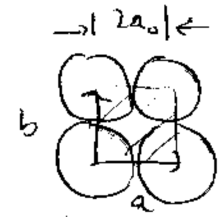
Mid-term solution

1 A (a)

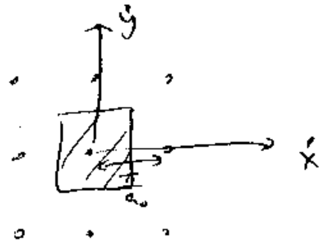


$\vec{a} = 2a_0 \hat{x}$
$\vec{b} = 2a_0 \hat{y}$

(b) size $= (2a_0)^2 = 4a_0^2$

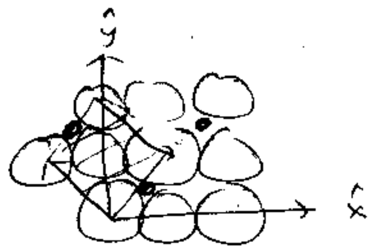
(c) one

(d) $\vec{a}^* = \frac{2\pi}{(2a_0)} \hat{x} = \frac{\pi}{a_0} \hat{x}$ $\vec{b}^* = \frac{\pi}{a_0} \hat{y}$



note: this picture is in RECIPROCAL SPACE

1 B (a)



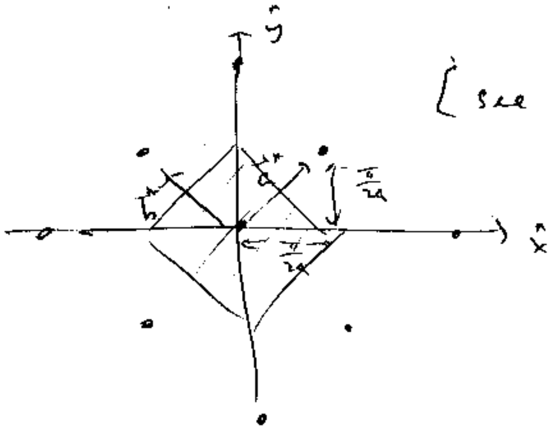
$\vec{a} = 2a_0 (\hat{x} + \hat{y})$
$\vec{b} = 2a_0 (-\hat{x} + \hat{y})$

(b) $8a_0^2$ $[\ = |\vec{a} \times \vec{b}|\ ]$

(c) two

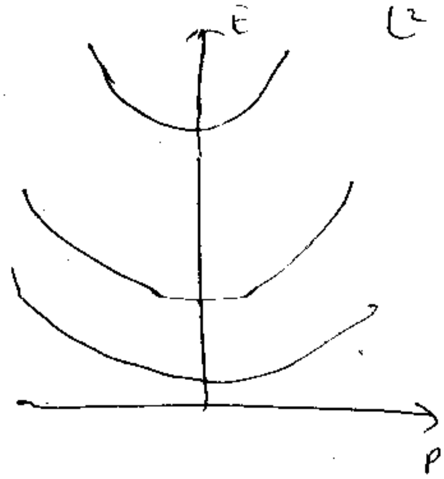
(d) $\vec{a}^* = 2\pi \frac{2a_0 (\hat{x} + \hat{y}) \times \hat{z}}{8a_0^2} = \frac{\pi}{2a} (\hat{x} + \hat{y})$

$\vec{b}^* = \frac{\pi}{2a_0} (-\hat{x} + \hat{y})$



[see also HW 4]

2 Let $\epsilon_0 = \frac{\pi^2 \hbar^2}{2mL^2}$



(a) energy bands.

$$E_1 = \epsilon_0 + \frac{p^2}{2m}$$

$$E_2 = 4\epsilon_0 + \frac{p^2}{2m}$$

$$E_3 = 9\epsilon_0 + \frac{p^2}{2m}$$

(b) First note that if a band in $\underline{\underline{2D}}$ is occupied to p_{max}, then number of electrons per unit $\underline{\text{area}}$ is

$$\underset{\substack{\uparrow \\ \text{spin}}}{2} \frac{\pi p_m^2}{(2\pi\hbar)^2} = \frac{1}{2\pi}\left(\frac{p_m}{\hbar}\right)^2$$

