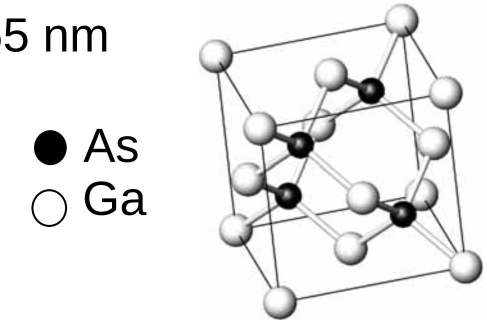
Introduction to Nanotechnology

Chapter 9 Quantum Wells, Wires and Dots

Surface issue:

For a nano-meter cube, the surface to volume ratio increases with decreasing size

For GaAs, a=0.565 nm



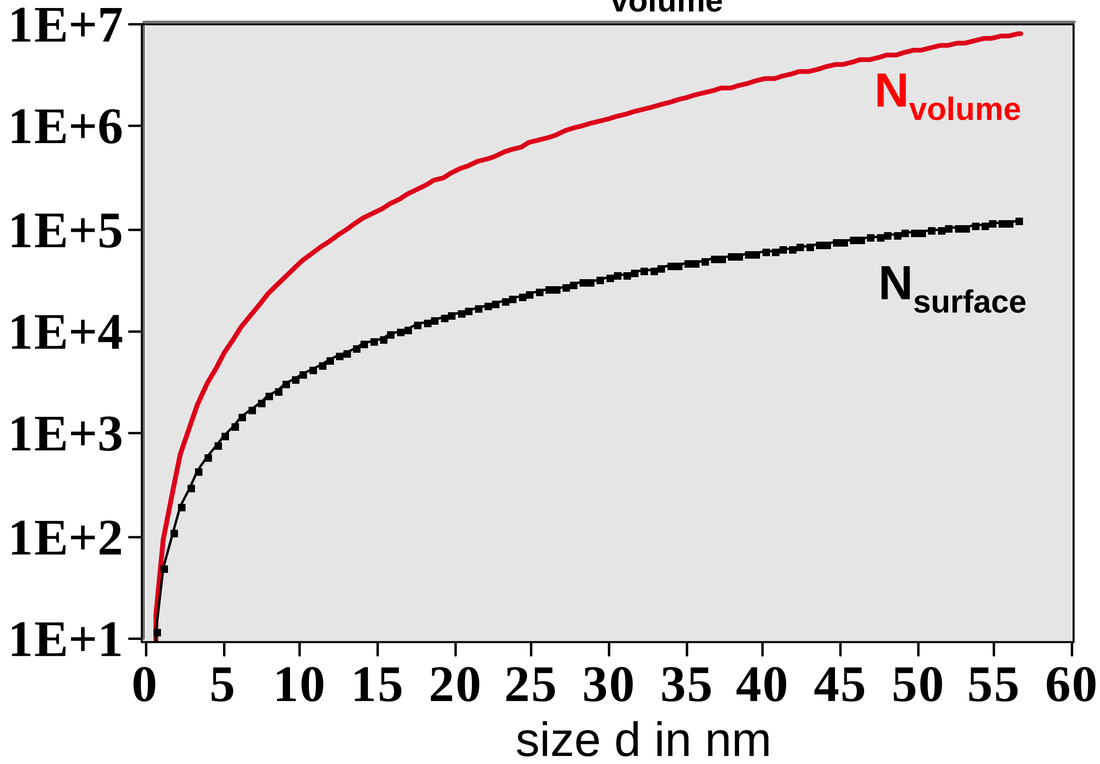
$1nm^3$ contains 22 Ga and 22 As

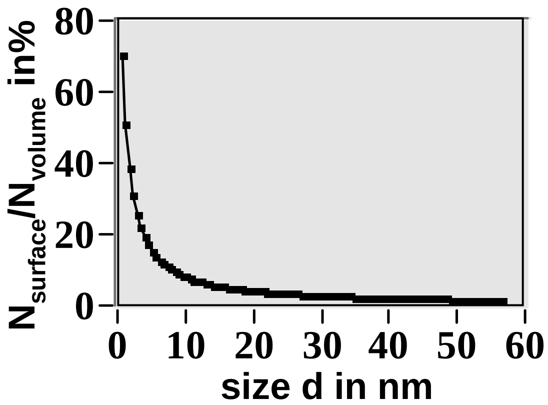
For an FCC cubic: $N_{surface} = 12\,n^2$

$N_{volume} = 8n^3 + 6n^2 + 3n$

n = number of atoms along edges

d = na, a = lattice constant





Sec. 9.3.3 Fermi Gas and Density of States

Classical description: momentum $p = mv$**, kinetic energy** $E = mv^2/2 = p^2/2m$

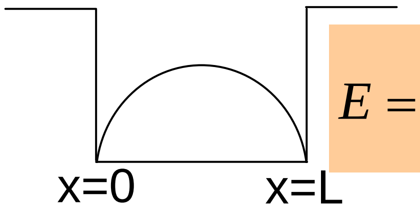
Quantum description: $p_x = \hbar k_x$

All conduction electrons are equally spread out in the ***k*** **space (reciprocal space)**

