Final Examination for “Solid State Physics”

09:10 pm - 12:00 pm, December 26, 2023

ID Number:__________________ Name:______________________________

NOTE: This is an open-book exam but net surfing is not allowed.

Q1. This is a problem related to semiconductor physics. (15%)

Small-gap semiconductors, such as InAs (E_g = 0.35 eV) and InSb (E_g = 0.18 eV) have dielectric constants ε = 15 for InAs and ε = 16 for InSb and effective masses m_e = 0.02 m for InAs and m_e = 0.014 m for InSb. Calculate

a) the donor ionization energy; (5%)

b) the radius of the ground state orbit. (5%)

c) At what minimum donor concentration will appreciable overlap effects between the orbits of adjacent impurity atoms occur? (5%)

Solution:

a) The donor ionization energy (Ed) for a small-gap semiconductor can be calculated using the equation: $E_d = \frac{e^4 m_e}{2\varepsilon^2 \hbar^2} = \left(\frac{13.6}{\varepsilon^2}\frac{m_e}{m}\right) eV$

For InAs: $E_d = \frac{13.6}{15^2} 0.02 = 0.00121\, eV$ = 1.21 *meV*

For InSb: $E_d = \frac{13.6}{16^2} 0.014 = 0.000744$ = 0.744 *meV*

b) The radius of the ground state orbit (a_d) can be calculated using the formula:

$$a_d = \frac{\varepsilon \hbar^2}{m_e e^2} = \left(\frac{0.53\varepsilon}{m_e / m}\right) \text{Å}$$

For InAs: $a_d = \frac{0.53*15}{0.02} = 397.5\,\text{Å}$

For InSb: $a_d = \frac{0.53*16}{0.014} = 605.7\,\text{Å}$

c) In the scenario of a uniform distribution of donor atoms, appreciable overlap occurs when more than one atom is present within a sphere of radius a_d. Hence, there is overlap when $n*\left(\frac{4}{3}\pi a_d^3\right)\geq 1$, so the minimum donor concentration $n\approx\frac{1}{\frac{4}{3}\pi a_d^3}$.

For InAs: $n\approx\frac{1}{\frac{4}{3}\pi(397.5)^3}=3.801*10^{-9}$ Å^{-3} = $3.801*10^{15}$ cm^{-3} = $3.801*10^{21}$ m^{-3}

For InSb: $n\approx\frac{1}{\frac{4}{3}\pi(605.7)^3}=1.074*10^{-9}$ Å^{-3} = $1.074*10^{15}$ cm^{-3} = $1.074*10^{21}$ m^{-3}

