What is the definition of nanomaterials??

(i) **Original:** “quantum size effect” where the electronic properties of solids are altered with great reductions in particle size

(ii) **New (by European Union):** On 18 October 2011, the European Commission adopted the following definition of a nanomaterial:[2]

A natural, incidental or manufactured material containing particles, in an unbound state or as an aggregate or as an agglomerate and where, for 50% or more of the particles in the number size distribution, one or more external dimensions is in the size range $1 \text{ nm} - 100 \text{ nm}$.

**Quantum confinement effect**

\[
E(R) = E_g + \frac{\hbar^2 \pi^2}{2R^2} \left[ \frac{1}{m_x} + \frac{1}{m_y} \right] - \frac{1.8 e^2}{\varepsilon R} \]

$m_e$ and $m_h$: effective masses
$\varepsilon$: bulk optical dielectric coefficient

Need to consider the effect of Quantum Mechanics !!
**Elementary Quantum Mechanics**

1. Matter Wave (by de Broglie)
2. Schrödinger equation
3. Particle in a box
4. Hydrogen atoms

**Matter Wave**

**Duality of Light: Wave or Particle**

**Wave: Diffraction, EM wave**

(c) X-ray diffraction involves constructive interference of waves being "reflected" by various atomic planes in the crystal.

**Particle: Photoelectric effect, Blackbody radiation, Compton effect**

The PE of an electron inside the metal is lower than outside by an energy called the work function of the metal. Work must be done to remove the electron from the metal.
What did Plank do ??? (Quantization of energy OR Quanta)

Classical: \( \bar{E} = kT \) (continuous)

Planck: \( E = nhf \) (discrete), \( n = 0, 1, 2, 3, \ldots \)

The result is consistent with experimental observation !!!

Planck’s law: \( E = nhf \) (discrete), \( n = 0, 1, 2, 3, \ldots \)

Duality of Matters:

1. Matter Wave (by de Broglie)
   For a photon, \( E = hf = pc \), \( c = f \lambda \)
\[ p = \frac{h}{\lambda} \] (wave → particle)

\[ \lambda = \frac{h}{p} \] (particle → wave)

The particle nature of matter (old quantum mechanics)

Thomson’s model

\[
\frac{e}{m} = \frac{V\theta}{B^2 \ell d}
\]
Rutherford’s model of the atom

In the $\alpha$ particle scattering experiment, a large angle of scattering is observed, which
cannot be observed by Thomson’s model.

All the mass and position charge $Ze$ were concentrated in a minute nucleus of the
atom of diameter $10^{-14}$ m and $Z$ electrons must circle the nucleus in some way.

Problem of stability ( planet model )

Accelerating electrons $\rightarrow$ electromagnetic radiation $\rightarrow$ lose energy $\rightarrow$ atoms will
collapse to nuclear dimensions ???

Why are atoms stable ??? $\rightarrow$ Bohr’s model
Atomic spectra:

\[ \frac{1}{\lambda} = R \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right), \quad R = 1.0973732 \times 10^7 \text{m}^{-1} \]
Bohr’s model

Figure 3.21 Diagram representing Bohr’s model of the hydrogen atom.

\[ L = mvr = n\hbar = \frac{nh}{2\pi}, \Rightarrow 2\pi r = \frac{nh}{mv} = \frac{nh}{p} = n\lambda \]

Bohr’s postulate

(1) \( e^-, \) nucleus \( \rightarrow \) Coulomb force.

(2) \( L = nh, \ h = \frac{\hbar}{2\pi} \) (angular momentum quantized)

(3) an electron moving in such an orbit doesn’t radiate EM \( \rightarrow \) total energy is conserved.

(4) Atoms can exist only in certain state, and the frequency \( f \) of an emitted photon is equal to \( (h\nu = E' - E) \).