Ch 9.3.2, Page 242, table 9.5, A2

dimensions	coordinate region	spacing in *k*	Fermi region	number of electron $N(E) \equiv$ Fermi region/spacing in *k*	Density of states $D(E) \equiv d\,N(E)/dE$
1	Length L	$2\pi/L$	$2k_F$	$\frac{4k_F}{2\pi/L} = \frac{2L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2} E^{1/2}$	$\frac{L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2} E^{-1/2}$
2	Area L^2	$(2\pi/L)^2$	πk_F^2	$\frac{2\pi k_F^2}{(2\pi/L)^2} = \frac{L^2}{2\pi}\left(\frac{2m}{\hbar^2}\right) E$	$\frac{L^2}{2\pi}\left(\frac{2m}{\hbar^2}\right)$
3	Volume L^3	$(2\pi/L)^3$	$\frac{4}{3}\pi k_F^3$	$\frac{2(4\pi k_F^3/3)}{(2\pi/L)^3} = \frac{L^3}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{3/2}$	$\frac{L^3}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}$

Confined electron wavefunction in a infinite square well



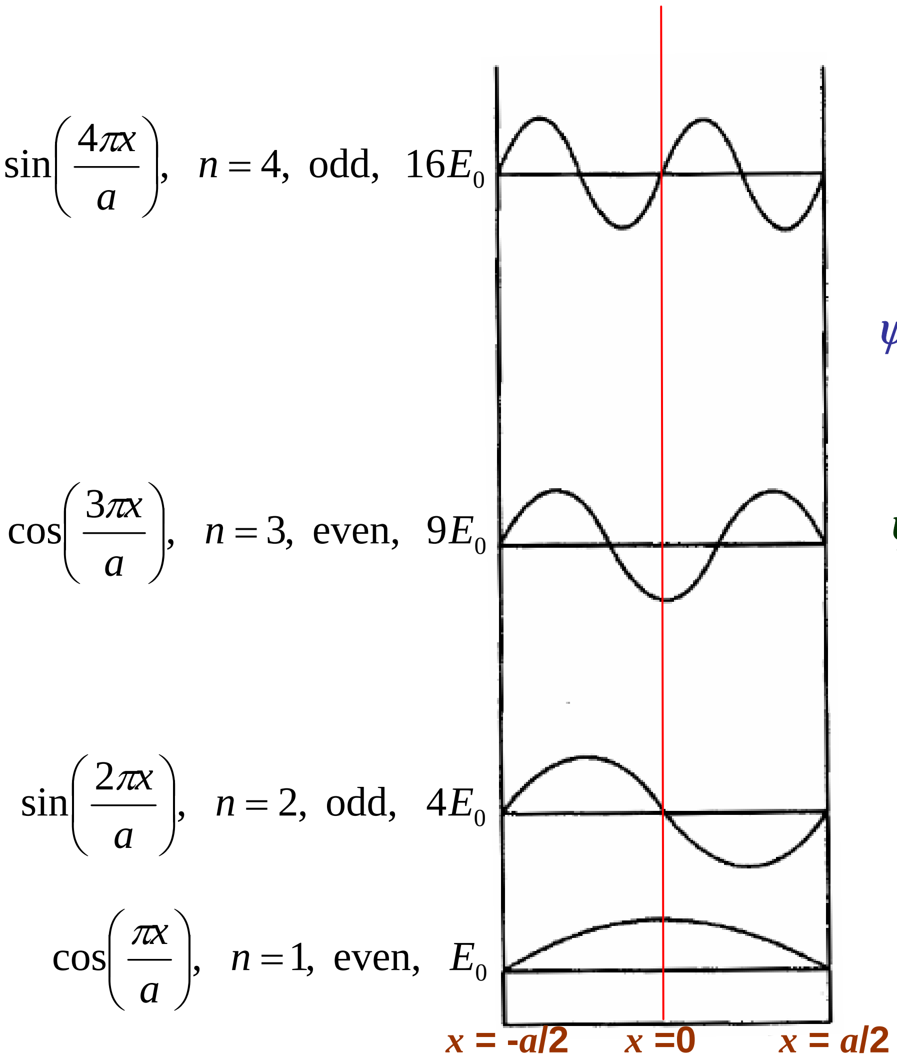
$$E = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$$ Eq. 2.1.15

Let L=a, not to consider spin

$$E_n = \left[\frac{\pi^2 \hbar^2}{2ma^2}\right] n^2$$

$$n = \frac{2k_F}{2\pi/a} = \frac{a}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2} E_n^{1/2}$$

c.f. 1D in Table A2



ψ_n = cos($n\pi x/a$) n = 1,3, 5,… even parity

$\psi_n(-x) = \psi_n(x)$

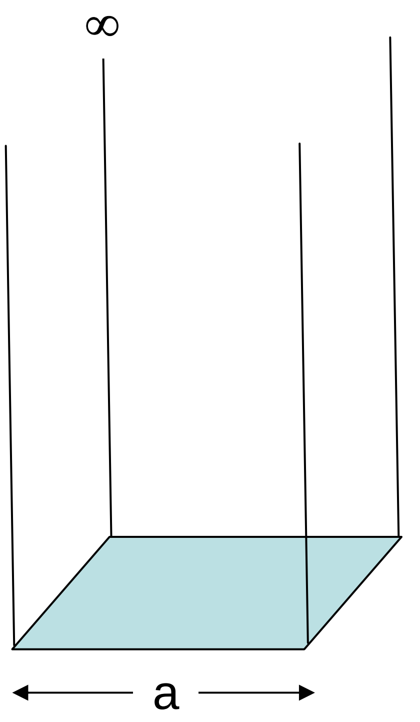
ψ_n = sin($n\pi x/a$) n = 2,4, 6,… odd parity

