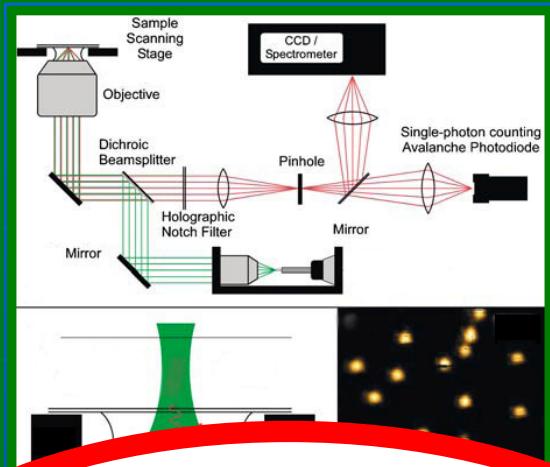
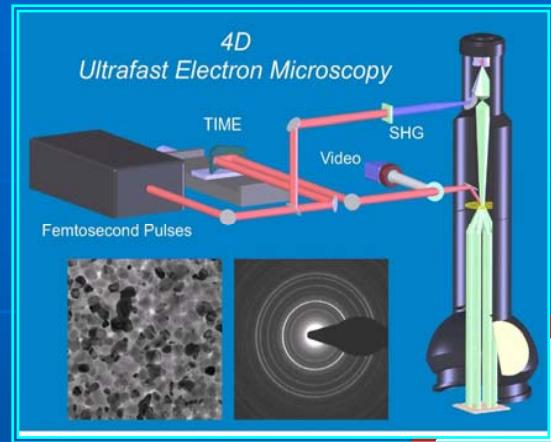


Studies of light-induced stochastic and coherent processes in nano-structured materials using single-molecule techniques and femtosecond time-resolved electron diffraction

Jau Tang 湯 朝 曄

*Research Center for Applied Sciences,
Academia Sinica, Nankang, Taiwan
and*

*Department of Photonics
NCTU, Hsinchu, Taiwan*



Quantum dots, Nanorods,
Biomolecules,
Metallic & Semiconductor films & NCs
Organic conductors, photovoltaics
Light-induced phase transitions



Structure
Dynamics
Functionality



Outline

Time-resolved electron diffraction and microscopy

Impulse-induced structural dynamics & its effects on diffraction patterns

Coherent acoustic wave excitation

Nano-scale heat transfer of laser-heated metals

Light-induced insulator-metal phase transitions

Confocal microscopy and single-molecule techniques

Fluorescence intermittency of illuminated nano-particles

Photo-induced charge transfer

Enzymatic reactions and intermittency of single biomolecules

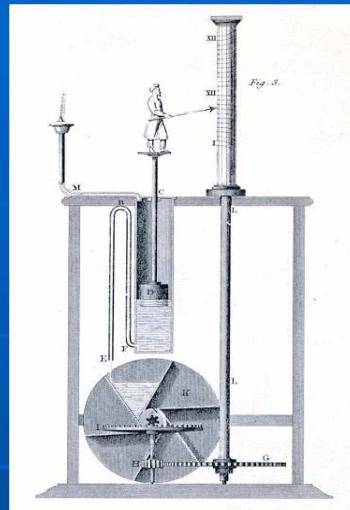
Förster's fluorescence resonance energy transfer

water clock (1500 BC)



Early water clock

Egyptian water clock (300 BC)



Sun dial clock (300 BC)



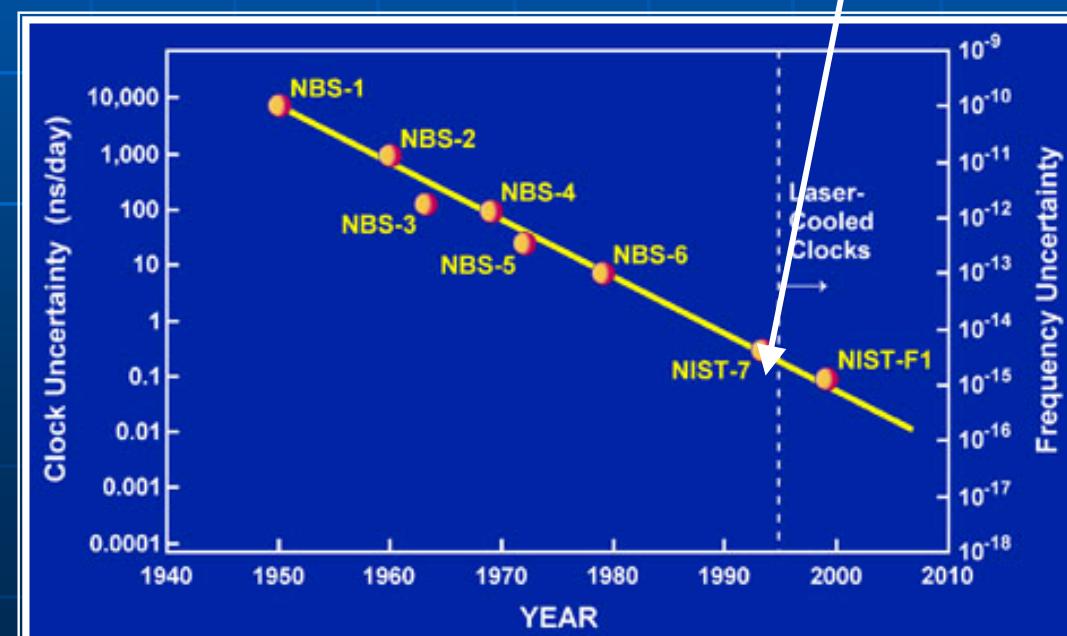
Pendulum clock (1500 AD)

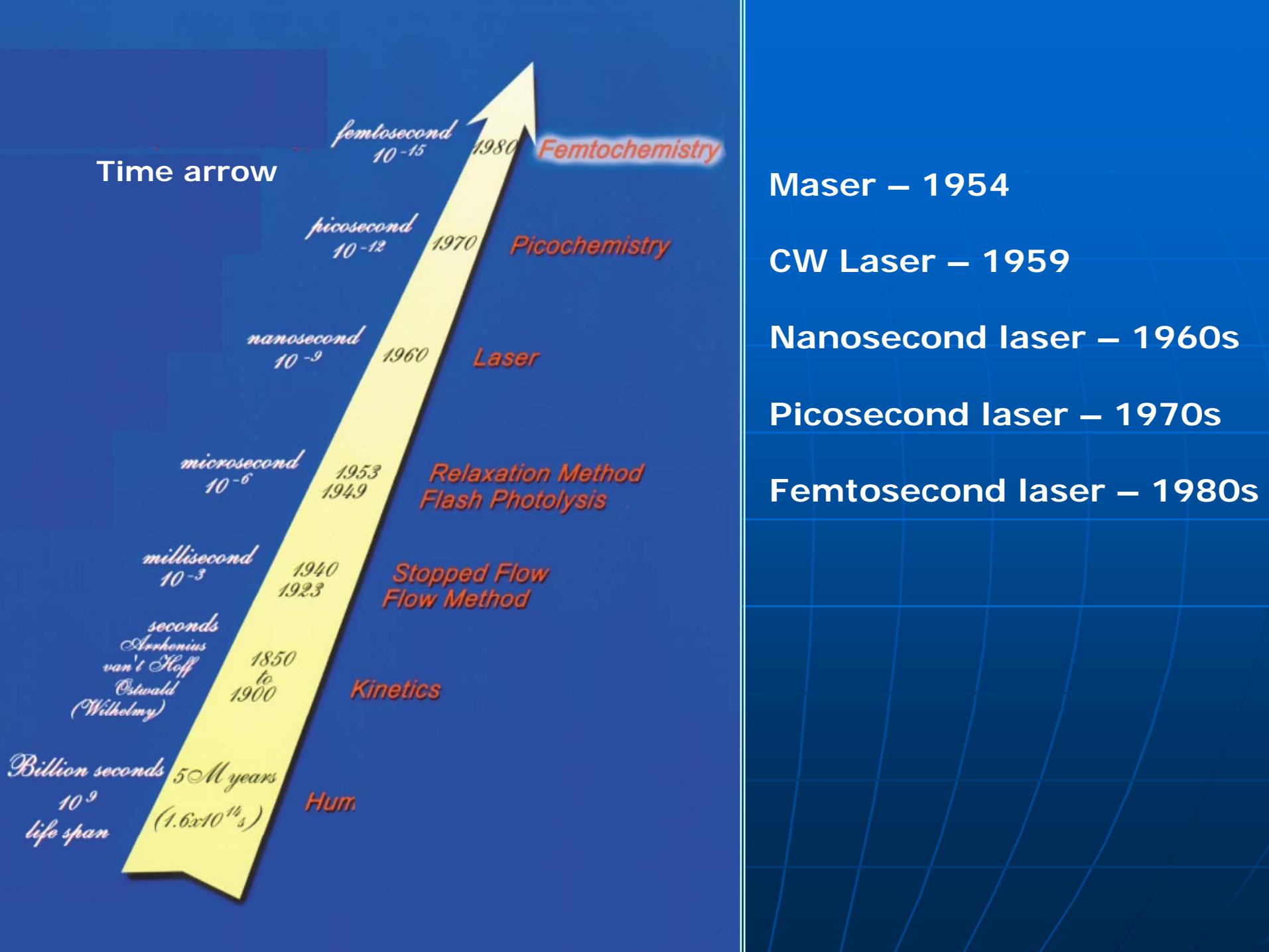
NIST F-1, (cesium fountain clock)



**NIST F-1
Atomic Clock**

accuracy: 10^{-15}





Maser – 1954

CW Laser – 1959

Nanosecond laser – 1960s

Picosecond laser – 1970s

Femtosecond laser – 1980s

A macro, micro and nano view of a human hand, 10 times magnification per frame.

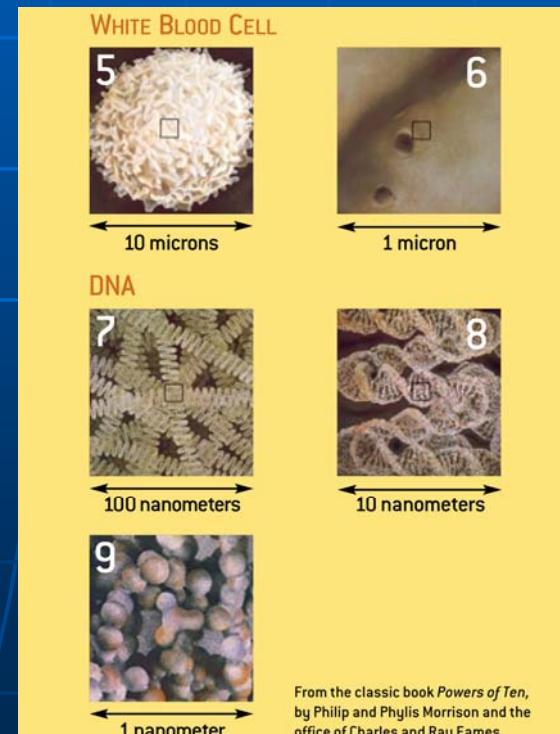


Janssen, Leeuwenhoek
(16th century)



Knott, Ruska
(1930)

(magnifying glasses invented 1st century)



From the classic book *Powers of Ten*,
by Philip and Phylis Morrison and the
office of Charles and Ray Eames.

maximum theoretical resolution
~ 1/2 wavelength

radios ~ miles to mm
infrared ~ mm to 800nm
light ~ 400-800nm
electrons ~ 20kV 0.008 nm

under the best of conditions:

human eye ~ 0.2 mm

light scope ~ 0.2 um

SEM ~ 3 nm

TEM ~ 0.2 nm

- ◆ Electron Microscopes were not developed until the Twentieth Century

Transmission Electron Microscope



- ◆ The first Transmission Electron Microscope (TEM) was built in 1932

Scanning Electron Microscope



- ◆ The first Scanning Electron Microscope (SEM) was built in 1942.

Scanning Tunneling Microscope



- ◆ The Scanning Tunnelling Microscope was developed in 1982.

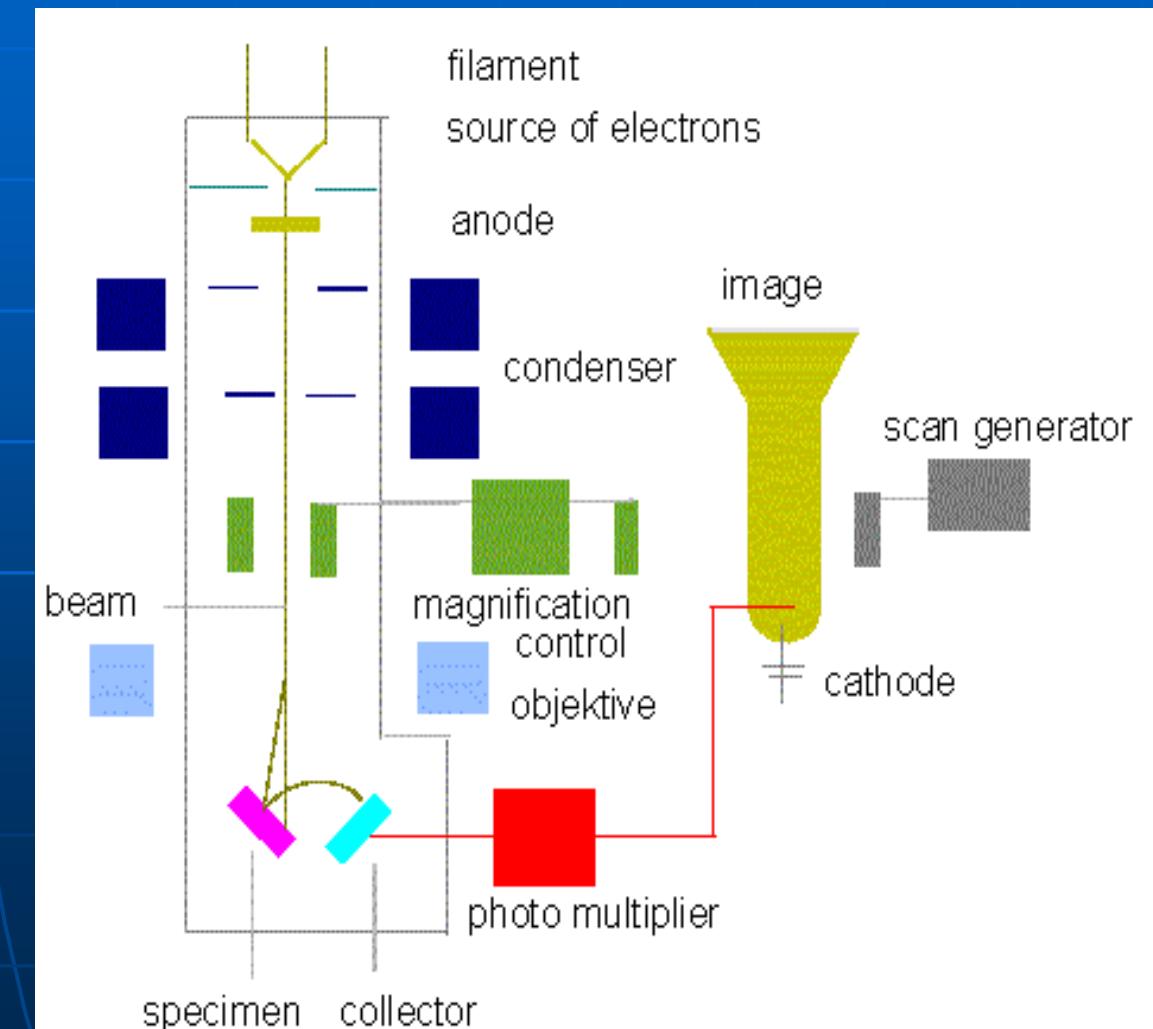
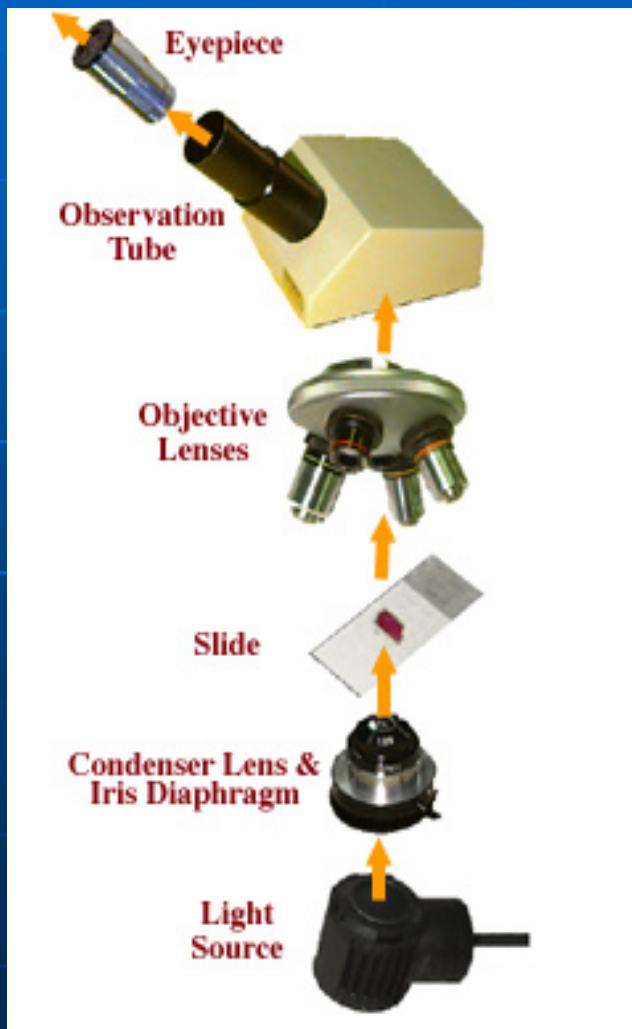
Atomic Force Microscope

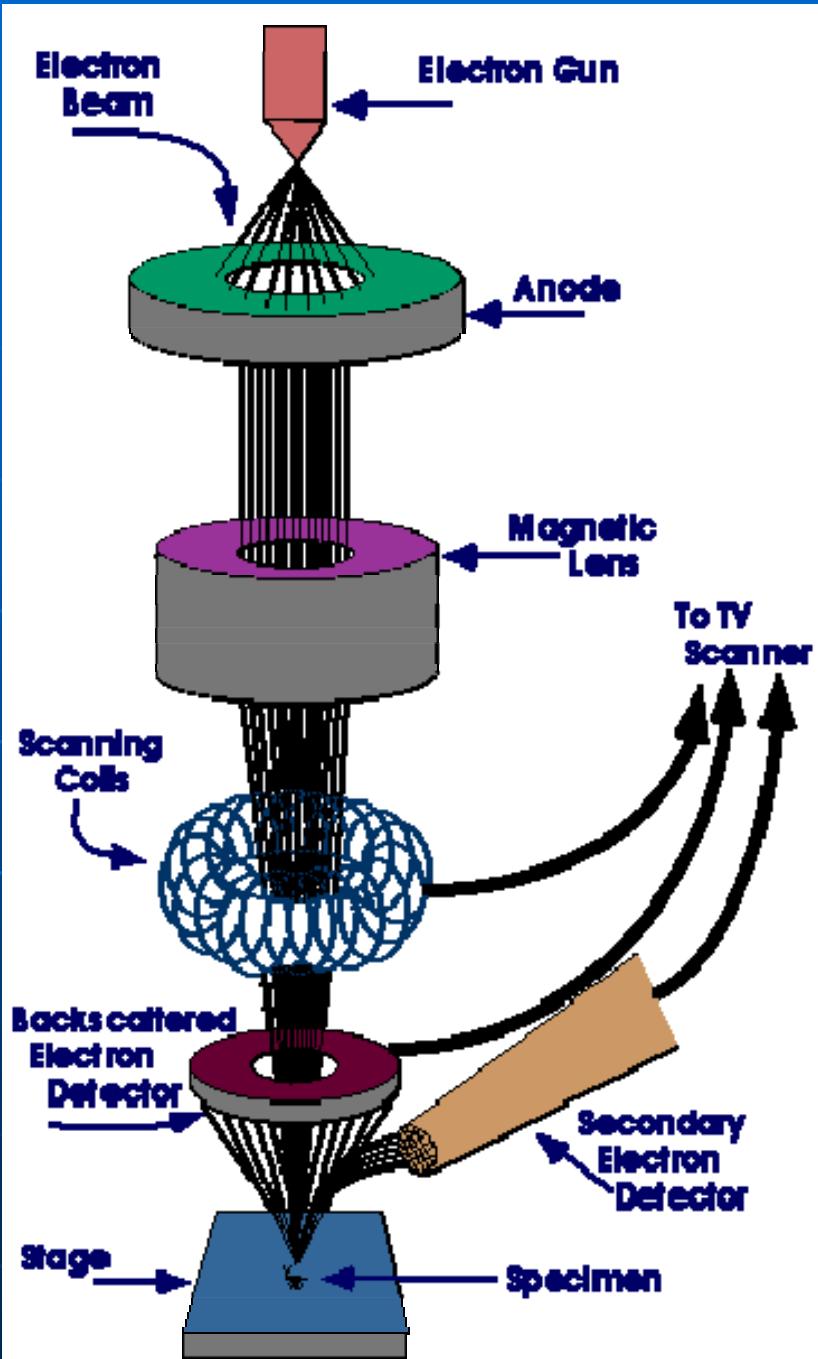


- ◆ The Atomic Force Microscope was developed in 1985.

