A Lattice Study of Quark and Glue momenta and angular momenta in the Nucleon

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χQCD Collaboration

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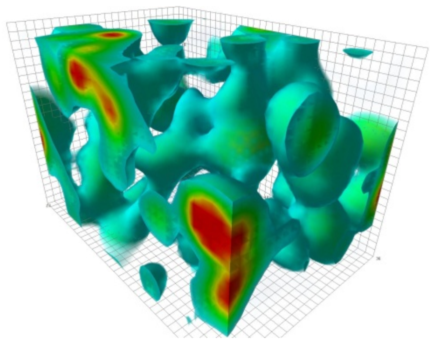
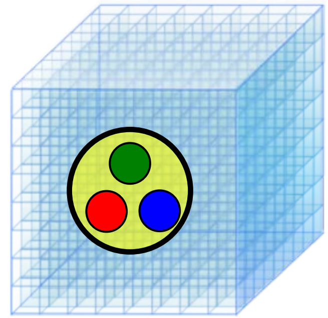
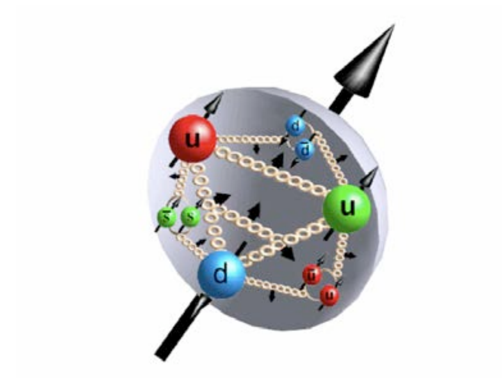
PRD91(2015)014505
(arXiv:1312.4816)







- <u>Outline</u>
  - – Introduction
  - – Lattice QCD framework
    - Challenges: Disconnected Insertion and Glue
  - – Lattice QCD results
  - – Summary & Prospects

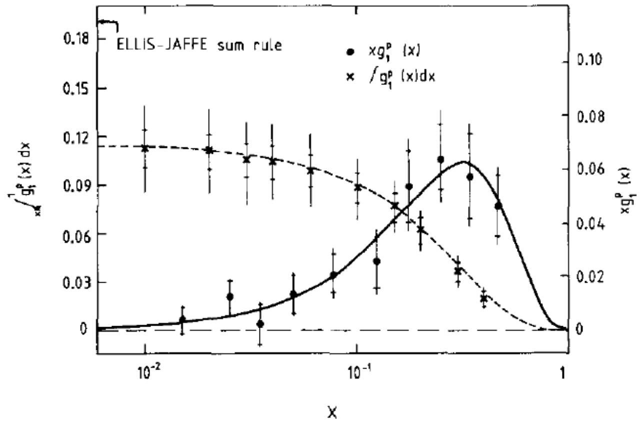
(Figs by D.B.Leinweber, K.Murano and STAR Coll.)

Puzzles in Nucleon structure

- Spin (axial vector)
  - **“Spin crisis”**
    - quark spin is small !

$$\Delta\Sigma = \sum_q [\Delta q + \Delta\bar{q}] = 0.2\text{–}0.3$$

EMC(1988)

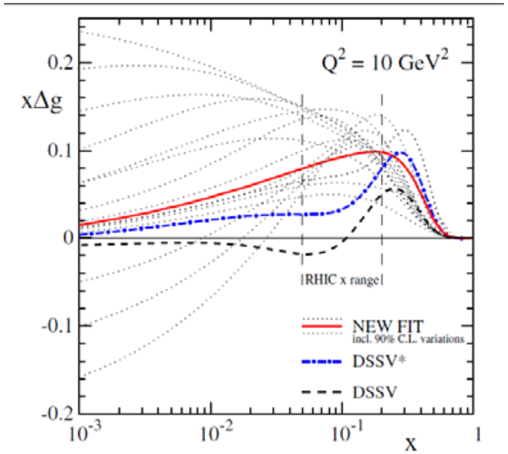


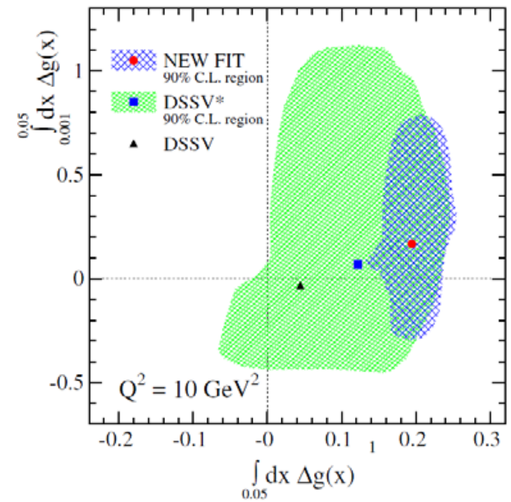
$$g_1(x) \simeq \frac{1}{2}\sum_q e_q^2[\Delta q(x) + \Delta\bar{q}(x)]$$

- Glue ?

$$\int_{0.05}^{0.2} \Delta g(x)dx = 0.1\pm^{0.06}_{0.07}$$

RHIC Spin: arXiv:1304.0079



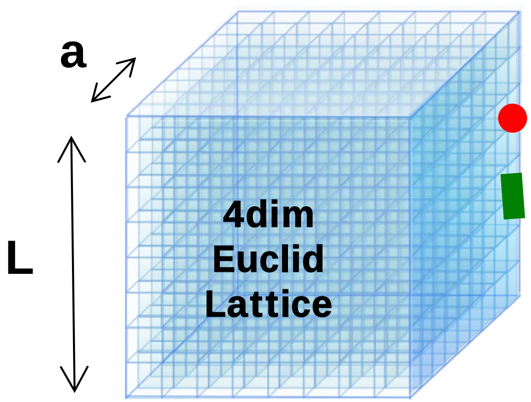


2015/10/05

De Florian et al., PRL113(2014)012001

Lattice QCD
First-principles calculation of QCD

$$Z = \int dU\, dq\, d\bar{q}\ e^{-S_E}$$



quarks on sites

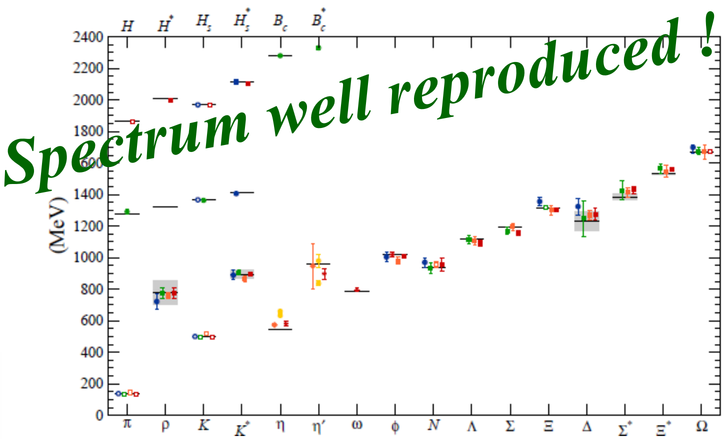
$$q(x)$$

gluons on links

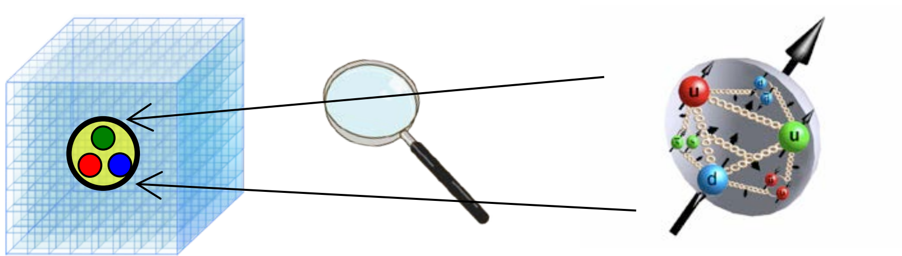
$$U_\mu(x, x+\mu) = \exp[-iaA_\mu]$$

K.G. Wilson

- **Well-defined reguralized system**
- **Gauge-invariance manifest**
- **Fully-Nonperturbative**
- **DoF ~ 10^9 ➔ Monte-Carlo w/ Euclid time**