$\psi_n(-x) = -\psi_n(x)$

Probability of finding an electron at a particular value of x = $|\Psi_n(x)|^2$

Degeneracy:

Energy of a 2D infinite rectangular square



$$E_n = \left(\frac{\pi^2\hbar^2}{2ma^2}\right)\left(n_x^2 + n_y^2\right) = E_0 n^2$$ eq. 9.9

Degeneracy (including spin states) :

n	degeneracy	n_x, n_y	n_x, n_y	n_x, n_y	n_x, n_y
1	4	0,1	1,0		
2	4	0,2	2,0		
3	4	0,3	3,0		
4	4	0,4	4,0		
5	8	0,5	5,0	3,4	4,3

Energy levels for a 1D parabolic potential well

$$V(x)=\frac{1}{2}kx^2$$

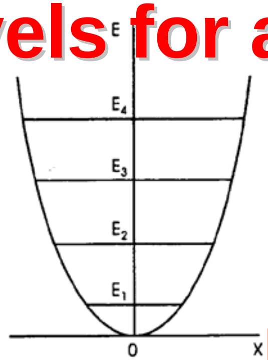
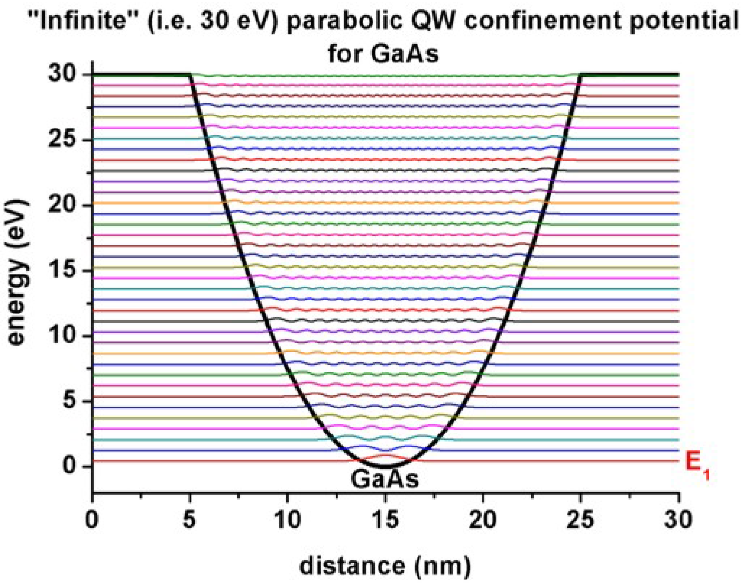
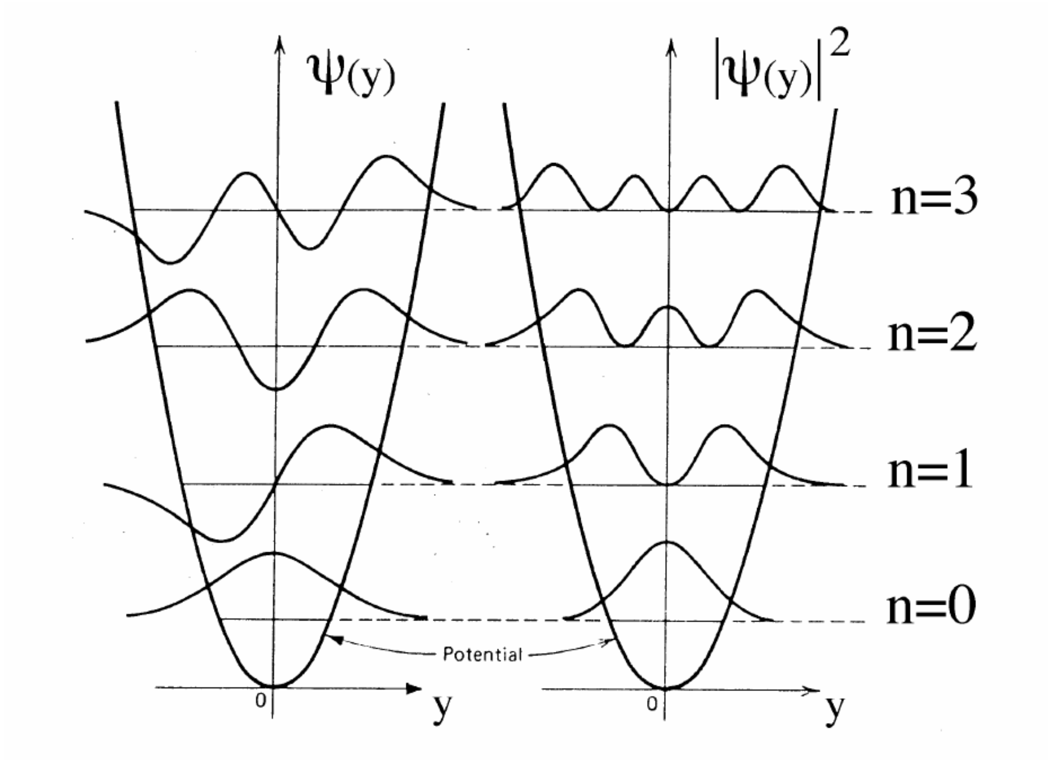


