Quantum Statistics

There are two main differences between quantum statistics and classical statistics:

1.In quantum statistics the energy levels are nearly always discrete. In classical statistics they are assumed to be continuous.

2. The counting of the number of arrangements of the elements is different in the two types of statistics. A dominant reason for this is the indistinguishability of the elements in quantum mechanics (resulting directly from the uncertainty relation). This is by far the most important effect.

When counting the number of arrangements of N elements in quantum statistics, we assume:

1.The elements are indistinguishable.

2. For Bose–Einstein statistics, any number of elements in the same state are allowed. For Fermi–Dirac statistics, at most 1 element in each state is allowed.

The Boltzmann Law

$$N_i = \frac{g_i}{e^{(\epsilon_i - \mu)/kT}} = N\frac{g_i e^{-\epsilon_i/kT}}{Z}$$

1. The elements are distinguishable.

2. Applied to Maxwell's distribution for gas velocity.

Bose–Einstein distribution

$$n_i = \frac{g_i}{e^{(\varepsilon_i - \mu)/kT} - 1}$$

$$f_{BE}(E) = \frac{1}{A\exp(E/k_B T) - 1}$$

1. The elements are indistinguishable, not obeying exclusion principle.

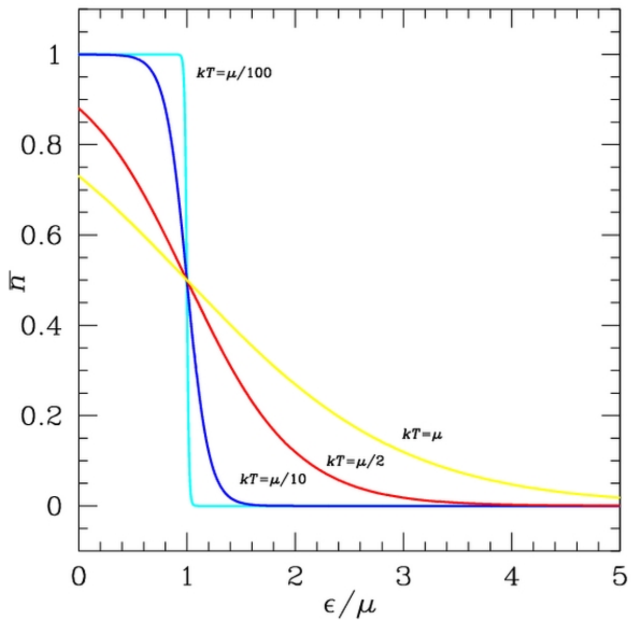
2. Applied to photon and phonon gases.

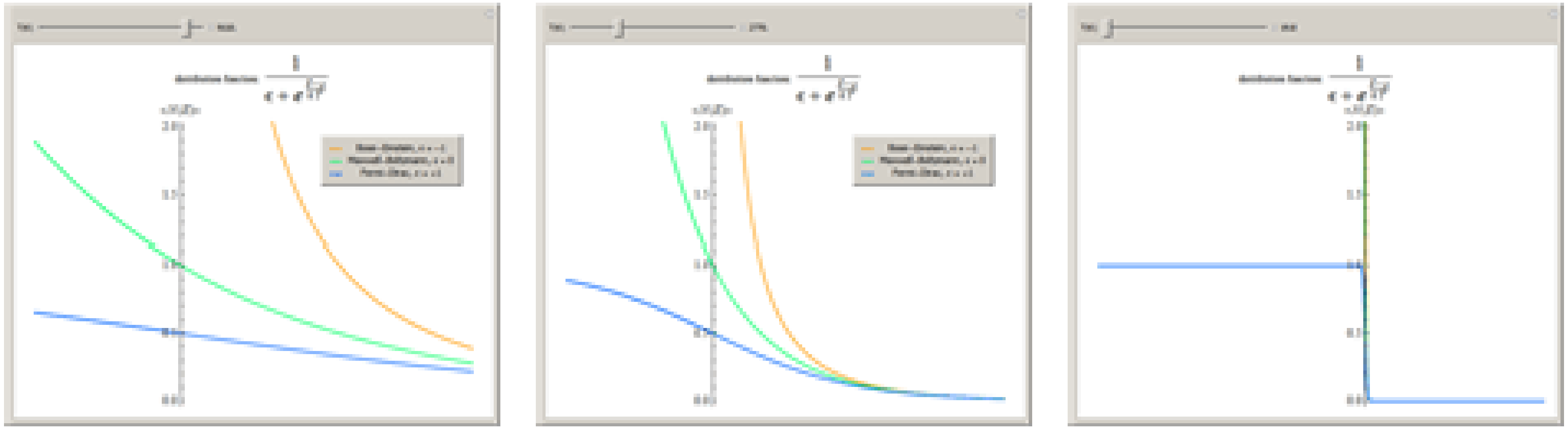
Fermi–Dirac (F–D) distribution

$$\bar{n}_i = \frac{1}{e^{(\epsilon_i - \mu)/kT} + 1}$$

1. The elements are indistinguishable, obeying exclusion principle.

2. Applied to electron gas.





T high T medium T low

B-E

Boltzmann

F-D

Homework#5 (Nov. 23, 2009):

(a)Determine the order of magnitude of the fraction of hydrogen atoms in a state with principle quantum number n = 2 to those in state n = 1 in a gas at 300 K.
(b)Take into account the degeneracy of the states corresponding to quantum numbers n = 1 and 2 of atomic hydrogen and determine at what temperature approximately one atom in a hundred is in a state with n = 2.