

HW 3

Solution

A 1. (a) $E_{(n_x, n_y, n_z)} = \underbrace{\frac{\pi^2 \hbar^2}{2m L^2}}_{\epsilon_0} \left(n_x^2 + n_y^2 + \left(\frac{n_z}{2}\right)^2 \right)$

$$\epsilon_0 = 6.02 \times 10^{-19} \text{ erg}$$

lowest energy state $(1, 1, 1)$ $E = (1 + 1 + \frac{1}{4}) \epsilon_0$

next $(1, 1, 2)$ $E = (1 + 1 + 1) \epsilon_0$

difference = $\frac{3}{4} \epsilon_0$

$$= 4.5 \times 10^{-19} \text{ erg} = 2.8 \times 10^{-7} \text{ eV} = 3.27 \text{ mK}$$

(b) ratio of probabilities = $e^{-\Delta E/k_B T} \approx \boxed{} e^{-0.0327}$

$$\approx 0.968$$

B since $E \propto \frac{1}{L^2}$, then get immediately $[\Delta E = 10^2 \times \text{previous answer}]$

$$\Delta E = 4.5 \times 10^{-17} \text{ erg} = 2.8 \times 10^{-5} \text{ eV} = 327 \text{ mK}$$

ratio of prob = $e^{-3.27} = 0.038$

(much smaller !!)

2.

$$E(n_x, n_y, k_z) = \underbrace{\frac{\pi^2 \hbar^2}{2mL^2}}_{\epsilon_0} \left(\left(\frac{n_x}{2} \right)^2 + 1 \right) + \frac{\hbar^2 k_z^2}{2m}$$

lowest $(1, 1)$: $E = \frac{5}{4} \epsilon_0 + \frac{\hbar^2 k_z^2}{2m}$

next $(2, 1)$ $E = 2 \epsilon_0 + \frac{\hbar^2 k_z^2}{2m}$

3rd $(3, 1)$ $E = \frac{13}{4} \epsilon_0 + \frac{\hbar^2 k_z^2}{2m}$

$\{ (1, 2) \text{ has } \frac{17}{4} \epsilon_0 + \frac{\hbar^2 k_z^2}{2m} \text{ is higher than } (3, 1) \}$