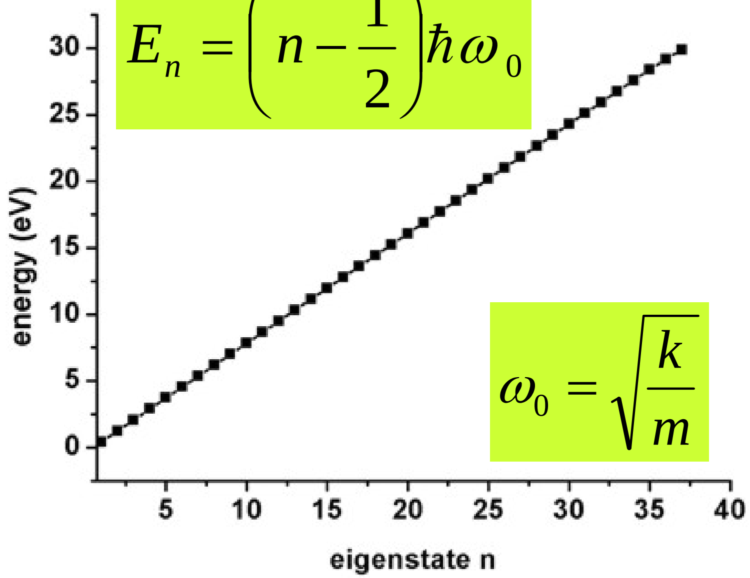
Fig. 9.14

$$\psi_n(x)=H_n(x)\,e^{-\frac{\alpha x^2}{2}}$$





$$E_n=\left(n-\frac{1}{2}\right)\hbar\omega_0$$



$$\omega_0=\sqrt{\frac{k}{m}}$$

Ch 9.3.4, Page 240

$N(E)$ and $D(E)$ in 0D, 1D, 2D and 3D with subbands

Ch 9.3.6, Page 360, table A3

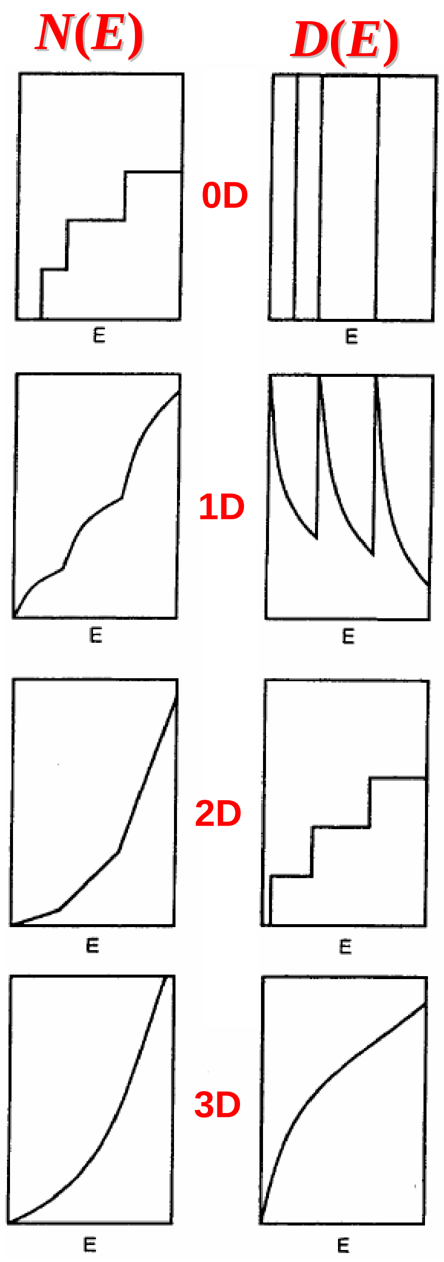
i = subband index

Ch 9.3.6, Page 243

dimensions	type	number of electrons $N(E)$, d_i = degeneracies	Density of states $D(E)$
0	dot	$2\Sigma d_i \Theta(E - E_{iw})$	$2\Sigma d_i \delta(E - E_{iw})$
1	wire	$\frac{2L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2} \Sigma d_i (E - E_{iw})^{1/2}$	$\frac{L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2} \Sigma d_i (E - E_{iw})^{-1/2}$
2	well	$\frac{L^2}{2\pi}\left(\frac{2m}{\hbar^2}\right) \Sigma d_i (E - E_{iw})$	$\frac{L^2}{2\pi}\left(\frac{2m}{\hbar^2}\right) \Sigma d_i$
3	bulk	$\frac{L^3}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{3/2}$	$\frac{L^3}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}$

$\Theta(x) = 1 \text{ if } x \geq 0; \qquad \Theta(x) = 0 \text{ if } x < 0$

$\delta(x) = \infty \text{ if } x = 0; \qquad \delta(x) = 0 \text{ if } x \neq 0; \qquad \int_{-\infty}^{\infty} \delta(x)dx = 1$



Crossover from 2D to 3D

2D film with thickness L_Z; k_Z quantized in unit of $2\pi/L_Z$

energy quantized in unit of $\varepsilon_Z = \hbar^2\pi^2/2m^*L_Z^2$

The 2D density of states: $D_{2DS}(E) = D_{2D}\sum_p \Theta(E - p^2\varepsilon_Z)$

