Electrical conduction in solids

Electrical conduction is the movement of electrically charged particles through a conductor or semiconductor, which constitutes an electric current. The charge transport may result as a response to an electric field (**E**) or by diffusion. The current density (**j**) is then:

$$\mathbf{j} = \sigma \mathbf{E} + qD\nabla n$$

with σ conductivity, q the elementary charge, D diffusion constant, and n the carrier density.

Equations of motion

$$\mathbf{F} = d\mathbf{P}/dt = m d\mathbf{v}/dt = \hbar d\mathbf{k}/dt = q(\mathbf{E} + \frac{1}{c}\,\mathbf{v} \times \mathbf{B})$$

Electrical conduction for metals

Drude Model (free electron gas model)

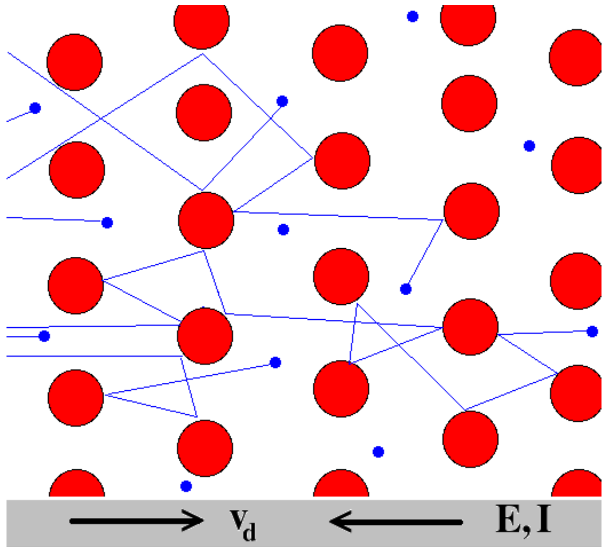
Paul Drude in 1900 explained the transport properties of electrons in metals. The model, which is an application of kinetic theory, assumes that the microscopic behavior of electrons in a solid may be treated classically and looks much like a pinball machine, with a sea of constantly jittering electrons bouncing and re-bouncing off heavier, relatively immobile positive ions.

1. Electronic equation of motion

$$\frac{d}{dt}\mathbf{p}(t) = q\mathbf{E} - \frac{\mathbf{p}(t)}{\tau},$$

2. Ohm's law

$$\mathbf{J} = \left(\frac{nq^2\tau}{m}\right)\mathbf{E} = \sigma E$$



Physical origins of electrical resistance

An exactly periodic lattice of positive ion cores does NOT cause scattering.

Electrons are scattered by:

1) Deviations from strict periodicity in the lattice:

a) Defects in the lattice (such as vacancies, dislocations, impurities, etc): usually fixed in space and time and temperature insensitive.

b) Lattice vibrations: varying in time and depending strongly on temperature.

2) Electron-electron collisions:

Normally this term is insignificant comparing with the above factor at room temperature and below.

If there is no scattering, electrons are moving under a constant **E** field, then

$$\hbar d\mathbf{k}/dt = -e\mathbf{E} \quad \Rightarrow \quad \hbar\mathbf{k}(t) = m\mathbf{v}(t) = -e\mathbf{E}t, \ \text{ assume } \mathbf{E}(0) = 0$$

If the collision time is τ, then $\mathbf{v}(\tau) = -e\mathbf{E}\tau/m$

$$\Rightarrow \quad \mathbf{j} = nq\mathbf{v} = ne^2\mathbf{E}\tau/m = \sigma\mathbf{E}$$

⇨ Electrical conductivity

$$\sigma = ne^2\tau/m$$

The collision time τ can, to a good approximation, be independently attributed to two factors, the phonons (τ_{ph}) and impurities (τ_i), and

$$1/\tau = 1/\tau_{ph} + 1/\tau_i$$

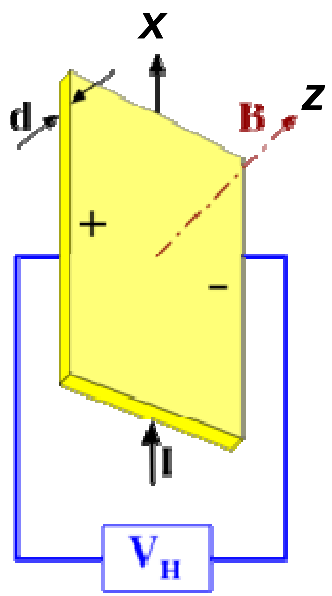
It is thus a function of temperature T. For example, for copper (Cu),

