Ratcheting and Transitions: Short Granular Chain in a Gradient of Vibration

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We report our experimental work on a one-dimensional gradient of vibration with a short granular chain. The system exhibits transitions of ratcheting dynamics from passive monotonic creeping against the gradient, to rapid stochastic head-swinging with a reversed bias in its direction, and to seemingly random fluctuations. The spontaneously emerged spatial pattern reflects bifurcations of the state of the chain. Evidence from counterpart experiments using uniform vibrations confirms a non-monotonic development of accessible modes behind the transitions, whereas the reversed ratcheting reflects an interesting dialogue between the size of the object and the spatial gradient.

A spatial gradient of excitation can create a variety of migrations in response to this broken symmetry. For molecules, thermophoretic activities refer to transports driven by temperature gradient [1] which in some cases result in distinct patterns [2]. For live organisms, migrations induced by uneven temperature or nutrients are coined as thermotaxis or chemotaxis [3, 4] with a long tradition of scholarly discussions. Vibrations have often been used as an excitation for macroscopic objects. But despite several in-depth studies in recent years on the directional movement induced by uniform vibrations, such as that of a rigid dimer which exhibits a spontaneous ratcheting associated with a special mode of bouncing [5] and that of a liquid drop along a tilted substrate or with coupled oscillations [6], the roles of a spatial gradient on the migration remain unexplored. In this Letter, we report our studies of a macroscopic object in response to a linearly distributed intensity of vibration. The use of such an extended gradient leads to the discovery of the transition of response modes that, in certain conditions, creates a distinct spatial divide over the smooth change of excitation intensity along the substrate. We show that the spatial pattern reflects an interesting interplay between the object size and the spatial gradient, a phenomenon that has not yet been reported in previous studies of thermal or non-thermal systems, to the best of authors knowledge.

We use a short granular chain as our minimal model to study the response of a flexible and highly dissipative object in vibration. Shown as in Fig. 1(a), the chain consists of N metal spheres with the spacing between adjacent ones being limited but otherwise free except with a maximal bending/deflection between adjacent spheres; the chain has also been studied in many previous works [7]. We build a shaker with a long substrate. Extra cares are taken on the rigidity of the substrate so that we are able to impose a sinusoidal vibration with its amplitude varying linearly with x, and the origin marks the location of a frictionless bearing where the vibration vanishes [8]. We use $\Gamma(x)$ to represent the amplitude of the vertical acceleration of the substrate as a function of x, which is defined by the peak value of the oscillation in units of

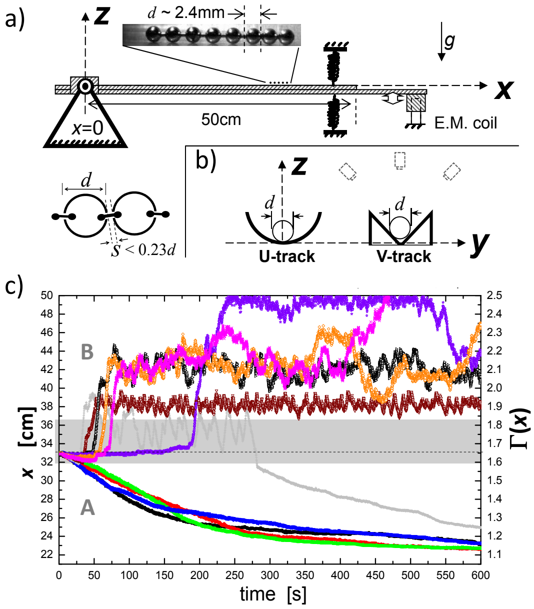

FIG. 1. (a) Side-view of the linear shaker and the granular chain– the use of an electromagnetic coil minimizes the unintended coupling along the x-direction. (b) The cross-section of the two types of track with schematics showing angles of the camera(s). (c) Sample trajectories of the center of the chain at a typical vibrational strength. With all events initiated at x=33cm, they demonstrate an intriguing split of behaviors and a transitional zone. (25Hz, V-track, N=8)

gravity g. The chain is free to move along x but partially restricted in the transverse directions by either type of the track (Fig. 1b) made of anodized aluminium . We track the migration of the center of the chain and, in selected cases, also the 3D movements of individual spheres with high-speed cameras zoomed-in from orthogonal directions. Figure 1(c) shows sample trajectories captured from multiple runs under identical conditions. With the same initial position, the chain can stochastically choose to migrate monotonically into region A, or to make rapid moves in the opposite direction across the shaded zone and reach region B where the motion appears random. These rapid moves are strongly biased towards $+x$ so