For thick films (large L_Z), ε_Z is small, and $D_{2DS}(E)$ approaches $D_{3D}(E)$ with volume L^2L_Z

for p=1, $E=\varepsilon_Z$

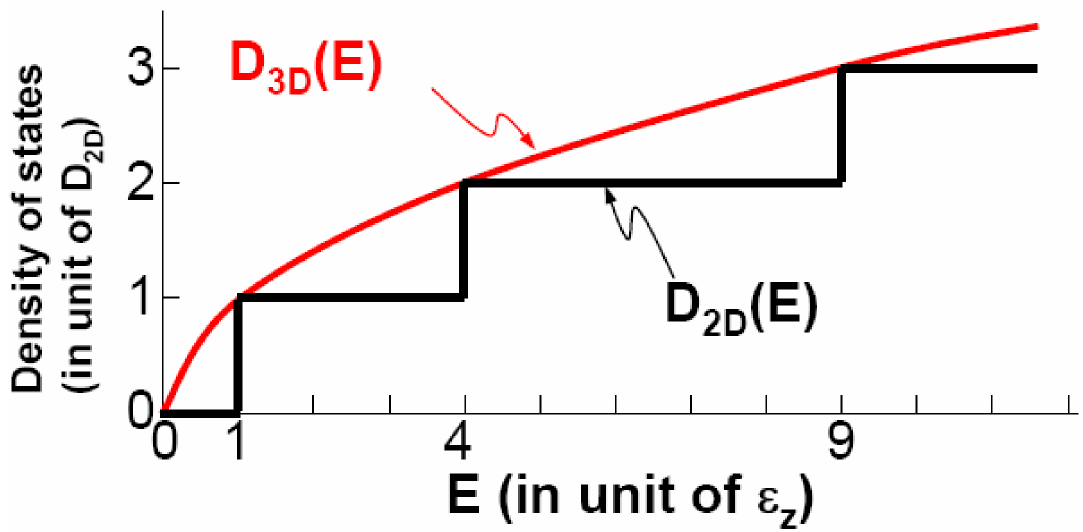
$$D_{2DS}(\varepsilon_Z) = D_{3D}(\varepsilon_Z) = \frac{L^2L_Z}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\varepsilon_Z^{3/2} = \frac{L^2L_Z}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left(\frac{\hbar^2\pi^2}{2mL_Z^2}\right)^{1/2} = \frac{L^2}{2\pi}\left(\frac{2m}{\hbar^2}\right) = D_{2D}$$

for p=2, $E=4\varepsilon_Z$

$$D_{2DS}(4\varepsilon_Z) = D_{3D}(4\varepsilon_Z) = \frac{L^2L_Z}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}2\varepsilon_Z^{1/2} = 2\frac{L^2L_Z}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left(\frac{\hbar^2\pi^2}{2mL_Z^2}\right)^{1/2} = 2\frac{L^2}{2\pi}\left(\frac{2m}{\hbar^2}\right) = 2D_{2D}$$

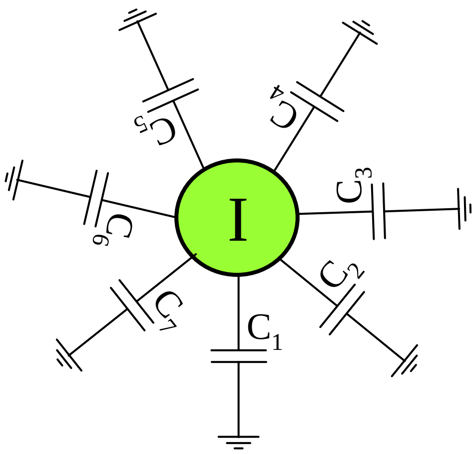
for p=3, $E=9\varepsilon_Z$

$$D_{2DS}(9\varepsilon_Z) = D_{3D}(4\varepsilon_Z) = \frac{L^2L_Z}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}3\varepsilon_Z^{1/2} = 3\frac{L^2L_Z}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left(\frac{\hbar^2\pi^2}{2mL_Z^2}\right)^{1/2} = 3\frac{L^2}{2\pi}\left(\frac{2m}{\hbar^2}\right) = 3D_{2D}$$