- Electrons can be focussed using magnetic lenses
- Shorter wavelength than visible light

SEM



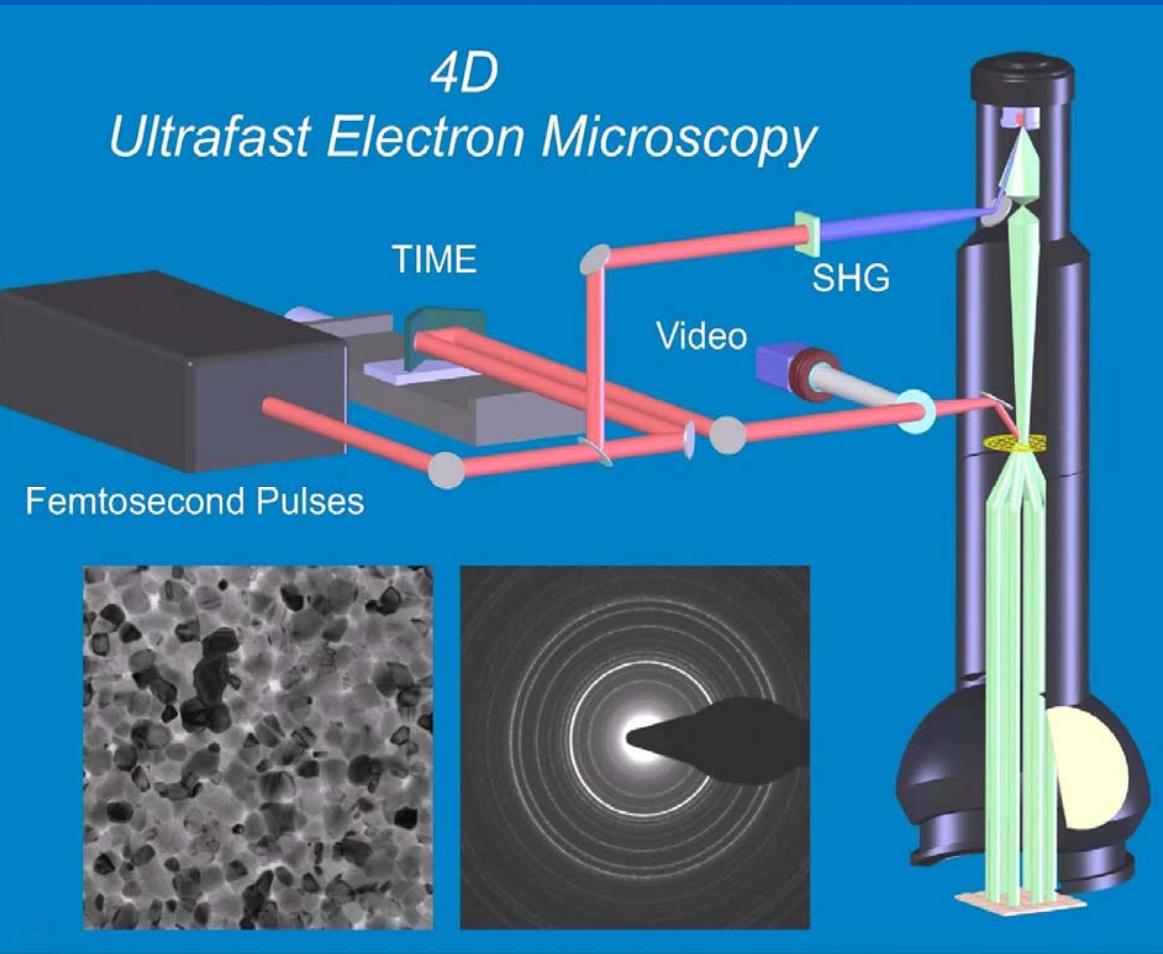


SEM

- Large depth of field
More of the sample is in focus at one time
- Higher resolution
Smaller features can be imaged
- Analysis
The electron beam interacts with the sample enabling information on composition to be collected using additional detectors

Studies of structural dynamics by Ultrafast Electron Diffraction and Microscopy

4D *Ultrafast Electron Microscopy*



nm spatial resolution

fs time resolution

Electron gun cathode
Lanthanum Hexaboride
LaB₆

Low work function ~ 2.5 eV

Melting pt.: 2530 °C

pulsed X-ray diffraction

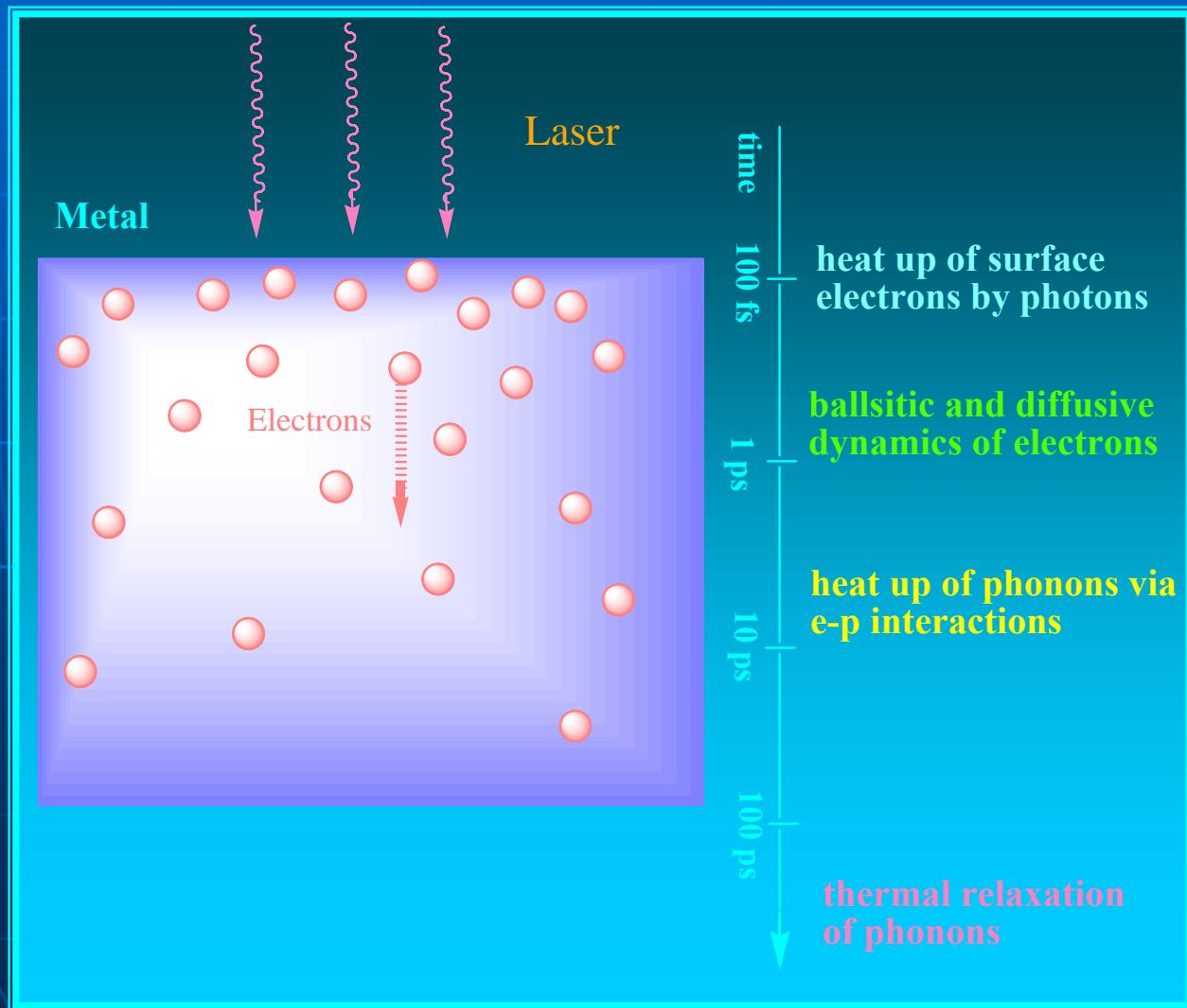
- Hugh synchrotron
- Small scatter cross section
- Deep penetration depth
- Thick and large sample
- Bulk materials

pulsed electron diffraction

- Desk top operation
- Larger (10^6) scatter cross section
- Shallow penetration depth
- Thin and small samples
- nano-structured materials and surface science

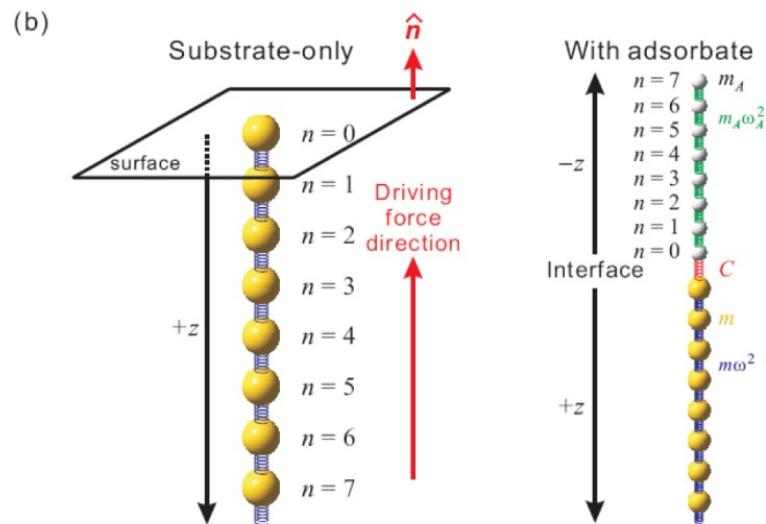
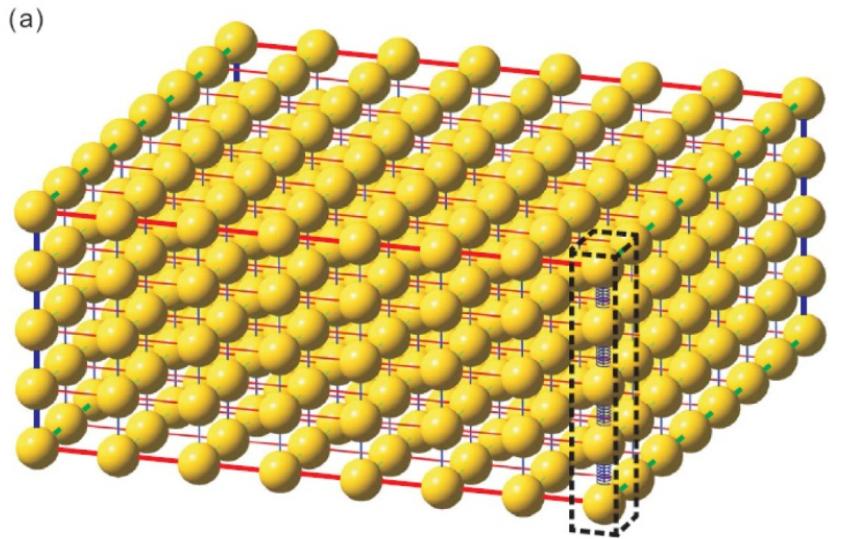
Ultrafast Electron Crystallography of Laser Heated Metals

J. Tang, D.-S. Yang and A. H. Zewail, J. Phys. Chem. C, (2007)

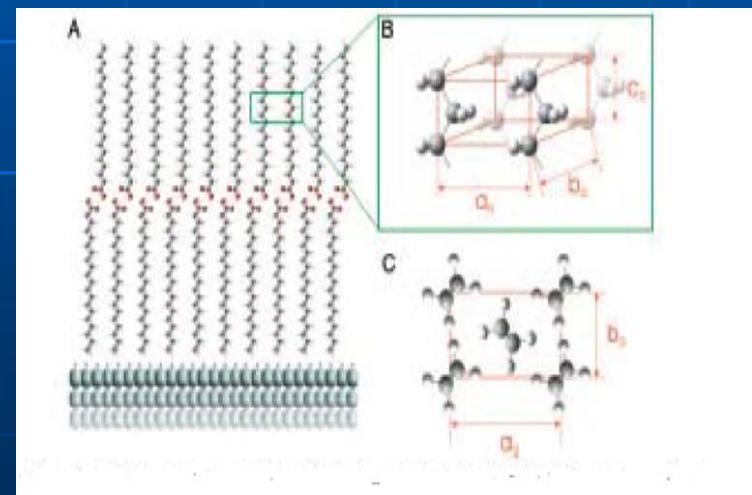


Fermi-Pasta-Ulam model

Anharmonic chain

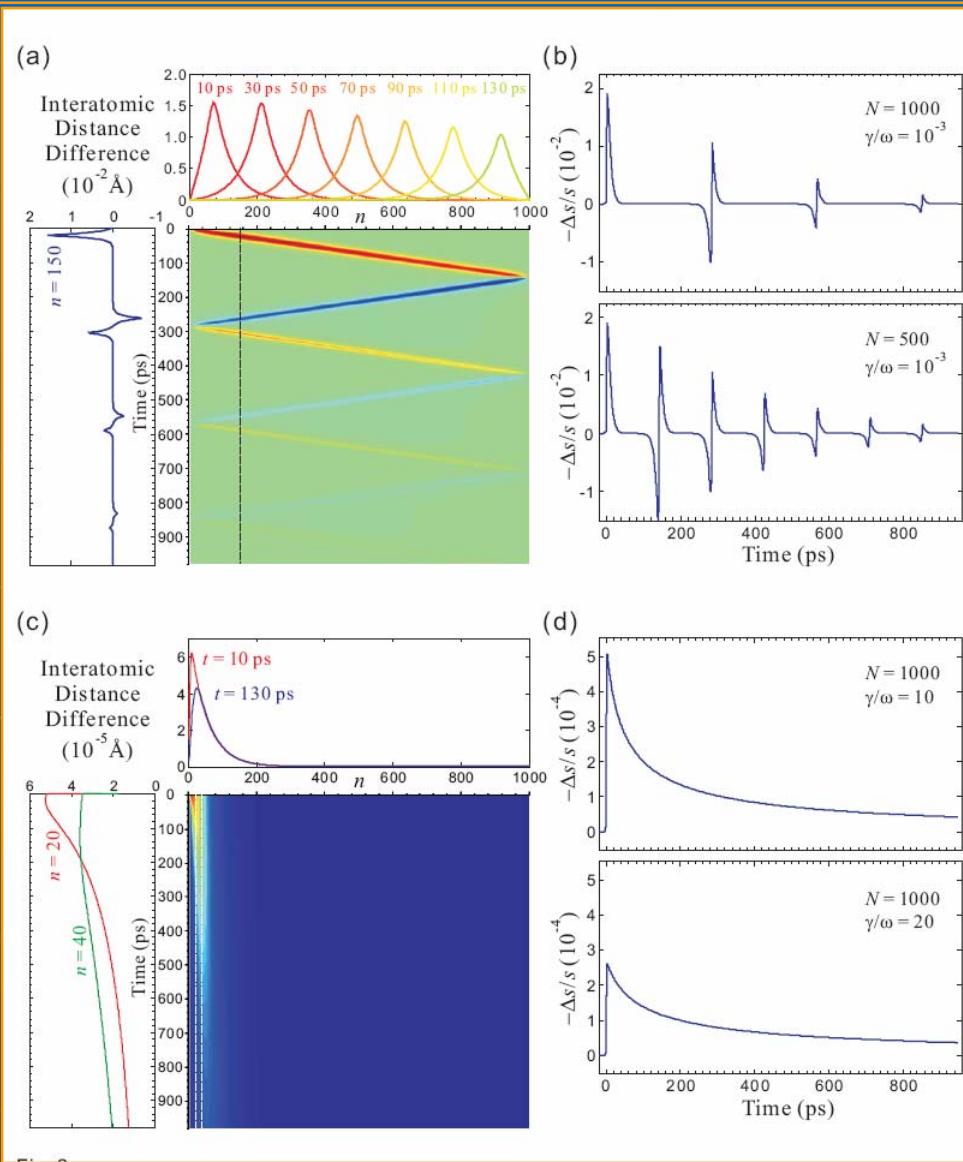


$$H = \sum_{k=0}^{N-1} \frac{p_k^2}{2m_k} + \sum_{k=0}^{N-2} \frac{m_k \omega_k^2}{2} (q_k - q_{k+1})^2 + \sum_{k=0}^{N-2} \alpha_k \frac{m_k \omega_k^2}{3} (q_k - q_{k+1})^3$$



UEC of Gold Film

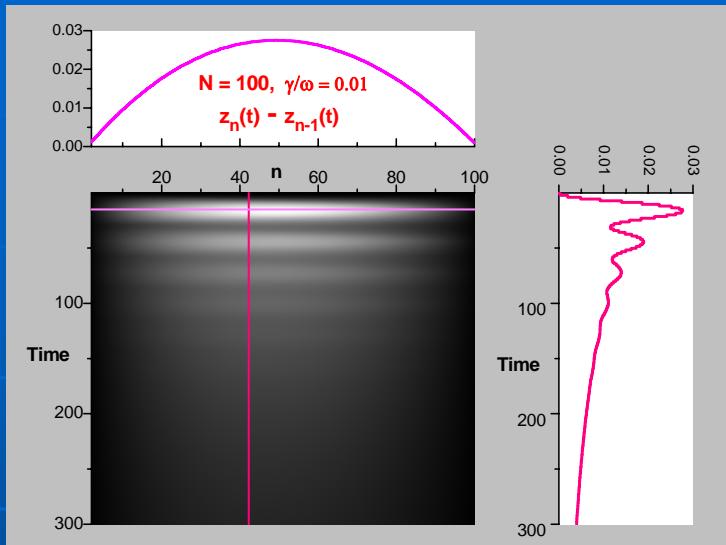
Shallow impulse, N = 1000



shallow impulse



Inter-atomic spacing



UEC of a thin gold film

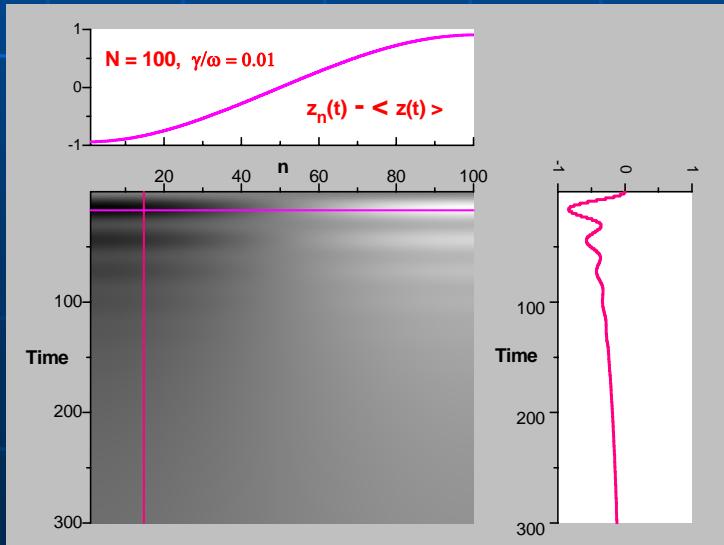
N = 100

Deep impulse

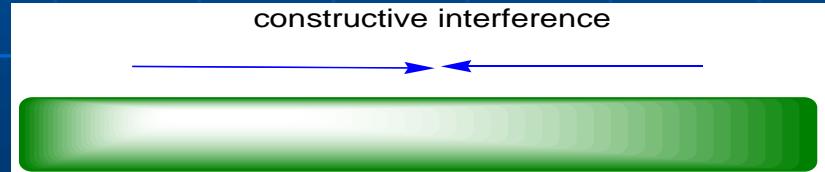
“breathing” phenomenon

Wavelength / 2 = slab thickness
Period / 2 = thickness / sound speed

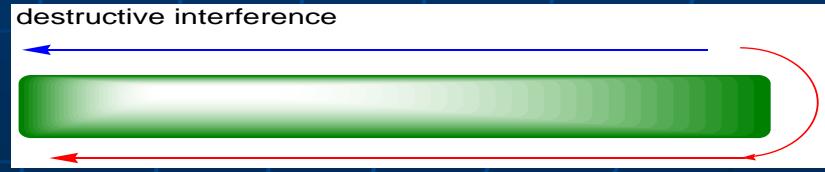
Position shift w.r.t. center of mass



Largest expansion in the middle

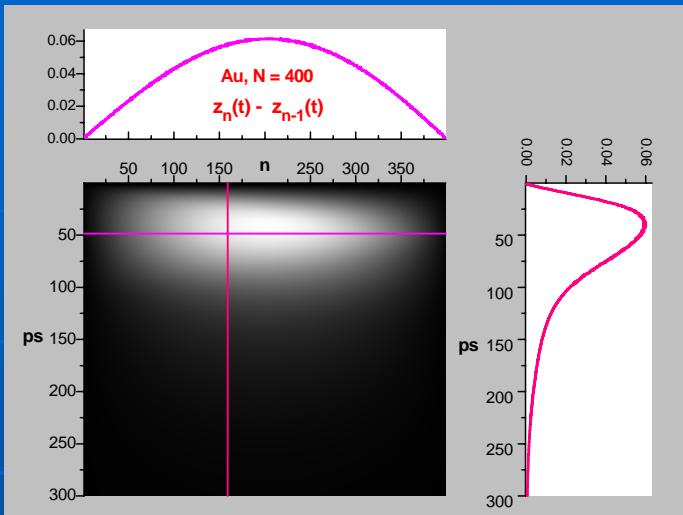


Smallest expansion on both surfaces

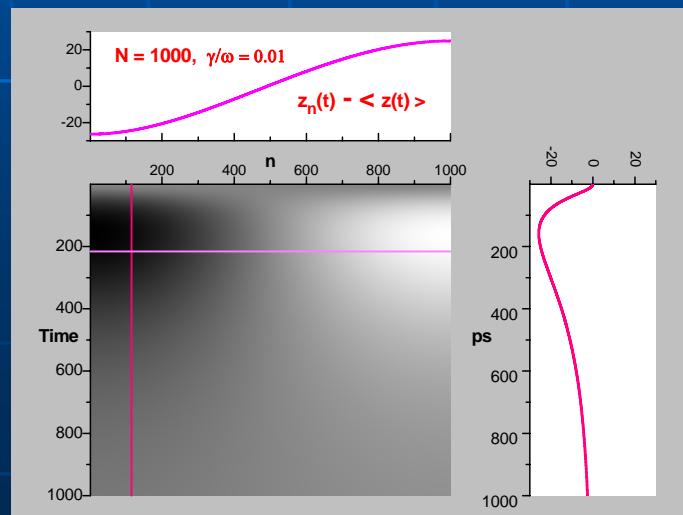


UEC of Gold Film N = 400

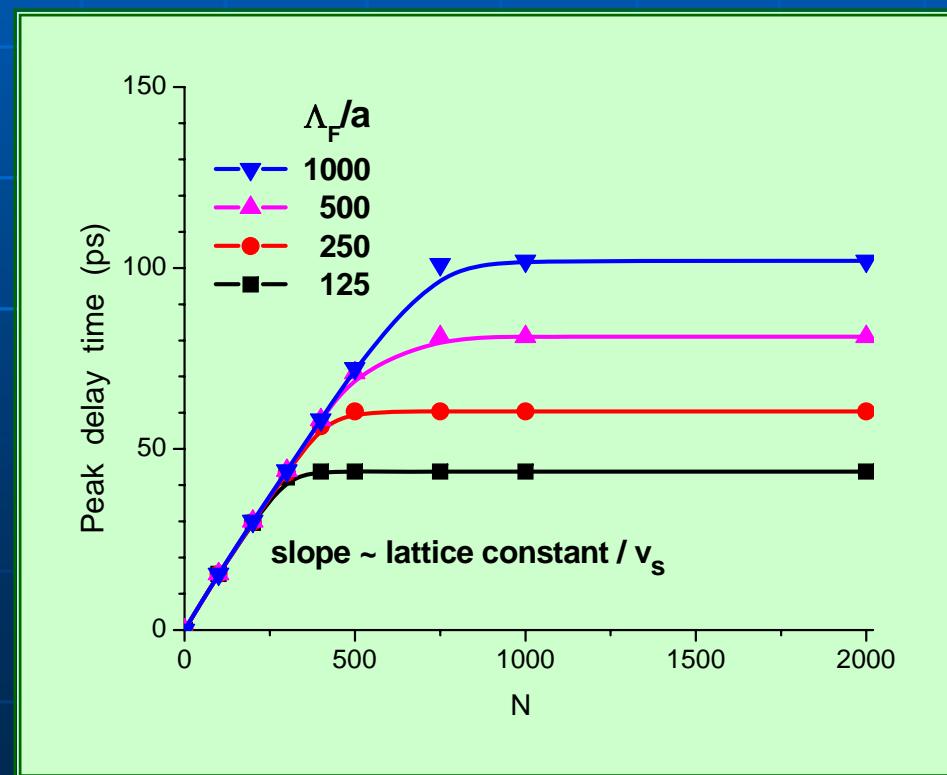
Inter-atomic spacing



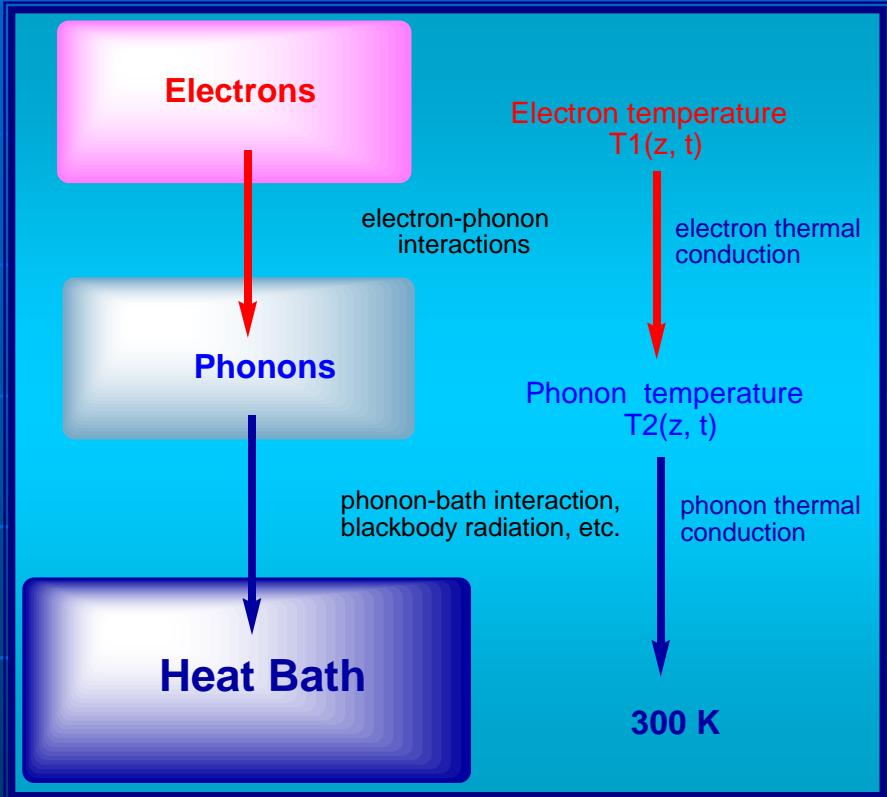
Position shift w.r.t. center of mass



Dependence of the peak delay time on the thickness of a substrate



Laser heating of metals and two-temperature model



$$\frac{\partial}{\partial t}[C_e(T_1)T_1(z, t)] = \frac{\partial}{\partial z} \left(\kappa_e(T_1) \frac{\partial}{\partial z} T_1(z, t) \right) - g(T_1(z, t) - T_2(z, t)) + S(z, t)$$

$$C_2 \frac{\partial}{\partial t} T_2(z, t) = \kappa_L \frac{\partial^2}{\partial z^2} T_2(z, t) - g(T_2(z, t) - T_1(z, t))$$