If only first band is occupied, then $E_{max} = 4\epsilon_0$

$$\epsilon_0 + \frac{p_m^2}{2m} = 4\epsilon_0 \qquad p_m^2 = 2m \cdot 3\epsilon_0$$

$$\therefore \text{density} = \frac{1}{2\pi} \frac{2m\cdot 3}{\hbar^2} \cdot \frac{\pi^2\hbar^2}{2mL^2} = \frac{3}{2}\frac{\pi}{L^2}$$

(c) if only 1st + 2nd band occupied, then

band 1: $\epsilon_0 + \frac{p_1^2}{2m} = 9\epsilon_0 \qquad p_1^2 = (2m)(8\epsilon_0)$

band 2: $4\epsilon_0 + \frac{p_2^2}{2m} = 9\epsilon_0 \qquad p_2^2 = (2m)(5\epsilon_0)$

$$\text{total density} = \frac{1}{2\pi}\frac{(p_1^2 + p_2^2)}{\hbar^2} = \frac{1}{2\pi}\frac{(2m)}{\hbar^2}(8+5)\cdot\frac{\pi^2\hbar^2}{2mL^2}$$

$$= \frac{\pi}{2}\cdot 13 \cdot \frac{1}{L^2}$$

alternatively, density of states per spin is $\frac{m}{2\pi\hbar^2}$

$\therefore$ if occupied only have 1,
then energy between ϵ_0 & $4\epsilon_0$

(a): total : $\underset{\substack{\uparrow \\ \text{spin}}}{2} \frac{m}{2\pi\hbar^2}(3\epsilon_0) = 2 \cdot \frac{m}{2\pi\hbar^2} \cdot 3 \cdot \frac{\pi^2\hbar^2}{2mL^2}$

$= \frac{3\pi}{2L^2}$ as before

(b) $2 \cdot \frac{m}{2\pi\hbar^2}\left[\underbrace{(9\epsilon_0 - \epsilon_0) + (9\epsilon_0 - 4\epsilon_0)}_{13\epsilon_0}\right]$

$= \frac{13\pi}{2L^2}$.

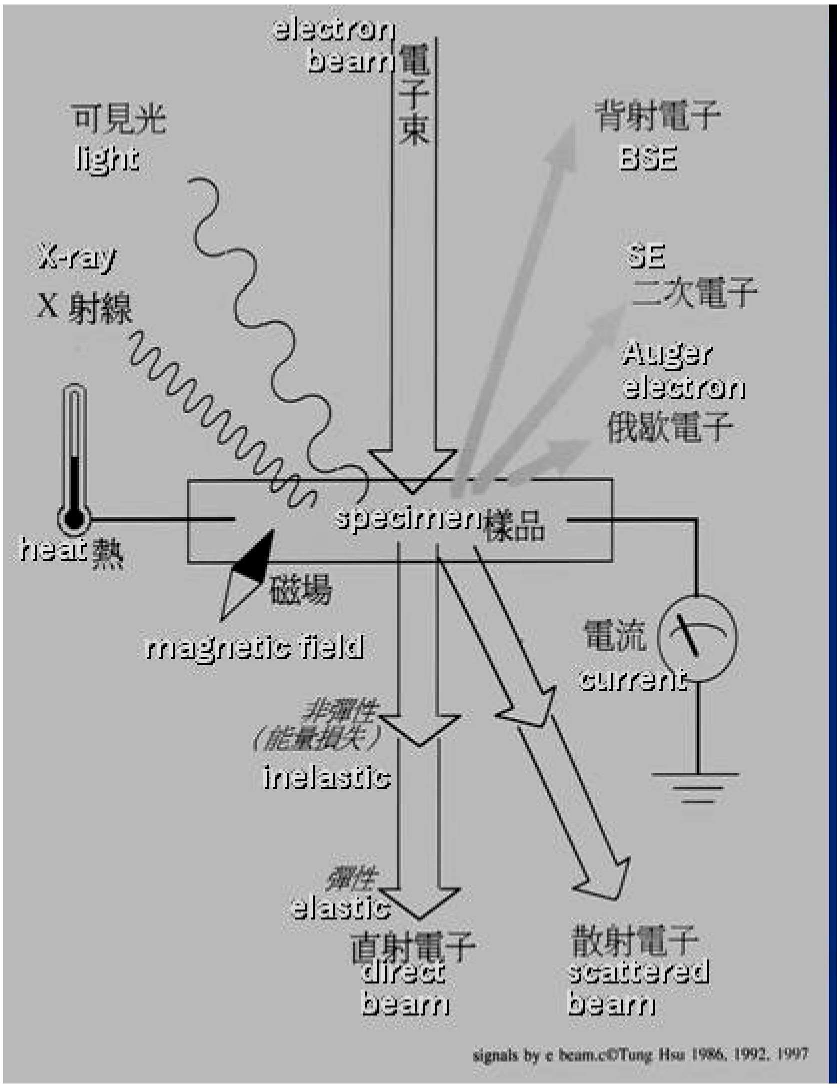
3 possible answers: (consult textbooks for details)

