DIFFERENTIAL SCHROEDINGER EQUATION

Argument leading to the equation:

1. Consistent with the de Broglie-Einstein postulates

$$\lambda = h/p \qquad \nu = E/h$$

2. Consistent with the energy equation

$$E = p^2/2m + V$$

3. Linear in wavefunction

if Ψ_1 and Ψ_2 are the solutions of the equation,
so is $\Psi = c_1\Psi_1 + c_2\Psi_2$

Interpretation of Wave Functions

$$P(x,t)\,dx = \psi^*(x,t)\psi(x,t)\,dx$$

At a given t, if a measurement is made to locate a particle associated with a wave function $\psi(x,t)$, then the probability $P(x,t)dx$ of finding the particle between x and $x + dx$ is equal to $\psi^*(x,t)\psi(x,t)dx$

Expectation Values

$$<x> = \int \psi^*(x,t) x \psi(x,t)\, dx$$

$$<p> = \int \psi^*(x,t)\left(-i\hbar\frac{\partial}{\partial x}\right)\psi(x,t)\, dx$$

$$<E> = \int \psi^*(x,t)\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t)\right)\psi(x,t)\, dx$$

TIME INDEPENDENT SCHROEDINGER EQUATION

WHEN POTENTIAL V(x,t) IS ONLY A FUNCTION OF x , ie. V(x)

THEN SCHROEDINGER EQUATION

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+V(x)\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

BY SEPARATION OF VARIABLES,
ASSUME SOLUTION

$$\Psi(x,t)=\psi(x)\Phi(t)$$

$\Psi(x,t)$: **WAVE FUNCTION**

ψ (x) : **EIGEN FUNCTION**
$\Phi(t)$: **TIME DEPENDENCE OF WAVE FUNCTION**

$$-\frac{\hbar^2}{2m}\Phi(t)\frac{\partial^2\psi(x)}{\partial x^2}+V(x)\psi(x)\Phi(t)=i\hbar\,\psi(x)\frac{\partial\Phi(t)}{\partial t}$$

⇓

$$-\frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2\psi(x)}{dx^2}+V(x)=i\hbar\frac{1}{\Phi(t)}\frac{d\Phi(t)}{dt}$$

⇓

$$\Phi(t)=e^{-i\omega t}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x) \qquad E=\hbar\omega$$