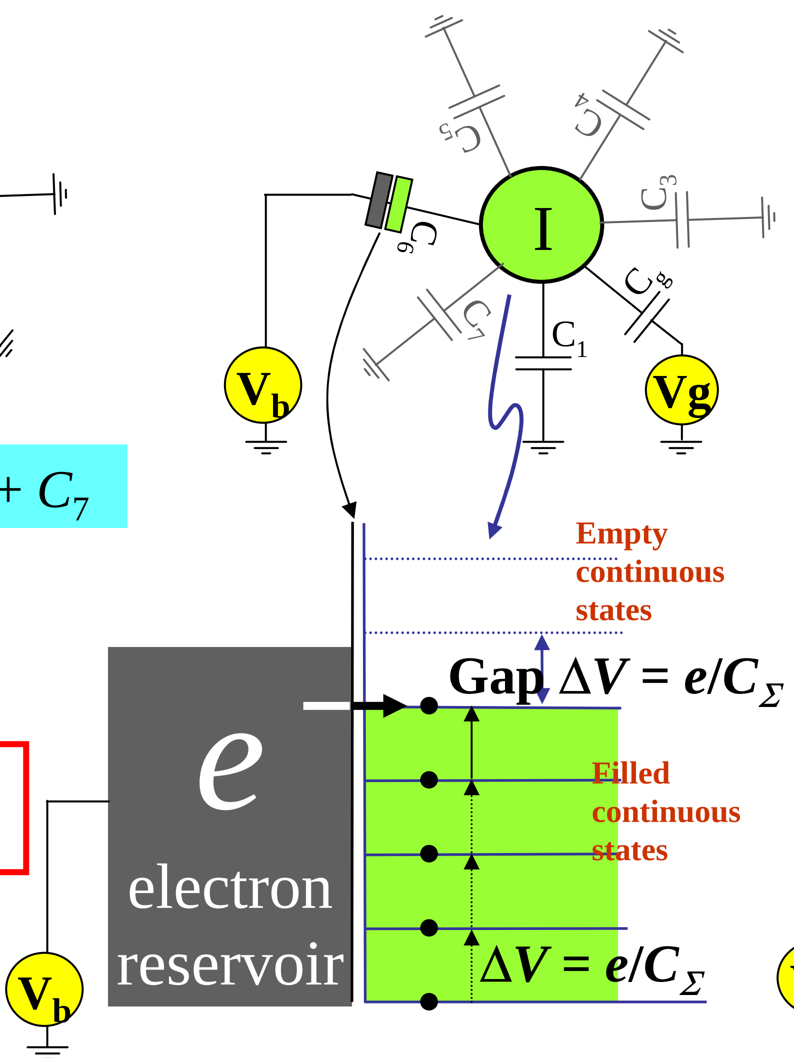
Single electron tunneling

A neutral isolated dot



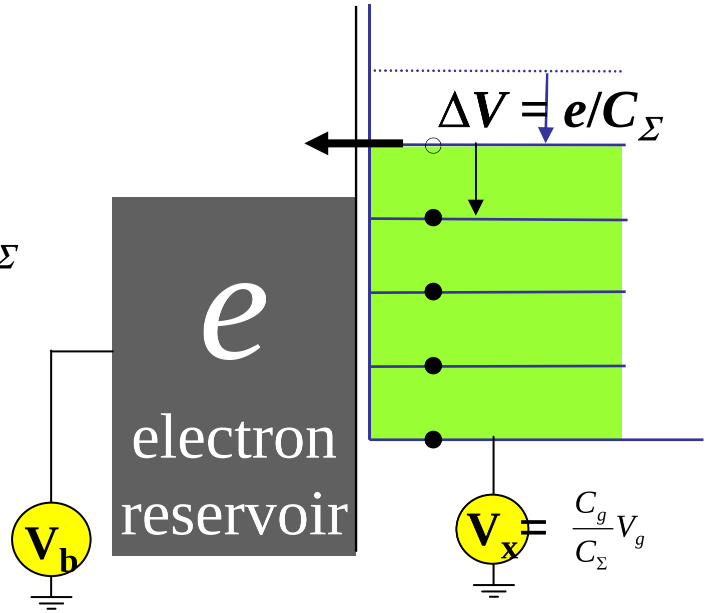
$$C_\Sigma \equiv C_1 + C_2 + \cdots + C_7$$

Electron Box

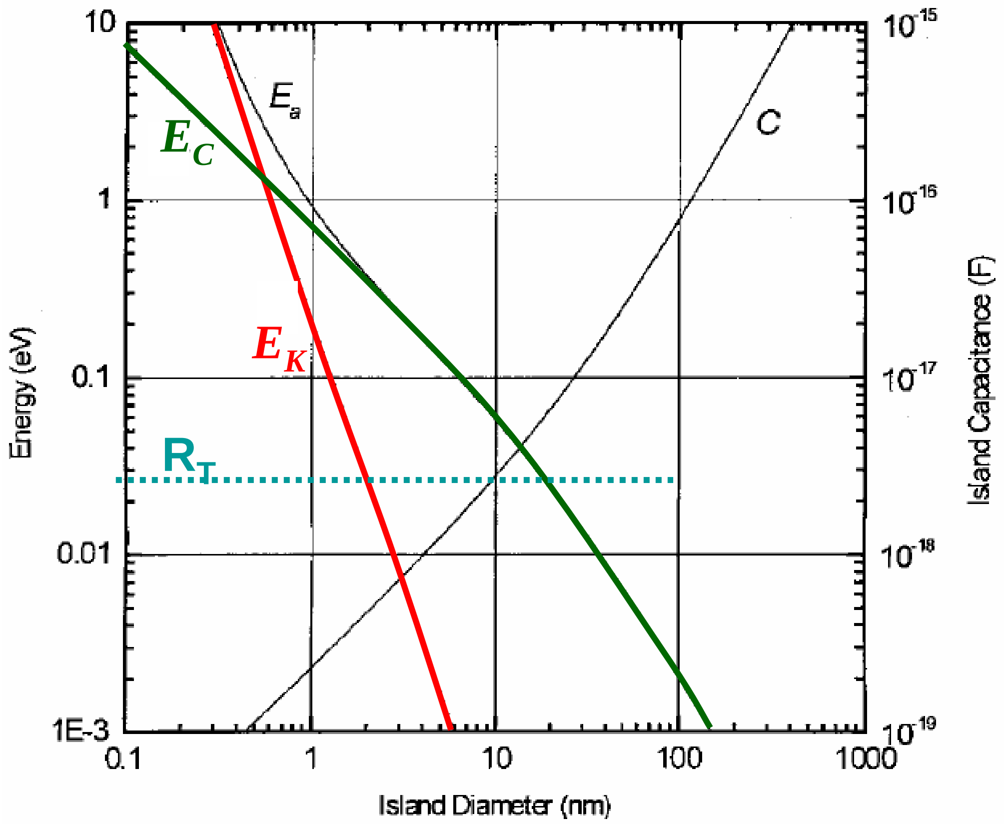


Inject electrons via a tunnel junction

An applied gate voltage can lift up island potential



Additive energy for a nanoparticle



$E_C = e^2/2C; \quad C = \pi\varepsilon_0\varepsilon_r D$

E_K = level spacing = $1/D(E_F)V$

$n = 10^{22}$ cm^{-3}

$m = m_e$

$\varepsilon_r = 4$

10% tunnel barrier=2nm

Likharev review article, Fig. 2

Criteria for Coulomb blockade behavior

Coulomb blockade $\longleftrightarrow$ isolated object

Charging energy ($e^2/2C$):
Electrostatic energy associated with charging/discharging an isolated object