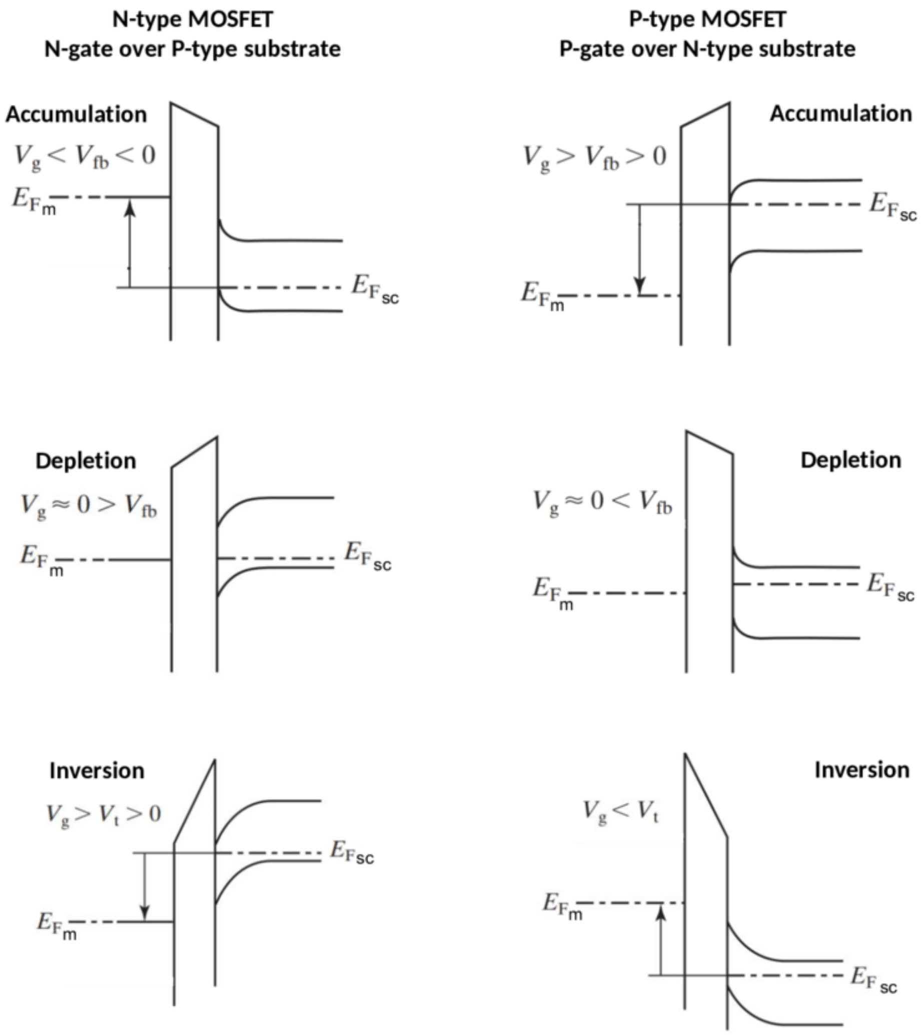
Q2. This is a problem related to the semiconductor device. (20%)

(a) Sketch the energy band diagram and the charge distribution in an MOS structure under basing conditions corresponding to carrier accumulation, depletion, and strong inversion. (10%)

(b) Sketch a typical CMOS inverter and describe the operation principle with the voltages applied to the input and the subsequent output voltages. (10%)

Solution:

(a) Energy Band Diagram and Charge Distribution in MOS Structure:



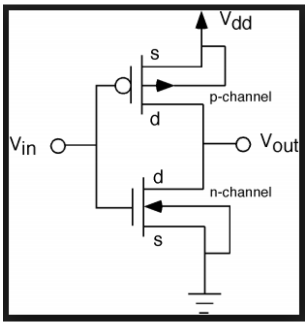
1. **Carrier Accumulation:** In accumulation, there is a buildup of majority carriers (electrons for n-type, holes for p-type) at the interface due to an applied voltage. The energy band diagram shows a flat band condition near the interface. The charge distribution reveals an excess of electrons (for n-type) or holes (for p-type) at the surface.

2. **Depletion:** Under depletion, there is no applied voltage, and the majority carriers are repelled away from the interface, leaving behind ionized acceptor or donor atoms creating a depletion region. The energy band diagram shows a bending of bands due to the electric field in the depletion region. The charge distribution indicates a decrease in carriers near the surface.
3. **Strong Inversion:** In strong inversion, a sufficiently high positive voltage is applied to the gate, attracting electrons (for n-type) or holes (for p-type) into the semiconductor surface, forming a conducting channel. The energy band diagram shows a **<u>significant bending of bands</u>**. The charge distribution shows a highly populated channel with majority carriers.

(b) CMOS Inverter and Operation Principle:

A CMOS inverter consists of a p-type MOSFET (PMOS) and an n-type MOSFET (NMOS) connected in series between the power supply (VDD) and ground (GND). The input is applied to both gates, and the output is taken from the common drain connection.

Operation Principle:



In	Out
1	0
0	1

1. **Input High (Logic 1):** When a logic high input voltage is given to the CMOS inverter, the PMOS (p-type) is switched OFF, providing a high-resistance path to VDD whereas the NMOS (n-type) is switched ON, creating a low-resistance path to ground. Thus, the output is pulled down to ground, resulting in a low logic output voltage.
2. **Input Low (Logic 0):** When the low input voltage is given to the CMOS inverter, the PMOS is switched ON, creating a low-resistance path to VDD. The NMOS transistor is switched OFF state, providing a high-resistance path to ground. Consequently, the output is pulled up to VDD, resulting in a high logic high output voltage.