Phonons: thermal capacity
thermal conduction
electrical resistivity by electron-phonon scattering
optical properties

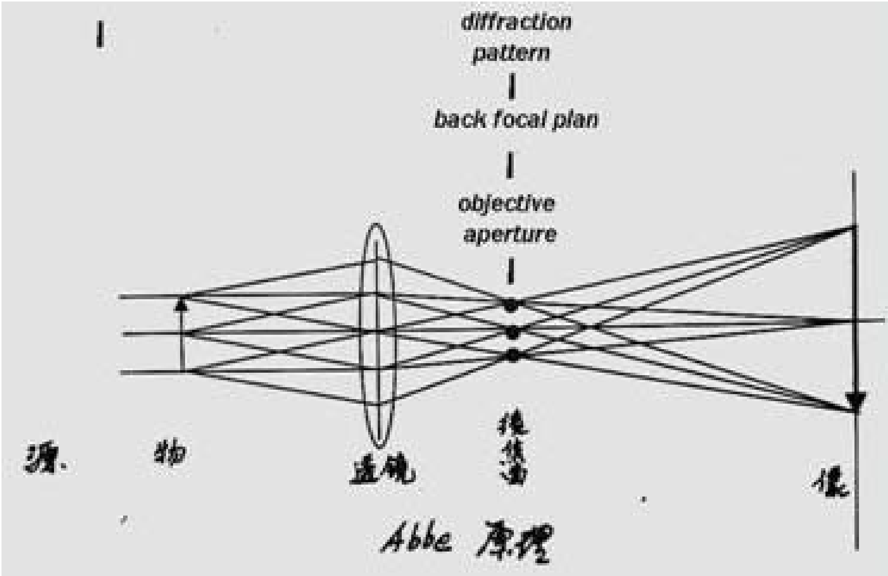
Defects: color centers
electrical resistivity by scattering in metals
electrical conductivity in semiconductors by doping

Keys to Midterm Exam:

Q4: Signals emitted from a solid specimen when it is bombarded by a high energy electron beam.