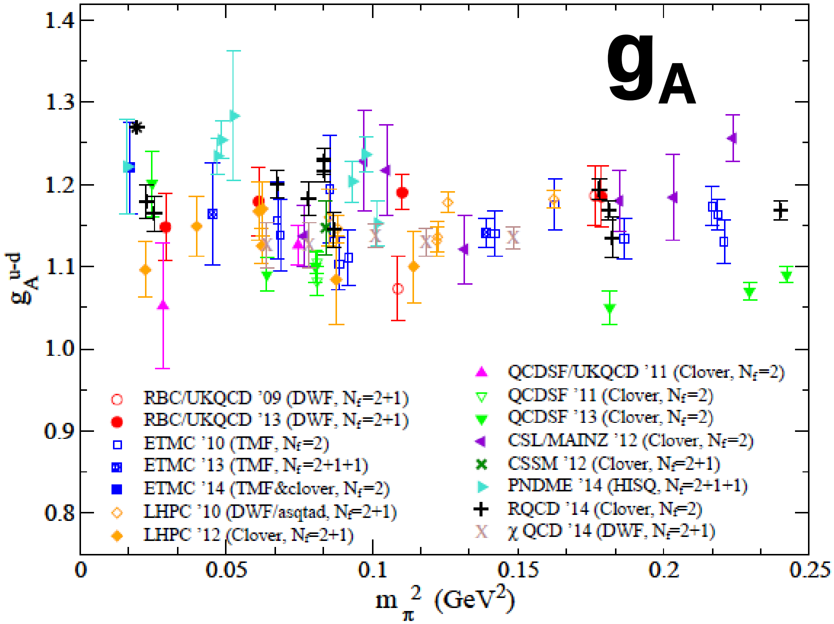


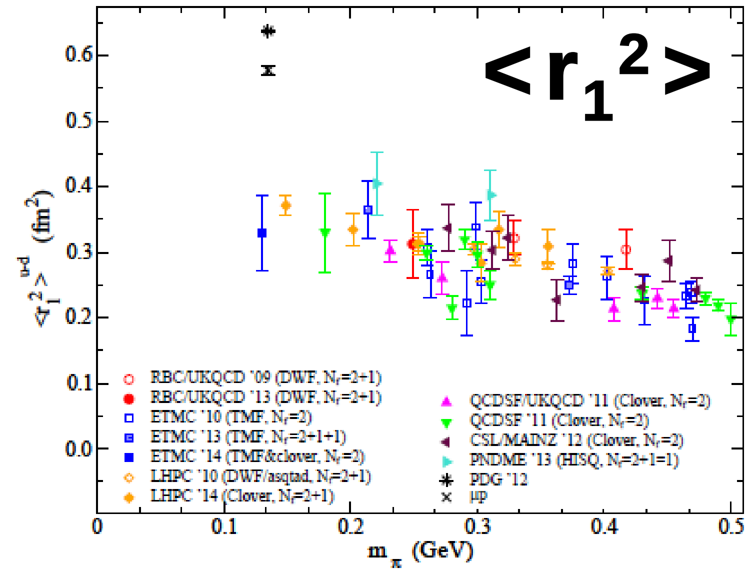
Summary by Kronfeld, arXiv:1203.1204



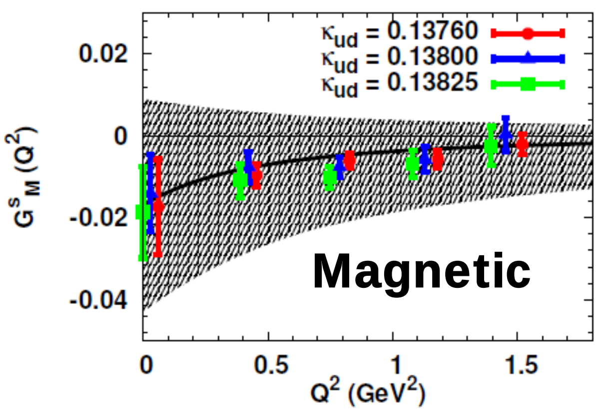
Isovector matrix elements

M. Constantinou @ Lat14



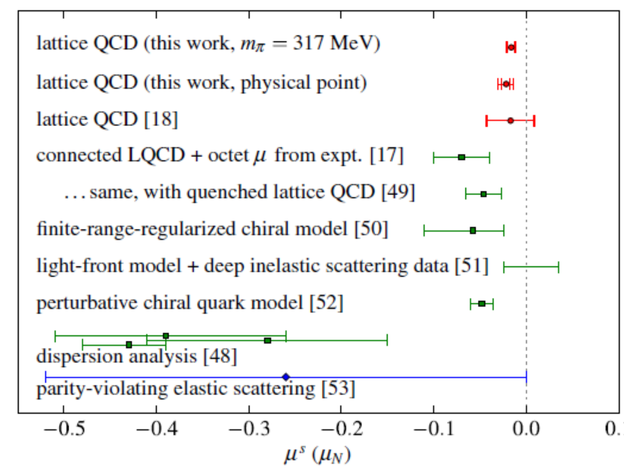


Strangeness EM form factor



$$G^s_M(0) = -0.017(25)(07)$$

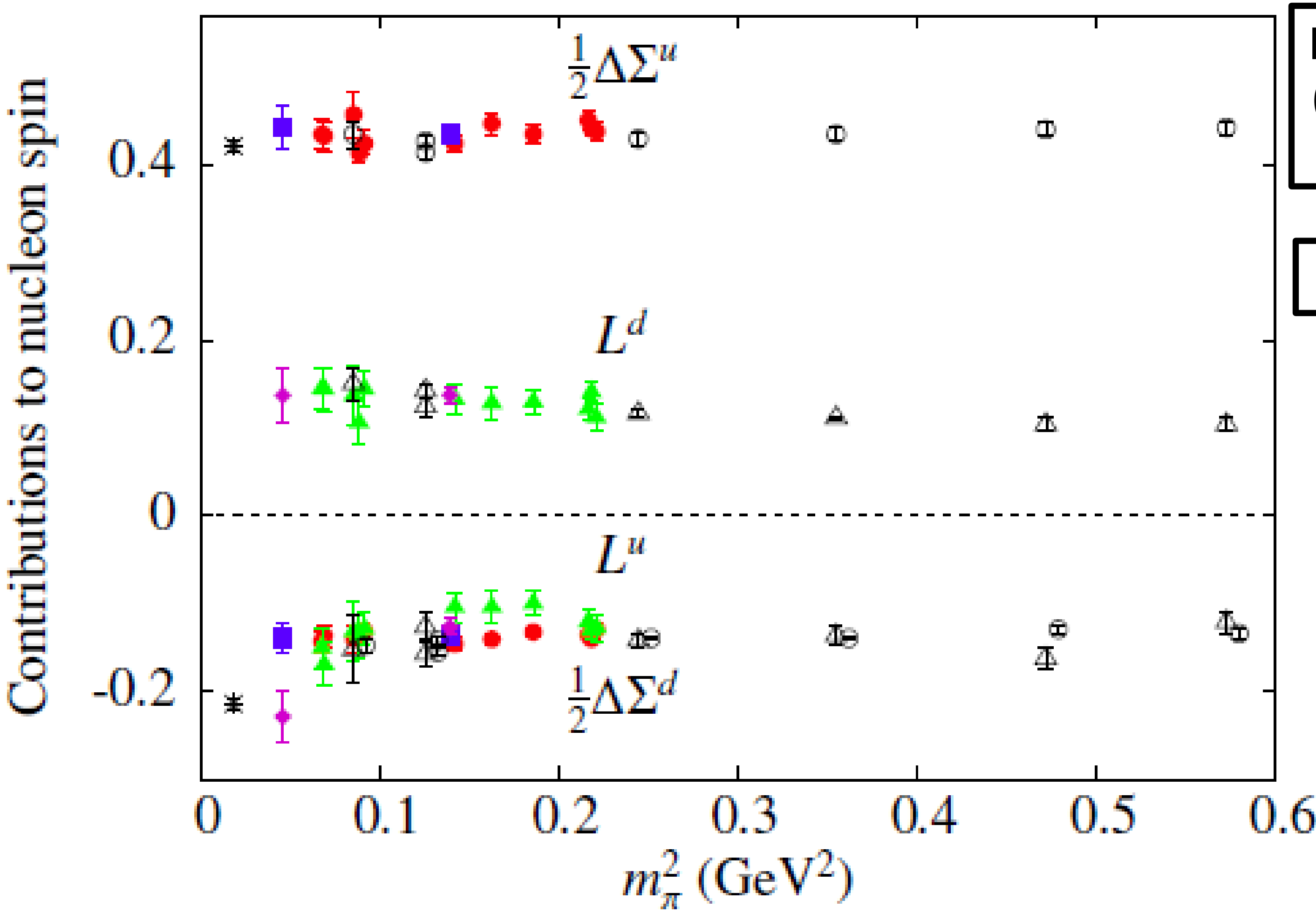
TD et al., PRD80(2009)094503



$$G^s_M(0) = -0.022(4)(4)(6)$$

J. Green et al., PRD92(2015)031501

How about proton spin ?