From the postulate:

\[ F = -\frac{Ze^2}{4\pi\varepsilon_0 r^2} = -m\frac{v^2}{r} \]

\[ L = mvr = nh \]

\( \Rightarrow v_n = \frac{Ze^2}{4\pi\varepsilon_0 nh} \), (speed quantized)

\( \alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137} \) (dimensionless) \( \Rightarrow v_n = \frac{Z\alpha c}{n} \)
Atomic radius \( r_n = \frac{\hbar}{mv_n} = \frac{\hbar}{m(\frac{Z\alpha c}{n})} = \frac{n^2}{Z} \left( \frac{\hbar}{mc\alpha} \right) \) (quantized)

If \( Z = 1, n = 1 \)

\[ r = \frac{\hbar}{mc\alpha} = 0.53 \, \AA = a_0 \] (Bohr’s radius)

\[ r_n = \frac{n^2}{Z} a_0 \]

Total energy of an atomic electron moving in one of the allowed orbits.

\[ E = K + V, \quad K = \frac{1}{2} m v_n^2 \]

\[ V = -\frac{1}{4\pi \varepsilon_0} \frac{Ze^2}{r_n} \]

\[ \therefore E_n = \frac{1}{2} m v_n^2 - \frac{1}{4\pi \varepsilon_0} \frac{Ze^2}{r_n} \]

\[ = \frac{1}{2} m \left( \frac{Z\alpha c}{n} \right)^2 - \frac{Ze^2}{4\pi \varepsilon_0} \left( \frac{Zmc\alpha}{n^2} \right) = \frac{1}{2} m \left( \frac{Z\alpha c}{n} \right)^2 - m \left( \frac{Z\alpha c}{n} \right)^2 \]

\[ = -\frac{1}{2} m \left( \frac{Z\alpha c}{n} \right)^2 \]

For ground state of hydrogen, \( Z = 1, n = 1 \)

\[ -\frac{1}{2} mc^2 \alpha^2 = -13.6 eV \]

\[ \therefore E_n = -13.6 \frac{Z^2}{n^2} (eV), \] (Hydrogen-like atomic energy level)

\[ \therefore a_0 = \frac{\hbar}{mc\alpha} \]

\[ \therefore E_n = -\frac{\hbar^2}{2ma_0^2} \frac{Z^2}{n^2} \]

The frequency of the EM radiation emitted when the electron makes a transition:

\[ f_{n_2 \rightarrow n_1} = \frac{E_{n_2} - E_{n_1}}{h} = \frac{m(Z\alpha c)^2}{2h} \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \]

\[ f = \frac{c}{\lambda}, \quad \frac{1}{\lambda_{n_2 \rightarrow n_1}} = \frac{m(Z\alpha c)^2}{2hc} \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = R_y Z^2 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \]

\[ R_y = \frac{m(\alpha c)^2}{2hc} = \frac{mc^2 \alpha^2}{4\pi hc} = 1.10 \times 10^7 \, \text{m}^{-1} \] (Rydberg constant)

The Bohr’s model can explain the atomic spectra successfully !!
De Broglie’s postulate of matter waves

For a photon, \( E = hf = pc \), \( c = f\lambda \) \( \Rightarrow \lambda = \frac{h}{p} \)

De Broglie: (Matter wave)
\[
p = \sqrt{2mE} = mv
\]
\[
\lambda = \frac{h}{p} = \frac{h}{mv}
\]
\( \Rightarrow \) Wave-like properties of particles (Matter Wave)

Revisit Bohr’s model
\[
L = mvr = nh = \frac{nh}{2\pi} \quad \Rightarrow \quad 2\pi r = \frac{nh}{mv} = \frac{nh}{p} = n\lambda
\]

Two slit interference experiment
One photon at a time reaches the detector.

3. Heisenberg's Uncertainty principles:

The more precisely the position is determined, the less precisely the momentum is known: $\Delta x \Delta p \geq \hbar$. 