Temperature-induced impulsive forces

$$\begin{aligned} F_k(t) &= -\gamma_{2,G} C_2 (T_{2,k}(t) - T_0) \ell^2 \\ &\quad - \gamma_{1,G} C_1 T_{1,k}(t) (T_{1,k}(t) - T_0) \ell^2 \end{aligned}$$

Spatial and temporal
dependence
of electron and
phonon temperatures

→ Impulsive force
on each atom →

Transient atomic
Displacement &
Bragg peak shift

Aluminum: $\gamma_{2,G} \sim 2.2$ phonons
 $\gamma_{1,G} \sim 1.6$ electrons

Heat capacity of electrons ~ T

Fermi electron gas model

$$C_e(T) \sim (\pi^2 N k_B / 3T_F) T, \quad T_F = E_F / k_B$$

Fermi temperature: Au - 6.4×10^4 , Al - 13.6×10^4 , Cu - 8.2×10^4

Heat capacity of phonons ~ constant in T

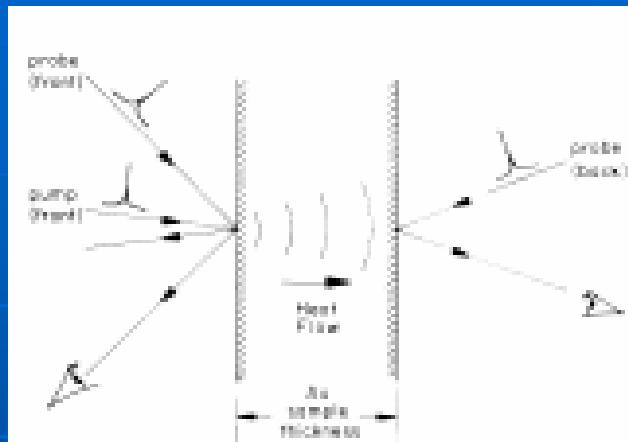
$$C = \frac{\partial}{\partial T} \sum_k \frac{\hbar \omega_k}{\exp(\hbar \omega_k / k_B T) - 1} \approx 3N_A k_B \quad \text{Dulong-Petit value}$$

Electronic thermal conductivity ~ T

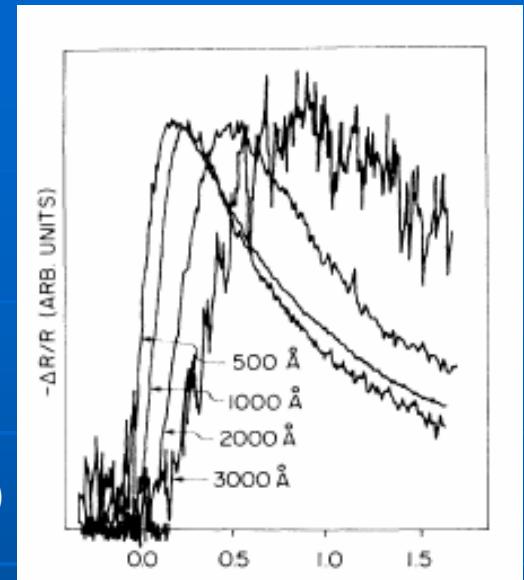
$$\kappa_e = \frac{\tau^2 k_B^2 n_e \tau}{3m_e} T, \quad \tau \text{ scattering relaxation time}$$

$$\Lambda = v_F \tau \quad \text{mean free path}$$

Transient optical reflectivity of a gold film

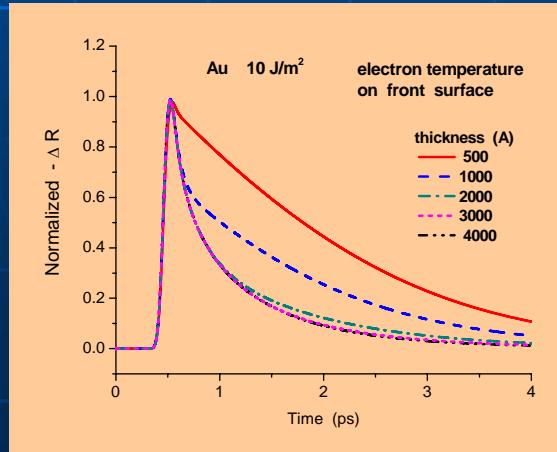


(rear surface)

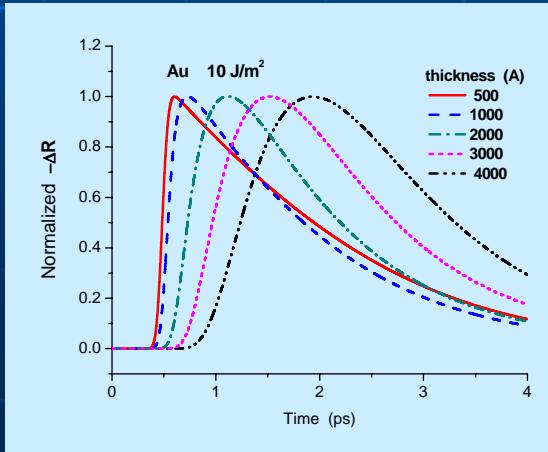


E. P. Ippen et al. Phys. Rev. Lett. 59, 1962, (1987)

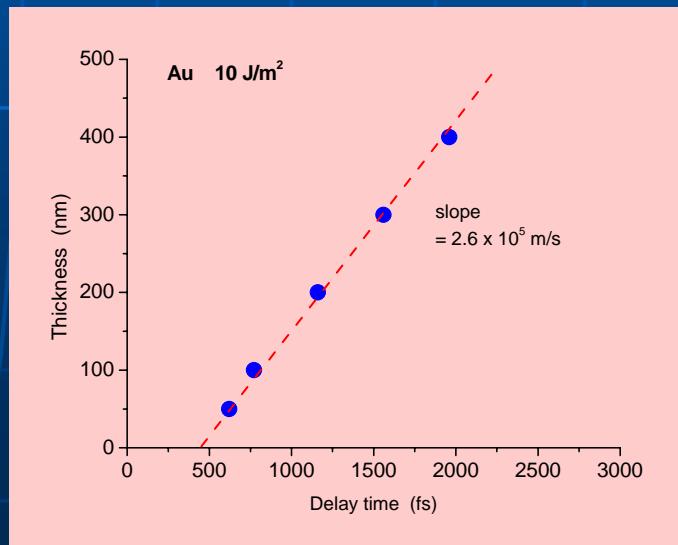
TTM (front surface)

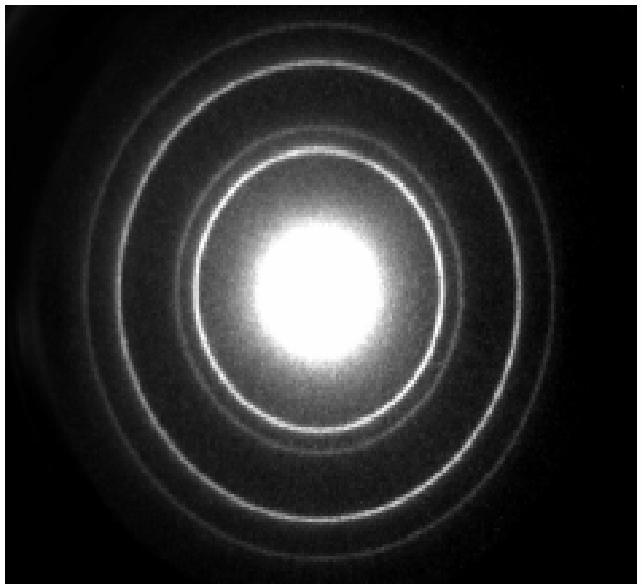


(rear surface)



Supersonic hot electrons

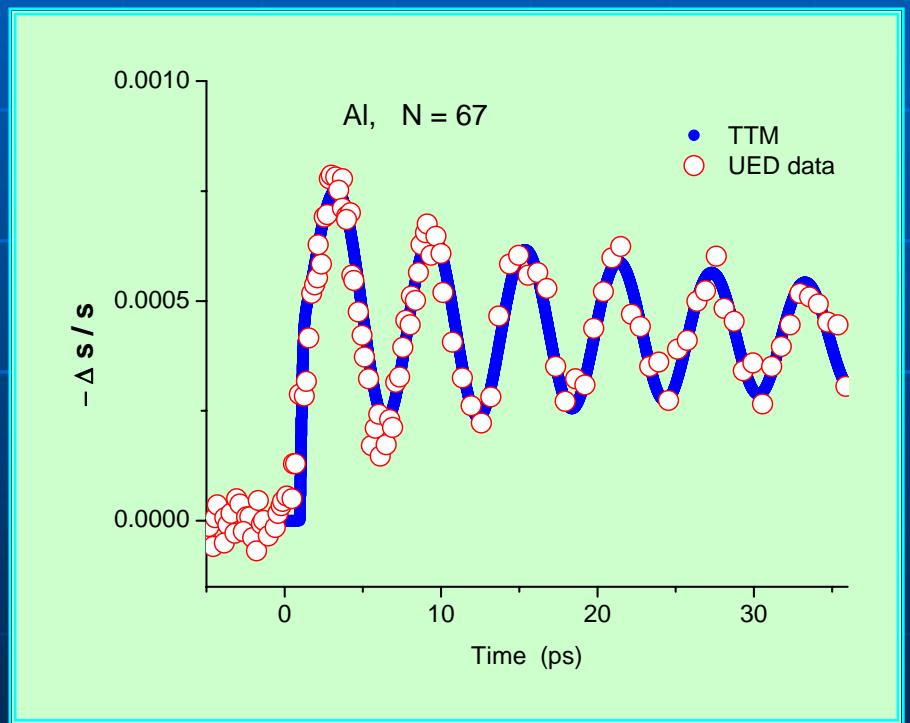
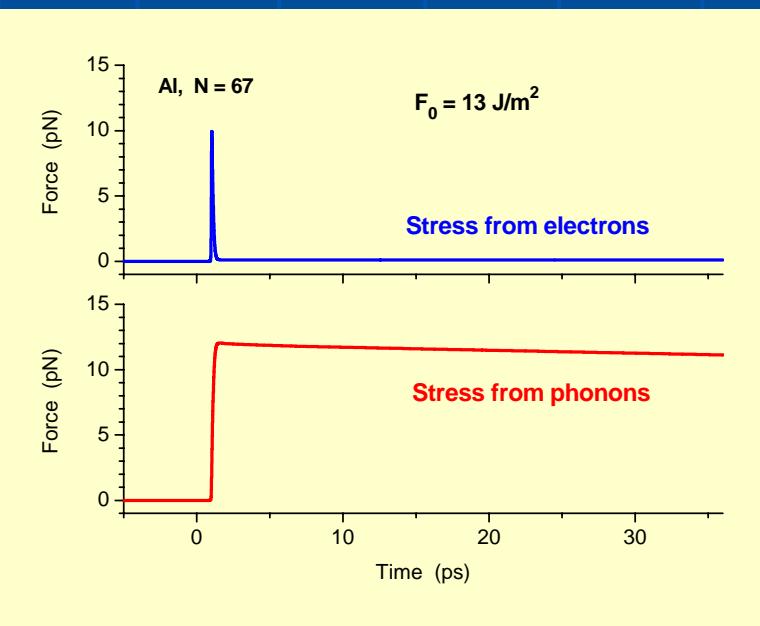




UEC of Aluminum Film

J. Cao et al. Phys Rev. Lett. (2006)
(20 nm thickness, fluence 13 J/m²)

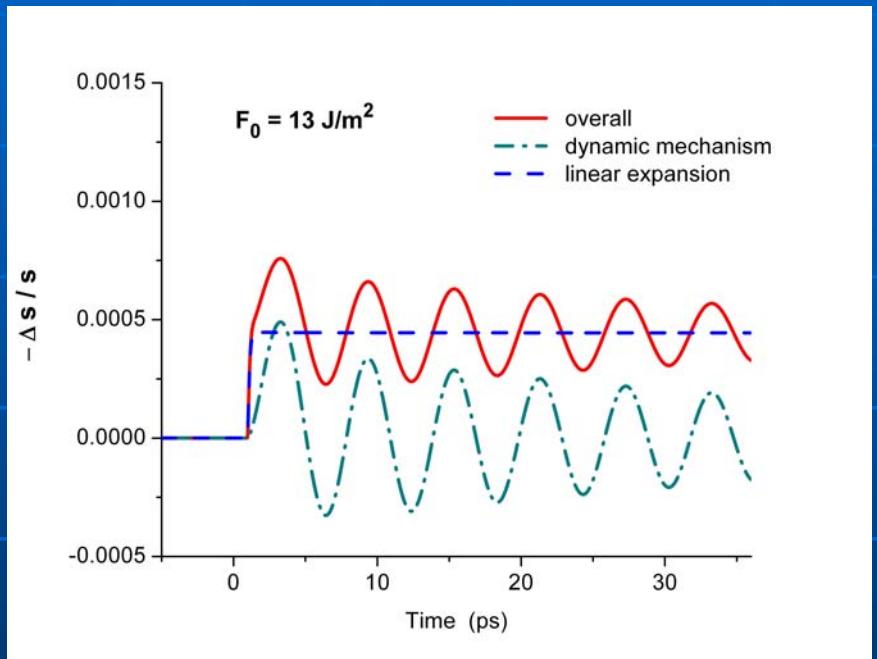
Acoustic wave excitation



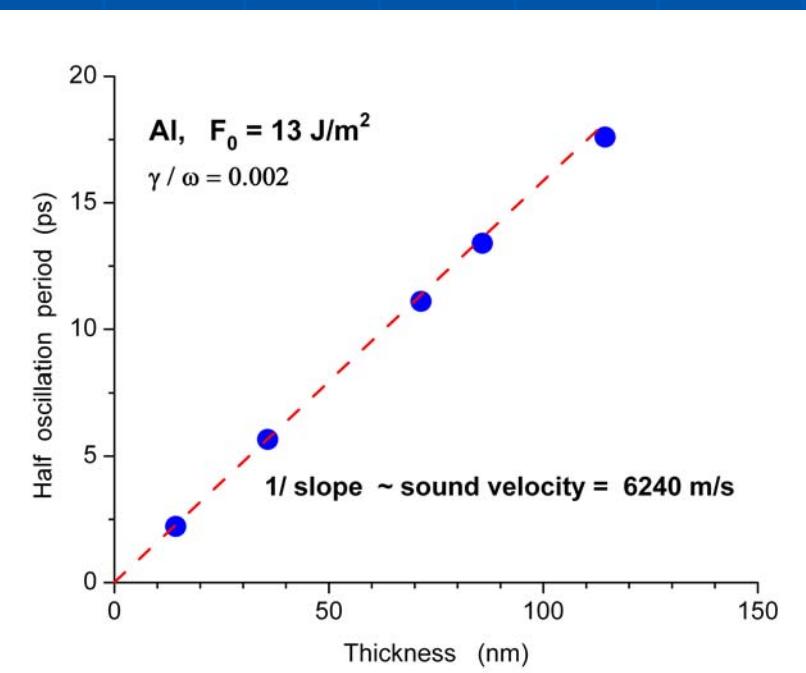
J. Tang, PRL (submitted)

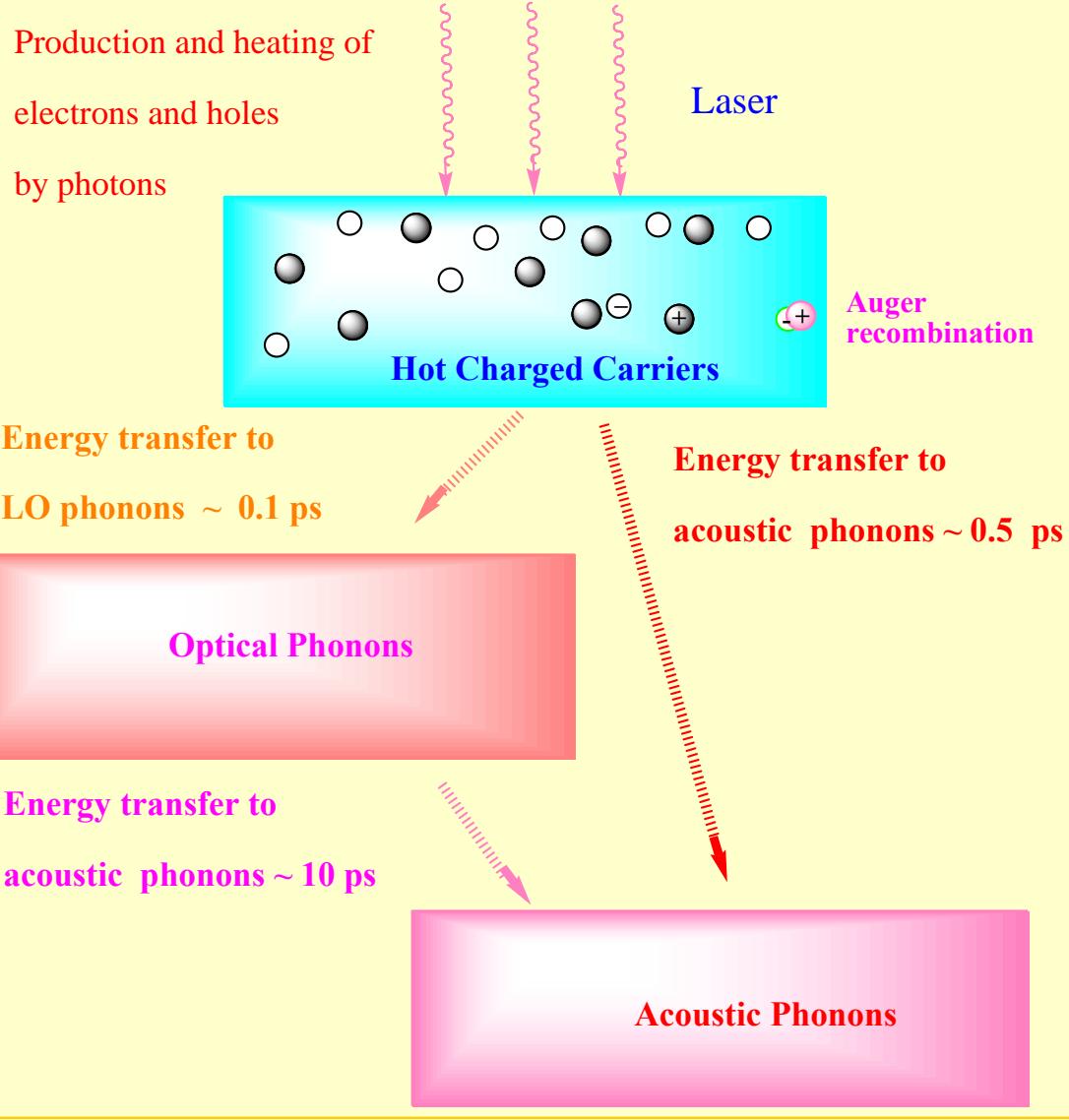
Dynamic mechanism

Static linear thermal expansion



Half oscillation period vs. thickness





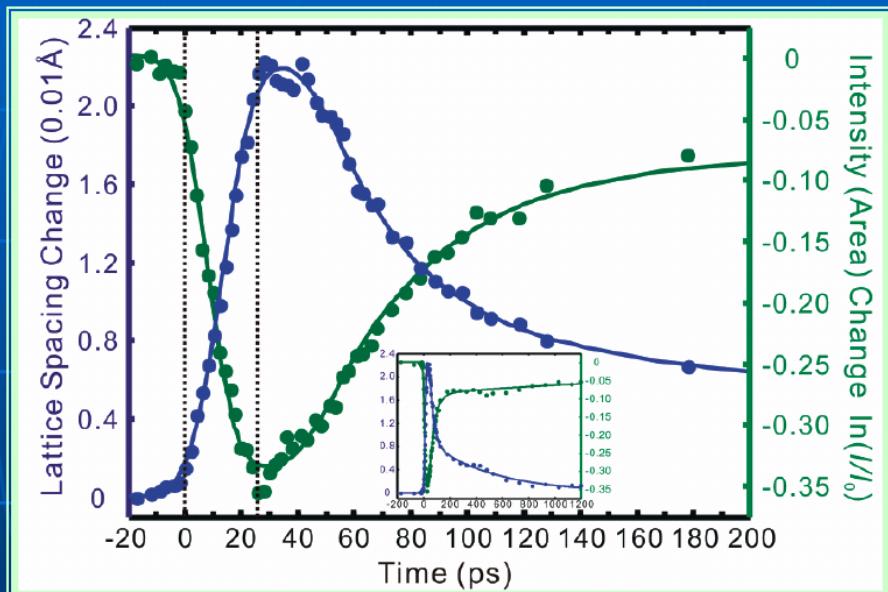
Laser heating of Si & GaAs semiconductors and three-temperature model

Charged carriers

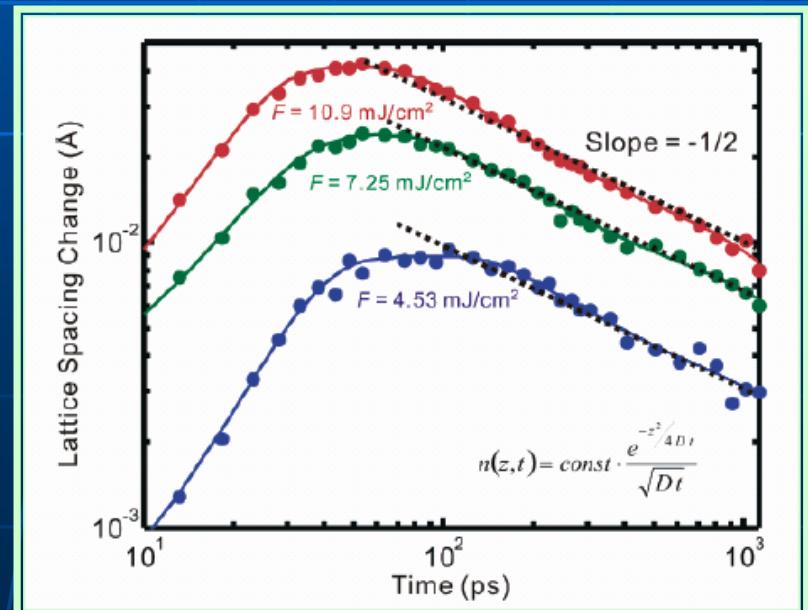
Optical phonons

Acoustic phonons

UEC of GaAs thin film



Yang, Gedik and Zewail, JPC (2007)