$\tau \sim 2 \times 10^{-9}$ s at 4K and $\tau \sim 2 \times 10^{-14}$ s at 300K

Hall effect

The **Hall effect** is discovered by Edwin Hall in 1879. It is the production of a voltage difference across an electrical conductor, transverse to an electric current in the conductor and a magnetic field perpendicular to the current.

At steady state, $m d\mathbf{v}/dt = m\mathbf{v}/\tau = -e(\mathbf{E} + \frac{1}{c}\, \mathbf{v} \times \mathbf{B})$



$v_x = -e\tau E_x/m - \omega_c \tau v_y$

$v_y = -e\tau E_y/m + \omega_c \tau v_x$

$v_z = -e\tau E_z/m$

Cyclotron frequency

$\omega_c = eB/mc$

If $v_y = 0$, then $E_y = -\omega_c \tau E_x = -eB\tau E_x/mc$

The Hall coefficient is defined as the ratio of the induced electric field (E) to the product of the current density (j) and the applied magnetic field (B).

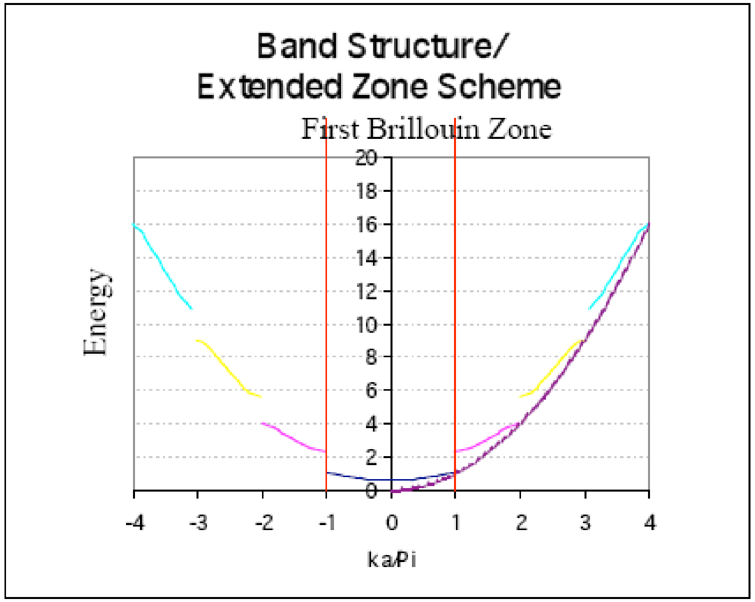
$$R_H = \frac{E_y}{j_x B} = \frac{dV_H}{IB} = -\frac{1}{nec}$$

Expression of band structure in different schemes

Bloch functions

$$\psi_{n\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{n\mathbf{k}}(\mathbf{r})$$

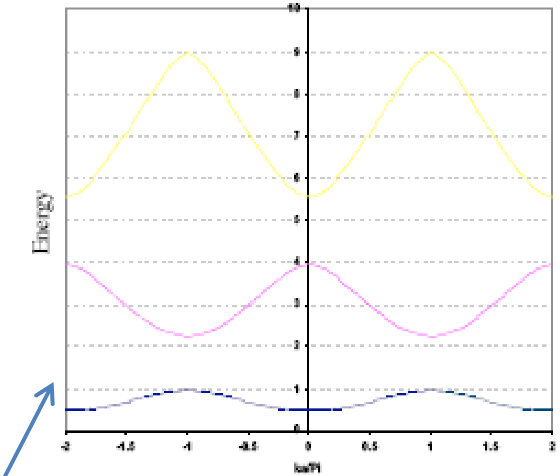
$$u_{nk}(\mathbf{r}+\mathbf{T}) = u_{nk}(\mathbf{r})$$