Criteria (for a well defined charge number) :

1. **to surmount thermal fluctuations**

$$\frac{e^2}{2C} >> k_B T \rightarrow \text{Small C or Low T}$$

C = total capacitance seen from the object

2. **to surmount quantum fluctuations**

electrical paths to charge/discharge the object

$$\underbrace{\left(\frac{e^2}{2C}\right)}_{\Delta E}\underbrace{(RC)}_{\Delta t} \geq h \Rightarrow R \geq R_K \equiv \frac{h}{e^2} \approx 26k\Omega$$

R **= total resistance seen from the object**

Parallel R

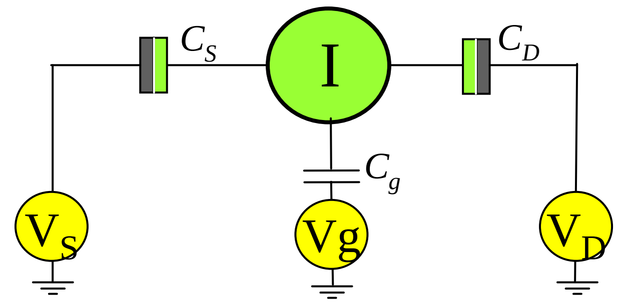
Material:
any conducting materials, including metal, semiconductor, conducting polymer, carbon nanotube, ...

Structure:
any structures with isolated objects accessible through tunnel junctions

The simplest structure:

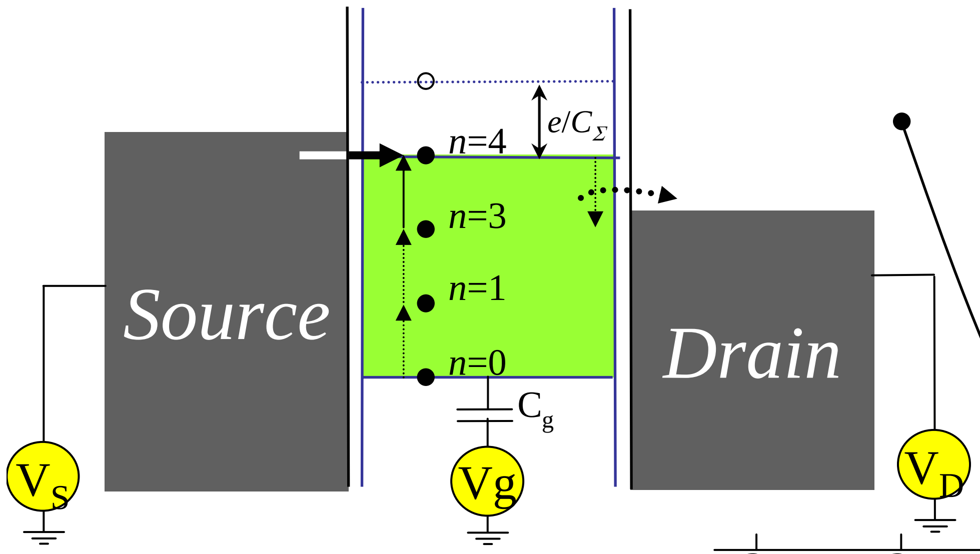
Single Electron Transistor (SET)