![Graphs showing the uncertainty principle](image)
4. Schrodinger equation (1-D)

\[ E_{\text{total}} = K(\text{kinetic}) + V(\text{potential}) \]

\[ E\psi(x) = -\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) \]

\[ (|\psi|^2 \, dx = \text{probability}) \]

**Time independent Schrodinger Equation:**

\[ H = \frac{p^2}{2m} + V = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \]

\[ \hat{H}\psi_E = E\psi_E \]

\[ \langle \text{Energy} \rangle = \int_{-\infty}^{\infty} \psi_E^* \hat{H} \psi_E \, dx \]

\[ = \int_{-\infty}^{\infty} \psi_E^* E \psi_E \, dx = E \int_{-\infty}^{\infty} \psi_E^* \psi_E \, dx = E \]

\[ \langle \text{Energy}^n \rangle = \langle \hat{H}^n \rangle \]

\[ = \int_{-\infty}^{\infty} \psi_E^* \hat{H}^n \psi_E \, dx = E^n \]

\[ \sigma = \sqrt{\langle \hat{h}^2 \rangle - \langle h \rangle^2} = \Delta \] (Standard deviation)

\[ \Rightarrow \Delta E = \langle \text{Energy}^2 \rangle - \langle \text{Energy} \rangle^2 = 0 \] (Time independe Sch. Eq.)

→ **Eigenvalue and eigenfunction**

**Solution of Schrodinger Equation:**

1. **Infinite potential well**
\[ V(x) = \begin{cases} 0 & \text{for } 0 < x < L \\ \infty & \text{otherwise} \end{cases} \]

\[ 0 < x < L \Rightarrow -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x) = E \psi(x) \, , \, \psi(0) = \psi(L) = 0 \]

\[ \Rightarrow \psi_n(x) = A \sin \frac{n \pi}{L} x \, , \, \text{Normalize } \int_0^\infty \psi_n^* \psi_n dx = 1 \Rightarrow A = \sqrt{\frac{2}{L}} \]

\[ \therefore \psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n \pi}{L} x \, , \, E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} = \frac{n^2 \hbar^2}{8mL^2} \, , \, n = 1, 2, 3, \ldots \]

\[
\begin{align*}
\langle x \rangle &= \frac{2}{L} \int_0^L x \sin^2 \frac{\pi x}{L} dx = \frac{L}{2} \\
\langle p \rangle &= \frac{2}{L} \int_0^L \frac{\sin \pi x}{L} (-i \hbar \frac{d}{dx}) \sin \frac{\pi x}{L} dx = 0 \\
\langle p^2 \rangle &= \frac{2}{L} \int_0^L \frac{\sin \pi x}{L} (-i \hbar \frac{d}{dx})^2 \sin \frac{\pi x}{L} dx = \frac{\pi^2 \hbar^2}{L^2} \\
\Delta p &= \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \frac{\pi \hbar}{L} \, , \, \Delta p \Delta x \geq \frac{\hbar}{2} \\
|\psi_n(x)|^2 &= \frac{2}{L} \sin^2 \frac{n \pi}{L} x = \frac{2}{L} \left( 1 - \cos \frac{2n \pi}{L} x \right) = \frac{1}{L} - \cos \frac{2n \pi}{L} x
\end{align*}
\]
**Remark:** Comparison with classical results:

(1) Potential step

\[ V(x) = \begin{cases} V_0 & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases} \]

**Case I** \( E > V_0 \)
Region I:

\[ \psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x} , \quad k_1 = \frac{\sqrt{2mE}}{\hbar} \]

\[ \therefore |\psi_{\text{incident}}|^2 = A^*A = |A|^2 , \quad |\psi_{\text{reflected}}|^2 = B^*B = |B|^2 \]

Region II:

\[ \psi_2(x) = Ce^{ik_2x} + De^{-ik_2x} , \quad k_2 = \frac{\sqrt{2m(E-V_0)}}{\hbar} \]