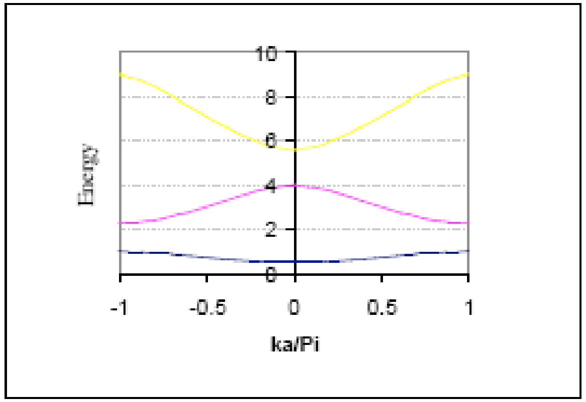
$$\psi_n(\mathbf{k}) = \psi_n(\mathbf{k} + \mathbf{G})$$

$$\epsilon_n(\mathbf{k}) = \epsilon_n(\mathbf{k} + \mathbf{G})$$

Periodic Zone Scheme



Reduced Zone Scheme

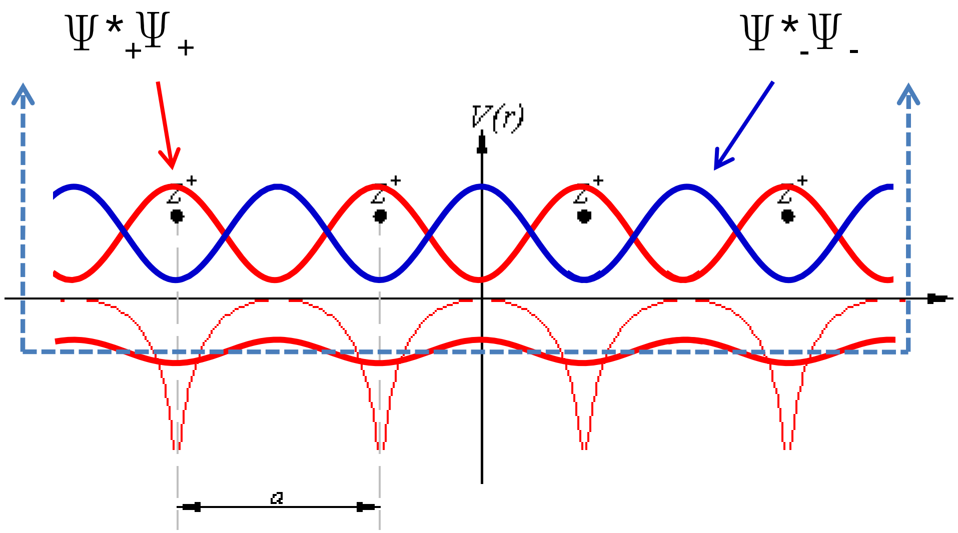


•Bands folded into First Brillouin Zone

At k = $\pm \pi/a$

$$\Psi_+(x) = e^{ikx} + e^{-ikx} = 2\cos(\pi x/a)$$

$$\Psi_-(x) = e^{ikx} - e^{-ikx} = 2i\sin(\pi x/a)$$



$$U(x) = 2U\cos(2\pi x/a)$$

$$E_g = \int_0^1 dx\, U(x)\, [\Psi^*_+\Psi_+ - \Psi^*_-\Psi_-] = U$$

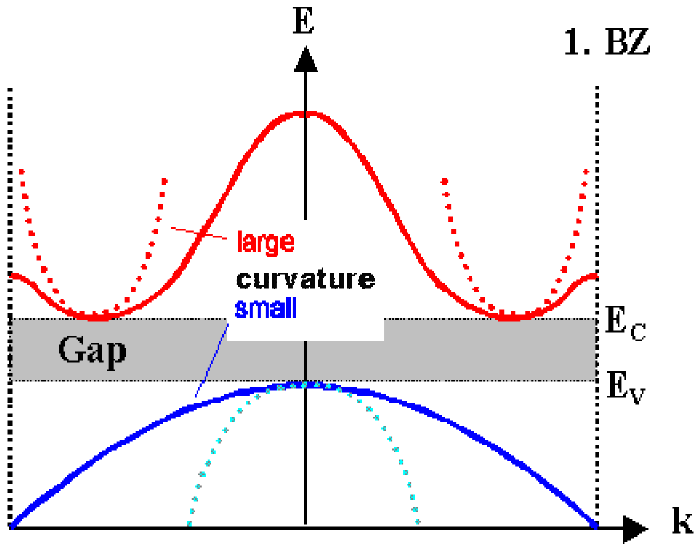
Effective mass

A particle's **effective mass (*m**)** is the mass it seems to carry in the *semiclassical model* of transport in a crystal. In a simplified picture that ignores crystal anisotropies, electrons or holes behave as free particles in a vacuum, but with a different mass. This mass is usually stated in units of the ordinary mass of an electron m_e (9.11×10^{-31} kg). In these units it is usually in the range 0.01 or 10.