Q3. This is a problem related to the superconductivity. (15%)

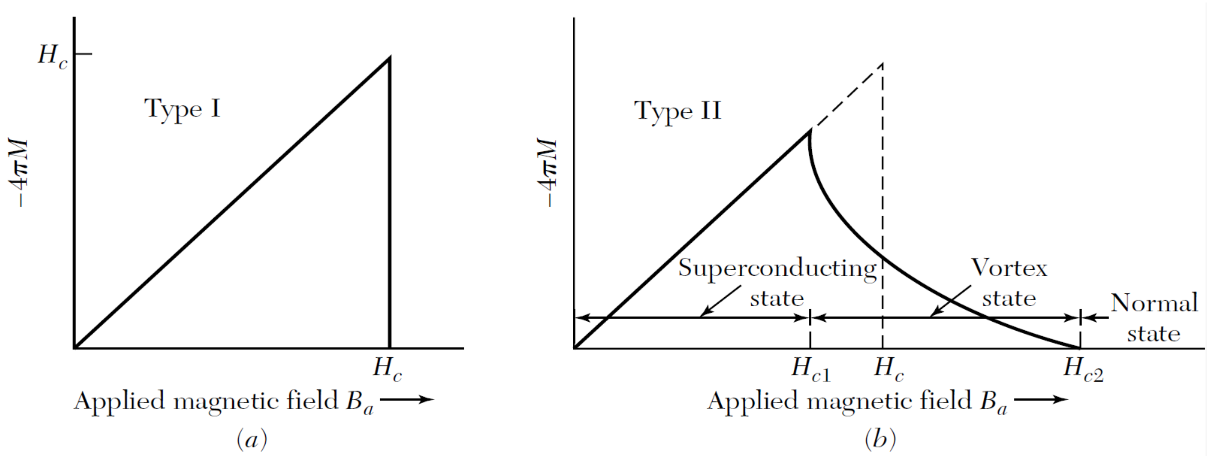
There are two types of superconductors, type I and type II.

(a) Describe their difference in response to an applied magnetic field. (7%)

(b) Express and discuss their difference in terms of penetration depth λ, coherence length ξ, and mean free path ℓ. (8%)

Solution:

a) Difference in response to an applied magnetic field:



Type I Superconductors: Type I superconductors exhibit a complete expulsion of magnetic field (Meissner effect) below a critical temperature. Below the critical temperature, these superconductors abruptly transition from a normal to a superconducting state, causing the magnetic field to be pushed out of the material. Type I superconductors have a single critical magnetic field strength (H_c), beyond which they return to the normal state.

Type II Superconductors: Type II superconductors can tolerate a certain range of magnetic fields, and they undergo a mixed state or the vortex state. In this state, some magnetic flux penetrates the material in the form of vortices or flux lines, but superconductivity is not completely destroyed. Type II superconductors have two critical magnetic field strengths: an upper critical field (H_{c2}), beyond which superconductivity is destroyed, and a lower critical field (H_{c1}), below which magnetic flux penetration is minimal.

(b) Difference in Terms of Penetration Depth (λ), Coherence Length (ξ), and Mean Free Path (ℓ):

1. **Penetration Depth (λ):**

   Type I Superconductors: In type I superconductors, the penetration depth is relatively large. It represents the characteristic distance over which the magnetic field is exponentially attenuated within the superconductor.

Type II Superconductors: In type II superconductors, the penetration depth is smaller compared to type I. It describes the distance to which magnetic flux penetrates the superconductor in the mixed state.

2. **Coherence Length (ξ):**

   Type I Superconductors: Type I superconductors have a well-defined coherence length, which is the characteristic length over which the superconducting wavefunction is correlated.