Part of u/d quark (Connected Insertion Only)

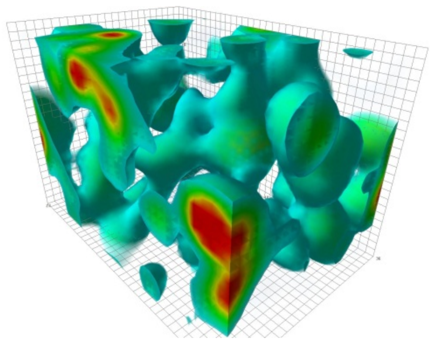
No Glue calc

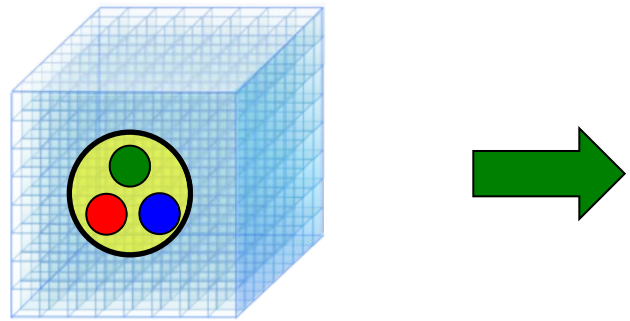
Open symbols: Asktad+DWF (LHPC)

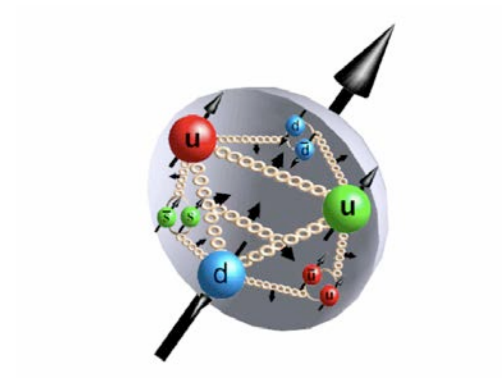
Filled symbols: TMF (ETMC)

(MSbar, mu=2GeV)

Fig from C. Alexandrou et al., PRD88(2013)014509







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Formulation on the Lattice

- 1st-moment <x> and spin J studied simultaneously
- Matrix elements of energy-momentum tensor

Gauge invariant decomposition

X.Ji (1997)

$$T_q^{\mu\nu} = \frac{i}{4}\left[\bar{q}\gamma^\mu \overrightarrow{D}^\nu q - \bar{q}\gamma^\mu \overleftarrow{D}^\nu q + (\mu \leftrightarrow \nu)\right]$$

$$T_g^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^\nu{}_\alpha$$

$$T_q^{\mu\nu} \rightarrow \bar{q}\vec{\gamma}\gamma_5 q + \bar{q}[\vec{x} \times (-i\vec{D})]q$$

$$T_g^{\mu\nu} \rightarrow \vec{x} \times (\vec{E} \times \vec{B})$$

Recent developments:
Chen et al., Wakamatsu, Hatta, Leader & Lorce, ...

- Nucleon matrix elements

$$\langle p, s|T^{\mu\nu}|p', s'\rangle = \bar{u}(p,s)\left[\boxed{T_1(q^2)}\gamma^\mu \bar{p}^\nu + \boxed{T_2(q^2)}\bar{p}^\mu i\sigma^{\nu\alpha}/2m + T_3(q^2)(q^\mu q^\nu - g^{\mu\nu}q^2)/2m + T_4(q^2) g^{\mu\nu} m/2\right] u(p', s')$$

$$\boxed{\langle x\rangle = T_1(0)} \qquad \boxed{J = \frac{1}{2}[T_1(0) + T_2(0)]}$$

(angular) momentum sum rules

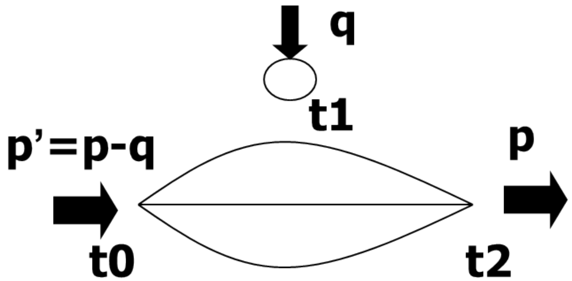
$$\langle x\rangle_q + \langle x\rangle_G = 1 \qquad J_q + J_G = 1/2$$

Formulation on the Lattice

- Calculate 3pt (& 2pt) ➔ matrix elements

$$\Pi^{3pt}(\vec{p}, t_2; \vec{q}, t_1) = \sum_{\vec{x}_2, \vec{x}_1} e^{-i\vec{p}\cdot\vec{x}_2} e^{+i\vec{q}\cdot\vec{x}_1} \langle 0 | \mathrm{T}\left[J_N(\vec{x}_2, t_2) T^{\mu\nu}(\vec{x}_1, t_1) \bar{J}_N(0) \right] 0 \rangle$$

$$(T^{\mu\nu} = T^{4i})$$



  – Typical examples:

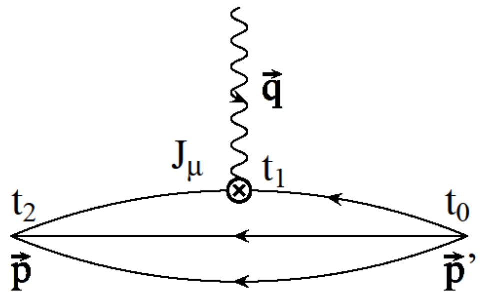
$$\mathrm{Tr}\left[\Gamma_e \Pi^{3pt}_{\boxed{T_{4i}}}(\vec{p}, t_2; \vec{q} = \vec{0}, t_1) \right] = C \cdot e^{-m(t_2 - t_1)} e^{-Et_1} \left[2p_i \cdot \boxed{T_1(0)} \right]$$

$$\mathrm{Tr}\left[\Gamma_m \Pi^{3pt}_{\boxed{T_{4i}}}(\vec{p} = \vec{0}, t_2; \vec{q}, t_1) \right] = C \cdot e^{-m(t_2 - t_1)} e^{-Et_1} \left[-i\epsilon_{ijm} q_j \boxed{(T_1(-q^2) + T_2(-q^2))} \right]$$

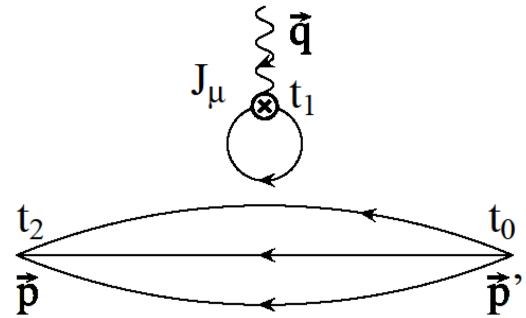
  – Other momentum combinations are calculated and T_1, T_2, (T_3) are determined simultaneously

Challenges in Lattice QCD
(1) Disconnected Insertion (DI)

- Two kinds of calc in Lattice:



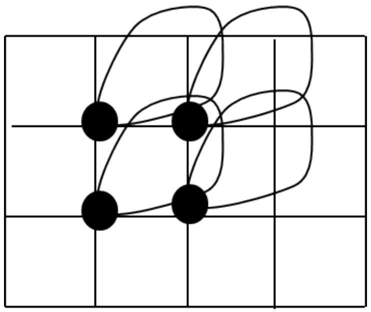
Connected Insertion (CI)