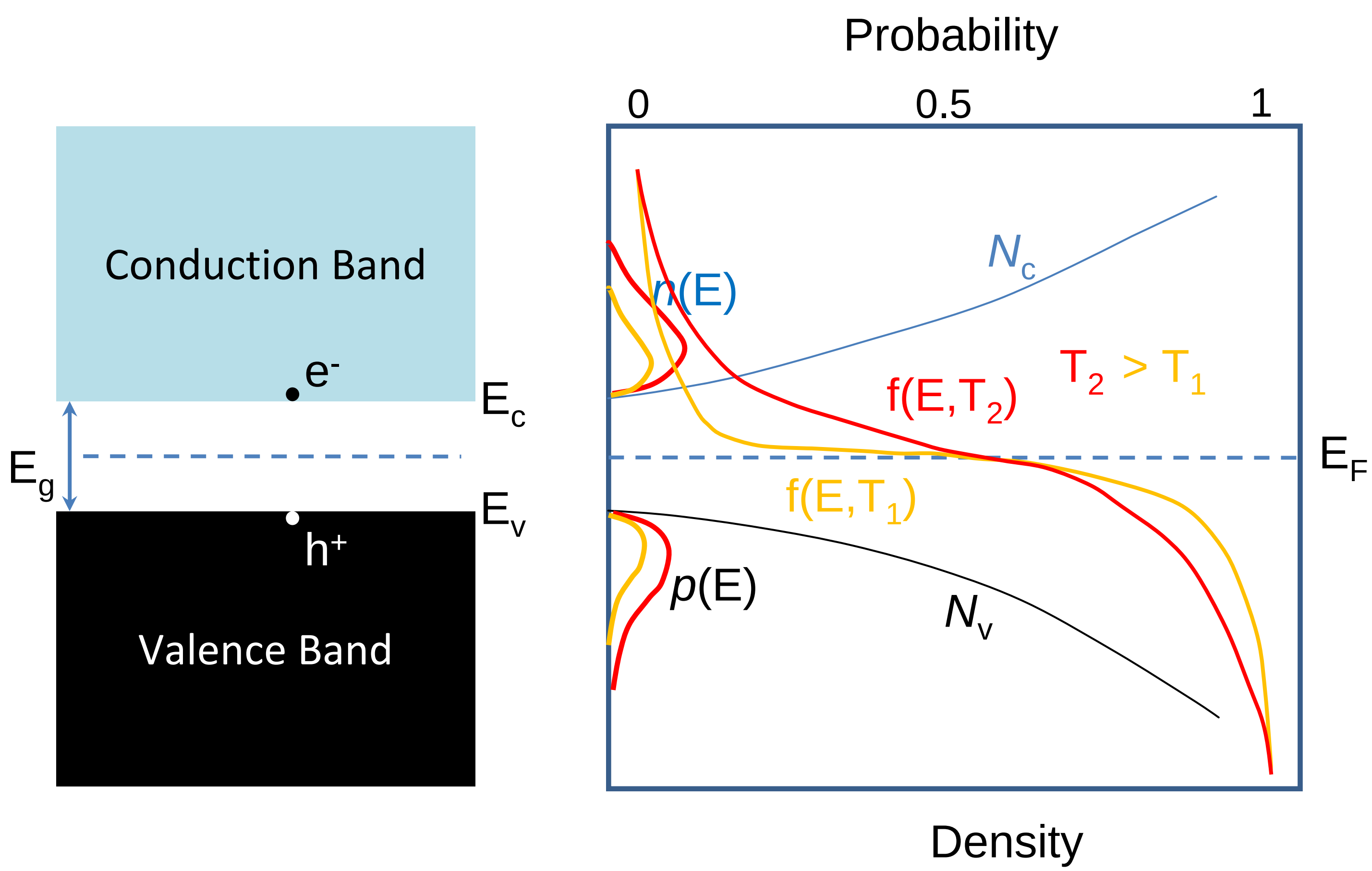
Electrical conductivity

$\sigma = ne^2\tau/m^*$ $\quad m^* = \hbar^2 \cdot \left[\frac{d^2\varepsilon}{dk^2}\right]^{-1}$.

The decisive factor for the effective mass is thus the *curvature of the dispersion curve at the extrema*, as