Dynamic vs. static mechanisms

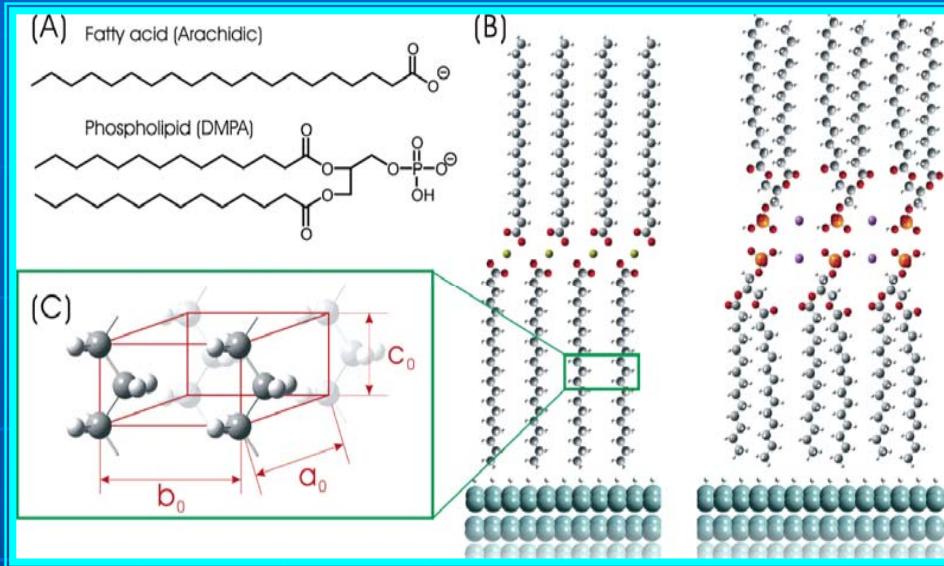
Dynamic

- Periodic expansion and contraction (thin film) or pure expansion (thick film)
- Non-local effect
- coherent (wave propagation)
- The magnitude of expansion is larger
- $\Delta s/s$ proportional to N (not saturated)
- $\Delta L/L \sim L^2$ (middle atoms expand more)

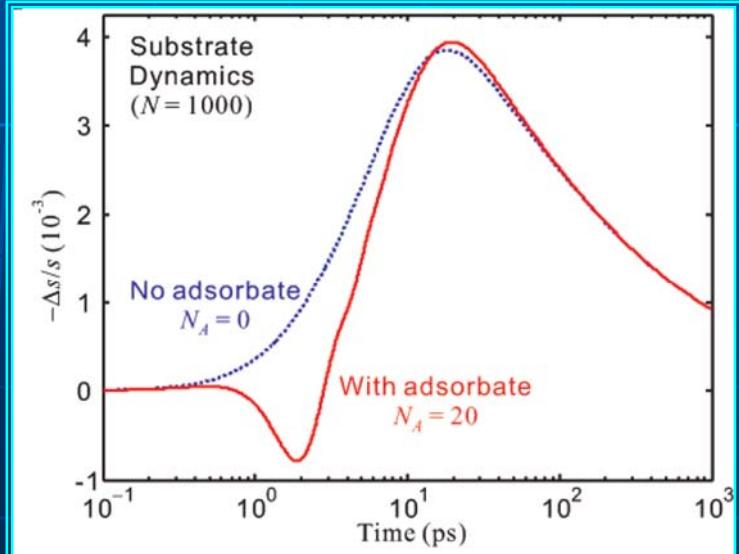
Static

- Only expansion occurs
- Local effect
- Non-coherent
- The magnitude of expansion is smaller
- $\Delta s/s$ independent of N
- $\Delta L/L \sim \Delta T$ independent of L

UEC of adsorbates & interfacial effects



Time dependence of
Bragg-spot shift



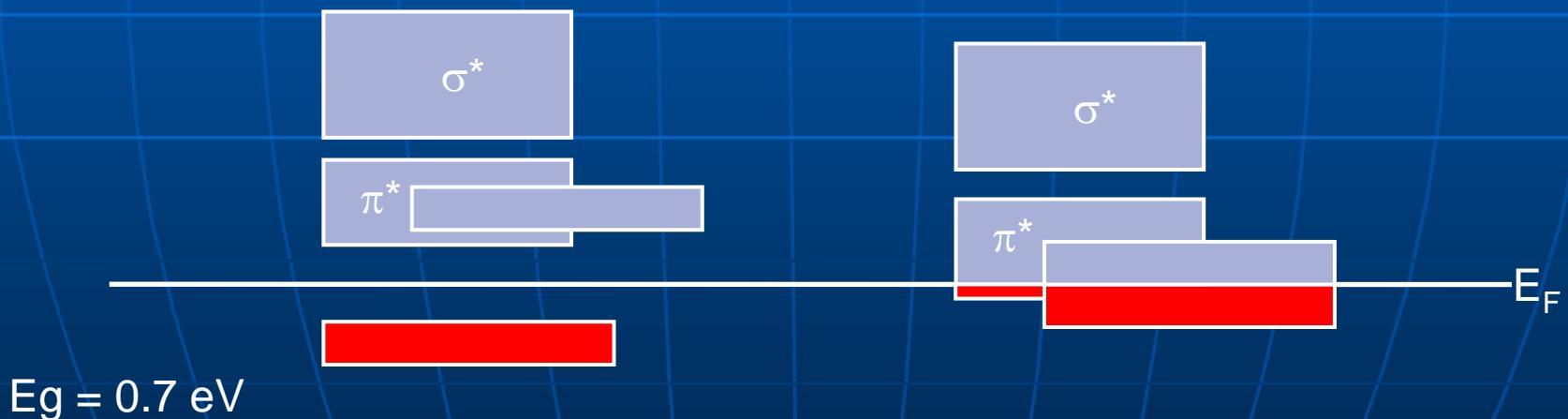
Laser-induced phase transition in VO₂

Insulator

↔

Metallic

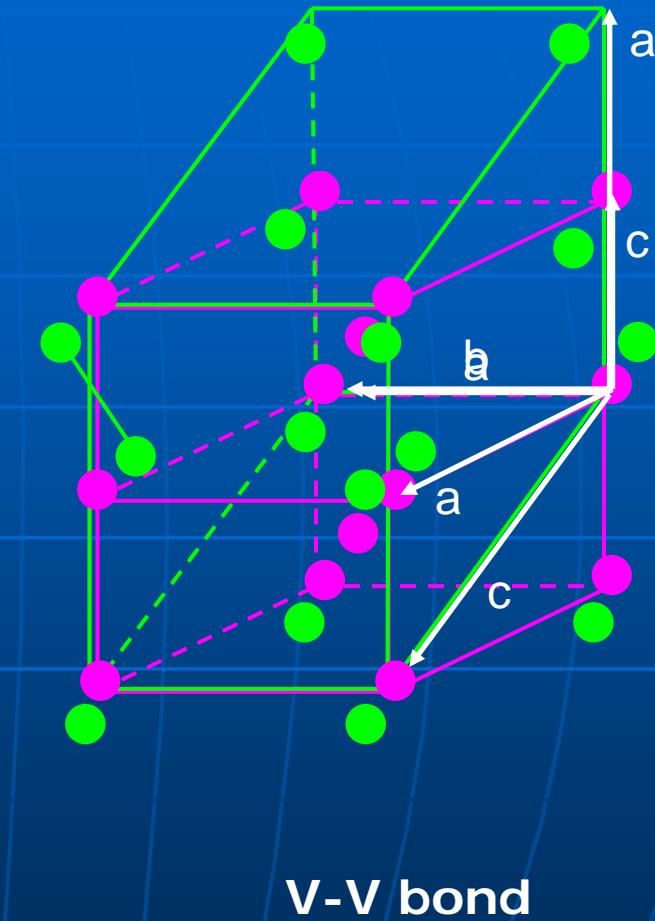
$T_c \sim 340$ K



Phase transition in VO_2

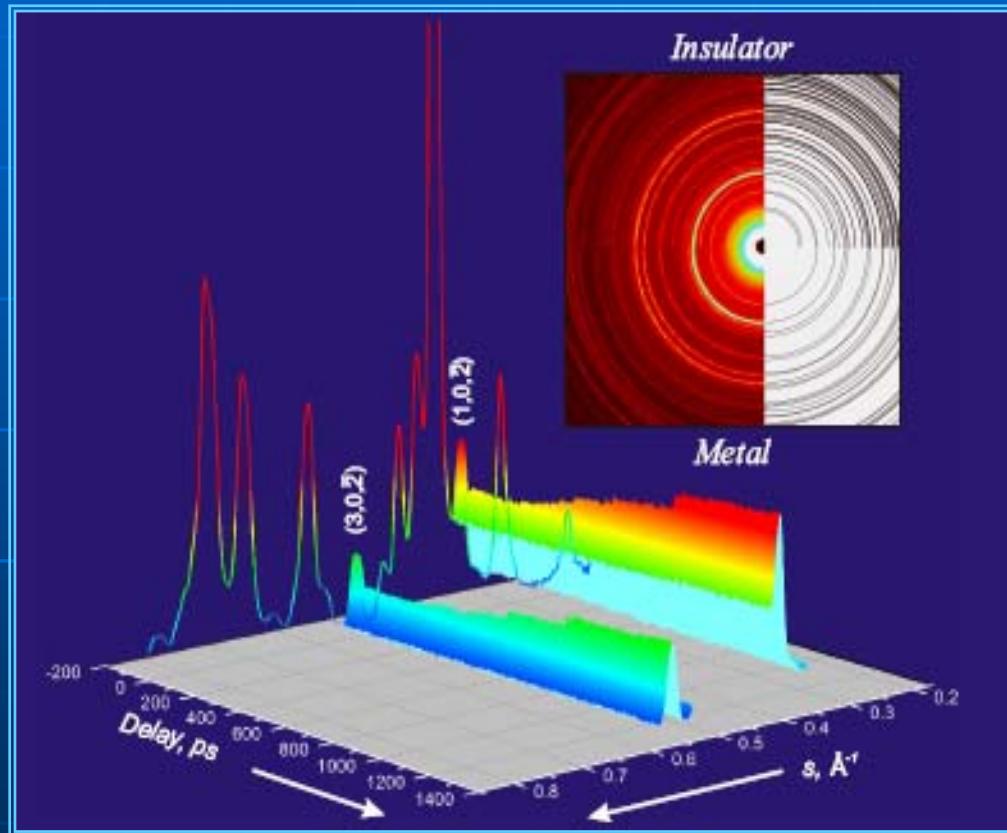
Insulator \leftrightarrow Metallic
Monoclinic Tetragonal (Rutile)

$$\begin{aligned} a_m &= 2c_r \\ b_m &= a_r \\ c_m &= a_r - c_r \end{aligned}$$

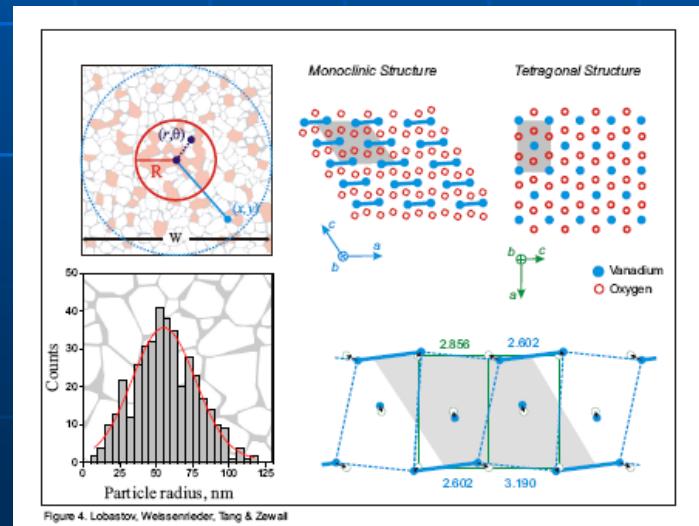


Electron diffraction pattern

Measured and simulated electron diffraction patterns

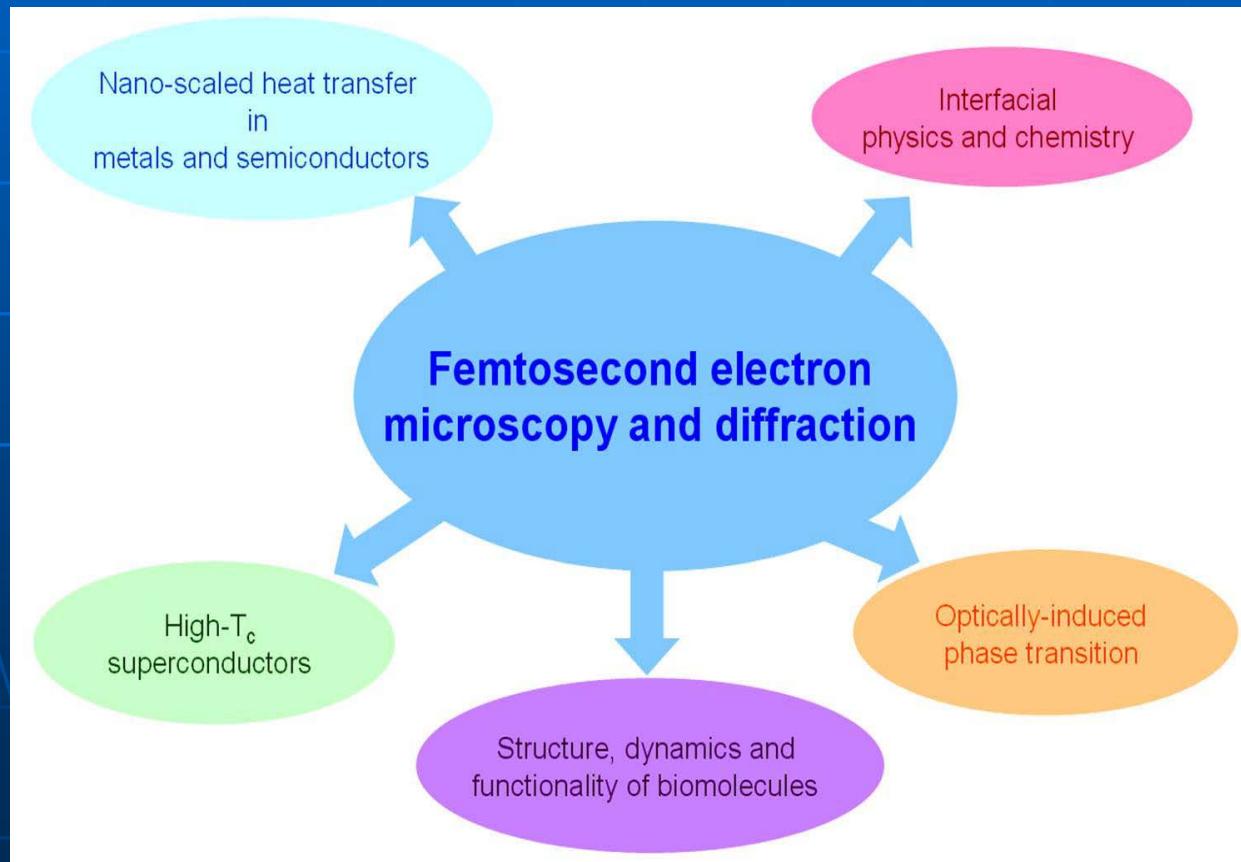


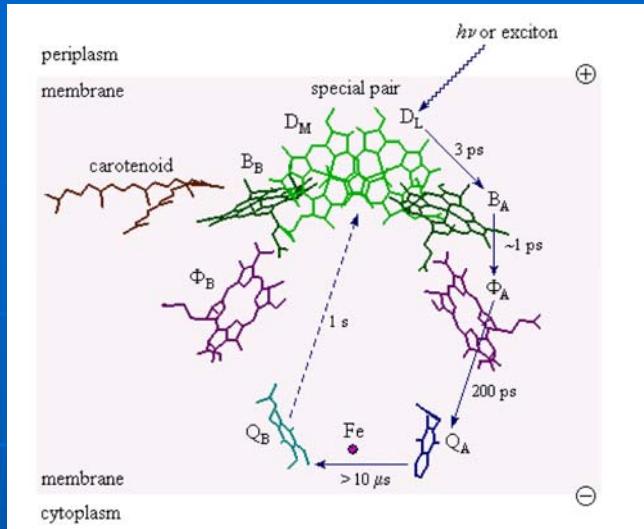
Electron gun: 120 keV
Laser : 776 nm
Fluence: 0.05 mJ/cm²



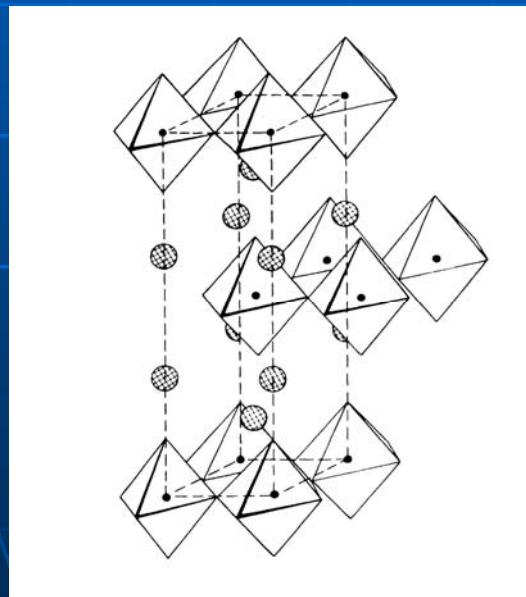
Natural Sciences: interfacial properties (surface physics, chemistry), nanoparticles, self-assembled monolayers, high T_c superconductors bond-breaking, conformation changes, etc.

Life Sciences: membrane protein, DNA, molecular recognition, hydration, charge transport, photosynthesis, etc.

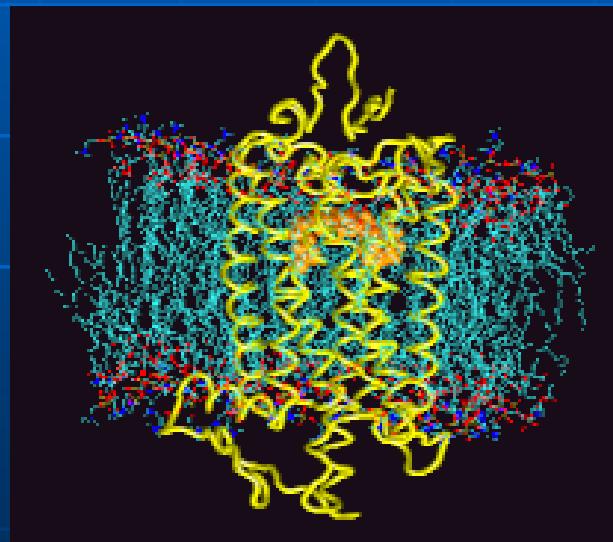




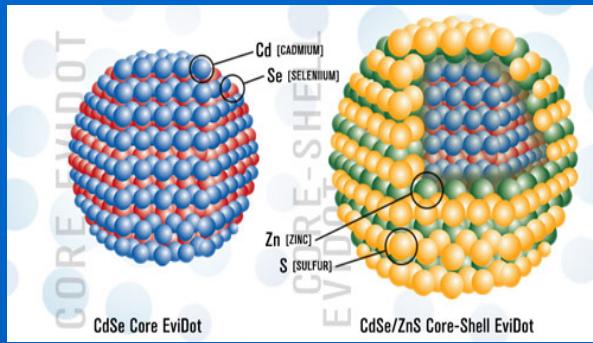
Photosynthetic RC



High Tc superconductor



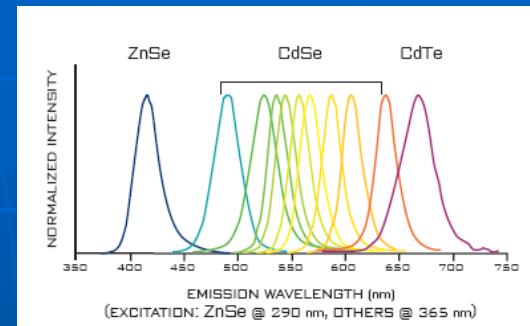
Rhodopsin



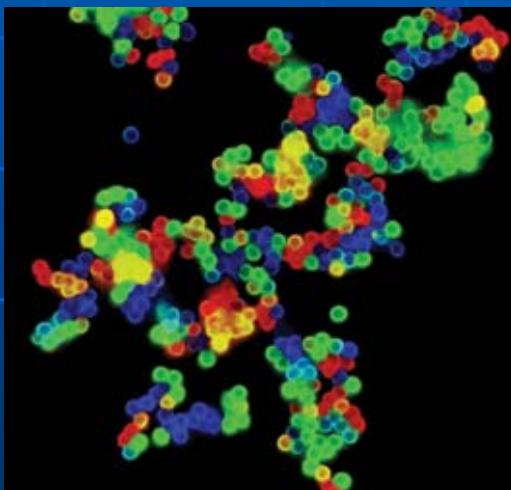
Quantum dots



Emission spectra of QDs

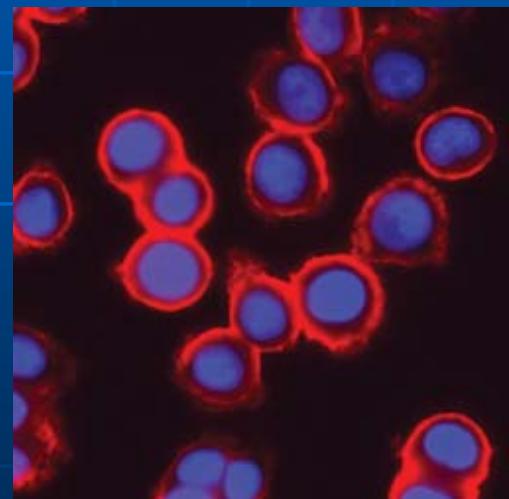
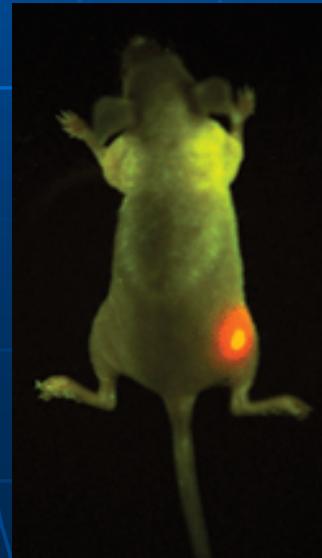


Polymer beads embedded with quantum dots fluoresce in five different colors.



labeling tags

Red QDs injected into a live mouse mark the location of a tumor.
C. Seydel, *Science*, 2003, 300, 5616.



Red light-emitting quantum dots tag proteins on the surfaces of breast cancer cells.