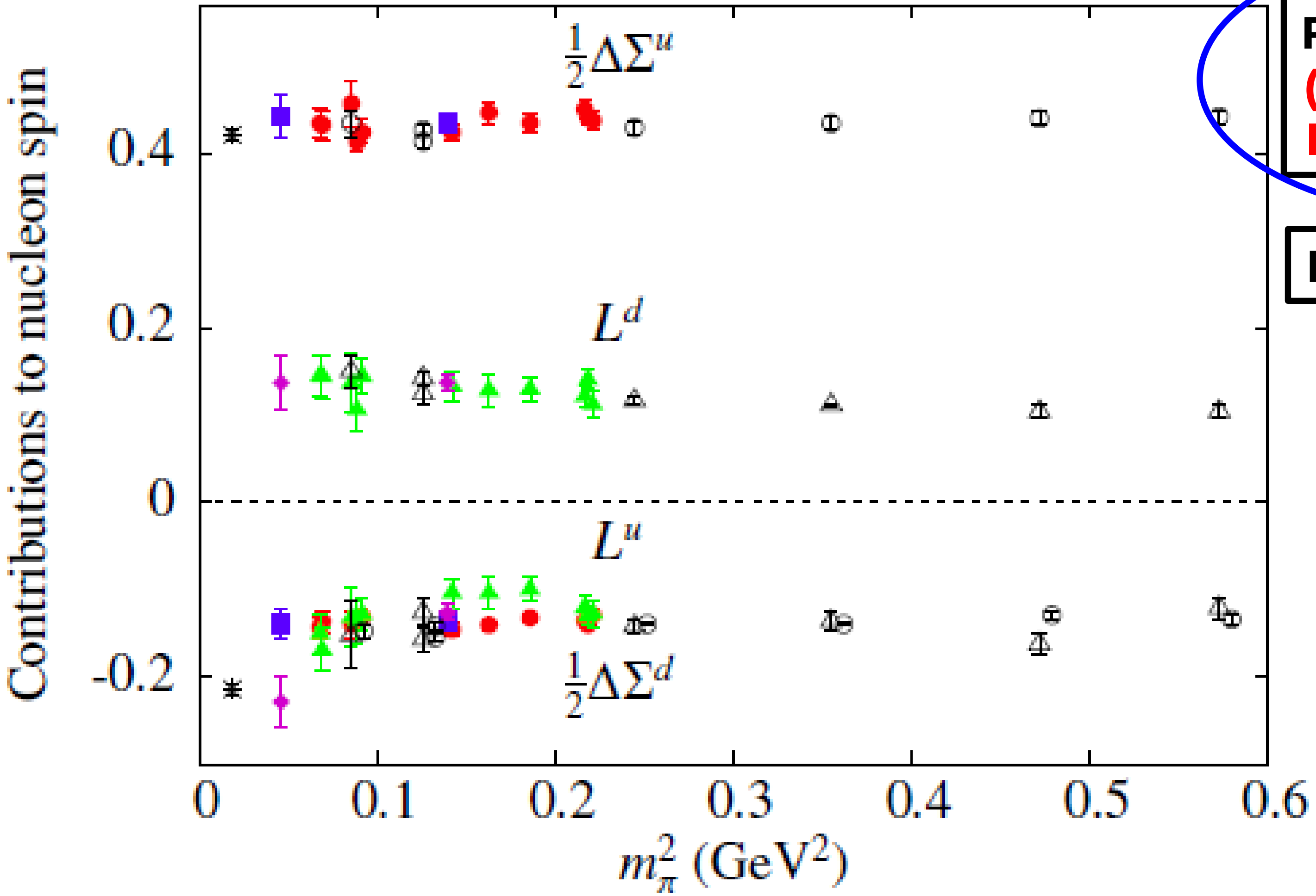
Disconnected Insertion (DI)

- DI is inevitable for flavor singlet quantities, but…
  - **All(source)-to-all(sink)** propagator is necessary
  - Straightforward calculation **impossible**
    - **O(10^5) inversions** for O(10^6) x O(10^6) matrix

$$\mathrm{Tr}[\Gamma M^{-1}] = \boxed{\sum_x} \mathrm{Tr}^{\mathrm{spin}}_{\mathrm{color}}[\Gamma M^{-1}\underline{(x,x)}]$$



How about proton spin ?



Part of u/d quark (Connected Insertion Only)

No Glue calc

Open symbols: Asktad+DWF (LHPC)

Filled symbols: TMF (ETMC)

(MSbar, mu=2GeV)

Fig from C. Alexandrou et al., PRD88(2013)014509

The approach for disconnected insertion

- Stochastic Method for DI
  - Use Z(4) (or Z(N)) noises such that

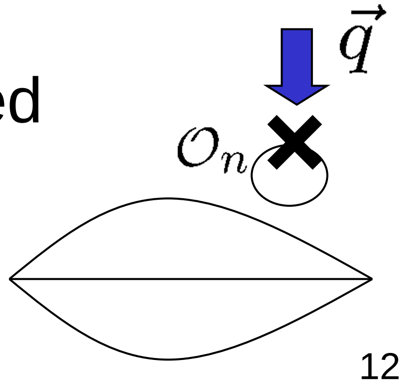
$$\lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\eta_i^{l\,\dagger}\eta_j^l=\delta_{ij}$$

  - DI loop can be calculated as

$$\mathrm{Tr}[\Gamma M^{-1}]=\lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\eta^{l\,\dagger}(\Gamma M^{-1}\eta^l)$$

S.-J.Dong, K.-F.Liu, PLB328(1994)130

- Introduce new source for noises (“off-diagonal” part)
  - ➔ Unbiased subtraction using hopping parameter expansion (HPE)
  - Off-diagonal contaminations are estimated in unbiased way



c.f. other approaches
All-to-all (Foley et al., 2005)
CAA/AMA (Blum et al., 2012)

Stochastic method for DI

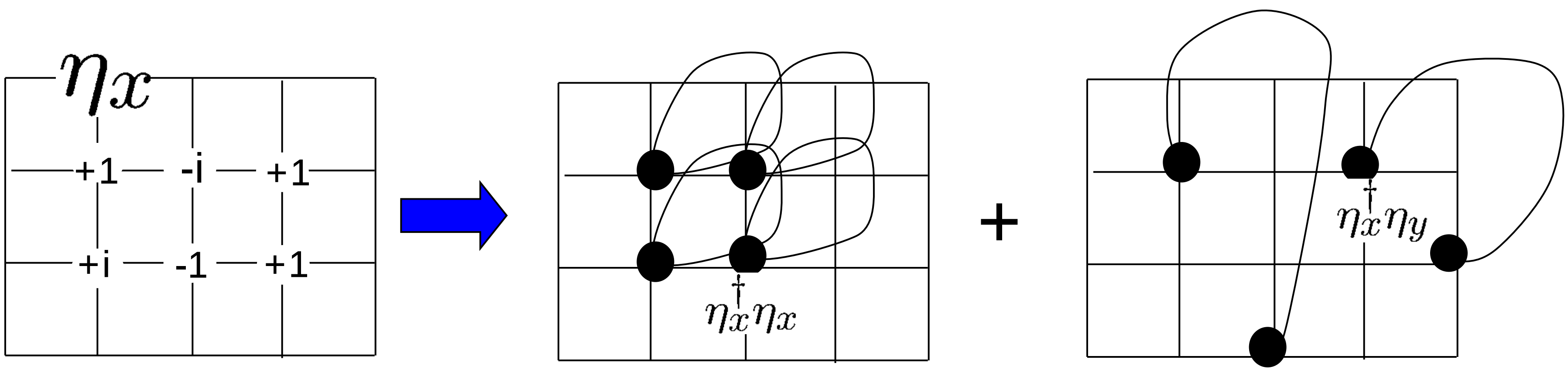
- Stochastic Method for DI
  - Noise

$$\lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\eta_i^{l\,\dagger}\eta_j^{l}=\delta_{ij}$$

  - DI loop

$$\mathrm{Tr}[\Gamma M^{-1}]=\lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\eta^{l\,\dagger}(\Gamma M^{-1}\eta^{l})$$

S.-J.Dong, K.-F.Liu, PLB328(1994)130

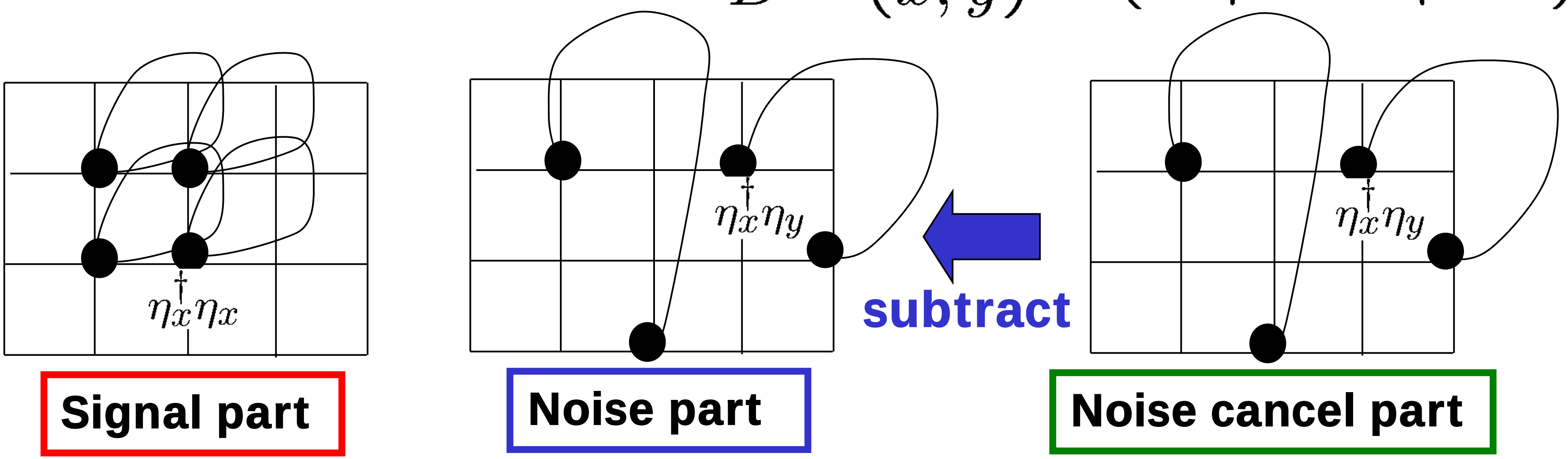


Stochastic source **Signal part** **Noise part**

Improvement of DI calc

- The **unbiased subtraction** using **hopping parameter expansion (HPE)** to eliminate off-diagonal noises