expressed in the second derivative. Large curvatures (*small* radius of curvature) give small effective masses, and vice versa.



Thermal excitation of semiconductors



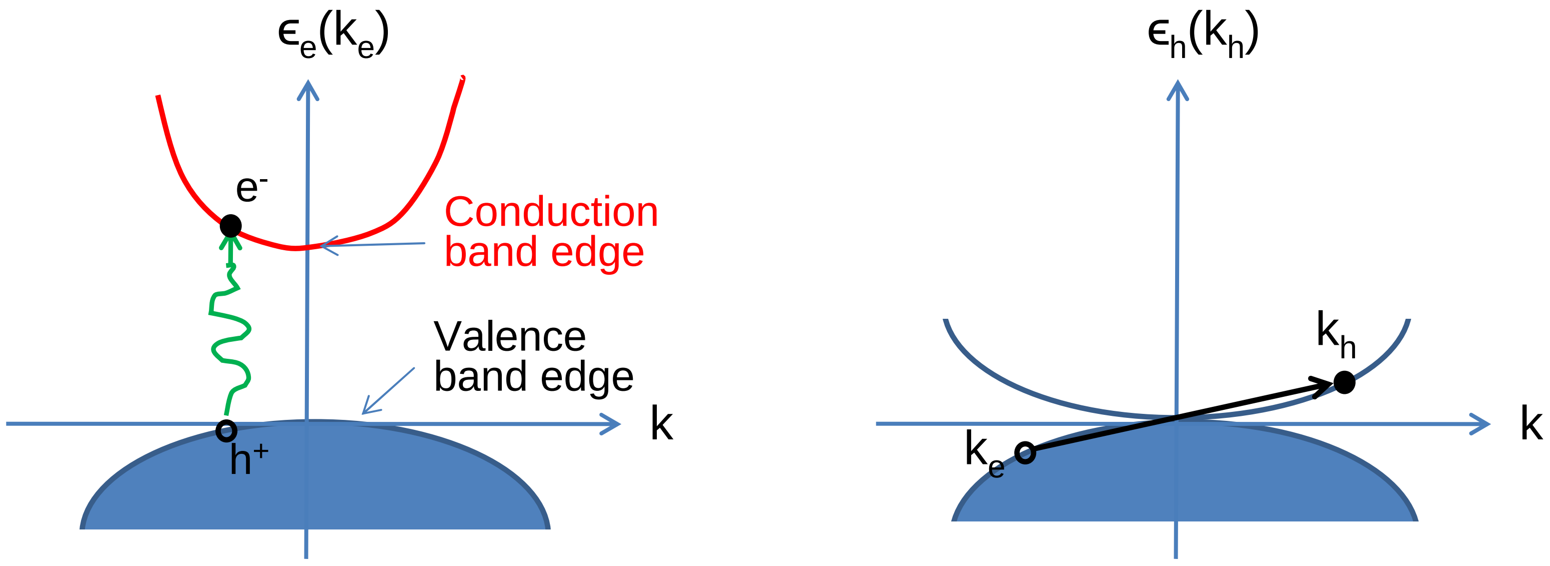
$$n(E) = \int N_c(E) f(E)\, dE \qquad p(E) = \int N_v(E)[1-f(E)]\, dE$$

