Emerging material : Graphene

- Single atomic layer graphite

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LETTERS

Two-dimensional gas of massless Dirac fermions in graphene

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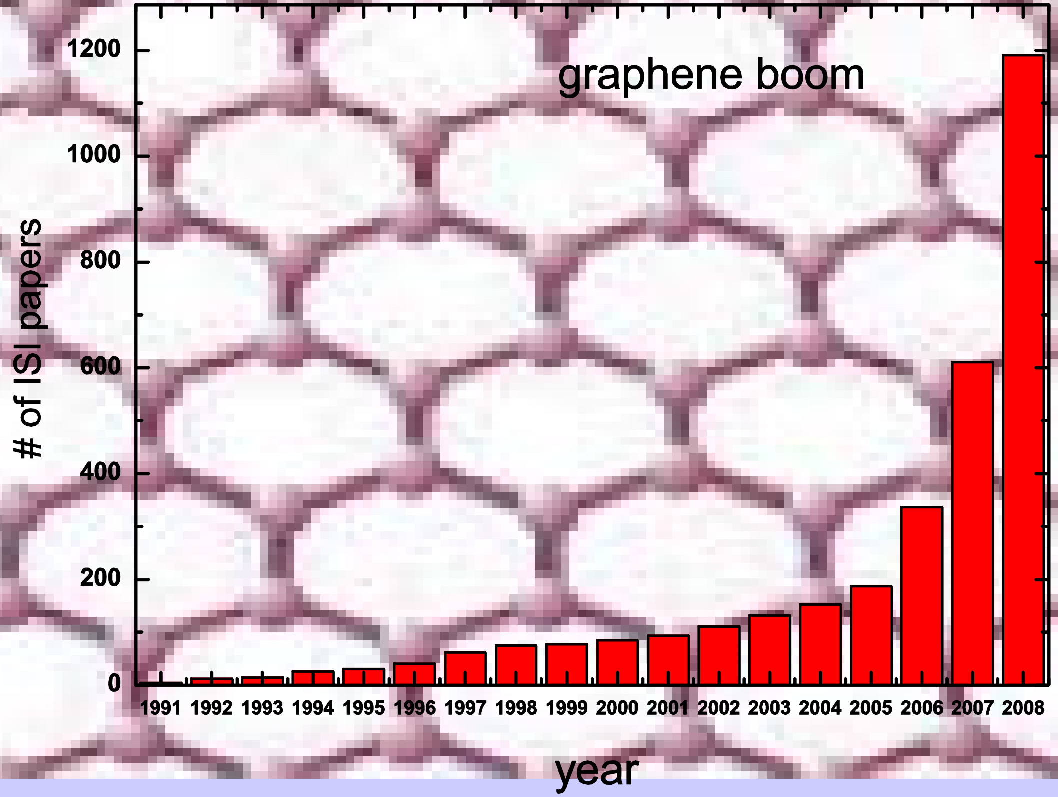
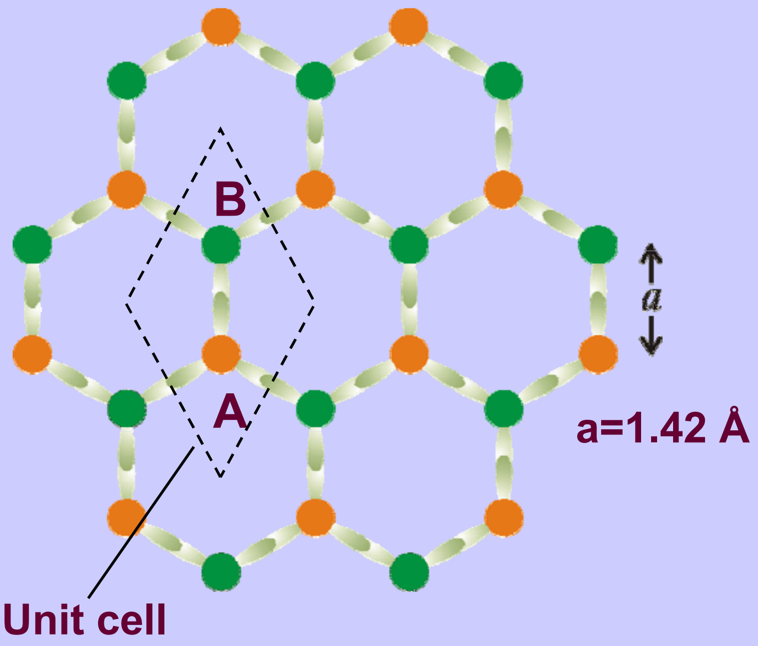


- ISI citation number : 972 (since 2005)
- Graphene focus session in APS March meeting 2007!

Nature podcast

Basics of graphene



Honeycomb structure

- Condensed-matter systems usually described accurately by the Schrödinger equation.

- Electron transport in graphene is governed by Dirac's (relativistic) equation.

- Charge carriers in graphene mimic relativistic particles with zero rest mass and effective speed of light $v_F \approx 10^6$m/s.

- Variety of unusual phenomena associated with massless Dirac fermions.

Dispersion relation

Dirac's (relativistic) Hamiltonian

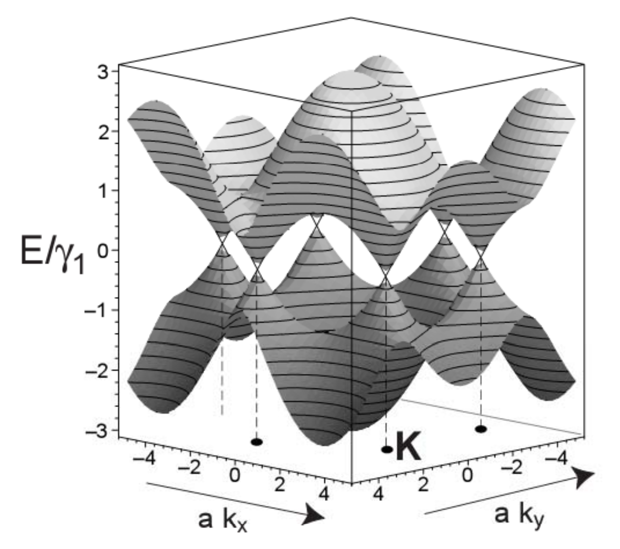
$$\mathrm{H} = \hbar \upsilon_{\mathrm{F}} \vec{\sigma} \cdot \vec{k} = \hbar \upsilon_{\mathrm{F}} \begin{pmatrix} 0 & k_x - ik_y \\ k_x + ik_y & 0 \end{pmatrix}, \;\; \mathrm{H} \begin{pmatrix} \psi_{\mathrm{A}} \\ \psi_{\mathrm{B}} \end{pmatrix} = E \begin{pmatrix} \psi_{\mathrm{A}} \\ \psi_{\mathrm{B}} \end{pmatrix}$$