$$D^{-1}(x,y) \qquad D^{-1} \simeq (1+\kappa D+\cdots)$$

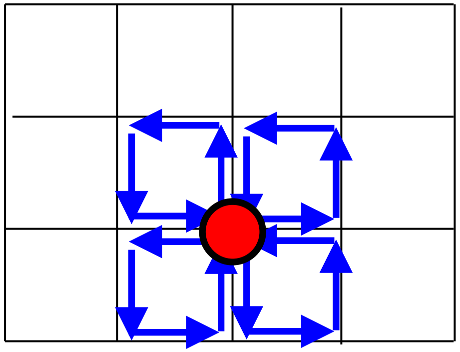


➔The error reduces by a factor of 2 or more

Challenges in Lattice QCD
(2) gluon matrix elements

- Gluon operator

$$T_G^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}{}_{\alpha}$$



  – Implementation is simple w/ link variables

  $F\mu\nu$ ←→ clover term w/ link $U\mu$

  – In practice, S/N is known to be notoriously noisy
    - Gluon DoF fluctuate too much in high-freq mode

M. Gockeler et al.,
Nucl.Phys.Proc.Suppl.53(1997)324

The approach for Glue

- Field tensor constructed from overlap operator

$$F_{\mu\nu}(x) \longleftarrow \text{Tr}_{\text{(spinor)}}\left[\sigma_{\mu\nu} D_{ov}(x,x)\right]$$

$(a \to 0)$

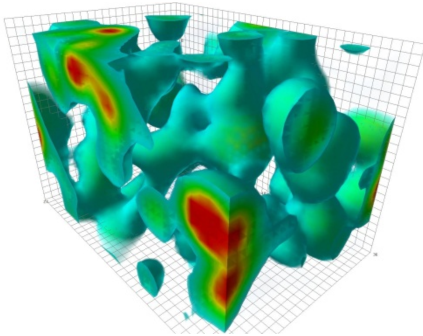
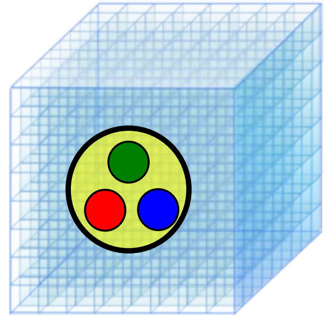
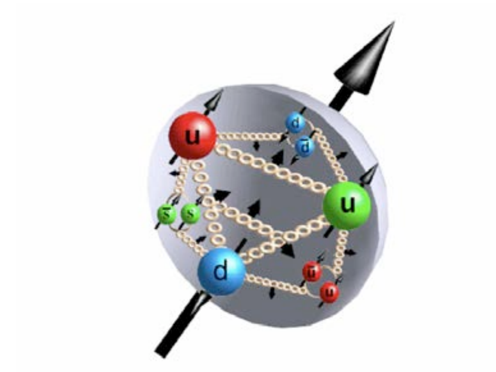
K.-F.Liu, A.Alexandru, I.Horvath
PLB659(2008)773

$$D_{ov} = \rho\left(1 + X\frac{1}{\sqrt{X^\dagger X}}\right), \; X = -\rho + D_W$$

  - Ultraviolet fluctuation is expected to be suppressed (automatic smearing)
  - In order to estimate $D_{ov}(x,x)$, stochastic method is used w/ color/spinor & (some) spacial dilution

$$D_{ov}(x,x) \Leftarrow \langle \eta_x^\dagger (D_{ov}\eta)_x \rangle$$

c.f. other approaches
Smearing (Meyer et al., 2008)
Change Action & response (Horsley et al., 2012)
Wilson-Flow (H.Suzuki, 2013)

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Lattice Setup

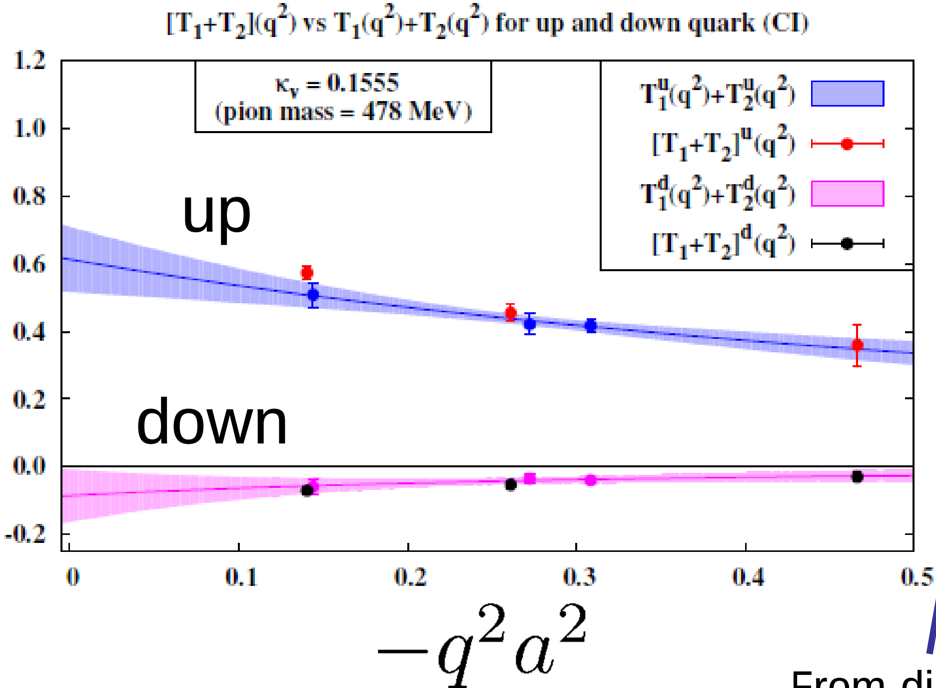
- Wilson Fermion + Wilson gauge Action
  - 500 configs with Quenched approximation
  - 1/a=1.74GeV, a=0.11fm (beta=6.0)
  - 16^3 x 24 lattice, L=1.76fm
  - kappa(ud) = 0.154, 0.155, 0.1555
    - m(pi) = 0.48, 0.54, 0.65 GeV
    - m(N) = 1.09, 1.16, 1.29 GeV
    - kappa(s)=0.154 , kappa(critical)=0.1568

Lattice Setup (cont'd)

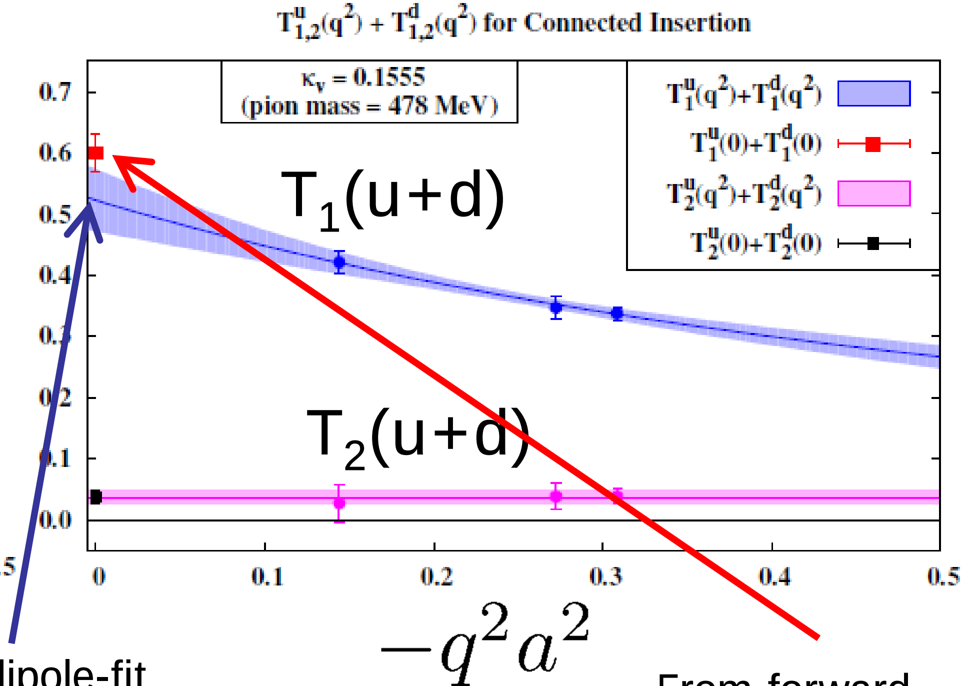
- Disconnected Insertion (DI)
  - Z(4) stochastic method, #noise=500
  - Unbiased subtraction w/ up to 4th HPE
- Glue matrix element
  - Overlap operator $D_{ov}(x,x)$
  - Z(4) stochastic method, #noise=2, w/ color/spinor dilution + spacial dilution (d=2 & even/odd ➔ taxi-distance=4)
- Improvement
  - Many nucleon sources, #src=16
  - CH, H and parity symmetry:
    - (3pt)=(2pt) X (loop)➔(3pt) = Im(2pt) X Re(loop) + ~~Re(2pt) X Im(loop)~~