Electron and hole

In solid state physics, an **electron hole** (usually referred to simply as a **hole**) is the absence of an electron from the otherwise full valence band. The concept of a hole is essentially a simple way to analyze the electronic transitions within the valence band.

Property	Electron	Hole
Charge	q_e	$q_h = -q_e$
Effective mass	m_e	$m_h = -m_e$
Velocity	$\mathbf{v}_e$	$\mathbf{v}_h = \mathbf{v}_e$
Wavevector	$\mathbf{k}_e$	$\mathbf{k}_h = -\mathbf{k}_e$
Energy	$\epsilon_e(\mathbf{k}_e)$	$\epsilon_h(\mathbf{k}_h) = -\epsilon_e(\mathbf{k}_e)$

Hole band



$$v = d\omega/dk = \frac{1}{\hbar} d\epsilon/dk$$

Electrical conduction for semiconductors

The total current density is the summation of the amounts contributed by different types of charge carriers.

$$\mathbf{j} = \Sigma n_i q_i \mathbf{v}_i = \Sigma \sigma_i \mathbf{E}$$

For an intrinsic semiconductor,

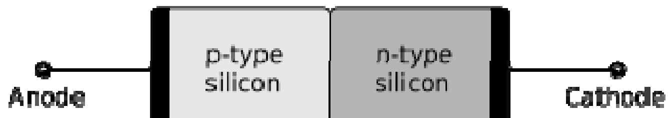
$$\mathbf{j} = -ne\mathbf{v}_n + pe\mathbf{v}_p = ne\mu_n\mathbf{E} + pe\mu_p\mathbf{E} = \sigma\mathbf{E}$$

$\mu_{n(p)} \equiv$ mobility of electron (hole)

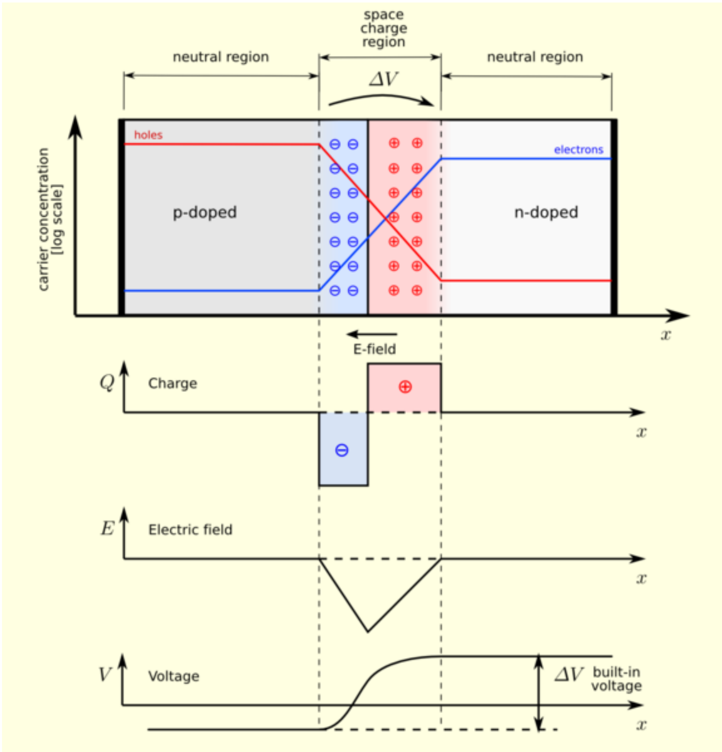
Electrical conductivity for semiconductor

$$\sigma = (ne\mu_n + pe\mu_p)$$

p-n junctions



$$\mathbf{j} = \mathbf{j}^{drift} + \mathbf{j}^{diff} = \sigma\, \mathbf{E} + qD\nabla n$$