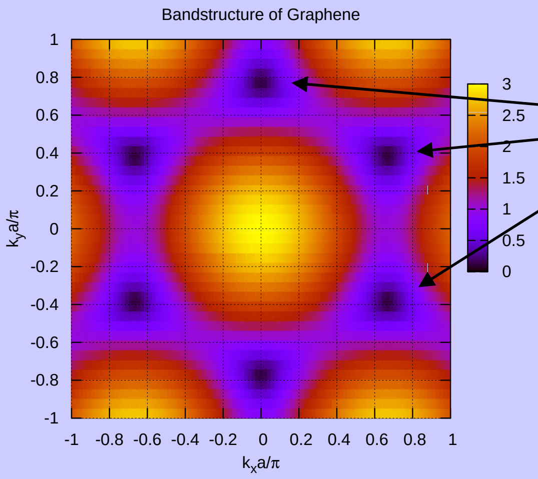
Pauli matrices for the sublattice index (A,B)

"Pseudospin" state

$$E(\vec{k}) = \pm \upsilon_F \hbar \left|\vec{k}\right|, \;\; \upsilon_F \approx 10^6 ms^{-1}$$

Massless quasiparticle



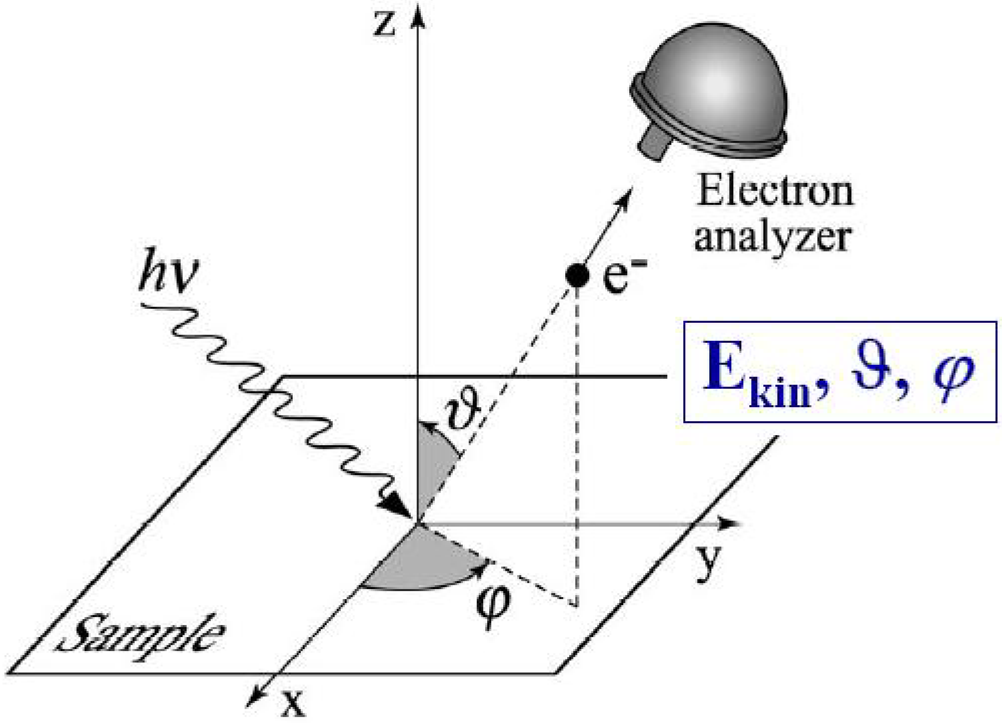


$$E(\vec{q} = \vec{K}) = 0$$

Dirac points

In a perfect graphene sheet μ crosses the Dirac points

Angle-resolved photoemission spectroscopy (ARPES)



$$\mathbf{K} = \mathbf{p}/\hbar = \sqrt{2m\boldsymbol{E_{kin}}}/\hbar$$

$$\begin{aligned} K_x &= \frac{1}{\hbar}\sqrt{2mE_{kin}}\sin\vartheta\cos\varphi \\ K_y &= \frac{1}{\hbar}\sqrt{2mE_{kin}}\sin\vartheta\sin\varphi \\ K_z &= \frac{1}{\hbar}\sqrt{2mE_{kin}}\cos\vartheta \end{aligned}$$

- **E_{kin}, K can be measured in UHV**
- **Conservation law : $E_{kin}=h\nu - \varphi - E_B$**

  $k_f - k_i = k_\nu$
- **E_B and k in solid can be determined**

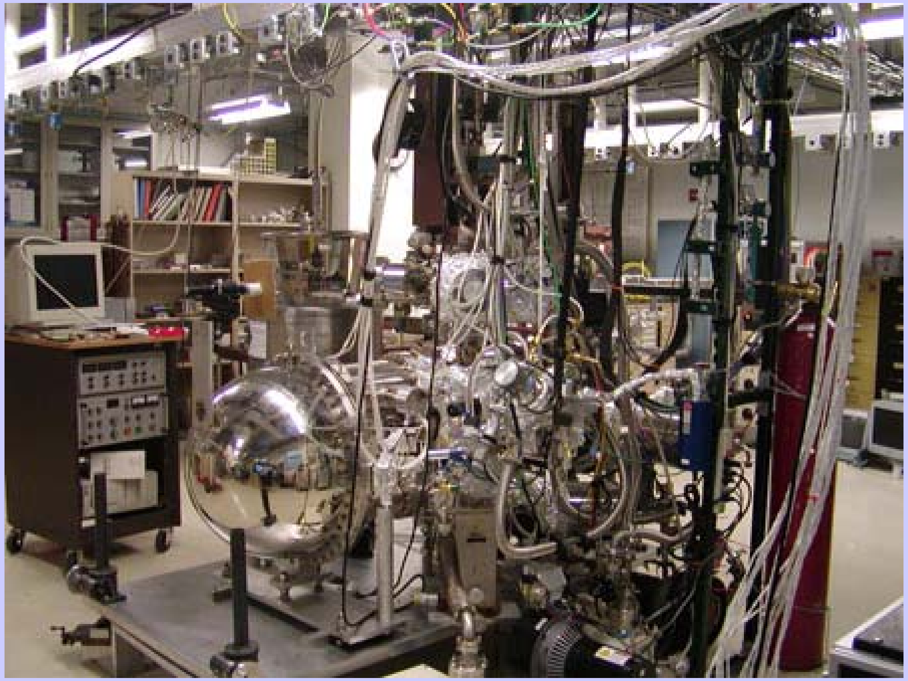
  ⇨ direct probe for dispersion relation in solids