   Type II Superconductors: Type II superconductors have two coherence lengths: the Ginzburg-Landau coherence length (ξGL) and the London penetration depth (λ). ξGL is related to the size of the superconducting vortex cores, and it is generally larger than the coherence length in type I superconductors.

3. **Mean Free Path (ℓ):**

   Type I Superconductors: The mean free path in type I superconductors is relatively long. It represents the average distance a conduction electron can travel without scattering.

   Type II Superconductors: The mean free path in type II superconductors is shorter compared to type I. The presence of vortices and flux lines contributes to increased scattering of electrons.

In summary, type I superconductors expel the magnetic field completely, have a larger penetration depth, longer mean free path, and a single coherence length. Type II superconductors tolerate magnetic fields to some extent, have smaller penetration depth, shorter mean free path, and two coherence lengths associated with the mixed state. The characteristics of type II superconductors make them suitable for applications in the presence of magnetic fields.

Q4. This is a problem related to the magnons and magnetic resonance. (15%)

(a) A Bloch wall in a crystal is the transition layer that separates adjacent domains magnetized in different directions. Explain the existence of Bloch walls in a ferromagnetic material. (8%)

(b) Taking the effective fields on the two-sublattice model of an antiferromagnetic as

$$B_A = B_a - \mu M_B - \epsilon M_A \ ; \quad B_B = B_a - \mu M_A - \epsilon M_B \ ,$$

show that Neel temperature can be expressed as $\frac{\theta}{T_N} = \frac{\mu + \epsilon}{\mu - \epsilon}$. (7%)

Solution:

(a) The existence of Bloch walls can be explained through the balance between competing energy considerations. Within a ferromagnetic material, there is a tendency for magnetic moments to align parallel to each other due to exchange interactions. However, this alignment can be disrupted by other factors, such as the shape anisotropy and external magnetic fields, leading to the formation of domains with different magnetization directions.

At the Bloch wall:

- **Exchange Energy:** This energy favors the alignment of neighboring magnetic moments.
- **Anisotropy Energy:** This energy prefers a specific direction for magnetic moments, which may not align with neighboring domains.
- **Zeeman Energy:** The interaction with an external magnetic field can influence the orientation of magnetic moments.

To minimize the total energy, a compromise is reached at the Bloch wall. The magnetization rotates smoothly from one domain to another, creating a transitional region. This rotation is described by the Bloch function, and the structure of the Bloch wall is such that the net magnetization is zero at the center of the wall.

(b) This question corresponds to problem 12.3 in Chapter 12 of the Kittel book.

B_a = applied field,
$$M_A T = C\left(B_a - \mu M_B - \varepsilon M_A\right)$$
$$M_B T = C\left(B_a - \varepsilon M_B - \mu M_A\right)$$

Non-trivial solution for B_a = 0 if

$$\begin{vmatrix} T+\varepsilon C & \mu C \\ \mu C & T+\varepsilon C \end{vmatrix} = 0 \Rightarrow T_N = C(\mu-\varepsilon) \qquad (1)$$

Rearrange the given equations

$$\begin{aligned} M_A T &= C\left(B_a - \mu M_B - \varepsilon M_A\right) \\ M_B T &= C\left(B_a - \varepsilon M_B - \mu M_A\right) \end{aligned} \Rightarrow \begin{aligned} M_A\left(T + C\varepsilon\right) + \mu M_B C &= B_a C \\ M_B\left(T + C\varepsilon\right) + \mu M_A C &= B_a C \end{aligned}$$

Now, sum both terms of the equations above, leading to

$$\begin{aligned} &\left(M_A + M_B\right)\left(T + C\varepsilon\right) + \left(M_A + M_B\right)\mu C = 2B_a C \\ &\Rightarrow \left(M_A + M_B\right)\left(T + C\varepsilon + \mu C\right) = 2B_a C \\ &\Rightarrow \frac{\left(M_A + M_B\right)}{B_a} = \frac{2C}{T + C\left(\varepsilon + \mu\right)} \end{aligned}$$