Potential applications

Labeling tags

Solar cells

White LEDs

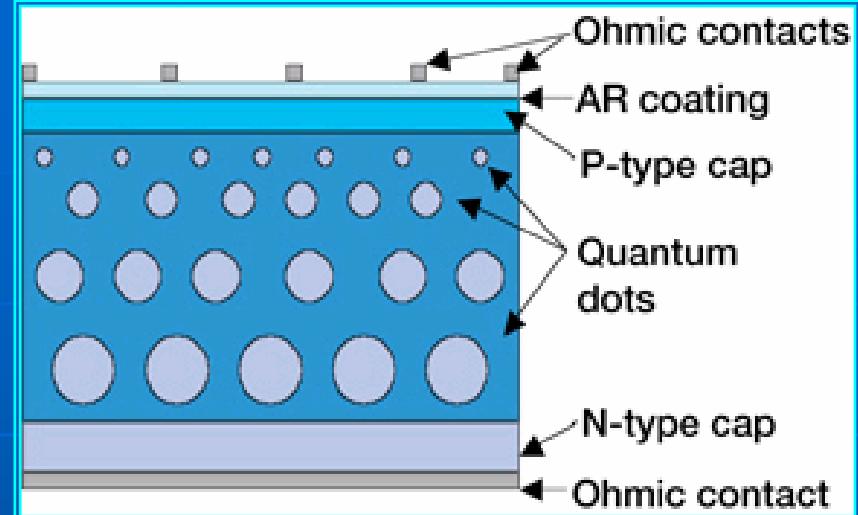
(LED Wavelength Converter)

Military Applications (Infrared Paints)

InAs/GaAs-based quantum dot laser

High density memory

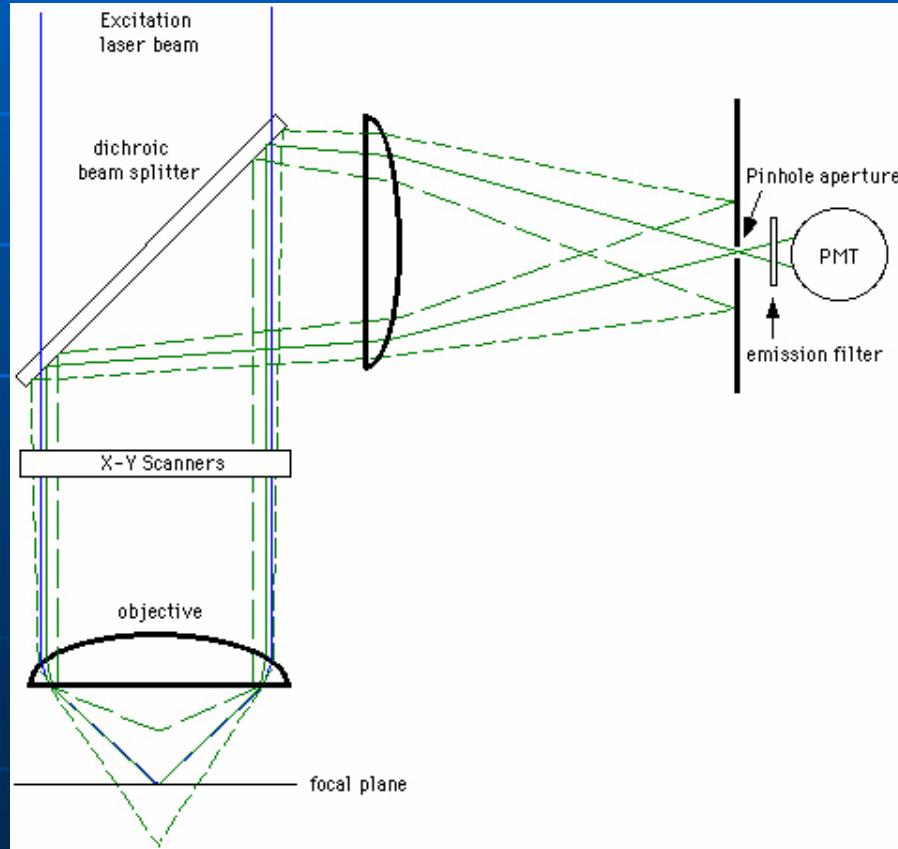
Quantum computing



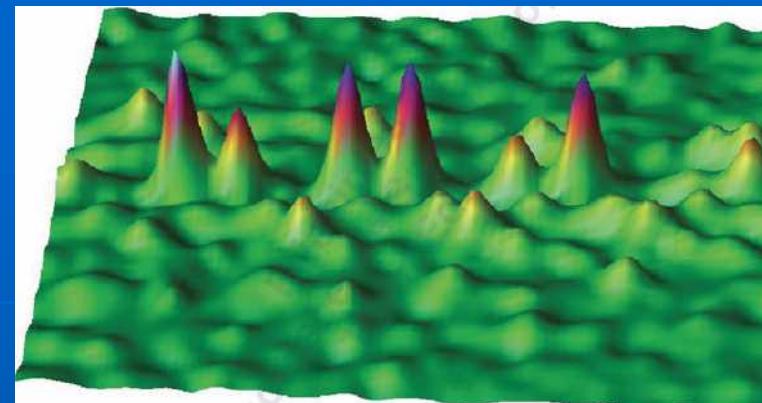
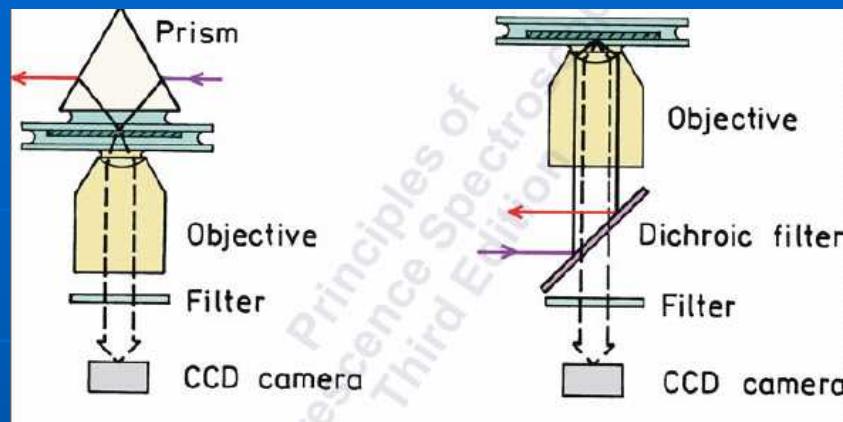
Intermediate-bandgap solar cell

Confocal microscopy

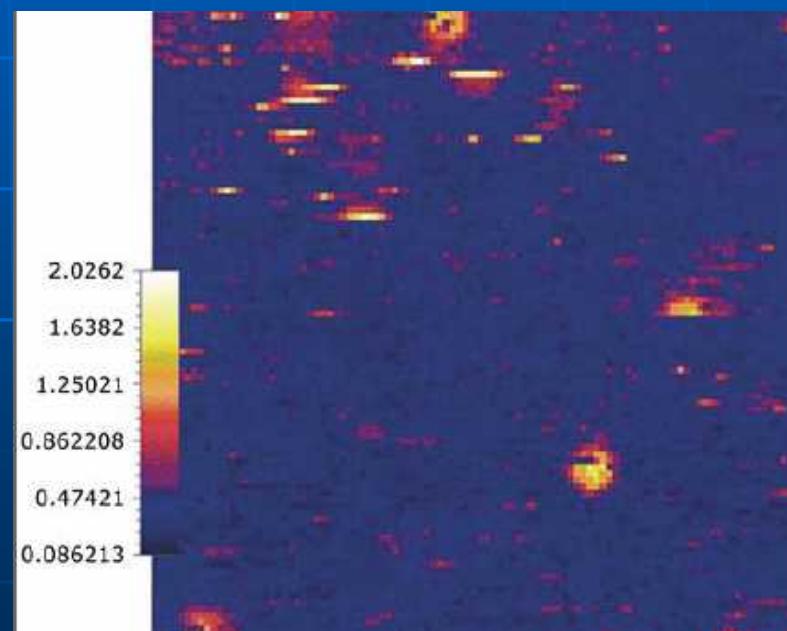
The microscope is able to filter out the out-of-focus light from above and below the point of focus in the object. A "pinhole" situated in front of the image plane which acts as a spatial filter and allows only the in-focus portion of the light to be imaged



Inverted microscope



Green fluorescent protein



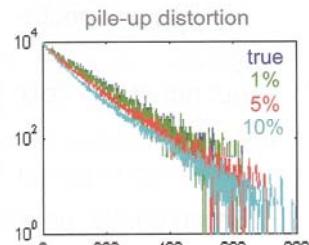
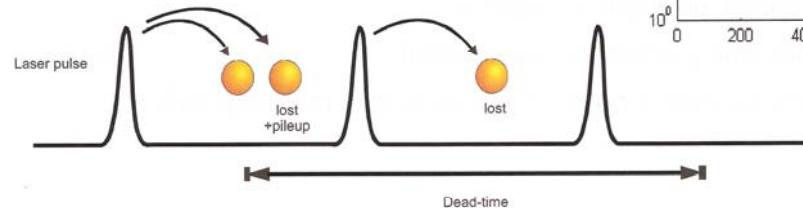
$10 \mu\text{m} \times 10 \mu\text{m}$

TCSPC Disadvantages and Difficulties

- Need pulsed light source and fast electronics (no problem today)
- Need fast single photon detector (can be difficult in the IR)
- Considered complicated & expensive (not true today)

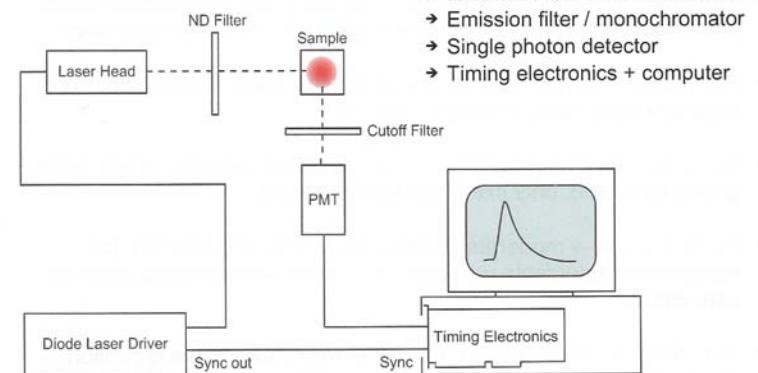
Specific Difficulties:

- Pile-up → keep count rate < 1..2%, use high laser pulse rate
- Dead-time → keep count rate < 1..2%, correct intensities



9

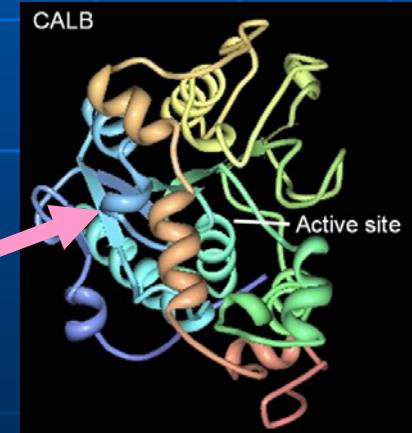
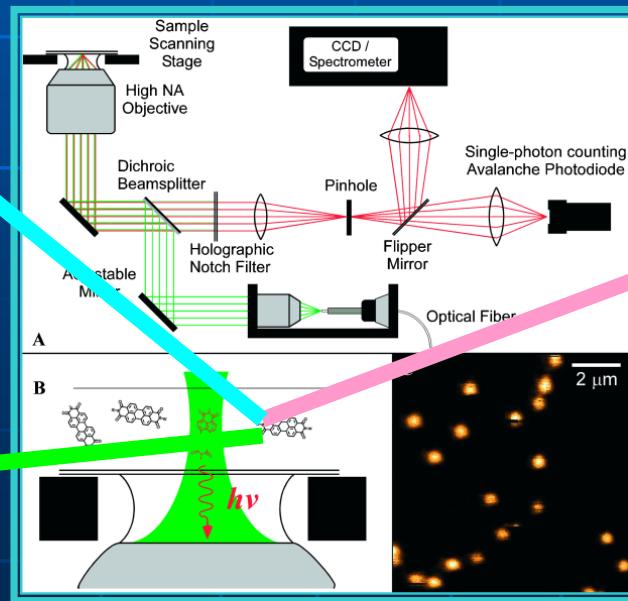
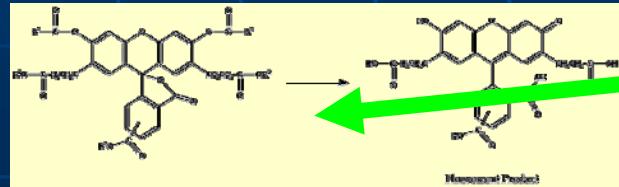
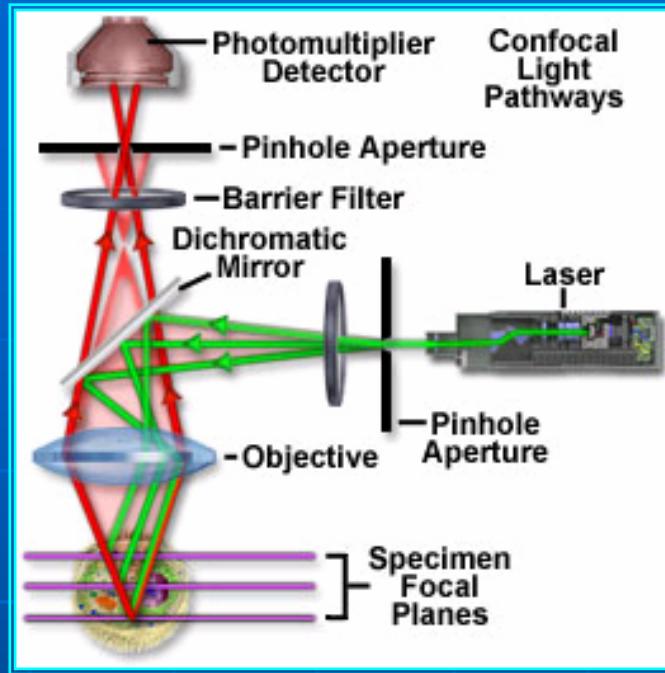
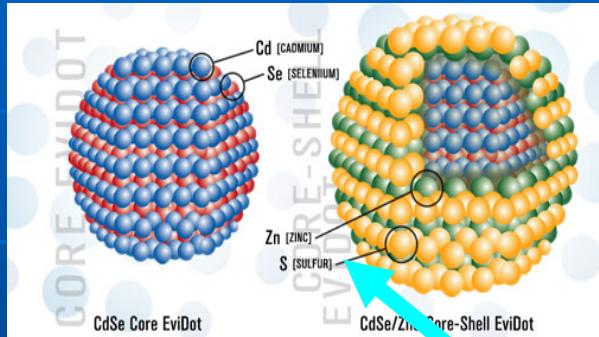
Basic TCSPC / TR-Fluorescence Setup



10

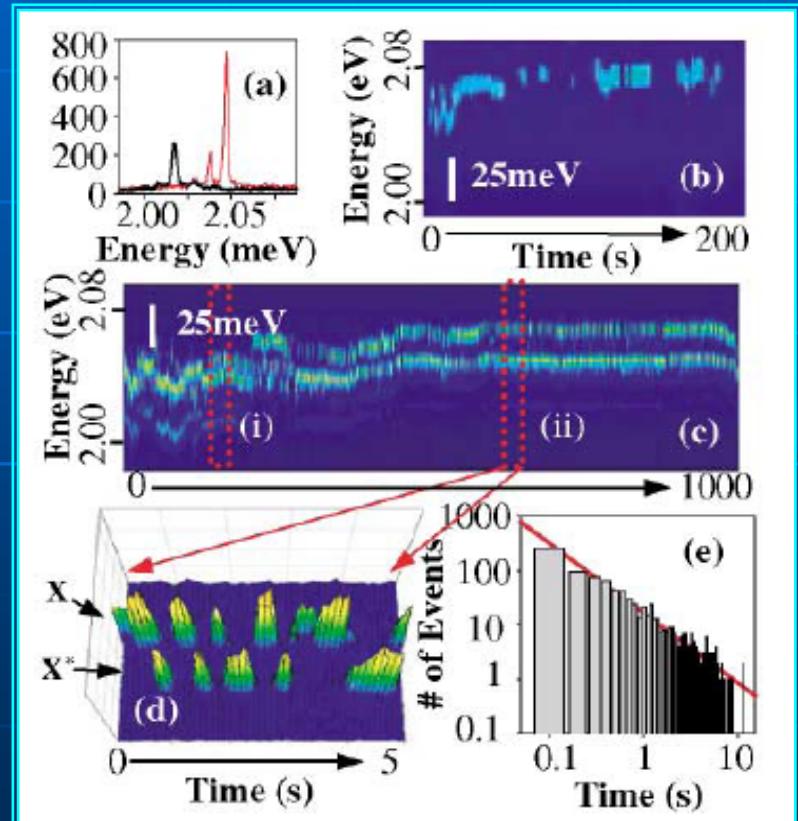
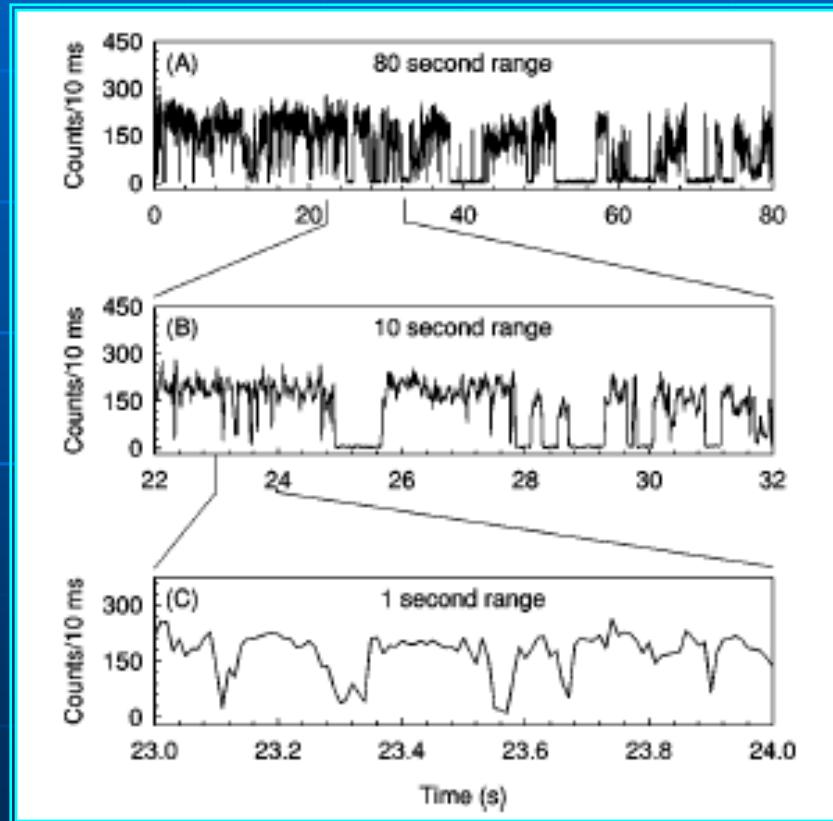


Confocal microscopy & Single-Particle Intermittency



Fluorescence intermittency (blinking)

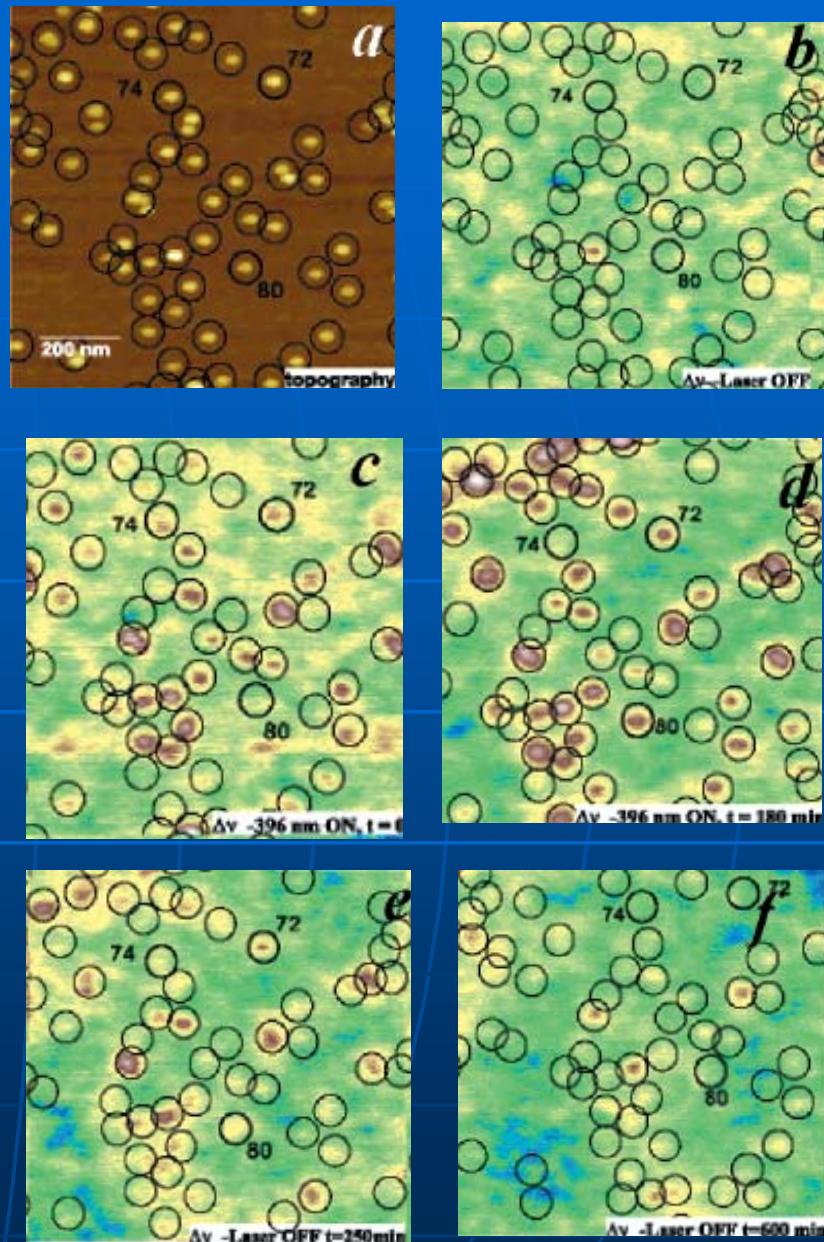
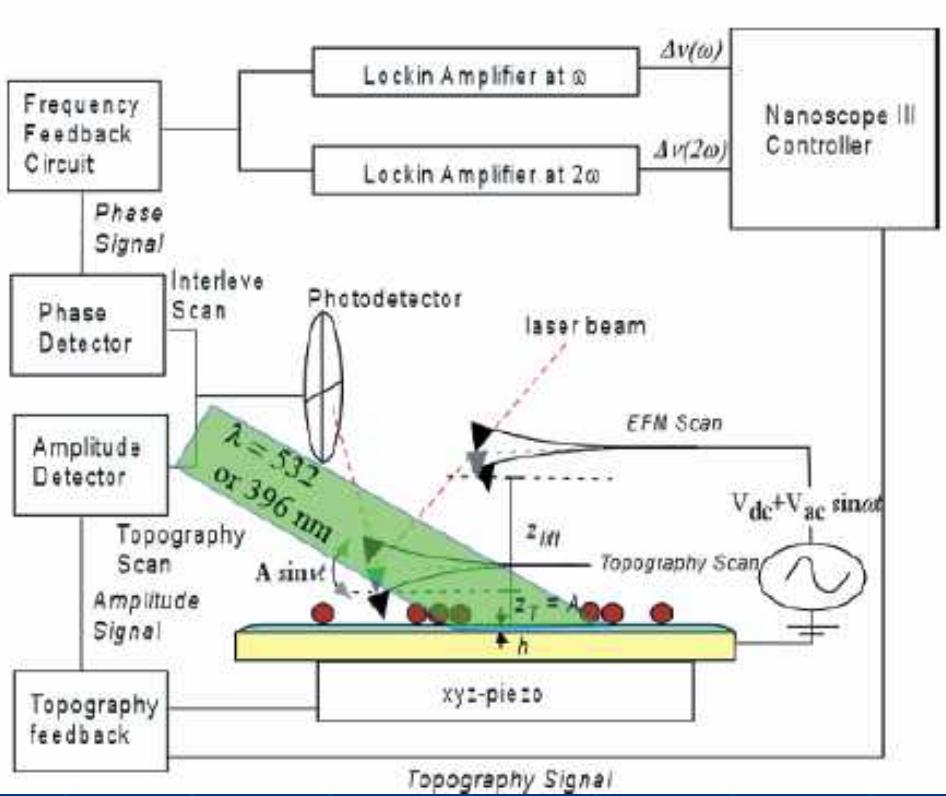
histogram



D. Nesbitt *et al*; *J. Chem. Phys.* 2000

M. G. Bawendi., *et al*. *PRL* 2002

Electrostatic force microscopy of CdSe



a) Topography of CdSe QDs

Charge map $10^3 \times 10^3 \text{ nm}^2$

b) Before illumination

c-d) *light-on*

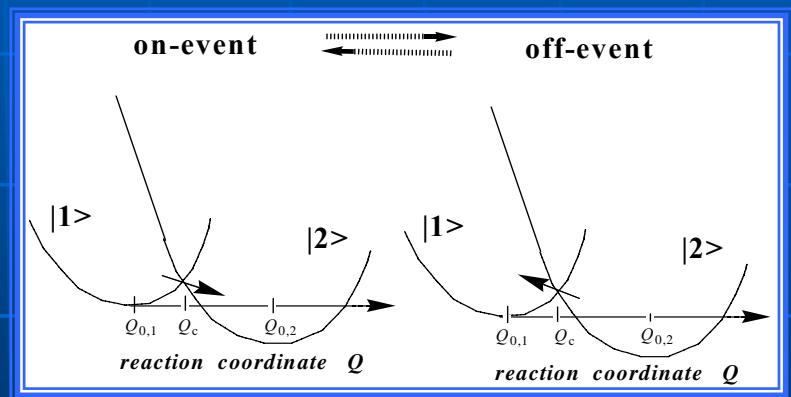
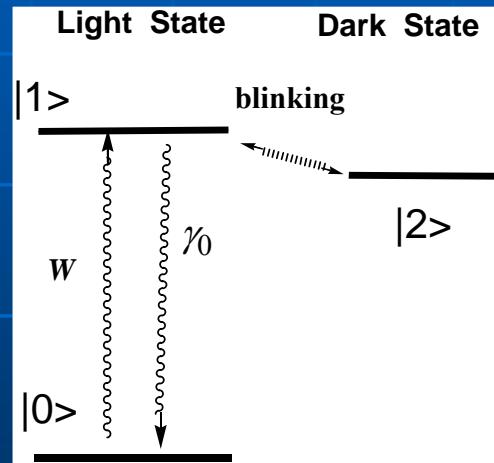
e) 4 hrs f) 10 hrs after light -off

- Why does a QD blink ?
- What causes a power law distribution (t^{-m}) ?
- Why is the exponent m close to $3/2$?