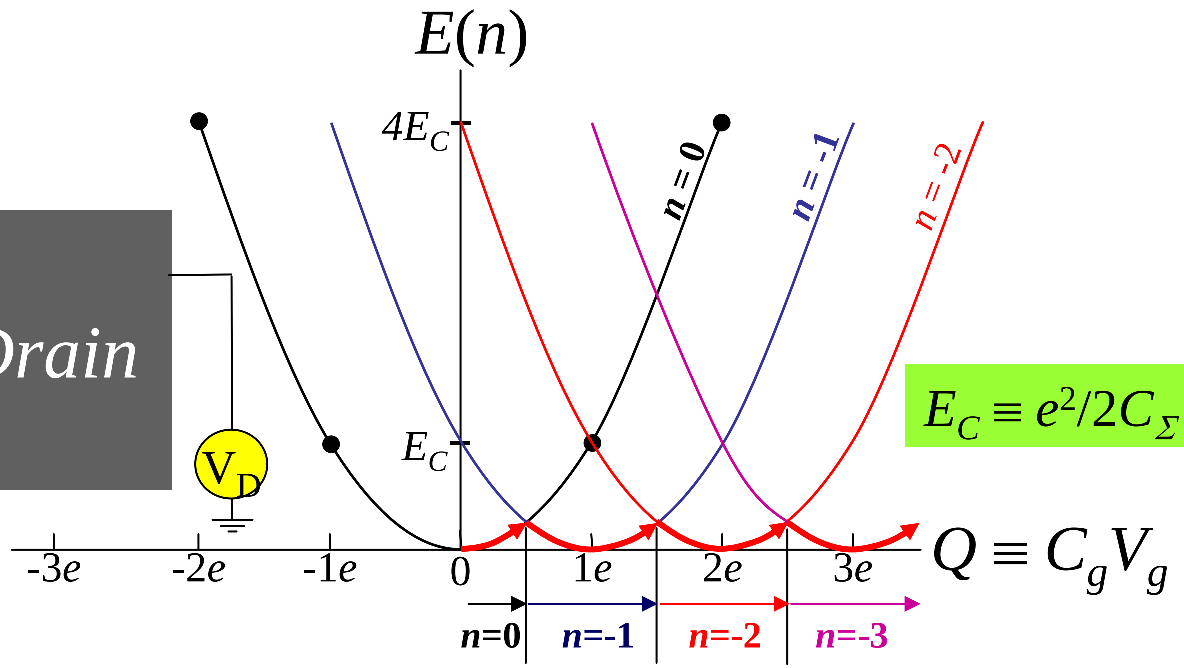
Single Electron Transistor



Electrostatic energy $E(n) = (ne + C_g V_g)^2 / 2C_\Sigma$

$C_\Sigma = C_S + C_D + C_g$



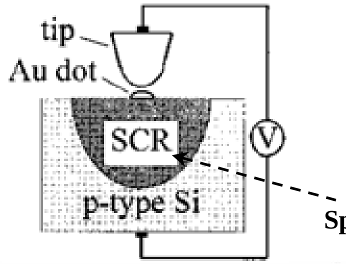


$E_C \equiv e^2 / 2C_\Sigma$

Various Single electron transistors:

(3) STM: on *Au* particle

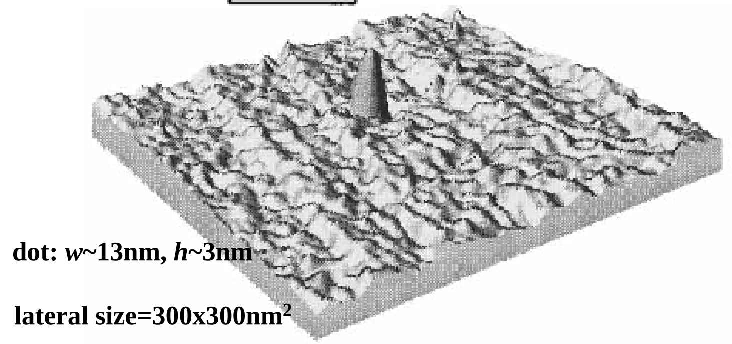
(STM=Scaning Tunneling Microscope)



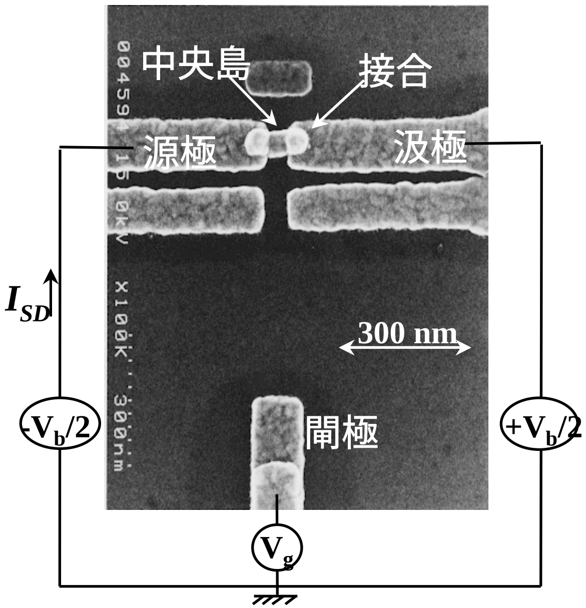
P. Radojkovic *et al.*

J. Vac. Sci. Technol. B 14(2), 1229 (1996)

(Technical Univ. of Munich)

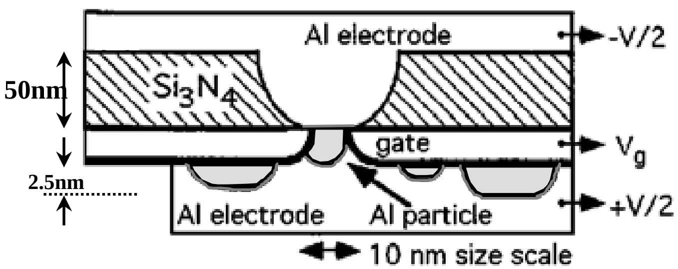


(1) All metal transistors: *Al* island



our work (and many other groups)

(2) Sandwich structure: *Al* particle

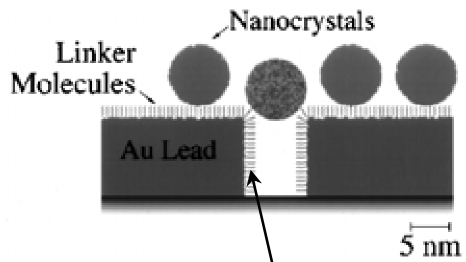


Al particles are isolated by thin Al_2O_3 layer

D.C.Ralph, C.T.Black and M. Tinkham

PRL, 78, 21 p. 4087 (1997) (Harvard Univ)

(4) Colloidal: *Au* or *CdSe* particles



self-assmbled molecular of dithiol molecules

David L. Klein *et al.*

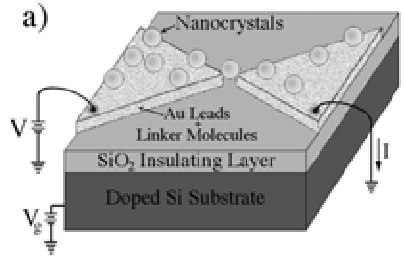
Nature, 389, p.699 (1997)

(Lawrence Berkeley National Lab.)

Ronald P. Andres *et al.* Science 722,1323 (1996)

(Purdue Univ.)

a)



b)