Results for CI: q^2-dependence

T_1+T_2



$T_1(u+d)$, T_2 $(u+d)$



From dipole-fit

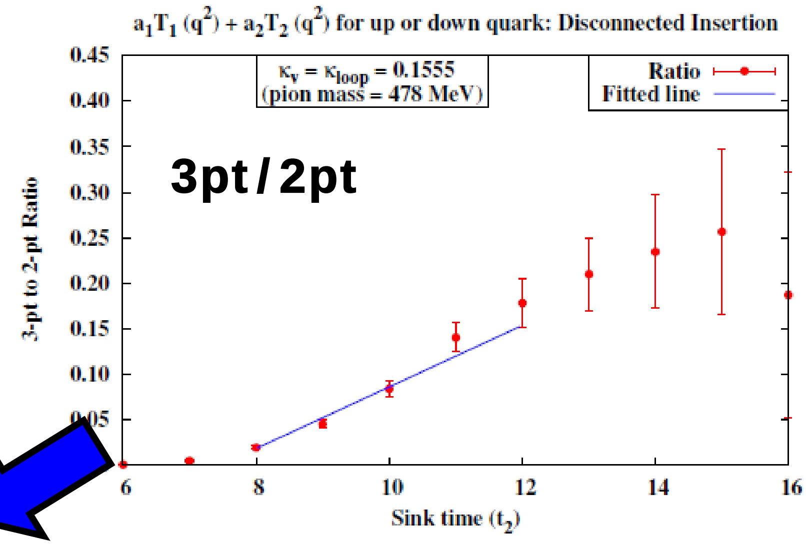
From forward matrix element

<u>Dipole-fit performed</u>

m_π =0.48GeV

Different q^2 extrapolation
(1) $T_1(q^2) + T_2(q^2)$
(2) $[T_1+T_2](q^2)$
gives consistent results

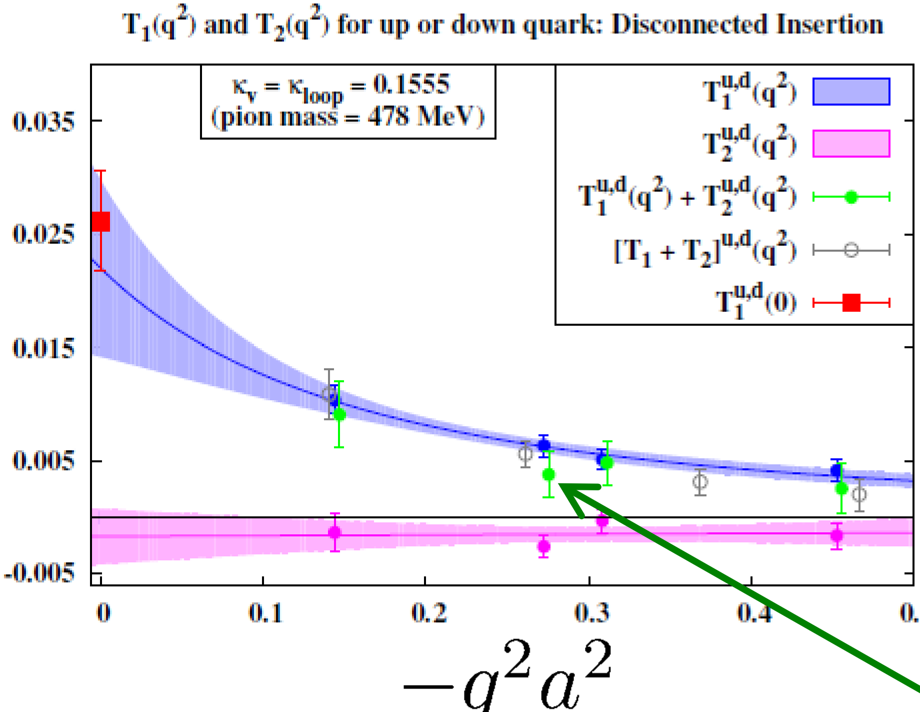
Results for DI: q^2-dependence



Slope ↔ Signal

(summation method)

T$_1$, T$_2$ (for u/d)



T_1

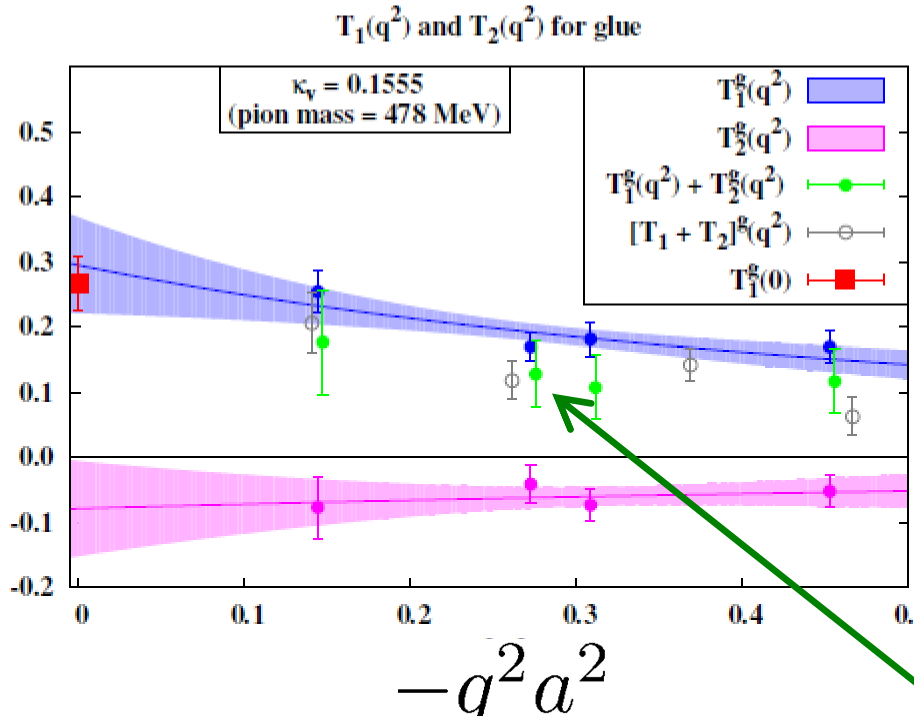
T_2

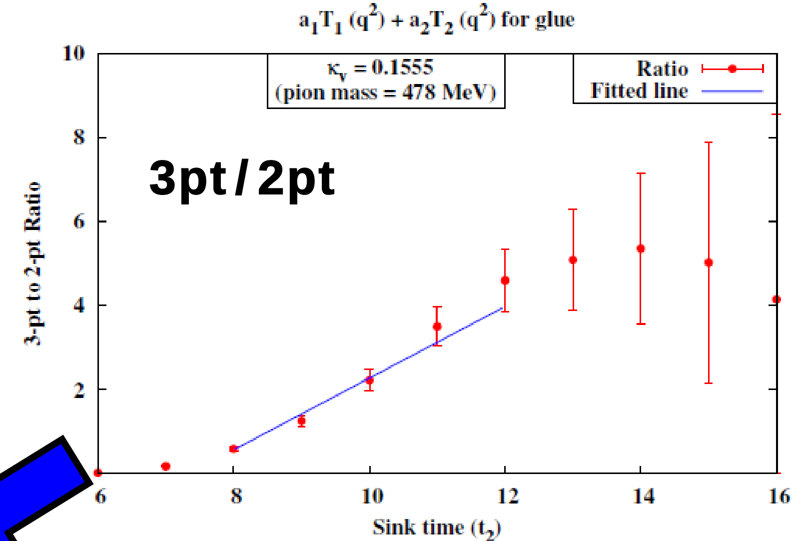
T_1+T_2

$m_\pi = 0.48$GeV

Results for Glue: q^2-dependence

T$_1$, T$_2$ (for glue)