Diffusion-controlled electron transfer (DCET) model

(Tang & Marcus, PRL and JCP , 2005)

Neutral light state \longleftrightarrow Positively charged dark state



1-D diffusion in energy space

$$\frac{\partial}{\partial t} \rho_1(Q, t) = L_1 \rho_1(Q, t) - C_1 \rho_1(Q, t)$$

$$\frac{\partial}{\partial t} \rho_2(Q, t) = L_2 \rho_2(Q, t) - C_2 \rho_2(Q, t)$$

$$L_k = D_k(t) \frac{\partial}{\partial Q} \left(\frac{\partial}{\partial Q} + \frac{1}{k_B T} \frac{\partial}{\partial Q} U_k(Q) \right)$$

$$C_k = \frac{2\pi |V_k|^2}{\hbar} \delta(U_{12}(Q))$$

DCET model for power-law intermittency

Blinking statistics $P(t)$: waiting time distribution for “on” or “off” events

$$\overline{P}_k(s) = - \int_0^\infty dt e^{-st} \frac{d}{dt} \left(\int_{-\infty}^\infty dQ \rho_k(Q, t) \right) = \frac{A_k \overline{G}_k(Q_c, Q_c; s)}{1 + A_k \overline{G}_k(Q_c, Q_c; s)}$$

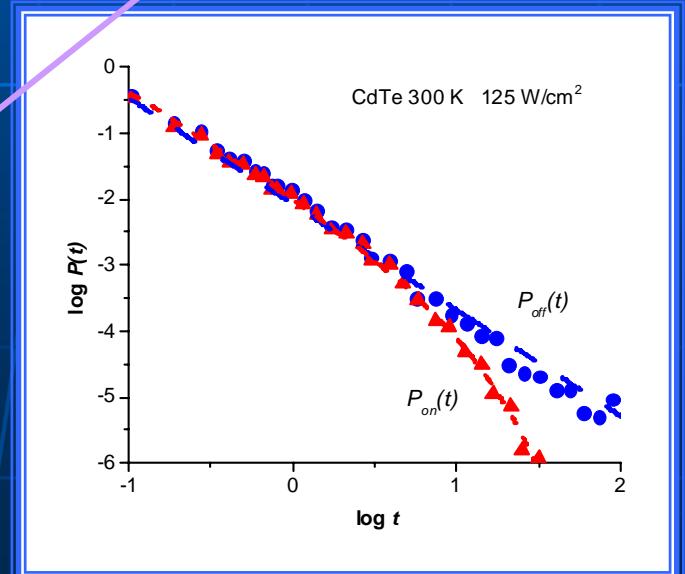
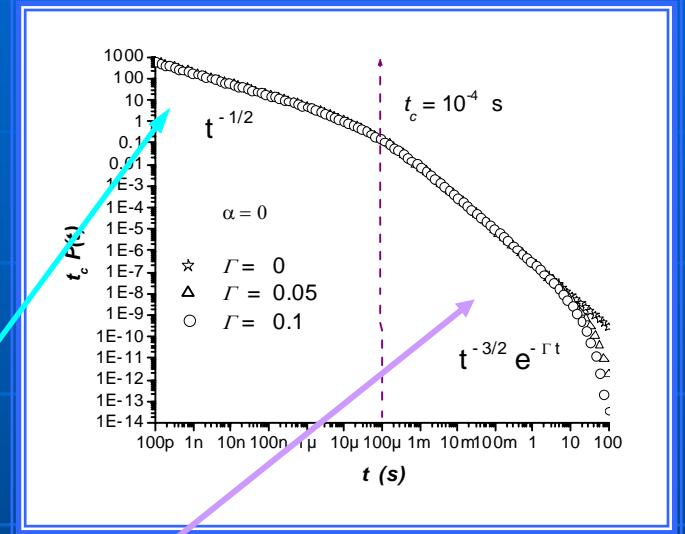
$$A_k = \frac{2\pi}{\hbar} |V_k|^2 \left| \partial(U_1(Q) - U_2(Q)) / \partial Q \right|_{Q=Q_c}$$

Debye medium (D is a constant)

$$P_k(t) \sim \frac{1}{\sqrt{\pi t_{c,k}}} t^{-\frac{1}{2}} \quad \text{if } t \ll t_{c,k}$$

$$P_k(t) \sim \frac{\sqrt{t_{c,k}}}{2\sqrt{\pi}} t^{-\frac{3}{2}} \exp(-\Gamma_k t) \quad \text{if } t_{c,k} \ll t \ll \tau_{L,k}$$

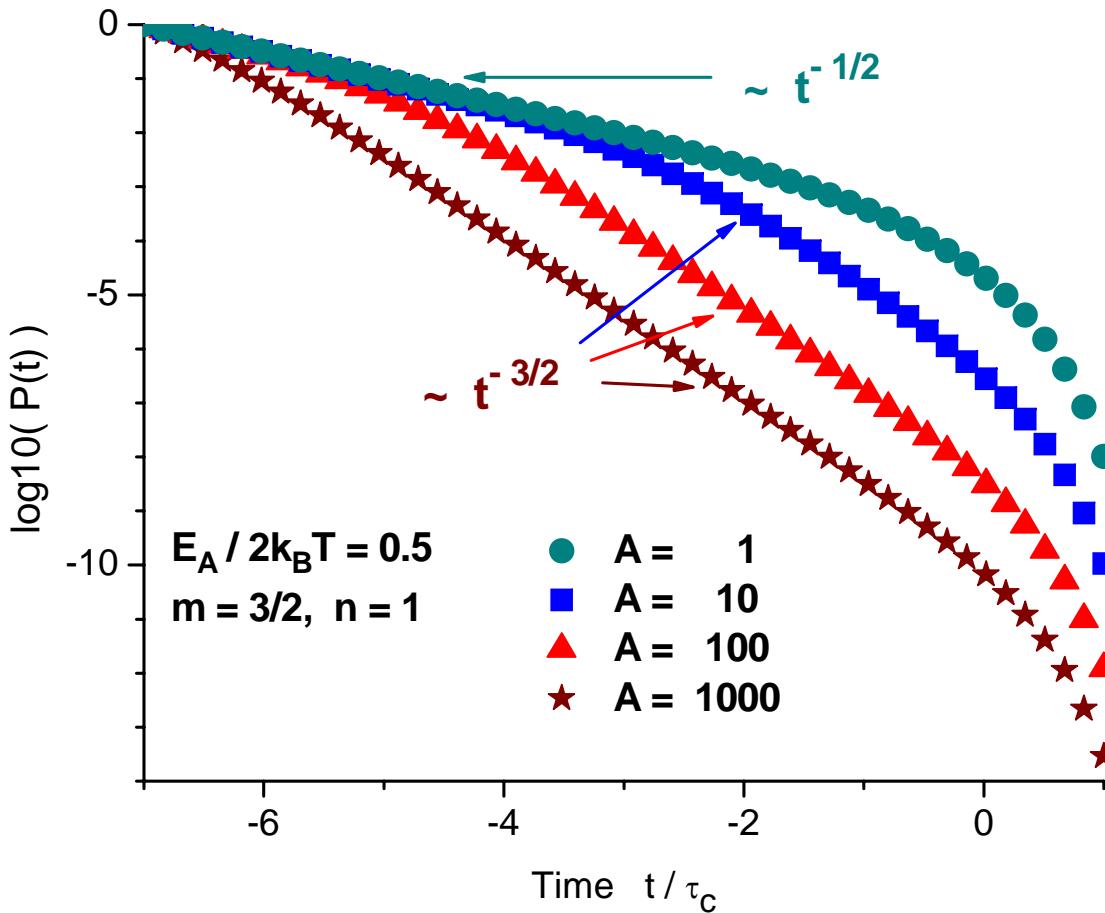
CdTe



$$P_k(t) = \frac{1}{\sqrt{\pi t_{c,k} t}} \left[1 - \sqrt{\pi t / t_{c,k}} \exp(t / t_{c,k}) \operatorname{erfc}\left(\sqrt{t / t_{c,k}}\right) \right] \exp(-\Gamma_k t), \quad (\text{DCET model})$$

Small A:
weak coupling

Large A:
strong coupling

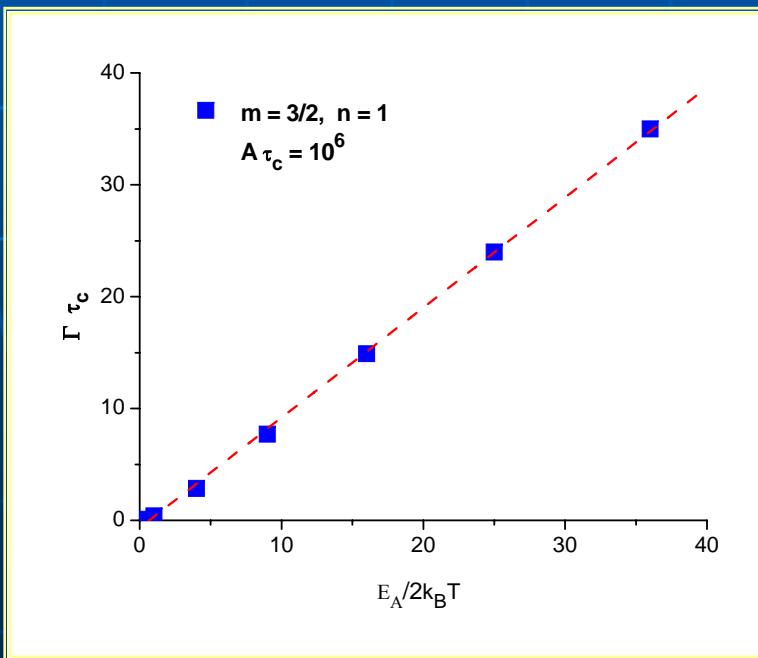


Bending rate Γ at longer times

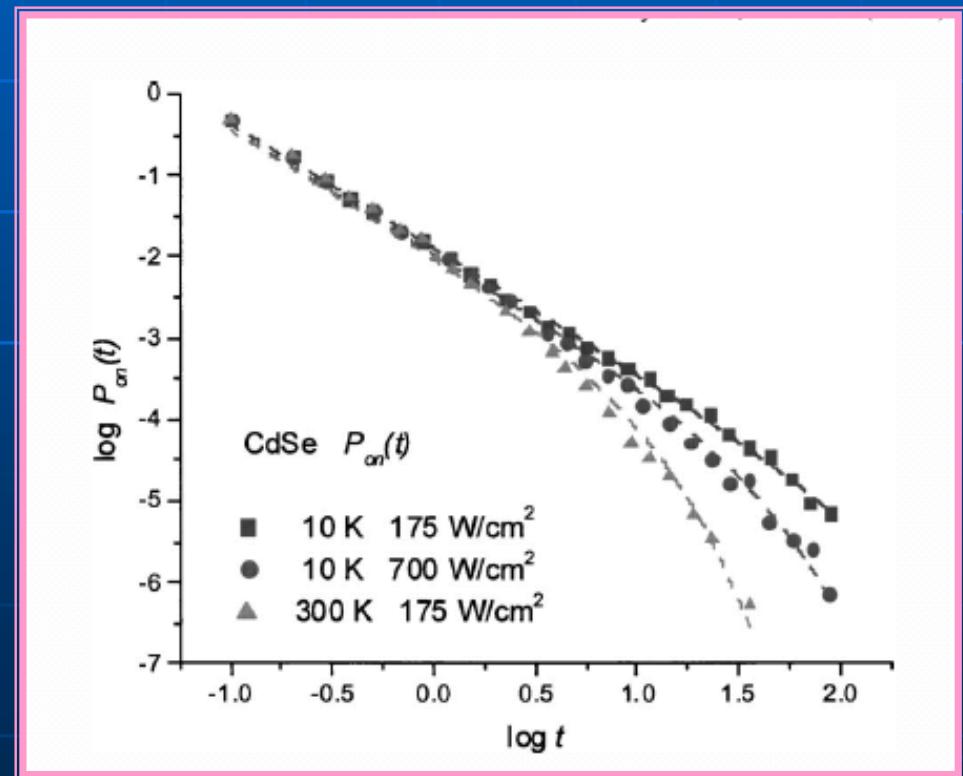
DCET model

$$\Gamma_k \tau_k \equiv \frac{E_{A,k}}{2k_B T},$$

$$E_{A,1} \equiv \frac{(\lambda + \Delta G^0)^2}{4\lambda}, \quad E_{A,2} \equiv \frac{(\lambda - \Delta G^0)^2}{4\lambda}$$



Bending rate and diffusion rate increase with:
Light intensity and QD size
Temperature (faster diffusion)



Normal vs. anomalous diffusion

$$\sigma^2(t) \sim t^{\beta_{CD}}$$

$$L_k = D_k(t) \frac{\partial}{\partial Q} \left(\frac{\partial}{\partial Q} + \frac{1}{k_B T} \frac{\partial}{\partial Q} U_k(Q) \right)$$

Debye dielectric medium

$$\bar{\chi}(s) = 1/(1+s \tau_D)$$

$$\bar{\tau}_{L,k}(s) = \tau_{L,k} \equiv \tau_{D,k} \epsilon_\infty / \epsilon_0,$$

$$\Theta_k(t) = \exp(-t/\tau_{L,k})$$

$$D_k(t) = \Delta_k^2 / \tau_{L,k}$$

$$\sigma^2(t) = \langle (Q(t) - \langle Q(t) \rangle)^2 \rangle \approx 2(\Delta^2 \tau_D / \tau_L) t$$

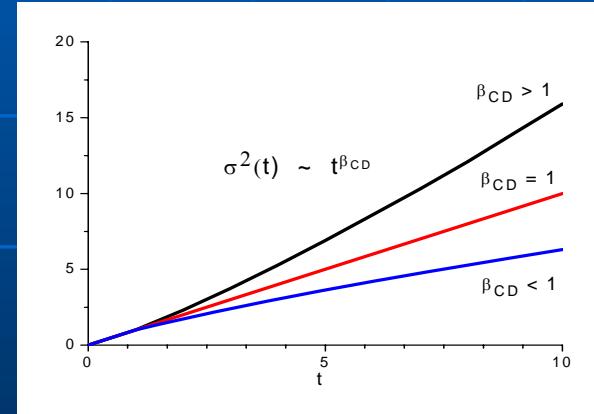
$$D_k(t) = -\Delta_k^2 \frac{d}{dt} \ln \Theta_k(t), \quad \Theta_k(t) \equiv L^{-1} \left(\frac{1}{s + \frac{1}{\tau_{L,k}(s)}} \right)$$

$$\bar{\tau}_{L,k}(s) = \frac{\epsilon_\infty}{\epsilon_0} \frac{1 - \bar{\chi}_k(s)}{s \bar{\chi}_k(s)}, \quad \bar{\chi}_k(s) = \frac{\bar{\epsilon}_k(s) - \epsilon_\infty}{\epsilon_0 - \epsilon_\infty}.$$

$\beta_{CD} < 1$ (subdiffusion)

$\beta_{CD} = 1$ (normal diffusion)

$\beta_{CD} > 1$ (superdiffusion)



Cole-Davison medium

$$\bar{\chi}(s) = 1/(1+s \tau_D)^{\beta_{CD}}$$

D(t) is time-dependent

More general anomalous diffusion

J. Tang, PR B (2007)

$$\frac{\partial}{\partial t} G(Q, Q'; t) = \frac{\partial}{\partial Q} \left(D_2(t) \frac{\partial}{\partial Q} + D_1(t) Q \right) G(Q, Q'; t) .$$

$$D_1(t) = - \frac{d}{dt} \ln \langle Q(t) \rangle$$

$$G(Q, Q'; t) = \frac{1}{\sqrt{2\pi \Delta_2(t)}} \exp \left[- \frac{(Q - Q' \Delta_1(t))^2}{2\Delta_2(t)} \right]$$

$$D_2(t) = D_1(t) \Delta_2(t) + \frac{1}{2} \frac{d}{dt} \Delta_2(t) .$$

$$\Delta_1(t) \approx 1 - (t/\tau_c)^\nu$$

Kohlrausch-Williams-Watts mode
(Havriliak-Negami dielectric function)

$$\Delta_2(t) \approx \Delta_2(\infty) (t/\tau_c)^\mu$$

$$\varepsilon(\omega) = \varepsilon_\infty + \Delta\varepsilon / \left[1 + (i\omega\tau_{HN})^\alpha \right]^\gamma$$

Ornstein-Uhlenbeck process: $\mu = \nu = 1$; D_1, D_2 : constant

Non-Debye medium (anomalous diffusion)

J. Tang and R. A. Marcus, PRL and JCP (2005)

$$\bar{P}_k(s) \approx \frac{1}{1 + ((s + \Gamma_k) t_{c,k})^{1 - \beta_{CD}/2}}$$

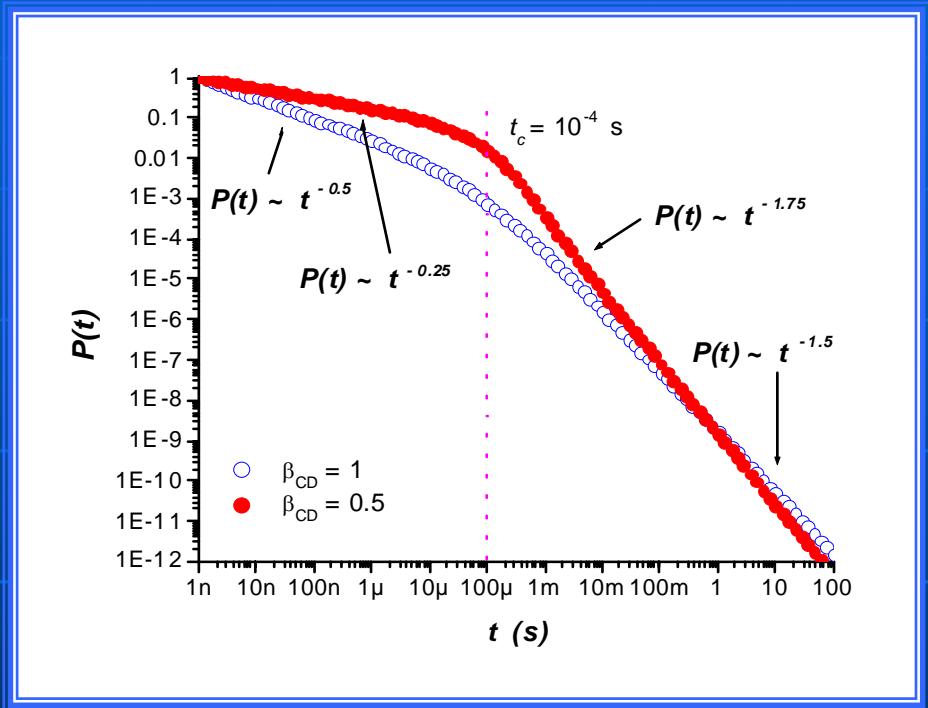
Mittag-Leffler function

$$E_a(z) = \sum_{n=0}^{\infty} z^n / \Gamma(na+1).$$

$$P_k(t) \sim \frac{d}{dt} [E_{-1+\beta_{CD}/2}(-(t/t_{c,k})^{-1+\beta_{CD}/2})] \exp(-\Gamma_k t) \quad \text{if } t \leq 1/\Gamma_k < \tau_{L,k}$$

$$P_k(t) \sim (t/t_{c,k})^{-\beta_{CD}/2} / \Gamma(1-\beta_{CD}/2) t_{c,k}, \quad \text{if } t < t_{c,k}$$

$$P_k(t) \sim (t/t_{c,k})^{-2+\beta_{CD}/2} \exp(-\Gamma_k t) / \left| \Gamma(\beta_{CD}/2 - 1) \right| t_{c,k} \quad \text{if } t_{c,k} < t \leq 1/\Gamma_k < \tau_{L,k}.$$