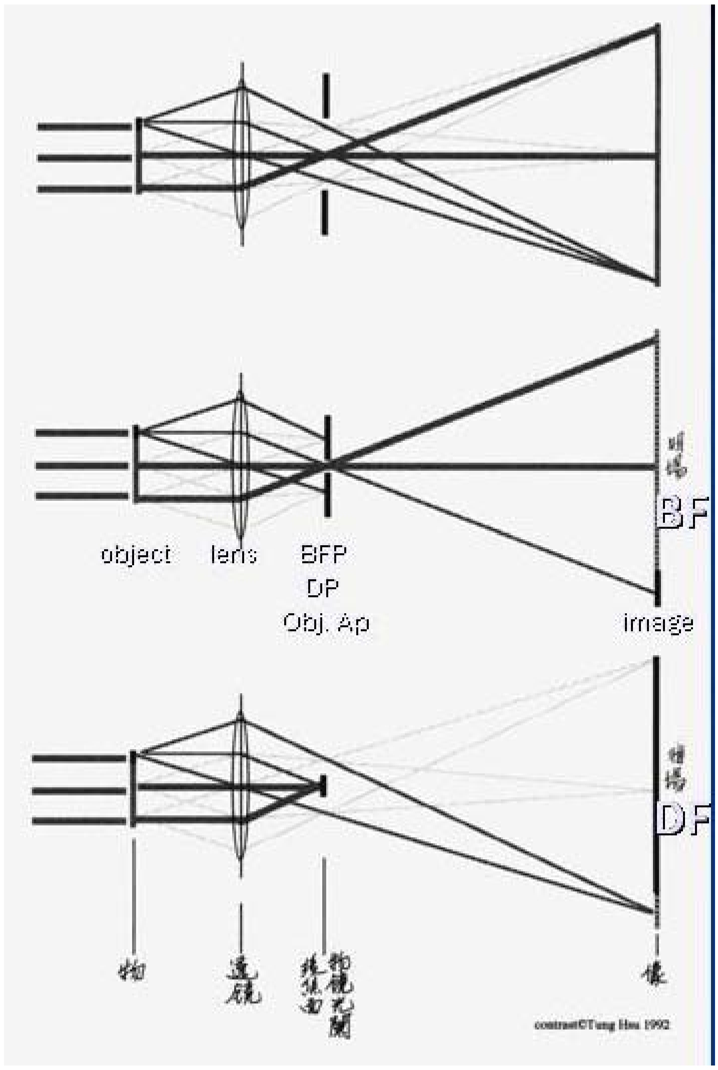
Q4.(a) and (b): Abbe's principle of imaging.



|← ~3 cm →|← ~2cm→ |

(c) The magnification of the image is approximately 2, or more specifically, -2.

Q6: BF and DF images. The specimen is a thin low atomic number film with a small high atomic number spot on it. An objective aperture is used to select the transmitted (or direct beam) beam to form the bright field image, or to block the transmitted beam to form the dark field image. In practice the aperture selects one scattered beam while block the transmitted beam and all other scattered beams.



Q7. This is a problem related to the scanning tunneling microscopy (STM).

(a) Describe how the piezoelectric tube scanner works in the STM? (5%)

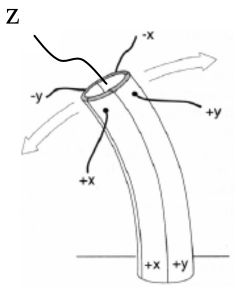
(b) Write down and sketch the relation of the tunneling current with the bias voltage applied between the tip and sample. (5%)

(c) What part of the sample's electronic structure is acquired with a positive bias applied to the tip? what about if the bias voltage is reversed? (5%)

Ans:

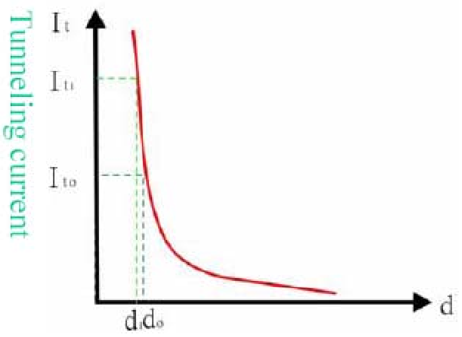
(a)

A tube made of PZT, metallized on the outer and inner surfaces, is poled in the radial direction. The outside metal coating is sectioned into four quadrants, which are designated to +x, +y, -x, -y, counterclockwise, as shown in the figure. The inner coating is connected to z. A $+V_x$ and $-V_x$ voltages are applied on the opposite x quadrants to bend the tube in the x direction. Likewise, the bending in the y direction is operated in the similar way. Since the voltage sum on the outer quadrants is kept at zero, the motion in z direction is adjusted by varying the voltage applied to the inner z electrode.