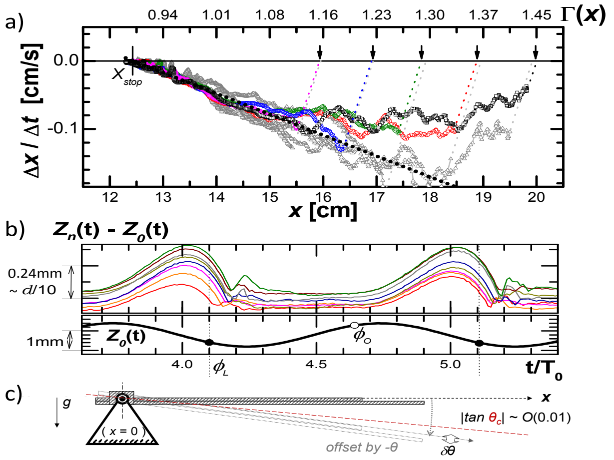

FIG. 2. (a) Migration velocity $\Delta x/\Delta t$ plotted against the instantaneous position of the center of the chain x, with five initial positions marked by vertical arrows. The velocity is computed at a timescale Δt=2s using central difference, while the thick dotted line showing a linear fit that defines a stall position at a long-time limit. Data using V-track are in solid symbols and U-track in gray, both driven at 20Hz. (b) The vertical movements of individual spheres resolved over the vibration period T_0 during such migration, with the motion of the substrate $Z_0(t)$ subtracted. ϕ_0 and ϕ_L indicate the theoretical predictions for the take-off and landing of a non-cohesive point mass starting at rest with $\Gamma = 1.357$, from elementary calculations. (c) Schematics for measuring the tilt-resistance of this monotonic creeping, with further details available on-line.

that going back from region B to A is exceptionally rare. In this work, we characterize the phenomena both below and above the split, and conclude with our understanding of the transition.

Ground state and regular ratcheting (the creeping) – In the low-excitation region, the chain exhibits a persistent one-way migration towards the decrease of Γ. Figure 2(a) shows that at a fixed driving condition, the coarse-grained velocity converges and the behavior is insensitive to a considerable range of initial excitations Γ. We have checked that this migration is neither an artifact due to the asymmetry of the chain (by switching its head and tail), nor the effect from the unintended residual vibrations along x [8]. The appearance of this monotonic creeping is robust for a wide range of chain length (up to $N = 20$) and on different surfaces. In addition, we zoom in and resolve the 3D motion of individual spheres within the vibration cycle, as shown by Fig. 2(b). Along the chain, the vertical movements (relative to the substrate) of all particles follow a similar pattern, with their amplitudes being in the same order that is small compared to both the particle diameter d and substrate displacement. The trajectories reveal that the take-off and landings of these particles exhibit just small lags from the theoretical predictions for a point mass, and that dissipation repeatedly resets the chain to a quiescent state before the next vibration cycle.

Remarkably, we also find that this one-way creeping can survive a considerable tilting against it [10]: we measure the critical slope $|\tan\theta_c|$ for the angular offsets, shown schematically by Fig. 2(c), and find an interesting correlation to the intensity of gradient as discussed in our on-line supplements. For all cases, the measured values significantly exceed the angular amplitude $\delta\theta$ of the substrate. This naturally relieves the suspicion that the periodic alternation of the substrate orientation might be the main mechanism causing the steady ratcheting.