Blinking statistics of nanorods and size dependence

J. Tang, JPC (2007)

$$P(t) \approx P_0 (t / \tau_c)^{-2+\mu/2} \exp(-(\Gamma t)^{2\nu-\mu})$$

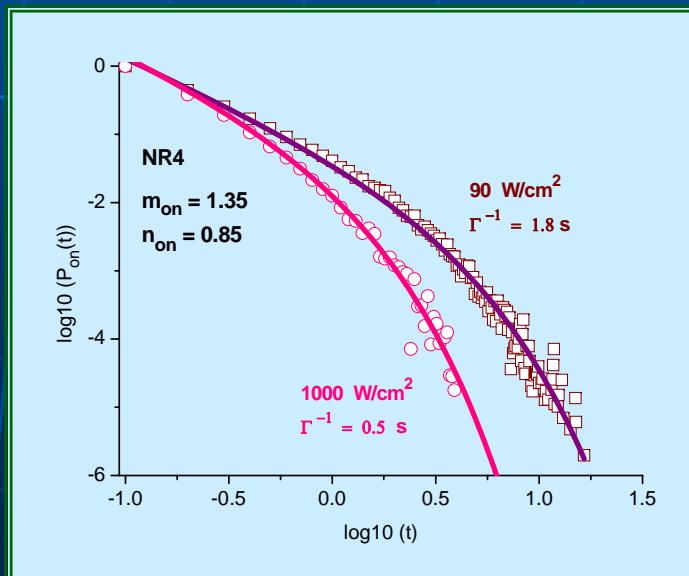
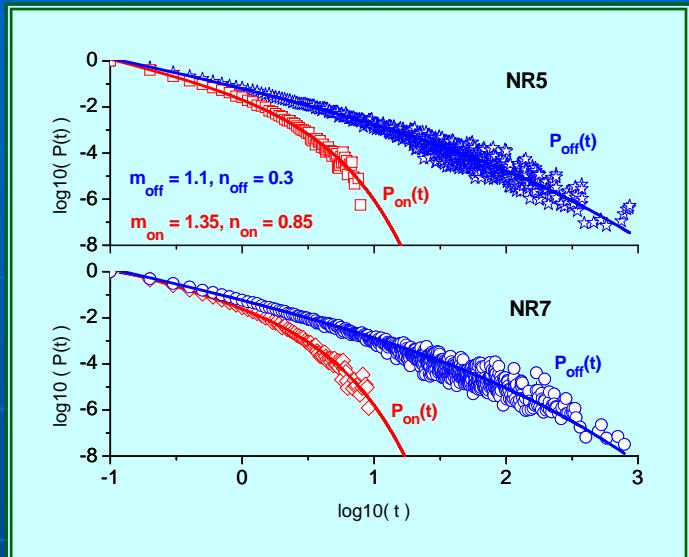
$$\propto t^{-m} \exp(-(\Gamma t)^n),$$

$$m = 2 - \mu/2$$

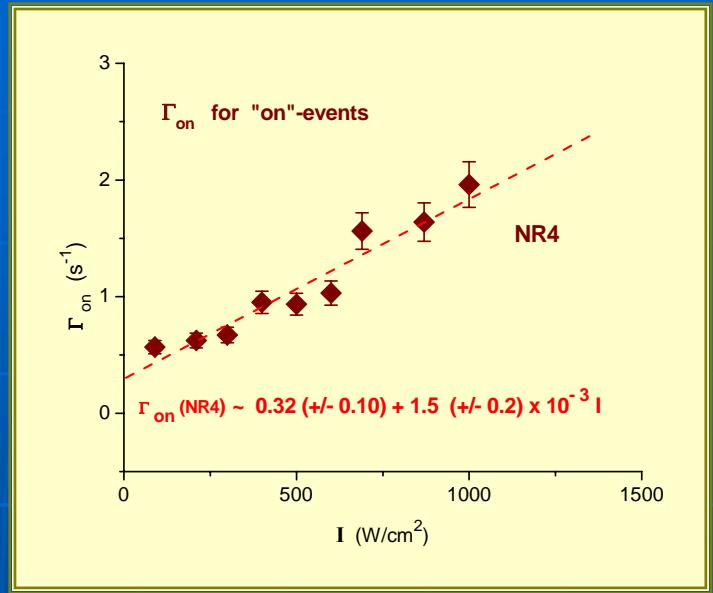
$$n = 2\nu - \mu$$

Size dependence

Intensity dependence

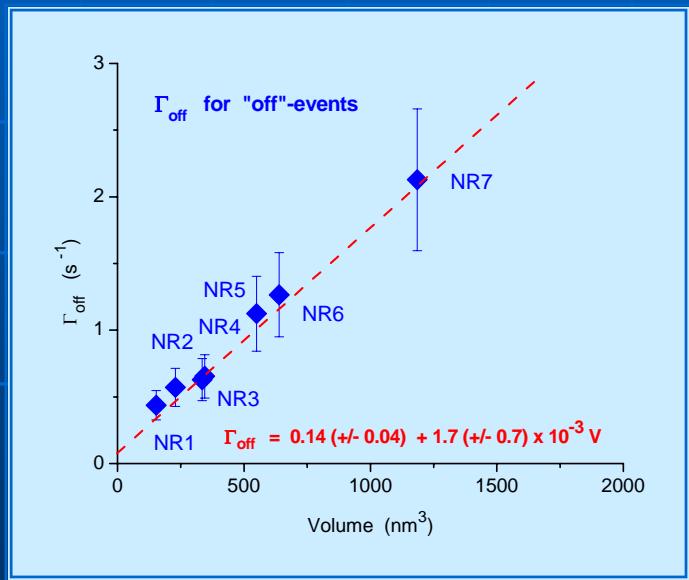


Light NRs



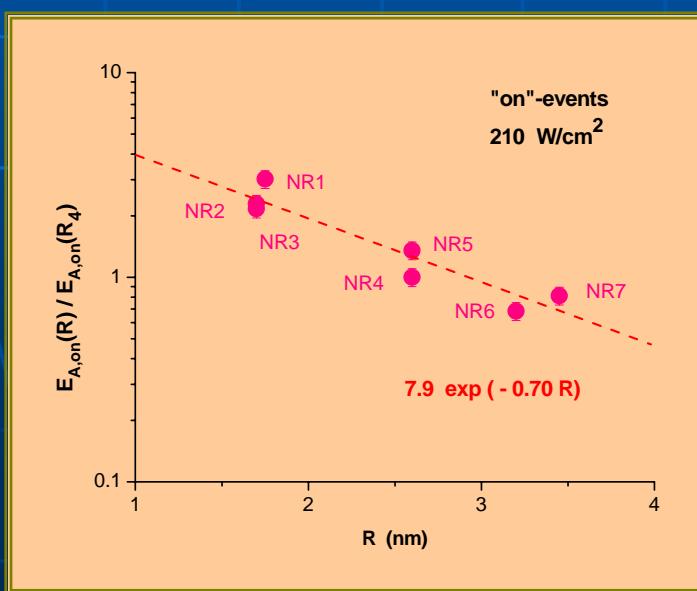
Intensity dependence

Dark NRs

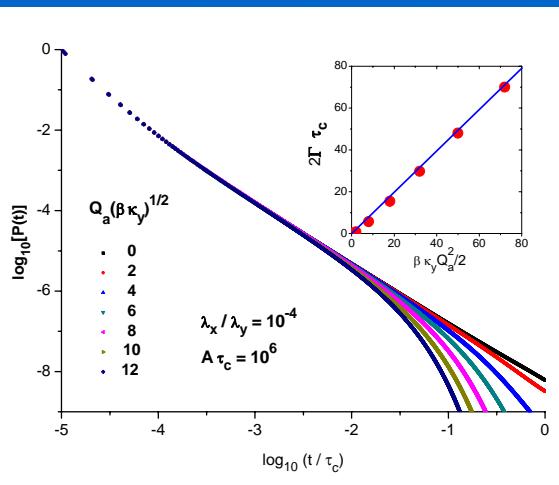


Volume dependence

Radius dependence

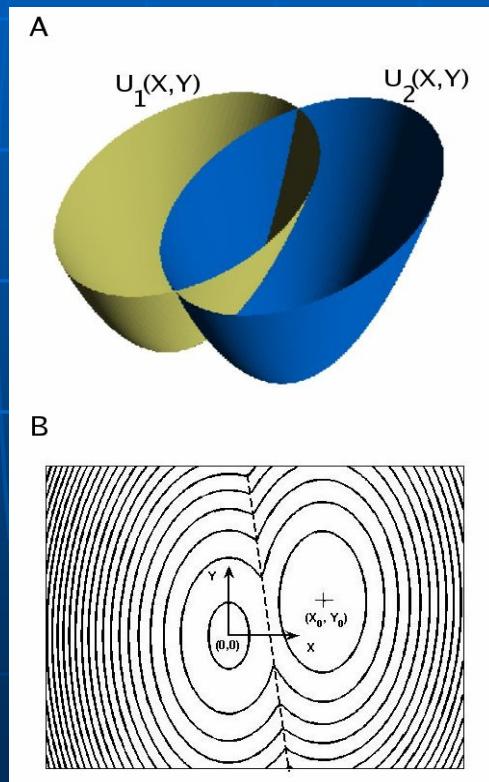
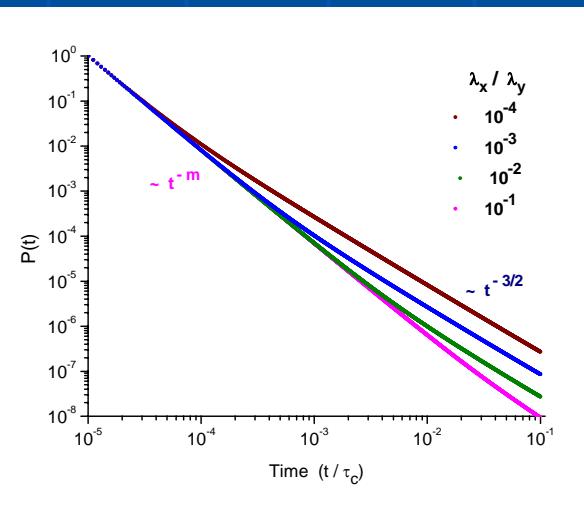


Exotic power law for Si blinking



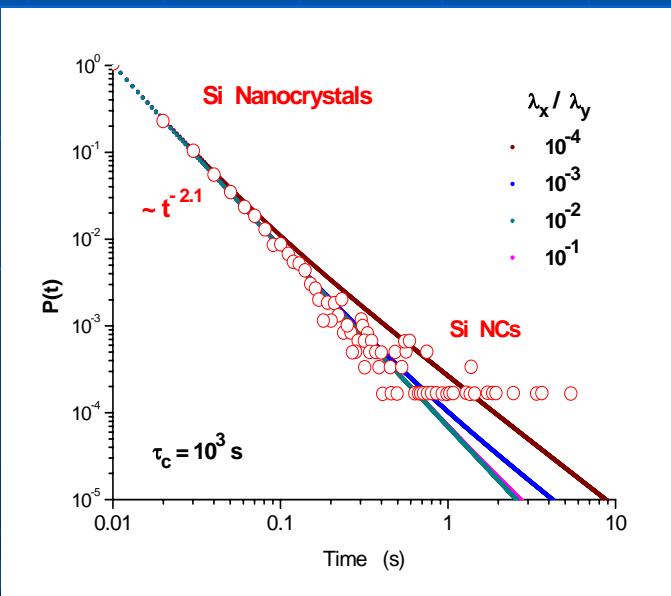
J. Tang, JCP (2007)

2-D model
Fast and slow
reaction coordinates



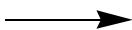
Crossing point is not like
A δ -function

Power-law exponent
Could be greater than 2

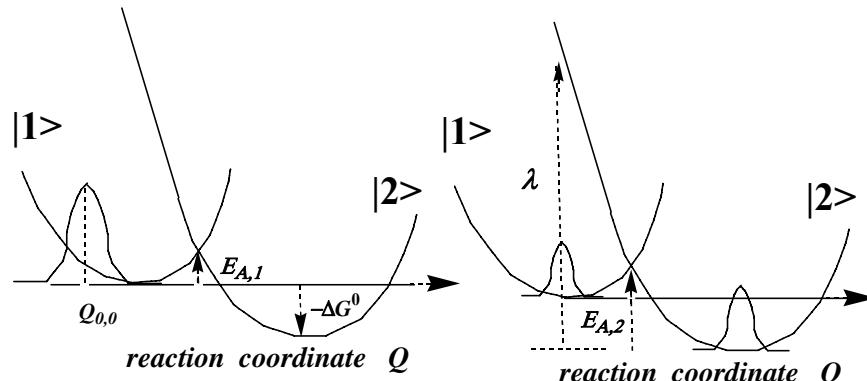


Ensemble fluorescence intensity decay

short time

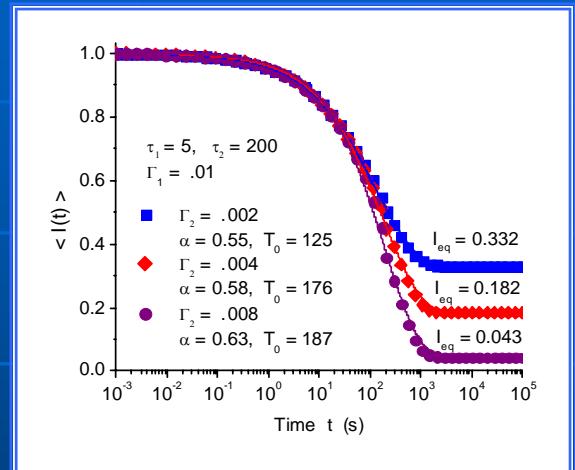


long time



stretched exponential decay

$$I_{eq} + (1 - I_{eq}) \exp(-(t/T_0)^\alpha)$$



Single particle

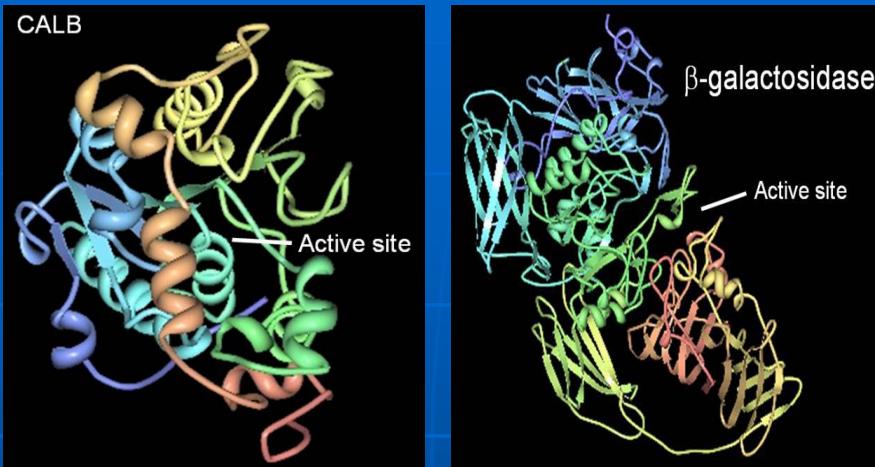
vs.

ensemble

- Power-law decay
- distinguishable
- at the crossing initially
- nonergodic

- Stretched exponential decay
- indistinguishable
- in the ground state initially
- ergodic

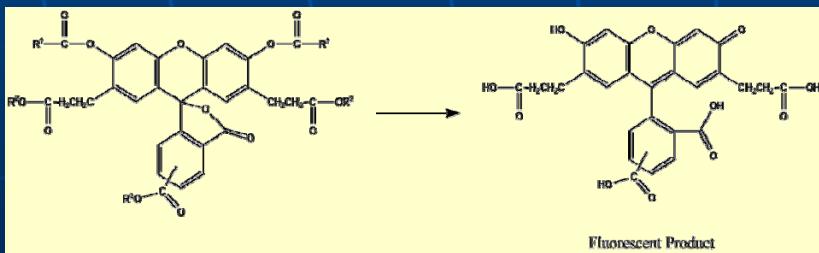
Intermittency of single-enzyme activity



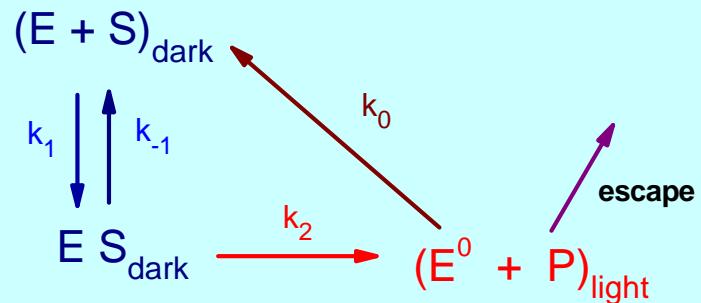
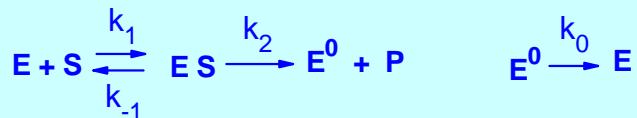
X. S. Xie, et al. Nature Chem. Bio. 2, 87 (2005)

O. Flomenbom, et al., PNAS 102, 2368 (2005)

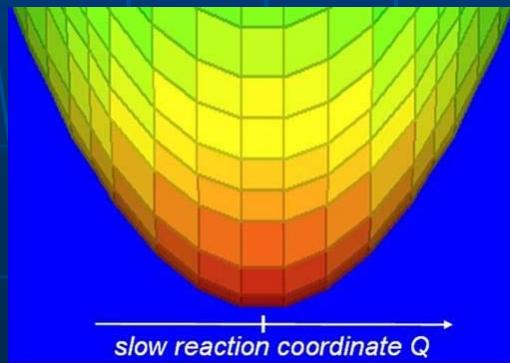
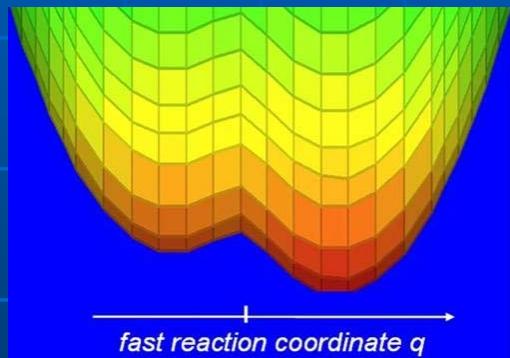
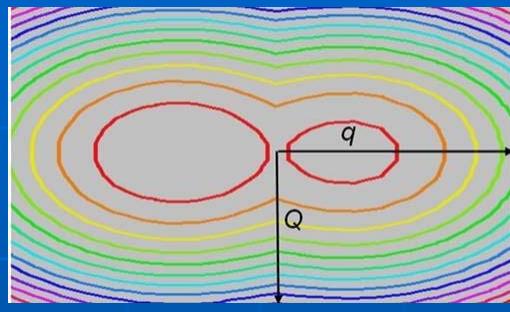
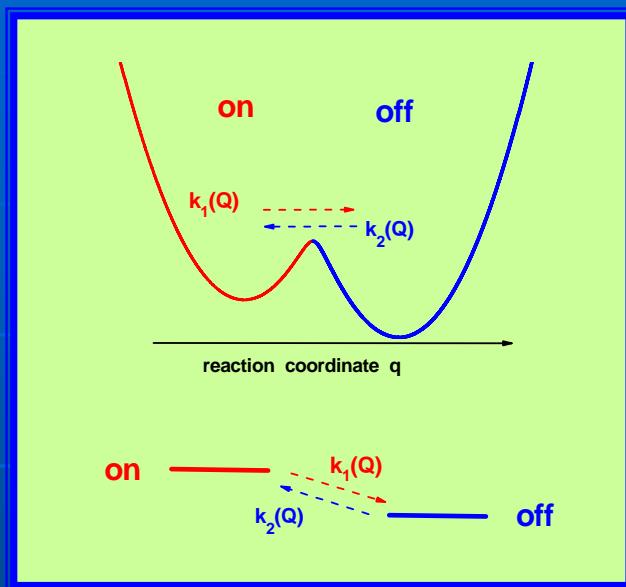
J. Tang, JPC (submitted)



Michaelis-Menten Scheme

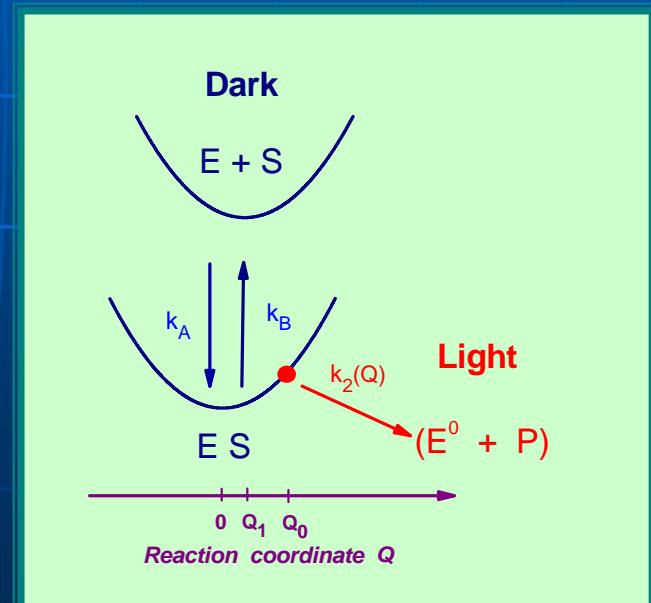


Michaelis-Menten scheme for enzyme reactions

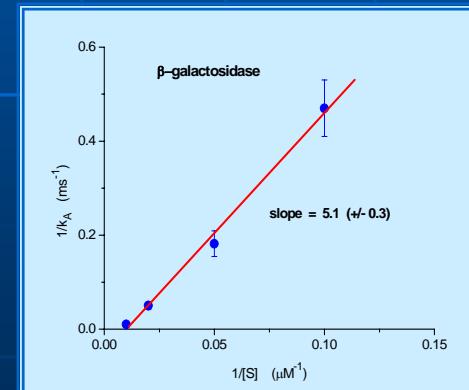
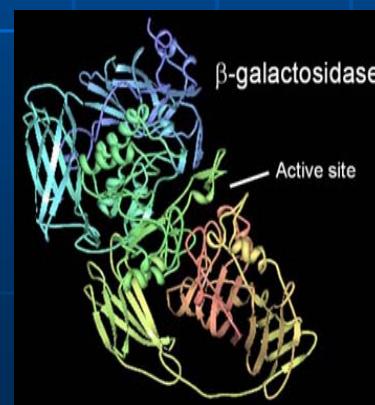
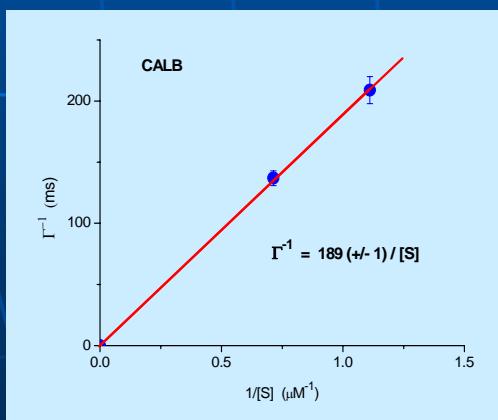
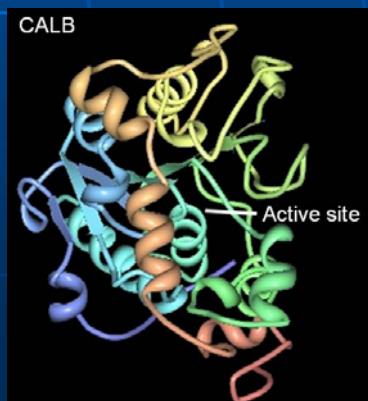
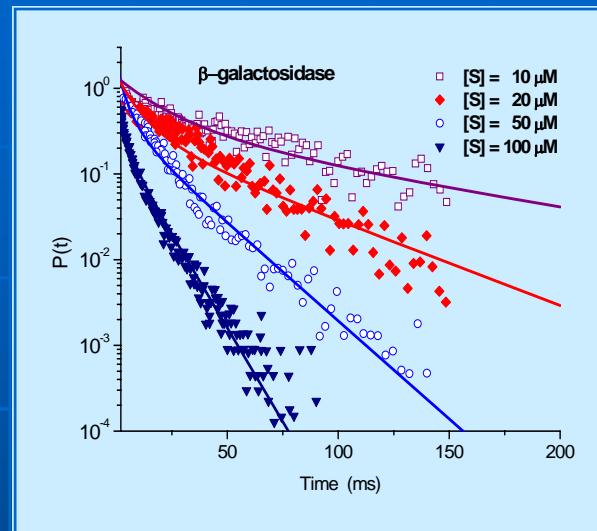
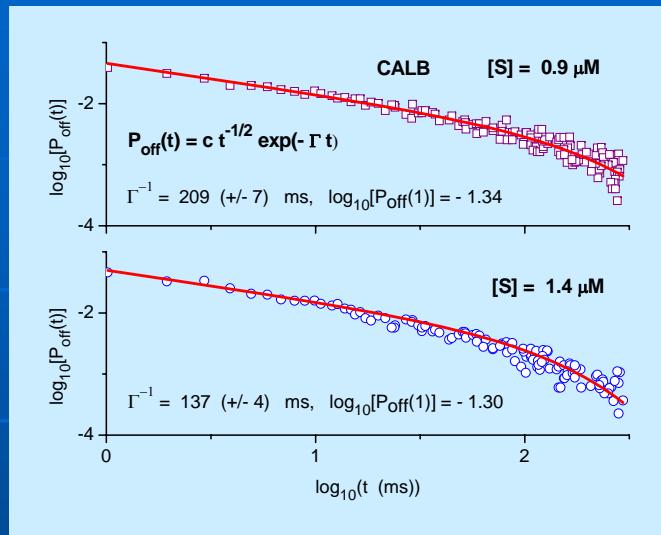


$$P(t) = \frac{k_A k_2}{\lambda_2 - \lambda_1} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t} \right)$$

$$\lambda_{1,2} = k_A + k_B + k_2 \mp \sqrt{(k_A + k_B + k_2)^2 / 4 - k_A k_B}.$$