⇓

$$\Psi(x,t)=\psi(x)e^{-i\frac{E}{\hbar}t}$$

Required Properties of Eigenfunctions

1) Both ψ(x) and $d\psi$(x)/dx must be *finite*

2) Both ψ(x) and $d\psi$(x)/dx must be *single valued*

2) Both ψ(x) and $d\psi$(x)/dx must be *continuous*

Qualitative Description of Eigenfunctions

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x)$$

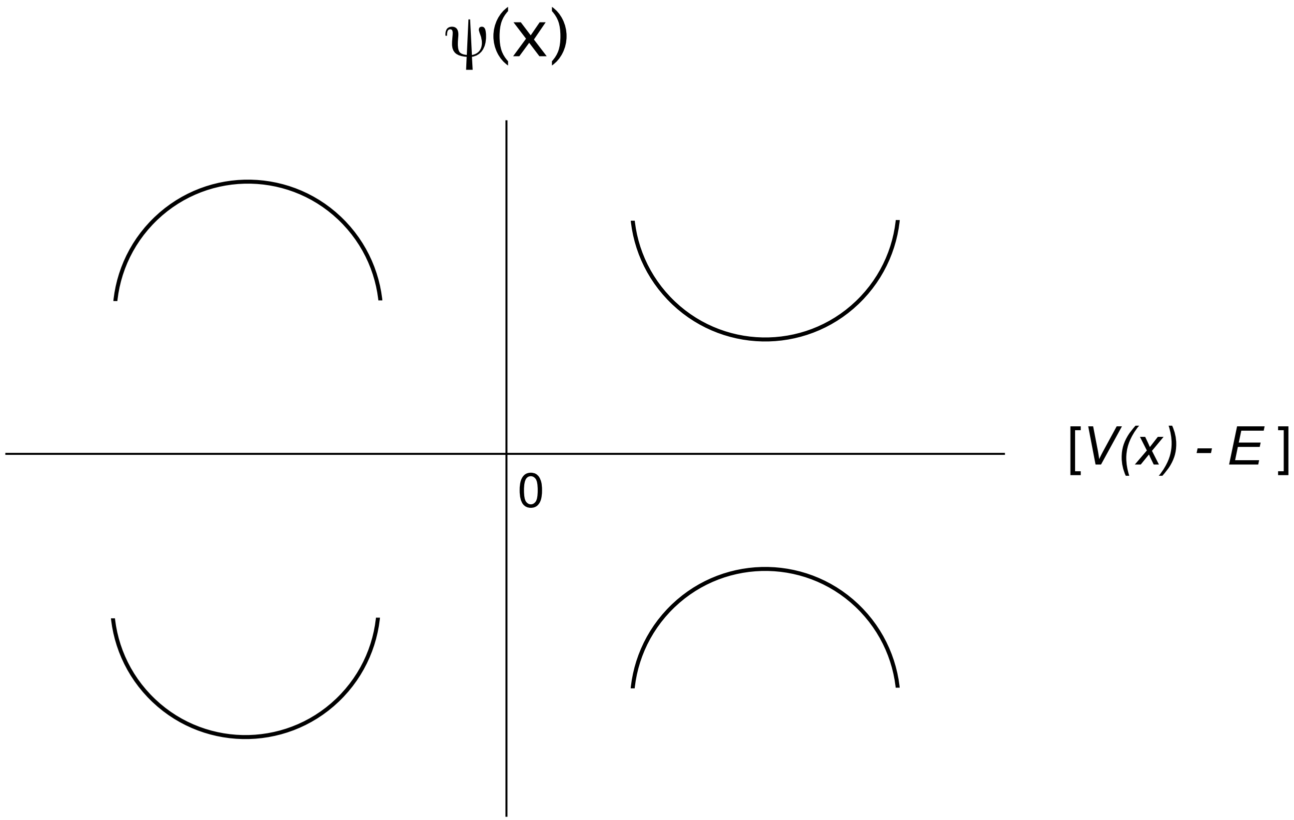
$$d^2\psi(x)/dx^2 = 2m/\hbar^2\,[V(x) - E]\,\psi(x)$$

The sign of $d^2\psi(x)/dx^2$ depends on both the signs of $[V(x) - E]$ and $\psi(x)$

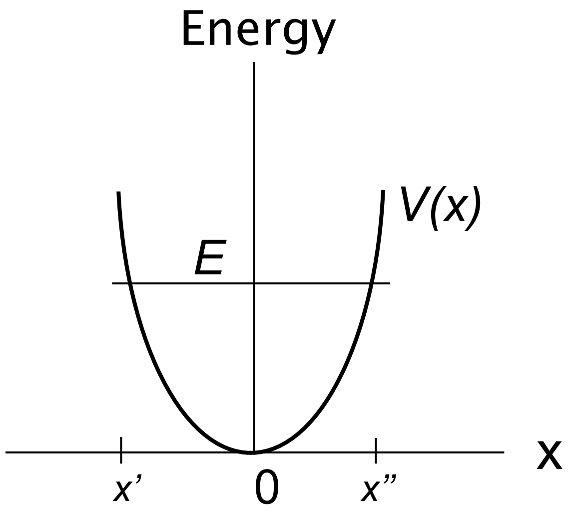
1) Positive $d^2\psi(x)/dx^2$ represents a $\psi(x)$ *concave upwards*

2) Negative $d^2\psi(x)/dx^2$ represents a $\psi(x)$ *concave downwards*

The sign of $d^2\psi(x)/dx^2$



Example: Simple Harmonic Oscillator



Homework#3 (Sept. 27, 2010):

Consider a particle moving under the influence of the potential $V(x) = C\,|x|$, where C is a constant.

(a) Use qualitative arguments to make a sketch of the first and of the tenth eigenfunctions for the system.

(b) Sketch both of the corresponding probability density functions.

(c) Then use the classical mechanics to calculate the probability density functions predicted by the theory.

(d) Plot the classical probability density functions with the quantum mechanical probability density functions, and discuss briefly their comparison.