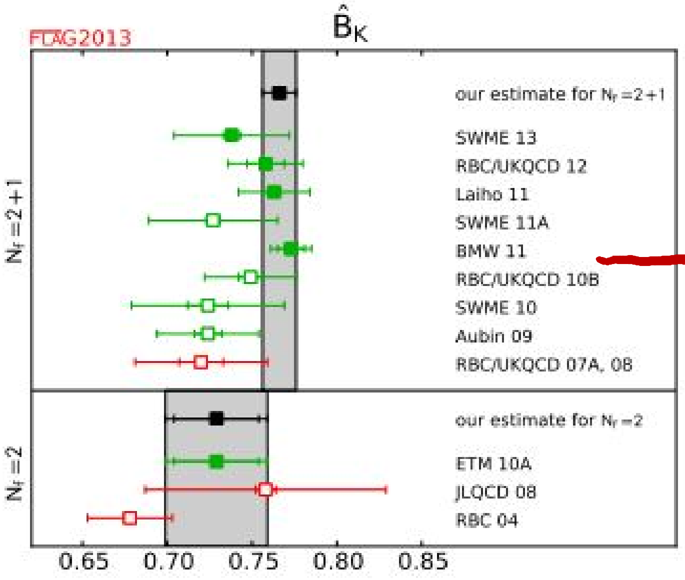
AB-initio Calculat

$$B_K \equiv \frac{\langle K | (\bar{s}\gamma_\mu d)^2 | \bar{K}}{8/3 f_K^2 m_K^2}$$

Status before lattice 2014

Garron LAT14

FLAG [Aoki et al., '13-14]



FLAG 2013

$$N_f = 2+1: \qquad \hat{B}_K = 0.7661(99)\,,$$

~1.3%

2nd useful example A2

PRD'12

- 1st complete calculation with physical kinematics ~2012 with errors around 20%:

$32^3 \times 64 \times 32$

$a^{-1} = 1.364$

146 configs

$$\mathrm{Re}A_2 = 1.381(46)_{\mathrm{stat}}(258)_{\mathrm{syst}} 10^{-8}\ \mathrm{GeV},$$

$$\mathrm{Im}A_2 = -6.54(46)_{\mathrm{stat}}(120)_{\mathrm{syst}} 10^{-13}\ \mathrm{GeV}.$$

- PRD2015, Re & Im A2 in continuum limit with ~10% errors completely dominated by continuum perturbation theory errors on Wilson coefficients to NLO:
- $48^3 \times 96 \times 24$; 1.728 GeV

$64^3 \times 128 \times 12$; 2.357 GeV

76; 40 configs

$$\mathrm{Re}(A_2) = 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{syst}} \times 10^{-8}\ \mathrm{GeV};$$

$$\mathrm{Im}(A_2) = -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{syst}} \times 10^{-13}\ \mathrm{GeV}.$$

Outlook for ε'/ε calculation

- Proposal for four-fold increase in statistics in ~year; bring errors down to ~20% statistical; ~15% systematic
- ~ 5 years total errors should be ~10%
- Improved experimental determination worth considering

Summary + Outlook

- **Significant lattice progress in calculation of K=> π π**

- **Re and Im A2 finished recently with ~10% total error in each;**

and 15% on $\mathrm{Re}\left(\epsilon'/\epsilon\right)_{EWP}$

- **60 years old puzzle Delta I=1/2 resolved: its mostly a result of an unexpected significant cancellation among N^2 and N amplitudes contributing to A2**
- **Re and Im A0 1st computation at physical kinematics with errors around 40% stat and 30% systematics will be completed very soon and plans underway for improvement to 20% and 15% in ~ yr**