In thermal equilibrium, $\mathbf{j}_n = 0$

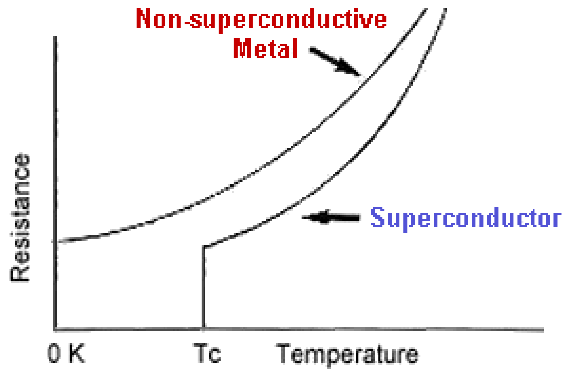
$$\sigma_n E_x = ne\mu_n(-dV_x/dx) = -eD_n dn/dx$$

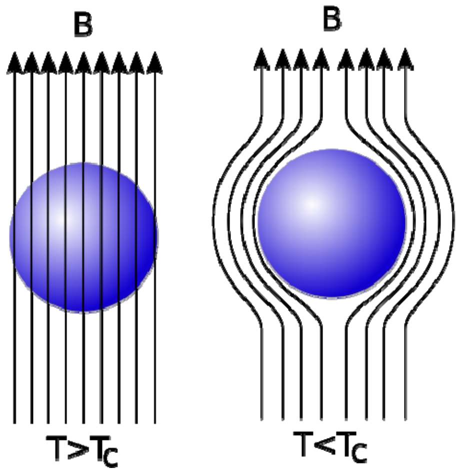
$$dn/dx = \frac{ne}{kT}(dV_x/dx)$$

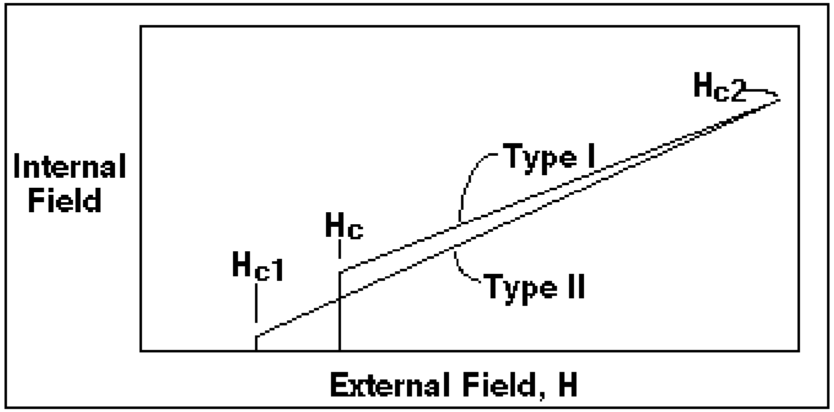
$$D_n = \frac{kT}{e}\mu_n \equiv \textit{Einstein relation}$$

Superconductivity

Superconductivity occurs in certain materials at very low temperatures. When superconductive, a material has an electrical resistance of exactly zero and no interior magnetic field (so-called the Meissner effect). It was discovered by Heike Kamerlingh Onnes in 1911.







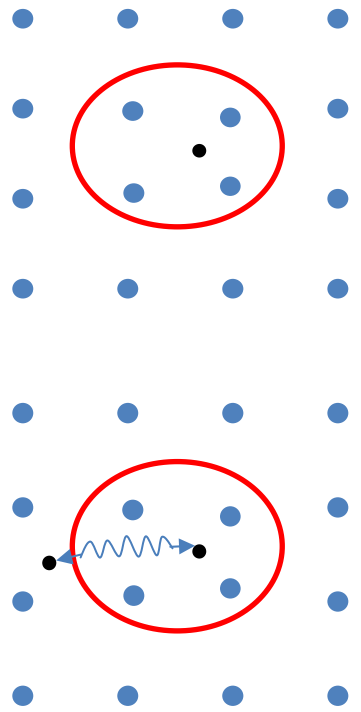
The Meissner effect is the expulsion of a magnetic field from a superconductor during its transition to the superconducting state.