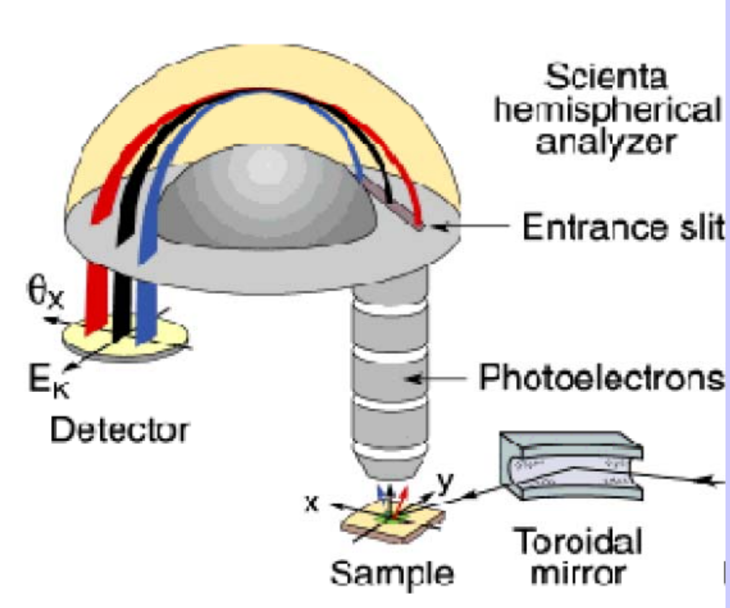
Angle-resolved photoemission spectroscopy (ARPES)

- State-of-art apparatus :

  2meV energy resolution and 0.2 degree angular resolution

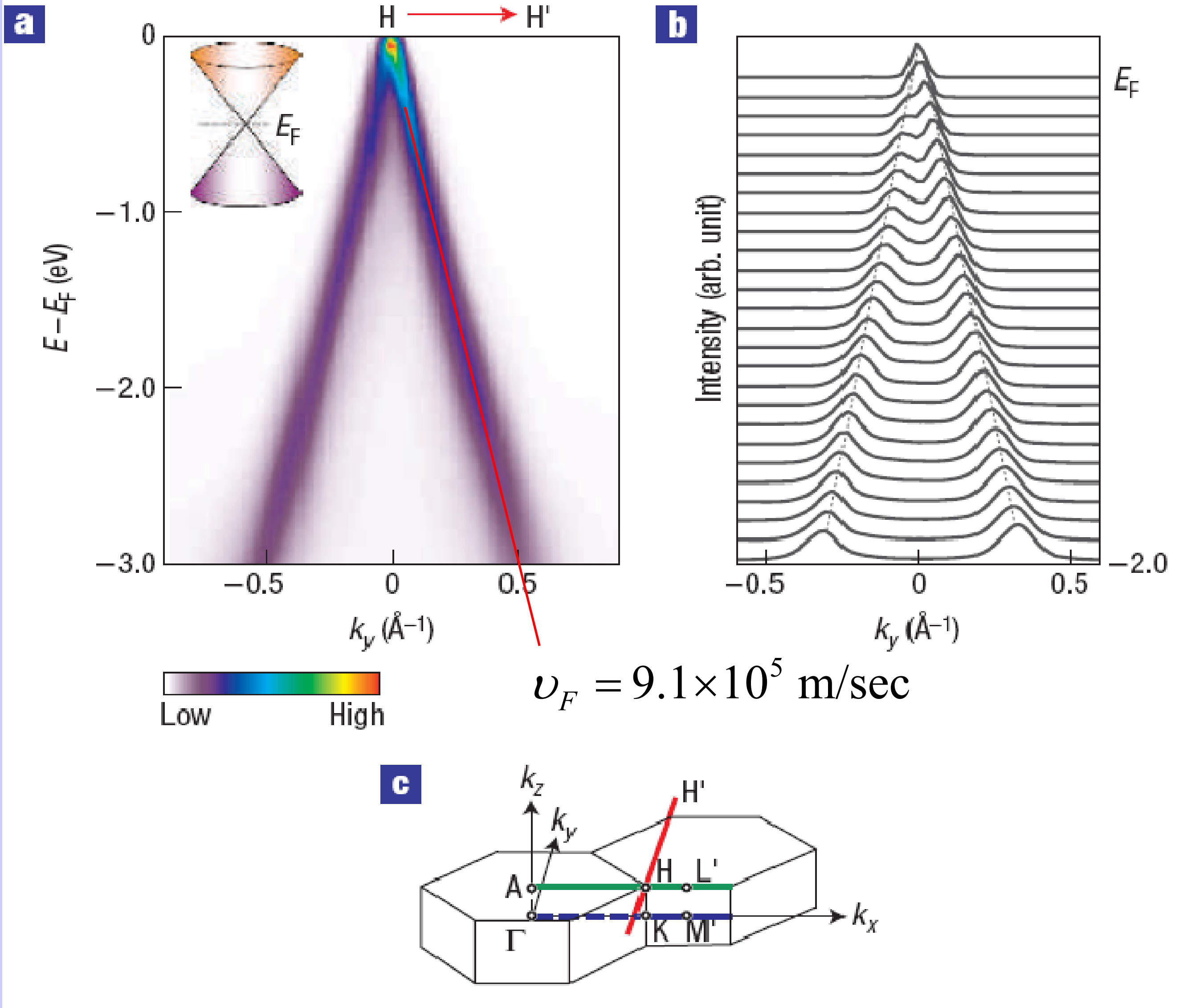
- Surface sensitive : only surface electrons carry inherent information without suffering complicated scattering





ARPES at Shen’s group at Stanford Univ.

Direct observation of Dirac Fermions



$$\upsilon_F = 9.1\times 10^5 \text{ m/sec}$$

J. Gonzalez, F. Guinea, M.A.H. Vozmediano, PRB ***59,*** *R2474 (1999)*

Electron-electron interactions

- How effective the screening of interactions in graphene?

In normal metal (Thomas-Fermi theroy),

$$\text{Potential} \sim \frac{1}{\mathrm{r}} e^{-k_0 r} \quad \text{(Yukawa potential)}$$

In graphene, DOS(E_F)=0 ⇒ *Interactions imperfectly screened*

- *Marginal* Fermi Liquid behavior

At T=0 K, the quasiparticle lifetime at low energies scales as

$$\tau_E \sim (E - E_F)^{-1}$$

Confirmed experimentally (ARPES): *S. Xu et al., PRL* ***76****, 483 (1996)*

[Usual Fermi Liquid scales as $\tau_E \sim (E-E_F)^{-2}$]