Mode transitions, with a reversed ratcheting – One can also manually relocate the chain to positions with higher values of Γ, and find that bifurcations occur as the ground-state creeping becomes unstable. Figure 3(a) shows that, with an initial value of Γ ∼1.65, the chain can stochastically depart from the creeping state. This bifurcation creates a transition zone in which the chain can migrate with a bias towards $+x$, at a much higher magnitude of speed than the ground state shown on the left. We also refer to this trend as a *reversed ratcheting* – until the chain reaches the far right of the diagram where this directionality is lost.

Figure 3(b) shows the vertical movements of individual spheres, and illustrates two typical modes in the transition zone. Both modes exhibit a highly uneven distribution of displacement amplitudes along the chain. In contrast to the creeping state in which the chain is repeatedly reset to quiescence within each vibration cycle (Fig. 2b), the trajectories demonstrate a surplus of kinetic energy[11]. In particular, Mode 1 is identified as the main mechanism responsible for the rapid migration across the transition zone. The cartoons on the right-hand side of Fig. 3(b) summarize our observations from high-speed imaging, illustrating a coherent sequence of the "sticking" of the main body at phase ϕ_1, the "forward spreading" of one swinging head until its end particle hits the substrate at ϕ_2, and the "free flight" starting at ϕ_O (and spanning over ϕ_3 and ϕ_4). The trajectory graph further reveals that ϕ_O almost coincides with ϕ_2, and that the process of the backward bending extends for about half of the vibration cycle and overlaps almost entirely with the free flight [9]. Friction acts asymmetrically on the chain, as it provides a stationary support for the forward spreading of the chain but is inactive during the backward bending during the flight. The repetition of this stick-spread-fly sequence establishes a coherent movement, resulting a ratcheting that is order-of-magnitude faster than the ground-state creeping, in the direction of the swinging head.

On the other hand, Mode 2 exhibits a tug-of-war between two ends of the chain, creating a "waiting time" with only slow drifts. Fig. 3(b) illustrates a typical event that Mode 2 precedes for a few seconds before the chain switches to Mode 1. But generally speaking, in this transition zone, either mode can emerge directly from the