Type I superconductors are comprised of pure metals, whereas Type II superconductors are comprised primarily of alloys or intermetallic compounds.

BCS theory

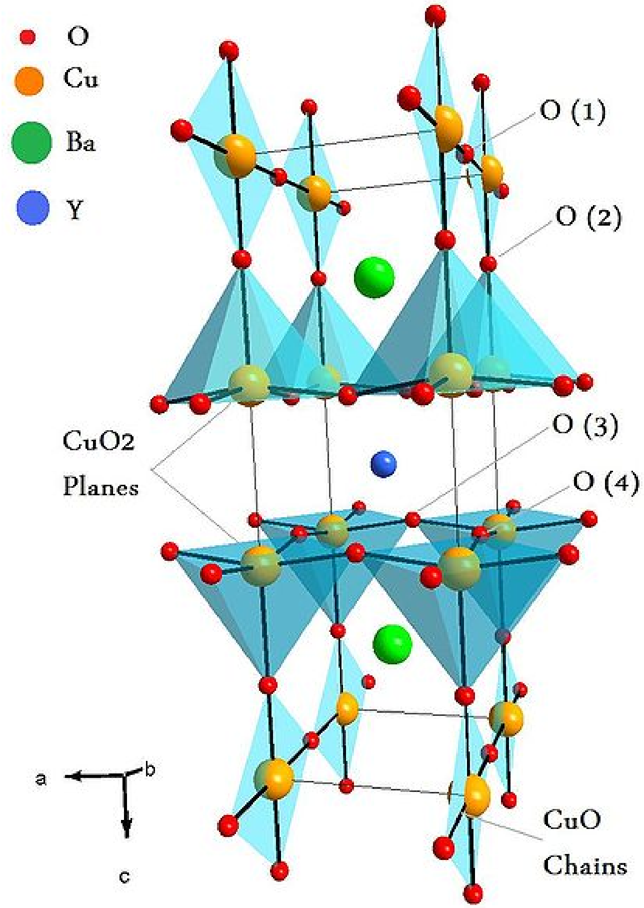
BCS theory is the first microscopic theory of superconductivity, proposed by Bardeen, Cooper, and Schrieffer in 1957. It describes superconductivity as a macroscopic effect caused by a "condensation" of pairs of electrons into a boson-like state.

An electron moving through a conductor will attract nearby positive charges in the lattice. This deformation of the lattice causes another electron, with opposite "spin", to move into the region of higher positive charge density. The two electrons then become correlated. There are a lot of such electron pairs (Cooper pairs) in a superconductor, so that they overlap very strongly, forming a highly collective "condensate". Breaking of one pair results in changing of energies of remained macroscopic number of pairs. If the required energy is higher than the energy provided by kicks from oscillating atoms in the conductor (which is true at low temperatures), then the electrons will stay paired and resist all kicks, thus not experiencing resistance. Thus, the collective behaviour of "condensate" is a crucial ingredient of superconductivity.



High T_c superconductors

In 1986, Bednorz and Müller discovered superconductivity in a lanthanum-based cuprate perovskite material, which had a transition temperature of 35 K (Nobel Prize in Physics, 1987). It was shortly found by Wu and Chu that replacing the lanthanum with yttrium, i.e. making YBCO, raised the critical temperature to 92 K, which was important because liquid nitrogen could then be used as a refrigerant. In February 2008, an iron-based family of high temperature superconductors was discovered by Hideo Hosono of the Tokyo Institute of Technology.



Homework#9 (Dec. 28, 2009):

Assuming concentrations n, p; relaxation time τ_e, τ_h; and masses m_e, m_h, show that the Hall coefficient in the drift velocity approximation is

$$R_H = (p-nb^2)/[ec(p+nb)^2] \quad ,$$

Where $b = \mu_e/\mu_h$ is the mobility ratio.