*Peres, Guinea, Castro-Neto, PRB **73,** 125411 (2006)*

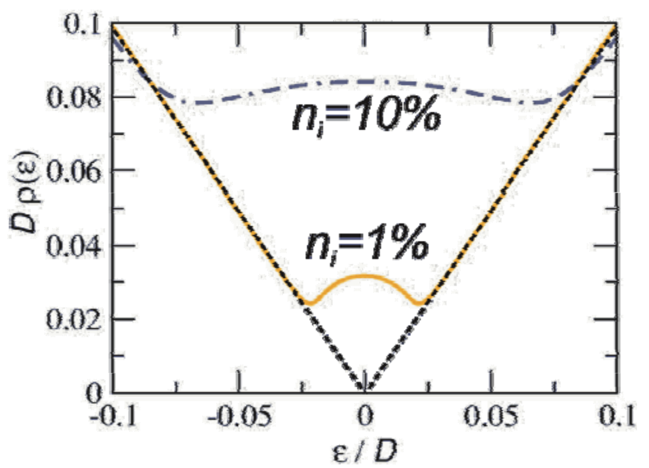
Disorder effect

- ✷ Long-range carbon order in graphene only possible at T=0K (Hohenberg-Mermin-Wagner theorem). At finite T: topological defects always present.
- ✷ Defects cannot be annealed away in 2D honeycomb lattice ("Kinetically constrained system": remnant disorder scales logarithmically with annealing time.)
- ✷ Still long electronic mean free paths (mobility $\mu > 10^4$ cm^2/V-sec)

So, what is the role of disorder?

A finite density of local defects give rise to a impurity band around E_F

n_i : impurity density



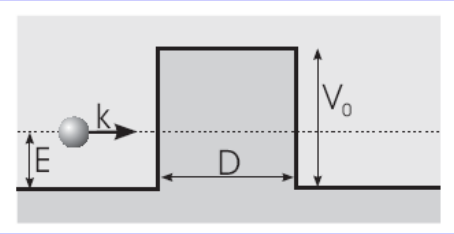
- Disorder modelled by long range (Coulomb) screened scatterers leads to:

$$\sigma_{\min} \approx c \cdot \left[\frac{e^2}{h}\right],$$

from experiment, $c \approx 4.$

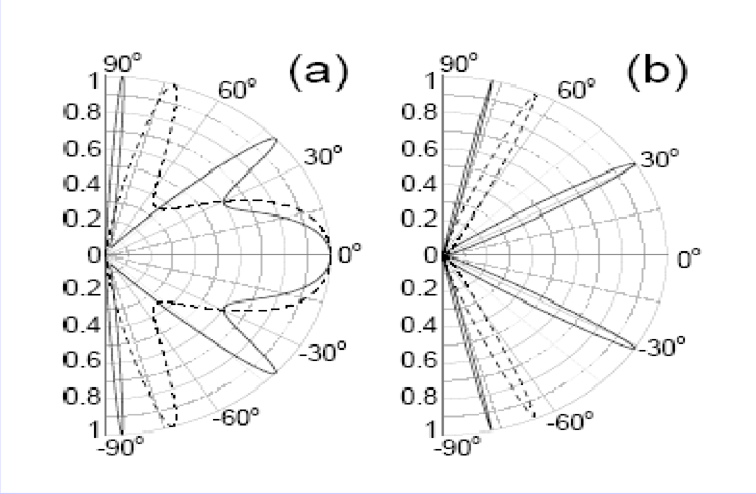
M. I. Katnelson, K. S. Novoselov, and A. K. Geim, cond-mat/0604323

Klein paradox in graphene



Klein paradox: **unimpeded penetration of relativistic particles through high and wide potential barriers - 1930**

Barrier *always* transparent for angles close to normal incidence !!



Monolayer

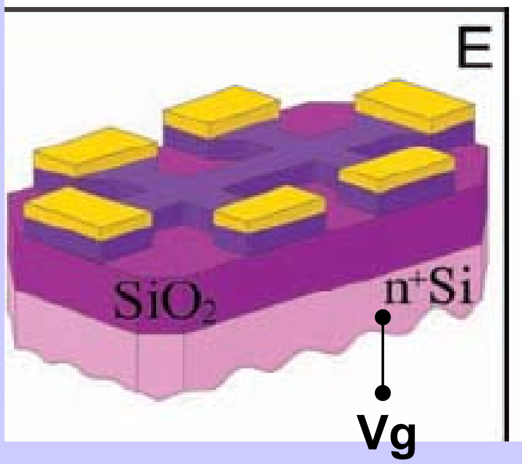
Bilayer

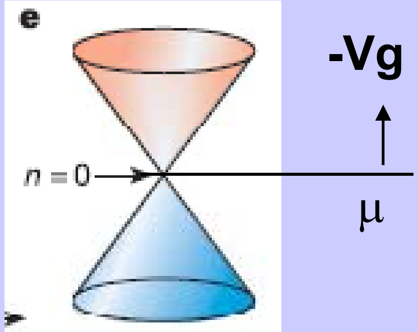
Massive fermions are reflected close to normal incidence!

Impurity scattering in the bulk of graphene is strongly suppressed !!!

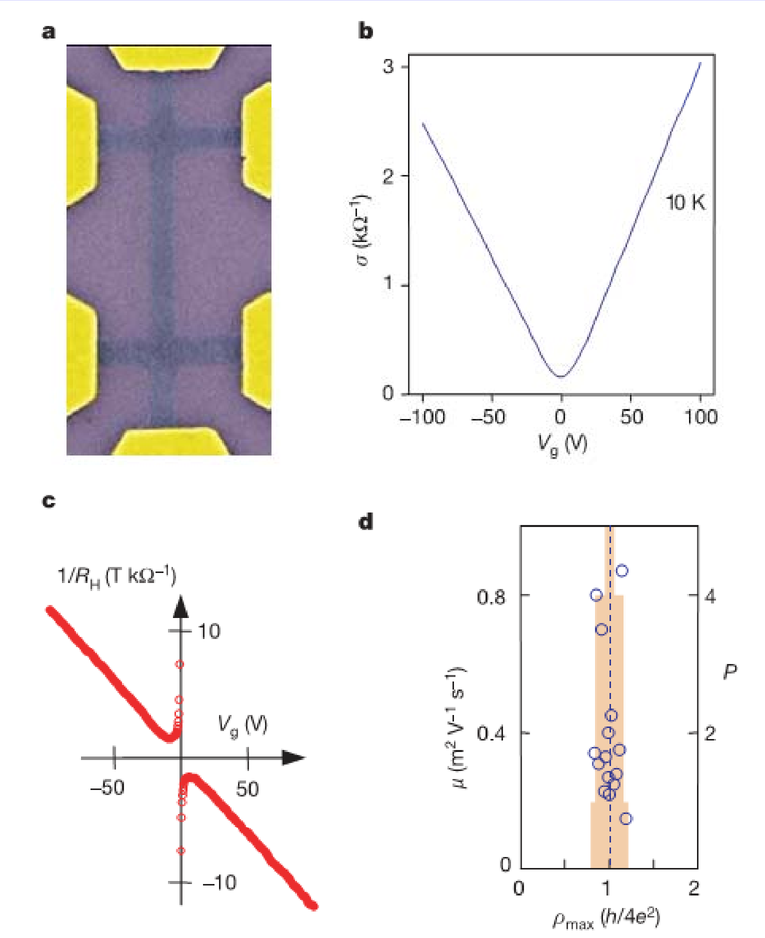
Transport in Graphene

Novoselov, et al., Science 04', Nature 05'