Diffusion-controlled Michael-Menten reaction

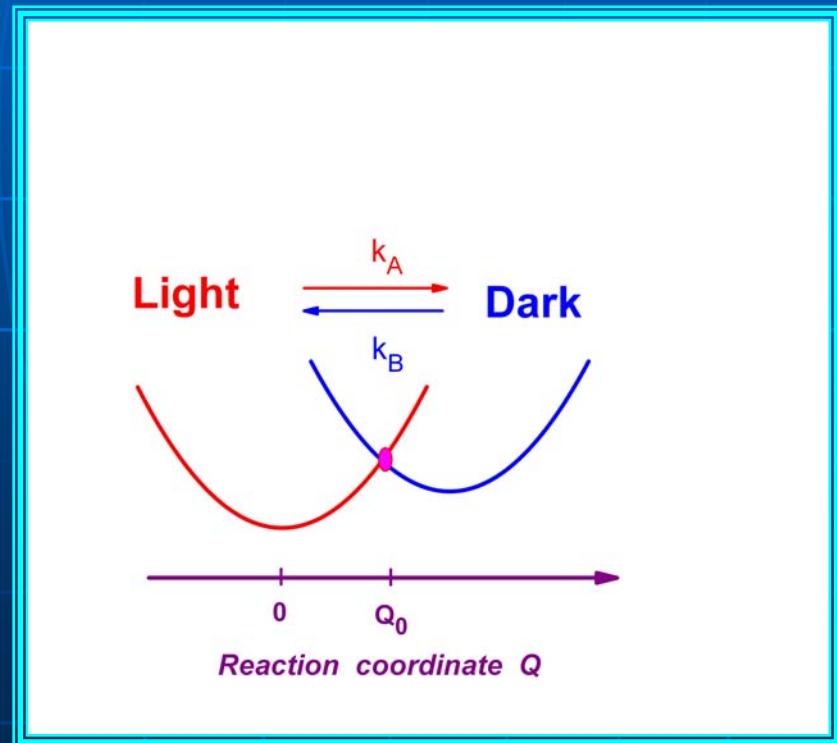


Fluctuations of potential barrier height

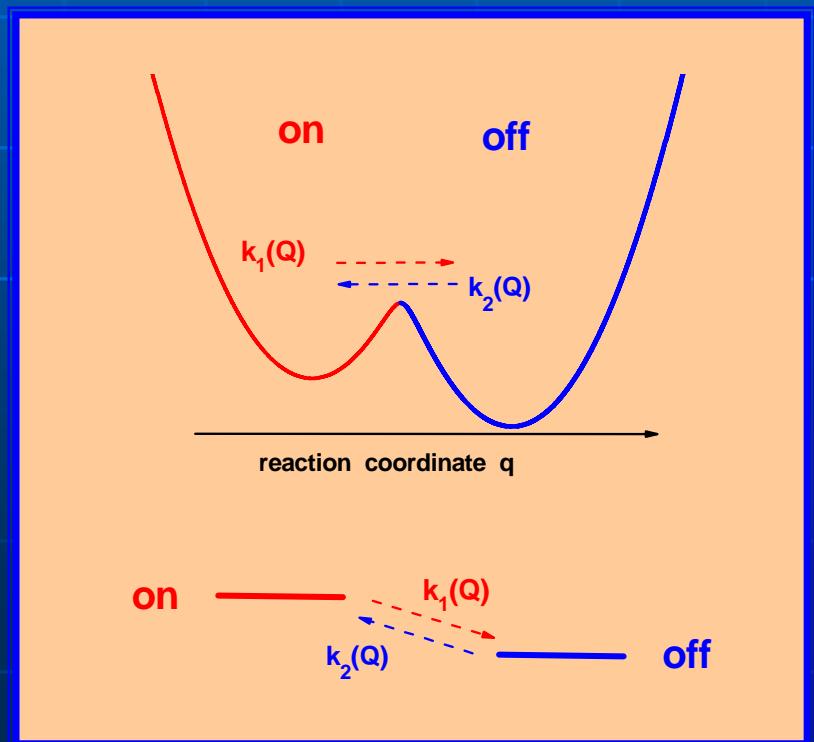


Non-exponential blinking statistics

Charge transfer reactions



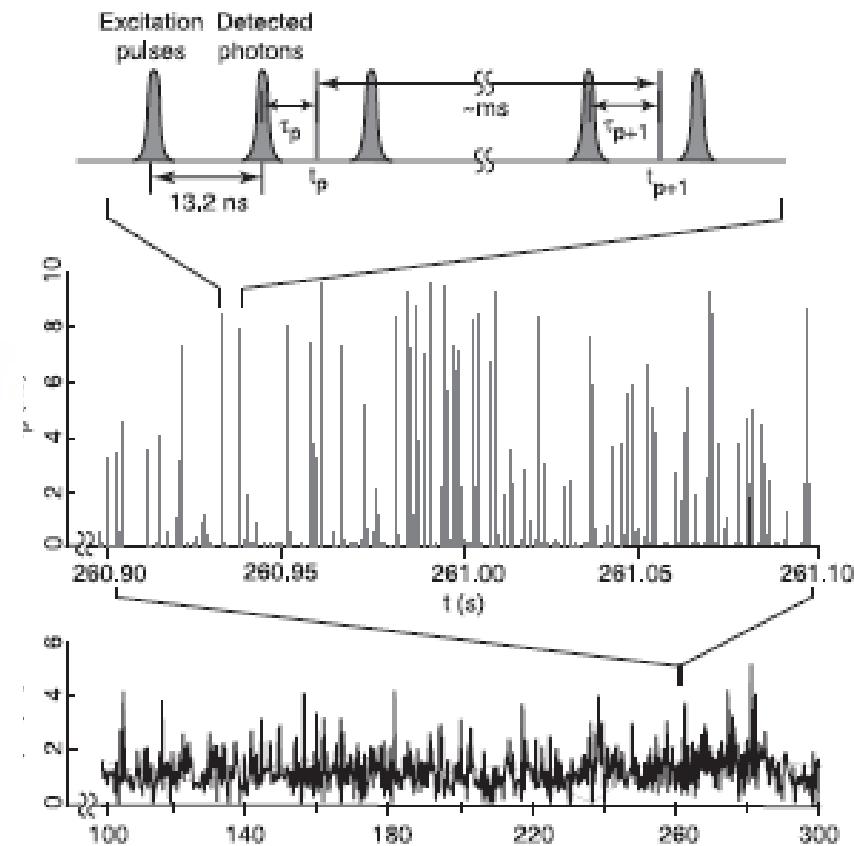
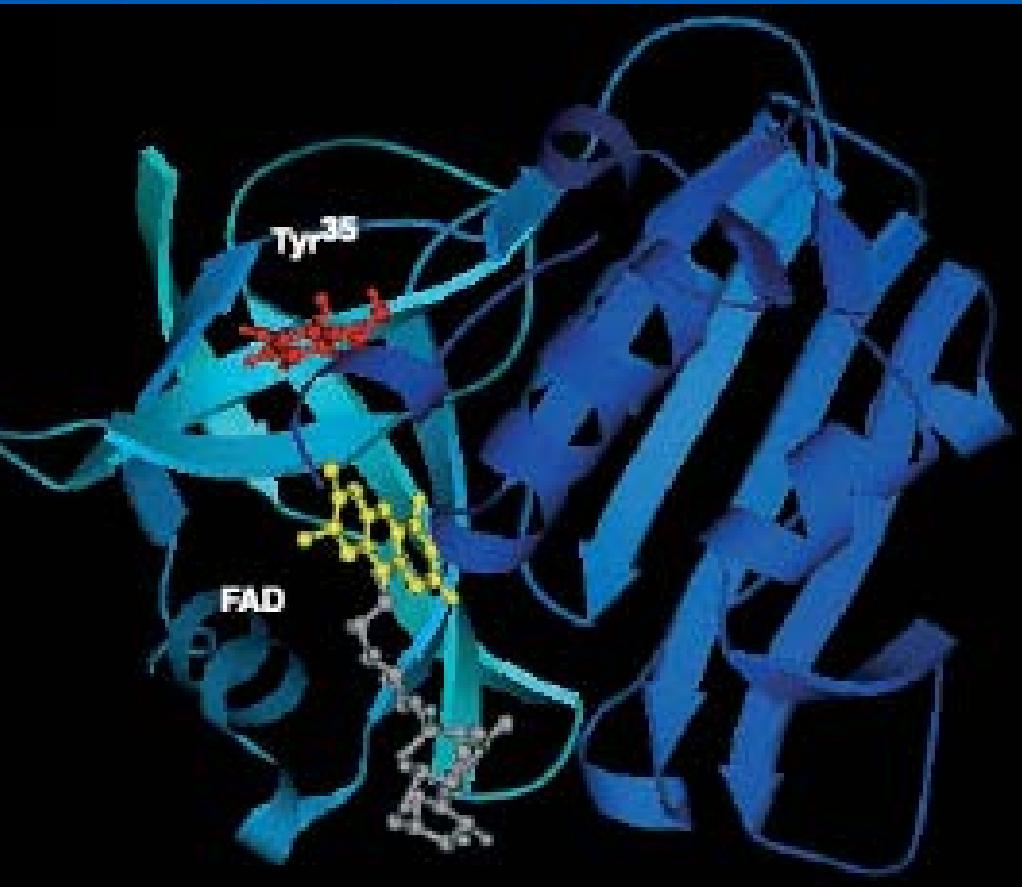
Catalytic reactions



Fluctuating electron transfer rates in single protein molecules

Fre/FAD complex

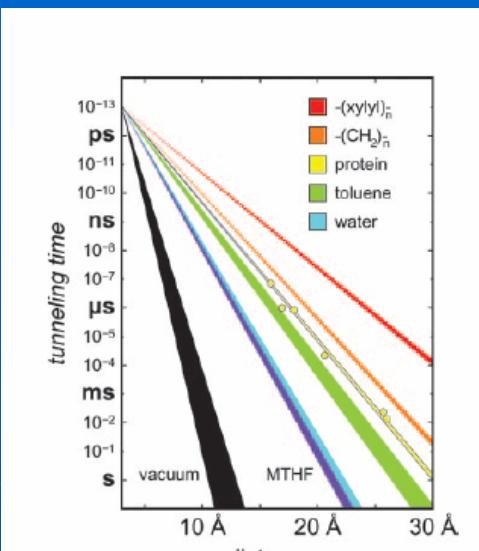
H. Yang *et al.* *Science* 302, 262 (2003)



Distance-fluctuation model

$$\gamma = \frac{2\pi |V_{ex}|^2}{\hbar \sqrt{4\pi \lambda k_B T}} \exp\left(-(\lambda + \Delta G^0)^2 / 4\lambda k_B T\right)$$

$$|V_{ex}|^2 \sim \exp(-\beta_{DA} \ell(t))$$



If ET is activationless $\Delta G^0 = -\lambda$

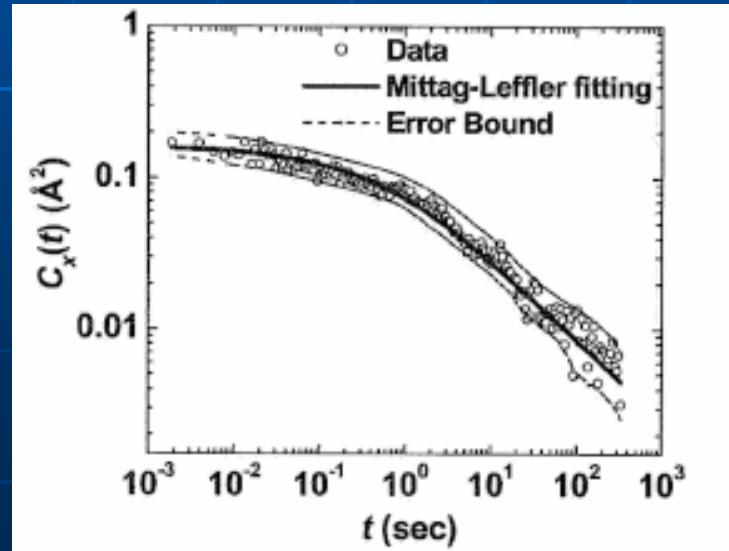
$$C_2(t) \equiv \left\langle \delta \gamma^{-1}(t) \delta \gamma^{-1}(0) \right\rangle / \left\langle \gamma^{-1}(t) \right\rangle \left\langle \gamma^{-1}(0) \right\rangle$$

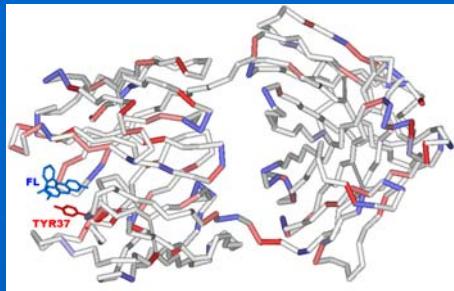
$$ET \text{ rate} \sim \exp(-\beta_{DA} \ell)$$

$$C_2(t) \equiv \exp(\beta^2 C_Q(t)) - 1$$

$$C_Q(t) \equiv \left\langle \delta \ell(t) \delta \ell(0) \right\rangle,$$

$$\delta \ell(t) = \ell(t) - \left\langle \ell(t) \right\rangle$$

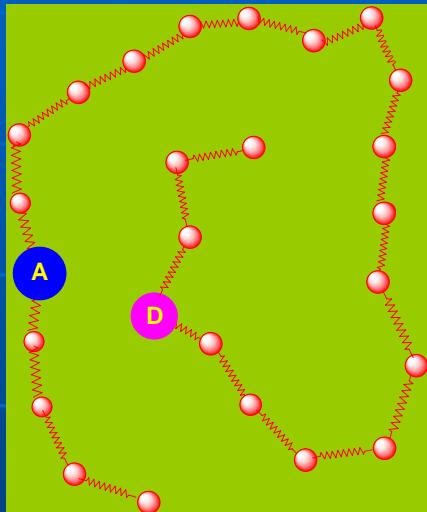




Distance-fluctuation model of a Rouse chain

J. Tang and R. A. Marcus, PR E (2006)

Polymer, DNA, protein



- Neutron scattering, NMR

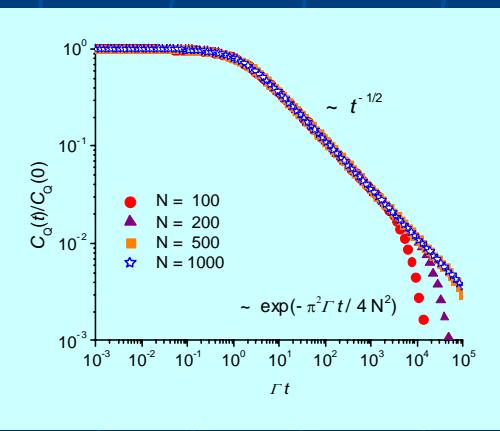
$$\frac{d}{dt} \begin{pmatrix} Q_0 \\ Q_1 \\ \vdots \\ Q_{N-1} \end{pmatrix} + \frac{\omega^2}{\zeta} \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} Q_0 \\ Q_1 \\ \vdots \\ Q_{N-1} \end{pmatrix} = \begin{pmatrix} F_0(t)/m\gamma \\ F_1(t)/m\gamma \\ \vdots \\ F_{N-1}(t)/m\gamma \end{pmatrix}$$

$$\zeta \frac{d}{dt} Q(t) + \omega^2 \mathcal{R} Q(t) = F(t)/m$$

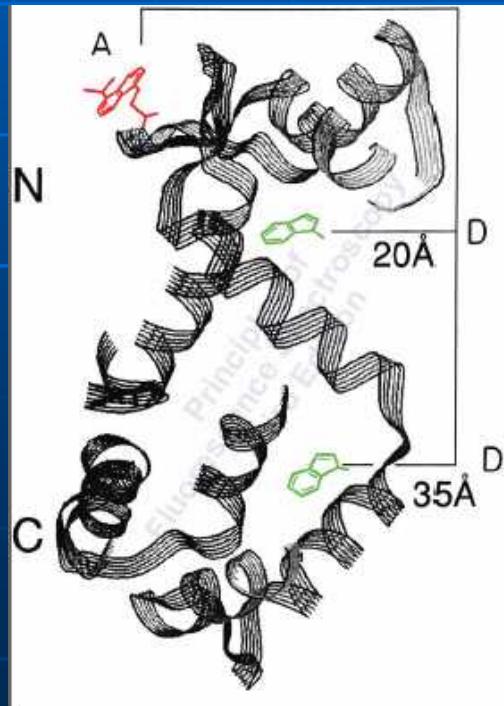
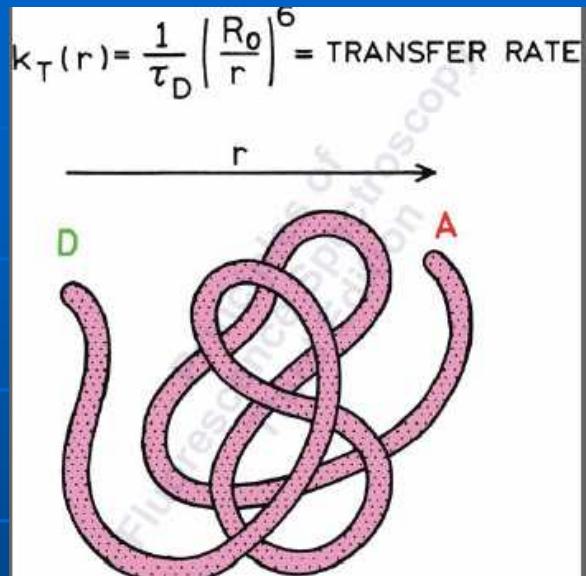
$$\langle F_i^\mu(t) \rangle = 0$$

$$\langle F_i^\mu(t) q_j^\nu(t) \rangle = 0$$

$$\langle F_i^\mu(t) F_j^\nu(\tau) \rangle = 2m k_B T \zeta \delta(t-\tau) \delta_{ij} \delta^{\mu\nu}$$

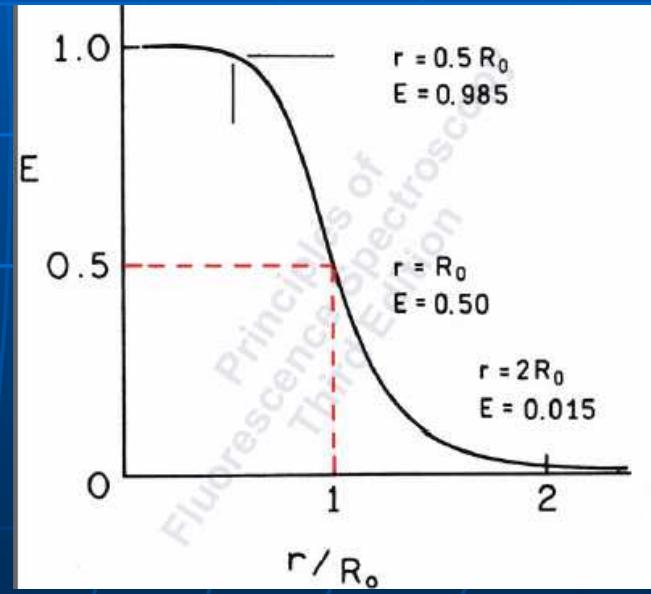


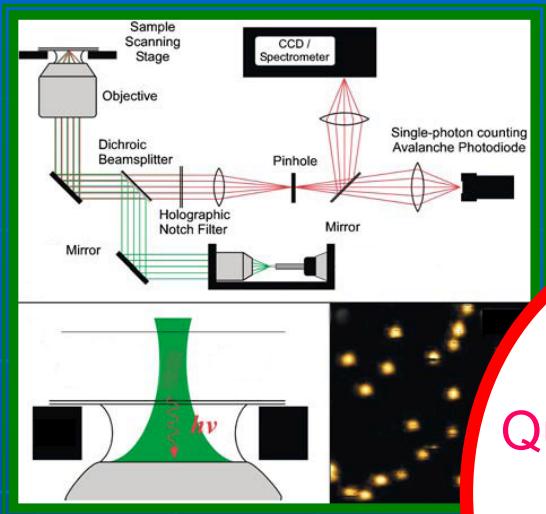
Förster's Fluorescence resonance energy transfer (FRET)



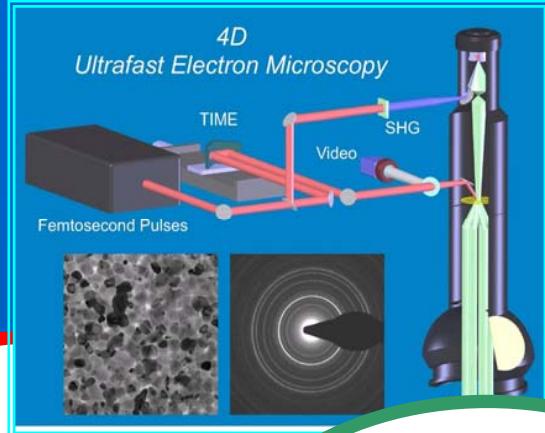
Efficient of energy transfer

$$E = \frac{k_T(r)}{\tau_D^{-1} + k_T(r)}$$

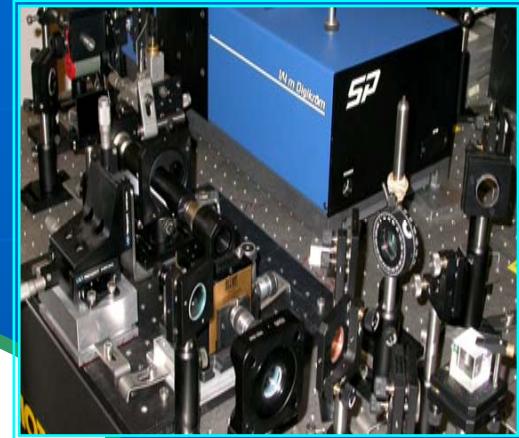




Quantum dots,
Nanorods,
Biomolecules



Photochemistry,
Photovoltaics



Light-induced
phase transitions

Protein dynamics &
Biomed applications