T_1

T_2

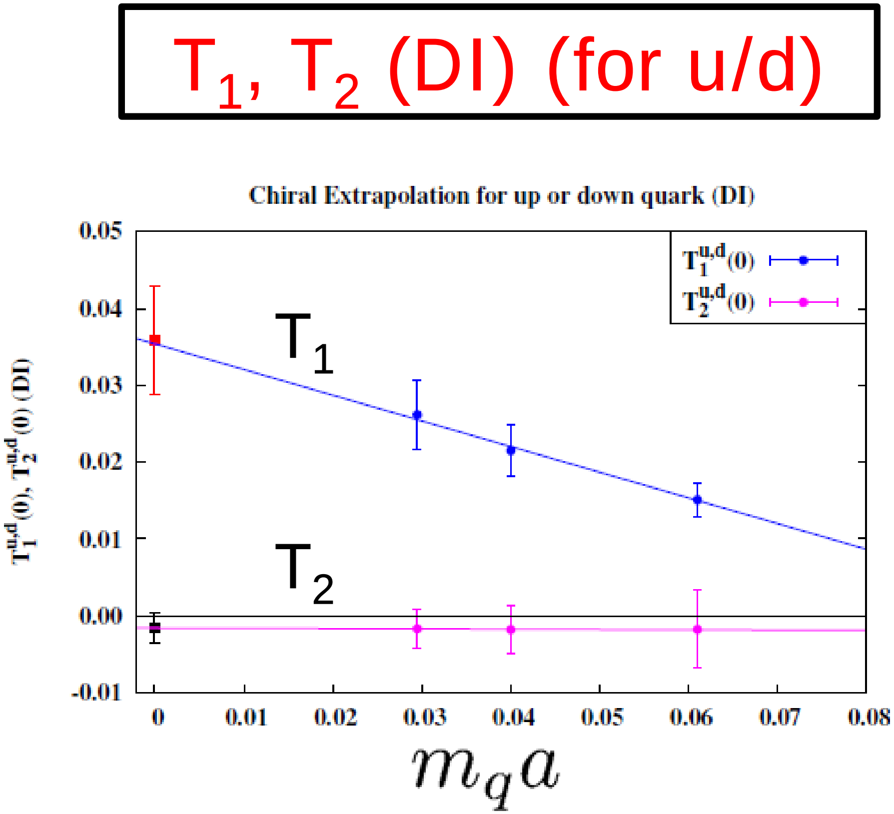
T_1+T_2

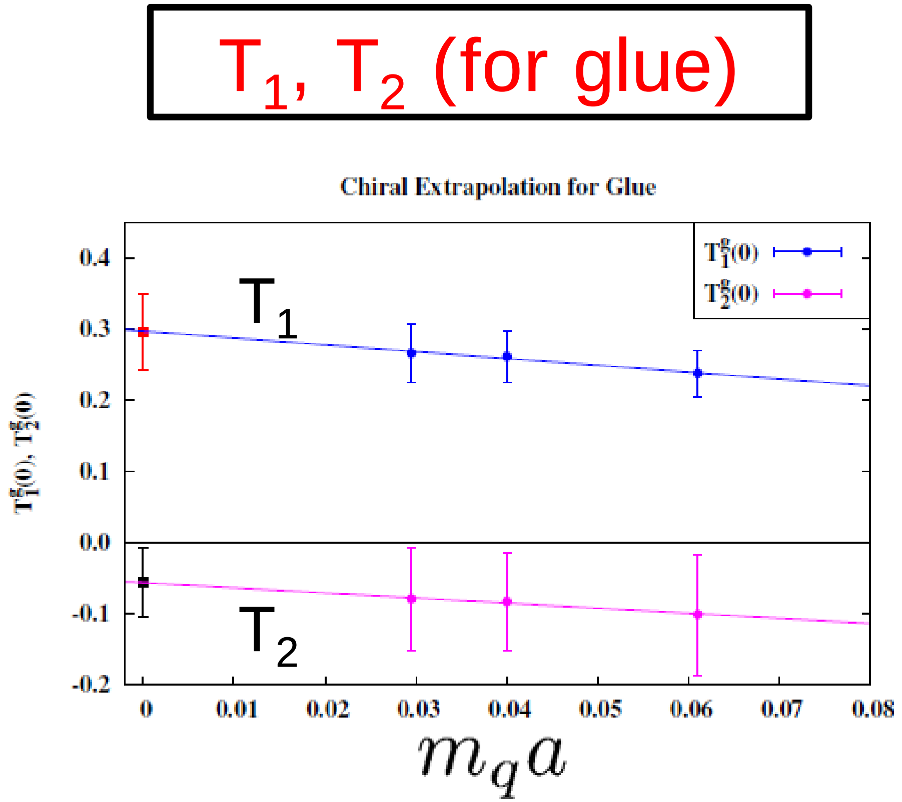
Slope ←→ Signal

(summation method)

m_π = 0.48GeV

Chiral Extrapolation





Simple Linear-extrapolation is performed

Renormalization

- Quark-glue mixing

$$\left(\begin{array}{c} \langle x\rangle_q^{\overline{MS}}(\mu) \\ \langle x\rangle_G^{\overline{MS}}(\mu) \end{array}\right) = \left(\begin{array}{cc} Z_{qq}(a\mu, g_0) & Z_{qG}(a\mu, g_0) \\ Z_{Gq}(a\mu, g_0) & Z_{GG}(a\mu, g_0) \end{array}\right) \left(\begin{array}{c} \langle x\rangle_q^{lat} \\ \langle x\rangle_G^{lat} \end{array}\right)$$

Check on Momentum sum rules for lat results

$$\langle x\rangle_q^{lat} + \langle x\rangle_G^{lat} = 0.95(7)$$

$$2(J_q^{lat} + J_G^{lat}) = 0.95(9)$$

$Z_{qG} = 0$
(quenched)

$$Z_{qq} = 1 + \frac{g_0^2}{16\pi^2} C_F \left(\frac{8}{3}\log(a^2\mu^2) + f_{qq}\right),\ Z_{qg} = -\frac{g_0^2}{16\pi^2}\left(\frac{2}{3}N_f \log(a^2\mu^2) + f_{qg}\right),$$

$$Z_{gq} = -\frac{g_0^2}{16\pi^2} C_F \left(\frac{8}{3}\log(a^2\mu^2) + f_{gq}\right),\ Z_{gg} = 1 + \frac{g_0^2}{16\pi^2}\left(\frac{2}{3}N_f \log(a^2\mu^2) + f_{gg}\right).$$

Lat PT calc (one-loop)

← M.Glatzmaier, K.-F.Liu, M.Ramsey-Musolf, arXiv:1403.7211

$$\begin{array}{ll} f_{qq} = -7.60930 & f_{qG} = 0 \\ f_{Gq} = -2.37600 & f_{GG} = -3.76900 \end{array}$$

$$\frac{1}{\sqrt{X^\dagger X}} = \int_{-\infty}^{\infty} \frac{d\sigma}{\pi} \frac{1}{\sigma^2 + X^\dagger X}$$

(Integral form for glue op.)

Renormalization

- "Sum-rule improved" version

$$\langle x\rangle_q^{lat,S} + \langle x\rangle_G^{lat,S} = 1 \qquad 2(J_q^{lat,S} + J_G^{lat,S}) = 1$$

"normalization-improvement" by imposing sum-rules to account for latt systematics

$$\langle x\rangle_q^{lat,S} = Z_q^L \langle x\rangle_q^{lat}$$

$$\langle x\rangle_G^{lat,S} = Z_G^L \langle x\rangle_G^{lat} \quad \text{etc.}$$

  - We also have to modify matching coeffs

$$\begin{pmatrix} \langle x\rangle_q^{\overline{MS}}(\mu) \\ \langle x\rangle_G^{\overline{MS}}(\mu) \end{pmatrix} = \begin{pmatrix} Z_{qq}(a\mu, g_0) & Z_{qG}(a\mu, g_0) \\ Z_{Gq}(a\mu, g_0) & Z_{GG}(a\mu, g_0) \end{pmatrix} \begin{pmatrix} \langle x\rangle_q^{lat,S} \\ \langle x\rangle_G^{lat,S} \end{pmatrix}$$

"Sum rule constraint" $Z_{qq} + Z_{Gq} = 1, \quad Z_{Gq} + Z_{GG} = 1$

➡ $\tilde{f}_{qq} = \tilde{f}_{Gq} = (f_{qq} + f_{Gq})/2 \qquad \tilde{f}_{qG} = \tilde{f}_{GG} = (f_{qG} + f_{GG})/2$