Electric field effect in graphene

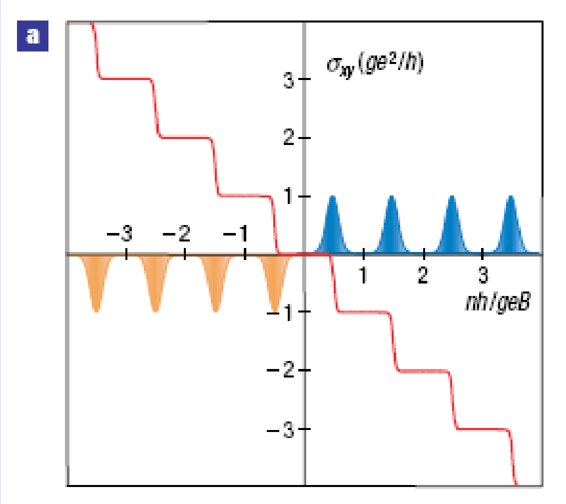


- Chemical potential tuned by $V_g \sim n_c$
- Ambi-polar field effect
- Robust minimal conductivity ?

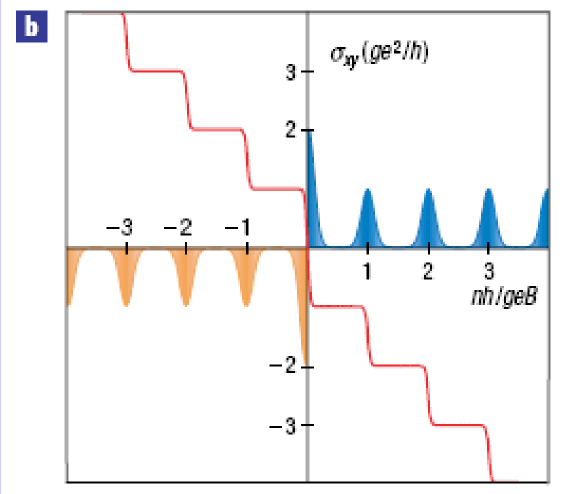
$\sigma_{min} = 4e^2/h$, at Dirac point

Integer QHE in Graphene

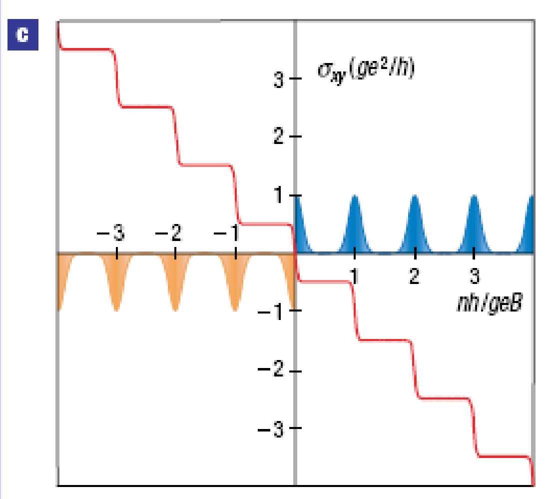
Novoselov, et al., Nat. Phy. 06’



2-DEG
free-Fermion



Bilayer graphene
Berry’s phase 2π



Single-layer graphene
Berry’s Phase π

- **For a given B, D.O.S. at each Landau level = gB/Φ_0**
- **Anomaly at lowest Landau level in graphene**
- **Internal field (Berry’s phase) ⇨ non-zero QHE in zero external field***

* Haldane, et al., PRL 88’

Basic formalism of Berry's phase

Berry, PRSLA'84

Hamiltonian $H(\vec{R})$

$$H\left|n(\vec{R})\right\rangle = E_n\left|n(\vec{R})\right\rangle$$

Adiabatic change in $\vec{R}$,

$$\left|\psi(\mathrm{t})\right\rangle = e^{i\gamma_n}\left[e^{-i\int_0^t E_n dt'}\left|n(\vec{R})\right\rangle\right]$$

γ_n can be determined by requiring

$$H(\vec{R})\left|\psi(\mathrm{t})\right\rangle = i\hbar\frac{\partial}{\partial t}\left|\psi(\mathrm{t})\right\rangle$$

Along a closed path C in $\vec{\mathrm{R}}$ space

$$\boxed{\gamma_n(C) = \int_C X(\vec{R})\cdot d\vec{R}, \quad X(\vec{R}) \equiv \left\langle n(\vec{R})\right| i\nabla_R \left| n(\vec{R})\right\rangle}$$

Berry's phase

Berry's vector potential

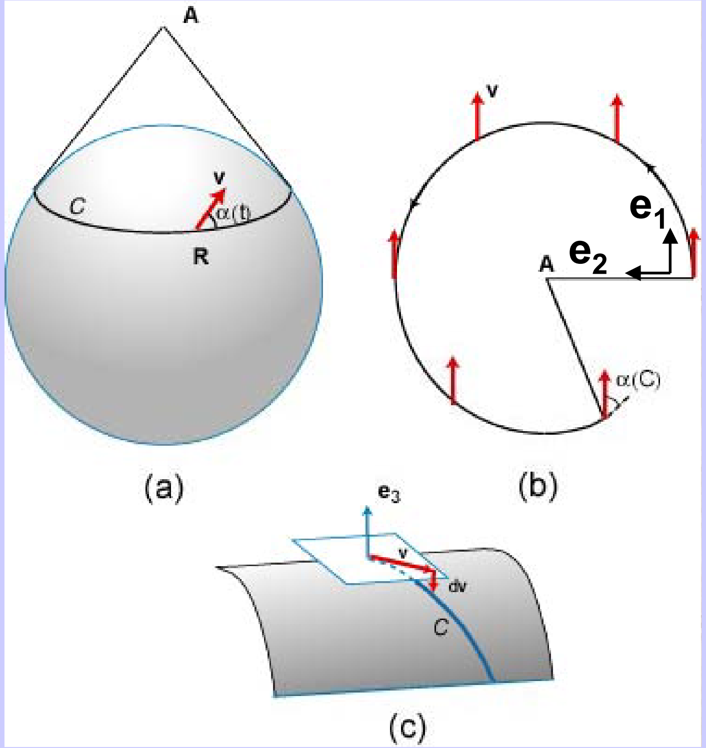
" remarkable and rather mysterious results "

- Berry 1983

" ….. is essentially that of the holonomy which is becoming quite familiar to theoretical physicists "