No reflected wave \( \to D = 0 \)

\[ |\psi_{\text{trans}}|^2 = C^*C = |C|^2 \]

Boundary conditions:

1. \( \psi_I(0) = \psi_{II}(0) \Rightarrow A + B = C \)
2. \( \frac{\partial \psi_I}{\partial x} \bigg|_{x=0} = \frac{\partial \psi_{II}}{\partial x} \bigg|_{x=0} \Rightarrow k_1(A - B) = k_2C \)

\[ |\psi|^2 : \text{probability length} \propto \frac{\text{number length}}{\text{number length}} \]

Reflection and transmission \( \propto \frac{\text{number distance}}{\text{time}} = \frac{\text{number distance}}{\text{time}} \propto |\psi|^2 \nu \) (Probability flux)

\[ T = \frac{|\psi_{\text{trans}}|^2 v_I}{|\psi_{\text{in}}|^2 v_I} = \frac{|\psi_{\text{ref}}|^2 k_2}{|\psi_{\text{in}}|^2 k_1} = \frac{C^*Ck_2}{A^*Ak_1} \]

\[ R = \frac{|\psi_{\text{ref}}|^2 v_I}{|\psi_{\text{in}}|^2 v_I} = \frac{|\psi_{\text{ref}}|^2}{|\psi_{\text{in}}|^2} = \frac{B^*B}{A^*A} \]

\[ \begin{cases} A + B = C \\ A - B = \frac{k_2}{k_1}C \end{cases} \Rightarrow C = \frac{2k_1}{k_1 + k_2}A , \quad B = \frac{k_1 - k_2}{k_1 + k_2}A \]

\[ \Rightarrow T = \frac{4k_1k_2}{(k_1 + k_2)^2} \]

\[ R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2} \]

\( \Rightarrow T + R = 1 \), \( R \neq 0 \), wave property
Remark:

Classical: no reflection for $E > V_0$ (R=0)

Quantum: $R \neq 0$ for $E > V_0$, (wave property)

Case II $E < V_0$ (bound)

Region I: $\psi_I(x) = Ae^{ik_1x} + Be^{-ik_1x}$, $k_1 = \frac{\sqrt{2mE}}{\hbar}$

Region II: $\psi_{II}(x) = Ce^{\alpha x} + De^{-\alpha x}$, $\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$

$\psi_{II}$ cannot diverge $\Rightarrow C = 0$

Boundary conditions:
(1) $\psi_I(0) = \psi_{II}(0) \Rightarrow A + B = D$
(2) $\frac{\partial \psi_I}{\partial x} \bigg|_{x=0} = \frac{\partial \psi_{II}}{\partial x} \bigg|_{x=0} \Rightarrow ik_1(A - B) = -\alpha D$

$\Rightarrow B = \frac{-\alpha + ik}{\alpha - ik} A$
\[ |B| = \sqrt{B^*B} = \sqrt{\left(\frac{-\alpha + ik}{\alpha - ik} A\right)^* \left(\frac{-\alpha + ik}{\alpha - ik} A\right)} = \sqrt{A^*A} = |A| \]

\[ R = \frac{B^*B}{A^*A} = 1 \quad , \quad T = 0 \]

\[ \delta = \frac{1}{\alpha} = \frac{1}{\sqrt{2m(V_0 - E)}} \] (penetration depth)

**(II) Tunneling barrier**

\[ V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \quad (I) \\ V_0 & \text{for} \quad 0 < x < a \quad (II) \\ 0 & \text{for} \quad x > a \quad (III) \end{cases} \]

**Case I** \( E < V_0 \)

Region I : \( \psi_I(x) = Ae^{ik_1x} + Be^{-ik_1x} \), \( k_1 = \frac{\sqrt{2mE}}{h} \)

Region II : \( \psi_{II}(x) = Ce^{-\alpha x} + De^{\alpha x} \), \( \alpha = \frac{\sqrt{2m(V_0 - E)}}{h} \)