At $T > T_N$, the susceptibility $\chi = \frac{M_A + M_B}{B_a} = \frac{2C}{T + C\left(\varepsilon + \mu\right)}$

Also, in an antiferromagnet the susceptibility above the Néel temperature has the form

$$\chi = \frac{2C}{T + \theta}$$

So $\theta = C\left(\mu + \varepsilon\right)$ (2)

From (1) and (2), we obtain $\frac{\theta}{T_N} = \frac{C\left(\mu+\varepsilon\right)}{C\left(\mu-\varepsilon\right)} = \frac{\mu+\varepsilon}{\mu-\varepsilon}$

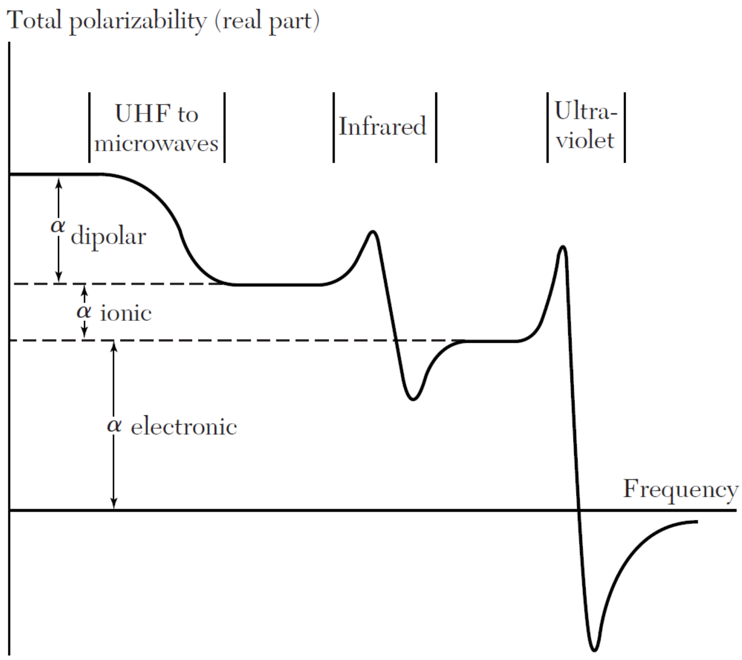
Q5. This is a problem related to the dielectrics and ferroelectrics. (15%)

(a) Sketch the dipolar, ionic, and electronic contributions to the polarization of a material in different frequency ranges upon application of an external electric field. (7%)

(b) Assume an electron is tied to a positive nucleus with a spring of force constant k = 0.911 *N/m*. Derive the frequency dependence of the electron's polarizability under an *ac* EM field of $E\sin(\omega t)$. The electron mass m_e = 9.11 x 10^{-31} *kg*. At what frequency the system will show the maximum polarizability? (8%)

Solution:

(a) Kittel, Introduction to Solid State Physics, Fig. 8 in Chapter 16. Schematic of the frequency dependence of the several contributions to the polarizability.



In response to an external electric field, the polarization of materials unfolds across different frequency domains. At low frequencies, dipolar polarization takes precedence, characterized by the alignment and relaxation of permanent dipoles. Transitioning to intermediate frequencies, ionic polarization becomes prominent, depicting the movement of charged ions in response to the alternating electric field. At higher frequencies, electronic polarization prevails, showcasing the displacement of electrons within atoms or molecules due to the rapidly changing external field.

(b) This question is about the Lorentz oscillator model, also known as the Drude-Lorentz oscillator model, which involves modeling an electron as a driven damped harmonic oscillator. The goal of this model is to (a) use Newton's Second Law to obtain the motion of the electron, which (b) can then be used to obtain expressions for the dipole moment, polarization, susceptibility, and dielectric constant.

The equation of motion in the local electric field $E\sin(\varpi t)$ is

$$m\frac{d^2x}{dt^2}+m\varpi_0^2x=-eE\sin(\varpi t),$$

so that, for $x=x_0\sin(\varpi t)$, $m\left(-\varpi^2+\varpi_0^2\right)x_0=-eE$.