- Simom 1983

Parallel transport of vector v on curved surface



Constrain **v in local tangent plane;**
no rotation about $\mathbf{e}_3$
$[\mathbf{e}_1, \mathbf{e}_2]$: local tangent plane

Parallel transport

$$\boxed{\mathbf{e}_3 \times d\mathbf{v} = 0}$$

v acquires geometric angle α relative to local $\mathbf{e}_1$

complex vectors $\hat{\boldsymbol{\psi}} = (\mathbf{v} + i\,\mathbf{w}) / \sqrt{2}$ $\hat{\mathbf{n}} = (\mathbf{e}_1 + i\,\mathbf{e}_2) / \sqrt{2}$

angular rotation is a phase $\hat{\boldsymbol{\psi}} = \hat{\mathbf{n}}\, e^{i\alpha}$

$$\boxed{d\alpha = -\hat{\mathbf{n}} \cdot i d\, \hat{\mathbf{n}}}$$

cf. $X(\vec{R}) \equiv \left\langle n(\vec{R}) \middle| i\nabla_R \middle| n(\vec{R}) \right\rangle$

Berry's phase and Geometry

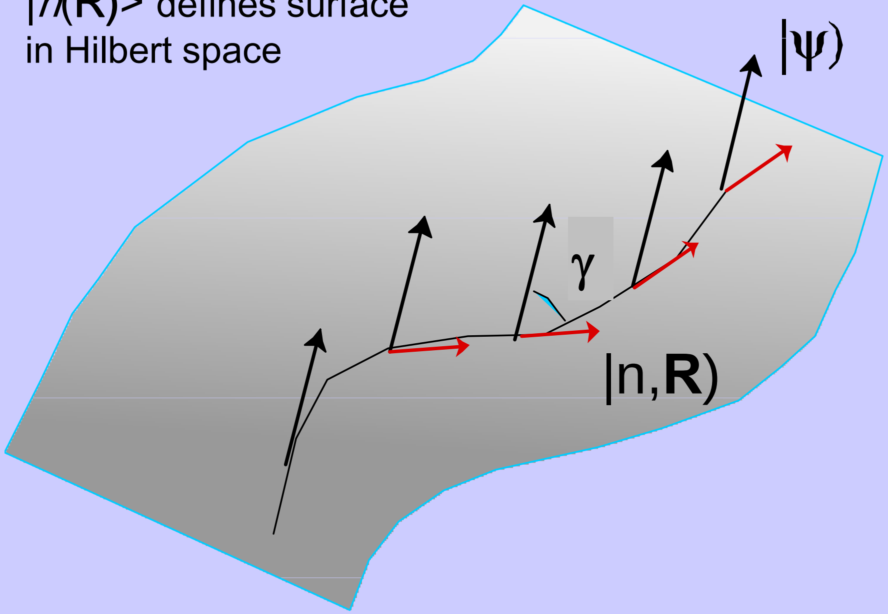
Change Hamiltonian H (**R**) by evolving **R**(t) adiabatically

Constrain particle to remain in one state $|n(\mathbf{R})>$

Simon, PRL '83

Ong and Lee, cond-matt '05

|*n*(R)> defines surface in Hilbert space



$$|\psi\rangle = |n(\mathrm{R})\rangle\, \mathrm{e}^{i\gamma}$$

wavefcn, *evolving on* surface |n R), acquires Berry phase γ

$$\gamma = \int d\,\mathrm{R}\cdot \mathrm{X}(\mathrm{R})$$

(holonomy)

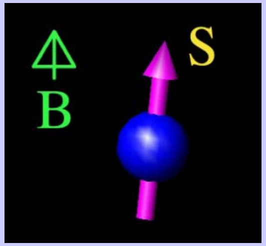
$$\boxed{\mathrm{X}(\mathrm{R}) \equiv < n(\mathrm{R})\,|\,i\nabla_{\mathrm{R}}\,|\,n(\mathrm{R}) >}$$

⇨ **Berry vector potential**

$$\mathbf{\Omega}(\mathrm{R}) \equiv \nabla_{\mathrm{R}} \times \mathrm{X}(\mathrm{R})$$

⇨ **Berry curvature**

A particle with spin s in magnetic field



Hamiltonian

$$H(\vec{B}) = -g\mu_B \vec{s}\cdot\vec{B}, \text{ with eigenvalues } E_n = g\mu_B B n \quad (n = -s, -s+1, .. + s)$$

$$H(\vec{B})\left|n(\vec{B})\right\rangle = E_n\left|n(\vec{B})\right\rangle,$$

Berry's curvature

$$\boldsymbol{\Omega}_n(\vec{B}) = \nabla_{\vec{B}} \times \left\langle n(\vec{B})\right| i\nabla_{\vec{B}} \left|n(\vec{B})\right\rangle = \mathrm{Im}\sum_{m\neq n} \frac{\left\langle n(\vec{B})\right|\nabla_{\vec{B}} H\left|m(\vec{B})\right\rangle \times \left\langle m(\vec{B})\right|\nabla_{\vec{B}} H\left|n(\vec{B})\right\rangle}{(E_n - E_m)^2}$$

With $\nabla_{\vec{B}} H = g\mu_B \vec{s}$,

$$\boxed{\boldsymbol{\Omega}_n(\vec{B}) = n\,\vec{B}/B^3}$$

Gauge field results from a monopole n at the origin of ***B*** space

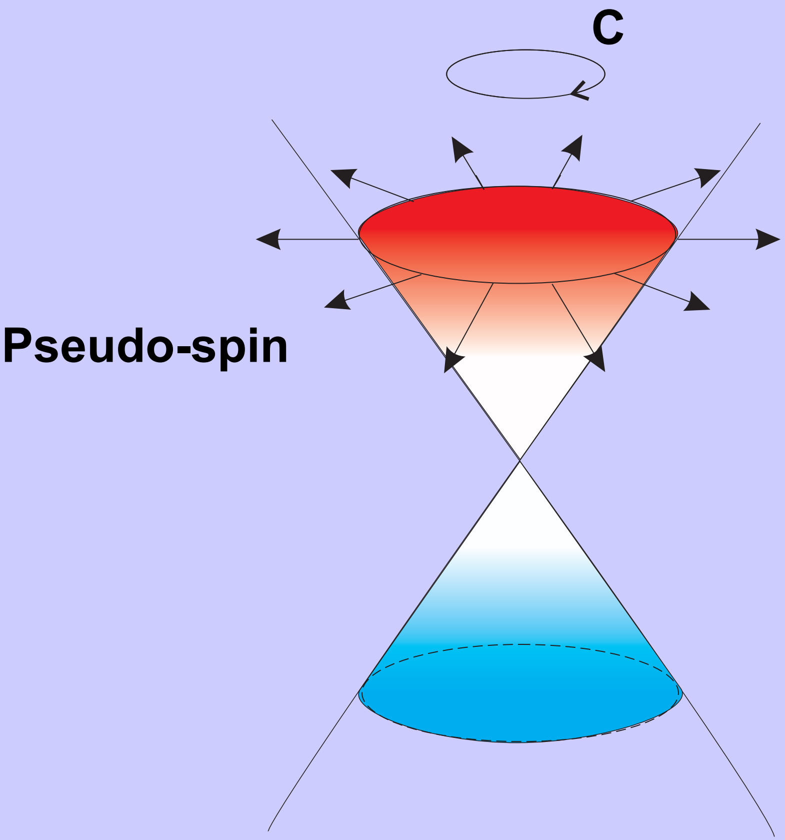
Berry's phase with adiabatic variation of $\vec{B}$ around a loop C

$$\boxed{\gamma_n(C) = -\iint_C \boldsymbol{\Omega}_n(\vec{B})\cdot d\vec{S} = -n\,\Omega(C)}$$