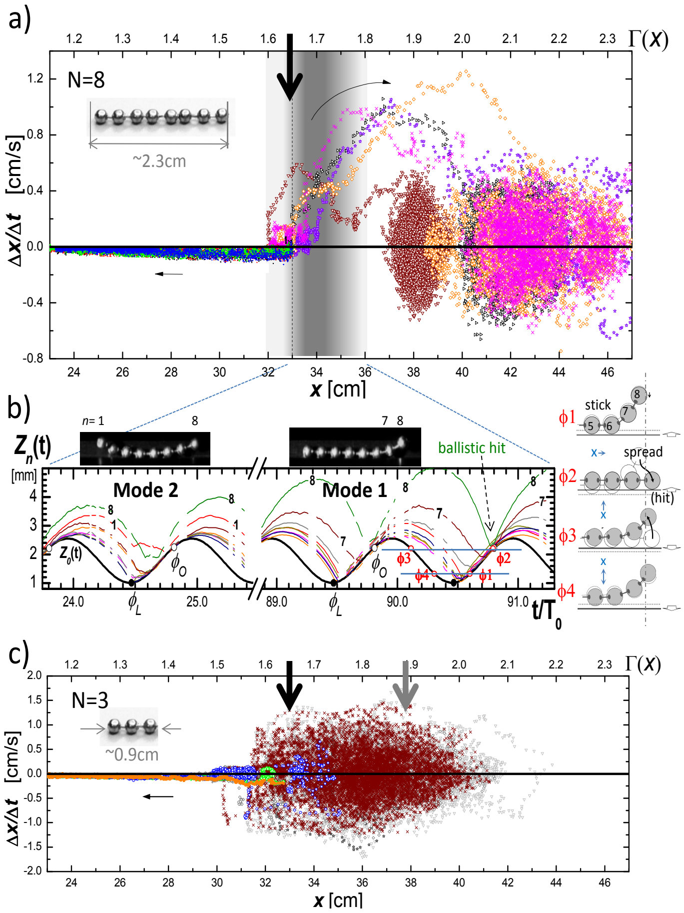

FIG. 3. (a) Overlay of 12 typical events, independently initiated with the center-of-mass at x=33cm, under the same conditions (25Hz, V-track, N=8). The transition zone, which shows a strong bias in velocity that creates a spatial divide, is marked in shade. (b) Vertical movements of individual spheres, $Z_n(t)$, for a chain crossing the transition zone. $Z_0(t)$ represents the movement of the substrate. $T_0 = 1/(25\text{Hz})$. ϕ_0 and ϕ_L mark the theoretical predictions of the take-off and landing for a non-cohesive point mass starting at rest with $\Gamma = 1.705$. The recording shows a tug-of-war (Mode 2) followed by the asymmetric head-swinging (Mode 1). The cartoons illustrate four representative phases of Mode 1, with the cross-and-arrow indicating the movement of the center-of-mass between adjacent phases. (c) Overlay of 8 events for N=3, with half of them initiated at x=33cm (in solid colors) and others at x=38cm (in gray), showing no obvious divide on the phase space.

ground state in a stochastic fashion.

One may speculate that, in Mode 1, the swinging head of the chain could have pointed in either direction equally. However, our experiments show that, in the shaded zone marked on Fig. 3(a), this mode and the movements are strongly biased toward the direction of $+x$, and appears to fence off most stochastic attempts of invasion from the right. Nevertheless, we also find that such bias can be weakened as the chain length decreases. Figure 3(c) shows that, for N=3, the chain in the same range of Γ would explore its neighborhood in both directions almost equally, with no significant bias. The reason behind such intriguing dependence on N will come clear with further information to follow.

Roles of Γ *and the chain length* – We have also set up counterpart experiments with uniform vibrations, shown as in Fig. 4, to study the intrinsic dynamics of the chain in the *absence* of the spatial gradient. To facilitate statistical studies, these experiments are performed on a large circular track, using the same V-shape cross-section and aluminium as in the main experiments. The long-time observations on the tangential displacements Δx first provides a stringent criterion for the symmetry of the chain to be qualified for our experiments. Moreover, the long-time recording also allows us to verify that, at low excitations ($1 < \Gamma < 1.5$), the chain takes off and lands periodically but is unable to produce a net migration, offering a direct confirmation that the ground-state ratcheting is indeed driven by the gradient, as one might also extrapolate from the empirical relationship between the tilt-resistance and the intensity of gradient [9, 10].

The statistics of $< |\Delta x|^2 >$ in Fig. 4 exhibit an interesting non-monotonic dependence on Γ, while sample trajectories of the center-of-mass are shown as insets. As Γ goes from 1.65 to 1.7 [12], $x(t)$ reveals distinct "steps" (up to nearly 3cm in size, in stochastic directions) that are associated with increasingly frequent Mode 1 events, as confirmed by high-speed imaging. Our high-speed imaging also confirms that, upon further increase of Γ, both excited modes would disappear, as one might also anticipate [9]. Here we use Δt=1s in computing Δx, which would be the displacement accumulated over 25 vibration cycles. Therefore the decline of $< |\Delta x|^2 >$ captures the breakdown of Mode 1, because the accumulated displacement decreases considerably as the movements between successive bouncing become uncorrelated. For a range of chain length N except 2 [9], we have found the similar correlation of this non-monotonicity of $< |\Delta x|^2 >$ to Mode 1 activities.

Focusing on the case for N=8, we note that the onset of Mode 1 takes place in a range of Γ that corresponds to a spatial region about 2cm wide in our main experiments with gradient – a width that is comparable to the size of the chain. This offers a plausible explanation for the directional bias: in the transition zone shown by Fig. 3(a), the $+x$ side of the chain is significantly more excited than the other, therefore is likely to become the "leading" side. However, as the ratcheting proceeds, this directionality would persist only for a limited distance because, once the chain reaches the region with much higher Γ, the coherent mode itself is destroyed by excessive excitations and the migration becomes random.