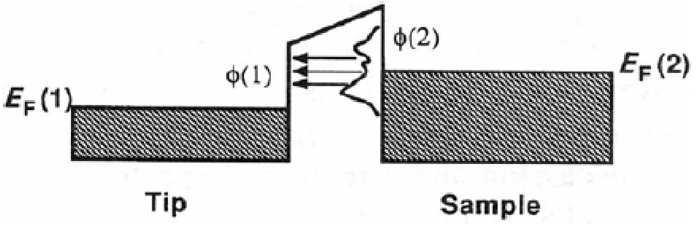
(b)

$$I_t \propto (V/d)\exp(-A\phi^{1/2}d)$$

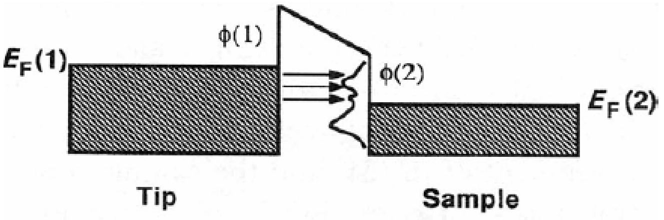


(c)

When the tip is biased positively, the filled density of states of the sample is probed.

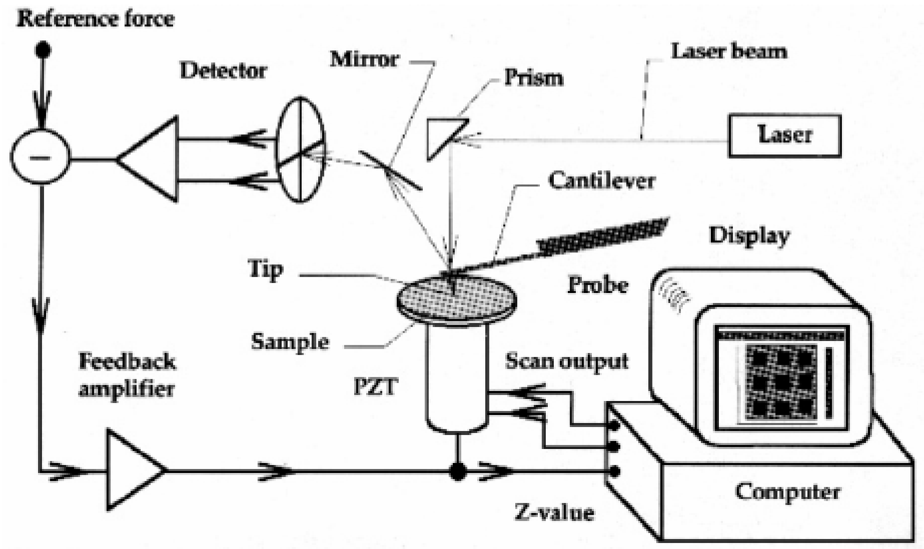


When the tip is biased negatively, the empty density of states of the sample is probed.



Q8. Sketch the core components of an atomic force microscope including the cantilever probe, sample, scanner, laser, photodiode detector, computer and feedback controller, and briefly describe how the beam deflection mechanism works. (10%)

Ans:



When the AFM probe (tip and cantilever) is scanned against the sample, the angular deflection of the cantilever causes a twofold larger angular deflection of the laser beam shone on its backside. The reflected laser beam strikes a position-sensitive photodetector consisting of two side-by-side photodiodes. The difference between the two photodiode signals indicates the position of the laser spot on the detector and thus the angular deflection of the cantilever.