Gauge flux through the loop C

Solid angle that *C subtends at origin*

Massless Dirac Fermion and π Berry's phase

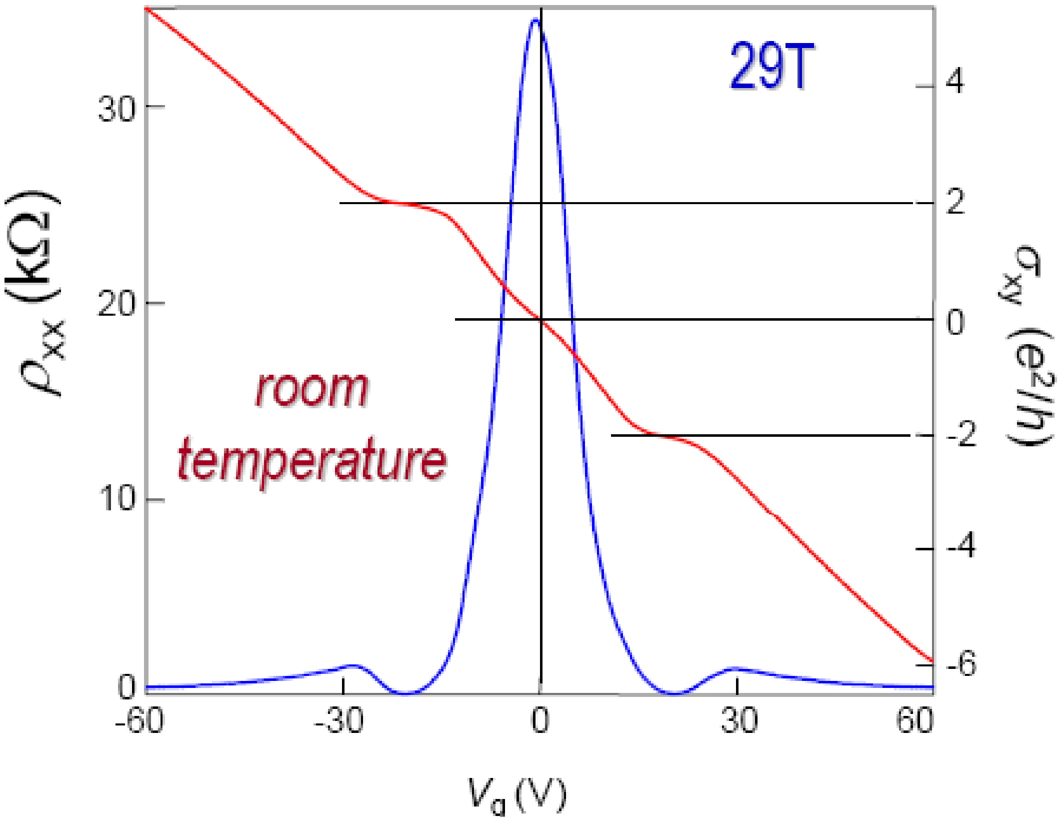


- Pseudospin eigenstate along $\vec{k}$
- Closed contour C in k space associated with cycltron path
- Berry's phase acquired along path C

$$\gamma(\mathrm{C}) = -\iint_C \mathbf{\Omega}\cdot d\vec{S} = -\frac{1}{2}\Omega(C) = -\pi$$

Solid angle

Quantum Hall effect at room temperature !



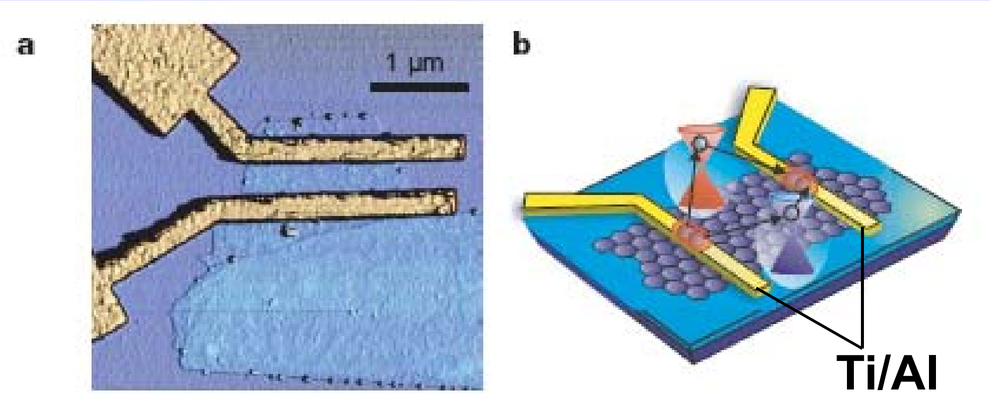
$$E_n = sign(n)(2n)^{1/2}\frac{\hbar\upsilon_F}{\ell_B}$$

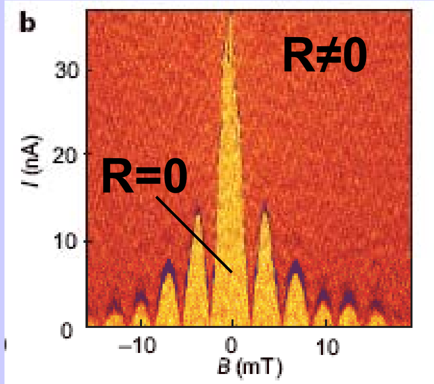
$$\text{For B} = 29\ \text{Tesla}, \mathrm{E}_1 - \mathrm{E}_0 = 0.196\text{eV} = 2271\text{K !!}$$

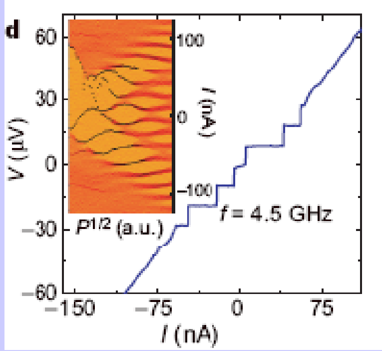
$$cf: \quad E_n = (n+\frac{1}{2})\hbar\omega_C,\ \Delta E = \hbar\omega_C = 3.36meV = 39K$$