Similarly, the reason behind why the directional bias declines at smaller N (Fig. 3c) is now clear: Figure 4 reveals that reducing N from 8 to 3 almost doubles the width of transition and consequently the spatial width

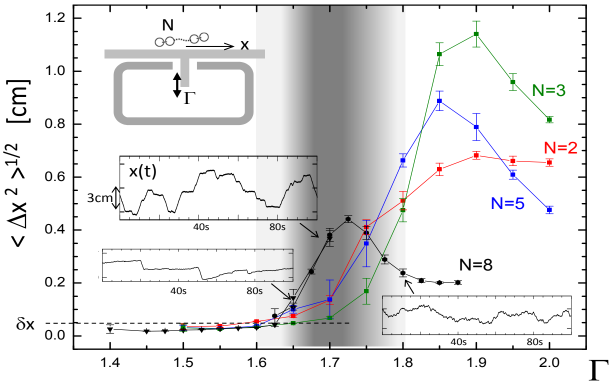

FIG. 4. Root-mean-squared displacement $< |\Delta x|^2 >^{1/2}$ at the timescale Δt=1s as Γ varies, for different chain length N. Statistics are collected over an one-hour period with uniform vibrations at 25Hz. The insets display trajectories at three different values of Γ for N=8. The shade marks the range of Γ corresponding to that in Fig. 3(a).

for the corresponding transition in the main experiments. Since the chain length has been decreased by more than a factor of two, this distance is then too short for acquiring a significant change of state in the same setting. Therefore the Mode 1 activities in this case would not exhibit a sufficiently strong bias in directions. This completes our explanation on why for N=8 we see the reversed ratcheting and subsequently a spatial divide, whereas for N=3 the chain appears to move diffusively upon transition and shows no "gap" on the phase space.

In conclusion, we have characterized the ratcheting of a macroscopic object along a gradient of vibrations, using a short chain as an example. The spatial patterns reflect bifurcations of dynamics. The reversal of ratcheting with a dramatic spatial divide underscores the role of object size in relation to the spatial gradient, and a particular mode of coherent movements is identified for resulting the rapid ratcheting. Our observations provide several targets for theoretical work, such as proper order parameters for describing the bifurcations, the prediction of transition thresholds, and models to determine the directional bias or an expected flux along the gradient. Important mysteries remain with the robust ground-state creeping: the small and nearly in-sync movements of connected masses is rectified into a persistent ratcheting against the gradient, and such migration survives a considerable degree of tilting. Decoding how a flexible and heavily-damped object interacts collectively [13] with the spatial gradient and produces the rectification can be an interesting challenge.

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[8] We attach accelerometers to multiple locations along the substrate to verify both the vibration being sinusoidal and its linearity with a deviation less than 3 percent.

[9] High-speed videos and further details are available at http://www.phys.sinica.edu.tw/jctsai/gradient/, with partial contents forthcoming at EPAPS-no-xxxxxxx. We have also included a prediction for the breakdown of the periodic head-swinging at much higher excitations, based on that ϕ_2 almost coincides with ϕ_O at $\Gamma \sim 1.7$.

[10] This is done by adjusting the height difference between the spring suspension and the roller bearing. Changing control parameters to produce creeping at different spatial zones allows our deduction of how the tilt-resistance varies with the intensity of gradient $\nabla\Gamma/\Gamma = 1/x$ around the chain. Data and further details are presented through our on-line supplements and a subsequent publication.

[11] The accumulation of kinetic energy at the end(s) of the chain is essential for sustaining these excited modes– we find that replacing the aluminium with a more dissipative

material such as acrylic can prevent these modes.
[12] This onset of Mode 1 can be adjusted by our deliberately suppressing the bending– see our on-line supplements.
[13] The ground-state creeping along the gradient might appear slow compared to Mode 1, but its efficiency is remarkable: numerical analyses in our on-line supplements show that simple models using randomized momentum impacts would have severely under-estimated the persistence and speed of the creeping seen in our experiments.