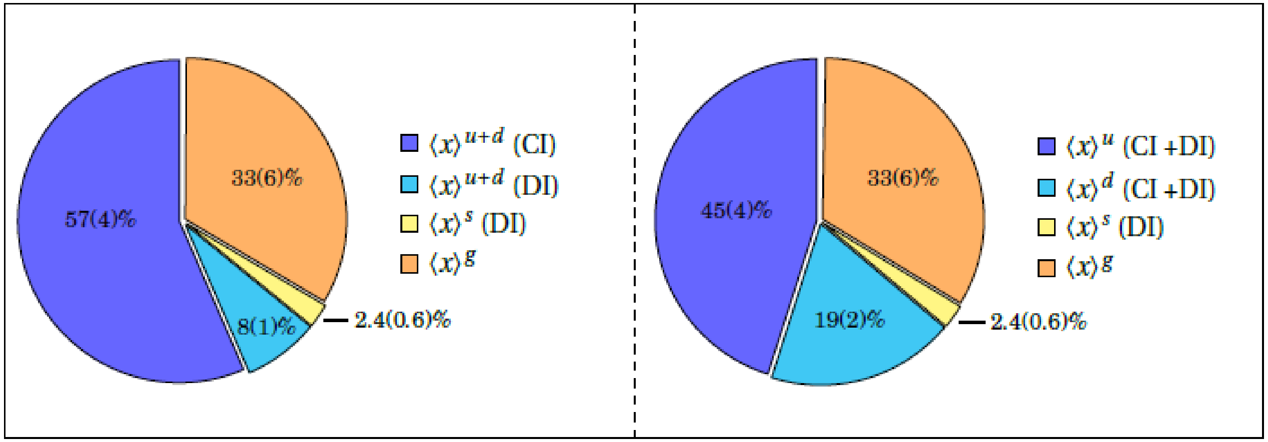
$$Z = \begin{pmatrix} 0.9641 & 0.0119 \\ 0.0359 & 0.9881 \end{pmatrix}$$

(ad-hoc solution w/ ~1% sys err)

(to MSbar mu=2GeV)

Results

<x>



	CI(u)	CI(d)	CI(u+d)	DI(u/d)	DI(s)	Glue
$\langle x \rangle$	0.413(38)	0.150(19)	0.565(43)	0.038(7)	0.024(6)	0.334(55)
$T_2(0)$	0.286(108)	-0.220(77)	0.062(21)	-0.002(2)	-0.001(3)	-0.056(51)
$2J$	0.700(123)	-0.069(79)	0.628(49)	0.036(7)	0.023(7)	0.278(75)
g_A	0.91(11)	-0.30(12)	0.62(9)	-0.12(1)	-0.12(1)	–
$2L$	-0.21(16)	0.23(15)	0.01(10)	0.16(1)	0.14(1)	–

$\overline{MS},\ \mu = 2\ \mathrm{GeV}$ **(Stat. Error Only)**

Results

J

Spin = 25(12)%

Glue = 28(08)%

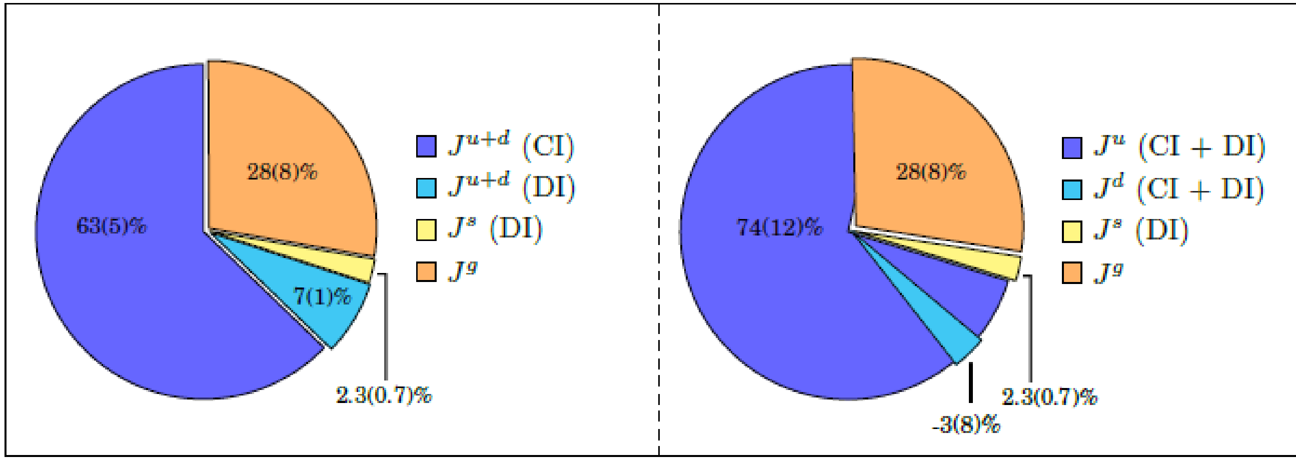
Orbital = 47(13)%

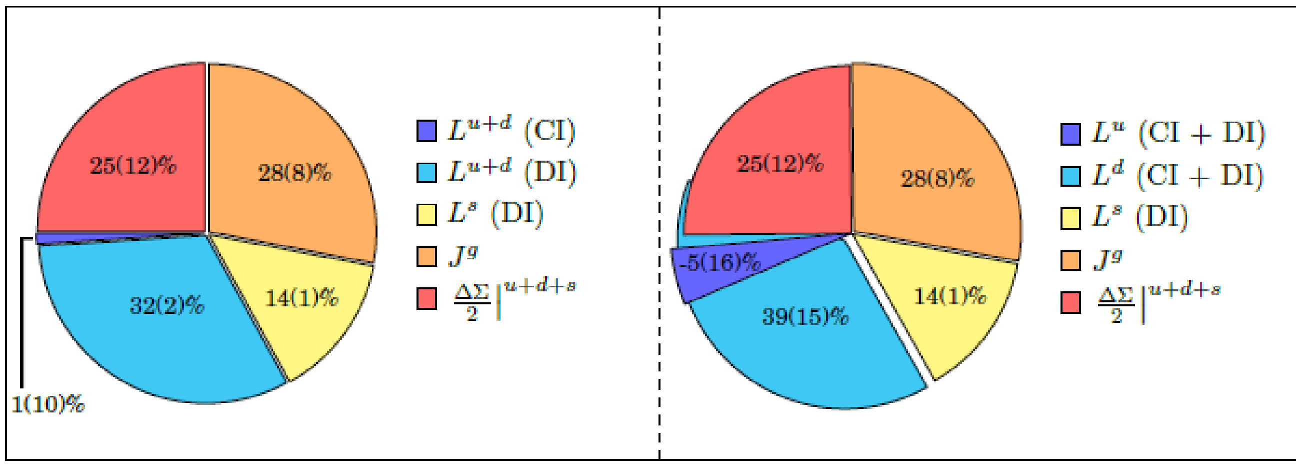
DI part is important

L(u) + L(d) [CI] ~= 0

J(u) >> J(d) [CI] ~= 0

(observed in other Lat)





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From our old results:
Dong et al., PRL75(1995)2096

Systematic errors to be explored

- Dynamical quark effect
  - This is quenched calc.
- Uncertainty in (long) chiral extrapolation
  - m(pi) = 0.48--0.65 GeV in this calc
- Contamination from excited states
  - Sys error could be large (quite common in N on lat)
- Finite volume artifact, discretization artifact
  - m(pi) L >~ 4, a = 0.11fm
- Renormalization
  - Perturbative vs. non-perturbative, etc.

Comparison

- quark spin

Quenched calc (1995) $\Delta\Sigma^{u,d}\ (\mathrm{DI}) \simeq \Delta\Sigma^{s}\ (\mathrm{DI}) \simeq -0.12$

Recent dynamical clac $\Delta\Sigma^{u,d}\ (\mathrm{DI}) \sim -0.05$

(Boston, QCDSF, Engelhardt, ETMC,CSSM/QCDSF,...) $\Delta\Sigma^{s}\ (\mathrm{DI}) \sim -0.03$

$$L = J - \Delta\Sigma/2$$

➔ **Large orbital mom by large negative DI in quenched**

➔ **Smaller orbital mom by going to full QCD ?**

HOWEVER:

$$g_A^0 = (\Delta u + \Delta d)[CI] + (\Delta u + \Delta d + \Delta s)[DI] \sim 0.25$$
$$g_A^8 = (\Delta u + \Delta d)[CI] + (\Delta u + \Delta d - 2\Delta s)[DI] = 0.579(25)$$

➔ **Large DI & larger orbital favored ?**

Close-Roberts (1993)

SU(3) breaking effect change situation ?

Lattice calc (Lin et al. (2009), Sasaki et al.(2009), Erkol et al. (2010)) suggests small SU(3) breaking

Summary & Prospects

- The first study of **complete calc** of proton spin
  - Connected (CI), Disconnected (DI) & Glue
  - DI: stochastic method + unbiased subt. w/ HPE
  - Glue: overlap operator to improved S/N
- Quenched calc at heavy quark mass
  - J (u+d): 70(5)%, J(s): 2.3(7)%, J(glue): 28(8)% where L(u+d+s): 47(13)%
- **Future:**
  - Full QCD calc at lighter mass
  - New decomposition, ΔG
  - (Wilson-Flow for EM-tensor op., etc.)