S/Graphene/S Josephson Junction

Heersche, et al., Nature 07’





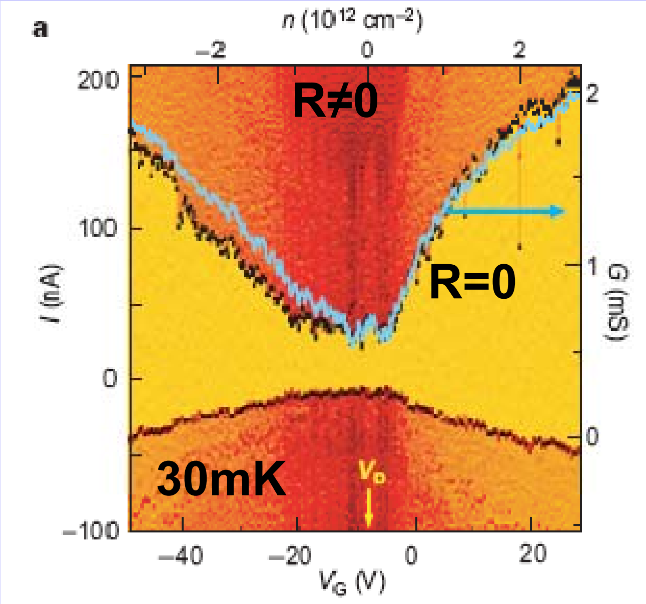


Shapiro steps

- S electrodes spaced by graphene
- DC and AC Josephson effect
- Phase coherent transport at Dirac point

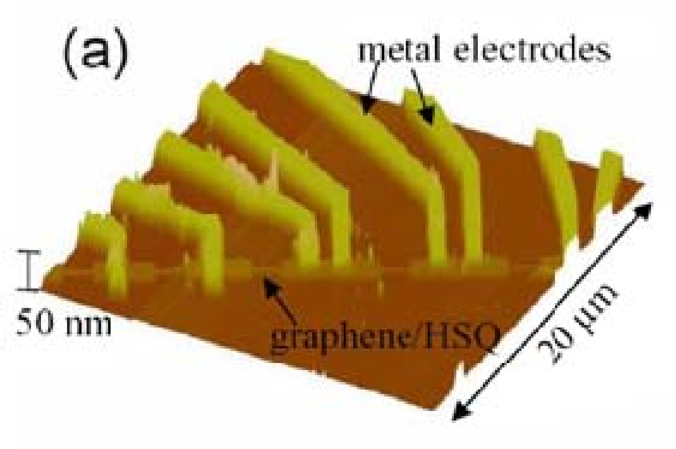
DC Josephson :

$$I_C \propto \frac{\sin(\pi\Phi/\Phi_0)}{\pi\Phi/\Phi_0},\ \Phi = \text{total magnetic flux}$$



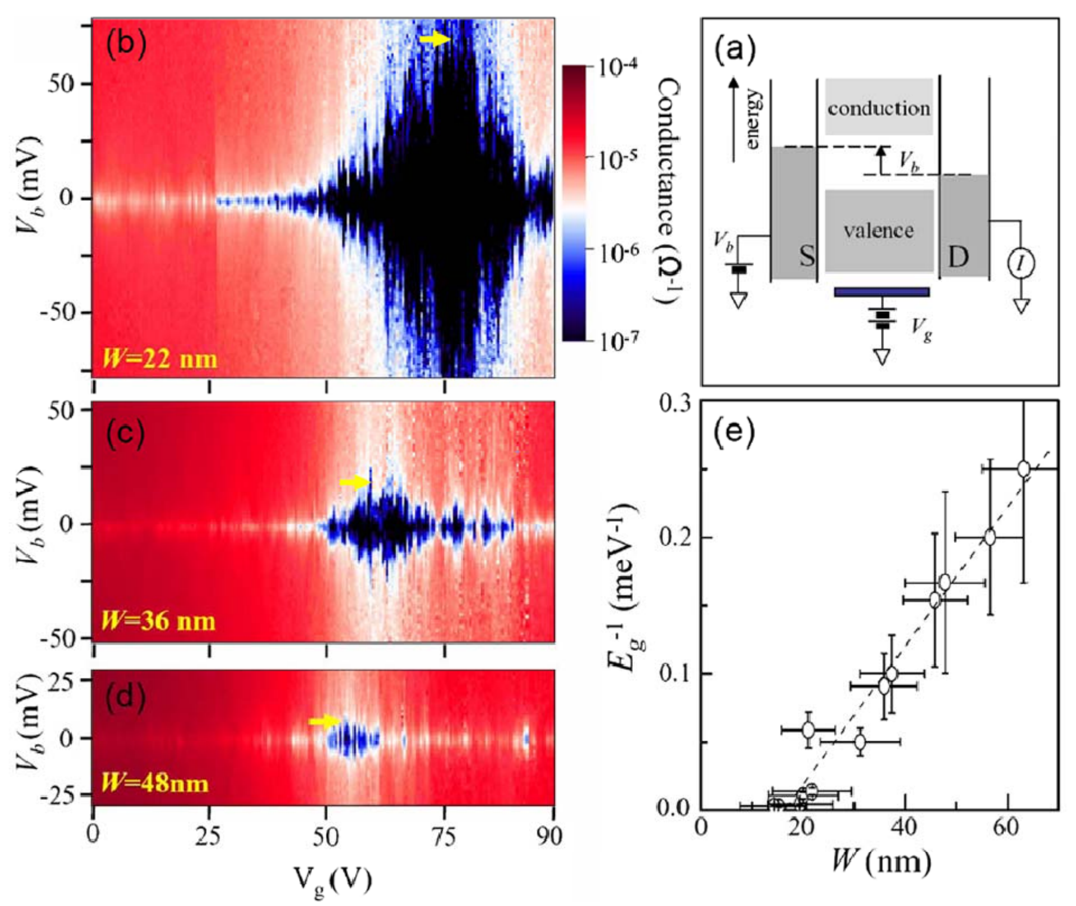
Graphene nano-ribbon : Energy gap engineering

- Gap opening due to quasi-1D confinement of the carriers



$$E_g = \frac{\alpha}{(W - W^*)}$$

$$\alpha \sim 0.2\,eV \cdot nm, W^* = 16nm$$



Concluding Remarks

- **Massless Dirac Fermion and insensitive to impurity scattering**
- **Marginal Fermi-liquid behaviour**
- **Unavoidable defects and disorder in 2-D graphene**
- **Exhibit robust minimal conductivity and shifted IQHE**
- **Phase coherent transport at the Dirac point**
- **Appear of band gap in graphene nanoribbon**