DIFFERENTIAL SCHROEDINGER EQUATION

Argument leading to the equation:

1. Consistent with the de Broglie-Einstein postulates

$$\lambda = h/p \qquad \nu = E/h$$

2. Consistent with the energy equation

$$E = p^2/2m + V$$

3. Linear in wavefunction

if Ψ_1 and Ψ_2 are the solutions of the equation, so is $\Psi = c_1\Psi_1 + c_2\Psi_2$

TIME INDEPENDENT SCHROEDINGER EQUATION

WHEN POTENTIAL V(x,t) IS ONLY A FUNCTION OF x , ie. V(x)

THEN SCHROEDINGER EQUATION

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+V(x)\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

BY SEPARATION OF VARIABLES,
ASSUME SOLUTION

$$\Psi(x,t)=\psi(x)\Phi(t)$$

Ψ *(x,t) :* ***WAVE FUNCTION***

ϕ *(x) :* ***EIGEN FUNCTION***

Φ *(t) :* ***TIME DEPENDENCE*** **OF WAVE FUNCTION**

$$-\frac{\hbar^2}{2m}\Phi(t)\frac{\partial^2\psi(x)}{\partial x^2}+V(x)\psi(x)\Phi(t)=i\hbar\,\psi(x)\frac{\partial\Phi(t)}{\partial t}$$

⬇

$$-\frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2\psi(x)}{dx^2}+V(x)=i\hbar\frac{1}{\Phi(t)}\frac{d\Phi(t)}{dt}$$

⬇

$$\Phi(t)=e^{-i\omega t}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\,\psi(x)$$ $E = \hbar\omega$

⬇

$$\Psi(x,t)=\psi(x)e^{-i\frac{E}{\hbar}t}$$

One-dimensional system: free particle

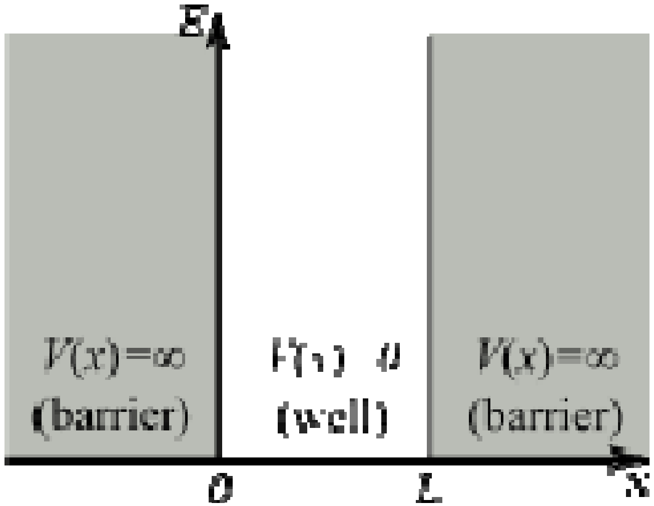
V(x) = 0

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

$$\psi(x) = e^{ikx}$$

$$E = \frac{\hbar^2 k^2}{2m} = \hbar\omega$$

One-dimensional system: particle in a box



Inside the box

$$\psi(x) = A\sin(kx) + B\cos(kx),$$

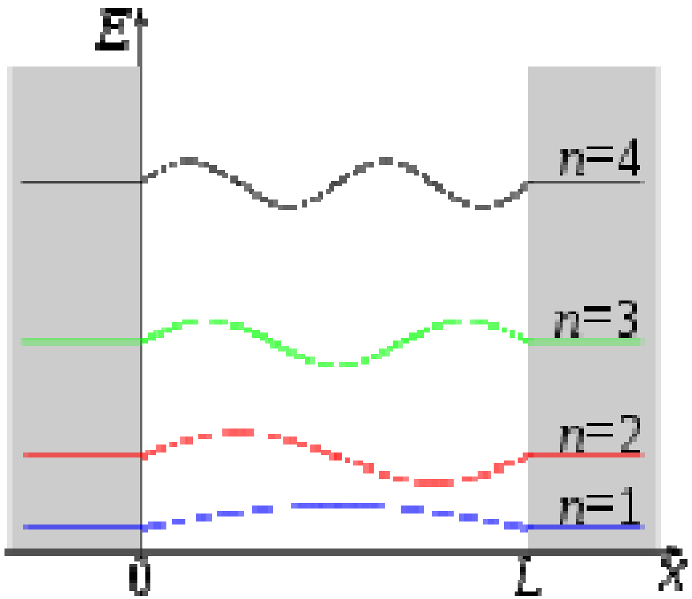
Outside the box

$$\psi(x) = 0$$

$$E = \frac{k^2 \hbar^2}{2m}.$$

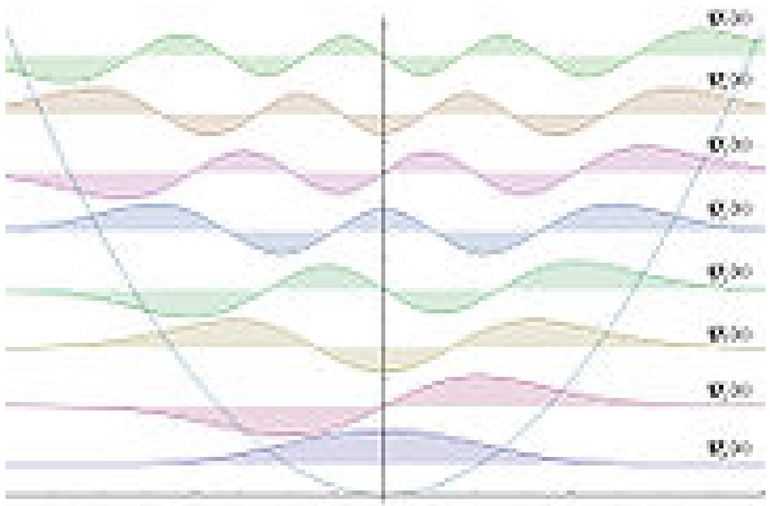
The wavefunction must be continuous at the interfaces, meaning that $\phi(0) = \phi(L) = 0$.

$$k_n = \frac{n\pi}{L}, \quad \text{where} \quad n \in \mathbb{Z}^+$$



One-dimensional system: simple harmonic oscillator

$$V(x) = ½ (m\, w^2\, x^2)$$



Wavefunction representations for
the first eight bound eigenstates, n = 0 to 7

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

Homework#1:

Consider a particle approaching a rectangular potential barrier V(x).
Here V(x) =0 for x<-(1/2)a and x> (1/2)a; V(x) = V_o for |x| ≤ a.
(a)Write down the solution for the eigenfunctions and eignvalues for E < V_0.
(b)Also find the solution for the eigenfunctions and eignvalues for E > V_0.