Region III: \( \psi_{III}(x) = Fe^{ik_1x} + Ge^{-ik_1x} \), \( k_1 = \frac{\sqrt{2mE}}{h} \)

B.C. \( \psi(x), \frac{\partial}{\partial x} \psi \) must be continuous at \( x = 0, x = a \) and let \( k = k_1 \)

\[ \Rightarrow \begin{cases} A + B = C + D \\ ikA - ikB = \alpha D - \alpha C \\ Ce^{-\alpha x} + De^{\alpha x} = Fe^{ik_1x} \\ (\alpha D)e^{\alpha x} - (\alpha C)e^{-\alpha x} = ikFe^{ik_1x} \end{cases} \]
\[ T = \frac{FF^*}{AA^*} = \left[ 1 + \frac{1}{4} \left( \frac{V_0^2}{E(V_0 - E)} \right) \sinh^2(\alpha a) \right]^{-1} \]
\[ = \left[ 1 + \frac{1}{16} \frac{V_0^2}{E(V_0 - E)} \left( e^{2\alpha a} - 2 + e^{-2\alpha a} \right) \right]^{-1} \]
\[ \approx 16 \frac{E}{V_0} (1 - \frac{E}{V_0}) e^{-2\alpha a} \quad \text{for} \quad \alpha a \gg 1 \]

Example: Scanning Tunneling Microscopy (STM)

From IBM.STM Gallery
3. Case 3. Particle in a box (3-D)

Electron confined in three dimensions by a three dimensional infinite "PE box". Everywhere inside the box, \( V = 0 \), but outside, \( V = \infty \). The electron cannot escape from the box. What is the energy and wavefunction of the electron?

Schrödinger's equation for three dimensions

\[
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0
\]

if \( L_x = L_y = L_z = L \) \( \Rightarrow (n_x, n_y, n_z) \) correspond to one state

\[E = \frac{\pi^2 \hbar^2}{2m} \left( \frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2} \right)\]

(if \( L_x = L_y = L_z = L \) \( = (n_x^2 + n_y^2 + n_z^2) \frac{\pi^2 \hbar^2}{2mL^2} \)

Hydrogen Atom:
Three quantum number $(n, l, m_l)$

$$\psi(r, \theta, \phi) = R_{n,l}(r)Y_{l,m_l}(\theta, \phi), \quad Y_{l,m_l}(\theta, \phi) = \Theta(\theta)\Phi(\phi)$$

$R(r)$: radial wavefunction, $Y$: spherical wavefunction

Electron energy in the hydrogen atom is quantized.

$n$ is a quantum number, 1, 2, 3,

$$E_n = -\frac{m_e^4}{8\varepsilon_0 \hbar^2 n^2} = -\frac{(13.6 eV)}{n^2}$$

$$a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_e^2} = 0.0528 nm \quad \text{--> Bohr radius}$$

<table>
<thead>
<tr>
<th>$n$</th>
<th>Principal quantum number</th>
<th>$n = 1, 2, 3, ...$</th>
<th>Quantizes the energy of the electron, $E = -(13.58 \text{ eV})\hbar$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell$</td>
<td>Orbital angular momentum quantum number</td>
<td>$\ell = 0, 1, 2, ... (n-1)$</td>
<td>Quantizes the magnitude of orbital angular momentum, $L$. $L = \hbar [\ell(\ell + 1)]^{1/2}$</td>
</tr>
<tr>
<td>$m_r$</td>
<td>Magnetic quantum number</td>
<td>$m_r = 0, \pm 1, \pm 2, ... , \pm \ell$</td>
<td>Quantizes the orbital angular momentum component along a magnetic field, $B_z$. $L_z = m_r \hbar$</td>
</tr>
<tr>
<td>$m_s$</td>
<td>Spin magnetic quantum number</td>
<td>$m_s = \pm \ell/2$</td>
<td>Quantizes the spin angular momentum component along a magnetic field $B_z$. $S_z = m_s \hbar$</td>
</tr>
</tbody>
</table>
Table 3.1  Labelling of various $n\ell$ possibilities.