The dipole moment has the amplitude $p_0=-ex_0=\dfrac{e^2E}{m\left(\varpi_0^2-\varpi^2\right)}$,

from which $\alpha(electronic)=p/E=\dfrac{e^2/m}{\varpi_0^2-\varpi^2}$.

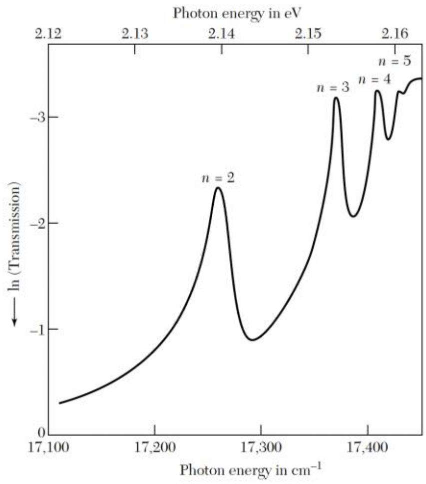
The resonance absorption will be obtained at a frequency

$$\varpi_0=\left(k/m_e\right)^{1/2}=\sqrt{\frac{0.911}{9.11*10^{-31}}}=1*10^{15}\,Hz$$

Q6. This is a problem related to the optical properties of solids. (20%)

(a) The dielectric function $\epsilon(\omega)$ can be used to describe the responses of a material to the EM waves. Please write down the major processes with the value of $\epsilon(\omega)$ and the quasiparticles generated for conductors, semiconductors, and insulators. (10%)

(b) The following diagram is an optical transmission spectrum taken on Cu_2O at 77K. The energy gap for Cu_2O is 2.17 eV. Discuss the origin of the peaks in the diagram and try to derive a formula to interpret the energy spacings between these peaks. (10%)



Solution:

(a) For conductors, the dielectric function takes the form $\varepsilon(\varpi)=1-\frac{\varpi_p^2}{\varpi^2}$, describing the process of free electron oscillation or plasmon generation, with ϖ_p being the plasma frequency.

In semiconductors, the dielectric function is given by $\varepsilon(\varpi)=\varepsilon_\infty-\frac{\varpi_p^2}{\varpi^2-i\gamma\varpi}$, representing the excitation of charge carriers across the band gap. Here, ε_∞ is the high-frequency dielectric constant and γ is the damping constant. The bound state of the electron and hole is referred to as an exciton. Excitons are quasiparticles, and their formation is a fundamental process in the optical properties of semiconductors.

For insulators, the dielectric function is expressed as $\varepsilon(\varpi)=\varepsilon_\infty-\frac{\varpi_p^2}{\varpi^2}$, depicting the excitation of bound electrons across the band gap.

(b) The energy spacings between these peaks can be interpreted using the energy-band model for semiconductors. In a semiconductor, the energy levels are quantized, and transitions between these levels result in absorption or emission peaks in the optical spectrum. The energy spacing between consecutive peaks can be related to the photon energy using the formula: $(\Delta E) = \hbar \varpi$, where $\hbar$ is the reduced Planck constant. Additionally, for a semiconductor with an energy gap *Eg*, the relationship between the photon energy and the energy gap is given by: $\hbar \varpi = E_g$. Therefore, the energy spacing between peaks in the optical transmission spectrum is directly related to the energy gap of the semiconductor material.

Consider an electron in the conduction band and a hole in the valence band. The pair is weakly bound and attracts each other by the coulomb potential $U(r) = -e^2 / \varepsilon r$, where *r* is the distance between the particles and *e* is the appropriate dielectric constant. This is similar to the hydrogen atom problem and the energy levels referred to the top of the valence band are given by $E_n = E_g - \frac{\mu e^4}{2\hbar^2 \varepsilon^2 n^2}$.

Here *n* is the principal quantum number and *μ* is the reduced mass: $\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_h}$.