<table>
<thead>
<tr>
<th>$n$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1s</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2s</td>
<td>2p</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3s</td>
<td>3p</td>
<td>3d</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4s</td>
<td>4p</td>
<td>4d</td>
<td>4f</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5s</td>
<td>5p</td>
<td>5d</td>
<td>5f</td>
<td>5g</td>
</tr>
</tbody>
</table>

Table 3.2
The radial and spherical harmonic parts of the wavefunction in the hydrogen atom. ($a_0 = 0.0529\text{nm}$)

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\ell$</th>
<th>$R(r)$</th>
<th>$m$</th>
<th>$Y(\theta,\phi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>$\left(\frac{1}{a_0}\right)^{1/2}2\exp\left(\frac{-r}{a_0}\right)$</td>
<td>0</td>
<td>$\frac{1}{2\pi}$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$\left(\frac{1}{2a_0}\right)^{1/2}(2\frac{r}{a_0})\exp\left(\frac{-r}{2a_0}\right)$</td>
<td>0</td>
<td>$\frac{1}{2\pi} \cos \theta$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>$\left(\frac{1}{2a_0}\right)^{1/2}(\sqrt{3}\frac{r}{a_0})\exp\left(\frac{-r}{2a_0}\right)$</td>
<td>1</td>
<td>$\frac{3}{2\pi} \sin \theta \cos \phi$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-1</td>
<td>$\frac{3}{2\pi} \sin \theta e^{i\phi}$</td>
</tr>
</tbody>
</table>

Correspond $m = -1$ and

$\{\cos \sin \phi \}$. 

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(a) Radial wavefunctions of the electron in a hydrogenic atom for various \( n \) and \( l \) values. (b) \( r^2|R_n,l|^2 \) gives the radial probability density. Vertical axis scales are linear in arbitrary units.

(a) The polar plots of \( Y_{n\ell}(\Theta,\Phi) \) for 1s and 2p states. (b) The angular dependence of the probability distribution which is proportional to \( |Y_{n\ell}(\Theta,\Phi)|^2 \).
Oritabl angular momentum and Space quantization

(a) The electron has an orbital angular momentum which has a quantized component, \( L_z \), along an external magnetic field, \( \mathbf{B}_{\text{external}} \). (b) The orbital angular momentum vector \( \mathbf{L} \) rotates about the \( z \)-axis. Its component \( L_z \) is quantized and therefore the orientation of \( \mathbf{L} \), the angle \( \theta \), is also quantized. \( \mathbf{L} \) traces out a cone. (c) According to quantum mechanics, only certain orientations (\( \theta \)) for \( \mathbf{L} \) are allowed as determined by \( \ell \) and \( m_\ell \).

Orbital Angular Momentum and Space Quantization

**Orbital angular momentum**

\[
L = \hbar \left[ \ell \left( \ell + 1 \right) \right]^{1/2}
\]

where \( \ell = 0, 1, 2, \ldots n-1 \)

**Orbital angular momentum along \( B_z \)**

\[
L_z = m_\ell \hbar
\]

**Selection rules for EM radiation absorption and emission**

\[
\Delta \ell = \pm 1 \quad \text{and} \quad \Delta m_\ell = 0, \pm 1
\]
Electron Spin and Intrinsic Angular Momentum $S$

Electron spin

$$S = \hbar \left[ s(s + 1) \right]^{1/2} \quad s = \frac{1}{2}$$

Spin along magnetic field

$$S_z = m_s \hbar \quad m_s = \pm \frac{1}{2}$$

the quantum numbers $s$ and $m_s$ are called the spin and spin magnetic quantum numbers.

Spin angular momentum exhibits space quantization. Its magnitude along $z$ is quantized so that the angle of $S$ to the $z$-axis is also quantized.